IHS Economics Series
Working Paper 37
September 1996

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Reihe Ökonomie / Economics Series No. 37

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Abstract

An oligopolistic market with vertical product differentiation is parametrized in cost parameters. This allows me to study the impact of the technology of the firms (cost parameters) on market structure, conduct, and performance. Firms which differ only by the order of the sequential move to choose a quality use sophisticated entry deterring or entry accommodating strategies.

I show that infinitesimal changes in the cost parameters can generate discontinuities in market profits. In particular, higher entry costs can lead to a more competitive outcome.

Keywords
Entry deterrence, vertical product differentiation, sequential quality choice, barriers of entry

JEL-Classifications
L13

Comments
Financial support from the Instituto Valenciano de Investigaciones Económicas (IVIE) and EU Human Capital and Mobility Program is gratefully acknowledged.

1 Introduction

How do barriers of entry affect the outcomes of an industry? In the empirical literature the importance of sunk cost as barriers of entry is emphasized. Since the book by Bains (1956) entry barriers are seen as an important factor of the competitiveness of a market. This leaves open the question as to whether incumbent firms actively think of entry deterrence or they simply benefit from blockaded entry. Work by Smiley (1988) and Bunch and Smiley (1992) suggests that an important number of firms try to choose strategies in order to discourage further entry and that these firms primarily use credible strategies.

Entry deterring strategies seem to be particularly relevant in highly concentrated markets. Advertising, protection against imitators, and entry deterring product specifications seem to be the most often used instruments.

The paper contributes to the theoretical literature on the product specification of firms under the threat of entry. Entry-deterring strategies have received some attention in the recent literature (for early work see Hay, 1976, and Schmalensee, 1978). Entry deterrence and accommodation is a strategic decision of a firm which makes it necessary for the firm to make predictions about hypothetical behavior of established and potential competitors. In order to have clear-cut results, firms' decision making is formulated as if it is very sophisticated (see Prescott and Visscher, 1977, and Eaton and Wooders, 1985), which is formalized by the notion of subgame perfect equilibrium of an extensive game.

In models of product differentiation firms' strategies of product specification may be modified when an entry threat is introduced in the model. This affects the degree and nature of competition. Brand proliferation and product relocation by a monopolist under the threat of entry are analyzed by Schmalensee (1978) and Bonanno (1987) (see also Canoy and Peitz, 1995) who argue that the number and the positioning of the goods in the product space can be affected by entry considerations. However, in the case of an imitating entrant, the monopolist may have an incentive to withdraw the imitated product when the possibility of exit is introduced into the model (Judd, 1985).

Alternatively, not a single firm but a sample of single-product firms may choose their strategies in order to deter further entry (Hung and Schmitt, 1988, Donnenfeld and Weber, 1995).[1] Here, imitators are no threat because the imitated firm has no reason

[1] An analysis with multi-product oligopolists seems desirable, see Champsaur and Rochet (1989) for duopoly models under blockaded entry. In the model which I have chosen firms do not want to offer quality ranges in a duopoly under blockaded entry (see Champsaur and Rochet, 1990) but might do so in order to deter entry if exit is prohibited.

to withdraw its imitated product. Hung and Schmitt (1988) stressed the importance of entry costs in order to deter entry. This issue has been further investigated by Donnenfeld and Weber (1995).[2]

Following Donnenfeld and Weber (1995), I will also analyze a model in which two single-product incumbents face the threat of entry in a vertical market. My focus is on the impact of the cost structure on the outcome of the game with sequential quality choice. Imposing sequential quality choice results in unique equilibrium outcomes. I will look at cost structures which are ruled out in models with natural oligopolies. Allowing for variable costs of quality makes the model much richer than previous work because variable costs of quality represent how symmetric or asymmetric a market is. The asymmetry of the market relates to the relative profitability of the two sides of the market. The model is fully specified and completely solved.

In line with the literature I obtain that high entry costs discourage entry. As in Donnenfeld and Weber (1992), incumbency does not necessarily lead to higher profits in comparison to the entrant. Entry deterrence can only be achieved by reduced product differentiation which affects the competitiveness of the market (holds also in Donnenfeld and Weber, 1995). A new result is that, under accommodated entry, incumbents can possibly increase profits when they cooperate in their product specification.

The main result of the paper is to establish that small changes in the cost parameters can explain large changes in aggregate and average profits. This can be shown for changes of the variable cost of quality which may not affect the number of active firms.[3] The most surprising result of the paper seems to be that higher entry costs can lead to a more competitive outcome, i.e. an increase in the sunk cost which leads to deterred entry causes lower price-cost margins for both incumbents as well as average profits if the market is sufficiently asymmetric. This contradicts the view that higher barriers of entry necessarily hurt competition.

Underlying this result is the strategic reasoning of the firms who, in equilibrium, are not going to obtain the same profits because the market is asymmetric. Incumbent 2 as the strategically strong player can impose his will to deter entry on incumbent 1 by producing a close substitute to the good of incumbent 1. This drives prices down for both incumbents but is profitable for incumbent 2 because he can largely increase his market share compared to the constellation under accommodated entry.

[2] In similar models, work has also looked at blockaded (Shaked and Sutton, 1982) and free entry (Donnenfeld and Weber, 1992, among others). All these analyses are placed in the framework of natural oligopolies.

[3] The result holds also when changing the sunk cost such that the number of active firms is affected - a rather trivial result. Alternatively, also small changes in demand parameters can generate large changes in aggregate and average profits.

Section 2 presents the model, in Section 3 I will provide some helpful lemmas on two-stage games, i.e. also the product specification is simultaneous. Section 4 analyzes the extensive game of sequential product specification and simultaneous price setting. In addition, it provides an interpretation as a delay supergame which is played in real time so that the comparative statics results of the extensive game translate into dynamics of an industry in the delay supergame. This interpretation seems to be helpful because in the "real" world an entry threat persists over time and long-term decisions (product specification) are taken while already competing in the market.

2 The Model

2.1 The Game between the Incumbents and the Potential Entrant

Consider a market with three firms. Two firms will be called the *incumbents* because they have already entered the market, i.e. set up business. The third firm will be called the *potential entrant* because entrance will depend on the parameters of the model. The incumbents will be indexed by $I1$ and $I2$, respectively, while the potential entrant is indexed by E. I will analyze a market in which firms sequentially choose a quality level q_n, $n = I1, I2, E$, of their single good. Assuming sequential quality choice of the incumbents will result in unique equilibrium outcomes. Entering the market, a firm has to pay a fixed cost K which is sunk and independent of quality. I assume that entering and choosing a quality are simultaneous decisions for the entrant, i.e. a firm which enters has to decide upon the product specification at this point.

Firms, which are in the market with a chosen quality of their good, simultaneously set the price of their respective goods. I will analyze an extensive game with perfect information and simultaneous moves of the following form:

(1) Incumbents' quality choice.
(1A) Firm $I1$ chooses q_{I1}.
(1B) Firm $I2$ chooses q_{I2}.
(2) The potential entrant decides whether to enter. If she enters she chooses q_E and incurs the fixed cost K.
(3) Firms set prices (p_{I1}, p_{I2}, p_E).

A delay supergame interpretation of this game will be given at the end of Section 4. Next I will describe the vertical market. This will allow me to determine the payoffs for any strategy profile.

