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# One-Way Spillovers, Endogenous Innovator/Imitator Roles and Research Joint Ventures

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## Abstract

We consider a two-period duopoly characterized by a one-way spillover structure in process R&D and a very broad specification of product market competition. We show that *a priori* identical firms always engage in different levels of R&D, at equilibrium, thus giving rise to an innovator/imitator configuration and ending up with different sizes. In view of this endogenous firm heterogeneity, the social benefits of, and the firms' incentives for, research joint ventures are somewhat different from the case of *ex post* firm symmetry. The key properties of the game are submodularity (R&D decisions are strategic substitutes) and lack of global concavity.

## Keywords

Oligopolistic R&D, one-way spillovers, research joint ventures, submodularity

## JEL-Classifications

C72, L13, O32

### **Comments**

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# 1 Preliminaries

## 1.1 Introduction

This paper develops a simple two-stage strategic model of process R&D/product market competition characterized by a stochastic directed R&D spillover structure. Know-how may only flow from the more R&D intensive firm to its rival (in a duopoly setting). Leakages in the reverse direction are simply ruled out. This spillover specification suggests that the associated R&D process can be suitably approximated by a one-dimensional representation. This is discussed in Section 2 where we provide precise and independent interpretations of the spillover parameter as (i) the probability that a full spillover occurs, (ii) an inverse measure of patent length, or (iii) an inverse measure of imitation lag.

We derive two different sets of results. First, with a relatively wide scope of generality in the product market specification, we establish the existence of subgame-perfect equilibrium, as well as some general characterization of its properties. In particular, we show that no equilibrium can be symmetric although the two competing firms are (*ex ante*) identical. This suggests that inter-firm differences can emerge naturally through the very process of technological progress.

In view of the resulting (inherent) asymmetry between the two firms, it is reasonable to question the validity of the general results from the theory of research joint ventures (henceforth RJV's), since these results were derived in a framework of *ex ante* and *ex post* identical firms: See d'Aspremont-Jacquemin (1988, 1990) and Kamien-Muller-Zang (1992). Our second set of conclusions confirms the superior performance of the joint lab or cartelized RJV, established by Kamien-Muller-Zang (1992), but only under some additional assumptions, the analogs of which were not needed for the analysis under full symmetry.

Thus, as outlined above, the present work relates to two different research areas in applied microeconomics, one dealing with intra-industry firm heterogeneity and the other with strategic R&D and RJV's. In order to relate our work more precisely to these two independent areas, we begin with a literature summary for each..

### 1.1.1 Inter-Firm Heterogeneity

Variability across firm characteristics within a given industry is a ubiquitous phenomenon. Firms tend to differ in several ways including product niches, advertising strategy, corporate culture, organizational forms, incentive/compensation schemes, R&D strategy ... More obviously, they often differ in their size, market conduct and overall performance. Economists have long sought to reconcile this observed heterogeneity with conventional economic wisdom. In providing a brief review of the numerous attempts to fill this gap, it is instructive to distinguish between the emergence of intra-industry heterogeneity and its persistence over time. A thorough yet concise overview is given by Röller and Sinclair-Desgagné (1996) who also develop a simple model showing that initial differences among firms may either amplify or die down depending on market characteristics.

A standard explanation of intra-industry heterogeneity postulates that firms have different production technologies and inputs available to them, or face different factor prices, Katz-Rosen (1994). Among many evolutionary studies, Alchian (1950) emphasizes the key role (historical) random events may play in shaping the relative fortunes of firms in an industry. Similarly, Barney (1986), argues that firms can (consistently and systematically) read their common business environment differently, and thus behave differently. This may also happen in game-theoretic models of multi-stage competition with firms making different conjectures about their rivals' types and moves in a framework of rational behavior and incomplete information [see Kreps-Spence (1985)]. Another strand of the literature on the variety of firms' beliefs is based on bounded rationality on the part of firms (operating in a highly complex environment) and evolutionary theory [see, e.g., Nelson-Winter (1982)].

Since firms are generally able to observe the relative performance of their rivals over time, and since superior technological know-how and organizational forms cannot be kept secret from one's rivals for long, explaining the long-run persistence of firm heterogeneity necessarily entails identifying barriers to convergence and imitation. Among others, Caves-Porter (1977) and Sutton (1991) provide various arguments along these lines within the traditional paradigm of industrial economics.

Management theorists and business strategists have also convincingly argued for some time that organizational factors often play a fundamental role in explaining the absence of convergence. Teece (1980) reported that the diffusion of administrative innovation (such as the “*M* form” organization) can take two to five times longer than the diffusion of technological innovation (for which Mansfield (1985) estimates an upper bound of 15 months for US firms). Rumelt (1995) provides an extensive list of reasons why organizations might resist change, accounting for inertia factors and corporate culture. Hermalin (1994) develops a model capturing the relation between the internal control structures of (*ex ante* identical) firms and their strategic interaction in the product market, and leading to asymmetric equilibria.

In the context of industry dynamics, some models have established that exogenous idiosyncratic random shocks lead to persistent differences among firms in a competitive industry: Jovanovic (1982), Hopenhayn (1992), Lambson (1992), Fishman-Rob (1995). In a deterministic infinite-horizon setting, Flaherty (1980) analyses process R&D competition among a fixed number of firms using open-loop strategies. She finds that, under reasonable assumptions, only industry steady-states with different market shares across firms can be locally stable. A related model is studied by Spence (1984) who also initiated the standard way imperfect appropriability of R&D is modelled via constant multidirectional spillover rates.

### 1.1.2 Research Joint Ventures

The central aim of the literature on RJV’s is to provide a performance comparison between various R&D cooperation scenarios, ranging from full cooperation as in a cartelized RJV to pure (strategic) competition, among firms which remain competitors in the product market. See Katz (1986), d’Aspremont-Jacquemin (1988, 1990) and Kamien-Muller-Zang (1992). The main result is that the cartelized RJV, which may be viewed as a situation where firms run one joint R&D lab at equal cost to each, yields the best performance among all scenarios considered, in terms of R&D propensity, consumer surplus and producer surplus. Several studies have built on the results of d’Aspremont-Jacquemin (1988) and Kamien-Muller-Zang (1992) to address

related questions, including Suzumura (1992), DeBondt-Slaets-Cassiman (1992) and Salant-Shaffer (1992).

More recently, another set of papers dealing with similar questions in somewhat modified models only partially confirms the above (and related) conclusions: Reynolds-Isaac (1992) and Stenbacka-Tombak (1995) consider models with stochastic R&D processes, and Amir (1995a) deals with the standard model, but with R&D returns that are not strongly decreasing (thus changing a crucial assumption of previous work). Under these modified specifications, the latter studies report a reduced scope of validity for the superiority of the cartelized RJV over the other scenarios.

We are now ready to resume the introduction of the present study.

### 1.1.3 The Present Paper

We consider the standard two-period model of process R&D/product market competition modified only in the way imperfect appropriability of R&D results is modelled. We assume that R&D spillovers are uni-directional and stochastic. They can only take place from the firm with higher R&D activity to its rival (but never vice-versa) in a binomial fashion: With probability  $\beta$ , full spillover occurs and with probability  $(1 - \beta)$  no spillover occurs. A detailed justification and interpretation of this specification is provided in Section 2.

Due to the one-way nature of the spillover process at hand, the overall payoff structure of a firm changes, depending on whether it is receiving or giving away R&D leakages, i.e., depending on whether we are above or below the diagonal in R&D space. Nonetheless, it turns out that the payoff functions inherit the strategic substitutes<sup>1</sup> property of the equilibrium profit function from the product market competition. Through the very nature of the spillover process, it follows that a firm would never find it optimal to exactly reproduce its rival's level of R&D activity. Together, these two properties of the model lead to existence of only asymmetric subgame-perfect equilibria for the symmetric two-stage game at hand, thereby yielding endogenous

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<sup>1</sup>In other words, the marginal payoff to increasing a firm's own R&D expenditure is decreasing in the rival's expenditure.

roles of R&D innovator (the more R&D intensive firm) and R&D imitator. Furthermore, the innovator ends up with a lower unit (production) cost than that expected by the imitator, and thus with a higher market share in the product market. Since the innovator also invests more in R&D, and the imitator receives some R&D for free (in expected terms), the equilibrium profit comparison is generally ambiguous.

Thus, our model postulates *ex ante* identical firms and yields only equilibria with (endogenously) heterogeneous firms: In size (which in a Cournot or Bertrand framework is tied to unit production cost) and in R&D intensity (which might involve R&D strategy, lab type and size, the composition of R&D, etc., if the R&D process were explicitly modeled). In view of the level of generality of our model formulation and the simplicity of the underlying asymmetry-generating mechanism, the present analysis offers an interesting new perspective on both the emergence and the persistence of intra-industry heterogeneity, for industries characterized by nearly one-dimensional R&D processes (again, this is discussed in detail in Section 2). According to this perspective, the mere knowledge that R&D leakages flow (in a stochastic sense) only from the more R&D intensive firm to the rival, leads firms to (endogenously) settle for innovator and imitator roles, thereby trading off profits in the product market and R&D costs in complementary ways.

The second part of the paper deals with the usual performance comparison of RJV scenarios, applied to our new framework. Of particular interest here is the comparison between a cartelized RJV (denoted Case *J*), which may be viewed as a joint lab run at equal cost by the firms together, and the purely noncooperative case (denoted Case *N*). The reason for questioning the robustness of the RJV analysis of previous studies (all based on multi-directional spillovers and symmetric outcomes) lies in Lemma 3.6<sup>2</sup> once one has observed that Case *J* has *ex post* firm symmetry as a built-in feature while Case *N* always leads to *ex post* asymmetry. Naturally, the appeal to Lemma 3.6 here is only to provide a sense of intuition, since in both

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<sup>2</sup>This is an intrinsic property of Cournot and Bertrand competition, which is stated precisely in Lemma 3.6, and holds under a broad specification of these models. It roughly states that total equilibrium profits, given a fixed total unit cost  $K$  is lowest with equal unit costs for each firm and highest with one firm having cost  $K$  and the other 0.

