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# Potential Output, the Natural Rate of Unemployment, and the Phillips Curve in a Multivariate Structural Time Series Framework

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## **Abstract**

We propose a bivariate structural time series framework to decompose GDP and the unemployment rate into their trend, cyclical, and irregular components. We implement Okun's law by a generalised version of the common cycles restriction allowing for a phase shift between the two cycles and add a price-wage block to the system. We estimate by maximum likelihood Phillips curve-type equations, where the particular cycles enter the wage and price equations in levels though the trends are modelled as non-stationary stochastic processes. The extended models provide an improved estimate of the current cyclical position, compared to univariate estimates and the HP filter.

## **Keywords**

Structural time series model, trends and cycles, Phillips curve

## **JEL-Classifications**

C22, E30

**Comments**

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## 1. Introduction

The decomposition of macroeconomic time series into their long-run trend and cyclical components has become a matter of lasting concern. Among many uses one important application is the assessment of the fiscal and monetary policy stance from the cyclical position of GDP and the unemployment rate. It has recently gained renewed attention for the goal of cyclical adjustment of government budget balances in the course of the consolidation efforts of EU member states (Giorno et. al., 1995; European Commission, 1995b, Barrell et. al., 1994). The traditional theoretical foundations for the dichotomy between trend and cyclical components lie in the neo-classical synthesis and aggregate demand-supply models, respectively. The long-run trend in GDP is determined by a production function, while cyclical deviations occur due to price- and wage rigidities translating demand shocks into output fluctuations. Price and wage behaviour is traditionally described by Tobin's wage-price mechanism giving rise to the well-known unemployment-inflation trade-off. At potential output and the natural rate of unemployment, respectively, there occurs no inflationary pressure from the labour market.

There are numerous methods for the extraction of trend and cyclical components (see, e.g. Canova, 1993). Among the currently most widely used are non-parametric smoothing techniques, i.e., the Hodrick-Prescott (HP) filter (Hodrick and Prescott, 1980), the Beveridge-Nelson-decomposition (e.g., King et. al., 1991; Evans and Reichlin, 1994; Karras, 1994; Sefton, 1995), and the production function approach (e.g., Torres and Martin, 1990; Giorno et. al., 1995). These methods differ widely in the utilisation of economically motivated restrictions as outlined above. While smoothing techniques are atheoretical by their nature, the production function approach makes explicit use of the close co-movement of GDP and unemployment cycles and the property of inflation neutrality of potential GDP. Potential output is defined as the outcome of a production function, i.e.,  $PO = q^{PO} f(K, L^{PO})$ , where the factor inputs and total factor productivity are at their potential levels. Total factor productivity at potential output  $q^{PO}$  is found by smoothing techniques, while potential employment  $L^{PO}$  is calculated from the smoothed labour supply and an estimate of the unemployment rate that is consistent with non-accelerating inflation (e.g., Layard, Nickell and Jackman, 1991). The third approach, the multivariate Beveridge-Nelson-decomposition, based on vector autoregressions, uses the information contained in the co-movement of GDP with various cyclical indicators and/or common trend restrictions in order to decompose GDP into a random walk and a stationary component. However, there are clear limitations to imposing restrictions related to the above cyclical relationships. In particular, as the VARs are specified in first differences, Phillips curve type relationships relating prices and wages to cycles in levels cannot be modelled explicitly.



From an econometric viewpoint, smoothing techniques and the production function approach suffer from several shortcomings. As concerns the HP filter, it has been criticised for various deficiencies (e.g., King and Rebelo, 1993), the most important being the arbitrariness of the smoothing parameter and its tendency to produce spurious cycles (Harvey and Jäger, 1992; Boone and Hall, 1995). Since the HP filter is also used in the production function approach (Giorno et. al., 1995) for smoothing labour supply and total factor productivity, its shortcomings also apply to a somewhat less extent to the latter.<sup>1</sup> Moreover, as a general feature, smoothing techniques based on two-sided symmetric moving averages of past and future observations give rise to a substantial so-called end-point bias, i.e., biased estimates at the end of the sample (Barrell et. al., 1994). For policy purposes, however, the estimate of the current cyclical position is certainly the most important outcome of the whole exercise.

The present paper proposes an approach that allows for explicitly modelling cyclical relationships in levels and estimation within a maximum likelihood framework. It is based on a multivariate extension of structural time series (STS) models, advocated in a series of papers by Harvey (Harvey, 1985, 1989; Harvey et. al. 1986; Harvey and Koopman, 1992; Harvey and Jäger, 1993). STS models are designed to decompose a time series into several unobserved components, i.e., a non-stationary trend, a cycle, and an irregular term. The particular components are specified as separate parametric stochastic processes. This specific feature allows for imposing restrictions on the particular components. We construct a bivariate STS model for GDP and the unemployment rate and impose a close co-movement of the two cycles. We also add wage and price equations to the system with the cycles and provide a full maximum likelihood estimate of a Phillips curve in its traditional form, where prices and wages are related to the cyclical components in levels though the trends follow non-stationary processes. The wage-price block allows for a misspecification test for the property of inflation neutrality of the particular trends, as extracted by the model. After an outline of our approach we present results for Austrian quarterly data. We will compare estimates from the HP filter and univariate STS models with the multivariate approach.