2.2 The Model of the Vertical Market

Goods are differentiated by quality q. Firm n sets its quality q_n and its price p_n. There are up to 3 firms in the market. The quality of a good is measured by a real number $q_n \in [\underline{q}, \bar{q}] \subset [0, \infty)$. The goods are ordered such that $q_1 < q_2 < q_3$. In such a market there are customers who only differ in the income they have. First, individual customers are analyzed, and then market demand is derived.
A customer of type $\theta \in [\underline{\theta}, \bar{\theta}] \subset [0, \infty)$ has a conditional utility function for good n

$$u_n = x_0 + \theta(q_n - \bar{q})$$

where the Hicksian composite commodity is denoted by x_0; its price is normalized to 1. If a customer does not buy in the market his utility is $u_0 = x_0 - R - \theta\bar{q}$. The price vector is denoted by p, the vector of qualities by q. From the context it will be clear when the vectors are points in $\Re^2$ or $\Re^3$.
Income of a customer of type θ is defined as $y = R + \theta\bar{q}$ which is the reservation price of a customer of type θ for a unit of the good with the highest possible quality. Thus customers with a higher income care more about quality, which is expressed by the valuation of quality θ and there is a linear mapping from the income of a customer to its type. Note that y can more generally be interpreted as a share of income. It is only important that the relevant budget for the customer is greater or equal to $R + \theta\bar{q}$. Consequently, one has the indirect utility function

$$v(p, y; \theta) \quad = \quad \max[0, \max_n v_n(p, y; \theta)]$$

where $v_n(p, y; \theta) = R - p_n + \theta q_n$ is the conditional indirect utility function of type θ. This kind of indirect utility function was first proposed by Mussa and Rosen (1978) and has been used in several papers on vertical product differentiation (see e.g. Anderson, de Palma, and Thisse, 1992). The utility presentation of such a function is analyzed in Peitz (1995).[4]
The description of a good by (p_n, q_n) can be translated into the (x_0, q)-space because $x_0 = y - p_n$ if a customer buys good n and spends all his income. Figure 1 represents the indifference curves of a customer of type $\theta = 1$ and of type $\theta = 0$. Good n is chosen if the Euclidean distance between origin and indifference line is maximal on the set of goods.

[4] An alternative description of customers is provided by Gabszewicz and Thisse (1979, 1980) and Shaked and Sutton (1982,1983). Tirole (1988) uses the specification above with $R = 0$.

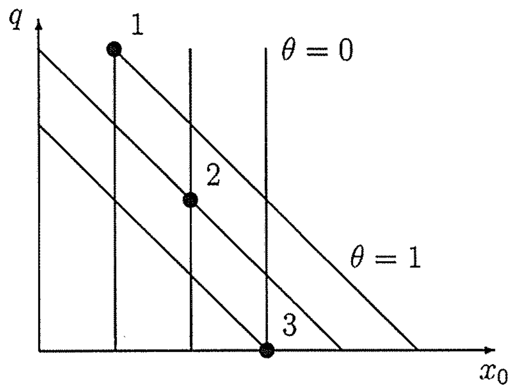

Figure 1: Preferences in a Vertically Differentiated Market.

If there is heterogeneity among customers, there is positive demand for all goods in the differentiated market on a set of price vectors with non-empty interior.
The whole population of customers is assumed to be of mass 1. Customers are assumed to be uniformly distributed over θ with density g. The support of the density is $[\underline{\theta}, \bar{\theta}] = [0,1]$. This reduces the number of parameters without affecting any of the qualitative results. A customer of type θ is indifferent whether to buy good n or to buy good $n+1$ if

$$-p_n + \theta q_n = -p_{n+1} + \theta q_{n+1}.$$

The marginal customer between good n and good $n+1$, $n = 1, 2$, is denoted by θ^n where

$$\theta^n = \frac{p_{n+1} - p_n}{q_{n+1} - q_n}.$$

Customers between θ^1 and θ^2 buy good 2. When every customer buys one unit and when there is strictly positive demand for each good n, $n = 1, 2, 3$, market demand is of the following form:

$$X_1(p,q) = \int_{\underline{\theta}}^{\theta^1} g(\theta)\, d\theta = \frac{p_2 - p_1}{q_2 - q_1}, \tag{1}$$

$$X_2(p,q) = \int_{\theta^1}^{\theta^2} g(\theta)\, d\theta = \frac{p_3 - p_2}{q_3 - q_2} - \frac{p_2 - p_1}{q_2 - q_1}, \tag{2}$$

$$X_3(p,q) = \int_{\theta^2}^{\bar{\theta}} g(\theta)\, d\theta = 1 - \frac{p_3 - p_2}{q_3 - q_2}. \tag{3}$$

Firm n produces its good at unit costs cq_n, $c \in [0,1]$. Hence goods are more costly in production when they are of higher quality. The restriction $c \in [0,1]$ implies that,

for any number of firms with different qualities, each firm has a positive market share when prices equal marginal costs. This is a property shared by models of horizontal product differentiation and distinct from models with natural oligopolies.[5] At $c = \frac{1}{2}$, both sides of the market have the same profitability and the market is symmetric. The market becomes more asymmetric when $|c - \frac{1}{2}|$ increases. $|c - \frac{1}{2}|$ is taken as the measure of the asymmetry of the market.
Firms maximize profits $\pi_n = (p_n - cq_n)X_n(p,q)$. Note that an entrant has to pay the fixed costs K from these profits. The equilibrium concept which will be used is subgame perfect Nash in pure strategies.

3 Simultaneous Move Games

3.1 Duopoly Equilibrium in Prices for Given Qualities

For two firms in the market there exists a marginal customer θ^1 who is indifferent between good 1 and good 2.

$$\theta^1 = \frac{p_2 - p_1}{q_2 - q_1}$$

Market demand for interior points is equal to

$$\begin{aligned} X_1(p) &= \theta^1 - \max\{0, \frac{p_1 - R}{q_1}\} \\ X_2(p) &= 1 - \theta^1 \end{aligned}$$

The demand function for the low-quality good is kinked. Only customers with low income may not buy in the market. With the following lemma the existence of a unique equilibrium in prices is established. The lower bound on R guarantees that $p_1 < R$ holds in equilibrium. The other cases $p_1^* > R$ and $p_1^* = R$ are analyzed in Canoy and Peitz (1995).

Lemma 1.
For $R \geq \frac{1}{3}(q_2 - q_1) + \frac{1}{3}c(q_2 + 2q_1)$ there exists a duopoly equilibrium in prices for $q_1 < q_2$ which is uniquely determined by

$$p_1^* = \frac{1}{3}(q_2 - q_1) + \frac{1}{3}c(q_2 + 2q_1)$$

[5] The necessary and sufficient condition for natural oligopolies in the model is $c \notin [\underline{\theta}, \bar{\theta}] = [0,1]$. Hence for $c > 1$ only a finite number of firms can be supported in the market independent of the fixed cost. See Shaked and Sutton (1983) on natural oligopolies and Anderson, de Palma, and Thisse (1992) for a discussion. Cremer and Thisse (1991) showed that every model of horizontal product differentiation can be rewritten as a model of vertical product differentiation.

$$p_2^* = \frac{2}{3}(q_2 - q_1) + \frac{1}{3}c(2q_2 + q_1)$$

Proof. Follows from straightforward computation. □

3.2 Quality Choice in Duopoly

For later use it will be helpful to analyze the following two-stage game:
Stage 1: Duopolists simultaneously choose qualities.
Stage 2: They compete in prices.
Profit functions at stage 1 depend on q_1 and q_2 and are denoted by $\tilde{\pi}_1(q_1, q_2)$ and $\tilde{\pi}_2(q_1, q_2)$.

$$\tilde{\pi}_1(q_1, q_2) = \frac{1}{9}(1 + c)^2(q_2 - q_1) \tag{4}$$

$$\tilde{\pi}_2(q_1, q_2) = \frac{1}{9}(2 - c)^2(q_2 - q_1) \tag{5}$$

As it turns out, continuation profits at stage 1 only depend on the quality difference $q_2 - q_1$ between the two goods. It will be assumed that the market is covered in equilibrium in prices for any quality choice.

(A) $R > \frac{1}{3}(\bar{q} - \underline{q}) + \frac{1}{3}c(\bar{q} + 2\underline{q})$.

The standard result in models of vertical differentiation is maximal differentiation of duopolists when all customers buy in the market, i.e. $(q_1, q_2) = (\underline{q}, \bar{q})$.

Lemma 2.
If (A) is satisfied and $q_1 < q_2$ duopolists choose maximal differentiation $(\underline{q}, \bar{q})$ in the unique subgame perfect equilibrium.
For $c < 0.5$, $\bar{q}$ is more profitable,
for $c = 0.5$, both sides of the market have the same profitability,
for $c > 0.5$, $\underline{q}$ is more profitable.

Proof. At stage 1 firm 1's profit function $\tilde{\pi}_1(q_1, q_2)$ is decreasing in q_1. The maximum is achieved at $q_1 = \underline{q}$ for given q_2 under the restriction $q_1 < q_2$. Analogously, for firm 2. The restriction $q_1 < q_2$ only selects the index. Straightforward computations show which side of the market is more profitable. □

3.3 Oligopoly Equilibrium in Prices for Given Qualities

In this subsection the equilibrium in prices is determined when there are three firms with given qualities $q_1 < q_2 < q_3$.[6] Market demand is given by equations (1) to (3). Since competition is more intense than in the duopoly, the market will always be covered in equilibrium if (A) is satisfied.