Cases  $N$  and  $J$ , total unit cost is clearly endogenous. Moreover, even if total profits improve through cooperation, an asymmetric outcome in Case  $N$  also means different incentives for the firms to cooperate in R&D.

We find that a strengthening of the usual assumption of strongly diminishing returns to R&D is crucially needed to restore the validity of the central conclusion from previous work on RJV's. Thus, our initial premise - that symmetry of outcomes plays an important role in the analysis of RJV's - is founded.

## 1.2 Summary of Submodular Optimization/Games

Here, we define all the notation and state all the results from submodular optimization needed in our analysis, in the simplest (but self-contained) form. Let  $I_1, I_2$  be compact real intervals and  $F : I_1 \times I_2 \rightarrow \mathfrak{R}$ .

$F$  is submodular [strictly submodular] if for all  $x_1 > x_2$  in  $I_1$  and all  $y_1 > y_2$  in  $I_2$ , we have  $F(x_1, y_1) - F(x_1, y_2) \leq [<] F(x_2, y_1) - F(x_2, y_2)$ . The following result is a special case of Topkis's Monotonicity Theorem (Topkis (1978)).

**Theorem 1.1.** *If  $F$  is continuous in  $y$  and submodular [strictly submodular] in  $(x, y)$ , then  $\arg \max_{y \in I_2} F(x, y)$  has maximal and minimal [all of its] selections nonincreasing in  $x \in I_1$ .*

The next result identifies an easy test for submodularity, and is often called Topkis's Characterization Theorem:

**Theorem 1.2.** *If  $F$  is twice continuously differentiable,  $F$  is submodular iff  $F_{12}(x, y) = \frac{\partial^2 F(x, y)}{\partial x \partial y} \leq 0$ . Furthermore,  $F_{12}(x, y) < 0$  implies strict submodularity.*

Finally, we need the following definition and existence result. A two-player game is submodular if both payoff functions are submodular and both action spaces are compact real intervals.

**Theorem 1.3.** *A two-player submodular game possesses a pure strategy Nash equilibrium.*

Topkis (1979) proved this result for ( $n$ -player) supermodular games ( $F$  is supermodular iff  $-F$  is submodular). Vives (1990) extended it to *two-player* submodular games. See also Milgrom-Roberts (1990). For  $n > 2$ , Theorem 1.3 is not valid in general. As a closing remark, the more general (ordinal) versions of Theorem 1.1 and 1.3 developed by Milgrom-Shannon (1994) are not applicable to our model.

## 2 The Noncooperative Model

### 2.1 The Model

Consider an industry composed of two *a priori* identical firms, each with initial unit cost  $c$ , engaged in the following two-stage game. In the first stage, Firms 1 and 2 decide on unit cost reduction  $x$  and  $y$ ,  $x, y \in [0, c]$ , on the basis of a known R&D cost schedule  $f(\cdot)$ . In the second stage, upon observing the new unit costs, the firms compete in the product market by choosing outputs (i.e., Cournot competition) or prices (i.e., Bertrand competition). There is no need to specify the mode of competition as the equilibrium profits of the second stage are modeled by a general function which has both models as special cases.

While this two-stage framework is standard in the recent R&D literature, our set-up departs from previous ones in the way imperfect appropriability of R&D is modelled. We consider R&D processes where leakages flow only from the more R&D-active firm to the rival in an all-or-nothing probabilistic fashion. Specifically, our stochastic one-way spillover process may be described as follows: Given that the autonomous cost reductions by Firms 1 and 2 are  $x$  and  $y$ , respectively, with (say)  $x \geq y$ , the effective (or final) cost reductions are given by  $X$  and  $Y$ , respectively, with

$$X = x \text{ and } Y = \begin{cases} x & \text{with probability } \beta \\ y & \text{with probability } 1 - \beta. \end{cases} \quad (2.1)$$

In view of the central role of the spillover process in our model, we provide a detailed discussion/justification for it in the next subsection.

We restrict attention to subgame-perfect equilibria only. A (pure) strategy for Firm  $i$  is a pair  $(x_i, a_i)$  where  $x_i \in [0, c]$  and  $a_i$  is a map from  $[0, c]^2$  to the set of product market decisions (outputs or prices). The overall payoff to a firm is simply its second-stage profit minus its first-stage R&D cost.

The following basic assumptions are in effect throughout the paper (interpretations and illustrative examples are given below).

**(A1)** For every pair of R&D decisions  $(x, y) \in [0, c]^2$ , the second-stage game (product market game) has a unique Nash equilibrium, with corresponding payoffs (i.e., profits) given by a function  $\Pi$  of the two firms' post R&D unit costs. Here,  $\Pi(\cdot, \cdot)$  denotes the Nash profits of the firm whose unit cost is the first argument.

**(A2)** (i)  $\Pi : [0, c]^2 \rightarrow \Re$  is continuous, and strictly submodular.

(ii)  $\Pi$  is nonincreasing (nondecreasing) in its first (second argument).

(iii)  $\Pi(c_1, c_1) < \Pi(c_2, c_2)$  if  $c_1 > c_2$ .

**(A3)**  $f$  is nondecreasing.

These conditions can be interpreted as follows. (A1) allows for a broad scope of product market competition modes, including in particular Cournot and Bertrand specifications. The equilibrium uniqueness assumption is convenient and not particularly restrictive. For the Cournot model, for instance, Amir (1996b) shows that it holds whenever  $P(\cdot) - c_i$  is a log-concave function, where  $P(\cdot)$  is the inverse demand function and  $c_i$  the unit cost of Firm  $i$ ,  $i = 1, 2$ . This is implied, in particular, by  $P(\cdot)$  itself being log-concave, and is thus quite general. Milgrom-Roberts (1990) give a uniqueness argument with examples for Bertrand competition with differentiated products.



(A2)(i) may be viewed, in the present context, as a (negative) complementarity condition as it holds that the improvement in a firm's profits resulting from a unit drop in own costs increases with the unit cost of the rival firm. (A2)(ii) is self-explanatory: a firm's profits decrease with own cost, but increase with rival's cost. (A2)(iii) says that in a symmetric duopoly, a unit drop in both firms' costs raises their profits. Put differently, own cost effects dominate rival's cost effects on profit.

With (A3) clearly being a natural assumption, we now argue that this set of assumptions yields a rather general framework, which, as noted earlier, leaves open the possibility that the second-stage game may encompass other modes of competition (in addition to Cournot and Bertrand). First, note that all three parts of (A2) are satisfied in the case of

- (i) Cournot competition with linear demand  $P(Q) = a - bQ$  and unit costs  $k_1$  and  $k_2$ , which leads to equilibrium profits (say) for Firm 1 given by  $\Pi(k_1, k_2) = (a - 2k_1 + k_2)^2/9b$ , and
- (ii) Bertrand competition with differentiated products, linear demand  $q_i = a - p_i + bp_j$ ,  $0 < b < 1$ ,  $i, j = 1, 2$ ,  $i \neq j$ , and units costs  $k_1, k_2$ , which leads to equilibrium payoff for Firm 1 equal to  $\Pi(k_1, k_2) = [(2+b)a - (2-b^2)k_1 + bk_2]^2$ .

While precise assumptions on inverse demand in a Cournot duopoly (say) implying (A2)(i) are not known, this condition is widely accepted since it is satisfied in the most commonly chosen specifications. In particular, (A2)(i) holds when demand is given by  $P(Q) = a - bQ^2$ ,  $Q \leq \sqrt{a/b}$ , the tedious derivation being left to the reader.

Finally, we observe that  $\Pi$  is ordinaly submodular (i.e.,  $-\Pi$  satisfies the single-crossing property, Milgrom-Shannon (1994)), as a consequence of Assumption (A2)(ii). However, it turns out that our analysis requires the stronger notion of cardinal submodularity (or (A2)(i)).

Some of our results require the following smoothness assumption (with Part (ii) being a minor strengthening of (A2)(iii) given (A2)(ii)).

(A4) (i)  $\Pi$  and  $f$  are twice continuously differentiable.

(ii)  $|\Pi_1(z, z)| > |\Pi_2(z, z)|$ , for all  $z \in [0, c]$ .

We now complete the description of the two-stage game by deriving the payoff functions. Observe that the game is perfectly symmetrical, i.e., independent of a relabeling of the players. The expected payoff to Firm 1 (say) is, with  $x, y \in [0, c]$ ,

$$F(x, y) = \begin{cases} \beta\Pi(c - x, c - x) + (1 - \beta)\Pi(c - x, c - y) - f(x) & \text{if } x \geq y \\ \beta\Pi(c - y, c - y) + (1 - \beta)\Pi(c - x, c - y) - f(x) & \text{if } x \leq y. \end{cases} \quad (2.2)$$

The expected payoff to Firm 2, defined similarly, is given by  $F(y, x)$ , in view of the symmetry of the game. The expressions in (2.2) reflect the facts that the firms get (i) the same second-stage profits corresponding to the larger cost reduction for both, with probability  $\beta$ , (ii) the profits corresponding to their autonomous cost reductions with probability  $(1 - \beta)$ , and (iii) pay for their autonomous cost reduction only.

It is easy to see that  $F$  inherits the continuity property of  $\Pi$  (from Assumption (A2)(i)). It turns out that  $F$  also inherits the submodularity of  $\Pi$  (also in (A2)(i)), but not the differentiability of  $\Pi$  and  $f$ , which fails along the diagonal of  $[0, c]^2$ , nor the concavity of each line in (2.2) assumed below for some of our results. This is intended only as a preview here and will be established later.

