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Abstract

If some of the returns to migration accrue from return migration, the optimal duration of migration may be shorter than the feasible duration of migration. We develop a model that provides and highlights conditions under which return migration takes place even though a reversal of the inter-country wage differential does not occur. In particular, we consider the higher purchasing power of savings (generated from work abroad) at home than abroad as a motive for return migration. Inter alia, our model produces a negative relationship between the optimal duration of migration and the purchasing power differential and in some (but not all) cases, a negative relationship between the optimal duration of migration and the wage abroad. In addition, and contrary to our prior anticipation, our utility maximization analysis suggests that East-West migration will tend to be temporary while inter-European Community (or intra-West European) migration will likely be permanent.

Zusammenfassung

Sofern die Vorteilhaftigkeit einer Auswanderungsentscheidung von einer späteren Rückkehr in das Herkunftsland beeinflusst wird, kann die optimale Aufenthaltsdauer im Zielland unter der maximal möglichen Aufenthaltsdauer liegen. Wir leiten Bedingungen ab, unter denen Auswanderung in Verbindung mit einer späteren Rückwanderung eine optimale Strategie darstellt, obwohl keine Umkehr des positiven Lohndifferentials zwischen dem Zielland und dem Herkunftsland erfolgt. Insbesondere untersuchen wir die Rolle einer höheren Kaufkraft von im Ausland akkumulierten Ersparnissen im Herkunftsland in ihrer Bedeutung für Remigration. Unter anderem besteht eine negative Beziehung zwischen der optimalen Aufenthaltsdauer und der Höhe des Kaufkraftdifferentials sowie in einigen (aber nicht allen) Fällen eine negative Beziehung zwischen der optimalen Aufenthaltsdauer und dem Lohnniveau im Zielland. Die Ergebnisse unseres Modelles deuten unerwarteterweise darauf hin, daß Ost-West-Wanderungen tendenziell temporärer Natur sein sollten, während Wanderungen innerhalb der Europäischen Union (bzw. innerhalb Westeuropas) tendenziell permanenten Charakter haben sollten.

Keywords

Return migration, migrants' savings, purchasing power differentials

JEL-Classifications

F22, J61

1. Introduction

The theoretical literature on labor migration pays relatively little attention to return migration. Yet there is considerable evidence that the duration of migration is shorter than the duration of life or of working life. Even in the case of the United States, ordinarily viewed as a country of destination, not departure, the numbers involved are large. For example, until 1957 the United States collected information both on the arrival of migrants and on their departure. Between 1908 and 1957, of the 15.7 million migrants admitted to the United States 4.8 million departed - nearly one third (LaLonde and Topel, 1997).

With conventional migration theory attributing migration to a positive wage differential, a conventional explanation of return migration is, not surprisingly, a negative wage differential. While this explanation might be correct, considerable return migration takes place in the absence of a reversal of the relative wages of the sending and receiving countries.

Speaking generally, the typical explanations are that migrants return because of a failure, or because of success: if reality does not tally with expectations or the draw from a mixture of good draws and bad draws (random shocks) is bad, migrants may return. Alternatively, migrants whose returns to human or financial capital are higher at home than abroad may find it optimal to return. For example, the recent insightful study by Borjas and Bratsberg (1996) gives precisely these two reasons for return migration.

The set of reasons is richer, however. We offer two examples. First, a member of a family may migrate in order to diversify the familial income earning portfolio. If income away from home and income at home do not covary fully, and there is a post-migration pooling and sharing of income, the family's risk is lowered. Just as bearing one risk makes agents less willing to bear another risk, *not* bearing that one risk makes agents more willing to bear another risk (Pratt and Zeckhauser, 1987, Kimball, 1993). This then allows for experimentation at home with a relatively high risk - high return option, for example, a high yield seed variety in agriculture. When such an experiment is successful, the need for a migration-provided insurance ceases. Thus, the reason for return migration is not that the *migrant* accumulated capital with an expected high return at home

but rather that his or her migration *facilitated* a high return investment at home by others (Stark, 1991). Second, if migration takes place under asymmetric information, return migration could arise from information becoming symmetric. Consider the case where employers at destination offer the same wage to observationally indistinguishable workers whose individual skills and productivities differ. If all these workers are offered a wage equal to the average product of the group of migrant workers, and if this wage is higher than the home country wages of these workers, migration will take place. If, as a by-product of employers' monitoring and coordinating activities associated with the process of production, individual productivities are identified and wages are adjusted accordingly, the low skill workers may return home. Absent pooling, which conferred a high average wage, return migration could be preferable to continued migration for these workers (Stark, 1995).

In this paper we propose and explore in considerable detail yet another motive for return migration: a higher purchasing power of savings (generated from work abroad) at home than abroad. Specifically, we investigate the role of deviations from purchasing power parity in rendering given amounts of savings facilitate higher levels of consumption at home than abroad. While earlier work considered the effect of the probability of return migration on migrants' optimal savings (Stark, 1991, chapter 27), the current paper develops a reverse line of inquiry: how migrants' savings determine the optimal timing of return.

In exploring the role of a higher purchasing power of savings at home than abroad, a difficulty arises because of the likelihood that wages at home are lower than wages abroad. Even if consumption of a given good or service is more enjoyable at home than abroad, an optimizing agent needs weigh more, and more enjoyable, consumption against the opportunity cost of higher wages which in turn would have entailed even higher consumption. To see through this tension we proceed in stages. First, we formulate a benchmark model in which we net out the influences of differentials in purchasing power parity and in preferences for consumption by location. Inter alia, we pose and answer the questions whether (upon optimization) life-long migration will dominate life-long non-migration, and whether return migration will take place at all. We further inquire, for the case of a possible single split between migration and home stay, which of these two comes first, and whether there can be a sequence of several migratory and return moves. We

find that if the planning period were to include a migration spell, the spell has to occur at the beginning of the planning period, and that migration will be continuous. Our final conclusion though is that the set of feasible and optimal migration strategies includes only two elements: the strategy of no migration and the strategy of permanent migration.

We next formulate a richer model in which we assume, as in the benchmark model, that wages abroad are higher than wages at home while, unlike in the benchmark model, savings generated from work abroad purchase more goods and services at home than abroad, and consumption of a given good or service at home confers a higher utility than consumption of the same good or service abroad. Interestingly, several results derived for the benchmark model carry through, for example, that if migration takes place it must be continuous and occur early on. However, unlike in the benchmark model, the set of feasible and optimal strategies now admits return migration. To allow us determine the optimal *duration* of migration and investigate how this duration responds to different levels of model parameters we employ a specific form of the utility function. We derive several novel results including, for example, an inverse relationship between the optimal duration of migration and purchasing power parity (for the case of an interior solution) and an inverse relationship between the optimal duration of migration and the wage abroad (for some border solution cases). We provide a complete list of our results in the concluding section where we also draw several implications for research and for policy.

