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Abstract

The present paper shows how cointegration analysis within a multivariate framework may be applied for the estimation of energy demand elasticities in order to account for the non–stationarity of the time series used. The dynamic modelling approach followed is one based on general–to–specific modelling within a system. The case is for annual Austrian residential energy demand over the period 1970 to 1993. The explanatory variables used are real energy price, real disposable income, and the temperature variable heating degree days. The results indicate that there is one cointegrating vector only. The long–run energy demand elasticities derived are -0.02 for price, +1.13 for income, and +0.77 for temperature. The long–run system of energy demand makes sense from an economist's point of view and provides evidence that the aggregate price elasticity may be much lower than commonly assumed, and suggested by many other, more traditional empirical studies in this field. In the short–run analysis, the error–correction term turns out to play an important role, thereby clearly disqualifying a traditional VAR model in first differences investigated as a rival model.

Keywords

energy demand elasticities, cointegration, system analysis

JEL-Classifications

C51, C52, H31, Q43

Comments

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1 Introduction

Over the past decade, one of the most important developments in empirical modelling has certainly been the introduction of cointegration analysis in time series econometrics. This has given rise to a renewed interest in the well–known problem that traditional estimation of time series econometric models may, in the presence of unit roots in the time series data, lead to nonsense or "spurious" regression results, or – in case of taking differences – at least to the loss of important long–run information about the underlying data generating process (DGP).

In the energy economics literature, the penetration of aggregate demand studies that use methods of cointegration analysis is still remarkably modest. Outstanding examples are the relatively early study by Hunt and Manning [21] and the more recent work done by Yu and Jin [42], Bentzen and Engsted [4], Engsted and Bentzen [13], and Bentzen [3]. Among these studies, however, only Bentzen and Engsted [4] and Engsted and Bentzen [13] have applied cointegration analysis within a multivariate framework, and none has explicitly employed a progressive modelling strategy like Hendry's general–to–specific approach. Moreover, none of the studies mentioned has focused on the residential sector of energy demand, where conservation potentials are generally regarded to be largest and thus reliable demand elasticity estimates of particularly great importance to energy policy planners and decision–makers, respectively. Lastly, in my opinion these studies neglect to clearly emphasize the importance of starting from the joint density function and testing for weak exogeneity of the variables before using any type of single–equation model.

The present paper, based essentially on more extensive work done in Madlener [31] [32], is designed to fill this gap. It shows in some detail how cointegration analysis within a multivariate framework and in the presence of an naturally exogenous variable may be applied for an improved way of estimating energy demand elasticities. The case is for the log of annual Austrian residential energy demand, denoted by q, over the time period 1970 to 1993. Four explanatory variables, also in logs, are employed: real energy price p, real disposable income of the private households y, and a temperature variable "heating degree days" that enters the model both in levels (h) and in first differences (denoted Δh; cf. Section 3 for a detailed data description). In order to avoid the worst features of data mining (see, for example, Lovell [29]), the dynamic modelling approach followed is one based on general–to–specific modelling within a system. This methodology, originally introduced by Sargan [39], is today most often associated with Hendry (see Hendry [17] for a thorough treatment), and — for historical reasons — sometimes referred to as the "LSE–methodology" (cf. Mizon [33] for a recent discussion of the history of the approach).

In this context, a "system" denotes a system of linear dynamic equations, designed to represent the joint density function of a set of related variables. Thus a system approach is generally a much more comprehensive analysis of the relationships between the variables employed than a single–equation analysis and, as a consequence, also involves a larger modelling burden. Nowadays, however, with the availability of easy–to–use econometrics computer packages for PCs, it is certainly fair to argue that in principle *"... computational problems no longer provide an excuse for avoiding system methods"* (Hendry and Doornik [18], p.5).

As a matter of fact, several very important advantages are linked to a system approach, as compared to more traditional single–equation studies of, say, energy demand (for the case of Austria, Wohlgemuth [41] has recently provided a detailed study of this kind). By modelling the joint distribution of the variables, regime shifts and structural breaks, for example, may be investigated and also their links to changes in the exogeneity status of the variables. Another advantage is the possibility of testing for the precise form of the relationships between the variables. Finally, cointegration is essentially a system property in the sense that the determination of a matrix β of cointegration vectors requires system analysis and is linked to the issue of testing for weak exogeneity.

The structure of the paper is as follows: In Section 2, the stage for the investigation is set up by discussing the system of equations employed and by briefly outlining Johansen's approach to cointegration analysis. Next, in Section 3, the data are presented and inspected, covering some unit root testing. In Section 4, firstly, the general unrestricted system of the three endogenously treated variables q, p, and y is estimated, analyzed, and tested.[1] Secondly, a detailed cointegration analysis and some testing for

[1] Note that the variable h is a–priori regarded to be a natural exogenous variable, determined by

weak exogeneity and (over)identification is pursued. Thirdly, the system is mapped into I(0) space by use of a vector error–correction form and subsequently simplified in order to end up with a satisfactory structural model. Lastly, Section 5 contains a summary of the findings and some conclusions drawn from the analysis.

2 The System

2.1 Introduction of the Concept Used

The class of system considered in this study is a three–dimensional vector autoregressive (VAR) model of the form

$$\mathrm{x}_t = \sum_{j=1}^{k} A_j \mathrm{x}_{t-j} + \Phi D_t + \epsilon_t, \qquad (1)$$

in which $\mathrm{x}_t = (q_t, p_t, y_t)'$ and $D_t = (d_{1,t}, \ldots, d_{s,t})$ is a set of deterministic conditioning variables (such as constant, dummy variables, and trend) and the stochastic temperature variable h, assumed to be both stationary and exogenous, and entered in levels and first differences for reasons that will become clearer later on. ϵ_t is a three–dimensional error vector, independently distributed with mean zero and covariance matrix Σ, i.e. $\epsilon_t \sim$ IN(0,Σ).

Four a–priori assumptions are being imposed in order to obtain a general congruent linear VAR model that represents a valid basis for the inference procedures adopted later on. Firstly, in order to exclude explosive roots, we assume that none of the roots of $det(I - \sum_{j=1}^{k} A_j L^j) = 0$ lie within the unit circle. Secondly, in order to exclude moving average error processes from the category of model being considered, we further assume that k is finite. Thirdly, the initial values x_{1-k}, x_{2-k}, ..., x_0 are assumed to be fixed. Fourthly, the parameters $(A_1, \ldots, A_k, \Phi, \Sigma)$ are required to be constant.

processes outside the system under study, thereby assuming no relevant loss of information by conditioning on it. In other words, as the variable h is *not* caused by human economic behaviour (and only altered remarkably over a much longer time horizon than considered here), its treatment as an exogenous variable is considered to be justified. For a discussion of Hendry's views on "not caused" and "exogenous", respectively, see Hendry [17], Ch.5, esp. p.157. In fact, he uses the interesting example of "energy from the sun impinging on earth".

The following very useful reformulation of Eq. (1) represents an observationally equivalent reparameterization of the VAR system into a vector error–correction (VECM) form (see, inter alia, Hendry, Pagan, and Sargan [20], Engle and Granger [12], Johansen [23], or Banerjee et.al. [1]):

$$\Delta \mathbf{x}_t = \sum_{j=1}^{k-1} \Pi_j \Delta \mathbf{x}_{t-j} + \Pi \mathbf{x}_{t-k} + \Phi D_t + \epsilon_t, \tag{2}$$

where $\Pi_j = (\mathbf{I} - \sum_{i=1}^{j} A_i)$ for $j = 1, 2, \ldots, (k-1)$ are the "interim multiplier matrices", characterizing the short–run behaviour of the system and $\Pi = (\mathbf{I} - \sum_{j=1}^{k} A_j)$ is the matrix of static long–run responses. This transformation imposes no further restrictions. Note, however, that in case $\mathbf{x}_t \sim I(1)$, then $\Delta \mathbf{x}_t$ is I(0). Hence the system specification is balanced only if $\Pi \mathbf{x}_{t-k}$ is I(0). Obviously, in such a case Π cannot have full rank n, since that would contradict the assumption $\mathbf{x}_t \sim I(1)$. Therefore, let $rank(\Pi) = p < n$, and let further α and β be $n \times p$ matrices of rank p such that $\Pi = \alpha\beta'$ and the linear combinations $\beta'\mathbf{x}_t$ are I(0). This yields the following VECM of reduced rank:

$$\Delta \mathbf{x}_t = \sum_{j=1}^{k-1} \Pi_j \Delta \mathbf{x}_{t-j} + \alpha\beta' \mathbf{x}_{t-k} + \Phi D_t + \epsilon_t \tag{3}$$

The linear combinations $\beta'\mathbf{x}_t$ comprise the p cointegrating I(0) relations, while the "loadings matrix" α contains the adjustment coefficients (cf. Johansen [23] [24] and Hylleberg and Mizon [22]). In this sense $\Pi = \alpha\beta'$ defines the short–run adjustment α to the steady–state relations β.

