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Abstract

This paper analyzes the impact of public investment on the dynamics of private capital formation in an intertemporal optimizing market-clearing framework. The key feature characterizing the analysis is that the public good is treated as a durable capital good, subject to congestion. We show how in the presence of congestion the effect of government investment on private capital formation involves a tradeoff between the degree of substitution between private and public capital in production and the degree of congestion. Both lump-sum and distortionary tax financing are considered, with this tradeoff being tightened in the latter case.

Keywords
Congestion, public capital

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E62, H41

1. Introduction

The effect of public investment on private capital formation is a crucial public policy issue. Empirical research into this question was stimulated by Aschauer (1989a, 1989b) who suggested that public capital has a powerful impact on the productivity of private capital. Aschauer's results were controversial and have generated substantial empirical research directed at determining the robustness of his position. While the evidence is mixed, there seems to be a consensus generally supporting the productivity of public investment, although suggesting that its impact is somewhat weaker than that originally proposed by Aschauer.[1]

The theoretical analysis of the productivity of public investment proceeds by introducing government expenditure as an argument in the production function, to reflect, among other reasons, an externality in production. Two formulations can be identified. Most of the existing literature treats the current *flow* of government expenditure as the source of contribution to productive capacity. For example, Aschauer and Greenwood (1985), Aschauer (1988), Barro (1989), and Turnovsky and Fisher (1995) do so in a neoclassical Ramsey framework. Barro (1990), and Turnovsky (1995) employ a simple "A-K" endogenous growth model. While the flow specification has the virtue of tractability, it is open to the criticism that insofar as productive government expenditures are intended to represent public infrastructure, such as roads and education, it is the accumulated *stock*, rather than the current flow, that is relevant.

Despite this criticism, few authors have adopted the alternative approach of specifying productive government expenditure as a stock. Arrow and Kurz (1970) were the first authors to formulate government expenditure as a form of investment. More recently, Baxter and King (1993) study the macroeconomic implications of increases in the stocks of public goods. They

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[1]See Gramlich (1994) for a comprehensive survey of this empirical literature, most of which is for the United States. Lynde and Richmond (1993) provide evidence suggesting that public capital has played an important role in enhancing the productivity growth of UK manufacturing.

derive the transitional dynamic responses of output, investment, consumption, employment, and interest rates to such policies by calibrating a real business cycle model. Futagami, Morita and Shibata (1993) extend the Barro (1990) A-K growth model to include government capital.

The theoretical model developed in this paper analyzes the impact of the *stock* of government infrastructure expenditure -- public capital -- on the accumulation of private capital in a Ramsey type framework. A key feature of the analysis is the central role assigned to congestion. The few existing models that do introduce public capital, treat it as a pure public good, and thus fail to take account of the congestion typically associated with public capital. Yet, as Barro and Sala-i-Martin (1995) have argued, virtually all public services are characterized by some degree of congestion. Even national defense, sometimes cited as the purest of public goods, is not congestion-free.[2] These considerations suggest that the incorporation of congestion is an important consideration in assessing the relationship between public and private capital formation.

In contrast to the usual specification of congestion in macro models, which is typically to normalize aggregate government expenditure by the size of the economy, we introduce a more general parameterization, adapting a form of congestion function from the public goods literature.[3] This is important since our results highlight the existence of an important tradeoff between: (i) the degree of congestion and, (ii) the substitutability between public and private capital in production, in determining the impact of public investment on the rate of accumulation of private capital.

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[2]This was originally argued by Thompson (1976).

[3]See e.g. Edwards (1990).

Two alternative modes of government financing are considered. The assumption made by Barro (1990) and others of a continuously balanced budget in which the only tax is an income tax, the (endogenous) revenue of which is spent on productive capital, is restrictive. It makes the economy behave essentially like a centrally planned economy in which the income tax provides the mechanism whereby the central planner appropriates the resources from the private sector. Thus, insofar as the resources withdrawn from the private sector are reinvested productively by the government, raising the income tax has both a contractionary effect and a stimulating effect. By contrast, the introduction of lump-sum taxation (or equivalently government debt) in addition to the distortionary income tax enables the effects of the distortionary tax and the expenditure itself to be decoupled, thereby clarifying the respective roles played by each in the capital accumulation process.

2. Macroeconomic Equilibrium with Congestion

To analyze congestion it is important to distinguish between individual quantities, and the corresponding aggregate quantities. The economy consists of N identical individuals, each of whom has an infinite planning horizon, possesses perfect foresight, and faces a perfect capital market. With all agents being identical, all aggregate private quantities are simply multiples of individual quantities. We denote individual quantities by lower case letters and aggregate quantities by corresponding upper case letters, so that $X = Nx$. We shall express the equilibrium dynamics of the economy in terms of the aggregate stocks of private and public capital, though care must be used in deriving these equilibrium conditions from the individual behavioral relationships.