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<sup>1</sup> Röger (1994) has pointed out that the application of the HP-Filter to all factor inputs is equivalent to its application to GDP itself. Moreover, work by the European Commission (1995a) indicates that traditional NAIRU estimates based on wage and price equations exhibit a high degree of uncertainty, with their confidence bounds up to  $\pm 3$  percentage points. Thus, in fact, the NAIRU estimates are often subject to judgemental revision.

## 2. Structural Time Series (STS) Models

The following model has been proposed by Harvey (1985) for the decomposition of a macroeconomic time series  $y$  into a non-stationary trend  $y^{\text{tr}}$ , a stationary cycle  $\varphi$ , and an irregular component  $v$ .

$$(1) \quad y_t = y_t^{\text{tr}} + \varphi_t + v_t$$

The trend follows a so-called local linear trend, that is a random walk with stochastic drift  $\mu_t$  which, in turn, again is specified as random walk, i.e.,

$$(2) \quad \begin{aligned} \Delta y_t^{\text{tr}} &= \mu_{t-1} + \eta_t^{(1)} \\ \Delta \mu_t &= \eta_t^{(2)} \end{aligned}$$

where  $\eta_t^{(1)}$  and  $\eta_t^{(2)}$  are white noise. This is an ARIMA(0,2,2) process. If  $\sigma_1^2 = \text{var}(\eta_t^{(1)}) = 0$  the model reduces to a random walk with drift. If  $\sigma_1^2 = 0$ , but  $\sigma_2^2 = \text{var}(\eta_t^{(2)}) > 0$ , the trend is still a process integrated of order 2, i.e.,  $\Delta^2 y_t^{\text{tr}} = \eta_t^{(2)}$ . Such a trend tends to be relatively smooth compared to a random walk, with a more or less slowly changing slope.

The cyclical component  $\varphi_t$  is specified as a so-called stochastic cycle,

$$(3) \quad \begin{bmatrix} \varphi_t \\ \varphi_t^* \end{bmatrix} = \begin{bmatrix} \rho \cos \lambda & \sin \lambda \\ -\sin \lambda & \rho \cos \lambda \end{bmatrix} \begin{bmatrix} \varphi_{t-1} \\ \varphi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \eta_t^{(\varphi)} \\ \eta_t^{(\varphi)*} \end{bmatrix}, \quad \text{with } |\rho| < 1,$$

which is derived from a dampened cosine wave of fixed length  $\lambda$ , subject to shocks  $\eta_t^{(\varphi)}$  and  $\eta_t^{(\varphi)*}$ . In its ARIMA form it turns out to be a stationary ARMA(2,1) process, whose AR-polynomial is restricted to have two conjugate complex roots thereby generating a cyclical impulse response to both innovations (Harvey, 1989). The whole model can be written in state-space form with the state vector  $\mathbf{a}_t = (y_t^{\text{tr}}, \mu_t, \varphi_t, \varphi_t^*)'$ , and the vector of innovations  $\mathbf{h}_t = (\eta_t^{(1)}, \eta_t^{(2)}, \eta_t^{(\varphi)}, \eta_t^{(\varphi)*})'$ . For identification, the restrictions  $E v_t \eta_t^{(i)} = 0$  and  $E \eta_t^{(i)} \eta_t^{(j)} = 0$ , for  $i \neq j$ , have to be imposed, thus giving a diagonal variance-covariance matrix  $\text{cov}(\mathbf{h}_t)$ . Also, the restriction  $\sigma_\varphi^2 = \text{var}(\eta_t^{(\varphi)}) = \text{var}(\eta_t^{(\varphi)*})$  is usually imposed. Finally, the set of hyperparameters ( $\rho, \lambda, \sigma_v^2, \text{cov}(\mathbf{h}_t)$ ) may be estimated by maximum likelihood (ML) using the Kalman filter. Once the hyperparameters have been estimated, the Kalman filter gives the optimal *filtered* estimates  $\mathbf{a}_{t|t}$  of the state vector  $\mathbf{a}_t$  at time  $t$ , given *past and current* observations  $(y_1, \dots, y_t)$ . The subsequent application of a smoothing algorithm provides optimal *smoothed* estimates  $\mathbf{a}_{t|T}$  for  $\mathbf{a}_t$  given *all* available observations  $(y_1, \dots, y_T)$ .