2.2 Cointegration Analysis

In the late 1980s and early 1990s cointegration analysis, originally introduced in the literature by Engle and Granger [12], whose 2-step method for the bivariate case was later on extended by Johansen [23] [24] for the multivariate case, has become a rapidly developing subject and probably even led to something like a "new era" in econometrics. While in the past a great many econometricians have applied ordinary least squares estimation on rather simple log–linear single–equation models, implicitly assuming that the variables under consideration are stationary stochastic processes. More recent research, however, has provided evidence that the majority of economic time series appear to exhibit stochastically changing time trends (cf. the seminal paper by Nelson and Plosser [34]).

In general, non–stationarity in levels has strong implications for the estimation of parameters like, as in our case, energy demand elasticities, because of the rather high probability that the occurring relationships between the variables are of a nonsense (or spurious) nature only (the historically seminal works on this topic are Yule [43] and Granger and Newbold [14]; for more recent discussions see Phillips [36] or Charemza and Deadman [7], among others).

A common way of handling problems of non–stationarity in levels and multicollinearity has been the differencing of the variables before running estimations. This approach, however, suffers from the important drawback that lon–run properties of the data are lost, thereby restricting the models to the explanation of purely short–run effects. The crucial point is that if the long–run information should be retained, then one has to ensure that the existing common (but unrelated) stochastic trends can be separated from the co–movements of the variables due to any prevailing equilibrating forces in the economy (see Harris [16] for a nice and very recent discussion of this problem).

In other words, the central issue in any discussion of this kind is (weak) stationarity of the variables involved. A stochastic process is called *weakly stationary* if its mean and variance are time–independent (i.e. constant), and the autocovariance dependent on the time lag only (i.e. on the gap between the periods and not on the actual point in time at which the autocovariance is being considered). As already mentioned, however, most economic time series simply do not possess this property.

The introduction of the cointegration concept by Engle and Granger [12] provided a potential solution to the problem of non–stationarity in time series econometrics. The idea of this concept is that although the variables employed are individually non–stationary I(1) processes, linear combinations of the variables may nevertheless be stationary, i.e. I(0) processes. If this is the case, the variables are said to be "cointegrated". Formally, an $n \times 1$ vector $\mathbf{x}_t$ of variables is cointegrated if a linear function $z_t = \beta' x_t$

exists that is I(0) for some non–zero $n \times 1$ vector β.

In sum, therefore, the proper way to estimate time series data in the sense of cointegration analysis is the following: First, the time series properties in terms of integration and cointegration of all the variables involved in the model or system, respectively, have to be evaluated (for a discussion see Banerjee and Hendry [2]). Secondly, provided the variables are indeed non–stationary but cointegrated, long–run elasticities may be estimated by cointegration methods and short–run elasticities may be derived by employing a (parsimonious) vector error–correction model (VECM) in which the cointegration relations are explicitly included.[2] In addition, if any variables turn out to be weakly exogenous, then conditioning on these variables is possible, leading to a conditional VECM model.

The Johansen maximum likelihood procedure followed in this study (Johansen [23] [24], modified in Johansen and Juselius [27] to allow for dummy variables and in Johansen [25] for partial systems), enables the empirical determination of the reduced rank p of the system (i.e. the dimension of the cointegrating space), provided that the systems in Eq. (1) and Eq. (2), respectively, are well specified and, in particular, have constant coefficients and homoskedastic innovation errors.

3 The Data and Some Data Analysis

3.1 Description and Sources

Austrian annual data for the four variables log of energy consumption q, log of real energy price p, log of real disposable income y, and log of heating degree days h, are employed for the time period 1970 to 1993.

The consumption variable used is that of final energy consumption of the private household sector, which in 1992, for instance, accounted for remarkable 41.5% of the total final energy consumption in Austria. The figures used are published annually by the Austrian central statistical office ÖSTAT in *Statistische Nachrichten* under the title "Energieaufkommen und -verwendung in der österreichischen Volkswirtschaft 19xx" and

[2] As Engle and Granger [12] have shown, a cointegrated system can always be represented by a valid dynamic ECM and vice versa.

comprise energy consumption for all major end uses (i.e. also private automobile fuel consumption).

The price variable used is an aggregate real energy price index (deflated by the consumer price index) for the private household sector, calculated by the Austrian energy agency "Energieverwertungsagentur" (EVA) from the official ÖSTAT consumer energy expenditure statistics (i.e. price indices and weight factors). This index is essentially unpublished but can be obtained upon request either from the author or directly from EVA (EVA, c/o Mr Fickl, Linke Wienzeile 18, A-1060 Vienna, Austria).

The income variable used is real disposable income of the private households, taken from the Economics Database of the Austrian Institute of Economic Research WIFO (WIFO, P.O. Box 91, A-1103 Vienna, Austria) and indexed at constant 1976 prices.

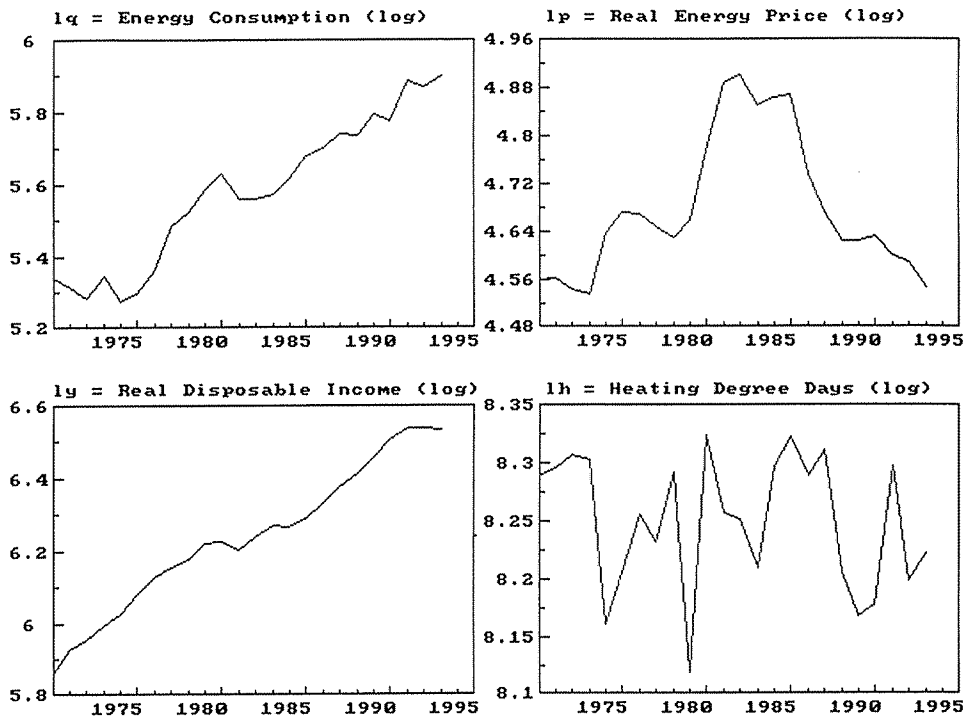
Finally, the variable heating degree days is the sum of annual heating degree days, defined as $HDD = \sum_n (BT - T_n)$, and published annually by ÖSTAT in *Energieversorgung Österreichs - Jahresheft 19xx*. BT denotes a constant ambient temperature of 20 °C and T_n the average outdoor air temperature of the day. Days are only counted as heating degree days (by means of the counting index n) if the outdoor air temperature of that day is below or equal to an assumed marginal heating temperature of 12 °C (cf. Austrian standard ÖNORM B8135). The heating degree days reported are average values, calculated as a weighted arithmetic mean of the sums of heating degree days in the nine Austrian provinces, and the weights employed being 1991 census data.[3]

3.2 Visual Inspection of the Time Series

Visual inspection of the graphs in Figure 1 indicates that both q and y have strong upward trends of a similar magnitude. Hence they might be modelled as stationary deviations from a linear deterministic trend or, alternatively, as variables with stochastic trends within a cointegrated system.

