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Vacancy Durations - A Model for Employer's Search

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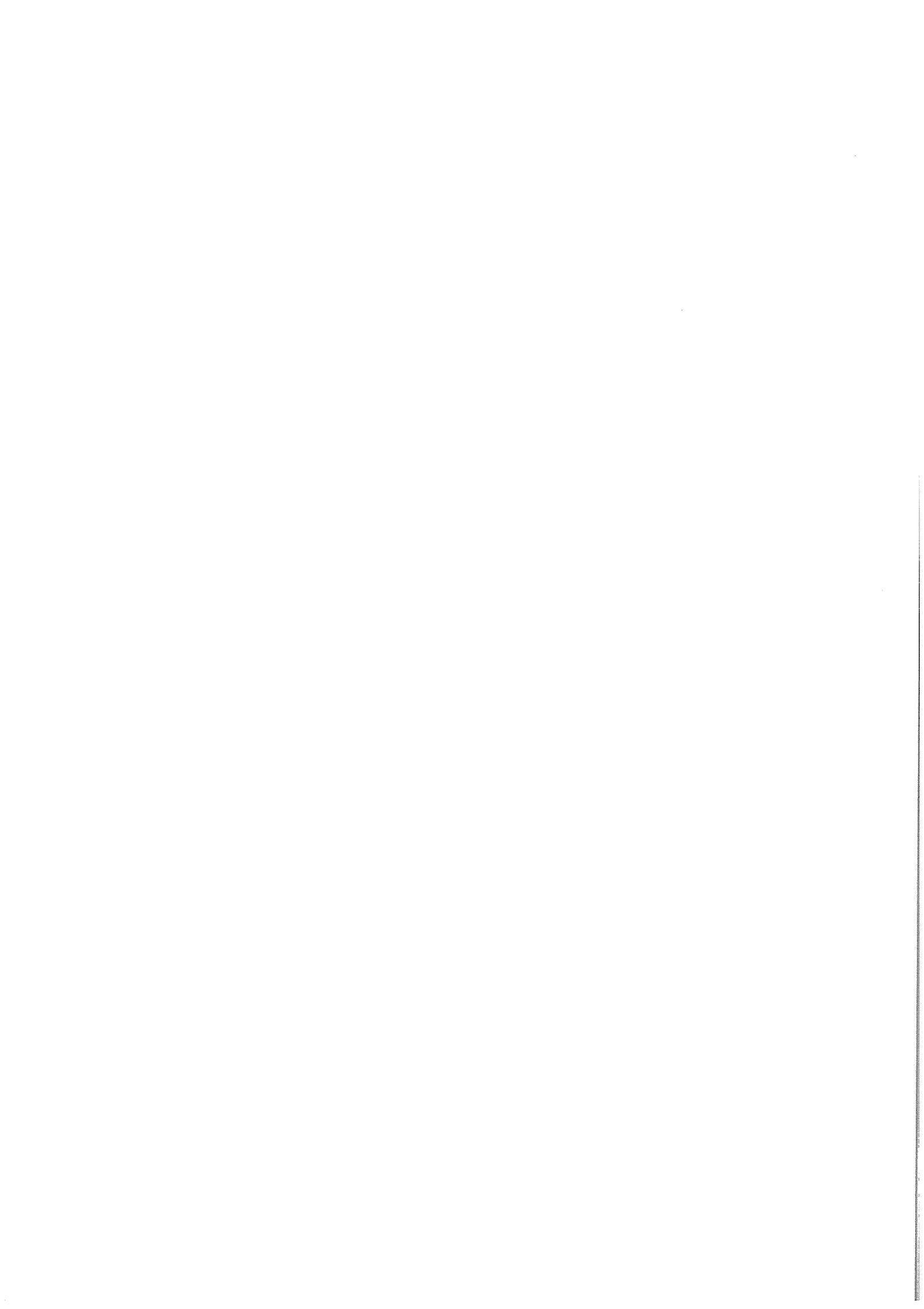
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Abstract

This paper investigates a Dutch data set on vacancy durations and numbers of applicants to inquire employer's search strategies. A non-sequential search process assumes that most vacancies are filled from a pool of applicants, which is formed shortly after the posting of the vacancy. The time spent on recruiting applicants and the duration of the selection process are estimated with a proportional hazard model, via the arrival - and attrition rates of applicants.