## 2.2 The Spillover Process

As described by (2.1), the stochastic spillover process at hand is new and a thorough justification is warranted. The key feature of this process is that know-how may only flow from the more R&D intensive firm (the innovator) to the other firm (the imitator). While this terminology might be suggestive of some sequentiality in carrying out R&D, such a feature is not part of our model.

Furthermore, the effective cost reduction of the imitator (given by  $Y$  in (2.1)) is a binomial random variable with the spillover parameter  $\beta$  as success probability. Thus the spillover process only admits extreme realizations: Either full or no spillover will actually take place, even though  $\beta$  can assume any value in  $[0, 1]$ . The boundary values of  $\beta$  have the usual interpretations: A value of 0 means that R&D is perfectly

appropriable while a value of 1 means that R&D is a purely public good, both in a world of certainty.

We now define the certainty-equivalent spillover process for future reference. This is obviously a deterministic process, characterized by (assuming w.l.o.g. that  $x \geq y$ )  $X := x$  and  $Y := y + \beta(x - y) = \beta x + (1 - \beta)y$ , instead of (2.1). Here, the imitator ends up with his autonomous cost reduction plus a fraction (given by the spillover rate  $\beta$ ) of the difference in the two cost reductions. This is simply the expected cost reduction under the original stochastic spillover process.

In previous related studies, spillovers were always treated as a deterministic two-way process (in the duopoly case). Spence (1984), d'Aspremont-Jacquemin (1988), De Bond-Slaets-Cassiman (1992) and Kamien-Muller-Zang (1992)), as well as many others related studies, all assume that a fixed proportion (given by the spillover parameter) of every firm's R&D effort or benefit flows freely to (all) the rival(s).<sup>3</sup> As argued by Kamien-Muller-Zang (1992), the underlying R&D process in these studies is implicitly assumed to be a "multi-dimensional heuristic rather than a one-dimensional algorithmic process." Thus, it necessarily involves trial and error on the part of the firms which follow different sets of research paths and/or approaches. Further quoting Kamien, Muller, and Zang: "The spillover effect in this vision of the R&D process takes the form of each firm learning something about the other's experiences: which approaches appear more or less promising and which are 'dead ends'."

By contrast, the R&D process associated with the one-way (or uni-directional) spillover structure here is best approximated by a one-dimensional process. This need not mean that firms necessarily pursue only a single path or approach. Rather, in case of a multi-path R&D process, our spillover structure suggests the presence of a more or less natural order on the various steps to be performed. Consequently, it is appropriate in such a setting to postulate that the only spillover potential is from the firm with higher R&D activity to the laggard(s).<sup>4</sup>

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<sup>3</sup>Suzumura (1992) has a more general spillover structure, but know-how still flows both way in his set-up, and the focus is on symmetric equilibria as well. Amir (1995b) provides a critique of the notion of additive spillovers in cost reductions.

<sup>4</sup>Alternatively, the R&D process may be a "multi-dimensional heuristic" whereby firms commit

This justification of the one-way nature of the R&D leakages applies equally well to the stochastic and to the certainty-equivalent versions of the spillover process. However, the stochastic version considerably enlarges the scope of interpretation of the spillover effects, thereby allowing for interesting links to the broader R&D literature, as we now argue.

The spillover process given by (2.1) is a reasonable approximation for potential leakage effects in several different contexts. We present here the most prominent examples. The first and perhaps most natural interpretation of  $\beta$  is as the probability that R&D leakages will actually take place. In other words,  $\beta$  would represent here a measure of the ease of carrying out industrial espionage and/or reverse engineering efforts in the industry under consideration. This presumes the absence of effective patent protection of the differential in know-how between the innovator and the imitator.

The second possible interpretation of  $\beta$  is as the perceived probability (by the firms) that effective patent protection will not be granted to protect the innovation differential of the leading firm. This explanation presumes an environment with potential patent protection, and an R&D process with relatively easy reverse engineering opportunities (in the absence of patents), and *ex ante* uncertainty as to the success of patent applications.

The third possible interpretation of  $\beta$  relates it to the length of patent protection and the interest rate, as follows. Consider a situation where a patent of length  $T$  periods has been issued, thus allowing the innovator (or leading firm) to collect asymmetric per-period duopoly profits  $\Pi_A$  (corresponding to a larger market share) for the first  $T$  periods, and symmetric duopoly profits  $\Pi_S$  (corresponding to the same low unit cost for both firms) thereafter. Assume that, in this fictitious (infinite-horizon) dynamic scenario, firms discount future profits at a rate  $\delta$  ( $0 < \delta < 1$ ). Then the present value of the innovator's stream of profits may be written as (omitting R&D

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to certain paths and approaches and learn about their success only towards the end of the process. In this vision, continuous peeking does not help as the paths are assumed unrelated, and  $\beta$  is then simply the probability that a failing firm is able to change paths at the end and to imitate a successful firm.

costs):

$$\frac{\delta^T}{1-\delta}\Pi_S + \frac{1-\delta^T}{1-\delta}\Pi_A.$$

Multiplying across by  $(1-\delta)$  and comparing the outcome to the top line in (2.2) leads to the identification  $\beta = \delta^T$ , which suggests that  $\beta$  may also be taken as an inversely related proxy for patent length, which is quite intuitive. This analogy may serve as a bridge between the literature on process R&D and that on patent races and patent design (see, for instance, Gilbert-Shapiro (1990)).

A related alternative in an infinite-horizon setting is to think of  $T$  as the number of periods it takes for the imitator to successfully reproduce the innovator's advance, in the absence of patents. The above expression for the innovator's stream of profits is still valid here, with  $\beta$  being an (inversely related) measure of the imitation lag  $T$ . This interpretation connects the present work to the literature on R&D-based growth, see e.g. Grossman and Helpman (1991).

Another interpretation of  $(1-\beta)$  is as a measure of a firm's inertia, or its inability to effectively implement internal changes. In a world where process R&D is mainly organizational, in the sense that it amounts to improving management structure, compensation and incentive schemes, production technique ..., a firm may fail to effectively imitate a rival's superior organization form due to various internal and global factors such as managerial myopia, union policy, barriers to entry (i.e., weak competitive pressures)... This interpretation relates our model to the (business) strategy literature: See, e.g., Rumelt (1995) and Sinclair-Desgagné and Röller (1996).

While these (and other possible) interpretations distinguish our spillover process from the standard multi-directional structure prevalent in the process R&D literature so far, some common features between the two processes exist.  $\beta$  here is also a characteristic of the technological environment, assumed exogenous to the model. A constant value for  $\beta$  is a highly stylized simplification, for a firm's ability to absorb spillover (or imitate) ought to depend explicitly on its own R&D investment.

We now conclude this discussion with two final observations.  $\beta$  should be regarded throughout this paper as the perceived *ex ante* probability of full spillover taking place. Also, we point out that the certainty-equivalent version of the spillover

process leads to a (deterministic) model which is not as tractable as our model here. In particular, existence of a subgame-perfect equilibrium would entail one extra assumption restricting the value of  $\beta$  and depending on the properties of the product market profit function.

## 2.3 Properties of the Noncooperative R&D Model

Here, we state and interpret all the results pertaining to the two-stage game under consideration. Since the second-stage game admits a unique Nash equilibrium, every Nash equilibrium  $(x^*, y^*)$  of the game with payoffs (2.2) induces a subgame-perfect equilibrium of the two-stage game, and vice-versa. In view of this one-to-one correspondence, we use the two terminologies interchangeably.

We begin with the fundamental property of the game at hand (strategic substitutability), and a key structural characteristic of its equilibria (asymmetry).

**Theorem 2.1.** *Assume (A1)-(A2) hold. Then the following are true:*

- (i) *The game with payoffs (2.2) is submodular, and hence has a pure-strategy Nash equilibrium.*
- (ii) *Every interior Nash equilibrium is asymmetric if (A4) holds and  $\beta > 0$ .*
- (iii) *Every Nash equilibrium is asymmetric if, in addition to the hypothesis of (ii), the following holds (here subscripts denote partial derivative)*

$$f'(0) < -\beta\Pi_2(c, c) - \Pi_1(c, c) \text{ and } f'(c) > -(1 - \beta)\Pi_1(0, 0). \quad (2.3)$$

A discussion of these results is provided at the end of this subsection. The next result deals with uniqueness of equilibrium. Observe that in view of the symmetry of the game and the fact that no equilibrium can involve the firms taking the same decisions (Theorem 2.1), the sharpest uniqueness result would yield two equilibria.

**Theorem 2.2.** *Under Assumptions (A1)-(A4) and (2.3), the R&D game (with pay-offs (2.2)) has exactly two Nash equilibria, of the form  $(\bar{x}, \underline{x})$  and  $(\underline{x}, \bar{x})$ , with (say)  $\bar{x} > \underline{x}$ , if in addition  $\beta \in (0, 1]$  and*

$$f''(\cdot) > (\Pi_{11} - \Pi_{12})[c - (\cdot), z], \forall z \in [0, c] \quad (2.4)$$

and

$$f''(\cdot) > (\Pi_{11} + 2\Pi_{12} + \Pi_{22})[c - (\cdot), c - (\cdot)]. \quad (2.5)$$

Define the best-response correspondence in the usual way, i.e., (say) for Firm 1,  $r_1(y) = \arg \max\{F(x, y) : x \in [0, c]\}$ . The next result holds that  $r_1$  and  $r_2$  are essentially as depicted in Figure 1 (note that  $r_1 = r_2$ , by symmetry).

**Corollary 2.3.** *Under the hypothesis of Theorem 2.2 (i.e., (A1)-(A4), (2.3)-(2.5)),  $r_1$  and  $r_2$  are continuous nonincreasing functions everywhere in  $[0, c]$  except at one point  $d \in (0, c)$  where  $r_i(d^-) > d > r_i(d^+)$ .*

Figure 1 goes here.

The last results in this section deal with comparative statics of the equilibrium  $(\bar{x}, \underline{x})$  as  $\beta$  increases in  $(0, 1]$ .