It is also noteworthy that STS models encompass the HP filter. The HP filter can be characterised as the optimal estimator of the STS model  $y_t = y_t^{\text{tr}} + v_t$ , composed of a smooth trend ( $\sigma_1^2 = 0$ ) and an irregular component, but without a cycle. The smoothing parameter  $\tau_{\text{HP}}$

is given as the ratio of the variances of the irregular component and the change in the slope, i.e.,  $\tau_{HP} = \sigma_v^2 / \sigma_2^2$ . The HP filter estimate of the cycle is then nothing but the smoothed irregular component with the value of  $\tau_{HP}$  imposed rather than estimated (Harvey and Jäger, 1993).

## 2.1. Possible Restrictions on Cyclical Components

The extension of the above univariate STS model to the multivariate case is straightforward. The simplest case, in the context of a bivariate model for two series  $y_t$  and  $ur_t$ ,

$$(4) \quad \begin{aligned} y_t &= y_t^{tr} + \varphi_t^{(y)} + v_t^{(y)} \\ ur_t &= ur_t^{tr} + \varphi_t^{(ur)} + v_t^{(ur)} \end{aligned}$$

would be the simultaneous estimation of the two equations, possibly allowing for correlations between corresponding innovations  $\eta_t^{(y,i)}$  and  $\eta_t^{(ur,i)}$ . The bivariate model (4) can be easily stacked in one single state-space form. The literature so far suggests two ways to establish a link among different cyclical components. In the so-called *similar cycles* (SC) model the cycles  $\varphi_t^{(y)}$  and  $\varphi_t^{(ur)}$  are independent of each other with the only restriction that the cycle lengths are equal, i.e.,  $\lambda_y = \lambda_{ur}$ . The *common cycles* (CC) model incorporates only one cycle  $\varphi_t$  at which the particular series participate with different scaling parameters 1 and  $\theta$ , i.e.,  $\varphi_t^{(ur)} = \theta \varphi_t^{(y)}$ . Both restrictions seem not entirely satisfactory for the purpose of modelling the business cycle. In the SC model there is no direct link between the cycles and nothing prevents them from moving rather independently with, for instance, very different turning points. On the other hand, the CC model obviously is incapable of modelling possible leads or lags between different cycles. However, it is well-known that the unemployment cycle generally follows the output one with a time lag of up to several quarters. We therefore use a generalisation of the common cycles model (GCC), proposed by Rünstler (1996), where the cyclical component of  $ur_t$  is also linked to  $\varphi_t^*$  from equation (3).

$$(5) \quad \begin{aligned} y_t &= y_t^{tr} + \varphi_t + v_t^{(y)} \\ ur_t &= ur_t^{tr} + (\theta \varphi_t + \theta^* \varphi_t^*) + v_t^{(ur)} \end{aligned}$$

This simple generalisation has the appealing interpretation that  $(\theta \varphi_t + \theta^* \varphi_t^*)$  follows (or leads) the original cycle  $\varphi_t$  with a constant phase shift  $\omega = \lambda^{-1} \arctan(\theta^*/\theta)$ . Its variance relative to the variance of  $\varphi_t$  is given by the scaling factor  $\vartheta^2 = \theta^2 + \theta^{*2}$ . More precisely, any linear combination  $(\theta \varphi_t + \theta^* \varphi_t^*)$  of  $\varphi_t$  and  $\varphi_t^*$  follows the same autocorrelation function

$$\text{corr}(\theta \varphi_t + \theta^* \varphi_t^*, \theta \varphi_{t-s} + \theta^* \varphi_{t-s}^*) = \rho^s \cos(\lambda s),$$

independent of  $\theta$  and  $\theta^*$ , while the crosscorrelations between  $\varphi_t$  and  $(\theta \varphi_{t-s} + \theta^* \varphi_{t-s}^*)$  are given by

$$\begin{aligned} \text{corr}(\varphi_t, \theta\varphi_{t-s} + \theta^*\varphi_{t-s}^*) &= \rho^s (\theta^2 + \theta^{*2})^{-1/2} (\theta \cos(\lambda s) + \theta^* \sin(\lambda s)). \\ &= \rho^s \cos(\lambda(s - \omega)). \end{aligned}$$

Obviously, the crosscorrelation function reaches its local extremes at  $s - \omega = k \pi/2$  with the global maximum at  $s = \omega$ . Thus  $\omega$  gives the phase shift of  $(\theta\varphi_t + \theta^*\varphi_t^*)$  with respect to  $\varphi_t$ . Also, as there is a *pair* of shocks  $\eta_t^{(\varphi)}$  and  $\eta_t^{(\varphi)*}$ , the relationship between the two cycles is not deterministic. The bivariate GCC model might be regarded as a restricted bivariate AR(1) process with conjugate complex roots. This can be seen from the fact that the two cyclical components,  $\varphi_t^{(y)}$  and  $\varphi_t^{(ur)}$ , are linear transformations of  $\varphi_t$  and  $\varphi_t^*$ .