Keywords

Vacancy durations, numbers of applicants, attrition

JEL-Classifications

C2, C5, J6

Comments

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1 Introduction

Search theory in labour markets goes back to the sixties (Stiegler, 1962). The literature in this field is very elaborate on job search by unemployed workers (for surveys see e.g. (Mortensen, 1986), (McKenna, 1990)), whereas there exists little literature on employer's search. This is mainly due to the lack of data. There are of course a couple of reasons, why the interest in employer's search and vacancy durations should be stressed.

Inspections of the unemployment/vacancies relation show a striking shift in unemployment at given vacancies, which was detected in nearly all advanced countries. In traditional u/v analysis (Blanchard and Diamond, 1989), changes in aggregate demand lead to movements along a given u/v curve: a high demand is compatible with many vacancies and few unemployed and vice versa. If the u/v curve shifts outward at a given level of aggregate demand, this indicates an increasing inflexibility or maladjustment in the labour market: there are more unemployed at the same level of job vacancies. Hence there is strong theoretical evidence, that shifts in the u/v curve reflect the efficiency with which the labour market matches unemployed workers to job vacancies. For empirical evidence on shifts in the u/v curve in international comparison see for example (Jorgen and MacFarlan, 1993).

The lower efficiency in the matching process can either be a result of workers becoming more choosy about taking jobs or of firms becoming more choosy about hiring workers. The level of social benefits and high unemployment may have undermined work habits and changed public attitude towards claiming benefits and thus decreased the search effort of unemployed. Another possibility is the introduction of redundancy payments and employment protection. It would discourage firms from getting rid of workers and increase the choosyness of employers faced with a given number of applicants for a given vacancy.

With the above considerations it is reasonable to investigate the search behavior of both parts in the matching process, the unemployed as well as the employers.

In research on employers' behavior a pioneering study by (Holt and David, 1966) was followed by papers by (Barron and Bishop, 1985) and (Barron et al., 1985), who investigate employer's search efforts to fill a vacancy. They examine effects of factors as employer size, unionization, dismissal costs, capital and labour market conditions on intensive and extensive search and hiring costs. Intensive employer's search is measured by the average number of hours the employer spends on recruiting, screening and interviewing per applicant. Extensive search is measured by the average number of applicants and the number of interviewed per employment. (Beaumont, 1978), (Roper, 1988), (Renes, 1989) and (van Ours, 1989) study vacancy durations. They relate the vacancy duration to various characteristics of the vacancy and the employer. Renes and van Ours, who use hazard models, also address the question how the rate at which vacancies are filled

depends on the elapsed duration of the vacancy. Papers by (van Ours and Ridder, 1992) and (van Ours and Ridder, 1993) combine examinations of search strategies used by employers and vacancy durations.

Most models for search of unemployed use the idea of sequential search. In these models job offers arrive sequentially at a point process (e.g. a Poisson process). At each offer the unemployed has to decide whether to accept it and stop searching, or to reject it and continue the search. The job offer is modeled as an independent draw from a given wage distribution and the optimal decision whether to take the job or not maximizes the searcher's expected utility. The optimal strategy depends on the knowledge of the wage offer distribution, the arrival rate of jobs, the stationarity of the search environment, the utility function and the searcher's time horizon and it is characterized by a reservation wage. The probability that an unemployed job seeker finds a job in a short time interval, given that he is still unemployed at the beginning of the interval - the hazard rate of leaving unemployment - is equal to the product of the arrival rate and the probability of accepting an offer.

A similar model for employer's search was set up by (Lippman and McCall, 1976). We can think of it as applicants arriving according to a Poisson Process at a firm and the firm deciding to employ the applicant according to the maximal profit for the firm. The firm knows the arrival rate of applicants and the distribution of characteristics of potential applicants. The optimal strategy maximizes the expected profit and is specified by a reservation productivity or reservation values of observed productivity related characteristics. In this sequential employer's search model the hazard rate of filling a vacancy is equal to the product of the arrival rate of applicants and the probability that the applicant is acceptable.

Whereas we can imagine that an unemployed's strategy follows a sequential search, it is not very plausible that firms, especially large firms, do not try to get an overview of all possible applicants. So we would rather assume that employers try to attract a number of applicants and choose the suitable employee out of this pool. We call this a non-sequential search strategy.

The papers preceding this one deal with model specification for employer's search: In their paper van Ours and Ridder (van Ours and Ridder, 1992) show that employers use a non-sequential search strategy: applications arrive shortly after the vacancy has been posted, and the rest of the vacancy duration is used to select a new employee from the pool of applicants. In a second paper (van Ours and Ridder, 1993) they assume that the vacancy duration is divided in an application period, where applicants arrive, and a selection period, where no new applications are taken, and estimate mean durations of application- and selection periods with a proportional hazard model.