Lemma 3.
If (A) is satisfied there exists a three-firm equilibrium in prices for $\underline{q} \leq q_1 < q_2 < q_3 \leq \bar{q}$ which is uniquely determined by

$$p_1^* = \frac{1}{2}c(q_1 + q_2) + \frac{(q_3 - q_2)(q_2 - q_1)}{6(q_3 - q_1)} \tag{6}$$

$$p_2^* = cq_2 + \frac{(q_3 - q_2)(q_2 - q_1)}{3(q_3 - q_1)} \tag{7}$$

$$p_3^* = \frac{1}{2}c(q_2 + q_3) + \frac{(q_3 - q_2)(q_2 - q_1)}{6(q_3 - q_1)} + \frac{q_3 - q_2}{2} \tag{8}$$

Proof. Remark first that in equilibrium each firm has positive demand. The argument is as follows: assume two firms have positive demand. In order to be profit maximizing, prices must be equal to or above marginal cost. But then the third firm can set a price above marginal costs selling to a set of customers of positive demand. Consequently, an equilibrium must satisfy the first-order conditions of profit maximization where demand is given by equations (1) to (3). This leads to the following equation system.

$$\begin{array}{llll} -2p_1 & +p_2 & & = -cq_1 \\ (q_3 - q_2)p_1 & -2(q_3 - q_1)p_2 & +(q_2 - q_1)p_3 & = -(q_3 - q_1)cq_2 \\ & p_2 & -2p_3 & = -(q_3 - q_2) - cq_3 \end{array}$$

For the unique candidate of an equilibrium in prices, prices as given by (6) to (8) follow. Since firms profit functions are quasi-concave in its own price and second-order conditions are satisfied, (p_1^*, p_2^*, p_3^*) is the unique maximizer. □

3.4 Quality Choice in Oligopoly

Again I analyze the quality-then-price game. Lemma 4 states the best responses at stage 1 for each firm given the quality choices of the competitors. Lemma 5 characterizes the unique equilibrium of the two-stage game under the strategy restriction

[6] The existence of a unique equilibrium in prices can be shown provided that the customer density is log-concave (see Caplin and Nalebuff, 1991). However, in this paper I want to work with an analytic solution for the equilibrium prices and restrict therefore the customer density to be uniform.

$q_1 < q_2 < q_3$ and Lemma 6 provides the marginal costs of quality for which this is an equilibrium of the unrestricted game.
Profits at stage 1 are

$$\tilde{\pi}_1(q_1, q_2, q_3) = (q_2 - q_1)\left(\frac{1}{2}c + \frac{q_3 - q_2}{6(q_3 - q_1)}\right)^2 \quad (9)$$

$$\tilde{\pi}_2(q_1, q_2, q_3) = \frac{1}{9}\frac{(q_3 - q_2)(q_2 - q_1)}{q_3 - q_1} \quad (10)$$

$$\tilde{\pi}_3(q_1, q_2, q_3) = (q_3 - q_2)\left(\frac{1}{2} - \frac{1}{2}c + \frac{q_2 - q_1}{6(q_3 - q_1)}\right)^2 \quad (11)$$

Lemma 4.
Suppose (A) is satisfied and qualities have to satisfy $q_1 < q_2 < q_3$.
(1) Given q_2, q_3 firm 1 sets $q_1 = \underline{q}$.
(2) Given q_1, q_3 firm 2 sets $q_2 = \frac{1}{2}(q_1 + q_3)$.
(3) Given q_1, q_2 firm 3 sets $q_3 = \bar{q}$.

Proof. Since $\frac{\partial \tilde{\pi}_1}{\partial q_1} < 0$ and $\frac{\partial \tilde{\pi}_3}{\partial q_3} > 0$, (1) and (3) hold. $\frac{\partial \tilde{\pi}_2}{\partial q_2} = 0$ is equivalent to $q_2 = \frac{1}{2}(q_1 + q_2)$. This is a global maximizer on $[q_1, q_3]$. □

Lemma 5.
Suppose (A) is satisfied and qualities have to satisfy $q_1 < q_2 < q_3$.
The three firms choose $(q_1, q_2, q_3) = (\underline{q}, \frac{1}{2}(\underline{q} + \bar{q}), \bar{q})$ in the unique subgame perfect equilibrium.
Profits are ranked according to
$3 \succ 2 \succ 1$ when $c < \frac{1}{6}(2\sqrt{2} - 1)$,
$3 \succ 1 \succ 2$ when $\frac{1}{6}(2\sqrt{2} - 1) < c < \frac{1}{2}$,
$1 \succ 3 \succ 2$ when $\frac{1}{2} < c < \frac{1}{6}(7 - 2\sqrt{2})$,
$1 \succ 2 \succ 3$ when $\frac{1}{6}(7 - 2\sqrt{2}) < c$.

Proof. The quality choice follows from Lemma 4. Substituting the quality choices of parts (1) and (3) into part (2) of Lemma 4 gives $q_2 = \frac{1}{2}(\underline{q} + \bar{q})$. The profit ranking is obtained by straightforward computations. □

Remark that in Lemma 5 I studied the restricted game with $q_1 < q_2 < q_3$. In the unrestricted game each firm's profit function at stage 1 (continuation profits) is no longer quasi-concave in its quality. Only for intermediate values of c there exists a subgame perfect equilibrium of the unrestricted game. This implies that for three firms one cannot analyze natural oligopolies in a two-stage game with quality choice in the first stage and price setting in the second stage.

Lemma 6.
If (A) is satisfied and $\frac{1}{6} \leq c \leq \frac{5}{6}$ three firms choose $(\underline{q}, \frac{1}{2}(\underline{q}+\bar{q}), \bar{q})$ in the unique subgame perfect equilibrium. For $c \notin [\frac{1}{6}, \frac{5}{6}]$ there does not exist a subgame perfect equilibrium.

Proof. It follows from Lemmas 4 and 5 that $(\underline{q}, \frac{1}{2}(\underline{q}+\bar{q}), \bar{q})$ is the unique candidate to occur in a subgame perfect equilibrium of the unrestricted game. It remains to be shown that the firm which has chosen $\underline{q}$ cannot increase profits by producing a quality inside $[\frac{1}{2}(\underline{q}+\bar{q}), \bar{q}]$ and the firm which has chosen $\bar{q}$ cannot increase profits by producing a quality inside $[\underline{q}, \frac{1}{2}(\underline{q}+\bar{q})]$. The firm relocating from $\underline{q}$ to $(\frac{1}{2}(\underline{q}+\bar{q}), \bar{q})$ makes profits $\tilde{\pi}_2(\frac{1}{2}(\underline{q}+\bar{q}), q_2, \bar{q}) \leq \frac{1}{36}(\bar{q} - \frac{\underline{q}+\bar{q}}{2}) = \frac{1}{72}(\bar{q} - \underline{q})$. For $\underline{q}$ to be a global maximizer, $\tilde{\pi}_1(\underline{q}, \frac{1}{2}(\underline{q}+\bar{q}), \bar{q})$ must be at least as high.

$$\left(\frac{1}{6}+c\right)^2 \frac{1}{8}(\bar{q}-\underline{q}) \geq \frac{1}{72}(\bar{q}-\underline{q})$$

This reduces to $c \geq \frac{1}{6}$. Analogously, if $c \leq \frac{5}{6}$ the firm at $\bar{q}$ cannot increase its profits by producing a quality between the two other firms. □

The non-existence under simultaneous quality choice might seem disturbing. However, as it is shown in the subsection on accommodated entry, a subgame perfect equilibrium exists for $c \notin [\frac{1}{6}, \frac{5}{6}]$ when quality choice is sequential.

4 Main Results

This section contains the main results on the model presented in Section 2. It is always implicitly assumed that (A) is satisfied which implies that, in equilibrium, all customers buy in the market for any parameter constellation c, K.