The individual agent is endowed with a fixed stock of labor, $\bar{l}$. He is a representative consumer-producer and chooses his consumption, c, stock of private capital, k, to maximize the following concave intertemporal utility function

$$\max \int_{o}^{\infty} U(c)e^{-bt}dt \qquad U' > 0, \ U'' < 0 \qquad (1a)$$

subject to the income constraint

$$\dot{k} = (1-t)f(k,\bar{l},K_g^s) - c - d_k k - s \qquad (1b)$$

and initial stock of capital,

$$k(0) = k_0 \qquad (1c)$$

where: t = distortionary rate of income tax, s = lump-sum taxation, K_g^s = services obtained from the stock of public capital, b = constant rate of consumer time preference, and d_k = constant rate of physical depreciation of private capital, $0 < d_k < 1$.

A critical feature of the model concerns the specification of the productive services derived by the representative agent from government capital. These are represented by

$$K_g^s = K_g \left(k/K\right)^{1-s} \quad 0 \leq s \leq 1 \qquad (2)$$

where K_g denotes aggregate public capital, K denotes the aggregate private capital stock. In particular, (2) implies that in order for the level of public capital services, K_g^s, available to the individual firm to remain constant over time, given its individual capital stock, k, the growth rate of K_g must be related to that of K in accordance with $\dot{K}_g / K_g = (1-s)\dot{K}/K$ so that s parameterizes the degree of congestion associated with the public good.[4] The case $s = 1$ corresponds to a non-rival, non-excludable public capital good that is available equally to each

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[4]The function (2) is the standard specification of the median voter model of congestion; see e.g. Edwards (1990). It implies decreasing marginal congestion provided $s < 1$. The specification of government services by (2) implies that the use of public capital is congested only by the use of private capital. Other formulations of congestion are also possible; see e.g. Glomm and Ravikumar (1994). For example, public services might be congested by output or employment. Given that labor is supplied inelastically, (2) is a natural specification.

firm, independent of the size of the economy; there is no congestion. There are few examples of such pure public goods, so that this case should be treated largely as a benchmark. At the other extreme, if $s = 0$, then only if K_g increases in direct proportion to the aggregate capital stock, K, does the level of the public service available to the individual firm remain fixed. We shall refer to this case as being one of *proportional* congestion, meaning that the congestion grows in direct proportion to the size of the economy.[5] Road services and infrastructure that play a productive role in facilitating the distribution of the firm's output may serve as examples of public goods subject to this type of congestion. In between, $0 < s < 1$, describes partial congestion, where K_g can increase at a slower rate than does K and still maintain a fixed level of public services to the firm.[6]

Thus, using (2), the individual firm's production function can be written as

$$y = f\left[k, \bar{l}, K_g\left(k/K\right)^{1-s}\right].$$ (3)

Provided the public good is associated with some congestion, ($s \neq 1$), aggregate private capital is introduced into the production function of the individual firm, generating an externality in an analogous way to Romer (1986).

The production function is assumed to exhibit positive marginal productivity in all three factors, $f_k > 0, f_l > 0, f_g > 0$, (where to conserve notation we let f_g denote the derivative of the production function with respect to the services of public capital). In addition, we assume that the marginal physical product of both private factors are diminishing, $f_{kk} < 0, f_{ll} < 0$ and that there are non-increasing returns to scale in the two private factors, $k, \bar{l}$. Beyond that we shall

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[5]In the case $s = 0$ the good is like a private good in that the median voter receives his proportionate share.

[6]The case $s < 0$ can be interpreted as describing an extreme situation in which the services of public capital are congested more rapidly than the overall growth of the economy. See Edwards (1990) for evidence supporting this case. While we do not discuss it, one can easily interpret our results in this circumstance.

leave open the question of whether the productivity of the public good declines, and whether or not the production function is of constant returns to scale in the two private factors [as assumed by Turnovsky and Fisher (1995)] or constant returns to scale in all three factors [as assumed by Aschauer (1988)].

The optimality conditions for this representative agent's problem can be expressed as follows:

$$U'(c) = \boldsymbol{l} \tag{4a}$$

$$(1 - \boldsymbol{t}) \left[f_k + (1 - \boldsymbol{s}) \left(\frac{K_g}{k} \right) \left(\frac{k}{K} \right)^{1-s} f_g \right] - \boldsymbol{d}_k = \boldsymbol{b} - \frac{\dot{\boldsymbol{l}}}{\boldsymbol{l}} \tag{4b}$$

where $\boldsymbol{l}(t)$ is the shadow value corresponding to the budget constraint (1b). The optimality condition (4a) equates the marginal utility of consumption to the shadow value of wealth, while (4b) equates the net rate of return on investment in capital to the rate of return on consumption. The former, given by the left hand side of (4b) is equal to the after-tax marginal physical product of capital, net of depreciation. In the absence of congestion, the marginal physical product of private capital is, as usual, f_k. To the extent that there is congestion, this is augmented by the term $(1 - \boldsymbol{s})(K_g / k)(k/K)^{1-s} f_g$. This reflects the fact that an increase in the individual's private capital stock, k, given the aggregate stock of capital, K, raises the marginal benefits he derives from public capital. This externality is important in assessing the impact of government capital on the behavior of the economy. The final optimality condition is the transversality condition $\lim_{t \to \infty} \boldsymbol{l} k e^{-bt} = 0$, and ensures the intertemporal sustainability of the equilibrium.