**Theorem 2.4.** *Under (A1)-(A2), the following hold as  $\beta$  increases in  $(0, 1]$  :*

- (i) *Holding one firm's R&D level constant, the extremal best-responses of the other firm are nonincreasing (in other words,  $r_1$  and  $r_2$  shift down).*
- (ii) *The total equilibrium R&D level  $\bar{x} + \underline{x}$  (associated with the unique Nash equilibrium pair  $[(\bar{x}, \underline{x}) \text{ and } (\underline{x}, \bar{x})]$ ) decreases if, in addition, (2.4)-(2.5) hold.*
- (iii)  *$\bar{x}$  itself decreases if, in addition, (2.4)-(2.5) and the following holds*

$$f''(\cdot) \geq (1 - \beta) \left[ \Pi_{11}(c - (\cdot), c - z) + \frac{\Pi_1(c - (\cdot), c - z)\Pi_{12}(c - z, c - (\cdot))}{(\Pi_1 + \Pi_2)(c - z, c - z) - \Pi_1(c - z, c - (\cdot))} \right], \forall z > (\cdot). \quad (2.6)$$

We now provide a discussion of the results of this section. Theorem 2.1(i) suggests that our model is well-defined under remarkably general conditions. In particular, the absence of any concavity/convexity assumptions on the primitives of the model is new in the R&D literature. From the proof of Theorem 2.1, one easily sees that Assumption (A3), i.e., the monotonicity of  $f$  is not needed. For the overall payoff function  $F$  to inherit the submodularity property of the equilibrium profit function  $\Pi$ , Assumption (A2)(iii) turns out to be crucial. Submodularity of  $F$  here has the usual negative complementarity interpretation: The marginal returns to increasing a firm's R&D expenditure decrease with the rival's R&D expenditure, and this holds independently of whether the firm is receiving or giving away spillovers!

The asymmetric structure of equilibria is a central feature of our analysis. It says that it would never be optimal for a firm to exactly reproduce its rival's R&D behavior. Either the rival is conducting too little R&D so it pays for the firm to engage in strictly more R&D activity (even though it knows that it will potentially go to its rival), or the rival is conducting too much R&D so it pays for the firm to perform strictly less R&D and potentially acquire the R&D differential for free (i.e., imitate). Graphically, this property translates into the graph of  $F$  having a nonconcave kink along the diagonal in  $[0, c]^2$ .

In view of this asymmetry feature of equilibrium outcomes, our model is a natural candidate for explaining the ubiquitous inter-firm heterogeneity within most industries. The driving force behind this endogenous firm heterogeneity lies in the (probabilistic) anticipation of one-way flow of R&D spillovers from the leading firm to its rival. Under such a spillover structure (which is thoroughly discussed in the next subsection), firms endogenously emerge with different production cost structures through the very process of adopting (costly) technological progress. Thus, the competing firms end up with different levels of R&D activity (hence with different types of R&D strategy/labs), different firm sizes and market shares in the product market.

Theorem 2.2 is a convenient result as it allows for more straightforward analysis of equilibrium behavior, unencumbered with some of the difficulties associated with multiple equilibria. For instance, it is needed for clear-cut answers to the compar-



ative statics analysis of Theorem 2.4. Both theorems require assumptions on  $f''$ , which would translate into strong convexity of  $f$  since  $\Pi$  is typically convex in own costs or even jointly. Similar assumptions have always been made in related studies (e.g., d'Aspremont-Jacquemin (1988), Kamien-Muller-Zang (1992)), and are crucially needed to insure that payoffs are concave in own R&D decision. In our model, such assumptions can only yield concavity of each line in (2.2), but not of  $F$  itself.

While Theorem (2.4)(i)-(ii) are intuitively clear, (iii) is perhaps less so. The fact that each firm would decrease its R&D level as  $\beta$  increases, holding the rival's R&D level constant does not imply that, at equilibrium, both R&D levels go down.<sup>5</sup> In other words, there are two effects governing the response of  $\bar{x}$  (say) to changes in  $\beta$ . The first is captured in Part (i) and is rather intuitive: The leading firm (or innovator) would always cut down on R&D as the likelihood of full spillover to the rival increases, with the rival's R&D level constant. However, if the rival also decreases his R&D level, the other effect is that the firm under consideration will want to respond by increasing R&D activity. The overall effect on  $\bar{x}$  then depends on the relative strength of these two effects. The added condition (2.6) is thus needed to shift the balance towards a decline of  $\bar{x}$ . A similar argument would apply to  $\underline{x}$ . We leave this to the reader, but point out that Part (i) is also very intuitive for the lagging firm (or imitator): As the likelihood of the innovator's edge freely spilling over increases, it finds it advantageous to cut down on its own autonomous (costly) R&D activity.

### 3 Research Joint Ventures

Following d'Aspremont-Jacquemin (1988) and Kamien-Muller-Zang (1992), we consider here different R&D cooperation schemes among firms which remain competitors in the product market. These schemes are characterized by two key features: whether firms coordinate in choosing R&D expenditure (i.e., "collude" in the first-stage of the

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<sup>5</sup>In the language of supermodularity analysis, one cannot find orders on the two actions sets that would make each payoff supermodular in the two decisions and in the pair (own decision,  $\beta$ ). Hence, the comparative statics result for supermodular games cannot be invoked (Milgrom-Roberts (1990), Sobel (1988)).

game), and whether firms cooperate in the actual conduct of R&D (by increasing  $\beta$ ).

Here, we are mainly concerned with only one RJV scenario: the joint lab. This is characterized by the firms running one joint R&D facility at half the cost each, and will be denoted by  $J$ . We prove below that  $J$  is equivalent (for our model) to Kamien-Muller-Zang's case  $CJ$ , or cartelized RJV, whereby firms coordinate R&D expenditures in the first-stage and fully communicate during the R&D process (i.e. set the spillover rate equal to 1).

In the course of investigating the properties of Case  $J$ , it turns out that it is useful to also consider the following broader RJV specification. Let  $C_s$  denote the scenario whereby firms coordinate their R&D investments (so as to maximize total profits), while the spillover parameter is given by  $s \in [0, 1]$ . Thus, in particular,  $s = 0, \beta, 1$  stand for the cases where the spillover rate is reduced to 0, kept as it is<sup>6</sup>, and increased to 1 (its maximum value), respectively. Note here that the case  $s < \beta$  is not necessarily economically meaningful within the context of our model in the sense that spillovers are generally thought of as being unpreventable by the firms. Nonetheless, the case  $s = 0$  is particularly useful below for comparative purposes.

The joint objective function of the two firms in Case  $C_s$  (assuming w.l.o.g. that  $x \geq y$ ) is to maximize  $F(x, y) + F(y, x)$  over  $x, y$  in  $[0, c]$ , with  $\beta$  set equal to  $s$ , which reduces to

$$2s\Pi(c - x, c - x) + (1 - s)[\Pi(c - x, c - y) + \Pi(c - y, c - x)] - f(x) - f(y). \quad (3.1)$$

The single-firm objective in Case  $J$  is to maximize over  $x \in [0, c]$

$$\Pi(c - x, c - x) - \frac{1}{2}f(x). \quad (3.2)$$

Observe that (3.1) reflects the (potential) operation of two separate R&D labs by the cartel, with variable spillover parameter, while (3.2) reflects the operation of one joint lab with equal cost sharing. In both cases, the two firms face competition in the product market, as captured by the function  $\Pi$  (see Section 2). Thus, in particular, a

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<sup>6</sup>The case  $C_\beta$  here is clearly the analog of the second scenario analyzed in d'Aspremont-Jacquemin (1988).

symmetric outcome necessarily obtains in Case  $J$  (by construction). As will be seen below, this may or may not be true for Case  $C_s$ ,  $s \in [0, 1)$ .

Our central concern in this section is a performance comparison between the noncooperative model of Section 2 (to be denoted  $N$ ) and Case  $J$  (which we show below to be essentially equivalent to Case  $C_1$ ). The performance criteria of interest here are: propensity for R&D, firm profits, consumer and social welfare. The cases  $C_0$  and  $C_\beta$  are analyzed here only as useful intermediate steps in the overall analysis.

We first point out that Cases  $J$  and  $C_1$  are interchangeable in the following precise sense (proofs are in the next section).

**Lemma 3.1.** *Cases  $J$  and  $C_1$  are equivalent in the sense that they both lead to the same joint objective function, and hence to the same optimal R&D levels (one of which is equal to zero) and the same optimal total profits.*

In view of this lemma, one might wonder why two different definitions of the same cooperation scenario are provided here. The answer is that Case  $J$  offers the convenient feature of built-in symmetry, by its very definition, thus doing away with the need to discuss profit and R&D cost sharing. On the other hand, stating it as Case  $C_1$  is useful below through its properties as the limit case of  $C_s$  as  $s \rightarrow 1$ . Furthermore, it is as Case  $C_1$  that this scenario is established in Kamien-Muller-Zang (1992) as yielding the best performance (among three other possibilities) in market price, R&D propensity and social welfare.

Our first comparison of Cases  $J$  and  $N$  concerns R&D propensities. This requires, however, an intermediate lemma which is of independent interest, a comparison between Case  $J$  and Case  $N$  with  $\beta = 0$  (the latter is denoted  $N_0$  below). In dealing with this comparison, an additional assumption is now introduced as a new version of (A4). It quantifies the dependence of profits on own vs. cross cost reductions in a symmetric duopoly setting.

(A5)  $\Pi$  and  $f$  are twice continuously differentiable and  $|\Pi_1(z, z)| \geq 2|\Pi_2(z, z)|$ ,  $\forall z \in$

$[0, c]$ .