4.1 Blockaded Entry

Blockaded entry prevails when the potential entrant cannot make profits although the incumbents ignored the entry threat when they had to choose the quality of their goods.

Proposition 1.
If $K \geq \frac{1}{36}(\bar{q} - \underline{q})$ entry is blockaded. The potential entrant does not enter and $(q_{I1}, q_{I2}) = (\bar{q}, \underline{q})$ for $c < \frac{1}{2}$, $(q_{I1}, q_{I2}) \in \{(\underline{q}, \bar{q}), (\bar{q}, \underline{q})\}$ for $c = \frac{1}{2}$, and $(q_{I1}, q_{I2}) = \underline{q}, \bar{q})$ for $c > \frac{1}{2}$ in the unique subgame perfect equilibrium.

Proof. From Lemma 2 it follows that incumbents choose maximal differentiation when they ignore the entry threat. This result holds under sequential and simultaneous quality choice. In this case the potential entrant can at most make $\frac{1}{36}(\bar{q} - \underline{q})$ when she enters (see Lemma 4). Incumbent 1 chooses the side of the market which is more profitable. □

Note that the critical entry cost K above which entry becomes blockaded is independent of c. Consequently, the entry decision of the potential entrant can only be affected by the marginal cost of quality c if incumbents are using entry deterrents strategically.

4.2 Accommodated Entry

In this subsection I will describe the outcome in the case of sequential quality choice in the order $I1$, $I2$, E. This means it will be assumed that entry does occur. However, in this subsection it may well be that the equilibrium profit the entrant will make is greater than the fixed cost and still entry does not occur in the subgame perfect equilibrium because the incumbents are able and willing to deter entry (see below). But for the fixed costs sufficiently small, entry deterrence is impossible. Lemmas 5 and 6 can be used to show the following proposition. The profit ranking in Lemma 5 determines the position which firms will take in the quality space.

Proposition 2.
Suppose $K = 0$. For $c \in [\frac{1}{6}, \frac{5}{6}]$, the unique subgame perfect equilibrium with three firms and sequential quality choice is characterized by the qualities $\underline{q}, \frac{1}{2}(\underline{q} + \bar{q}), \bar{q}$ where $I1$ produces the most profitable and E the least profitable quality level, i.e. the profit ranking is $I1 \succ I2 \succ E$.

Proof. Lemma 6 states that the above characterization holds in the game with simultaneous quality choice. It also holds under sequential quality choice. Clearly, incumbent 1 as the first mover can guarantee himself the highest profit and the entrant will get the smallest profit. Lemma 5 provides the profit ranking which determines which firm produces which quality. □

Note that the profit ranking is always $I1 \succ I2 \succ E$ which is in contrast to the result by Donnenfeld and Weber (1992). It shows the importance of the asymmetry of the market for the relative performance of the firms in a market with sequential product

specification. I will now show that the profit ranking of Donnenfeld and Weber (1992), which is $I1 \succ E \succ I2$, holds in an "extremely" asymmetric market.
I will provide an answer to what happens in the case $c \notin [\frac{1}{6}, \frac{5}{6}]$. Suppose $c < \frac{1}{6}$. In the unique subgame perfect equilibrium either one of the two following quality choices will result.
Possibility 1: "fighting" incumbent 2. Incumbent 2 does not leave the center position for the entrant. To be able to keep this position incumbent 2 must reposition his good in the quality space, i.e. $q_{I1} = \bar{q}$, $q_{I2} > \frac{1}{2}(\underline{q} + \bar{q})$ such that the equilibrium profit the entrant makes at $(q_{I1}, q_{I2}, q_E) = (\bar{q}, q_{I2}, \underline{q})$ is equal to her profit at $(\bar{q}, q_{I2}, \frac{1}{2}(q_{I2} + \bar{q}))$, and $q_E = \underline{q}$. By equations (9) and (10), the quality of incumbent 2, q_{I2}, is determined by

$$\frac{1}{36}(\bar{q} - q_{I2}) = (q_{I2} - \underline{q}) \left(\frac{1}{2}c + \frac{\bar{q} - q_{I2}}{6(\bar{q} - \underline{q})} \right)^2$$

Define $\lambda = \frac{\bar{q} - q_{I2}}{\bar{q} - \underline{q}}$. Then the above equality reduces to

$$\lambda = (1 - \lambda)(3c + \lambda)^2$$

which can be easily solved for λ. For any c, there exists a solution. There is a unique admissible solution $\lambda \in [0, \frac{1}{2}]$ for $c \leq \frac{1}{6}$. Remark that λ and hence q_{I2} increases with c.
Possibility 2: "passive" incumbent 2. Incumbent 2 accepts the least profitable position among the three firms, $q_{I1} = \bar{q}$, $q_{I2} = \underline{q}$, $q_E = \frac{1}{2}(\underline{q} + \bar{q})$.
There exists a critical c, which is denoted by c_0, below which $I2$ makes higher profits in possibility 2 than under possibility 1. Hence the result of Donnenfeld and Weber (1992) that $E \succ I2$ holds in the model for $c \in [0, c_0)$. The critical c_0 is approximately 0.067. For larger c which remain below $\frac{1}{6}$, incumbent 2 will produce some intermediate quality above $\frac{1}{2}(\underline{q} + \bar{q})$ such that the entrant does not want to produce a higher quality than incumbent 2. See Figure 2 for the chosen qualities of incumbent 2. Analogous results are obtained in the case $c > \frac{5}{6}$. The critical marginal cost of quality which separates the profit ranking $I2 \succ E$ from $E \succ I2$ is denoted here by c^0.
The findings of this subsection are summarized by Figure 2 which represents the quality choice of each of the firms depending on the marginal cost of quality. For $c \in [c_0, c^0]$ the profit ranking is $I1 \succ I2 \succ E$. For $c \in [0, c_0) \cup (c^0, 1]$, it is $I1 \succ E \succ I2$. Remark that $c = 0, 1$ are the borders to markets with natural oligopolies. With this it is shown that the result by Donnenfeld and Weber (1992) extends to models in which there are no natural oligopolies as long as the market is "extremely asymmetric". I call the market "moderately asymmetric" if $c \in [c_0, \frac{1}{6}) \cup (\frac{5}{6}, c^0]$. When $c \in [\frac{1}{6}, \frac{5}{6}]$ I speak of a "quite symmetric" market.
Incumbent 1's quality choice only depends on whether c is greater or less than $\frac{1}{2}$