Tests of the SC and GCC restrictions can be conducted using likelihood ratio (LR) tests. The test of the SC restriction,  $H_0: \lambda_y = \lambda_{ur}$ , is straightforward with the LR-statistics following a  $\chi_1^2$  distribution. Testing for the GCC restriction amounts to testing for the presence of a further stochastic cycle (3),  $\psi_t$ , in the GCC model. For identification reasons it is sufficient to add  $\psi_t$  to one of either equations.

$$(6) \quad \begin{aligned} y_t &= y_t^{tr} + \varphi_t + v_t^{(y)} \\ ur_t &= ur_t^{tr} + (\theta \varphi_t + \theta^* \varphi_t^*) + \psi_t \end{aligned}$$

As Harvey (1989) points out, the test for the presence of the cycle  $\psi_t$  is equivalent to imposing the restriction  $H_0: \rho_\psi = 0$  on the corresponding dampening factor. In this case  $\psi_t$  becomes white noise. In general, the LR-test of  $H_0$  is not feasible due to the complication that  $\lambda_\psi$  is no longer identified under  $H_0$ . In the context of testing the GCC restriction, however,  $\lambda_\psi$  might be set equal to the cycle length  $\lambda$  of  $\varphi_t$ . Also, since under  $H_0$   $\psi_t$  is not distinguishable from the irregular component  $v_t^{(ur)}$ ,  $\sigma_\psi^2$  is also unidentified. Thus,  $v_t^{(ur)}$  must be removed from the unrestricted model. Under  $H_0: \rho_\psi = 0$  equation (6) then nevertheless reduces to equation (5).<sup>2</sup>

## 2.2. Price and Wage Equations

Non-stationary trends, as extracted by the above methods, do not account for inflation neutrality. Since the technology for calculating the NAIRU requires an estimated relation between inflation and the unemployment rate, it is inextricably linked to dynamic Phillips curve equations for price and wage changes. We therefore add standard price and wage equations to the bivariate models, that is, we aim at estimating a model of the general structure

$$(7) \quad y_t = y_t^{tr} + \varphi_t^{(y)} + v_t^{(y)}$$

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<sup>2</sup> For the univariate model, Harvey (1989, p. 251) proposes an LM test for the presence of a further cyclical component in the series based on a regression of the periodogram on the elements of the spectral generating function.

$$\begin{aligned}
ur_t &= ur_t^{tr} + \varphi_t^{(ur)} + v_t^{(ur)} \\
\Delta p_t &= a_1(L) \Delta w_{t-1} + a_2(L) \Delta p_{t-1} + a_3(L) \Delta q_{t-1} + a_4 (s_{t-k} - \mu) + \beta_1 \varphi_{t-1}^{(y)} + \varepsilon_t^{(p)} \\
\Delta w_t &= b_1(L) \Delta w_{t-1} + b_2(L) \Delta p_{t-1} + b_3(L) \Delta q_{t-1} - b_4 (s_{t-k} - \mu) - \beta_2 \varphi_{t-1}^{(ur)} + \varepsilon_t^{(w)},
\end{aligned}$$

where either of the above restrictions on cyclical co-movements might be imposed. The general specification of the price and wage equations follows Franz and Gordon (1994) and Pichelmann (1993).  $p$ ,  $w$ , and  $q$  denote prices, wages, and labour productivity (in logs). The labour share of income  $s_{t-k}$  acts as an error-correction term accounting for the stationary long-run relationship between real wages and labour productivity. The constant  $\mu$  enters the error-correction term thereby implying the absence of a linear trend component in prices and wages.

Estimates of the NAIRU, as used in the production function approach, combine the wage-price mechanism with structural labour market indicators and hysteresis effects (e.g., Coe, 1985; Franz and Gordon, 1994) in order to account for NAIRU changes and the resulting instabilities in the unemployment-inflation trade-off. There might remain, however, some doubt whether these structural indicators sufficiently capture changes in the NAIRU. In seeking for a Phillips curve type relationship it therefore seems a promising alternative way to extract the UR and GDP cycles by a less structural approach and insert them as explanatory variables in a wage-price block, as in equations (7).<sup>3</sup> Strictly speaking, however, equations (7) do not impose inflation neutrality of  $y_t^{tr}$  and  $ur_t^{tr}$  in the sense of explicit orthogonality restrictions between prices or wages and the lagged trends. Therefore, they do not rule out possible longer-term trade-offs between inflation and unemployment. A misspecification test for the property of inflation neutrality might be conducted by adding the changes in the slopes of the GDP and UR trends,  $\Delta^2 y_{t-k}^{tr}$  and  $\Delta^2 ur_{t-k}^{tr}$ , as explanatory variables to the price and wage equations.