Clearly, (A5) is a stronger version of (A4). We now argue that (A5) is not as restrictive as it might appear at first. It is easily seen to be satisfied under Cournot competition with linear demand and costs, with strict inequality if products are differentiated and with equality for homogeneous products (see Section 2 for the expression of  $\Pi$ ). For Bertrand competition with differentiated products, (A5) can be seen to hold if and only if the cross-demand coefficient (denoted by  $b$  in the discussion of (A1)-(A3) in Section 2) is in the interval  $(0, \sqrt{3} - 1] \approx (0, .73]$ ,<sup>7</sup> i.e., as long as demand is somewhat away from the well-known case of homogenous products ( $b = 1$ ).<sup>8</sup>

**Lemma 3.2.** *Under Assumptions (A1)-(A3), (A5) and (2.4), we have:*

- (i) *In Case  $N_0$ , there is a unique and symmetric equilibrium  $(x_0, x_0)$ .*
- (ii) *The equilibrium R&D level of Case J,  $x_J$ , satisfies  $x_J \geq x_0$ .*

We are now ready for the comparison of R&D propensities (interpretations of the results are given later on).

**Proposition 3.3.** *Under Assumptions (A1)-(A3), (A6) and (2.4)-(2.6),  $x_J \geq \bar{x} \geq \underline{x}$  (with strict inequality throughout unless  $\beta = 0$ ).*

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<sup>7</sup>In their treatment of Bertand competition, Kamien, Muller and Zang (1992) give  $\frac{2}{3}$  as a *lower* bound for this critical value of  $b$ . Since the two models are equivalent when  $\beta = 0$  (and our model is specified as in theirs), the fact that our bound is sharper indicates that (A5) is tight (see also the proof of Lemma 3.2(ii)).

<sup>8</sup>It can easily be seen that the  $\Pi$  function corresponding to the case  $b = 1$ , given by

$$\Pi(c_1, c_2) = \begin{cases} (c_2 - c_1)D(c_2) & \text{if } c_1 < c_2 \\ 0 & \text{if } c_1 \geq c_2, \end{cases}$$

(where  $D(\cdot)$  is the demand function) is not submodular in  $(c_1, c_2)$ . Hence this case fails Assumption (A.2) anyway, and thus does not fit our model.

The next comparison deals with equilibrium total profits. Due to the asymmetric nature of the equilibria in Case  $N$ , single-firm profit comparisons do not seem possible at this level of generality.

**Proposition 3.4.** *Total equilibrium profits are higher in Case  $J$  than in Case  $N$ , provided that at least one of the following conditions holds:*

$$2\Pi(c_2, c_2) \geq \Pi(c_1, c_2) + \Pi(c_2, c_1), \text{ for all } c_1 \geq c_2. \quad (3.3)$$

$$f''(\cdot) > (\Pi_{11} - \Pi_{12})(c - (\cdot), z) + (\Pi_{22} - \Pi_{12})(z, c - (\cdot)), \text{ for all } z \in [0, c]. \quad (3.4)$$

The welfare comparison essentially follows from Proposition 3.3-3.4 once the following plausible assumption about consumer surplus is added (note that given the level of generality of the product market competition here, consumer surplus cannot be explicitly defined in the usual way).

**(A6)** Consumer surplus is decreasing in the firms' unit costs.

This assumption holds in most commonly used specifications of Cournot and Bertrand competition. In particular, it holds for the cases of linear demand reported in Section 2. For Cournot competition (with homogeneous products), it actually holds for any demand function, provided production costs are linear and a Cournot equilibrium exists (see Amir (1996b) for exact conditions). This is because total output at equilibrium (and hence price) depends only on total unit cost (Bergstrom-Varian (1985)).

**Proposition 3.5.** *Regardless of whether full or no spillover is realized, (ex-post) social welfare is higher under Case  $J$  than under Case  $N$ , assuming (A1)-(A6), (2.4), (2.5), and either (3.3) or (3.4).*

We now provide a discussion of the results of this section emphasizing their relationship to related work on RJV's, namely d'Aspremont-Jacquemin (1988, 1990) and

Kamien-Muller-Zang (1992). As discussed in the Introduction, the main motivation behind our investigation here is a sort of robustness analysis of the principal conclusion from previous work: That a joint lab or cartelized RJV dominates all other scenarios considered (including, in particular, the noncooperative case) in terms of equilibrium prices (and thus consumer welfare), firm profits, and hence social welfare. The reasons for questioning the validity of this conclusion in the present context are that (i) the firms have different equilibrium profits in Case  $N$ , and hence different incentives to engage in R&D cooperation, (ii) Case  $J$  yields symmetric outcomes as a built-in feature while Case  $N$  always leads to asymmetric equilibria (see Salant-Shaffer (1992)). This last feature is important as firms (jointly) prefer not to compete on equal terms in the product market in typical specifications of Cournot and Bertrand competition (see examples in Section 2.1), as we now show:

**Lemma 3.6.** *Let  $\Pi$  be jointly convex on  $[0, c]^2$ ,  $k > 0$  and consider the following objective (with constraint):*

$$\{\Pi(c_1, c_2) + \Pi(c_2, c_1) : c_1 + c_2 = k\}. \quad (3.5)$$

*Then the  $\arg \max$  of (3.5) consists of  $(0, k)$  and  $(k, 0)$ , while the  $\arg \min$  is  $(\frac{k}{2}, \frac{k}{2})$ .*

Roughly, the main finding here is that the central conclusion of the RJV literature is fairly robust to the change in spillover structure introduced here and to its associated asymmetry consequence discussed above. Nonetheless, additional assumptions are needed here to ensure the validity of this conclusion (see Propositions 3.3-3.5). Next, we discuss the meaning of these extra assumptions.

First, observe that for Case  $N_0$ , our model is equivalent to those of d'Aspremont-Jacquemin (1988) and Kamien-Muller-Zang (1992). Thus, our Lemma 3.2(ii) may be viewed as a generalization of their analogous result to a broader class of profit functions (instead of that corresponding to linear demand). Assumption (A5), crucial for this generalization, was discussed earlier.

Condition (2.6), needed for Proposition 3.3, was also discussed in Section 2.3. As suggested by the upcoming Example, (2.6) is not necessary for the comparison of R&D

propensities, since total effective R&D can be declining in  $\beta$  even if the innovator's level  $\bar{x}$  is not, in which case Proposition 3.3 would not need Lemma 3.2(ii).

Condition (3.3), rewritten as  $\Pi(c_2, c_2) - \Pi(c_1, c_2) \geq \Pi(c_2, c_1) - \Pi(c_2, c_2)$ , says that effects on profits of *any* discrete change in own cost exceeds those due to the same change in rival's cost, starting from a symmetric duopoly. Thus (3.3) strengthens (A.4)(ii) which says the same thing but only for infinitesimal changes. Also, (3.3) has a simple interpretation in the context of Cournot competition with linear demand. It is then equivalent to  $2a + 3c_2 - 5c_1 \geq 0$  (with  $c_2 \leq c_1$ ), and thus boils down to assuming high demand (relative to costs). Condition (3.4) is needed to ensure concavity of the joint objective in Case  $C_0$ , resulting then in a symmetric R&D choice. (3.3) or (3.4) is needed to guarantee that total profits are higher for Case  $C_1$  than for Case  $C_0$ . (3.4) works by removing the asymmetry bias captured in Lemma 3.6. The idea is that if symmetry prevails in both cases, the cartelized firms prefer full to no spillover. (Note that due to the linearity of profits in  $\beta$ , either  $\beta = 0$  or  $\beta = 1$  is always preferred to any other  $\beta$ .)

Finally, Proposition 3.5, being a corollary of Propositions (3.3)-(3.4), needs all of their assumptions.

## 4 An Example

Here, we provide a brief summary of an example that illustrates many of the general results of Sections 2-3 in addition to allowing for more precise versions of those results and for many new results. The latter do not seem tractable at the level of generality of the present paper. A detailed analysis of this example is presented in the companion paper Amir-Wooders (1996).

Let Cournot competition, with linear demand  $P(Q) = a - Q$  and common initial cost  $c$ , prevail in the second-stage. As in d'Aspremont-Jacquemin (1988), take as the R&D cost function  $f(x_i) = \frac{1}{2}\gamma x_i^2$ . Assuming  $a > 2c$  and  $9\gamma > 4(1 - \beta)\frac{a}{c} \vee (8 - 6\beta)$ , Amir-Wooders (1996) establish the following:

1. There exists a unique pair of equilibria  $(\bar{x}, \underline{x})$  and  $(\underline{x}, \bar{x})$  given in closed-form.

2. (i)  $x_J \geq \underline{x}$ , (ii)  $x_J \geq \bar{x}$  iff  $9\gamma \leq 4\frac{a}{c}$  or  $9\gamma \geq 16(1 - \beta)$  and (iii) Expected total cost reduction  $(1 + \beta)\bar{x} + (1 - \beta)\underline{x} \leq 2x_J$ .
3. Total profits are higher for Case  $J$  than for case  $N$  if (i)  $a \geq 2.5c$ , or (ii)  $9\gamma \geq 18$ , or (iii)  $9\gamma \geq 12 - 7\beta$  and  $(\bar{x}, \underline{x})$  is interior.
4. Consumer surplus is higher for Case  $J$  than for Case  $N$ .

We now comment on the necessity of the assumptions of Section 3 in light of this central example. We begin with Condition (2.6). With the specification of this example, it can easily be shown that  $9\gamma \geq 16(1 - \beta)$  is sufficient for (2.6). Since  $x_J \geq \bar{x}$  for  $\beta = 0$  (see Lemma 3.2),  $\bar{x}$  must decline in  $\beta$  for  $\beta$  near 0 in order to guarantee  $x_J \geq \bar{x}$ . So 2(ii) suggests that Condition (2.6) is tight. Note though that the Example yields  $9\gamma \leq 4\frac{a}{c}$  as an alternative condition. Also, no version of 2(iii) is developed in the present paper.