In this paper the likelihood-function for the whole problem will be specified and estimated, taking also into account that during the selection period the number of applicants

may decrease due to attrition. Further we estimate a second model, where no fixed application- and selection periods are given, but applicants arrive and hold their application for some time during the whole vacancy duration. Here it may be that employees withdraw their application, when they found another job, or very infeasible applications are not even submitted to the selection process, whereas applications are taken during the whole vacancy duration. Estimations are carried out for the Dutch data set, which van Ours and Ridder used, which is provided by the Organization of Strategic Labor Market Research (OSA) vacancy survey.

2 Data and Model

Our data set is a survey on enterprises conducted in two stages. In the first interview firms were asked for open vacancies, for the duration of the vacancy the number of applicants and some special characteristics, as the type of the job, required skills, recruitment and selection procedures used. In the second interview, conducted about four months later, firms were asked if the vacancy of the first interview was filled, the date at which it was filled and the number of applicants. There is no information on the characteristics of applicants or on wage offers. The sample contains 670 vacancies, of which 74% were filled at the time of the second survey. For the estimation of the model we had to exclude observations, where the number of applicants in the second survey was missing. The sample means of the reduced data set are given in table (1).

In the data set we often observe that the number of applicants at the second interview is lower than the number of applicants at the first interview. We try to explain this effect by the selection process, which has already eliminated some infeasible applications.

We call the numbers of applicants at the first and second interview k_1 and k_2 , the vacancy duration at the first interview t_1 and the duration between the first interview and the end of the vacancy t_2 . For unfilled vacancies, t_2 gives the time between the two interviews. For vacancies, which were filled at unknown date, we take an expected value for t_2 .

We suppose that the complete vacancy duration is an exponentially distributed random variable with parameter θ . The density function is given by

$$f(t) = \theta \exp -\theta t \tag{1}$$

The number of applicants at every time t is given by a random process (X_t) . We suppose that applicants arrive according to a Poisson process with parameter λ and the probability of leaving the pool of applicants is characterised by the parameter μ .

We relate the parameters to a vector of explanatory variables x , by

$$\lambda = \exp(x' \beta_l), \quad \mu = \exp(x' \beta_m), \quad \theta = \exp(x' \beta_t).$$

Table 1: Sample Means

	Filled	Open	All
Job requirements			
Education (level)	2.96	3.27	3.02
Experience (years)	1.21	1.73	1.35
Type of job			
Commercial	0.47	0.27	0.43
Industry	0.24	0.28	0.25
Recruitment channels			
Advertisement	0.60	0.61	0.60
Labour exchange	0.32	0.35	0.33
Firm characteristics			
Number of employees ($\times 1000$)	0.39	0.43	0.40
Psychological test	0.24	0.37	0.27
Elapsed duration (days)			
First interview	53.6	77.3	58.9
Second interview	160.0	205.2	170.1
Number of applicants			
First interview	10.1	6.45	9.27
Second interview	11.1	5.76	10.45
Number of observations	440	127	567
Education levels:			
1 =	primary		
2 =	extended primary		
3 =	secondary		
4 =	higher vocational		
5 =	university		

In the following we derive two models for estimating the parameters. In the first model we assume that application- and selection periods are strictly separated, there is no hiring during the application period and no applicants arrive during the selection period. This model only gives a reasonable likelihoodfunction, if we assume T , the duration of the application period, to be a nonrandom variable.

In the second model, applicants may arrive during the whole vacancy duration, but hold their application only for some time, which may either be due to other job offers or to selection by the firm. With these assumptions the number of applicants can be

modelled according to a ‘birth- and death- process’, which results in a less complicated likelihood-function.

2.1 Model 1: Fixed Application- and Selection Period

In this model we suppose that the vacancy duration is divided into an application period, with duration T , i.e. it lasts from $t = 0$ to $t = T$ and a selection period, with duration S , i.e. it lasts from $t = T$ till $t = S + T$. For the theoretical derivations we assume that T and S are random variables, with an exponential distribution. In the estimation part, we will take T as a nonrandom value, for computational simplicity.

There is no hiring during the application period and no applicants arrive during the selection period.

During the application period, for $t \in [0, T]$, applicants arrive according to a Poisson Process.