[3] Note that due to a change in calculation in the mid–1970s and also the measurement technique, a mark–up of 51.9% was applied to the pre–1977 values in order to align them with the post–1977 values (according to the old method, BT was set equal to 18 °C and T_n to 15 °C.

Figure 1: *Data plots, 1970–93 (in logs)*

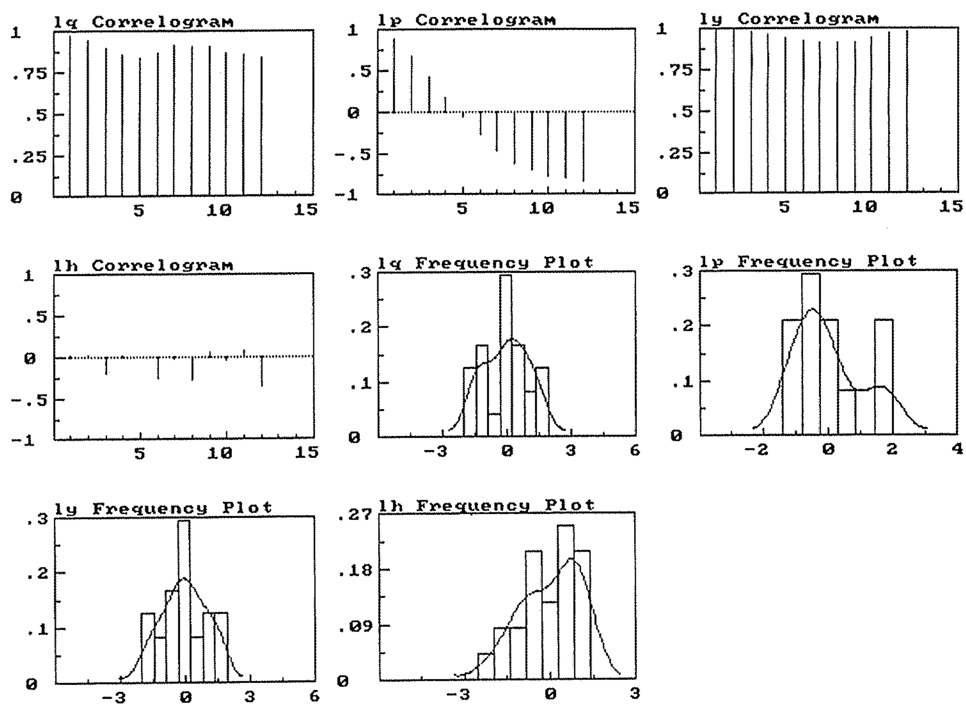


q reflects the dramatic impact of the two oil crises of 1973/74 and 1979/80 very well. y exhibits two interesting kinks, which could result from some structural change or regime shifts and will probably have to be modelled by one or more dummy variables. Moreover, the curve flattens out remarkably at the end of the time period studied. p shows mainly, as expected and in accordance with the developments of the residential energy demand curve, the dramatic effects induced by the two oil price shocks of the 1970s, as well as the energy price slump after 1985 back to the price level of the early 1970s (in real terms). Lastly and as expected, h obviously does not seem to follow any identifiable pattern, but depicts a distinctive trough in the year 1979, which is likely to play a role in the modelling and estimation outcomes, respectively.

Figure 2 allows an inspection of the correlograms, the densities, and the histograms of the four variables. The first order serial correlation coefficients are very high for q, p, and y, declining only very slowly with the higher order coefficients in the case of q and y, and declining, changing sign, and rising again in the case of p. As expected, the correlation coefficients are comparatively very low for h. These indications are consistent with the idea that each of the series q, p, and y is indeed non–stationary and probably integrated of order one, I(1), i.e. that differencing is necessary to remove their stochastic non–stationarity properties and make them I(0), whereas h seems to be stationary.

It should be kept in mind, however, that visual inspection only allows a first rough guess with respect to the data properties. We will (hopefully) gain more insights about the series' properties as the investigation proceeds.

Figure 2: *Residual correlograms, residual densities, and histograms, variables in levels (and logs), 1970–93*



3.3 Unit Root Testing

At this point we continue the analysis by using some univariate test statistics to test for unit roots in order to be able to determine the order of integration of the variables and to find some further evidence with respect to the stationarity properties of the data. The unit root tests have been pursued by using the econometrics computer package PcGive 8.0, developed by Doornik and Hendry [11].

The principle behind unit root testing is that the time series properties of each single variable may be determined by employing, for example, Dickey–Fuller (DF), Augmented Dickey–Fuller (ADF), or Phillips's Z–statistics (see Dickey and Fuller [8] [9] and Phillips and Perron [37], respectively). Note, however, that such univariate test results can provide no more than some coarse evidence with regard to the order of integratedness of the series, as they all suffer from some sort of size and/or power problem (very often there is a strong trade–off between the two). The main reason is that the associated tests are conditional on untested – and usually unlikely – auxiliary hypotheses concerning parameter constancy in the scalar representations. Furthermore, it should be pointed to the fact that the order of integration is being determined for a certain period in time and in this sense is not an "inherent property" of a time series (for example could one change the time period considered and well get a different result with respect to the integratedness of the series).

The analysis done here is restricted to the ADF test, at present probably the most popular and still widely regarded as being the most efficient test among the simple tests for integration. Table 1 depicts the values of the ADF test statistics, which take the form of the t-statistic for the hypothesis "$\phi = 0$" in the regression model

$$\Delta z_t = \phi z_{t-1} + \sum_{j=1}^{k} \phi_j \Delta z_{t-j} + \mu + \xi_t, \tag{4}$$

for any of the four variables, z_t, where Δ denotes first differences, μ is drift, and ξ_t an error term. Note that when $\phi = 0$, Eq.(4) is a regression in the differences Δz_t, corresponding to z_t being well modelled as an I(1) process and thus having a unit root in its autoregressive representation. Tables of critical values for the non–standard distribution for testing of the null hypothesis "$\phi = 0$" can be found, for instance, in Dickey and Fuller [9].

Table 1: *Augmented Dickey-Fuller test statistics*

Variable	t (ADF)	lag length	Variable	*t* (ADF)	lag length
q	-1.1187	3	Δ_q	-4.5751***	4
p	-1.3929	1	Δ_p	-2.7015*	1
y	-1.2062	3	Δ_y	-2.0404	3
h	-2.8186*	2	Δ_h	-4.4972***	2

(Notes: '*' denotes statistics significant at the 10 % level, The critical values used are those derived from the response surface in MacKinnon (1991); for variables in levels they are -2.66^{*}, -3.03^{**}, and -3.83^{***}, respectively, for variables in first differences -2.66^{*}, -3.04^{**}, and -3.86^{***}.)

Choosing the appropriate lag length is a discussion on its own. In the study presented here, I followed essentially the strategy suggested by Hendry and Doornik [18], i.e. to select the longest significant lag within the maximum lag length. Thus the reported values are for the t-statistic corresponding to the longest significant lag (at significance level $\alpha = 10\%$). In cases of no significant t-value amongst the lags, I have chosen the lag length with the highest (absolute) t-value or lowest t-probability, respectively. The maximum lag length for the calculation of these statistics was four.

On the basis of the ADF-test statistics, all variables apart from h seem to be non–stationary. y even appears to be I(2), which is rather implausible on economic grounds (and even more taking the visual inspection of the y-series into account). Although not reported in Table 1, an ADF-test on the second differences of y (i.e. testing for I(2)) yielded a result significant at the 1% level of -6.587 for y (critical value at 1% is -3.888, derived from the response surfaces in MacKinnon [30]). Hence unit–root testing provides some evidence that y could be I(2). Moreover, the additional inclusion of a linear trend in Eq. (4) showed some indication of trend–stationary attributes in both the sample of q (at the 10% significance level) and y (at the 1% level).

Based on the results of the visual inspection, the ADF-test statistics, and the knowledge that the ADF-test has a general tendency to under-(over-)reject the null when it is false (true) (i.e. its poor size and power properties), we will assume for the further analysis that h is stationary, and that – despite some ambiguity – that the three variables q, p, and y are all non–stationary I(1) variables. This seems to be reasonable, given the questionable robustness of the unit root test procedure used in providing a substantial method to discriminate between stationary and non–stationary processes.