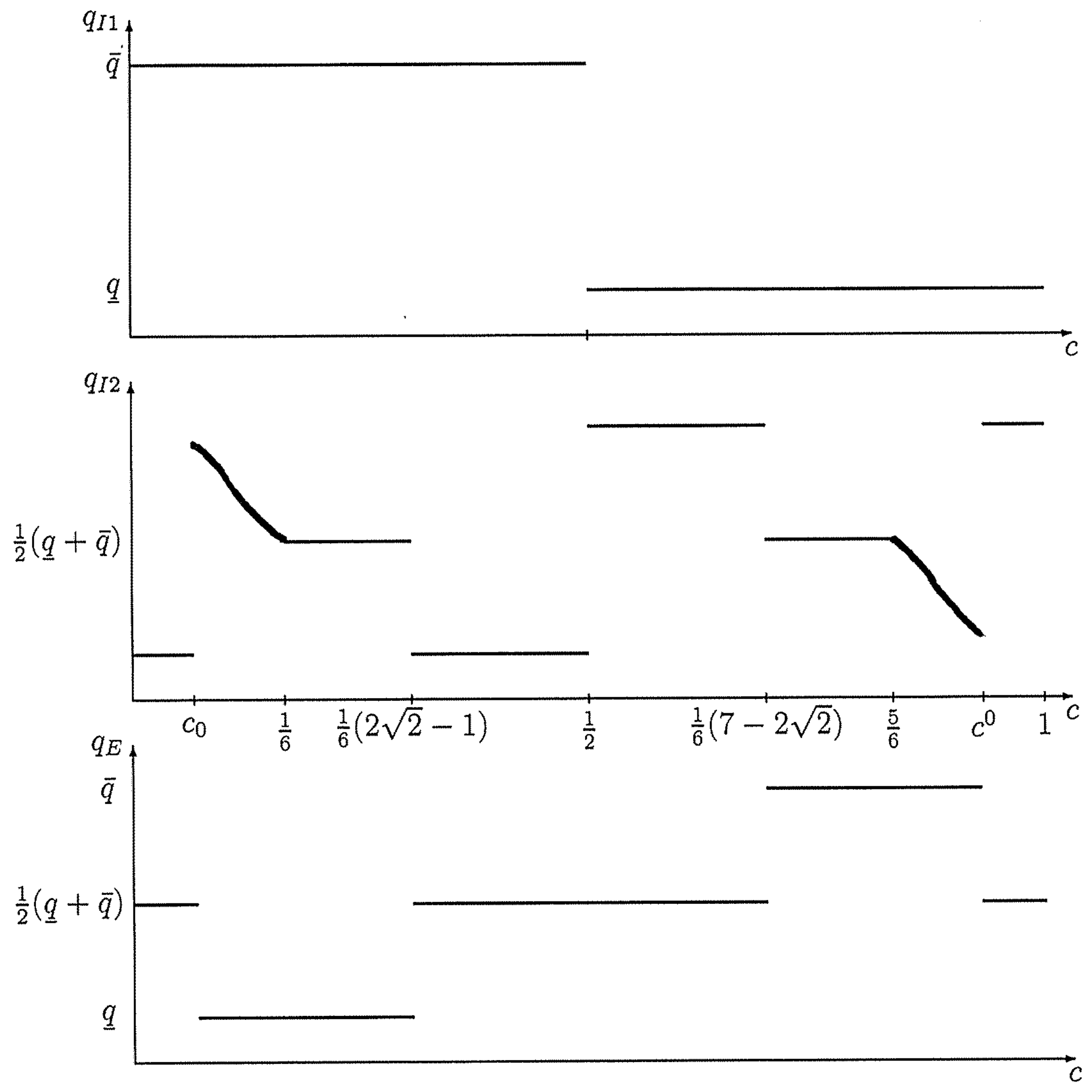

Figure 2: Quality Choice under Accommodated Entry.

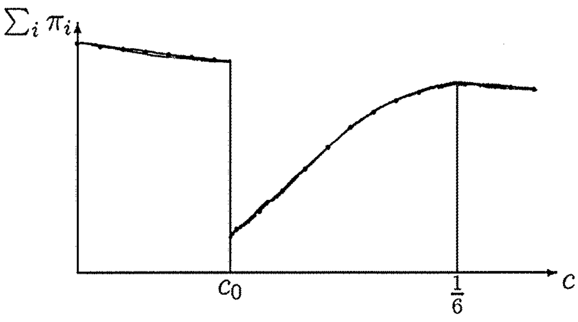

Figure 3: Aggregate Profits under Accommodated Entry.

and coincides with the quality choice under blockaded entry. Incumbent 2's quality choice coincides with the quality choice under blockaded entry on the interval $[\frac{1}{6}(2\sqrt{2}-1), \frac{1}{6}(7-2\sqrt{2}]$. In the interval $[c_0, c^0]$ he obtains the second highest profit. In order to achieve this he does not produce in the center between the two extreme qualities ("fighting" incumbent 2), whenever the market is moderately asymmetric. For $c \in [0, c_0) \cup (c^0, 1]$ incumbent 2 performs worse than the entrant but he cannot do better in absolute profits ("passive" incumbent 2) and ends up in the least profitable corner of the market.

These results lead to non-monotone comparative statics. Under accommodated entry aggregated (and mean) profits are discontinuous at the borders between extremely and moderately asymmetric markets, c_0 and c^0. When c rises above c_0, it becomes profitable for incumbent 2 to produce a quality close to $q_{I1} = \bar{q}$ and increased competition between the two incumbents depresses aggregate profits. Aggregate profits are plotted in Figure 3. It shows that a sudden change of profits does not necessarily mean that the behavior of the market participants or the market fundamentals (parameters of the model) changed "drastically".[7] Hence a puzzle that profits in an industry collapsed but behavior of the market participants and the market fundamentals seemed to be rather unchanged can be explained by strategic interaction. The dynamic interpretation of comparative statics results is presented below.

Incumbents could avoid their loss in aggregate profits when c exceeds c_0 or falls below c^0 if they cooperated at decision points (1A) and (1B). However, in order to cooperate they must be able to write a binding contract in which incumbent 1 compensates incumbent 2 for producing either $\bar{q}$ or $\underline{q}$.

[7] An example for a behavioral change would be the break-down of collusive behavior. A drastic change of the environment may be a supply or demand shock expressed by a discontinuous parameter change.

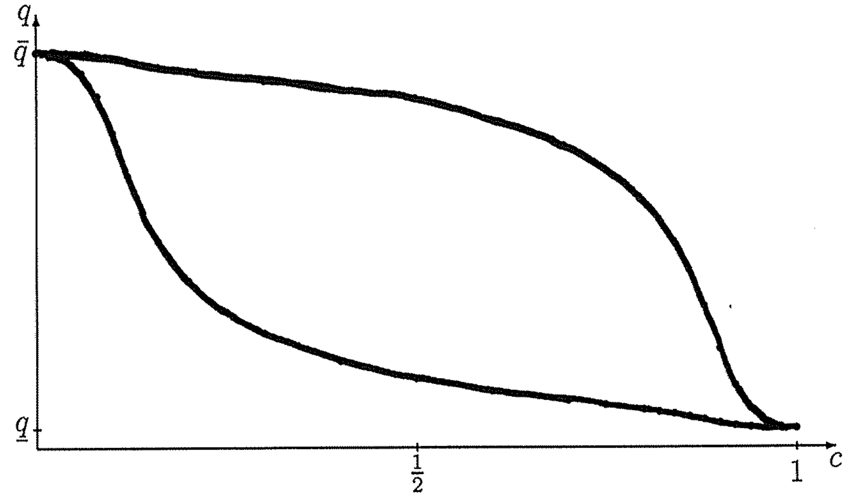

Figure 4: Maximal Squeezing of the Gap: Incumbents' Qualities

4.3 Entry Deterrence

When entry is not blockaded, i.e. $K < \frac{1}{36}(\bar{q} - \underline{q})$, it may be possible and profitable for the incumbents to deter entry by choosing qualities in the interior of $[\underline{q}, \bar{q}]$. Given q_{I1}, q_{I2}, the optimal quality choice of the entrant is either $\underline{q}$ or $\frac{1}{2}(q_{I1} + q_{I2})$ or $\bar{q}$ (see Lemma 4).

One has to distinguish between the possibility of entry deterrence and the profitability of such a strategy. In the case of natural oligopolies entry deterrence is always possible. In the boundary cases $c = 0$ and 1 both incumbents producing $\bar{q}$ or $\underline{q}$, respectively, destroy the possibility for the entrant to make positive profits in the subgame perfect equilibrium of any subgame. For $c \in (0, 1)$, entry deterrence is not always possible. Figure 4 shows incumbents' quality choices such that the entrant's profit is minimized, i.e. $\min_{q_{I1}} \min_{q_{I2}} \max_{q_E} \tilde{\pi}_E(q_{I1}, q_{I2}, q_E)$. This is what I call *maximal squeezing of the gap* (see also the appendix). Figure 5 then shows when entry deterrence is possible (region (1)-(4)). When fixed costs are below the lower line, entry deterrence is not possible and entry has to be accommodated (region (0)). Above the straight line entry is blockaded (region (5)).

However, even if entry can be deterred, it is not necessarily in the interest of the incumbents to do so. With the next proposition it is shown that in a quite symmetric or in a moderately asymmetric market both incumbents indeed have higher profits under entry deterrence than under accommodated entry whenever entry deterrence is possible.

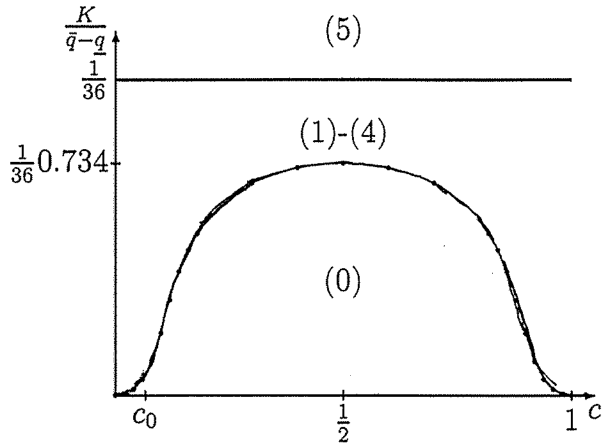

Figure 5: Entry Deterrence: (c, K)-diagram.