The probability that we observe k applicants at time t is given by

$$\mathbb{P}(X_t = k) = \frac{(\lambda t)^k}{k!} \exp(-\lambda t), \quad t \in [0, T]. \quad (2)$$

During the selection period, for $t \in (T, T + S]$ the number of applicants, who ‘survive’ the selection process is binomially distributed, with parameters n , which is the number of applicants at $t = T$, and $p = e^{-\mu t}$. The probability that we observe k applicants at time t , when the size of the pool of applicants is n , is given by

$$\mathbb{P}(X_t = k | X_T = n) = \binom{n}{k} e^{-\mu t k} (1 - e^{-\mu t})^{n-k}, \quad t \in (T, T + S]. \quad (3)$$

We derive the distribution of numbers of applicants during the whole vacancy duration, for $t \in (0, T + S]$ in the appendix. The probability that we observe k applicants, when the vacancy is filled, given we had 0 applicants when it was posted, is given by

$$\mathbb{P}(X_{T+S} = k | T, S, X_0 = 0) = \frac{(\lambda T \exp(-\mu S))^k}{k!} \exp(-\lambda T e^{-\mu S}), \quad (4)$$

and the probability that we observe k_2 applicants, when the vacancy is filled, given we had k_1 applicants when it was posted, is given by

$$\begin{aligned} \mathbb{P}(X_{T+S} = k_2 | T, S, X_0 = k_1) &= \\ &= \exp(-\lambda T e^{-\mu S}) \\ &\quad \sum_{n=\max\{0, k_1 - k_2\}}^{k_1} \frac{(\lambda T e^{-\mu S})^{k_2 - k_1 + n}}{(k_2 - k_1 + n)!} \binom{k_1}{n} e^{-\mu S(k_1 - n)} (1 - e^{-\mu S})^n. \end{aligned} \quad (5)$$

Now we consider the likelihood-function of one observation (k_1, k_2, t_1, t_2) , conditional on T , and decompose it, using Bayes’ Theorem

$$\begin{aligned}
 L(\lambda, \mu, \theta | k_1, k_2, t_1, t_2, T) &= \tag{6} \\
 &= f(k_1, k_2, t_1, t_2 | T, \lambda, \mu, \theta) \\
 &= \mathbb{P}(k_2 | k_1, t_1, t_2, T) \mathbb{P}(k_1 | t_1, T) f(t_1 + t_2 | t_1, T) f(t_1 | T)
 \end{aligned}$$

where $f(k_1, k_2, t_1, t_2 | T, \lambda, \mu, \theta)$ denotes the probability that we observe k_1 applicants at the first interview, after a duration of t_1 , and k_2 applicants after a duration of $t_1 + t_2$, which may be the complete vacancy duration, if the vacancy was filled at the time of the second interview. Implicitly we assume that time durations are independent of the numbers of applicants observed.

The parts of the likelihoodfunction are given by:

Likelihood Contributions of Numbers of Applicants:

The probability that we observe k_1 applicants at t_1 , is given by the Poisson distribution, if the application period is not finished, otherwise it is given by the formula of equation (4).

$$\begin{aligned}
 \mathbb{P}(k_1 | t_1, T) &= \tag{7} \\
 &= \begin{cases} \frac{(\lambda t_1)^{k_1}}{k_1!} e^{-\lambda t_1}, & t_1 < T \\ \frac{(\lambda T e^{-\mu(t_1-T)})^{k_1}}{k_1!} e^{-\lambda T e^{-\mu(t_1-T)}}, & t_1 \geq T \end{cases}
 \end{aligned}$$

The probability of k_2 is given by the Poisson distribution, if the application period is not finished at $t_1 + t_2$. It is given by equation (5), if the application period ended between the two interviews or it is given by the binomial distribution, if the application period ended before the first interview.

$$\begin{aligned}
 \mathbb{P}(k_2 | k_1, t_1, t_2, T) &= \tag{8} \\
 &= \begin{cases} \frac{(\lambda t_2)^{k_2-k_1}}{(k_2-k_1)!} e^{-\lambda t_2}, & t_1 + t_2 < T, k_2 \geq k_1 \\ \exp(-\lambda(T-t_1)e^{-\mu(t_1+t_2-T)}) \sum_{n=\max\{0, k_1-k_2\}}^{k_1} \dots, & t_1 \leq T \leq t_1 + t_2 \\ \binom{k_1}{k_2} e^{-\mu t_2 k_2} (1 - e^{-\mu t_2})^{k_1-k_2}, & T \leq t_1, k_2 \leq k_1 \end{cases}
 \end{aligned}$$

Likelihood Contribution of Time Durations:

$$f(t_1 + t_2|T) = \tag{9}$$

$$= \begin{cases} \theta \exp(-\theta(t_1 + t_2 - T)), & \text{for filled vacancies } (t_1 + t_2 = T + S) \\ \exp(-\theta(t_1 + t_2 - T)), & \text{for unfilled vacancies } (t_1 + t_2 < T + S) \end{cases}$$

2.2 Model 2: No Fixed Application- and Selection Period

In this model we assume that applicants arrive with a given rate according to a Poisson process and hold their application for some time. The duration of each application is given by an exponentially distributed random variable.