As a consequence of the non–stationarity assumption for q, p, and y, it will be important in the modelling procedure to choose models that can represent the non–stationarity of these variables. In other words, we look for the possibility that these variables form a cointegrating relationship, i.e. that there is some linear combination that is I(0), i.e. stationary. In actual fact and based on the economic reasoning that the temperature variable h should be included for establishing a sensible long–run relationship between the three non–stationary variables, I have allowed it to enter the cointegrating space (see Subsection 4.1 below). Note that a mix of I(1) and I(0) variables does not prevent cointegration from being present. However, what would be expected is that the number of cointegration relations increases by one (as each I(0) variable is stationary "in itself" and consequently should form a cointegration relation "in itself").

Finally, yet another characteristic of the variables energy consumption, energy price, disposable income, and heating degree days is worth mentioning, viz. that they are, in fact, non–negative. Consequently, the use of linear models in their logarithmic transformations is certainly data–admissible in the sense that they cannot produce negative fitted or predicted values.

4 Empirical Results

4.1 Testing the Initial General System

The analysis that follows may be seen as one of studying the demand for energy in the private household sector by commencing from the joint data density and testing the reductions required to eventually validate single–equation modelling. For the empirical investigation, I have employed the econometrics computer package FcFiml 8.0, as described in Doornik and Hendry [10]. In principle, the modelling sequence aspired to is the following: (i) VECM in I(1) space; (ii) VECM in I(0) space; (iii) parsimonious VECM; (iv) structural model; (v) conditional structural model in case of weakly exogenous variables.

We begin our analysis with the estimation of a VECM like the one introduced in

Eq. (3), re–written in an equivalent form as

$$\Delta \mathbf{x}_t = \sum_{j=1}^{k-1} \Psi_j \Delta \mathbf{x}_{t-j} + \alpha\beta' \mathbf{x}_{t-1} + \Phi D_t + \epsilon_t. \tag{5}$$

If we furthermore allow h_{t-1} to enter the cointegration relations (i.e. $\beta'\mathbf{x}_{t-1}$), we may reformulate Eq. (5) as

$$\Delta \mathbf{x}_t = \sum_{j=1}^{k-1} \Psi_j \Delta \mathbf{x}_{t-j} + \alpha\beta^{*'} \mathbf{x}^*_{t-1} + \Phi^* D_t + \epsilon_t, \tag{6}$$

serving as the benchmark model. Note that β^* becomes an $(n+1) \times p$ matrix and $\mathbf{x}^*_{t-1}$ an $(n+1) \times 1$ vector of variables. As the lack of observations narrows the choice of a common lag length k to be included, I have decided to moderately overparameterize the benchmark model with two lags on q, p, and y (i.e. $k = 2$). This seems to be a suitable compromise between whitening the residuals and allowing the short–run behaviour to be modelled by the $\Delta \mathbf{x}_{t-j}$ on the one hand, and saving on degrees of freedom on the other hand.

D_t contains an unrestricted constant, the temperature variable in first differences, Δh_t, assumed to be stationary, exogenous, and having a short–run influence only, as well as the two impulse dummy variables imp_{79} and imp_{86} (which take on the value of one for 1979 and 1986, respectively, zero otherwise). Preliminary analyses showed that including these two dummy variables leads to an improvement of the overall statistical properties of the system, whereas the inclusion of a deterministic trend did not. In particular, imp_{79} has been included to both proxy the apparent regime shift in the y–equation and to take care of the 1979 outlier in the h-variable, while imp_{86} seems to help modelling the enormous price slump after 1985 and the related consequences to the other variables. Note that both dummies have been entered unrestrictedly, since we can anticipate that they do not have any long–run effects on the modelled variables. The estimation period is 1972 to 1993.

The various test results displayed in Table 2 indicate that the benchmark system is indeed congruent. The hypothesis that the equations exhibit no serially correlated, heteroskedastic, or non–normal residuals are not being rejected. Moreover, no ARCH effects could be detected.

Table 2: *System diagnostic statistics*

Test statistic		Δq_t	Δp_t	Δy_t
Portmanteau	(3 lags)	2.4896	5.3267	0.8840
AR 1-1	F(1,10)	1.4562 [0.26]	0.0892 [0.77]	0.9757 [0.35]
Normality	χ^2 (2)	1.6515 [0.44]	1.1524 [0.56]	2.9570 [0.23]
ARCH 1	F(1, 9)	0.1490 [0.71]	0.4600 [0.51]	0.0121 [0.91]
Vector Portmanteau	(3 lags)	= 15.694		
Vector AR 1-2	F(9,14)	= 0.8467 [0.59]		
Vector Normality	$\chi^2(6)$	= 5.1890 [0.52]		

(Notes: p-values in [ ] where applicable; In PcFiml, diagnostic testing is performed both at the individual equations' and at the (multivariate) system level. Individual equation diagnostics take the residuals of each equation of the system in turn, and treat them as if they were from a single equation. The "Portmanteau statistic" is a degree of freedom-corrected version of the Box and Pierce (1970) statistic (sometimes also referred to as "Ljung–Box" or "Q*" statistic), designed as a goodness–of–fit test in stationary ARMA models. For a detailed description of the employed test statistics see Hendry and Doornik (1994).)

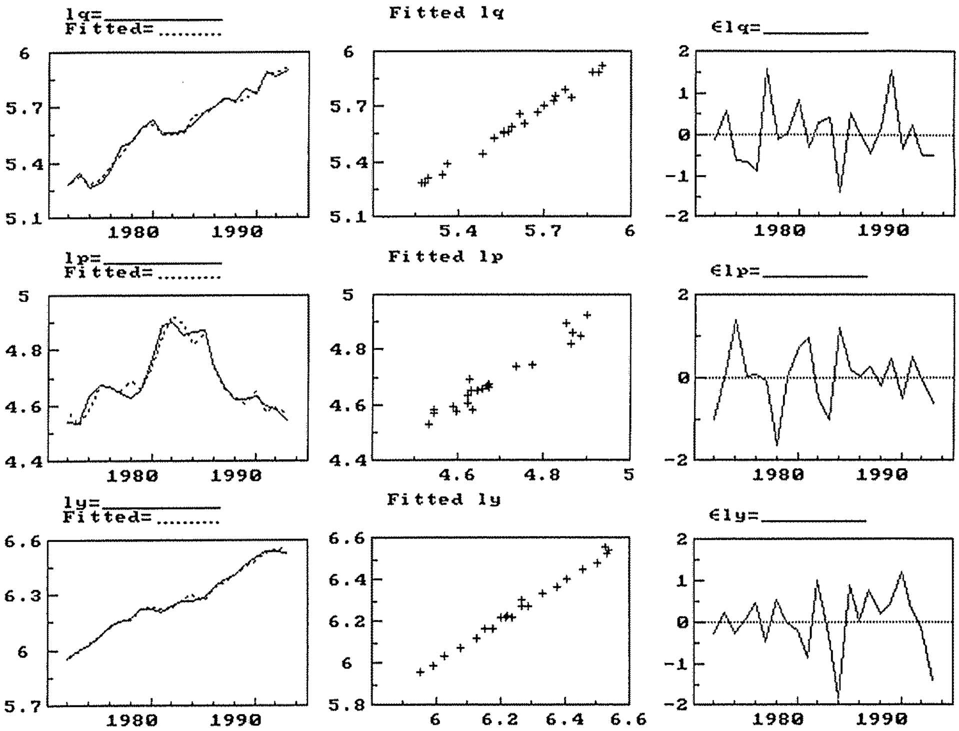
The correlation matrix of the residuals is reported in Table 3. As can be seen, there is one moderately large positive correlation between y_t and q_t residuals (+0.28) and one negative correlation of about the same size between y_t and p_t residuals (−0.29).

Table 3: *Residual correlations*

	q_t	p_t	y_t
q_t	1.0000		
p_t	-0.1137	1.0000	
y_t	0.2759	-0.2946	1.0000

Figure 3 shows the three sets of actual and fitted values, their cross plots, and the scaled residuals for each equation. It may be regarded as something like a "condensed view" of the descriptive power of the system. Despite the lack of fine detail in the plots, the differences in goodness–of–fit between the three equations are nonetheless quite obvious.