Proposition 3.
For $c \in [c_0, c^0]$, incumbents deter entry at any K for which entry deterrence is possible.

Proof. Computations show that duopoly profits under maximal squeezing of the gap are greater than profits under accommodated entry for both incumbents iff $c \in [c_0, c^0]$. □

If incumbents were choosing simultaneously (as in Donnenfeld and Weber, 1995), multiple equilibria with entry deterrence and accommodation exist. In addition, in the case of entry deterrence there would be a coordination problem among a continuum of equilibria for all c and for almost all K for which entry deterrence is profitable. In my model of sequential quality choice there is a unique subgame perfect equilibrium set, i.e. all equilibria are payoff equivalent. Hence one has a sharp separation between entry deterrence and accommodation. Also the continuum of entry deterring equilibria does not cause a coordination problem because incumbent 2 can base his decision on the quality choice of incumbent 1.
The payoff equivalence of all entry deterring equilibria for a given parameter constellation is due to the fact that continuation profits in the duopoly (equations (4) and (5)) and maximal continuation profits of the potential entrant with $q_E \in [\min\{q_{I1}, q_{I2}\}, \max\{q_{I1}, q_{I2}\}]$ (compare equation (10)) depend only on the quality difference $|q_{I1} - q_{I2}|$.
For $c \in [c_0, c^0]$, entry is deterred when $K > \pi_E$, which has been plotted in Figure 5. When the market is extremely asymmetric, it depends on the fixed costs K whether none, one, or both of the incumbents prefer entry deterrence to entry accommodation. For small K, the incumbents must produce close substitutes in order to deter entry.

c	(0)	(1)	(2)	(3)	(4)	(5)
0,1		+		+	+	+
$(0, 0.006] \cup [0.994, 1)$	+	+		+	+	+
0.0605, 0.9395	+	+	+	+	+	+
$[0.061, c_0) \cup (c^0, 0.939]$	+		+	+	+	+
$[c_0, c^0]$	+				+	+

Table 1: Marginal Cost of Quality and Competition.

This depresses duopoly profits. For parameters c, K such that at least one incumbent wants to deter entry, it is rather in the interest of incumbent 2 than of incumbent 1 to be in the single-product duopoly due to incumbent 2's poor performance under accommodated entry.

I distinguish the following six situations. Whenever entry deterrence is possible, one of situations (1) to (4) results in equilibrium:

(0) Entry deterrence is not possible and entry is accommodated.
(1) Both incumbents prefer not to deter entry and entry is accommodated.
(2) Incumbent 2 prefers entry deterrence, incumbent 1 entry accommodation. Incumbent 2 alone cannot deter entry and entry is accommodated.
(3) Incumbent 2 prefers entry deterrence, incumbent 1 entry accommodation. Incumbent 2 alone can deter entry and entry is deterred.
(4) Both incumbents prefer to deter entry and entry is deterred.
(5) Entry is blockaded.

In Appendix 1 I derive entry deterring qualities and the range of parameter values for which either of the situations results. For $c \in [0, 0.0605] \cup [0.9395, 1]$ situation (2) can be ruled out for any K. Hence incumbent 2 can impose his "will" on incumbent 1. Although incumbent 2 is the weak player in terms of profits, he is strategically the strong player.

In the neighborhood of $c = 0.0606$ depending on K each of the six situations can occur (for numerical values see Appendix 2).

For $c \in [0.0607, c_0) \cup (c^0, 0.9393]$ situation (1) can be ruled out for any K. For given c the sunk cost K which separate two situations from each other follow from the critical quality differences by straightforward calculations. Table 1 summarizes which situation can occur at c when varying K.

Remark that for any given parameter constellation all subgame perfect equilibria are payoff-equivalent, i.e. there is a unique subgame perfect equilibrium set. Furthermore, given any quality choice of incumbent 1 there exists a unique subgame perfect equilibrium of the subgame which follows. More importantly, for any parameter constellation there exists a subgame perfect equilibrium in which incumbent 1 chooses $\bar{q}$ if $c < c_0$ and $\underline{q}$ if $c > c^0$ (which is his quality choice under blockaded and accommodated entry) which implies that incumbent 1 does not need to do any strategic reasoning at all when choosing his quality: pick the more profitable corner of the market, wait, and see. Hence the observation made under accommodated entry, that it is only incumbent 2 who "often" has to revise his decisions when parameters change, holds also under deterred entry in an extremely asymmetric market. For $c \in [c_0, c^0]$, there exists a range of fixed cost such that both incumbents change product specifications.

Comparative statics properties are the following. Again I am interested in the profitability of the market. As a measure I take aggregated profits divided by the number of *active* firms, which is called average profits. I show that a smaller number of active firms does not imply less competition and higher average profits. For $c \in [0, c_0] \cup [c^0, 1]$, average profits drop discontinuously when the number of firms falls from 3 to 2. Hence higher sunk cost $K + \Delta K$ can lead to lower average profits. This result is due to the strategic strength of incumbent 2. Qualitatively the same result holds for average incumbents' profits. The sunk cost paid by the entrant is interpreted as the barrier of entry.

Theorem.

Whenever the market is extremely asymmetric, i.e. $c \in [0, c_0) \cup (c^0, 1]$, higher barriers of entry lead to lower average profits in the market on some range of values $K, \Delta K$.

Proof. For $c \in [0, c_0) \cup (c^0, 1]$, average equilibrium profits are

$$\frac{1}{N}\sum_{n=1}^{N} \pi_n = \begin{cases} \frac{1}{12}(c^2 - c + \frac{29}{36})(\bar{q} - \underline{q}) & \text{for } 0 \le K \le K^0 \\ 2((1+c)^2 + (2-c)^2)K & \text{for } K^0 < K < \frac{1}{36}(\bar{q} - \underline{q}) \\ \frac{1}{18}((1+c)^2 + (2-c)^2)(\bar{q} - \underline{q}) & \text{for } K \ge \frac{1}{36}(\bar{q} - \underline{q}) \end{cases}$$

where N is the number of active firms and K^0 is determined in Appendix 1. □

Remark that the price-cost margins $(p_n - cq_n)$ of both incumbents become smaller when the number of active firms falls from 3 to 2. This result holds since, when going to entry deterrence, incumbent 2 discontinuously increases his market share (scenario (3)). This is offset by a reduction in his price-cost margin. Due to closer substitutes incumbent 1 looses in market share and price-cost margin. Hence higher sunk cost can lead to more intense competition. Since prices and product specification have been increased nor necessarily all customers derive higher utility under more intense competition. The first example in Appendix 3 shows that at $c = 0.0606$ price-cost margins decrease when entry is deterred but there are customers of positive measure who obtain less utility. In markets close to markets with natural oligopolies all customers obtain higher utility. This is illustrated by the second numerical examples in Appendix 3 where $c = 0$. In this case higher barriers of entry lead to more intense competition which is beneficial to all customers.

4.4 The Delay Supergame Interpretation

In this subsection I will provide an interpretation for the extensive game in which decisions are made in real time. The game, to be played in discrete time, is called a delay supergame (see Selten, 1994). In a delay supergame all firms choose all their strategic variables simultaneously at each point in time. The choice of a particular variable at one point in time is called a decision. Each decision is implemented with a delay. At each point in time the full history of decisions is common knowledge.

Selten has provided an equivalence result between subgame perfect equilibria of multi-stage games (all players choose simultaneously at each stage) and corresponding delay supergames with finite horizons if all subgame perfect equilibria of the multi-stage game are payoff equivalent in each subgame. For the model of this paper I need a straightforward extension of Selten's result to extensive games. In order to avoid a lengthy formal analysis I will show the equivalence for the particular model informally. It is important to have a *unique subgame perfect equilibrium set* which holds when all subgame perfect equilibria are payoff equivalent.[8]

Let me supplement the model. I consider a market in which firms compete for customers in $T < \infty$ periods. Customers have been described in Section 2. In each period, there is a random draw of a large but finite number of customers out of the whole population. Since the random sample is large, the realization can be approximated by the original distribution function. Firms are assumed to know the distribution from which the sample is drawn but not the realization. Hence firms taking expected demand make on average only a small mistake. All decisions of the firms are based on the expectation. Since the random sample is of zero mass there are no intertemporal demand effects and one can stick to the atemporal derivation of aggregate demand in

[8]Note that Selten's work is related to e.g. Benoît and Krishna (1985) because a finitely repeated normal form game is a delay supergame of a one-stage game. Note also that I do not require payoff equivalence in each subgame.