So the probability that k applicants arrive up to time t is given by

$$\mathbb{P}(X_t = k) = \frac{(\lambda t)^k}{k!} \exp(-\lambda t)$$

and the density function of the duration of the application is given by

$$g(t) = \begin{cases} 0 & \text{for } t \leq 0, \\ \mu e^{-\mu t} & \text{else} \end{cases}$$

That means, we expect λ applicants to arrive in one day, and one applicant stays for $1/\mu$ days on average.

The probability that we observe k_2 applicants at time t , given that we had k_1 at time 0, is given by (Fisz, 1970) chapter 8.5, pp 342.

$$\mathbb{P}(X_t = k_2|X_0 = k_1) = \tag{10}$$

$$\exp\left(-\frac{\lambda}{\mu}(1 - \exp(-\mu t))\right) \sum_{j=0}^{\min\{k_1, k_2\}} \binom{k_1}{j} \left(\frac{\lambda}{\mu}\right)^{k_2-j} \frac{e^{-\mu t j} (1 - e^{-\mu t})^{k_1+k_2-2j}}{(k_2 - j)!}$$

This is the convolution of a Poisson distribution with parameter $\frac{\lambda}{\mu}(1 - \exp(-\mu t))$ and a Binomial distribution with probability $\exp(-\mu t)$. We can interpret this expression as the probability of all possible events, where j of the k_1 applicants stay, and $k_2 - j$ new applicants arrive.

The likelihood of one observation is then given by

$$\begin{aligned}
 L(\lambda, \mu, \theta | k_1, k_2, t_1, t_2) &= & (11) \\
 &= f(k_1, k_2, t_1, t_2 | \lambda, \mu, \theta) \\
 &= \mathbb{P}(X_{t_1+t_2} = k_2 | X_{t_1} = k_1) \mathbb{P}(X_{t_1} = k_1 | X_0 = 0) f(t_1 + t_2)
 \end{aligned}$$

And three parts of the likelihood function are given by:

Likelihood Contributions of Numbers of Applicants:

$$\begin{aligned}
 \mathbb{P}(k_2 | k_1, t_1, t_2) &= & (12) \\
 &= \exp\left(-\frac{\lambda}{\mu}(1 - \exp(-\mu t_2))\right) \\
 &\quad \sum_{j=0}^{\min\{k_1, k_2\}} \binom{k_1}{j} \left(\frac{\lambda}{\mu}\right)^{k_2-j} \frac{e^{-\mu t_2 j} (1 - e^{-\mu t_2})^{k_1+k_2-2j}}{(k_2-j)!},
 \end{aligned}$$

and

$$\mathbb{P}(k_1 | t_1) = \exp\left(-\frac{\lambda}{\mu}(1 - \exp(-\mu t_1))\right) \frac{\left(\frac{\lambda}{\mu}\right)^{k_1} (1 - e^{-\mu t_1})^{k_1}}{k_1!}, \quad (13)$$

Likelihood Contribution of Time Durations:

$$\begin{aligned}
 f(t_1 + t_2) &= & (14) \\
 &= \begin{cases} \theta \exp(-\theta(t_1 + t_2)), & \text{for filled vacancies} \\ \exp(-\theta(t_1 + t_2)), & \text{for unfilled vacancies} \end{cases}
 \end{aligned}$$

3 Estimation and Results

For maximizing the likelihood-function we use the GAUSS maximum likelihood routine MAXLIK.

3.1 Estimation Results of Model 1

In this model we take T , the duration of the application period as a given nonrandom value, and estimate the arrival rate λ , the attrition rate μ and the hazard rate of the vacancy duration θ .

In the estimation we fix the application period on $T = 21$ days. As the model supposes that applicants only arrive during the application period, the number of applicants cannot rise during the selection period. Therefore we have to adjust T for observations, where $t_1 > T$ and numbers of applicants are rising i.e. $k_2 > k_1$. In these cases we set $T = t_1 + t_2/4$. This problem arises in a large number of observations (about 50% of observations with $t_1 > T$), so we get a mean length of 47 days for the application period. This number is quite stable, if we vary T from 2 to 4 weeks, which suggests that the assumption ‘no applications are taken during the selection period’ might not be true for most firms. The same adjustment of T is made in cases, where the whole vacancy duration $t_1 + t_2$ is smaller than T .