Figure 3: *System actual and fitted values, crossplots, and scaled residuals, unrestricted VAR system in I(1) space, 1972–93*

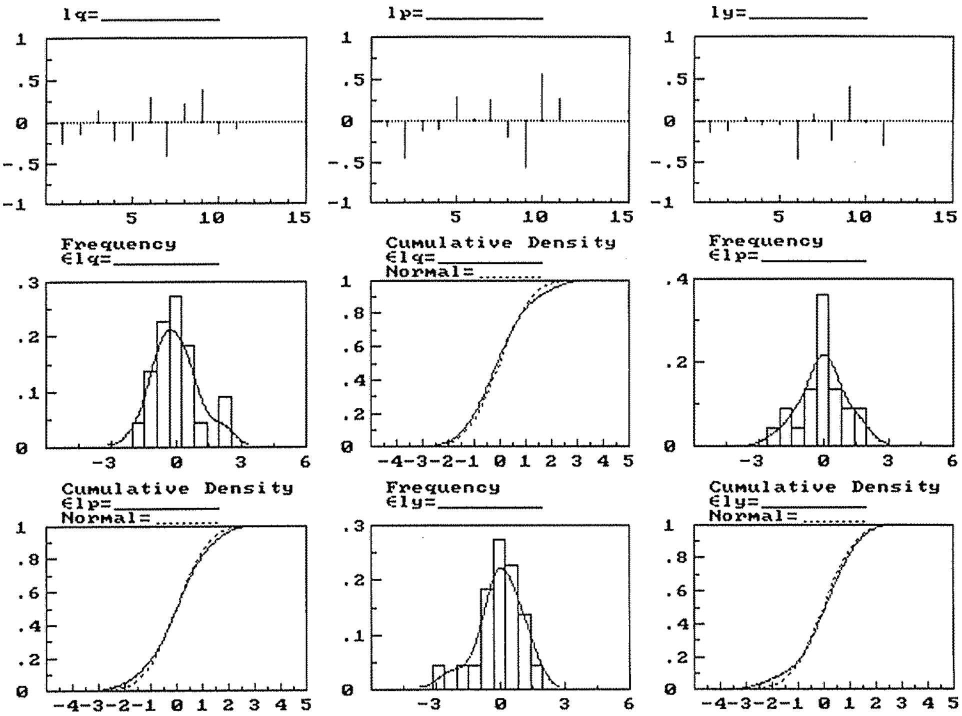


The correlations between the actual outcomes and the fitted values in each equation are given below. The very high values reflect the non–stationarity in the data and hence do not by themselves ensure a sensible model. Note that in a multivariate context, the squares of these correlations are the nearest equivalent to R^2 (cf. Hendry and Doornik [18]).

q	p	y
0.9942	0.9692	0.9969

The residual correlograms, density, histogram, and distribution plots of the individual equations are presented in Figure 4 and provide further indications for the approximate congruence of the system. Note that despite the fact that none of the specification tests could be rejected, there is some evidence of residual serial correlation in all three equations.

Figure 4: *Graphical diagnostics for the individual equations: redsidual correlograms, density, histogram, and distribution plots*



Next, the F-tests for the various variables in the system are considered. In Table 4, these are shown for the overall significance of each regressor in the system first, i.e. its contribution to all three equations taken together. Secondly, the result of the F-test against the four unrestricted regressors (i.e. the constant, Δh, and the two impulse dummies imp_{79} and imp_{86}) is reported.

Table 4: *F-tests on the retained and the unrestricted regressors*

F-tests on retained regressors, F(3, 9):			
q_{t-1}	3.2397 [0.07]	q_{t-2}	1.4505 [0.29]
p_{t-1}	10.2382 [0.003]***	p_{t-2}	1.5902 [0.26]
y_{t-1}	3.9585 [0.047]**	y_{t-2}	0.4746 [0.71]
h_t	2.5286 [0.12]		
F-test against unrestricted regressors, F(12,37) = 60.214 [0.00]***			

(Notes: p-values in brackets. '***' denotes significance at the 1% level, '**' at the 5% level. Variables entered unrestricted: (*i*) constant; (*ii*) imp_{79}, (*iii*) imp_{86}, (*iv*) Δh.)

As can be seen from the test outcomes (and as expected), the restricted regressors with lag one matter much more than those lagged two periods, although only p_{t-1} and y_{t-1} turn out to be significant at the 1% and 5% level, respectively. q_{t-1} and h_{t-1} only matter at the 15% significance level. However, as has already been noted, all regressors with lag two will be retained in the system in order to be able to model the short–run behaviour of the system. Finally, the hypothesis that the parameters of all unrestricted regressors are jointly equal to zero is clearly being rejected at the 1% level of significance (F(21,26) = 31.659 (0.00)).

Before turning to cointegration analysis, the introductory investigation into the general VECM in I(1) space is rounded up by reporting some dynamics features, viz. the long–run matrix $\hat{\Pi} = \hat{\pi}(1) - \mathbf{I}$, the long–run covariance matrix, the eigenvalues of $\hat{\Pi}$, and lastly the eigenvalues of the companion matrix (see Table 5).

Table 5: *Dynamic analysis of the VECM in I(1) space*

long-run matrix $\hat{\Pi} = \hat{\pi}(1) - \mathbf{I}$			
	q_t	p_t	y_t
q_t	−0.8277	−0.0237	0.8836
p_t	0.4758	−0.0361	−0.5966
y_t	−0.0941	0.0146	0.0578
long-run covariance:			
	q_t	p_t	y_t
q_t	0.1003		
p_t	−0.0407	0.5390	
y_t	0.0887	−0.0269	0.0795
eigenvalues of $\hat{\Pi} = \hat{\pi}(1) - \mathbf{I}$ *:*			
real		complex	modulus
− 0.6923		0.0000	0.6923
− 0.0568		0.0337	0.0661
− 0.0568		−0.0337	0.0661
eigenvalues of the companion matrix:			
real		complex	modulus
0.9208		0.0401	0.9217
0.9208		−0.0401	0.9217
− 0.0326		0.0000	0.0326
0.3074		0.2701	0.4092
0.3074		−0.2701	0.4092
0.3285		0.0000	0.3285

As indicated by the two small eigenvalues of $\hat{\Pi} = \hat{\pi}(1) - \mathbf{I}$, the rank of the long–run matrix seems to be less than three, which is consistent with the apparent non–stationarity of the data used. Note, however, that they are also greater than zero, suggesting some cointegration between the variables. The companion matrix exhibits no roots outside the unit circle, which is consistent with the idea of a non–explosive system. Also, the number of roots close to unity is not larger than the dimension of the long–run matrix, which is consistent with the idea of an I(1) system. Note that the fact of two moduli being close to unity suggests that there could possibly be two cointegrating relations.

4.2 Cointegration Analysis

In what follows, we will pursue a thorough cointegration analysis of our system of equations, using the maximum likelihood method of cointegration analysis developed by Johansen [23] [24].

First, in order to investigate the order of integration among the n variables, recall the reformulated VAR system introduced in Eq. (6):

$$\Delta \mathrm{x}_t = \sum_{j=1}^{k-1} \Psi_j \Delta \mathrm{x}_{t-j} + \alpha \beta^{*'} \mathrm{x}^*{}_{t-1} + \Phi^* D_t^* + \epsilon_t. \tag{7}$$

Note that although the error term $\epsilon_t \sim IN(0, \Sigma)$ and thus stationary, the $(n+1)$ variables comprised in $\mathrm{x}^*{}_{t-1}$ need not be so. The rank p of $\Pi = \alpha\beta'$ determines the number of stationary linear combinations of variables. In particular, the following three cases may be distinguished: (i) if $p = (n+1)$, (i.e., Π has full rank) then all variables in x_t have to be stationary for a "balanced" relation; (ii) if $p = 0$ (i.e., Π is the null matrix) , the model is expressed entirely in differences and there are no cointegrating relations; (iii) if $0 < p < (n+1)$, then there are p cointegrated (stationary) combinations of x_t. In other words, only case (iii) is really interesting with respect to cointegration.