With all agents being identical, aggregate and individual capital stocks are related by $K = Nk$, where N is the number of representative agents (firms). Thus in equilibrium, the individual output, y, and aggregate output, $Y = Ny$, may be expressed as

$$Y = Nf(k, l, K_g) = Nf(K / N, \bar{l}, K_g \boldsymbol{y}) \equiv F(K, \bar{l}, K_g \boldsymbol{y}) \tag{3'}$$

where the term $y \equiv N^{s-1} \leq 1$, reflects the net effect of congestion.[7] Congestion will be absent if either $N = 1$, or $s = 1$, in which case each agent will receive the full benefits of the public good; otherwise they will be scaled down by the factor $y \leq 1$.

The properties of the aggregate function *F* mirror those of the individual production function *f* In particular, the following relationships follow immediately from (3'):

$$F_k = f_k; \quad F_g = Nf_g; \quad F_{kk} = f_{kk}/N; \quad F_{kg} = f_{kg} \tag{3"}$$

use of which will be made in deriving the equilibrium dynamics. Since the labor supply is fixed, henceforth we shall simplify notation by suppressing it from the production function.

The government's behavior can now be described. It is assumed to set an exogenous rate of public investment in infrastructure, *G*, which, starting from an initial stock of public capital, $K_{g,0}$, leads to the accumulation of public capital in accordance with

$$\dot{K}_g = G - d_g K_g; \quad K_g(0) = K_{g,0} > 0 \tag{5a}$$

where government capital depreciates at the rate d_g per unit of time. The government finances its investment activities by using either distortionary or lump-sum taxation, in accordance with its flow budget constraint:

$$tF(K, K_g y) + S = G \tag{5b}$$

where *S* denotes aggregate lump-sum taxes.[8]

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[7]For notational convenience we delete *N* from the aggregate production function, *F*, and also from the aggregate consumption function, *C*, below.

[8]It is straightforward to introduce government debt. Since the conditions for Ricardian Equivalence hold, little is gained by doing so.

Aggregating (1b) over the *N* identical individuals and combining with (5b) yields the aggregate economy resource constraint

$$\dot{K} = F(K, K_g \boldsymbol{y}) - C - \boldsymbol{d}_k K - G \tag{6}$$

where $C \equiv Nc$ denotes aggregate consumption.

This macroeconomic system describes a standard perfect foresight equilibrium. Through the shadow value $\boldsymbol{l}$ it is forward looking, implying that changes in the steady state induce changes in the short run and in the transitional dynamics. The short-run solution for individual consumption, *c*, derived from (4a), and therefore aggregate consumption, *C*, is of the form

$$C = Nc = C(\boldsymbol{l}); \quad C_l < 0 \tag{7}$$

Substituting (7) into (6') and noting (3'), (3"), (4b), (5a), (5b) the macroeconomic dynamic equilibrium is summarized by the following three equations:

$$\dot{\boldsymbol{l}} = \boldsymbol{l}\left\{ \boldsymbol{b} + \boldsymbol{d}_k - (1 - \boldsymbol{t})\left[F_k + (1 - \boldsymbol{s})\frac{K_g \boldsymbol{y}}{K} F_g \right] \right\} \tag{8a}$$

$$\dot{K} = F(K, K_g \boldsymbol{y}) - C(\boldsymbol{l}) - \boldsymbol{d}_k K - G \tag{8b}$$

$$\dot{K}_g = G - \boldsymbol{d}_g K_g \tag{8c}$$

together with the flow government budget constraint (5d).

Given the degree of congestion, as parameterized by $\boldsymbol{y}$, and the scale of the economy, *N*, the system of equations (8a) - (8c) represents an autonomous dynamic system in the two private variables, $\boldsymbol{l}, K$, and the stock of government capital, K_g. The precise nature of the

equilibrium dynamics depends upon how the government chooses to finance its capital expenditures.