For Part 3., (i) and (ii) are easily verified to be (tight) sufficient conditions for (3.3) and (3.4) in the Example. On the other hand, 3.(iii) and 4. are obtained via direct calculation and have no counterpart in this paper. Combining 3. and 4. yields that social welfare is higher for Case  $J$  than for Case  $N$  if any one of the three conditions of 3. holds. Hence, the above remark on total profit applies to the social welfare comparison as well.

Finally, we report that a number of other results are derived in the context of the Example in Amir-Wooders (1996), including a three-way comparison between innovator, imitator and RJV participant of profits and markets shares, a complete characterization of the  $(\gamma, \beta)$  regions for which  $\frac{d\bar{x}}{d\beta} \geq 0$  and  $\frac{d\underline{x}}{d\beta} \geq 0$ , as well as other results.

## 5 Extensions

In view of the fact that one important dimension of value added in this paper is the level of generality of the analysis, we argue here that the present framework does extend to  $n$  firms. We assume that spillovers can occur only from the firm with



the highest R&D activity to all the other firms. The key observation here is that the equilibrium profit function  $\Pi$  for Cournot competition of a firm in the second-stage depends only on its own unit cost and on *the sum* of other firms' unit costs. Hence, the overall payoff of each firm in the two-stage game (analogous to (2.2)) depends only on the firm's own R&D decision and on the sum of other firms' R&D decisions. Furthermore, the game remains submodular (i.e., the marginal returns to increasing one's R&D level decrease as any rival(s) increase(s) R&D). Together, these two properties are sufficient to yield existence of a Nash equilibrium, using the backward mapping technique of Novshek (1985). This equilibrium would involve one "innovator" with a higher R&D level and  $(n - 1)$  imitators with the same lower R&D level.

Another extension of interest would be to develop a more general notion of spillover effects, characterized by *ex ante* symmetry but *ex post* potential asymmetry, that would yield endogenous emergence of innovator/imitator roles.

Finally, it can be shown that the game at hand has a symmetric mixed-strategy equilibrium. Under such a solution, the firms would still end up (endogenously) different with positive probability.

## 6 Proofs

This section provides all the proofs for the results given in the previous sections, in the order given. Recall that a brief review of the lattice-theoretic concepts needed here is provided in Section 1. We begin with some notation:

It is convenient to introduce the following sets and functions (note that, contrary to usual practice,  $x$  is along the vertical axis while  $y$  is on the horizontal axis below).

$$\begin{aligned}\Delta_u &\equiv \{(x, y) \in [0, c]^2 : x \geq y\}, \Delta_l \equiv \{(x, y) \in [0, c]^2 : x \leq y\} \\ U(x, y) &= \beta \Pi(c - x, c - x) + (1 - \beta) \Pi(c - x, c - y) - f(x) \\ L(x, y) &= \beta \Pi(c - y, c - y) + (1 - \beta) \Pi(c - x, c - y) - f(y)\end{aligned}$$

$U$  and  $L$  are respectively the top and bottom lines in the expression  $F(x, y)$  of

firm's payoff, as given by (2.2). By symmetry then, Firm 2's payoff is  $F(y, x) = L(y, x)$  if  $y \leq x$ , and  $U(y, x)$  if  $y \geq x$ .

Figure 2 goes here.

**Proof of Theorem 2.1.** (i) We show that  $F$ , as given by (2.2) is strictly submodular in  $(x, y)$ .<sup>9</sup> To this end, fix  $x_1, x_2, y_1, y_2$  in  $[0, c]$  with  $x_1 > x_2, y_1 > y_2$ . If all four points  $(x_1, y_1), (x_1, y_2), (x_2, y_1)$  and  $(x_2, y_2)$  lie in  $\Delta_u$  or in  $\Delta_l$ , strict submodularity of  $F$  follows directly from the strict submodularity of  $\Pi$  (i.e., Assumption (A2)(i)), since only the middle term of  $U$  and  $L$  depends on both  $x$  and  $y$ .

If some of the four points lie in  $\Delta_u$  and the rest in  $\Delta_l$ , it is easily seen that there are 4 different cases. It turns out that the proofs of strict submodularity of  $F$  are all similar, so we present the case depicted in Figure 2, i.e.,  $(x_1, y_1), (x_1, y_2), (x_2, y_2)$  are in  $\Delta_u$  while  $(x_2, y_1)$  is in  $\Delta_l$ . We must then show that  $U(x_1, y_1) - U(x_1, y_2) < L(x_2, y_1) - U(x_2, y_2)$ . We clearly have, given the location of the points,  $x_2 < y_1$ . Hence, by Assumption (A.2)(iii),

$$0 \leq \beta\Pi(c - y_1, c - y_1) - \beta\Pi(c - x_2, c - x_2). \quad (6.1)$$

Since  $\Pi$  is strictly submodular and  $f$  only depends on one variable,

$$\begin{aligned} & [(1 - \beta)\Pi(c - x_1, c - y_1) - f(x_1)] - [(1 - \beta)\Pi(c - x_1, c - y_2) - f(x_1)] \\ & \leq [(1 - \beta)\Pi(c - x_2, c - y_1) - f(x_2)] - [(1 - \beta)\Pi(c - x_2, c - y_2) - f(x_2)]. \end{aligned} \quad (6.2)$$

Adding up (6.1), (6.2) and the trivial equality  $\beta\Pi(c - x_1, c - x_1) - \beta\Pi(c - x_1, c - x_1) = 0$  and rearranging terms yields

$$\begin{aligned} & [\beta\Pi(c - x_1, c - x_1) + (1 - \beta)\Pi(c - x_1, c - y_1) - f(x_1)] \\ & - [\beta\Pi(c - x_1, c - x_1) + (1 - \beta)\Pi(c - x_1, c - y_2) - f(x_1)] \\ & < [\beta\Pi(c - y_1, c - y_1) + (1 - \beta)\Pi(c - x_2, c - y_1) - f(x_2)] \\ & - [\beta\Pi(c - x_2, c - x_2) + (1 - \beta)\Pi(c - x_2, c - y_2) - f(x_2)], \end{aligned}$$

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<sup>9</sup>Note that Topkis's Characterization Theorem cannot be used here, since  $F$  is not differentiable along the diagonal in  $[0, c]^2$ , but it can be used in the interior of  $\Delta_u$  and  $\Delta_l$  separately.

which says that  $F$  is strictly submodular for the 4-point choice of Figure 2. This last inequality is strict since (6.1) is strict if  $\beta > 0$  and (6.2) is strict if  $\beta < 1$ .

The argument for each of the remaining choices is similar, and thus left to the reader. Existence of a pure-strategy Nash equilibrium follows from Theorem 1.3.

(ii) Partial differentiation w.r.t.  $x$  yields

$$U_1(x, y) = -\beta[\Pi_1(c-x, c-x) + \Pi_2(c-x, c-x)] - (1-\beta)\Pi_1(c-x, c-y) - f'(x)$$

and  $L_1(x, y) = -(1-\beta)\Pi_1(c-x, c-y) - f'(x)$ . Along the diagonal  $x = y$ , the difference between these partials is  $U_1(x, x) - L_1(x, x) = -\beta[\Pi_2(c-x, c-x) + \Pi_1(c-x, c-x)] > 0$  (by (A4)(ii)). This implies that  $x$  can never be a best response to  $x$ , for any  $x \in (0, c)$ , since a necessary condition for that is  $U_1(x, x) \leq L_1(x, x)$ . (Note that the last inequality is simply a generalized first-order condition for a maximum in the absence of differentiability of  $F$ .) Hence no interior equilibrium can be symmetric.

(iii) In view of (ii), it remains to show that  $(0, 0)$  and  $(c, c)$  cannot be equilibria. To this end, consider  $U_1(0, 0) = -\Pi_1(c, c) - \beta\Pi_2(c, c) - f'(0) > 0$  by (2.3), and  $L_1(c, c) = -(1-\beta)\Pi_1(0, 0) - f'(c) < 0$  by (2.3). This implies that neither 0 nor  $c$  can be a best response to itself. Hence, all equilibria are asymmetric.

This completes the proof of Theorem 2.1.  $\square$

**Proof of Theorem 2.2.** Since the game is symmetric,  $(a, b) \in [0, c]^2$  is a Nash equilibrium whenever  $(b, a)$  is. Here, we show that when (2.4) holds, there is exactly one such pair of equilibria. We first show that  $r_1, r_2$  are as in Figure 1.

It is easily checked that (2.4) and (2.5) imply that  $U$  and  $L$  are strictly concave in  $x$  (for fixed  $y$ ), on  $\Delta_u$  and  $\Delta_l$ , respectively. Hence, if  $r_1(\cdot)$ , say, is discontinuous at some point  $y_0$ , then  $r_1(y_0^-)$  and  $r_1(y_0^+)$  cannot both lie in  $\Delta_u$  or both in  $\Delta_l$  (note here that  $r_1$  is an upper semi-continuous correspondence, due to the joint continuity of  $F$ , so that  $r_1(y_0^-)$  and  $r_1(y_0^+)$  are both in  $r_1(y_0)$ ). Furthermore, by Theorems 1.1 and 2.1(i), every selection from  $r_1$  is nonincreasing. Also, by Theorem 2.1,  $r_1$  cannot

intersect the 45° line. Therefore, there exists a unique point  $d \in (0, c)$  such that (i)  $r_1$  is discontinuous at  $d$ , with  $r_1(d^-) > d > r_1(d^+)$ , i.e.,  $r_1(d^-) \in \Delta_u$  and  $r_1(d^+) \in \Delta_l$ , (ii)  $r_1$  is continuous and lies in  $\Delta_u$  for  $y \in [0, d]$ , and (iii)  $r_1$  is continuous and lies in  $\Delta_l$  for  $y \in [d, c]$ . In other words,  $r_1$  and  $r_2$  are essentially as depicted in Figure 1.