We relate the parameters λ, μ, θ to the explanatory variables, by setting

$$\lambda = \exp(x'_\lambda \beta_\lambda), \quad \mu = \exp(x'_\mu \beta_\mu), \quad \theta = \exp(x'_\theta \beta_\theta),$$

and estimate $(\beta_\lambda, \beta_\mu, \beta_\theta)$.

As explanatory variables for the arrival rate of applicants λ , we take dummy variables for two sectors - commercial and industry, the required level of education, dummy variables for advertisement in newspapers and notification at the public employment office as well as firm size. We also include a constant term and interpret parameter values of the explanatory variables as deviations from the mean.

For the attrition rate μ , we take a constant, required level of education, firm size and a dummy variable for the use of a psychological test.

For θ we take the same explanatory variables as for λ .

Results:

Estimation gives similar results as (van Ours and Ridder, 1993), but a larger number of parameters attains values significantly different from 0. The results are summarized in table (2).

- effects of explanatory variables on λ and μ :

We observe a negative effect of required education on the arrival rate and even a larger negative effect on the attrition rate. This means that for a job with higher required level of education less people apply, but also the selection is more time consuming, as the attrition rate is lower.

The advertisement has a large effect on the arrival rate, whereas notification at the labor exchange has a much smaller effect.

In both service and industry sectors we observe larger arrival rates than in other sectors.

Firm size has a positive effect on both rates, larger firms get more applications, but also have a higher attrition.

The use of a psychological test has a negative effect on the attrition rate, which means that psychological tests are mainly used in longer selection procedures.

- effects of explanatory variables on the vacancy duration

Some parameters for the hazard rate have insignificant values, which may be due to the constant hazard model we use. Mean vacancy duration, which is given by $1/\theta$, is significantly shorter in the service sector. It also turns out that vacancy durations increase with required level of education.

- mean application and attrition rates:

The mean values for λ and μ (for the explanatory variables set at their mean values) are $\bar{\lambda} = 0.31$ and $\bar{\mu} = 0.003$, which means that we expect on average one applicant every three days, and an attrition rate, which is much smaller than the arrival rate.

The expected vacancy duration is with 203 days a bit higher than the mean of $t_1 + t_2$, because of the unfilled vacancies in the sample.

The expected number of applicants during the whole vacancy duration is 15.

Table 2: Results Model 1

	Parameter	Std. Error	P-value
Arrival rate (λ)			
Constant	-1.9020	0.0067	0.0000
Commercial	0.5573	0.0052	0.0000
Industry	0.3142	0.0071	0.0000
Education	-0.0175	0.0024	0.0000
Advertisement	0.7207	0.0063	0.0000
Labour exchange	-0.0451	0.0045	0.0000
Firm size	0.0694	0.0032	0.0000
Attrition rate (μ)			
Constant	-5.5642	0.0080	0.0000
Education	-0.2615	0.0074	0.0000
Firm size	0.2043	0.0070	0.0000
Psych. test	-0.1990	0.0172	0.0000
Hazard rate (θ)			
Constant	-5.0871	0.1596	0.0000
Commercial	0.3788	0.1470	0.0050
Industry	-0.0822	0.1867	0.3298
Education	-0.1978	0.0706	0.0025
Advertisement	-0.0497	0.1252	0.34550
Labour exchange	-0.0461	0.1288	0.3603
Firm size	-0.0102	0.0659	0.4384
Psych. test	-0.1976	0.1542	0.1000

Mean log.likelihood :	-15.3839
Number of observations :	567
Mean λ :	0.3119
Mean μ :	0.0036
Mean θ :	0.0064
Expected number of applicants :	15.11
Expected attrition :	5.01
Expected duration :	203.506
Mean application period :	48.44
Mean selection period :	121.69

3.2 Estimation Results of Model 2

We relate the parameters λ, μ, θ to the explanatory variables, by setting

$$\lambda = \exp(x'_\lambda \beta_\lambda), \quad \mu = \exp(x'_\mu \beta_\mu), \quad \theta = \exp(x'_\theta \beta_\theta),$$

and estimate $(\beta_\lambda, \beta_\mu, \beta_\theta)$.