Steady-state equilibrium occurs when $\dot{I} = \dot{K} = \dot{K}_g = 0$ and consists of the following relationships (where the tilde denotes steady-state values):

$$(1 - \tilde{t}) \left[F_k(\tilde{K}, \tilde{K}_g \boldsymbol{y}) + (1 - \boldsymbol{s}) \frac{\tilde{K}_g \boldsymbol{y}}{\tilde{K}} F_g(\tilde{K}, \tilde{K}_g \boldsymbol{y}) \right] = \boldsymbol{b} + \boldsymbol{d}_k \qquad (9a)$$

$$F(\tilde{K}, \tilde{K}_g \boldsymbol{y}) - C(\tilde{\boldsymbol{I}}) - \boldsymbol{d}_k \tilde{K} - G = 0 \qquad (9b)$$

$$\boldsymbol{d}_g \tilde{K}_g = G \qquad (9c)$$

together with the steady-state government budget constraint

$$\tilde{t} F(\tilde{K}, \tilde{K}_g \boldsymbol{y}) + \tilde{S} = G = \boldsymbol{d}_g \tilde{K}_g \qquad (9d)$$

The following long-run relationships are implied. The after-tax marginal physical product of private capital, inclusive of the spillover from the productivity of public capital, must equal its long-run real rental rate, the latter equaling the sum of the rates of time preference and depreciation. Equation (9b) represents steady-state market clearing, when net investment is zero, in which case output must be allocated either to consumption, or to replacing the depreciated capital stocks. Thus equation (9c) asserts that the (gross) steady-state flow of government infrastructure investment equals its rate of depreciation. Equation (9d) requires that in steady state that the total tax revenues collected by income and lump-sum taxes must equal the sum of government expenditure on depreciation.

Both the short-run and long-run behavior of the system depends upon the specification of government policy and two forms of finance shall be considered.

(i) *Government Investment Financed using Lump-sum Taxation*

In this case, the distortionary tax rate, t, is fixed exogenously. Given the fixed rate of government investment, G, equation (9c) determines the steady-state stock of government capital, $\tilde{K}_g$. Having obtained t and $\tilde{K}_g$, (9a) determines the steady-state stock of private capital, $\tilde{K}$, and given the degree of congestion, the level of output. Equation (9b) determines the equilibrium rate of consumption (marginal utility) that ensures steady-state product market equilibrium is met. With output and income tax revenues fixed, the steady-state government budget constraint (9d) determines the lump sum tax, $\tilde{S}$, necessary to maintain long-run government budget balance.

(ii) *Long-Run Government Investment Financed with Distortionary Taxes*

The second example we consider is where the government finances a given increase in its investment by a higher distortionary tax, raising the tax rate such that in the long-run, the additional tax revenues just finance the additional expenditure: that is

$$F(\tilde{K},\tilde{K}_g\mathbf{y})dt + t\left[F_k d\tilde{K} + F_g\mathbf{y}d\tilde{K}_g\right] = dG \tag{10}$$

Since output is changing along the transitional path, income tax revenues will not precisely finance the given increase in public investment. We assume then that any short-run imbalances are financed by a temporary adjustment in lump-sum taxes. As in case (i), $\tilde{K}_g$ is determined by (9c). Having determined this, (9a) and (10) jointly determine the required tax rate, $\tilde{t}$, and the stock of private capital, $\tilde{K}$. Consumption is then determined by the product market equilibrium (9b).

To determine the local dynamic behavior of the economy, we linearize the dynamic system (8) about the steady-state equilibrium (9), for set values of G and $\mathbf{y}$. The linearized dynamics are thus described by the following matrix differential equation in $\mathbf{1}$, K, and K_g:

$$\begin{pmatrix} \dot{I} \\ \dot{K} \\ \dot{K}_g \end{pmatrix} = \begin{pmatrix} 0 & -\tilde{I}(1-t)q_{12} & -\tilde{I}(1-t)q_{13} \\ -C_I & F_k - d_k & F_g y \\ 0 & 0 & -d_g \end{pmatrix} \begin{pmatrix} I - \tilde{I} \\ K - \tilde{K} \\ K_g - \tilde{K}_g \end{pmatrix} \tag{11}$$

where:

$$q_{12} \equiv F_{kk} + (1-s)\frac{K_g y}{K}\left[F_{kg} - \frac{F_g}{K} \right];$$

$$q_{13} \equiv y\left\{ F_{kg} + \frac{(1-s)}{K}\left[F_g + F_{gg}K_g y \right] \right\}$$

The determinant of the matrix in (11) is given by

$$\Delta \equiv d_g C_I \tilde{I}(1-t)q_{12}. \tag{12}$$

As usual, the two capital stocks K, K_g are assumed to evolve sluggishly from their respective

initial stocks, while the shadow value I is free to jump instantaneously in response to new

information. Thus in order for this dynamic system to be saddlepath stable, there must be two

stable and one unstable eigenvalues. Since one eigenvalue equals $-d_g < 0$ the remaining two

eigenvalues are $m_1 < 0$ and $m_2 > 0$. Furthermore, since the product of the three roots equals

Δ, it follows that $\Delta > 0$, thereby implying the restriction $q_{12} < 0$.[9]