2 A Benchmark Model of Migration

We build a benchmark model of migration by pooling together the following assumptions:

- (1) The individual has perfect foresight of his life expectancy, T , wages at home, w_H , and wages abroad, w_F . He is endowed with one unit of work per period which is inelastically supplied. Wages and consumption expenditures are expressed in nominal terms of the same currency (for example, US \$), and wages abroad exceed wages at home, $w_F > w_H$. Abroad and at home wages are constant over time. For the sake of simplicity, the rate of interest and the rate of time preference are assumed equal to zero.

- (2) At each point in time the individual's accumulated savings have to be non-negative; borrowing for consumption purposes against future wages is not possible.
- (3) All consumption goods are assumed to be locationally non-transferable, that is, at any point in time the location of earnings and the location of consumption expenditures are identical. Consumption at home or abroad is the sole argument in the individual's utility function.
- (4) The individual's utility function, $U(\cdot)$, is additively separable, well-behaved, and exhibits the usual concavity properties. Intertemporal utility, as modeled subsequently, is of the von Neumann-Morgenstern type. The same functional form holds for consumption abroad and consumption at home.
- (5) Purchasing power parity, E , is assumed to hold ($E = 1$), that is, the purchasing power of (a unit of) foreign currency at home is the same as it is abroad.

The individual maximizes the utility function

$$\tilde{U} = \int_0^T U(C(t)) dt \quad (1)$$

subject to

$$\int_0^t (w(t) - C(t)) dt \geq 0, \quad \forall t \in [0, T], \quad (2)$$

where $w(t)$ and $C(t)$ are, respectively, the periodic wage and consumption. In this general form, the set of optimal strategies includes several migratory moves during the individual's lifetime. The specific points in time at which these migratory moves take place are also subject to choice. According to the following three lemmas, the set of optimal strategies always includes the strategy of selecting to stay in the initial location at the beginning of life, and a subsequent return to that location in some future point in time t_F .

Lemma 1: For any concave utility function, it is optimal to maintain a constant level of consumption independent of location.

Proof: Proofs of Lemmas 1-3 and Corollaries 1 and 2 are in the appendix.

We denote by HF(t_F) the strategy of spending the first period of life, $[0, T-t_F]$, at home and the second period of life, $[t_F, T]$, abroad. FH(t_F) represents the strategy of spending the first period of life, $[0, t_F]$, abroad and the second period of life, $[t_F, T]$, at home.

Lemma 2: Suppose that the budget constraint (2) holds and that the intertemporal utility function is continuous and concave. Then, strategy FH(t_F) always dominates strategy HF(t_F).

Lemma 3: The strategy of spending the first period of life $[0, t_F]$ abroad, FH(t_F), weakly dominates all strategies in which the duration of staying abroad is split into two or more sub-intervals.

Note that lemmas (2) and (3) imply equivalence between the interpretation of t_F as the point of return or as the duration of staying abroad. Using lemmas (1) through (3), the budget constraint (2) reduces to constraints (3a) and (3b):

$$C_F t_F + C_H (T - t_F) - w_F t_F - w_H (T - t_F) \leq 0 \quad (3a)$$

$$C_F - w_F \leq 0. \quad (3b)$$

From the lemmas we know that the optimal migration strategy belongs to the class of FH(t_F) strategies for which the overall utility function (1) reduces to

$$U(C_F, C_H, t_F) = U(C_F) t_F + U(C_H) (T - t_F). \quad (4)$$

To maximize (4) subject to (3a) and (3b) we form the Lagrangian

$$\begin{aligned} \mathcal{L} = & t_F U(C_F) + (T - t_F) U(C_H) \\ & - \lambda [(C_F t_F + C_H (T - t_F) - w_F t_F - w_H (T - t_F))] - \mu (C_F - w_F). \end{aligned} \quad (5)$$

The necessary first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial C_F} = \frac{\partial U}{\partial C_F} t_F - \lambda t_F - \mu \leq 0 \quad (6)$$

$$\frac{\partial \mathcal{L}}{\partial C_H} = \frac{\partial U}{\partial C_H} (T - t_F) - \lambda (T - t_F) \leq 0 \quad (7)$$

$$\frac{\partial \mathcal{L}}{\partial t_F} = U(C_F) - U(C_H) - \lambda (C_F - C_H - w_F + w_H) \leq 0 \quad (8)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = [C_F t_F + C_H (T - t_F) - w_F t_F - w_H (T - t_F)] \leq 0 \quad (9)$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = C_F - w_F \leq 0 \quad (10)$$

$$C_F \frac{\partial \mathcal{L}}{\partial C_F} = 0 \quad (11)$$

$$C_H \frac{\partial \mathcal{L}}{\partial C_H} = 0 \quad (12)$$

$$t_F \frac{\partial \mathcal{L}}{\partial t_F} = 0 \quad (13)$$

$$\lambda [C_F t_F + C_H (T - t_F) - w_F t_F - w_H (T - t_F)] = 0 \quad (14)$$

$$\mu (C_F - w_F) = 0 \quad (15)$$

$$C_F, C_H, t_F, \lambda, \mu \geq 0. \quad (16)$$

According to the Kuhn-Tucker conditions either the Lagrange multipliers are 0 or the marginal conditions (9) and (10) hold by equality. Two particular cases for the duration of migration, t_F , are $t_F = 0$, which corresponds to the alternative of no-migration, and $t_F = T$, which represents the alternative of permanent migration.

Corollary 1: Since for no combination of $C_F, C_H > 0$ and $0 < t_F < T$ with any λ and μ the set of first-order conditions (6)-(16) is fulfilled, the alternative of temporary migration does not belong to the set of feasible migration strategies.

It remains to be shown that at least for some combinations of $C_F, C_H \geq 0$ and $t_F = 0$ or $C_F, C_H \geq 0$ and $t_F = T$ with one $C_i = 0$ ($i = F, H$) a solution exists. If the entire life span is spent abroad, the optimization is subject to both budget constraints (3a) and (3b). Since for permanent migration the two constraints are linearly dependent, both $\lambda > 0$ and $\mu > 0$.

Corollary 2: For the benchmark model the set of feasible and optimal migration strategies contains only two elements: the strategy of no-migration and the strategy of permanent migration. A comparison of the levels of consumption reveals that if migration takes place, it is from the low-wage economy to the high-wage economy.