Table 6 summarizes the results of the unrestricted cointegration analysis. It reports the eigenvalues μ_i, the log–likelihood values $l = -\frac{T}{2}\sum_{i=1}^{p} log(1-\mu_i)$, the associated maximum eigenvalues $Max = -Tlog(1-\mu_r)$, and the trace statistics $Trace =$

$-T\sum_{i=1}^{p} log(1-\mu_i)$. Also reported are the test statistics adjusted for degrees of freedom, i.e. by using $(T-mk)$ instead of T, following Reimers [38] (critical values are those from Osterwald–Lenum [35]; T denotes the sample size, m the number of variables in the model, and k the lag length used in estimating Eq. (6)).

Table 6: *Cointegration statistics*

μ_i		l_i				*rank p*
		241.111				0
0.7295		255.493				1
0.3246		259.810				2
0.0349		260.201				3
H_o: *"rank=p"*	*Max*	*Max(T-mk)*	*95%*	*Trace*	*Trace(T-mk)*	*95%*
p = 0	28.76***	20.92	21.0	38.18***	27.77	29.7
p ≤ 1	8.63	6.28	14.1	9.42	6.85	15.4
p ≤ 2	0.78	0.57	3.8	0.78	0.57	3.8
standardized estimated eigenvectors $\hat{\beta}_i'$:						
	q_t	p_t		y_t	h_t	
$i=1$	1.000	0.019		−1.117	−0.713	
$(i=2)$	11.05	1.000		5.462	58.92	
$(i=3)$	−0.999	−2.800		1.000	5.132	
standardized estimated adjustment coefficients $\hat{\alpha}_i'$:						
	$i=1$		$(i=2)$		$(i=3)$	
q_t	−0.8001		−0.0023		0.0022	
p_t	0.5303		0.0035		0.0152	
y_t	−0.0709		−0.0027		−0.0066	

(Note: '***' denotes significance at the 1% level.)

Note that, in general, the estimated αs and βs are identified only up to linear transformations, i.e. any non–singular matrix of full rank may be employed in order to get new αs and βs that together give the same matrix Π (in our case, the βs are standardized such that the dependent variable in each equation of the VECM is unity; the αs have been rescaled accordingly).

As can be seen from Table 6, the results from the unrestricted cointegration analysis formally support the hypothesis that, on the basis of the maximum eigenvalue (Max) and trace ($Trace$) test statistics, there is only one eigenvalue significantly different from zero (at the 1% level). Adjusted for degrees of freedom, i.e. including Reimers's small sample correction, no cointegrating vector can be detected (although it can be seen that the test outcome is indeed very close to the 5% level of significance). As it is still unclear, however, whether Reimers's correction should be the preferred solution or not (in particular, Kostial [28] has reported a tendency of Reimers' values to underestimate the dimension of the cointegrating space even when unadjusted), it appears to be justified to assume that the outcome determines the rank of Π to be equal to one. In other words, a single significant eigenvector reports the estimated cointegration vector as being representative for the cointegration space, which implies that the system is already identified. From Table 6 we can also see that the estimated cointegrating vector takes the form

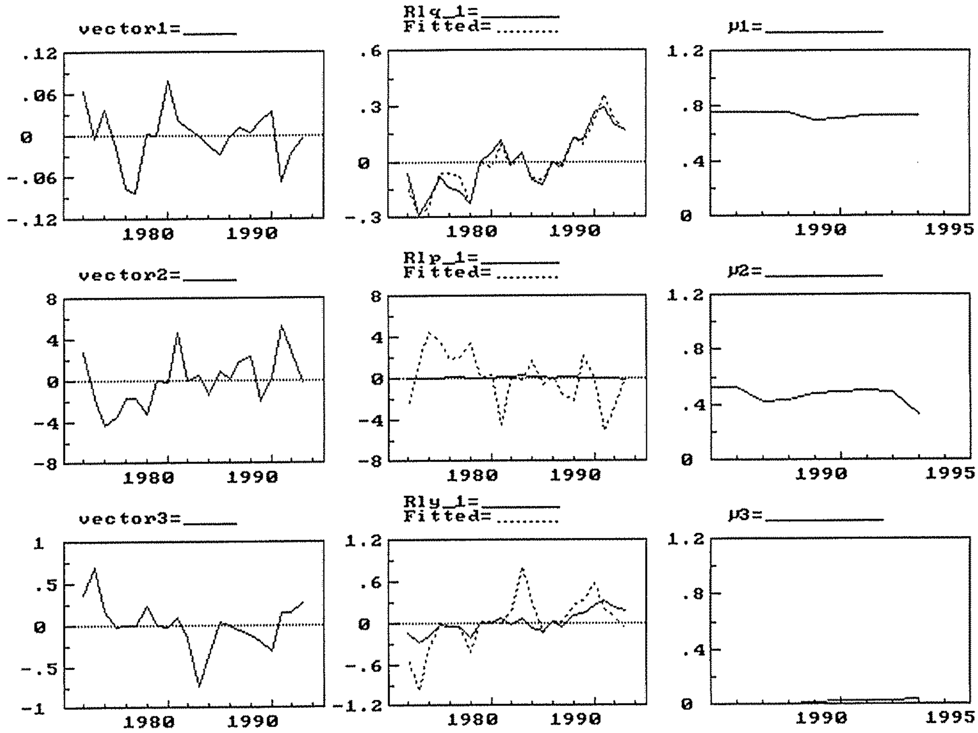
$$\hat{\beta}_1^{*\prime}\mathbf{x}^*{}_t = q_t + 0.019p_t - 1.117y_t - 0.713h_t. \quad (8)$$

In fact Eq. (8) may be read off directly as the lon–run energy demand relationship of the private households sector (i.e. $\mathrm{E}(\hat{\beta}_1' x_t) = 0$):

$$q_t = -0.019p_t + 1.117y_t + 0.713h_t \quad (9)$$

The estimated elasticities all have the expected signs and are of a quite reasonable magnitude (although the price elasticity estimate turns out to be unexpectedly low). Figure 5 provides another check for the adequacy of the model. It gives the plot of the time series of the cointegrating vectors $\hat{\beta}^{*\prime}\mathbf{x}^*{}_t$, the normalized variable x_{it} against the sum of the non–normalized coefficients $(-)\sum_{j\neq 1}\hat{\beta}_{ij}^{*\prime}x_{jt}^*$ (i.e. the long–run actual against the fitted) for $\mathbf{x}_t{}' = (q_t, p_t, y_t)$, and finally the recursively calculated eigenvalues (having partialled out both the full sample short–run dynamics and the unrestricted variables).

Figure 5: *Time series of the cointegrating vectors and recursive eigenvalues*



Only the first of the cointegrating vectors looks fairly stationary. Similarly, the actual and fitted values are close to each other for the first only, whereas the first and the third of the eigenvalues are reasonably constant at a non–zero value (in the case of μ_1) and at almost zero (in the case of μ_3). But again these judgements are basically no more than "visual guesses" (especially the estimation of the recursive eigenvalues should be called into question with regard to the lack of observations).

The single cointegrating vector ensures unique identification of every element such that the resulting characterization of the long run matches that of the DGP. Nonetheless, in order to learn more about the underlying DGP and also to test for weak exogeneity, I have imposed some overidentifying restrictions on the α_is and the cointegrating vector $\hat{\beta}_1^{*\prime}\mathbf{x}^*{}_t$ as a next step in the analysis. Before testing for weak exogeneity, note that theory suggests that q_t depends on p_t, y_t, and h_t; y_t probably depends on p_t.

The adjustment parameters α_{i1} determine the impact on the $\Delta\mathbf{x}_t$s in Eq.(7) when the cointegrating relationship is in disequilibrium. Thus a zero coefficient in any single

α–vector indicates weak exogeneity of the corresponding variable for the estimation of the long–run parameters in question. For example, if we reconsider the first α–vector reported in Table 6, corresponding to the first cointegrating vector, i.e. $\alpha = (-0.8001, 0.5303, -0.0709)'$, then we may conclude from the inspection of this vector that Δq_t responds to the disequilibrium changes represented by the long–run cointegrating relationship at a speed of 80%. Put differently, four fifths of the adjustment towards equilibrium is made within the same year. Note that the adjustment coefficients in the remaining two equations are very small, which clearly supports the weak exogeneity assumption (cf. Table 6).