As explanatory variables for the arrival rate of applicants λ , we take dummy variables for two sectors - commercial and manufacturing, the required level of education, dummy variables for advertisement in newspapers and notification at a public employment office and the firm size. We include a constant term and dummy variables for the duration of the vacancy at the time of the first interview - t_1 3-6 weeks and t_1 over 6 weeks, to check for time dependence.

For the attrition rate μ , we take a constant, the two sectors, required level of education, firm size and a dummy variable for the use of a psychological test.

For θ we take the same explaining variables as for λ , except the time dependence dummies.

The likelihood-function, as given above, consists of three parts: The first part is given by the number of applicants at the first interview, the second part by the number of applicants at the second interview, and the third part by the complete vacancy duration. As we suppose that the newspaper advertisement has no effect after over four months, we exclude it from the explanatory variables of λ for the calculation of the second part of the likelihood. For the same reason the time dependence parameters are excluded in the second part of the likelihood-function.

Results:

The effects of the explanatory variables on the parameters in this model coincide with the results of model 1. We are left to interpret the effects of time dependence and mean application and attrition rates. The results are summarized in table (3).

- effects of duration dependence:

The arrival rate of applicants decreases after 3 weeks and after 6 weeks, which goes along with the assumption that most applicants arrive shortly after the vacancy has been posted.

- mean application and attrition rates:

The mean values for λ and μ (for the explanatory variables set at their mean values) are $\bar{\lambda} = 0.26$ and $\bar{\mu} = 0.019$, which means that we expect on average one

applicant every four days, and each applicant stays for 52 days. The ratio of λ/μ is 14. That means the arrival rate is 14 times the attrition rate.

The arrival rate decreases rapidly with time. For the first three weeks we get a mean λ of 0.46, from the third to the sixth week mean λ of 0.33 and in the following weeks mean λ of 0.16.

The expected vacancy duration is with 216 days a bit higher than the mean of $t_1 + t_2$, because of the unfilled vacancies in the sample.

The expected number of applicants is with 13 during the whole vacancy duration equal to mean k_2 .

Table 3: Results Model 2

	Estimator	Std. Error	P-value
Arrival rate (λ)			
Constant	-2.0946	0.0122	0.0000
Commercial	0.7285	0.0140	0.0000
Industry	0.6520	0.0176	0.0000
Education	-0.0232	0.0067	0.0002
Advertisement	1.5117	0.0089	0.0000
Labour exchange	0.0873	0.0059	0.0000
Firm size	0.0431	0.0067	0.0000
$t_1 > 20$	-0.4776	0.0087	0.0000
$t_1 > 50$	-1.1291	0.0100	0.0000
Attrition rate (μ)			
Constant	-4.4026	0.0163	0.0000
Commercial	0.5938	0.0195	0.0000
Industry	0.7187	0.0231	0.0000
Education	-0.3049	0.0088	0.0000
Firm size	0.1791	0.0066	0.0000
Psych. test	0.0463	0.0087	0.0000
Hazard rate (θ)			
Constant	-5.4490	0.1770	0.0000
Commercial	0.3985	0.1725	0.0105
Industry	-0.0238	0.2167	0.4563
Education	-0.1916	0.0812	0.0092
Advertisement	-0.0344	0.1402	0.4032
Labour exchange	-0.0788	0.1475	0.2965
Firm size	0.0210	0.0765	0.3920
Psych. test	-0.1794	0.1724	0.1490
Mean log.likelihood :		-18.8769	
Number of observations :		567	
Mean λ :		0.2684	
Mean μ :		0.0191	
Mean θ :		0.0046	
Expected number of applicants :		13.79	
Expected duration :		216.83	
Mean λ , first 20 days :		0.467	
Mean λ , between 20 and 50 days :		0.334	
Mean λ , over 50 days :		0.164	

4 Conclusion

The estimation results of both models seem to justify the assumption of a non-sequential search and imply that application periods are short in relation to selection periods.

We find that arrival rates are high in comparison with attrition rates and a rapidly declining time dependence parameter on the arrival rate in the second model, which supports the hypothesis that most applicants arrive shortly after the vacancy has been posted, and that vacancy durations are mainly due to the selection of a suitable employee. These effects intensify for higher qualified jobs. For rising levels of required education we find smaller arrival rates, which means less applicants, smaller attrition, which implies a longer selection period, and longer mean vacancy durations.

If we compare both models the results of the second one seem to have a more plausible interpretation, as we can drop the arbitrarily fixed application period, and also explain the fact that applicants numbers are not automatically declining, if the vacancy has been open for a long time. In the second model we find that most applicants arrive during the first six weeks of the vacancy which suits well to the mean application period of 47 days in the first model and we can say that the application period is about one fifth of the whole vacancy duration. Even though it seems to be more likely that firms choose a fixed time only to collect applications and then select from this pool, the data suggest that this strategy is not strictly followed.