3 Migration if Purchasing Power Parity Does Not Hold

3.1 A General Model

To allow for a richer set of migration alternatives, we assume that the utility function of consumption at home differs from the utility function of consumption abroad. Even if the individual can purchase a consumption good for an identical price independent of the location of sale, utility may nevertheless be higher at the preferred place (in our context the home country) due to the presence of non-monetary (for example, cultural and social) utility components.

Consider then the utility function

$$U_M = \int_0^{t_F} U_F(C_F) dt + \int_{t_F}^T U_H(C_H) dt \quad (17)$$

where U_M = utility of a temporary migrant, U_F = utility in the foreign country, U_H = utility in the home country, T = the duration of life, fixed and known with certainty¹, and t_F = time spent

¹ Generally T may not be a given fixed number. To the extent that T depends on consumption, and to the extent that consumption, in turn, is determined by the location of stay, T will also depend on the decision whether or not to migrate, and if to migrate - for how long.

abroad. The migrant's budget constraints are identical to those in equation (3) above. For a nonmigrant, in equation (17), $t_F = 0$, whereas for a permanent migrant, in equation (17), $t_F = T$. In view of the observation of persistent purchasing power differentials across countries, we further study the case in which the purchasing power of one unit of foreign currency in the home country is E times higher than abroad ($E > 1$). By assuming that $U_F(C) = U(C)$ and $U_H(C) = U(E \cdot C)$ where $C \in \{C_F, C_H\}$, the notion of purchasing power differentials is explicitly introduced into the migrant's utility function.

From the concavity of U it follows immediately that there is no time-dependence of the consumption path.

Corollary 3: Lemmas 1 and 3 hold for the case $E \neq 1$ as well.

Overall utility can be split into two parts: utility derived from consumption at home and utility derived from consumption abroad. The following expression then gives the migrant's lifetime utility:

$$U_M = t_F U_F(C_F) + (T - t_F) U_H(C_H). \quad (18)$$

In addition to constraints (3a) and (3b) we add the time feasibility constraint (3c) $0 \leq t_F \leq T$.

The corresponding Lagrangian is given by

$$\begin{aligned} \mathcal{L} = & t_F U(C_F) + (T - t_F) U(E C_H) \\ & + \lambda [(w_F - C_F) t_F + (w_H - C_H) (T - t_F)] \\ & + \mu_1 (w_F - C_F) + \mu_2 (T - t_F) + \mu_3 t_F. \end{aligned} \quad (19)$$

The first order conditions are²

² For completeness, the remaining Kuhn-Tucker conditions are

$$C_F \frac{\partial \mathcal{L}}{\partial C_F} = 0, \quad C_H \frac{\partial \mathcal{L}}{\partial C_H} = 0, \quad t_F \frac{\partial \mathcal{L}}{\partial t_F} = 0,$$

with the non-negativity constraints given by

$$C_F \geq 0, \quad C_H \geq 0, \quad t_F \geq 0, \quad \lambda \geq 0, \quad \mu \geq 0.$$

$$\frac{\partial \mathcal{L}}{\partial C_F} = t_F U'_{C_F}(C_F) - \lambda t_F - \mu_1 \leq 0 \quad (20)$$

$$\frac{\partial \mathcal{L}}{\partial C_H} = (T - t_F) [U'(E C_H) \cdot E - \lambda] \leq 0 \quad (21)$$

$$\frac{\partial \mathcal{L}}{\partial t_F} = U(C_F) - U(E C_H) - \lambda (C_F - C_H - w_F + w_H) - \mu_2 + \mu_3 \leq 0. \quad (22)$$

From equations (20) and (21) we obtain the following expression for the Lagrange multipliers:³

$$\lambda = U'(E C_H) \cdot E \quad (23)$$

$$\mu_1 = t_F (U'_{C_F}(C_F) - \lambda), \quad (24)$$

and for t_F we get

$$t_F = T \frac{C_H - w_H}{C_H - w_H + w_F - C_F}. \quad (25)$$

If the duration of migration, t_F , falls in the feasible interval $[0, T]$, then $\mu_2 = \mu_3 = 0$. In this case, using (22) and (23), we obtain the following implicit functional relationship between C_F and C_H

$$U(C_F) - U(E C_H) + E U'(E C_H) (w_F - w_H - C_F + C_H) = 0. \quad (26)$$

If $\mu_1 = 0$, a migrant does not save and thereby in each point in time consumption expenditures are equal to his wage. From (26) we have

$$U(w_F) = U(E w_H). \quad (27)$$

As complementary slackness conditions we have

$$\lambda [(w_F - C_F) t_F + (w_H - C_H) (T - t_F)] = 0$$

$$\mu_1 (w_F - C_F) = 0$$

$$\mu_2 (T - t_F) = 0$$

$$\mu_3 t_F = 0.$$

³ A sufficient condition for equations (23) through (29) to hold as equalities is that $U(\cdot)$ belongs to a class of utility functions for which $U' = \infty$ in point $(C_F=0, C_H=0)$.

For any utility function this equation has a solution $E = w_F/w_H$. Thus, for each pair of wages at home and abroad there is exactly one purchasing power ratio E for which utilities at home and abroad are equal. Hence, there is an infinite number of solutions for the optimal duration of migration with respect to which the individual is indifferent.

If $w_F \neq E \cdot w_H$, then $U(w_F) \neq U(E \cdot w_H)$. In this case it is optimal to choose either no-migration or permanent migration. The choice is made depending on in which country the migrant's consumption is larger. If $\mu_1 > 0$, then from (23), $\lambda = EU'(EC_H)$, and from (24), $\lambda = U'(C_F)$, and hence

$$U'(C_F) = E U'(E C_H). \quad (28)$$

For any concave utility function, equation (28) implicitly defines C_H as a function of C_F , that is, $C_H = \Phi(C_F)$. Using (26), we thus obtain for the optimal consumption abroad that

$$U(C_F) - U(E \Phi(C_F)) + U'_{C_F}(w_F - w_H - C_F + \Phi(C_F)) = 0. \quad (29)$$

Substituting C_F (from (29)) and C_H (from (28)) into (25), the optimal duration of migration is defined. The optimal duration of migration parametrically depends on w_F , w_H , E , and U .

Proposition: For $E \neq 1$, the set of feasible and optimal migration strategies is richer than in the benchmark model (cf. Corollary 2). Any $t_F \in [0, T]$ may represent an optimal duration of staying abroad, that is, return migration may be preferable to no-migration or to permanent migration.