Both the overidentification restrictions imposed and the test outcomes are reported in Table 7. Since the hypotheses are linear on an I(0) parameterization of the system (i.e. in the case of cointegration, if $\mathbf{x}_t \sim I(1)$ then both $\Delta\mathbf{x}_t$ and $\beta'\mathbf{x}_t$ (and consequently $\beta^{*'}\mathbf{x}_t$) are I(0)), the test statistics are conventional likelihood ratio (LR) statistics with limiting χ^2–distributions and the degrees of freedom equal to the number of independent restrictions to be tested. In particular, I have tested whether α_{11}, α_{21}, or α_{31} are equal to zero, a necessary condition for q_t, p_t, and y_t, respectively, to be weakly exogenous for the parameters of the long–run energy demand equation (for detailed discussions on the issue of weak exogeneity see, inter alia, Boswijk [5], Hendry and Mizon [19], Mizon [33], Johansen [25] [26], and Urbain [40]). Moreover, I have tested for a unitary long–run income elasticity (which is equivalent to testing the hypothesis that $\beta_{13} = -1.0$).

Table 7: *Testing for identification and weak exogeneity*

H_o	*statistic*	*p-value*
$\beta_{31} = -1.0$	$\chi^2(0) =$ 1.86e-007	[1.00]
$\alpha_{11} = 0$	$\chi^2(1) =$ 17.941	[0.00]***
$\alpha_{21} = 0$	$\chi^2(1) =$ 7.788	[0.005]***
$\alpha_{31} = 0$	$\chi^2(1) =$ 0.624	[0.43]
$\alpha_{31} = 0$; $\beta_{13} = -1.0$	$\chi^2(2) =$ 0.624	[0.43]

(Note: '***' denotes significance at the 1% level.)

As can be seen from Table 7, the hypothesis that the long–run income elasticity is equal to unity cannot be rejected. Furthermore, the null hypotheses that income is weakly exogenous for the long run energy demand equation is not rejected (i.e. $\alpha_{31} = 0$). By contrast, testing for weak exogeneity of the price variable is clearly rejected. Note that this outcome provides evidence that — despite the outcome of a single cointegrating vector — single–equation modelling cannot be considered as being equivalent to system analysis. Finally, and as expected, the hypothesis that the energy demand variable is weakly exogenous in the long–run demand equation is rejected clearly as well (at the 1% level of significance).

4.3 Mapping into I(0) Space

For a system that incorporates non–modelled I(1) variables to be well defined in terms of I(0) modelled variables, all non–stationarity stemming from the non–modelled variables has to be mapped into cointegrating vectors. In cases where the locations of the unit roots and the cointegrating vectors are known, the system as a whole may be mapped into I(0). Thereafter, conventional asymptotics can be applied. On the contrary, in cases where the locations of the unit roots are unknown but the conditional distribution of the modelled variables given the non–modelled is stationary, the analysis proceeds as in Johansen [25] (cf. also Hendry and Mizon [19]).

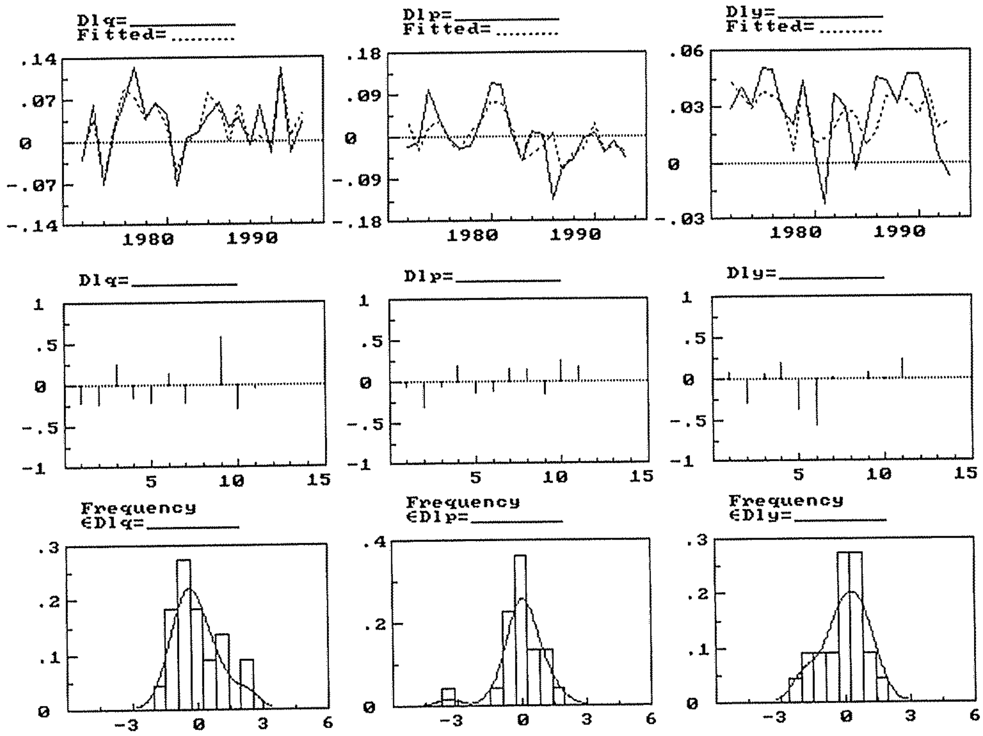
In particular, we can map the reduced rank system adopted (i.e. with $r = 1$ and $k = 2$) from I(1) into I(0) space by employing the following re–parameterization:

$$\Delta \mathrm{x}_t = \Psi_1 \Delta \mathrm{x}_{t-1} + \gamma ecm_{t-1} + \Phi^* D_t^* + \epsilon_t, \qquad (10)$$

where Ψ_1 is equal to Ψ_j in Eqs. (6) and (7) for $j = (k-1) = 1$, and $ecm_t = q_t - 0.0214p_t + 1.1310y_t + 0.7677h_t$ is the restricted cointegrating vector. γ measures the impact on $\Delta \mathrm{x}_t$ of being away from the long–run equilibrium or, put differently, to what extent households correct the errors of past decisions. Ψ_1 characterizes the short–run dynamics of the system.

Estimation and evaluation of this VECM in I(0) space (i.e. Eq. (10)) revealed that the original system's congruence is maintained. Next, both Δq_{t-1} and Δy_{t-1} (which turned out to be insignificant to the system) have been deleted, leading to a much stiffer competitor for any model developed thereof. Visual inspection of the graphs of actual and fitted values of Δq_t, Δp_t, and Δy_t, the residual correlograms, and the density and histogram plots depicted in Figure 6 confirms that the original system's congruency could still be maintained.

Figure 6: *Actual and fitted values, residual correlograms, and frequence plots, parsimonious VAR system in I(1) space*



Note that the imposition of a rank $r = 1$ on the cointegrating space (as indicated by the statistics reported in Table 6) and the deletion of both Δq_{t-1} and Δy_{t-1} already led to a reduction by 20 parameters, as compared to the original system. I will refer to this model as the parsimonious VECM (PVECM) system in I(0) space, despite the fact that there is still some remaining margin for further simplification (viz. Δp_{t-1} is only significant at the 15% level and could also be deleted if desired).

4.4 Empirical Simplifications of the PVAR

The first simplified model to be considered (referred to as SM I) is one in which all insignificant parameters have successively been eliminated. Altogether, another nine(!) restrictions have been imposed relative to the PVECM. Table 8 presents the FIML–estimates of this model.

Table 8: *FIML estimates, SM I*

$$\Delta q_t = -6.0009 - 0.7805 ecm_{t-1} + 0.5630 \Delta h_t + .1238 imp_{79}$$

(SE) (0.98) (0.13) (0.07) (0.03)

$$\Delta p_t = 3.3083 + 0.4280 ecm_{t-1} + 0.3606 \Delta p_{t-1} - 0.1311 imp_{86}$$

(SE) (1.44) (0.19) (0.15) (0.04)

$$\Delta y_t = -0.0036 ecm_{t-1}$$

(SE) (0.00)

$$ecm_t \equiv q_t + 0.0214 p_t - 1.1310 y_t - 0.7677 h_t$$

The specification test results reported in Table 9 do, apart from some problem indicated by the univariate normality test for Δy_t, not exhibit any misspecification problems, so that we can conclude that SM I essentially performs as well as the PVECM. Note that in SM I short–run changes in energy consumption are being explained by the error–correction term, temperature changes, and the impulse dummy imp_{79}, while energy price changes are explained by the error-correction term, lagged price changes, and the impulse dummy imp_{86}. Finally, income changes are solely explained by adjustments due to disequilibria from the long–run steady–state solution.