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<sup>3</sup> Several other works have tried to combine price-wage equations with various ways of extracting the cyclical components in GDP and the unemployment rate. Cote and Hostland (1994) used the cyclical components, as obtained by the HP-Filter, as lagged explanatory variables in a system of price and wage equations. The smoothing parameters were chosen so that the likelihood of the wage-price block was maximised. Adams and Coe (1990), built on the production function approach, however estimated a parametric equation for total factor productivity and thereby avoided extensive smoothing.

### 3. Results

We use Austrian quarterly data with the estimation period ranging from 1966q1 to 1994q3. In order to circumvent numerical instabilities that occur when a diffuse prior is used for the initial state vector  $a_0$  we applied the algorithm by Rosenberg (1973), which treats  $a_0$  as non-stochastic and estimates its elements explicitly.  $a_0$  can, however, be concentrated out of the likelihood function.<sup>4</sup>

As mentioned in section 1, the current estimate of the cycle, based on past and current observations only, is the relevant one for policy purposes. In evaluating the outcomes of different models it might therefore be more appropriate to regard the filtered instead of the smoothed estimate. The question remains, which benchmark one should choose for an evaluation of the former. One possible choice is the smoothed estimate itself, which has the particular advantage that each model is evaluated in itself. The smoothed estimate is by construction more efficient than the filtered one, as it is based on a larger information set.<sup>5</sup> In fact, it will turn out that the smoothed estimates generally are rather similar across various models, while the filtered estimates differ more widely. We will thus present some statistics for the difference  $F_t - S_t$  of the filtered,  $F_t = \varphi_{t|t}$ , and smoothed estimates,  $S_t = \varphi_{t|T}$ , of the cycle, i.e., the standard deviation of their difference, standardised by that of the smoothed estimate, and their correlation (*SF-indicators*). Clearly, these indicators will not be emphasised for model selection,<sup>6</sup> but are intended to give some evidence on the efficiency of the filtered estimate. Subsection 3.1 presents results for the HP filter and univariate STS models and compares them with bivariate models using the SC and GCC restrictions. In subsection 3.2 the preferred bivariate models are extended by a system of wage and price equations.

#### 3.1. Bivariate STS Models for Austrian GDP and UR

We start with estimating univariate (UV) models for GDP and UR, as described in equations (1) to (3). For the unemployment rate both the variances of the irregular and the random walk components are insignificant so that there remains a smooth trend with slowly changing slope and a cycle with a length of 28.4 quarters. For GDP the unrestricted model yields a local linear trend and an irregular component, but no cycle. If a smooth trend ( $\sigma_1^2 = 0$ ) is imposed the model finds a cycle of 13 quarters. This rather untypical feature of Austrian GDP has already been reported by Harvey and Jäger (1993). If, however, the two

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<sup>4</sup> Estimation of the various STS models was done using the optimisation procedures of GAUSS. Where possible, we also used the packages STAMP and STAMP II to verify the results. All data are from the Austrian Institute for Economic Research. GDP and UR have been seasonally adjusted with Census-X11.

<sup>5</sup> To find the filtered estimate of the HP-filter, we applied the HP-filter repeatedly for each time span 1 to  $t$ , took the estimated cycle  $\varphi_{t|t}$  from the final observation, and, finally, combined these estimates to one series  $F_t$ .

<sup>6</sup> Harvey (1989) has pointed out that the gain from smoothing generally rises with the variance of the innovations in the particular component. If therefore the estimate of the variance is biased, the SF-indicators might be misleading. Since the filtered and smoothed components are identical for the starting and end points by construction, the statistics is computed for a time period of 69q1 to 91q4.

univariate models are estimated simultaneously with smooth trends and the introduction of a correlation  $\gamma$  between the two pairs of cyclical innovations as a further parameter (model BV), a GDP cycle with a length of 25.9 quarters is extracted.<sup>7</sup> The correlation  $\gamma$  between cyclical innovations is estimated with -0.63. The application of the SC restriction  $\lambda_y = \lambda_{ur}$  in the model BV then gives a cycle length of 27.4 quarters. The restriction is easily accepted by the LR-test (Table 2). We also test whether the GDP cycle remains in the SC model, if the random walk component in the GDP trend is reintroduced (model SC<sub>RW</sub>). Contrarily to the univariate model, the random walk component virtually disappears and the cycle remains. Consequently, a smooth trend in GDP is accepted by the LR-test (Table 2). Finally, we apply the GCC restriction (5), that is, we impose one generalised common cycle for GDP and UR. Okun's law states as a rule of thumb that a 3% deviation of GDP from its potential is equivalent to a 1% change in the unemployment rate. The GCC model estimates Okun's coefficient  $1/\vartheta$  with 3.7, while the UR cycle lags the GDP one with 2.6 quarters. For the test of the GCC restriction, the GCC model extended with a second cycle  $\psi_t$  in the UR equation, imposing  $\lambda_\psi = \lambda$ , is compared with one extended by an irregular component in the UR equation (see equation (6)). This test rejects the GCC model at a significance level of 10 % (Table 2).