The degree of congestion is a factor in determining the stability of the system. In the absence

of congestion, $s = 1$, $q_{12} = F_{kk} < 0$, and saddlepoint stability clearly obtains. In the

presence of congestion, the condition $q_{12} < 0$ is equivalent to the condition that the gross

marginal product of private capital, inclusive of its effect that operates through the productivity

of public services, declines. This condition can be shown to hold if, for example, the

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[9]The eigenvalues are calculated from the characteristic equation of (7), namely

$$(m + d)\left[m^2 - (q_{11} + q_{22})m + (q_{11}q_{22} - q_{12}q_{21}) \right] = 0.$$

production function takes a linearly homogeneous CES form, such as that specified in (15) below. [10]

3. Dynamic Effects of Public Capital Expenditures

The formal solution to the linearized dynamic system, (11), is standard; see e.g. Turnovsky (1995, Chapter 9). We shall restrict our discussion of the dynamics to the adjustments of the capital stocks, since these are most central to the discussion of congestion. A brief Appendix provides the formal solutions for their time paths. Having determined these, the time paths of output, consumption, and other macroeconomic variables can be derived in a routine manner.

3.1 Steady-State Effects

In this section we analyze the impact of a permanent increase in government investment on the steady-state equilibrium of the economy, distinguishing between the two methods of financing the increase in public investment.

(i) Lump-sum Tax Financing

To determine the long-run effects of a permanent expansion in the rate of government investment, financed by lump-sum taxation (or debt), we differentiate the steady-state equilibrium conditions (9c) and (9a) with respect to *G*. This yields the following responses in the aggregate stocks of public and private capital:

$$\frac{d\tilde{K}_g}{dG} = \frac{1}{\boldsymbol{d}_g} > 0; \quad \frac{d\tilde{K}}{dG} = -\frac{\boldsymbol{q}_{13}}{\boldsymbol{d}_g \boldsymbol{q}_{12}} = -\frac{\boldsymbol{y}}{\boldsymbol{d}_g \boldsymbol{q}_{12}} \left[F_{kg} + \frac{(1-\boldsymbol{s})}{K} \left(F_g + F_{gg} K_g \boldsymbol{y} \right) \right] \quad (13a, 13b)$$

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[10] $\boldsymbol{q}_{12} < 0$ will also hold if the services of public capital are multiplicatively separable from the private factors of production, i.e. $Y = F(K, L)H(K_g \boldsymbol{y})$, and *F* is homogeneous of degree one in the private factors of production, *K* and $L = N\bar{l}$.

A permanent increase in the rate of government investment, G, simply expands long-run public capital stock, $\tilde{K}_g$, by a multiple equal to the inverse of the depreciation rate. The effect on the stock of private capital is of greater interest and depends crucially upon: (i) how the public capital interacts with private capital in production, and (ii) the degree of congestion associated with public capital. Suppose first that $s = 1$ so that there is no congestion. A larger stock of public capital will increase the level of public services, and as long as $F_{kg} > 0$, this will raise the marginal physical product of private capital and thereby stimulate the long-run accumulation of private capital, $\tilde{K}$. However, in the presence of congestion, a number of competing effects are in operation. As a simple example, suppose initially that the individual production function is of the form

$$y = F(k,\bar{l}) + aK_g^s = F(k,\bar{l}) + aK_g\left(k/K\right)^{1-s} \tag{14}$$

so that government services enter additively and linearly in the production function. In this case, the level of public services, K_g^s, has no impact on the productivity of private capital, as in Aschauer (1988). But as long as there is congestion ($s < 1$), the firm has an incentive to increase its stock of capital, and thereby increase the level of services it receives from public infrastructure. Thus, for a very different reason from that given above, an increase in public investment will lead to a long-run increase in private capital.

If the level of public services enters nonadditively and nonseparably into production, then both mechanisms we have just been describing are in operation. But to the extent that public capital, like private inputs, is subject to diminishing marginal physical productivity, these two expansionary effects will be offset by a contractionary effect. Indeed, if congestion is sufficiently severe we cannot rule out the possibility that the negative effect of the declining marginal physical productivity of public capital will overwhelm the positive effects described above so that an increase in public investment actually reduces the long-run stock of private capital.