Table 9: *Model diagnostic statistics, SM I*

		Δq_t	Δp_t	Δy_t
portmanteau	(11 lags)	14.967	39.771	31.031
AR 1-2	F(2,14)	2.7774 [0.10]	2.8409 [0.09]	2.8596 [0.09]
Normality	χ^2 (2)	1.9918 [0.37]	2.9474 [0.23]	6.2365 [0.04]**
ARCH 1	F(1,14)	0.6280 [0.44]	0.1256 [0.73]	0.0375 [0.85]
Xi^2	F(8, 7)	0.1988 [0.98]	1.2910 [0.37]	0.2511 [0.96]
Vector portmanteau	(11 lags)= 95.868			
Vector AR 1-2	F(18,31)= 1.1085 [0.39]			
Vector normality	χ^2 (6) = 7.7788 [0.25]			
Vector Xi^2	F(48,28)= 0.8636 [0.68]			

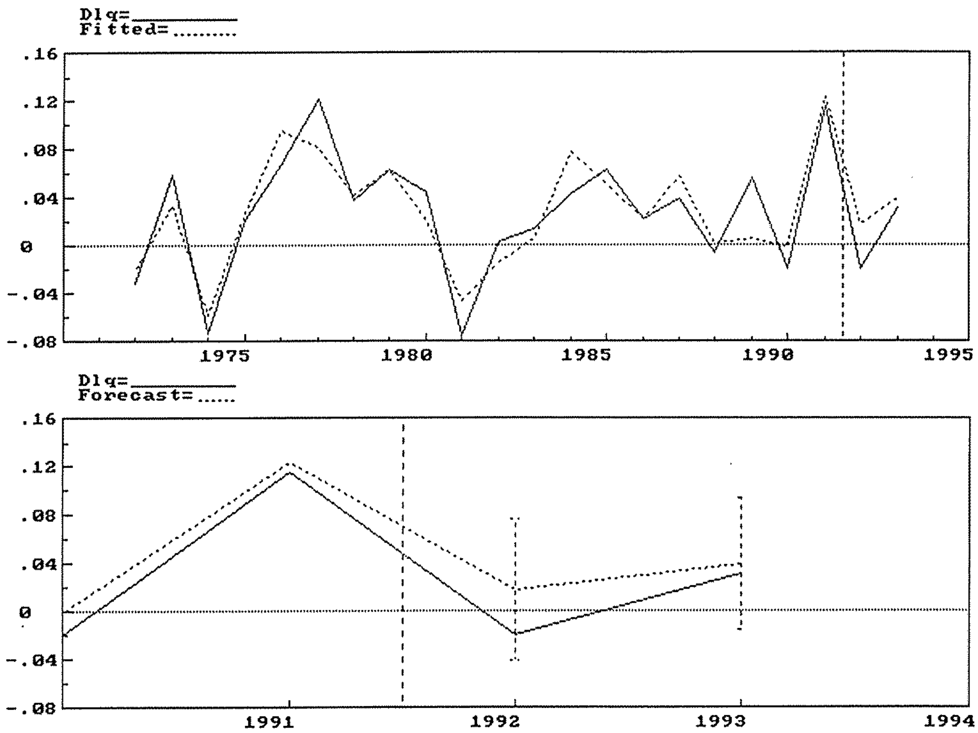
(Note: '**' denotes significance at the 5% level.)

The nine additional restrictions imposed are not rejected, as the outcome of the likelihood ratio test clearly indicates ($\chi^2(9) = 7.9625(0.54)$). Hence we may conclude that SM I parsimoniously encompasses the PVECM.

The second model considered for parsimonious encompassing within the PVECM framework is a VECM for the first differences of q_t, p_t, and y_t (i.e. Δq_t, Δp_t, and Δy_t). This model corresponds to Eq.(10), but ignores the *ecm*–term (i.e. $\gamma = 0$). As a system, a VECM in first differences looks basically like Eq. (2) with $\Pi = 0$, thereby ignoring the long–run information contained in the data. In fact, this class of model has historically been very popular in time series analysis of non–stationary data, especially since the publishing of the seminal paper by Box and Jenkins [6]. Note that the requirement $\gamma = 0$ implies three restrictions, as compared to the PVECM. I will refer to this second simplified model as SM II. A likelihood ratio test pursued at the 1% level of significance shows clearly, however, that SM II does *not* parsimoniously encompass the PVAR ($\chi^2(3) = 26.58(0.00)$). As a consequence, we may conclude that the zero frequency or long–run information contained in the cointegrating vector ecm_t indeed plays an important role in the modelling of q_t (and p_t) and once more demonstrates the superiority of the VECM approach chosen.

A next important step in the analysis would be to test for the parameter constancy of the various models introduced so far (see, for instance, Hansen [15] on this issue). However, due to the existing lack of observations, I have refrained from taking this step and have restricted myself to check the within–sample forecasting ability of SM I.

Figure 7: *Within-sample forecasting ability, SM I*



As can be seen from Figure 7, the forecasting performance of SM I is quite satisfactory. The actual and forecast values seem to diverge concertedly, and the 1-step ex–post forecasts are well within the 95% confidence intervals (error–bars denote ± 2SE and are based on the 1-step ahead forecast error variances, shown in relation to the realized values).

5 Summary and Conclusions

The aim of the underlying study was to demonstrate how system analysis and general–to–specific modelling of the Hendry–type may be applied to residential energy demand in order to avoid the problem of either nonsense regression outcomes or the loss of long–run information. It could be shown that for the application given some adaptations are necessary in order to align the desired economic model properties with those of the standard statistical model used this context.

In the analysis done I found some evidence for a single cointegrating vector which appeared to be fairly stationary. The long–run energy demand elasticities derived from the unrestricted general system were -0.02 for price, $+1.13$ for income, and $+0.77$ for temperature. The long–run system of energy demand makes sense from an economist's point of view, in that the long–run income elasticity is close to unity, temperature matters greatly, and price of energy obviously matters very little under a continued declining or low price regime.

Several overidentifying restrictions on the αs and βs have been tested in order to check for weak exogeneity and a unitary long–run income elasticity. The hypotheses of weak exogeneity for both price and income in the long–run demand equation was not rejected only for the latter, and the hypothesis of a long–run income elasticity of unity could – neither jointly with nor separately from the weak exogeneity hypothesis for income – be rejected, providing some evidence that a single–equation model is indeed not as appropriate as an analysis within a system.

The outcome of the investigation thereby confirms the justness of a system approach, in which the starting point of the modelling process is the joint density function of the variables, thereby taking more information about the underlying DGP into account than could be done within a single–equation framework.

Two simplified models have been derived from the parsimonious VAR model as short–run models rivalling each other, viz. SM I and SM II. While the former performed just as well as the congruent system and appeared to fit the data well, the latter – a VECM in first differences – did not parsimoniously encompass the PVECM. Lastly, we found that the error–correction term turned out to play an important role in the modelling of the short–run behaviour of q_t and p_t (and much less for y_t). In particular, after conditioning on y_t, it turned out that 80% of a disequilibrium in energy consumption from its long–run value are eliminated in the first year, whereas in case of a price shock 53% of the total effect are absorbed in the same year. The FIML estimates for the SM I showed that, apart from the error–correction term, only temperature changes and, surprisingly, the dummy variable imp_{79} seem to have a significant impact on changes in energy demand.

In sum, it has been demonstrated how to employ a VAR system and multivariate cointegration analysis for the modelling of residential energy demand, leading to a more useful treatment of the non–stationarity inherent in the data than commonly done within the framework of single–equation modelling.

Future research, however, will yet have to show whether the estimated low long–run price elasticity is attributable to the approach chosen, the particular case studied, substitutional price–demand effects amongst fuels not reflected in the aggregate data, etc. — or whether it is merely a result of including more recent data of the "low–price era" in the data sample than earlier studies in this field of research did. Personally, I have got the impression that the dramatic energy price developments over the past twenty–five years limit the feasibility of simple log–linear functional forms in this context, mainly due to the rather restrictive assumption of constant elasticities. In this respect, asymmetric models of energy demand that are capable of discriminating between price rises and falls, seem to be a promising tool for future studies (cf. Madlener [32] for a recent discussion, references, and an empirical application in a single–equation context).

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