The results for the various STS models, all estimated with a smooth trend, are summarised in Table 1. As a benchmark, Table 1 also presents HP filter estimates with the usual smoothing parameter of  $\tau_{HP} = 1600$ . Generally, apart from the univariate GDP model, the key parameters of the models correspond quite closely. The SF-indicators exhibit improved values for the SC and GCC models compared to the HP filter with the GCC model outperforming the SC one. The higher degree of correspondence between the smoothed and filtered estimates is also reflected in the graphs of the particular cyclical components, as shown in Fig. 1 to 3. In order to visualise the effect of the GCC restriction, Fig. 4 also shows the smoothed estimate of the UR cycle, reversed and standardised to the same variance as the GDP cycle.

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<sup>7</sup> More specifically, we comprise equations (4) in one single state-space form and introduce the covariances  $E(\eta_t^{(y,\varphi)} \eta_t^{(ur,\varphi)}) = E(\eta_t^{(y,\varphi)*} \eta_t^{(ur,\varphi)*})$  as off-diagonal elements in  $\text{cov}(\mathbf{h}_t)$ . All other off-diagonal elements of  $\text{cov}(\mathbf{h}_t)$  are restricted to zero.

Table 1: HP filter and STS models

	UR				GDP			
	HP	UV	SC	GCC	HP	UV	SC	GCC
RD <sup>2</sup>	---	0.18	0.19	0.18	---	0.14	0.15	0.12
cycle length $\lambda$	---	28.82	27.40	26.80	---	13.07	27.40	26.80
Std( $\sigma_v$ ) <sup>1)</sup>	---	0.00	0.00	0.00	---	6.09	5.36	5.82
Std( $\sigma_2$ ) <sup>1)</sup>	---	1.23	1.18	1.47	---	1.42	0.72	1.69
Std( $\sigma_\phi$ ) <sup>1)</sup>	---	13.50	13.45	---	---	1.87	5.36	4.12
Q(12)	---	28.14	15.77	18.50	---	11.06	10.68	13.48
SF-indicators								
Std( $S_t$ ) <sup>2)</sup>	0.30	0.38	0.40	0.36	1.18	0.67	1.10	1.07
Corr( $F_t, S_t$ )	0.49	0.72	0.71	0.69	0.52	0.54	0.60	0.81
Std( $F_t - S_t$ )/Std( $S_t$ )	1.02	0.69	0.72	0.74	0.99	0.83	0.81	0.59

Note: RD<sup>2</sup> denotes the ratio of explained to total variance of the differenced series.  
 $\sigma_v$ ,  $\sigma_2$ , and  $\sigma_\phi$  denote standard deviations of the irregular component and the innovations in the slope and the cycle.  
 Q(12) is the Ljung-Box statistics for autocorrelation in prediction errors ( $\chi_{12}^2$ )  
 $S_t$  and  $F_t$  denote smoothed and filtered estimates of the cycles.  
 1) values for UR multiplied by 100, for GDP by 1000.  
 2) values for GDP multiplied by 100.

While the near-rejection of the GCC restriction might stem from an under-parameterisation of the bivariate process for GDP and UR cycles, a more likely reason seems the sharp rise in the unemployment rate between 1981 and 1983. The models might tend to fix the UR trend somewhat too smooth in this period and attribute some part of the trend to the cyclical component in the unemployment rate. As a consequence, in the GCC model the size of the GDP cycle is then overestimated also.<sup>8</sup> In turn, potential output growth rates exhibit somewhat higher volatility, compared to the SC model. The rise in the unemployment rate in this period might also be the reason for some significant autocorrelations in the prediction errors for the unemployment rate that appear in all models (see the statistics Q(12) in Table 1).

<sup>8</sup> In order to address this point, we added a dummy variable to the UR trend allowing for two break points, i. e.,  $\Delta^2 ur_t^{tr} = d_t + \eta_t^{(ur,2)}$ , with the dummy  $d_t$  set to 1 for some  $t$ , (e. g., 1981q2) and to -1 for some  $t+k$  (e. g., 1982q3). For a reasonable range of different break points the dummy is highly significant and the GCC restriction is easily accepted (see Harvey and Koopman (1992) for the detection of outliers in STS models).