Next, we show that there is a unique equilibrium in the rectangle  $R := \{(x, y) : 0 \leq x \leq d \text{ and } d \leq y \leq c\} \subset \Delta_l$ . We do this by showing that  $r_1$  and  $r_2$  are (essentially) contractions in  $R$ . Whenever  $r_1$  is interior, the first-order condition  $L_1[r_1(y), y] = 0$ , the Implicit Function Theorem and (A4) yield that  $r_1$  is differentiable in  $R$  and

$$r'_1(y) = -\frac{L_{12}[r_1(y), y]}{L_{11}[r_1(y), y]} = \frac{(1 - \beta)\Pi_{12}[c - r_1(y), c - y]}{f''[r_1(y)] - (1 - \beta)\Pi_{11}[c - r_1(y), c - y]}.$$

Similarly, on  $R$ ,  $r'_2(x) = -\frac{U_{21}[x, r_2(x)]}{U_{22}[x, r_2(x)]}$ , and thus

$$r'_2(x) = \frac{(1 - \beta)\Pi_{12}[c - r_2(x), c - x]}{f''[r_2(x)] - (1 - \beta)\Pi_{11}[c - r_2(x), c - x] - \beta(\Pi_{11} + 2\Pi_{12} + \Pi_{22})[c - r_2(x), c - r_2(x)]}.$$

Straightforward computations show that  $r'_1(y) > -1$  iff

$$f''[r_1(y)] > (1 - \beta)(\Pi_{11} - \Pi_{12})[c - r_1(y), c - y] \quad (6.3)$$

and  $r'_2(x) > -1$  iff

$$f''[r_2(x)] > (1 - \beta)(\Pi_{11} - \Pi_{12})[c - r_2(x), c - x] + \beta(\Pi_{11} + 2\Pi_{12} + \Pi_{22})[c - r_2(x), c - r_2(x)]. \quad (6.4)$$

Clearly, (2.4)-(2.5) imply (6.3) and (6.4), and hence also imply that  $r'_i(\cdot) > -1$ ,  $i = 1, 2$ . Recapitulating, we have  $r'_i(\cdot) \in (-1, 0]$  in the interior of  $R$ ,  $i = 1, 2$ . Since  $r_1(d^+) < d$  and  $r_1$  is nonincreasing, whenever  $r_1$  is not interior in  $R$ , it must be that  $r_1 \equiv 0$ . Hence  $r'_i(\cdot) \in (-1, 0]$  in (all of)  $R$ . Then, uniqueness of equilibrium in  $R$  follows from a well-known argument (for a proof, see e.g. Amir (1996a), Lemma 2.3).

By symmetry, there must be exactly two Nash equilibria of the form  $(\bar{x}, \underline{x})$  and  $(\underline{x}, \bar{x})$ , and the proof of Theorem 2.2 is now complete.  $\square$

**Proof of Corollary 2.3.** This has already been proved in the first part of the proof of Theorem 2.2.  $\square$

**Proof of Theorem 2.4.** (i) Here, we want to show that  $r_1$  and  $r_2$  shift down as  $\beta$  increases. To this end, we need to show that each payoff is submodular in own decision and  $\beta$  (holding the rival's decision constant), and then invoke (Topkis's) Theorem 1.1. In view of the symmetry of the game and Corollary 2.3, it suffices to show submodularity in  $R$ , i.e. (in view of Theorem 1),  $L_{1\beta}(x, y) \leq 0$  and  $U_{1\beta}(y, x) \leq 0$ . We have  $L_{1\beta}(x, y) = \Pi_1(c - x, c - y) \leq 0$ , by (A.2)(ii), and

$$U_{1\beta}(y, x) = -\Pi_1(c - y, c - y) - \Pi_2(c - y, c - y) + \Pi_1(c - y, c - x) \leq -\Pi_2(c - y, c - y) \leq 0,$$

where the first inequality follows from (A2)(i) and the fact that  $y > x$  on  $R$ , and the second inequality follows from (A2)(ii). This completes the proof of Part (i).

(ii) This follows directly from Part (i).

(iii) Consider the (unique) equilibrium  $(\underline{x}, \bar{x})$  in  $R$ . If  $(\underline{x}, \bar{x})$  is not interior in  $R$ , we know from the (last part of) the proof of Theorem 2.2 that it must be the case that  $\underline{x} = 0$ . Then the fact that  $\bar{x}$  decreases in  $\beta$  follows directly from the fact that  $r_2$  shifts down (as  $\beta$  increases), i.e., Theorem 2.4(i).

If  $(\underline{x}, \bar{x})$  is interior in  $R$ , the following first-order conditions must hold:

$$-\beta[\Pi_1(c - \bar{x}, c - \bar{x}) + \Pi_2(c - \bar{x}, c - \bar{x})] - (1 - \beta)\Pi_1(c - \bar{x}, c - \underline{x}) - f'(\bar{x}) = 0, \quad (6.5)$$

and

$$-(1 - \beta)\Pi_1(c - \underline{x}, c - \bar{x}) - f'(\underline{x}) = 0. \quad (6.6)$$

Totally differentiating w.r.t.  $\beta$ , and collecting terms yields

$$\begin{aligned} & [\beta(\Pi_{11} + 2\Pi_{12} + \Pi_{22})(c - \bar{x}, c - \bar{x}) + (1 - \beta)\Pi_{11}(c - \bar{x}, c - \bar{x}) - f''(\bar{x})] \frac{d\bar{x}}{d\beta} \\ & + (1 - \beta)\Pi_{12}(c - \bar{x}, c - \underline{x}) \frac{d\underline{x}}{d\beta} = (\Pi_1 + \Pi_2)(c - \bar{x}, c - \bar{x}) - \Pi_1(c - \bar{x}, c - \underline{x}), \text{ and} \\ & (1 - \beta)\Pi_{12}(c - \underline{x}, c - \bar{x}) \frac{d\bar{x}}{d\beta} + [(1 - \beta)\Pi_{11}(c - \underline{x}, c - \bar{x}) - f''(\underline{x})] \frac{d\underline{x}}{d\beta} = -\Pi_1(c - \underline{x}, c - \bar{x}). \end{aligned}$$

Solving for  $\frac{d\bar{x}}{d\beta}$  (e.g. using Cramer's rule), we get  $\frac{d\bar{x}}{d\beta} \geq 0$  iff

$$f''(\underline{x}) \geq (1 - \beta) \left[ \Pi_{11}(c - \underline{x}, c - \bar{x}) + \frac{\Pi_1(c - \underline{x}, c - \bar{x})\Pi_{12}(c - \bar{x}, c - \underline{x})}{(\Pi_1 + \Pi_2)(c - \bar{x}, c - \bar{x}) - \Pi_1(c - \bar{x}, c - \underline{x})} \right],$$

which is clearly implied by (2.6). This completes the proof of Theorem 2.4.  $\square$

**Proof of Lemma 3.1.** From (3.1), the objective function for Case  $C_1$  reduces to (assuming w.l.o.g. that  $x \geq y$ )

$$\max\{2\Pi(c - x, c - x) - f(x) - f(y) : x, y \in [0, c]\}. \quad (6.7)$$

Hence, the optimal choice of  $y$  is clearly  $y = 0$ , and this objective then coincides with (3.2), the objective function of Case  $J$ . This completes the proof of Lemma 3.1.  $\square$

**Proof of Lemma 3.2.** (i) In the game  $N_0$ , the payoff function of Firm 1 (say) is

$$\Pi(c - x, c - y) - f(x). \quad (6.8)$$

This game is clearly submodular as a consequence of (A2)(i). Hence, it has a Nash equilibrium. Uniqueness follows from the proof of Theorem 2.2 since (2.4) is the same as (2.4) with  $\beta = 0$ . In other words, uniqueness follows here from the best-response having slopes in  $(-1, 0]$  as shown before (with  $\beta = 0$ ). Finally, symmetry of the unique equilibrium in  $[0, c]^2$  follows from the fact that the payoff (6.8) is strictly concave in  $x$  (implied by (2.4)), thus leading to continuous best-response functions which intersect at the 45° line.

(ii) Proceed by contradiction and assume that  $x_J < x_0$ . Assuming  $x_J$  and  $x_0$  are both interior, they satisfy the following first-order conditions:

$$-2(\Pi_1 + \Pi_2)(c - x_J, c - x_J) - f'(x_J) = 0, \quad (6.9)$$

and

$$-\Pi_1(c - x_0, c - x_0) - f'(x_0) = 0. \quad (6.10)$$

By (A2)(ii) and (A5),  $\Pi_1(c - x_J, c - x_J) + 2\Pi_2(c - x_J, c - x_J) \leq 0$ . Summing up this inequality and (6.9) yields

$$\begin{aligned} -\Pi_1(c - x_J, c - x_J) - f'(x_J) &\leq 0 \\ &= -\Pi_1(c - x_0, c - x_0) - f'(x_0), \quad \text{by (6.10)} \\ &\leq -\Pi_1(c - x_0, c - x_J) - f'(x_0), \end{aligned}$$

where the last inequality follows from (A2)(i) and the contradiction hypothesis  $x_J < x_0$ . Now, the inequality above (with the two outer terms) clearly contradicts the concavity of  $\Pi(c - (\cdot), c - x_J) - f(\cdot)$  which is itself implied by (2.4).