5 Appendix

Given, that applicants arrive according to a Poisson process, with parameter λt , for $t \in [0, T]$, and drop out binomially distributed, with probability $p = e^{-\mu t}$, for $t \in [T, T + S]$ we can calculate the probability, that we observe k applicants at time $T + S$, as the probability of all events, where $n \geq k$ applicants arrived up to T , and k applicants stayed until $T + S$.

$$\begin{aligned}
 P(X_{T+S} = k | T, S, X_0 = 0) &= \\
 &= \sum_{n=k}^{\infty} \frac{(\lambda T)^n}{n!} e^{-\lambda T} \binom{n}{k} e^{-\mu S k} (1 - e^{-\mu S})^{n-k} \\
 &= \frac{1}{k!} e^{-\mu S k} e^{-\lambda T} (\lambda T)^k \sum_{n=k}^{\infty} (\lambda T)^{n-k} \frac{1}{(n-k)!} (1 - e^{-\mu S})^{n-k} \\
 &= \frac{1}{k!} e^{-\mu S k} e^{-\lambda T} (\lambda T)^k \exp(\lambda T (1 - e^{-\mu S})) \\
 &= \frac{(\lambda T)^k}{k!} \exp(-\mu S k - \lambda T e^{-\mu S}) \\
 &= \frac{(\lambda T e^{-\mu S})^k}{k!} \exp(-(\lambda T e^{-\mu S}))
 \end{aligned}$$

This is a Poisson distribution with parameter $\lambda T e^{-\mu S}$

Next we calculate the probability of observing k_2 applicants, given we had $k_1 \neq 0$ at $t = 0$. We do this via induction and start with $k_1 = 1$, $k_2 = k \geq 1$

$$\begin{aligned}
 P(X_{T+S} = k | T, S, X_0 = 1) &= \\
 &= \sum_{n=k-1}^{\infty} \frac{(\lambda T)^n}{n!} e^{-\lambda T} \binom{n+1}{k} e^{-\mu S k} (1 - e^{-\mu S})^{n+1-k} \\
 &= \frac{1}{k!} e^{-\mu S k} e^{-\lambda T} (\lambda T)^{k-1} \sum_{n=k-1}^{\infty} \frac{n+1}{(n+1-k)!} (\lambda T)^{n+1-k} (1 - e^{-\mu S})^{n+1-k} \\
 &= \frac{1}{k!} e^{-\mu S k} e^{-\lambda T} (\lambda T)^{k-1} \sum_{m=0}^{\infty} \frac{m+k}{m!} (\lambda T)^m (1 - e^{-\mu S})^m \\
 &= \frac{(\lambda T)^{k-1}}{k!} e^{-\mu S k} e^{-\lambda T} (k + \lambda T (1 - e^{-\mu S})) \exp(\lambda T (1 - e^{-\mu S})) \\
 &= \exp(-\lambda T e^{-\mu S}) \frac{(\lambda T e^{-\mu S})^{k-1}}{(k-1)!} e^{-\mu S} \\
 &+ \exp(-\lambda T e^{-\mu S}) \frac{(\lambda T e^{-\mu S})^k}{k!} (1 - e^{-\mu S})
 \end{aligned}$$

Here either $k - 1$ applicants arrive according to the Poisson process derived before, and the applicant from the beginning survives, or k applicants arrive, and the applicant from the beginning drops out.

This suggests that the probability for general k_1 and k_2 is given by

$$\begin{aligned}
 P(X_{T+S} = k_2 | T, S, X_0 = k_1) &= \\
 &= \exp(-\lambda T e^{-\mu S}) \\
 &\quad \sum_{n=\max\{0, k_1 - k_2\}}^{k_1} \frac{(\lambda T e^{-\mu S})^{k_2 - k_1 + n}}{(k_2 - k_1 + n)!} \binom{k_1}{n} e^{-\mu S (k_1 - n)} (1 - e^{-\mu S})^n
 \end{aligned}$$

Every number n of the k_1 applicants of the beginning may survive, except in the case, where $k_2 < k_1$, the difference $k_1 - k_2$ cannot survive. (This explains why we sum over n from $\max\{0, k_1 - k_2\}$ to k_1 .) The residual $k_2 - (k_1 - n)$ applicants again arrive according to the Poisson process.

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