**Table 2: Likelihood Ratio-Tests<sup>9</sup>**

	$H_1$	$H_0$	LR	df	p
Correlation among cycl. shocks	BV	UV	19.85	1	<.001
SC restriction	BV	SC	0.03	1	.86
RW in GDP trend	SC <sub>RW</sub>	SC	0.12	1	.72
GCC restriction		GCC	3.52	1	.06
phase shift $\omega$	GCC	CC	6.91	1	.01

### 3.2. Price and Wage Equations

The order of integration of prices and wages is a matter of debate in the relevant literature (e.g., Baillie and Chung, 1996). While augmented Dickey-Fuller (ADF) tests cannot reject the hypothesis that prices and wages are integrated of order two also for Austrian data, for the purpose of modelling Phillips curve type relationships there are nevertheless arguments for using equations in first differences.<sup>10</sup> We will present results for equations in both first and second differences. The specification of the particular equations is based on SUR estimates using the filtered cycles, as obtained from ML-estimation, as explanatory variables. Since the system is recursive, this two-step estimation procedure provides consistent parameter estimates (and therefore good starting values for ML-estimation), though the standard errors are biased (Pagan, 1984). The wage equation in first differences is of standard kind. It is difficult to find a reasonably parsimonious price equation for Austria, however.<sup>11</sup> In equations in second differences, the only link between prices and wages is given in the wage equation by the labour share of income both in levels and first differences. This specification may be interpreted in terms of Johansen's (1995) second-order co-integration. Table 3 presents final ML-estimates for two models, i.e., the GCC model extended by the wage equation in first differences (GCC <sub>$\Delta w$</sub> ) with the GDP cycle as explanatory variable, and the SC model extended by equations in second differences (SC <sub>$\Delta^2 wp$</sub> ).

<sup>9</sup> The distribution of the LR-test of SC against SC<sub>RW</sub> is given by  $1/2 \chi_1^2$  (see Harvey, 1989).

<sup>10</sup> Both specifications are used in the literature. As examples for equations in first differences see Adams and Coe (1990), Franz and Gordon (1994), and Coe (1985). For equations in second differences see Cote and Hostland (1994).

<sup>11</sup> p, w, and q have been seasonally adjusted by forming fourth differences. While, in the wage equation, consumer prices ( $\Delta pc$ ) appear to have higher explanatory power than the GDP deflator ( $\Delta p$ ), the latter enters the equation through the labour share of income. The data also support a specification where  $\Delta^2 pc_{t-1}$  is used as a proxy for surprises in inflation (Layard, Nickell and Jackman, 1991). The failure in finding a parsimonious price equation has to do with the high dependency of Austrian on German prices. However, for several reasons we refrained from using German prices as explanatory variables.

**ML-estimates of Price and Wage Equations**

(t-values in parentheses)

GCC <sub>Δw</sub> : (R <sup>2</sup> = 0.89)						
Δw <sub>t</sub>	= 0.28 Δw <sub>t-1</sub>	+ 0.24 Δw <sub>t-2</sub>	+ 0.39 Δ <sup>2</sup> p <sub>c,t-1</sub>	+ 0.51 Δp <sub>c,t-4</sub>	+ 0.28 Δq <sub>t-4</sub>	- 0.21 (s <sub>t-8</sub> - 7.30) + 0.31 φ <sub>t-1</sub> <sup>(y)</sup>
	(3.83)	(3.74)	(2.76)	(5.68)	(4.71)	(5.24) (2.98)
SC <sub>Δ2wp</sub> :						
(R <sup>2</sup> = 0.42)						
Δ <sup>2</sup> p <sub>t</sub>	= - 0.57 Δ <sup>2</sup> p <sub>t-1</sub>	- 0.27 Δ <sup>2</sup> p <sub>t-2</sub>	- 0.25 Δ <sup>2</sup> p <sub>t-4</sub>	+ 0.21 φ <sub>t-1</sub> <sup>(y)</sup>		
	(7.08)	(3.28)	(3.46)	(1.99)		
(R <sup>2</sup> = 0.49)						
Δ <sup>2</sup> w <sub>t</sub>	= - 0.63 Δ <sup>2</sup> w <sub>t-1</sub>	- 0.46 Δ <sup>2</sup> w <sub>t-2</sub>	- 0.30 Δ <sup>2</sup> w <sub>t-3</sub>	- 0.26 Δs <sub>t-4</sub>	- 0.06 (s <sub>t-4</sub> - 7.28)	- 0.66 φ <sub>t-1</sub> <sup>(ur)</sup>
	(8.16)	(5.61)	(3.57)	(5.42)	(2.42)	(2.60)

**Table 3: Bivariate STS Models with Price and Wage Equations**

	GCC <sub>DW</sub>		SC <sub>D2WP</sub>	
	UR	GDP	UR	GDP
RD <sup>2</sup>	0.18	0.12	0.20	0.15
cycle length λ	27.86		25.56	
std(σ <sub>v</sub> ) <sup>1)</sup>	0.00	5.75	0.00	5.38
std(σ <sub>2</sub> ) <sup>1)</sup>	1.14	1.75	1.15	0.71
std(σ <sub>φ</sub> ) <sup>1)</sup>	-----	4.19	13.54	5.42
Q(12)	28.17	14.64	22.17	11.64
Std(S <sub>t</sub> ) <sup>2)</sup>	0.42	1.27	0.33	1.15
Corr(F <sub>t</sub> , S <sub>t</sub> )	0.90	0.92	0.81	0.70
Std(F <sub>t</sub> -S <sub>t</sub> )/Std(S <sub>t</sub> )	0.43	0.40	0.57	0.52