Without interiority, the only cases that might cause any difficulty are  $x_0 = c$  and  $x_J = 0$  (since we are trying to show that  $x_J \geq x_0$ ). First, we show that if  $x_0 = c$ , then  $x_J = c$  too. By (A2),  $-\Pi_1(c - x, c - x) \geq -\Pi_1(c - x, 0)$ , for all  $x \in [0, c]$ . Also, by (A5),  $-\Pi_1(c - x, c - x) - 2\Pi_2(c - x, c - x) \geq 0$ . Adding up the two inequalities yields  $-2\Pi_1(c - x, c - x) - 2\Pi_2(c - x, c - x) - f'(x) \geq -\Pi_1(c - x, 0) - f'(x)$ , which says that the derivative with respect to  $x$  of (6.7) is always higher than that of (6.8) with  $y = c$ . Since  $\Pi(c - (\cdot), 0) - f(\cdot)$  is concave by (2.4),  $x_0 = c = \arg \max \{\Pi(c - (\cdot), 0) - f(\cdot)\}$  implies that the latter maximand is nondecreasing. Hence, so is (6.7) since it has a larger derivative  $\forall x$ . Hence  $x_J = c$  too.

Next, we show that  $x_J = 0$  implies  $x_0 = 0$ . If  $x_J = 0$ , (6.9) becomes  $-2\Pi_1(c, c) - 2\Pi_2(c, c) - f'(0) \leq 0$ . By (A5),  $\Pi_1(c, c) + 2\Pi_2(c, c) \leq 0$ . Adding up yields  $-\Pi_1(c, c) - f'(0) \leq 0$ . Since  $\Pi(c - (\cdot), c) - f(\cdot)$  is concave by (2.4), we have  $x_0 = 0$ , and the proof of Lemma 3.2 is complete.  $\square$

**Proof of Proposition 3.3.** For extra clarity here, let us index the R&D equilibrium of Section 2 by the associated value of  $\beta$ , i.e. write  $\bar{x}_\beta$  for  $\bar{x}$  and  $\underline{x}_\beta$  for  $\underline{x}$ , for all  $\beta \in (0, 1]$ . For  $\beta = 0$ , we have  $\bar{x}_0 = \underline{x}_0 = x_0$  (from Lemma 3.2).

From Theorem 2.4(iii), we know that  $\bar{x}_\beta < \bar{x}_0 = x_0$ , for all  $\beta \in (0, 1]$ . Hence, from Lemma 3.2,  $x_J \geq x_0 > \bar{x}_\beta$ . This completes the proof of Proposition 3.3.  $\square$

**Proof of Proposition 3.4.** We first show that (3.3) is sufficient for the conclusion of this Proposition. To this end, note that in case  $C_\beta$ , the Nash equilibrium  $(\bar{x}, \underline{x})$  is

a feasible joint decision. Hence, equilibrium profits are no lower in Case  $C_\beta$  than in Case  $N$ . Next rewrite the joint objective (3.1), assuming w.l.o.g. that  $x \geq y$ , as

$$\Pi(c-x, c-y) + \Pi(c-y, c-x) + s[2\Pi(c-x, c-x) - \Pi(c-x, c-y) - \Pi(c-y, c-x)] - f(x) - f(y). \quad (6.11)$$

By (3.3), this objective is nondecreasing in  $s$ , for fixed  $(x, y)$ . Hence, optimal profits are higher for  $s = 1$ , i.e. for Case  $C_1$  or equivalently (Lemma 3.1) for Case  $J$ , than for any other  $s \in [0, 1]$ , in particular  $s = \beta$ . Thus, profits are higher in Case  $J$  than for case  $N$ .

We now show that (3.4) is also sufficient for the same conclusion. The joint objective for Case  $C_0$  is (from (3.1) with  $s = 0$ ):

$$G(x, y) \triangleq \Pi(c-x, c-y) + \Pi(c-y, c-x) - f(x) - f(y). \quad (6.12)$$

It can be verified that (6.12) is (jointly) strictly concave in  $(x, y)$  if (3.4) holds (to check this, one can see that  $G_{11} > G_{12}$  and  $G_{22} > G_{12}$  follow from (3.4)). Since (6.12) is also symmetric in  $(x, y)$ , there must be a unique arg max, which is also symmetric, i.e. of the form  $(x^*, x^*)$ . (Otherwise, if  $(a, b)$  is an arg max with  $a \neq b$ , then symmetry implies that  $(b, a)$  is also an arg max. With strict concavity, this leads to  $(\frac{a+b}{2}, \frac{a+b}{2})$  yielding a strictly higher value than the max itself, a contradiction.)

Consequently, one can restrict the maximization of (6.12) to choices on the diagonal, i.e. replace (6.12) with  $\max_x \{2\Pi(c-x, c-x) - 2f(x)\}$ , which is clearly below the joint objective in Case  $J$ , i.e. (from (3.2)),  $2\Pi(c-x, c-x) - f(x)$ . Hence, equilibrium profits are higher in Case  $J$  than in Case  $C_0$ . Now, since the objective function for Case  $C_s$  is linear in  $s$  (see (6.11)), total equilibrium profits are lower for Case  $C_\beta$  than for either  $C_0$  or  $C_1 \equiv J$ . Altogether then,  $C_1 \equiv J$  yields higher profits than all the  $C_s$ ,  $s \in [0, 1]$ , and thus also higher than Case  $N$  (recall that the latter has lower profits than  $C_\beta$ ). The proof is now complete.  $\square$

**Proof of Proposition 3.5.** Since Proposition 3.4 holds here, we know that producer welfare is higher in Case  $J$  than in Case  $N$ . From Lemma 3.2 and Theorem 2.4(iii), we know that  $x_J \geq x_0$  and that  $x_0$  dominates  $\bar{x}$ , the equilibrium R&D of the innovator.



The imitator's effective R&D level is  $\underline{x}$  with probability  $(1 - \beta)$  and  $\bar{x}$  with probability  $\beta$ , and is hence always below  $x_0$  too. Hence, by (A6), consumer welfare is higher in Case  $J$  than in Case  $N$ , and thus so is total welfare.  $\square$

**Proof of Lemma 3.6.** First, observe that both the objective and the constraint in (3.5) are symmetric in  $(c_1, c_2)$ . Hence, if  $(a, b)$  is an optimizer, so is  $(b, a)$ . Since the iso-profit curves are concave, the arg max must be a boundary choice. Therefore,  $(k, 0)$  and  $(0, k)$  must form the arg max. Analogous reasoning for the minimization case leads to the arg min being unique and equal to  $(\frac{k}{2}, \frac{k}{2})$ .  $\square$

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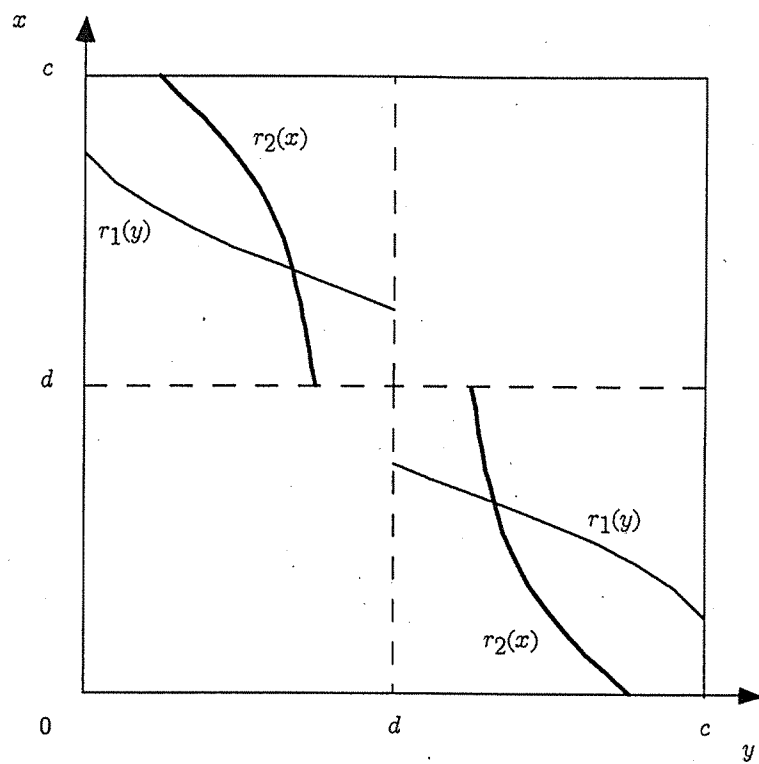


Figure 1

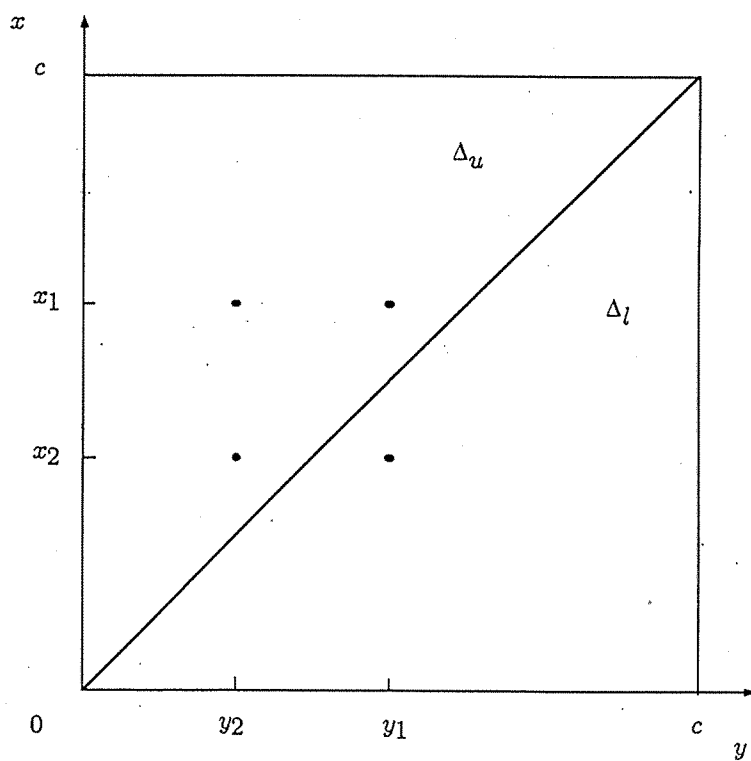


Figure 2



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