Though the problem is formally solved, a study of the dependence of the optimal duration of migration on the parameters of the model is impossible without assuming a specific form of the utility function. We employ such a specification next.

3.2 The Case of Logarithmic Utility Functions: Interior Solution

In order to obtain explicit results for the optimal duration of migration, we use in this and in the following sub-section a logarithmic specification of the utility function,⁴ that is, we assume that the utility function takes the form

$$U(C) = \ln C. \quad (30)$$

In order to secure comparability of the utility gained from consumption abroad and consumption at home, we measure consumption throughout in units of the currency of the foreign country. Due to higher purchasing power of the foreign currency at home ($E > 1$), a given level of utility can be secured with a lower nominal foreign currency expense.⁵ Hence,

$$U_H(C_H) = \ln(E C_H), \quad U_F(C_F) = \ln C_F. \quad (31)$$

For the functions in (31) we have

$$U'_{C_H} = \frac{1}{C_H}, \quad U'_{C_F} = \frac{1}{C_F}. \quad (32)$$

Here, unlike in the benchmark model, an equalization of consumption expenditures at home and abroad is not implied. Since an equal level of consumption at home and abroad confers equal marginal utility, by equation (32) we get

$$C_F = C_H. \quad (33)$$

However, the *level* of utility at home exceeds that reached abroad (as per (31) and (33) with $E > 1$). By substituting equations (30) through (33) into equation (26), we derive an equation for C_H :

$$C_H = \frac{w_F - w_H}{\ln E}. \quad (34)$$

As the purchasing power of the foreign currency at home increases, consumption at home (measured in foreign currency units) is reduced. By substituting equations (33) and (34) into

⁴ A logarithmic utility function represents a limiting case ($\alpha \rightarrow 0$) of the widely used, general class of utility functions which are represented by the power functions, $U(C) = C^\alpha/\alpha$ where $-\infty < \alpha < 1$. These functions are defined for all non-negative levels of consumption and are concave for the given range of the parameter α .

⁵ Note that the family of logarithmic functions $U_E(x) = \ln(E \cdot x)$ satisfies the condition that a transformation of one function into another by a scale transformation of an argument x entails a vertical shift of the curve.

equation (25), the optimal duration of staying abroad is given by

$$\frac{t_F}{T} = \frac{1}{\ln E} - \frac{w_H}{w_F - w_H}. \quad (35)$$

To obtain the utility of a temporary migrant conditional on an *optimal* duration of stay recall that the utility associated with such migration plan is given by equation (18). By equations (18), (31), and (33) through (35) we obtain

$$U_M = T \left[\ln \left(\frac{E \cdot (w_F - w_H)}{\ln E} \right) + \frac{w_H \ln E}{w_F - w_H} - 1 \right]. \quad (36)$$

Of course, we have to check whether the optimal duration is within the interval $0 < t_F < T$. Otherwise, either $t_F = 0$, or $t_F = T$, that is, temporary migration is not an optimal decision; choice is then confined to permanent migration or no migration at all and is determined by comparison of the utilities $U_{NM} = T \ln (E w_H)$ and $U_{PM} = T \ln w_F$, where the subscript NM stands for no migration, and the subscript PM stands for permanent migration.

Equation (35) gives rise to several interesting implications:

1. The partial derivative of the optimal duration of migration with respect to the foreign wage is

$$\frac{\partial t_F}{\partial w_F} = \frac{T w_H}{(w_F - w_H)^2} > 0, \quad (37)$$

that is, the optimal duration of migration is increasing in the wage abroad. A higher wage from work abroad allows for higher consumption both at home and abroad. Provided an interior solution exists, the dual rise in consumption induces prolongation of the time spent away and hence $\partial t_F / \partial w_F > 0$. However, as will be demonstrated below, this relationship is necessarily fulfilled only for interior solutions.

2. We further find that in the case of an interior solution, the optimal duration of migration is decreasing in the wage at home, that is,

$$\frac{\partial t_F}{\partial w_H} = - \frac{T w_F}{(w_F - w_H)^2} < 0. \quad (38)$$

3. If $E > 1$, buying power at home exceeds that abroad. In this case, the partial derivative of the optimal duration of migration with respect to E is

$$\frac{\partial t_F}{\partial E} = - \frac{T}{E \cdot \ln^2 E} < 0. \quad (39)$$

The optimal duration of stay abroad is *inversely* related to purchasing power at home.⁶

3.3 The Case of Logarithmic Utility Functions: Border Solution

To complete the analysis, we now turn to the border solution case. Here we add the restriction

$$C_F \geq C_{\min F}, \quad (40)$$

where $C_{\min F}$ is the minimal level of consumption abroad necessary to keep body and soul together. Without loss of generality we assume that for the case of the border solution $C_F \geq 1$. Imposing this restriction is equivalent to the consideration of a utility maximization problem for a discontinuous utility function $U_F(C_F)$ with the discontinuity occurring at $C_F = C_{F \min} = 1$.

Formally, we may assume that at consumption levels below C_F utility drops to a large negative constant. This implies infinite marginal utility to the left of $C_F = 1$.

The Lagrangian for the new problem with one additional restriction (as compared to (19)) is given by

$$\begin{aligned} \mathcal{L} = & t_F \ln C_F + (T - t_F) \ln (E C_H) \\ & + \lambda [(w_F - C_F) t_F + (w_H - C_H) (T - t_F)] \\ & + \mu_1 (w_F - C_F) + \mu_2 (T - t_F) + \mu_3 t_F + \mu_4 (C_F - 1). \end{aligned} \quad (41)$$

⁶ The qualitative results, that is, the signs of the partial derivatives of the optimal duration of migration as per the comparative statics performed above, hold for the class of power functions as well. Unlike in the case of the logarithmic utility function, in the case of power functions the optimal consumption expenditures at home and abroad are never equalized.