Section 2.[9]

In each period there is a decision point at which the incumbents decide upon the quality and the price of their product and the potential entrant decides upon entry, quality, and price. At each decision point when the potential entrant decides to enter she incurs a fixed cost K^t which she has to pay in money units of the period t when the decision becomes effective.[10] Hence K^t is not an entry cost which is paid once but a stand-by cost. All decisions at time t are a vector $r^t \equiv (e^t, q^t_{I1}, q^t_{I2}, q^t_E, p^t_{I1}, p^t_{I2}, p^t_E) \in \{0,1\} \times [\underline{q}, \bar{q}]^3 \times \Re^3_+$ where e is a decision of the potential entrant where $e = 1$ stands for entered (stand-by), $e = 0$ for not entered (off).

The order of moves in the extensive form game imposes an exogenous delay structure: price changes can be implemented fast, all with a delay of δ_3 periods. The decision to enter is made by setting quality q_E. Decisions e and q_E are implemented with delay δ_2. Incumbents differ from each other and from the potential entrant in the time they need in order to implement a quality change of their goods. Incumbent 1 has a delay δ_{1A} and incumbent 2 delay δ_{1B} with $\delta_{1A} > \delta_{1B} > \delta_2$. Furthermore, it is assumed that the time horizon is sufficiently long such that incumbent 1 can implement a quality change, i.e. $\delta_{1A} < T$.

Profits of firm n are the discounted value of all profits between $t = 1$ and T, $\pi_n = \sum_{t=1}^T (1-\gamma)^{t-1}\pi^t_n$, $0 \leq \gamma < 1$, $n = I1, I2$, and $\pi_E = \sum_{t=1}^T e^{t-\delta_2}(1-\gamma)^{t-1}(\pi^t_E - K^t)$. Profits at time t depend on the decisions which are implemented at t, denoted by $s^t \in \{0,1\} \times [\underline{q}, \bar{q}]^3 \times \Re^3_+$. When starting at $t = 1$ there exists a predetermined history for $t \leq 0$ which assigns initial values to all s^t. Whether they will prevail at time t depends on the delay structure. A decision r^t_n (the n-th component of r^t) assigns new values to all $s^{\tau+\delta}_n$ for $\tau \geq t$ when δ is the delay for decisions r_n.

A point in history is described by a vector of decisions r^t. A decision in the history with $t \leq 0$ is said to be relevant if it is implemented at $t \geq 1$. Since variables have an initial values one can speak of price and quality changes. I now provide the equivalence result.

Proposition 4.

For every predetermined history of all decisions $(s^t)_{t\leq 0}$ in a set which is of full measure on all relevant decision sets and any delay structure $\delta_3 < \delta_2 < \delta_{1B} < \delta_{1A} < T$ there exists a unique subgame perfect equilibrium set of the delay supergame. The subgame starting at period $\delta_{1A} + 1$ has a unique subgame perfect equilibrium set which is inde-

[9]For a positive mass of customers who buy in more than one period one would need to solve an intertemporal decision problem of the customers. Also there might be effects due to user-knowledge and other complementarities in time.

[10]In order to avoid problems of equilibrium existence for $c \notin [c_0, c^0]$ incumbents do not incur the fixed cost K^t although in a delay supergame this seems less satisfactory than in the extensive game.

pendent of the predetermined history. The vector of implemented decisions s^t in the periods $\delta_{1A} < t \leq T$ contains the same values as the outcome in the unique subgame perfect equilibrium set of the extensive game of Section 2.

Sketch of the Proof. First remark that one can ignore decisions which are irrelevant: I do not distinguish decisions of the potential entrant $(0, q_E, p_E)$ from $(0, \tilde{q}_E, p_E)$ because quality differences of non-existing goods are irrelevant. Analogously for implemented prices, when a good is not in the market. The uniqueness of the subgame perfect equilibrium (SPE) set of the extensive game has been shown above. Remark that at T decisions can be implemented which coincide with those taken in the SPE of the extensive game because $T > \delta_{1A}$.
Uniqueness is shown by backward induction. The price decisions which are implemented in the last period T have been made in $T - \delta_3$. For $T - \delta_2 < t \leq T - \delta_3$, only price decisions are implemented until T. The single period equilibrium price vector $p^*(q_{I1}^{t-\delta_{1A}+\delta_3}, q_{I2}^{t-\delta_{1B}+\delta_3}, q_E^{t-\delta_2+\delta_3})$ is implemented at $t + \delta_3$ when the entrant is active (trivial generalization of a finitely repeated normal form game). Otherwise, duopoly prices from Lemma 1 are implemented. At $T-\delta_2$ also q_E will be implemented until T if the entrant decides to become active. The decision $q_E^{T-\delta_2}$ affects the entrant's profits at T and the decision $p_E^{T-\delta_2}$ affects entrant's profits at $T - \delta_2 + \delta_3$. They are independent of each other. Hence under entry firms set prices $p^*(q_{I1}^{T-\delta_2-\delta_{1A}+\delta_3}, q_{I2}^{T-\delta_2-\delta_{1B}+\delta_3}, q_E^{T-2\delta_2})$ at $T - \delta_2$ and the entrant chooses quality q_E optimally given $q_{I1}^{T-\delta_{1A}}, q_{I2}^{T-\delta_{1B}}$. This argument can be applied recursively for $T-\delta_{1B} < t < T-\delta_2$. At $t-\delta_{1B}$ also the decision q_{I2} will be implemented until T. At $T - \delta_{1B}$ prices will be implemented in $T - \delta_{1B} + \delta_3$, q_E in $T - \delta_{1B} + \delta_2$ and q_{I2} in T. Incumbent 2 can only choose q_{I2} anticipating the optimal response of the entrant using continuation profits; a different decision in the hope for cooperation does not work because he cannot punish deviation from any cooperative behavior. Analogously to above, e and q_E is chosen in $T - \delta_{1B}$. The same reasoning gives the decisions in earlier periods. Now consider $t \leq T-\delta_{1A}$. Under entry deterrence incumbent 1 may be indifferent between a continuum of quality levels. However, the SPE of the subgame are payoff equivalent.
In the periods $\delta_{1A} < t \leq T$ outcomes are those of the SPE set of the extensive game. In the periods $0 < t \leq \delta_3$ they depend on predetermined values. Except for the possible indifference of the entrant between two points in the product space for some predetermined values of q_{I1} and q_{I2}, the SPE set of the delay supergame is unique. It is easy to show that this indifference occurs on a subset of the product of relevant decision sets of zero measure. In any period when all implemented decisions are endogenous the outcome coincides with the one in the unique SPE set of the extensive game. □

Results on the extensive game can now be interpreted in the delay supergame. The firm which takes longest to implement the decision of a quality change of its good will make the greatest profit in the market. In the case of accommodated entry it depends on c whether the most flexible firm (entrant) or incumbent 2 will be second in the profit ranking.
The comparative statics results of the previous subsections can be used to study dynamics in the delay supergame when the cost structure is time-dependent and its law of motion is common knowledge at every point in time. Firms react on parameter changes optimally and the comparative statics results from above translate into trajectories described by the equilibrium path. Alternatively, parameter changes can come by surprise affecting the results in transition periods.

5 Conclusion

In this paper I provided an example of an oligopolistic market with vertical product differentiation which is parametrized in cost parameters. This allowed me to study the impact of the technology of the firms (cost parameters) on market structure, conduct, and performance. Firms which differ only by the sequential move to choose a quality may use sophisticated entry deterring or entry accommodating strategies. Results depend critically on the cost parameters: drastic changes of the profitability of the market can be explained by small changes in the cost parameters.
The analysis suggests that allowing for strategic entry deterrence some intuitive relationships between barriers of entry, actual market profitability, and the degree of competitiveness do not necessarily hold. In particular, higher barriers of entry may have a positive effect on competition.