Note: for notation see Table 1

<sup>1)</sup> values for UR multiplied by 100, for GDP by 1000

<sup>2)</sup> values for GDP multiplied by 100

The coefficients of the cycles in price and wage equations are significant at the 5% level in all cases. As concerns the SC<sub>Δ2wp</sub> model, it is worth noting that, indeed, while GDP and UR cycles are significant in the price and wage equation respectively, this does not hold the other way round. While the smoothed estimates are affected only slightly, both models exhibit a higher correspondence of smoothed and filtered cycles, compared to the respective model without the wage-price block (Fig. 6). In particular, contrarily to the bivariate models, the 1979 boom is now largely detected already from the *filtered* estimate of the GCC<sub>Δw</sub> model. The

equations seem reasonably stable. CUSUM statistics do not indicate instabilities in either model at the 10% significance level (Fig. 7). We also added a random walk component to the wage equation in the  $GCC_{\Delta w}$  model, thereby allowing the constant term to change over time, but did not find evidence for time-variation. Finally, we tested for inflation neutrality in the  $GCC_{\Delta w}$  model as outlined in section 2.2. The changes in the slopes of the GDP and UR trends,  $\Delta^2 y_{t-k}^{tr}$  and  $\Delta^2 ur_{t-k}^{tr}$ , were added to the wage equations at lags 1 to 4. The LR-statistics ( $\chi_4^2$ ) yield insignificant values of 0.79 and 6.43 for GDP and UR trends, respectively. Thus the property of inflation neutrality of the particular trends is not rejected though the higher value for the unemployment rate indicates that there might exist possible longer-term trade-offs between unemployment and real wages.

## 4. Conclusions

Each approach to decompose a series into its long-run trend and cyclical components necessarily needs a set of identifying restrictions. For STS models these are basically given by assumptions on the nature of the stochastic processes underlying the particular components, together with orthogonality restrictions. The above results suggest that the implementation of economically motivated restrictions is feasible in multivariate STS models and that it might improve the estimate of the current cycle.<sup>12</sup> While the univariate STS model for GDP supported a local linear trend plus noise model, the utilisation of the cyclical co-movement of GDP and the unemployment rate revealed a pronounced cycle and a smooth trend. Moreover, we found reasonably stable relationships between price and wage inflation and the cycle. In sum, we imposed the basic cyclical relationships as used in the production approach, however with estimation and testing in a maximum likelihood framework.

More generally, the results appear promising with respect to the applicability of multivariate STS models for the investigation of cyclical relationships. While vector autoregressions may be regarded as an approximation to STS models (Harvey, 1989), the latter have the advantage of providing more direct ways for modelling cyclical relationships. Moreover, since first differencing removes a major part of business cycle frequencies from the data, STS models might in some cases also provide tests of higher power.<sup>13</sup> The higher flexibility of STS models might be illustrated by one further example, related to our work. Blanchard and Quah (1989) used GDP and the unemployment rate in a bivariate VAR and identified the trend

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<sup>12</sup> In fact, the finding that the utilisation of cyclical co-movements improves estimates of the GDP cycle is a rather general one. It is well-known that the Beveridge-Nelson decomposition gives reasonable results only in the multivariate case. Also, Adams and Coe (1990) reported a considerable improvement of their cyclical estimates when the information contained in price and wage movements was exploited in a full systems estimation..

<sup>13</sup> From the gain function of the first difference filter it is evident that it attaches a weight of less than 0.5 to frequencies of around 20-25 quarters. At the same time the variance of short term frequencies is increased with a factor of up to four. As, e. g., Baxter (1994) has pointed out and demonstrated with an empirical example, first differencing might therefore distort relationships at business cycles frequencies to a considerable extent.

component by some variant of the BN-decomposition. The generalised common cycles restriction is clearly similar with respect to the underlying rationale of utilising cyclical co-movements. However, the Blanchard and Quah (1989) approach requires UR to be stationary, while the STS approach does not. The latter might therefore be also applied to those European countries with high unemployment rates.

Finally, for our data the bivariate STS models support a smooth GDP trend with slowly changing slope. Such a trend is integrated of order 2. While this seemingly contradicts the results of unit root tests, various work has provided evidence for the view that it might be, in fact, consistent with them. Harvey and Jäger (1993) have shown by a simulation study that augmented Dickey Fuller tests are very unlikely to detect a possible second unit root. On the other hand, Perron (1989) has found that a unit root in US GDP is rejected, if one single appropriate break point is introduced in the deterministic trend component. In sum, this suggests that indeed a smooth trend might be a reasonable specification and that, in turn, the specification of the GDP trend as a random walk might provide overly volatile estimates.

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