If $C_F > 1$, we have the case of sub-section 3.2 (with the interior solution of the problem given there). For the border solution case we take $C_F = 1$ with (41) being transformed into

$$\begin{aligned} \mathcal{L} = & (T - t_F) \ln(E C_H) \\ & + \lambda [(w_F - 1) t_F + (w_H - C_H) (T - t_F)] \\ & + \mu_1 (w_F - 1) + \mu_2 (T - t_F) + \mu_3 t_F. \end{aligned} \quad (42)$$

The first order conditions are⁷

$$\frac{\partial \mathcal{L}}{\partial C_H} = (T - t_F) \left[\frac{1}{C_H} - \lambda \right] \leq 0 \quad (43)$$

$$\frac{\partial \mathcal{L}}{\partial t_F} = -\ln(E C_H) + \lambda (w_F - 1 - w_H + C_H) - \mu_2 + \mu_3 \leq 0. \quad (44)$$

If the condition $0 < t_F < 1$ is not satisfied for given solutions of C_H , we set $t_F = 0$ or $t_F = 1$. Since $\mu_2 = 0$ and $\mu_3 = 0$ unless $t_F = 0$ or $t_F = 1$, from the first order conditions we obtain

$$C_H \ln(E C_H) = -w_H + C_H - 1 + w_F. \quad (45)$$

For $C_F = 1$ we also have

$$t_F = T \frac{C_H - w_H}{C_H - w_H + w_F - C_F}. \quad (46)$$

Equation (45) cannot be solved analytically, but for nearly all parameter values it has at least one solution in the interval $[0, 1]$. The pair of equations (45) and (46) enables us to find these solutions numerically and hence trace the dependence of the optimal duration of stay abroad on the wage rate abroad. A detailed depiction of two cases illustrates.

Throughout the analysis that follows we normalize the expected duration of life such that $0 \leq t_F \leq T = 1$. We consider first the border solution for the case $\ln E = 0.5$ and $w_H = 1$. Figure 1 depicts the dependence of the optimal duration of staying abroad on the wage abroad. To keep the graphical presentation parsimonious, the horizontal lines corresponding to no-migration ($t_F = 0$)

⁷ Cf. footnote 2 for a complete set of analogous Kuhn-Tucker conditions.

and permanent migration ($t_F = 1$) are not drawn. While negative wages ($w_F < 0$) are irrelevant (recall assumption 2 of section 2), by assumption, wages in the interval $0 \leq w_F \leq 1$ also need not be considered as utility abroad remains negative even if the entire income is consumed. For wages in the interval $1 < w_F \leq 1.5$ no-migration ($t_F = 0$) is the dominant alternative. A positive wage differential thus need not entail migration. Only for wages abroad falling in the interval $1.5 < w_F < 2$ does the optimal duration of migration fall in the feasible region $0 < t_F < 1$, that is, temporary migration occurs only if wages abroad are in this interval. By contrast, for $w_F > 2$, there is no incentive to return, that is, the dominant alternative is permanent migration ($t_F = 1$). In the feasible interval $1.5 < w_F < 2$, the optimal duration of migration is positively related to the wage abroad. This replicates the result of equation (37). It is worth mentioning that imposition of the restriction $C_F \geq 1$ does not change these results. The reason is that for any $w_F \geq 1$, by implication, consumption abroad exceeds this minimum consumption level, that is, in this particular case with $w_H = 1$, the constraint is never binding.

Figure 2 depicts the border solution for a second case wherein $\ln E = 2$ and $w_H = 0.1$, a case in which a high purchasing power differential ($E = 7.39$) is matched with a low wage at home, w_H . For specific values of the wage abroad, w_F , there are two solutions to equation (45). By inserting these values into equation (46), we obtain two corresponding values for the optimal duration of staying abroad. For wages abroad between $0 \leq w_F \leq 1$ there is no solution to equation (45). However, for wages abroad between $1.0 < w_F < 1.1$ there are two border solutions. Since the first border solution requires an optimal duration of staying abroad that exceeds total life expectancy ($t_F > T = 1$, not represented in Figure 2), the second border solution is chosen [$t_F(\text{border})$]. For wages $w_F \geq 1.1$ there is only one border solution. In the interval $1.0 \leq w_F < \infty$, t_F is a monotonically *decreasing* function of w_F ($dt_F/dw_F < 0$). Thus, in this case of the border solution, the optimal duration of staying abroad is *negatively* related to the wage abroad. This result contrasts with the previous case of $\ln E = 0.5$ and $w_H = 1$ in which the optimal duration of staying abroad was *positively* related to the wage abroad. The reason underlying this result is that with a large purchasing power differential and a large and increasing wage differential, a prolonged stay abroad comes at the expense of a large and increasingly larger consumption at home. Optimization then mandates a shortening of the stay abroad.