To analyze the interaction between private and public capital further, it is useful to focus on the aggregate CES production function:

$$Y = \left[aK^{-r} + b\overline{L}^{-r} + c(\boldsymbol{\gamma}K_g)^{-r} \right]^{(1/r)} \tag{15}$$

where $\boldsymbol{h} \equiv 1/(1 + \boldsymbol{r})$ is the elasticity of substitution. Evaluating $\boldsymbol{q}_{13}$ for this function we find

$$\operatorname{sgn}\!\left(\frac{d\tilde{K}}{dG} \right) = \operatorname{sgn}(\boldsymbol{q}_{13}) = \operatorname{sgn}\!\left(s_k + (1 - \boldsymbol{s})s_g - (1 - \boldsymbol{s})(1 - \boldsymbol{h}) \right) \tag{16}$$

where $s_k \equiv F_k K / Y$, $s_g \equiv F_g K_g \boldsymbol{\gamma} / Y$ are the shares (elasticities) of output attributable to private capital and to the services of public capital, respectively.[11] Rewriting (16) in the form:

$$sgn\!\left(\frac{d\tilde{K}}{dG} \right) = sgn\!\left(\boldsymbol{h} + \frac{s_k}{(1 - \boldsymbol{s})} - (1 - s_g) \right) \tag{16'}$$

highlights the tradeoff that exists between the degree of congestion and the elasticity of substitution in order for government investment to raise the long-run stock of private capital.   In the absence of congestion $(\boldsymbol{s} = 1)$, or if the production function is Cobb-Douglas $(\boldsymbol{h} = 1)$, (16) implies $d\tilde{K}/dG > 0$. But if congestion is proportional, $(\boldsymbol{s} = 0)$, the expansionary effect will dominate if and only if the elasticity of substitution exceeds the share of output attributable to labor, $\boldsymbol{h} > s_l$.   For a Leontief production function $(\boldsymbol{h} = 0)$, the expansionary effect will dominate if and only if $\boldsymbol{s} > s_l/(1 - s_g)$. Thus, with a high degree of congestion, an expansion in government investment may quite plausibly crowd out private capital, even if the two types of capital are complementary in production, $(F_{kg} > 0)$.

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[11]Relevant properties of the CES function include: $s_k + s_l + s_g = 1$, where $s_l \equiv F_l \overline{L} / Y$,

$$F_{kk} = (1 + \boldsymbol{r})(s_k - 1)F_k / K;\ F_{gg} = (1 + \boldsymbol{r})(s_g - 1)F_g / \boldsymbol{\gamma}K_g,\ F_{kg} = (1 + \boldsymbol{r})s_k F_g / K$$

Combining these with the definitions of $\boldsymbol{q}_{12}$, $\boldsymbol{q}_{13}$, and the definition of $\boldsymbol{h}$, enables us to derive expressions (16), (16'), (16'').

Another way to assess how congestion affects the impact of government investment expenditure on private capital accumulation is to use (13b) to consider $\left(d\tilde{K}/dG\right)_{s=1} - \left(d\tilde{K}/dG\right)_{s<1}$. Intuitively, one might expect that government investment will have a smaller impact on private capital formation in the presence of congestion. This may, or may not, be the case. It is straightforward to show that if the production function is of the Cobb-Douglas form, then $\left(d\tilde{K}/dG\right)_{s=1} > \left(d\tilde{K}/dG\right)_{s<1}$ implying that congestion does impact adversely upon private capital formation. However, if the production function is of the linear additively separable form (14), we find $\left(d\tilde{K}/dG\right)_{s=1} < \left(d\tilde{K}/dG\right)_{s<1}$ and reach the counter-intuitive proposition that congestion enhances private capital formation. In the absence of congestion, for such a production function, public investment has no impact on the productivity of private capital so that $\left(d\tilde{K}/dG\right)_{s=1} = 0$. As discussed above, congestion induces the agent to increase his private investment, so as to enhance the services he derives from the public investment.

(ii) Distortionary Tax Financing

Suppose now that the permanent increase in government investment, G, is financed by a distortionary income tax. From the steady-state equilibrium condition (9c) we see that the long-run response of government capital stock, $\tilde{K}_g$, remains as given by (13a). From (9a), (9b) we can show

$$\left(\frac{\partial \tilde{K}}{\partial G}\right)_{Dist} - \left(\frac{\partial \tilde{K}}{\partial G}\right)_{Lump-sum} = \frac{\partial \tilde{K}}{\partial t}\left(\frac{\partial t}{\partial G}\right)_{Dist} = \frac{b + d_k}{q_{12}(1-t)^2}\left(\frac{\partial t}{\partial G}\right)_{Dist} \tag{17}$$

Thus we see that a distortionary tax-financed increase in government investment will have a less expansionary effect on the long-run private capital stock than a lump-sum tax financed increase if and only if the higher public investment requires a higher long-run tax rate, $\tilde{t}$, as will normally be the case. Financing using a distortionary tax tightens the tradeoff between the

degree of congestion and the elasticity of substitution that is consistent with a positive response of private capital. Analogous to (16') we now obtain:

$$sgn\left(\frac{d\tilde{K}}{dG}\right)_{dist} = sgn\left(\boldsymbol{h} + \frac{\left[s_k/(1-\boldsymbol{s}) - (1-s_g)\right]}{\left[1 - (g/s_g)(s_k/(1-\boldsymbol{s}) + s_g)\right]}\right) \tag{16''}$$

where $g \equiv G/Y = \boldsymbol{d}_g K_g/Y$. For example, in the case of a Cobb-Douglas production function, distortionary tax financing will ensure a positive long-run effect on the private capital stock if and only if the share of government capital in production exceeds the long-run claim of government investment in output, $s_g > g$.