Appendix

Appendix 1. In this appendix I first determine critical values λ_{ij} which depend on c and are decisive for the occurrence of situations (0) to (4). With $\lambda_{i,i+1} = \frac{|q_{I1}-q_{I2}|}{\bar{q}-\underline{q}}$ I denote the critical relative quality difference of the incumbents which separates situation (i) from situation $(i+1)$, $i, = 0, \ldots, 3$. If for $i, j = 0, \ldots 4$, $i < j$, $\lambda_{i,i+1} > \ldots > \lambda_{j-1,j}$ then $\lambda_{ij} \equiv \lambda_{i,i+1}$ and situations $(i+1), \ldots, (j-1)$ do not occur for given c. For situation (5), $|q_{I1} - q_{I2}| = \bar{q} - \underline{q}$.
Let me begin with situation (0). Whether entry deterrence is possible or not depends on $\tilde{q}_{I1}$ and $\tilde{q}_{I2}$ which are solutions to $\min_{q_{I1}} \min_{q_{I2}} \max_{q_E} \tilde{\pi}_E(q_{I1}, q_{I2}, q_E)$. This is equivalent to solving $\tilde{\pi}_E(q_{I1}, q_{I2}, \underline{q}) = \tilde{\pi}_E(q_{I1}, q_{I2}, \frac{1}{2}(q_{I1} + q_{I2})) = \tilde{\pi}_E(q_{I1}, q_{I2}, \bar{q})$ because of Lemma 4. This has been called maximal squeezing of the gap (see Figure 4 in the text). There exists a unique solution $\tilde{q}_1 \leq \tilde{q}_2$. Because of the sequential quality choice $\tilde{q}_2 = \tilde{q}_{I1}$ for $c < \frac{1}{2}$ and $\tilde{q}_1 = \tilde{q}_{I1}$ for $c > \frac{1}{2}$. Analogously for $\tilde{q}_{I2}$. For $c = \frac{1}{2}$, both ways of indexing are possible. I define $\lambda_{01} = \frac{|\tilde{q}_{I1}-\tilde{q}_{I2}|}{\bar{q}-\underline{q}}$. When $K < \tilde{\pi}_E(\tilde{q}_{I1}, \tilde{q}_{I2}, \bar{q})$ then entry deterrence is impossible.
Let $(q^*_{I1}, q^*_{I2}, q^*_E)$ denote equilibrium quality levels under accommodated entry (see subsection 4.2). Then entry deterrence is possible but not profitable for both incumbents iff $\tilde{\pi}_E(\tilde{q}_{I1}, \tilde{q}_{I2}, \bar{q}) < K < \tilde{\pi}_E(q^*_{I1}, q^*_{I2}, q^*_E)$. For $c \leq \frac{1}{2}$,

$$\begin{aligned} \lambda_{12} &= \min\left\{\lambda^*_1, \lambda^*_2 \;\middle|\; \lambda^*_1 \text{ solves } \tilde{\pi}_{I1}(q^*_{I1}, q^*_{I2}, q^*_E) = \frac{1}{9}(2-c)^2\lambda_1(\bar{q}-\underline{q}) \right. \\ &\qquad \left. \text{and } \lambda^*_2 \text{ solves } \tilde{\pi}_{I2}(q^*_{I1}, q^*_{I2}, q^*_E) = \frac{1}{9}(1+c)^2\lambda_2(\bar{q}-\underline{q})\right\}. \end{aligned}$$

The right-hand side of the equations follows from equations (4) and (5) of Section 3. As it turns out $\lambda_{12} = \lambda^*_2$, i.e. it is always incumbent 2 who first prefers entry deterrence when increasing K and hence separates situation (1) from situation (2).
λ^*_1 is the critical value above which both incumbents prefer entry deterrence. It separates situations (2) and (3) from situation (4), and I set $\lambda_{34} = \lambda^*_1$. Computations show that for $c \in [0, c_0)$

$$\begin{aligned} \lambda_{12} &= \frac{9}{8}\left(\frac{\frac{1}{6}+c}{1+c}\right)^2, \\ \lambda_{34} &= \frac{9}{8}\left(\frac{\frac{7}{6}-c}{2-c}\right)^2. \end{aligned}$$

Indices have to be reversed for $c \in (c^0, 1]$. Computations also show that for $c \in [c_0, c^0]$ situation (1), (2), and (3) cannot occur. Hence $\lambda_{04} = \lambda_{01}$ for $c \in [c_0, c^0]$, and I do not write down expressions for the other $\lambda_{i,i+1}$, $i = 1, 2, 3$, in this case.

To complete the analysis of critical λ, λ_{23} needs to be derived. For $c \in [0, c_0)$, it is the solution to $\tilde{\pi}_E(\bar{q}, \bar{q} - \lambda(\bar{q} - \underline{q}), \underline{q}) = \tilde{\pi}_E(\bar{q}, \bar{q} - \lambda(\bar{q} - \underline{q}), \bar{q} - \frac{1}{2}\lambda(\bar{q} - \underline{q}))$ which is equivalent to

$$\lambda = (1 - \lambda)(3c + \lambda)^2$$

(compare equations (9) and (10) and the subsection on accommodated entry). If $K > \tilde{\pi}_E(\bar{q}, \bar{q} - \lambda(\bar{q} - \underline{q}), \underline{q})$ incumbent 2 can deter entry on his own.

Now I can determine K^0. It is equal to the maximal continuation profit of the entrant when incumbent 1 chooses $\bar{q}$ and incumbent 2 a quality such that he prefers entry deterrence. Since for small c he can deter entry whenever he wants to do so

$$K^0 = \tilde{\pi}_E\left(\bar{q}, \bar{q} - \max\{\lambda_{12}, \lambda_{23}\}(\bar{q} - \underline{q}), \bar{q} - \frac{1}{2}\max\{\lambda_{12}, \lambda_{23}\}(\bar{q} - \underline{q})\right).$$

Analogously for $c \in (c^0, 1]$.

Appendix 2. Numerical values: At $c = 0.0606$ situation (0) occurs for $0 \leq K < 0.001431(\bar{q} - \underline{q})$, (1) for $0.001431(\bar{q} - \underline{q}) < K < 0.0014349(\bar{q} - \underline{q})$, (2) for $0.0014349(\bar{q} - \underline{q}) < K < 0.0014363(\bar{q} - \underline{q})$, (3) for $0.0014363(\bar{q} - \underline{q}) < K < 0.010164(\bar{q} - \underline{q})$, (4) for $0.010164(\bar{q} - \underline{q}) < K < 0.02778(\bar{q} - \underline{q})$, and (5) for $K > 0.02778(\bar{q} - \underline{q})$.

Appendix 3. Fix $c = 0.0606$, $\underline{q} = 0$, $\bar{q}$. (One may take any other parameters such that the market is extremely asymmetric.) Equilibrium prices under accommodated entry are $p_{I1} = 0.3371$, $p_{I2} = 0.0568$, and $p_E = 0.1136$. Hence, price-cost margins are 0.2765, 0.0568, and 0.0833 respectively. Under deterred entry at $K^0 (= 0.00144)$ equilibrium prices are $p_{I1} = 0.093$ and $p_{I2} = 0.075$. Both price-cost margins $p_{I1} - cq_{I1} = 0.032$ and $p_{I2} - cq_{I2} = 0.018$ are smaller than those under accommodated entry. Since p_{I2} under accommodated entry is the lowest price and there are customers of positive measure who consider quality improvements as "unimportant", these customers do not obtain less utility when entry is deterred.

In the second example I keep qualities unchanged and set $c = 0$. Since quality improvements are free prices and price-cost margins coincide in this example. Under accommodated entry, equilibrium prices are $p_{I1} = \frac{7}{24}$, $p_{I2} = \frac{1}{24}$, and $p_E = \frac{1}{12}$. Under deterred entry at $K^0 = \frac{1}{1152}$ equilibrium prices are $p_{I1} = \frac{1}{48}$ and $p_{I2} = \frac{1}{96}$ and qualities are $q_{I1} = 1$ and $q_{I2} = \frac{31}{32}$. Hence, customers buy goods of at least the same quality at lower prices.

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