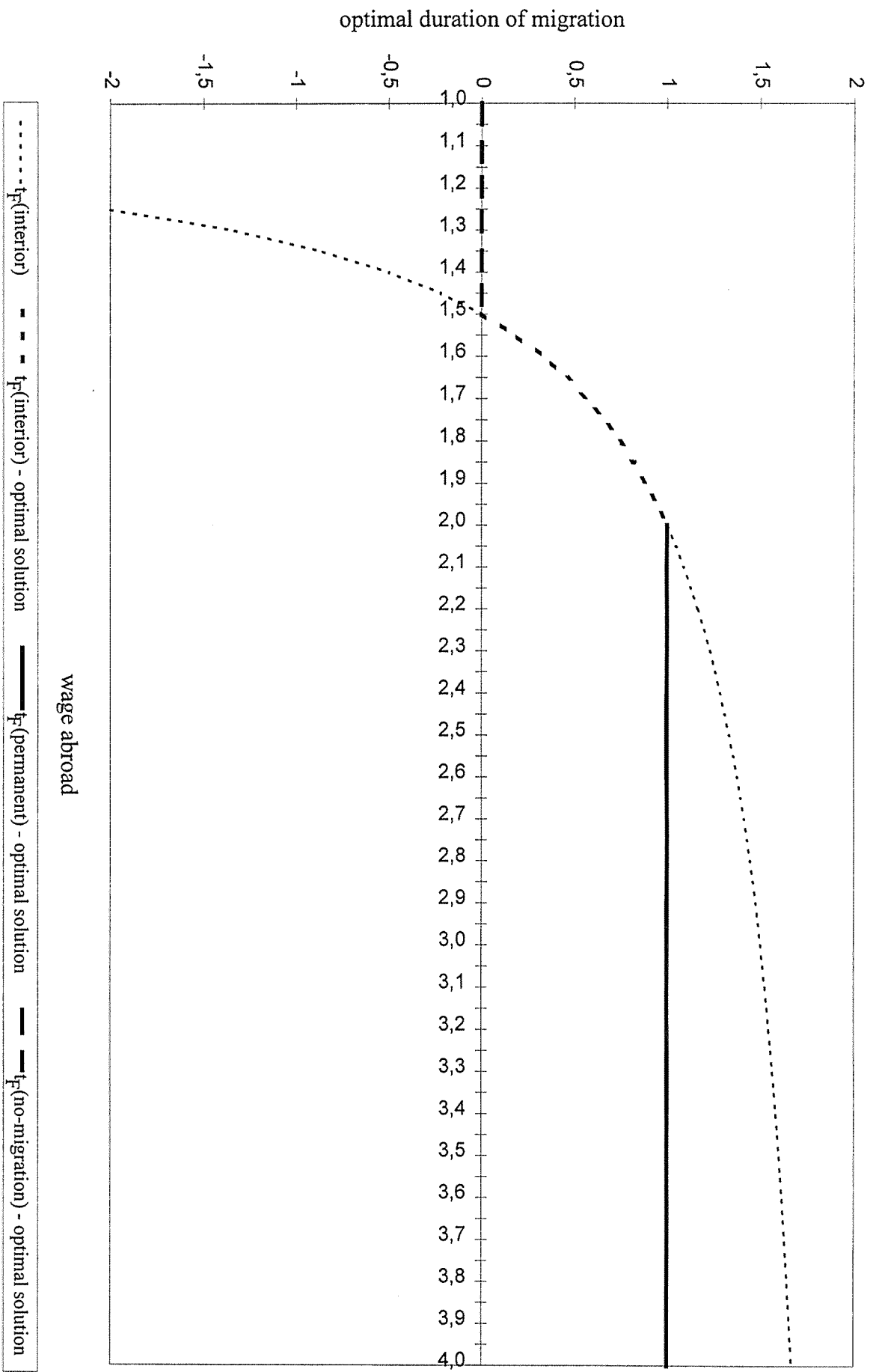


Figure 1: The Optimal Duration of Migration for $\ln E = 0.5$ and $w_H = 1$.

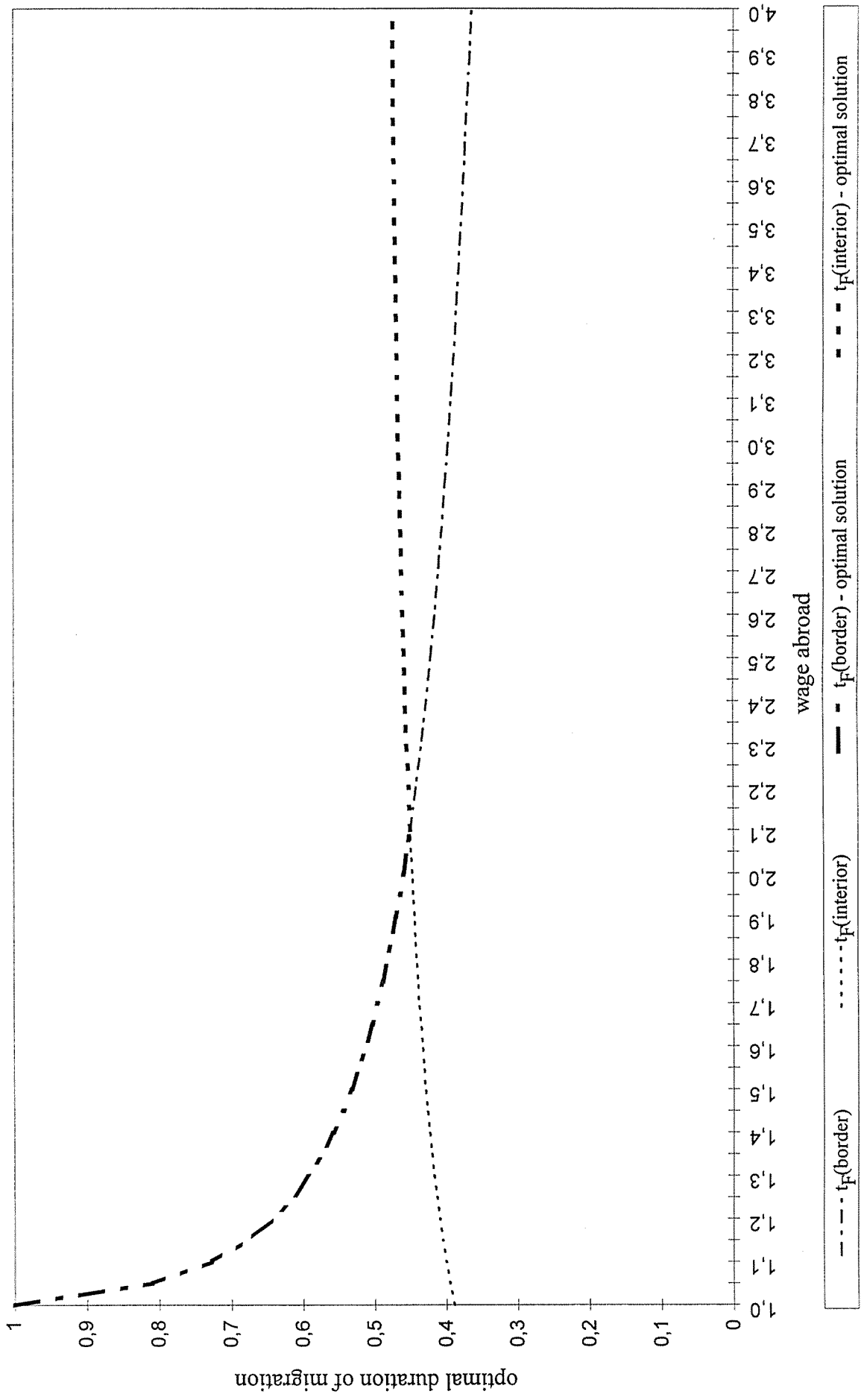


Figure 2. The Optimal Duration of Migration for $\ln E = 2$ and $w_H = 0.1$.

We next investigate the conditions of a transition from the border solution to the interior solution. The transition takes place when the constraint C_F ceases to bind, that is, when consumption abroad exceeds the baseline consumption level. In the case $\ln E = 0.5$ and $w_F = 1$, as pointed out above, the constraint $C_F > 1$ is never binding for any wage abroad larger than 1. As the feasible interval is $1.5 < w_F < 2$, the issue of transition need not be considered. By contrast, in the case $\ln E = 2$ and $w_H = 0.1$ the transition point where the border solution and the interior solution coincide is $w_F = 2.1$ (see Figure 2). For all values of w_F that exceed this value, the constraint C_F is no longer binding.

At $w_F = 2.1$, the optimal duration of staying abroad reaches its minimum. The interior solution and border solution curves depicting the optimal duration of staying abroad intersect at this wage since the optimal duration of staying abroad is negatively related to the wage abroad in the case of the border solution, while it is positively related to the wage abroad in the case of the interior solution. The optimal duration of staying abroad is thus depicted by the upper bold sections of the border solution and the interior solution curves in Figure 2 (where the interior solution is derived from equation (35)).