3.2 Transitional Effects

We now turn to the short-run dynamic responses to a permanent increase in the rate of public investment on the transitional behavior of the private capital stock. For simplicity, we shall focus on the case of lump-sum financing and assume that the income tax rate, t, is set to zero. The response of public capital to a permanent increase in G is described by (A.1a). Its behavior is very simple and is illustrated in Fig 1.A. Following an increase in G, K_g rises along a smooth monotonic path toward its larger long-run value, with the speed of adjustment and its steady-state increase being determined solely by the rate of depreciation, d_g.

The transitional behavior of the rate of private investment is of more interest. To investigate its initial response, we evaluate the time derivative of (A.1b) at $t = 0$, to obtain (with manipulation):

$$\frac{d\dot{K}(0)}{dG} = \frac{-C_I \tilde{I}(d_g + m_I)}{[F_k - (d_k + m_I)]z}\left[q_{13} + \frac{[F_k - (d_k + m_I)]}{C_I \tilde{I}(1-t)}F_g\right] \tag{18}$$

where z, defined in (A.2) in the Appendix is shown to have same sign as that of $(d_g + m_I)$. Assuming that private capital is productive, $(F_k > d_k)$ the coefficient on the term in parenthesis in (18) is positive. Consequently, the initial qualitative response of private investment to an increase in public investment depends upon the sign of the term in parenthesis. This represents two offsetting effects and is ambiguous. First, to the extent that $q_{13} > 0$, so that public capital leads to the long-run accumulation of private capital, private investment is stimulated in the short run. The stronger the interaction, the larger the long-run response of private capital, and the greater the incentive to invest in the short run, though the fact that public capital accumulates slowly initially dampens this positive effect on private investment. The second term in parenthesis, $[F_k - (d_g + m_I)]F_g\big/[C_I \tilde{I}(1-t)]$, represents the direct effects due to the positive marginal physical product of public investment and has a negative influence on private investment. A higher rate of return to public capital, leaving aside congestion effects, tends to depress the rate of private investment in the short run, because it

leads to an increase in wealth that initially causes the private sector to substitute consumption for savings and capital accumulation. Over time, as the positive output effects of the government capital stock come into operation, the expansionary effect dominates and the public investment increases the private capital stock, as we have noted.

Fig. 1.B illustrates the two alternative adjustment paths that the private capital stock may follow. Locus A depicts the case where the capital stock increases monotonically along the entire adjustment path, while Locus B illustrates the case where the rate of private investment initially falls. Which of the two cases is more probable is not clear. [12] By contrast, when government expenditure impacts on production as a *flow* -- as in Aschauer (1988) and Turnovsky and Fisher (1995) -- the additional government expenditure has an immediate expansionary effect on the rate of private capital accumulation. This is because the effects of the higher flow impact immediately on the productivity of private capital, rather than only gradually as the additional public capital stock is accumulated.

4. Concluding Remarks

The impact of public investment on private capital formation remains one of the important issues in macroeconomics. In this paper we have analyzed the subject in an intertemporal optimizing market-clearing framework. The key feature characterizing our analysis is that we have treated the public good as taking the form of a durable capital good, subject to congestion. This view of government expenditure provides a more realistic approach to analyzing the intertemporal tradeoffs that public expenditure policy imposes on the private sector. We have also attempted to take seriously the public finance aspects of this question

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[12]A similar balancing of expansionary and contractionary short-run responses of private investment is discussed by Baxter and King (1993). Their numerical calibration tends to suggest that only the contractionary effect exists when $F_{kg} = 0$. In our analysis the corresponding condition is $q_{13} = 0$ which reduces to $F_{kg} = 0$ in the absence of congestion, $s = 1$; see the definition of q_{13} below (11).

by considering the choice between lump-sum tax (bond) financing and distortionary tax financing.

Our main focus has been in determining the effect of an unanticipated permanent increase in the rate of government investment on the dynamics of private capital formation. We have shown that the effect on the long-run stock of private capital depends upon a number of factors: (i) How public capital interacts with private capital in production, (ii) the degree of congestion associated with public capital, and (iii) the mode of finance.

The case of lump-sum financing is treated as a benchmark case. In the absence of congestion a higher stock of public capital will lead to a higher stock of private capital if and only if the two factors are complements in production. In the presence of congestion the effect involves a tradeoff between the degree of substitution between private and public capital in production and the degree of congestion. The greater the degree of congestion, the larger must the elasticity of substitution be, in order for the private capital stock to increase. This is certainly met in the case of the Cobb-Douglas production function, but if the degree of congestion is large, an increase in public investment may ultimately reduce the private capital stock if the elasticity of substitution is sufficiently small. In plausible circumstances, the financing of government investment using a distortionary tax will almost certainly be less expansionary than when lump-sum tax financing is employed.