What are the utility implications of our results? Considering first the case $\ln E = 0.5$ and $w_H = 1$, we note that for the border solution, the utility of a temporary migrant is given by $U_M = (T - t_F) \ln(E \cdot C_H)$. (Recall (41) and that $\ln(C_F=1) = 0$.) Using this equation we can compare the utility of a temporary migrant with the utility of a permanent migrant as well as with the utility of a non-migrant. The utility maximizing strategy is contingent on the wage abroad. For the case of wages abroad falling in the interval $1 \leq w_F \leq 1.5$, the alternative of no-migration is dominant. Contrary to this, for wages in the interval $1.5 < w_F < 2$ temporary migration is chosen. For wages $w_F > 2$ permanent migration turns out to be the optimal alternative. Given the optimal duration of staying abroad, higher wages abroad either increase utility or leave it unaffected, but they are never negatively related to utility. Turning to the case $\ln E = 2$ and $w_H = 0.1$, for any wage abroad $w_F > 1$, the alternative of no-migration is dominated as the utility at home is negative.⁸ Recall that

⁸ Note that the negative value of the utility in the case of no-migration can be changed without affecting the essential message as it is always possible to add a constant thereby rescaling the unit of measurement without implying any behavioral change. (We treat the minimum consumption abroad necessary for survival as baseline that confers zero utility, and calculate other utility values from this reference point.)

for wages abroad $1 < w_F < 1.1$ there are two border solutions. The first border solution [not depicted in Figure 2] requires an optimal duration of staying abroad that exceeds total life expectancy ($t_F > T = 1$), while the second border solution [$t_F(\text{border})$ of Figure 2] is dominated by permanent migration which is the utility maximizing alternative. For $w_F \geq 1.1$ there is only one border solution. The corresponding utility arising from temporary migration is higher than the utility gained from permanent migration. At point $w_F = 2.1$, the transition from the border solution to the interior solution of temporary migration takes place, that is for wages abroad marginally higher than $w_F = 2.1$ the constraint on consumption abroad is no longer binding. In the interval $2.1 < w_F < \infty$ temporary migration dominates the other alternatives. Not surprisingly, the solution of both the bounded and the unbounded problems posits utility as a monotonically increasing function of w_F . But as we have clearly shown, an increasing utility as a function of rising w_F does not (necessarily) entail substitution of permanent migration for temporary migration. Under conditions as specified, temporary migration is, and continues to be, the strategy that confers the highest utility, even when w_F is rising.

The conclusion drawn from comparing the utility implications of both cases is that under some configuration of parameter values, temporary migration is superior to permanent migration; under other configurations the reverse holds. To the extent that Figure 2 reflects conditions pertaining to East European - West European migration, whereas Figure 1 reflects intra-West European or intra-European Union (EU) migration, our analysis suggests that East-West migration will tend to be temporary, while intra-EU migration is likely to be permanent. This prediction is contrary to our expectation on writing this paper.

4 Conclusions

This paper set out to identify conditions under which the optimal duration of migration is less than the maximal feasible duration of migration. This occurs if all or some returns to migration accrue from return migration. Without reversal of the inter-country wage differential, both migration and return migration can be optimal. When work and consumption are sequential, the individual will work where the wage is highest and consume where the price of consumption is lowest: if the wage abroad is higher than the wage at home and the price of consumption at home is lower than the price of consumption abroad, the optimal duration of migration is the duration

of work. Our analysis indicates that in the presence of a purchasing power differential ($E > 1$), migrants who return home and dissave for the purpose of discretionary consumption at home attain maximal utility. The optimal duration of migration (and hence the optimal point of return) depends, in a well-defined way, on the wage rates abroad and at home, on the consumption levels abroad and at home, on the capacity to accumulate savings abroad and transfer them home, and on life expectancy.

Our specific analysis, which recurses to logarithmic utility functions, produces precise formulas for optimal consumption and the optimal duration of migration for both the interior and the border solution cases. If purchasing power differentials exist and are sufficiently high (E is sufficiently larger than 1), the following holds: First, if wages abroad are sufficiently high with respect to the minimal level of consumption required abroad, the saving rate would also be high. Upon return, consumption at home, in nominal terms, will be the same as abroad, but the utility level obtainable will be higher. Second, if wages abroad are not sufficiently high, post-return consumption in nominal terms will be less than consumption abroad. Although lower in nominal terms, such consumption will still confer higher utility levels than could have been secured abroad. Third, the optimal duration of migration declines with a rise in the purchasing power differential. This result points to an interesting policy implication: both the country of origin and the country of destination can affect the value of migrants' savings and thereby return migration through exchange rate policies. Either devaluation of the country-of-origin currency or appreciation of the country-of-destination currency is likely to accelerate return migration. When wages abroad increase, in some fully defined cases, the duration of migration is prolonged, but in others it is shortened. Finally, under fully specified conditions, a positive and rising wage differential is consistent with temporary migration being and remaining the optimal migration strategy.

Our analysis also suggests that the standard method for calculating the differential in real wages between abroad and home requires revision: the wage abroad should not be deflated in its entirety by the country-of-destination price level - the amount that is saved should be deflated by the price level at the country of origin. This implies that (when $E > 1$) the effective wage differential between abroad and home is larger than the conventionally measured differential. Interestingly, this need not result in a larger stock of migrants in the high wage country. As demonstrated in this paper, the response to, hence the realization of, this "enhanced" differential is manifested through return migration, not through migration.

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Appendix

Proof of Lemma 1:

For any strictly concave utility function, any parameter $\zeta \in (0, 1)$, and any $C_1 \neq C_2$,

$$U[\zeta C_1 + (1 - \zeta) C_2] > \zeta U(C_1) + (1 - \zeta) U(C_2).$$

Hence, any split of intertemporal consumption into unequal levels is inferior to consumption at equal levels during the entire stay in a given location. ■

Proof of Lemma 2:

By assumption, $w_F > w_H$. Due to (2), this implies $C_H \leq w_H$. Since for an optimal strategy the lifetime budget constraint must be satisfied with equality, for strategy HF(t^*) we have $0 < C_H \leq w_H < w_F \leq C_F$. For strategy FH(t^*), it follows from (2) that $C_F \leq w_F$ and thus $C_H \geq w_H$.

For $T = 1$, the budget constraint for both strategies is

$$t^* C_F + (1 - t^*) C_H = t^* w_F + (1 - t^*) w_H = \text{const.}$$

Therefore, for any feasible $C_F, C_H \in [w_H, w_F]$ there exists an optimal C^* ,

$$C^* = t^* w_F + (1 - t^*) w_H, \quad \text{with } C^* \in [w_H, w_F].$$

Due to concavity we have

$$U(C^*) = U[t^* w_F + (1 - t^*) w_H] = U(t^* C_F + (1 - t^*) C_H) > t^* U(C_F) + (1 - t^*) U(C_H).$$

Note that strong inequality holds because under strategy HF(t^*) for any $w_F > w_H$, $C_F > C_H$. Hence, FH(t^*) > HF(t^*) for any t^* . ■

Proof of Lemma 3:

From Lemma 1 we know that at any location consumption is independent of location and is a constant function of time. Non-satiation and (2) valued at $t = T$ imply

$$w_F t_F + w_H (T - t_F) = C_F t_F + C_H (T - t_F).$$

Note that due to (2) a migrant has always non-negative savings in the foreign country and dissaves at home. For the optimal level of expenditures, if migration occurs more than once in

life, there exists a duration of staying at home between any two stays abroad which satisfies (2). However, for a sufficiently long duration of staying at home, the budget constraint binds, and hence strategy HF is not always feasible. This is equivalent to stating that strategy FH weakly dominates all other strategies. Furthermore, if, for example, some transportation costs exist, strategy FH becomes strongly dominant. ■

Proof of Corollary 1:

Considering all possible parameter constellations, we distinguish 32 cases.⁹ In order to demonstrate that the set of feasible migration alternatives contains exactly two elements, that is, the alternatives of no-migration and of permanent migration, it is, however, sufficient to consider the following four cases:

Case 1: $C_F, C_H > 0$ and $0 < t_F < T$; $\lambda = 0$ and $\mu = 0$

Due to non-satiation we have $(\partial U/\partial C_F) \geq 0$. But then (6), $t_F(\partial U/\partial C_F) \leq 0$, does not hold. Hence the parameter constellation of case 1 is not feasible.

Case 2: $C_F, C_H > 0$ and $0 < t_F < T$; $\lambda = 0$ and $\mu \neq 0$

From (7) together with (12) we obtain $(T - t_F)(\partial U/\partial C_H) = 0$. Since due to non-satiation $(\partial U/\partial C_H) > 0$, for this to hold it is required that $t_F = T$ which contradicts our assumption that $C_H > 0$.

Case 3: $C_F, C_H > 0$ and $0 < t_F < T$; $\lambda \neq 0$ and $\mu = 0$

From (7) together with (12) we obtain $(T - t_F)(\partial U/\partial C_H) = \lambda(T - t_F)$ which holds only if $t_F = T$ and/or $(\partial U/\partial C_H) = \lambda$. Since the alternative of permanent migration, $t_F = T$, does not conform to our assumptions, we proceed assuming that $(\partial U/\partial C_H) = \lambda$. From (6) we know that $(\partial U/\partial C_F) = \lambda$, so that $(\partial U/\partial C_F) = (\partial U/\partial C_H)$, and hence we have (7') $C_F = C_H$. Inserting (7') into (8), we get

⁹ There are eight combinations: $C_F, C_H, t_F > 0$; $C_F, C_H > 0$ and $t_F = 0$; $C_F, t_F > 0$ and $C_H = 0$; $C_H, t_F > 0$ and $C_F = 0$; $C_F > 0$ and $C_H, t_F = 0$; $C_H > 0$ and $C_F, t_F = 0$; $t_F > 0$ and $C_H, C_F = 0$; and $C_F, C_H, t_F = 0$. Each of these combinations has to be checked for four different sets of Lagrange multipliers: $\lambda = 0$ and $\mu = 0$; $\lambda \neq 0$ and $\mu = 0$; $\lambda = 0$ and $\mu \neq 0$; and $\lambda \neq 0$ and $\mu \neq 0$.

$\lambda(w_F - w_H) = 0$. For this to hold, it is necessary that $w_F = w_H$ since $\lambda = 0$ is ruled out by assumption. The condition of equal wages abroad and at home, however, is in contradiction to our assumption that $w_F > w_H$.

Case 4: $C_F, C_H > 0$ and $0 < t_F < T$; $\lambda, \mu \neq 0$

From (9) we have $(C_F - w_F)t_F + (C_H - w_H)(T - t_F) = 0$ and from (10) in conjunction with (15) we obtain $C_F = w_F$. Inserting (10) in (9) yields $(C_H - w_H)(T - t_F) = 0$. Since $t_F = T$ is not feasible, we are left with $C_H = w_H$. Substituting for $C_H = w_H$ and $C_F = w_F$ in (8), in conjunction with (13) we get $U(C_F) = U(C_H)$. Hence we have $C_H = w_H = C_F = w_F$ which again violates our assumption that $w_F > w_H$. ■

Proof of Corollary 2:

We need consider two remaining cases:

Case 5: $C_F = 0, C_H > 0$ and $t_F = 0$; $\lambda > 0, \mu \geq 0$

From (7) in conjunction with (12) we have (7'') $(\partial U / \partial C_H) = \lambda$ and from (8) we obtain $-U(C_H) - \lambda(-C_H - w_F + w_H) \leq 0$. From (9) we get $(C_H - w_H) \leq 0$. Since for $\lambda = 0$ (7'') would not hold, we assume $\lambda \neq 0$. From (9) in conjunction with (14) we know that (9') $C_H = w_H$. Substituting for $(\partial U / \partial C_H)$ in (8) using (7'') and (9'), we obtain the solution $(\partial U / \partial C_H)w_F \leq U(C_H)$.

If no-migration is the preferred alternative, the individual's lifetime budget constraint (3a) binds and hence $\lambda > 0$. Since the entire life span is spent at home, the individual's budget constraint abroad is, by implication, irrelevant for the optimization and thus $\mu \geq 0$.

Case 6: $C_F > 0, C_H = 0$ and $t_F = T$; $\lambda, \mu > 0$

From (6) in conjunction with (11) we have $[(\partial U / \partial C_F) - \lambda]T - \mu = 0$. Since $C_H = 0$ and recalling (13), (8) simplifies to $U(C_F) - \lambda(C_F - w_F + w_H) = 0$. Assuming $\lambda, \mu > 0$, from (10) together with (15) we obtain $C_F = w_F$.

Hence (8) reduces to $U(C_F) = \lambda w_H$ so that $\lambda = U(C_F) / w_H$ and $\mu = T[(\partial U / \partial C_F) - U(C_F) / w_H]$. ■

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