But even if public investment stimulates private capital in the long run, the short-run effects on the rate of private investment are unclear, as they depend upon two offsetting effects. First, to the extent that public capital enhances the productivity of private capital, private investment will be stimulated in the short run. But productive public investment also has a negative effect on private investment. This is because it generates an increase in wealth that initially causes the private sector to substitute consumption for capital accumulation. Indeed it is quite possible

for this negative effect to dominate suggesting that the short-run crowding out of private capital is perfectly plausible.

While our analysis has focused on the polar cases of lump-sum tax financing and distortionary-tax financing of government investment, neither of these policies is optimal. Congestion introduces a permanent externality in production that requires the long-run taxation of income for its elimination. Thus it is straightforward to show that the well known Chamley (1986)-Judd (1985) proposition asserting that the steady-state tax on capital should be zero applies only in the absence of congestion.

Appendix:

Stable Solutions for the Dynamics of Capital Accumulation

This appendix presents the formal solutions to the linearized dynamic system (11) for the case where the government employs lump-sum tax financing $(t = 0)$; the case of distortionary tax financing is essentially identical. The solution assumes that K and K_g starts out from initial steady-state values, $K_0, K_{g.0}$, respectively, so that

$(\tilde{K} - K_0) = -\left(q_{13}/d_g q_{12}\right)\left(\tilde{K}_g - K_{g,0}\right)$; see (13b).

Using standard solution methods, the stable time paths followed by $K_g, K,$ and l are:

$$K_g(t) = \tilde{K}_g + (K_{g,0} - \tilde{K}_g)e^{-d_g t} \tag{A.1a}$$

$$K = \tilde{K} + \frac{d_g}{z q_{12}}\left(\left[F_k - (d_k - d_g)\right]q_{13} - F_g y q_{12}\right)\left(\tilde{K}_g - K_{g,0}\right)e^{m_1 t}$$

$$+\frac{1}{z}\left[F_k y d_g - C_l \tilde{l} q_{13}\right]\left(\tilde{K}_g - K_{g,0}\right)e^{-d_g t} \tag{A.1b}$$

$$l = \tilde{l} - \frac{\tilde{l}}{z m_1}\left(\left[F_k - (d_k - d_g)\right]q_{13} - F_g y q_{12}\right)\left(\tilde{K}_g - K_{g,0}\right)\left(d_g e^{m_1 t} + m_1 e^{-d_g t}\right) \tag{A.1c}$$

where q_{12}, q_{13} are defined by equations (11); $m_1 < 0, -d_g < 0$ are the stable eigenvalues of (11), and

$$z \equiv d_g[F_k - (d_k - d_g)] - C_l \tilde{l} q_{12} = (d_g + m_1)[(F_k - d_k) + (d_g - m_1)] \tag{A.2}$$

In the second inequality defining z, we have used the fact that m_1, being an eigenvalue, satisfies:

$$m_1[(F_k - d_k) - m_1] = -C_l \tilde{l} q_{12} \tag{A.3}$$

Equations (A.1) provide the basis for the analysis of the dynamic adjustment of the economy in response to an increase in the rate of government investment, undertaken in Section 3.
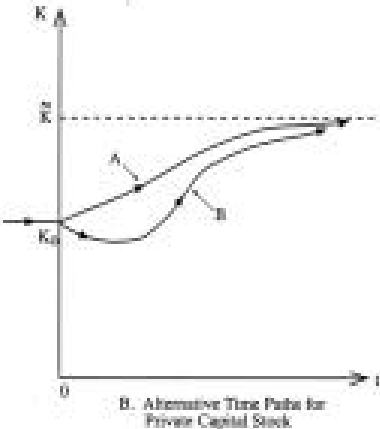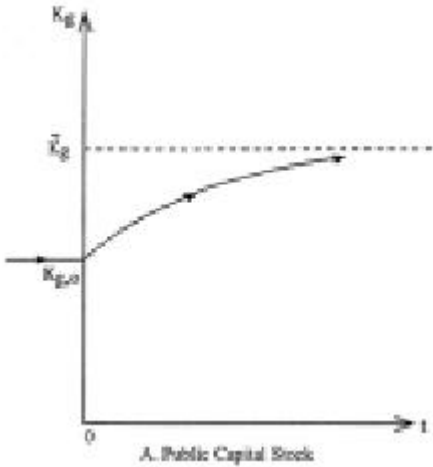
A. Public Capital Stock

B. Alternative Time Paths for
Private Capital Stock

Fig. 1 Time Paths for Capital Stocks

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