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Abstract

Based on the Heckscher-Ohlin-Vanek (H-O-V) theory, the paper develops theoretical models that lead to estimating cross-industry equations in a proper way, when allowing for departures from some of the strong assumptions of the H-O theory, such as perfect competition, equal factor unit requirements and factor prices across countries, and internationally immobile factors. Based on these theoretical models we try to address properly the issue of empirical estimation of the H-O-V equations, as well as to reformulate the rank hypotheses that allow for direct tests of the H-O-V theory when some of the assumptions of the original factor-proportions theory are relaxed.

Keywords

International trade, comparative advantage, increasing returns to scale, product differentiation, factor-content

JEL-Classifications

F11, F12, F14

Comments

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Contents

1 Introduction

The trade literature offers an impressive number of studies based on the *Heckscher – Ohlin* ($H - O$) factor-proportions theory, according to which goods are really indirect traded factors, otherwise internationally immobile. The $H-O$ theory derives the determinants of comparative advantage in a "two-ness" world (two-good, two-factor, two-country setting), predicting that each country will export that good produced using relatively intensively the country's relatively abundant factor. The $H-O$ theory is based on a number of very strong assumptions, like identical production functions, factor unit requirements and factor prices across countries, perfect competition in the goods and factors market, perfect mobile production factors within countries, while completely immobile across countries, identical and homothetic preferences in consumption.

The main questions addressed by the existing literature refer to the following issues: the appropriate formulation of the $H - O$ theorem in a multi-factor, multi-good, and multi-country framework, proper tests of the $H - O$ theory, proper links of the theory to empirical tests.

The relevance of the $H - O$ theory started to be questioned as important facts of modern international trade proved to be inconsistent with its theoretical framework. A first doubt whether actual trade patterns and factor endowments are related as predicted by theory is built around the *Leontief Paradox* and other related studies. Leontief (1953) tried to test the factor-proportions theory using US data for 1947, and he found that the US, expected to be the most capital-abundant country in the world, had its exports more labour-intensive than its imports. He compared the capital per man embodied in \$1 million worth of imports with the capital per man embodied in \$1 million worth of exports and he found that the US was in 1947 a net exporter of labour services. His unexpected results led to doubt about the usefulness of the $H - O$ theorem and brought many controversial discussions in regard to the proper empirical implementation of the factor-proportions theory. One response to the *Leontief Paradox* suggests that the US trade may be better explained in terms of factors other than labour and capital. Leamer (1980) shows that Leontief's comparison does not reveal the relative abundance of capital and labour in a multi-factor world, and therefore there is no paradox if the computations are conceptually correct. Since Leontief's findings, a significant amount of research has been oriented towards establishing the general validity of the $H - O$ theorem. In general, the effort was unsuccessful in reversing the *Leontief Paradox*.

As Leamer (1980) shows, the two-factor, two-good version is not a proper representation of the theory. The reconsideration of the *Leontief Paradox* rests on the *Heckscher – Ohlin – Vanek* ($H - O - V$) theorem (Vanek, (1968)), which is a generalisation of the $H - O$ theory. Vanek (1968) is the first to offer a restatement of the $H - O$ theorem in the multi-factor, multi-good case, where there is no unique ordering of technologies according to relative factor intensities, and hence one cannot state it with respect to the commodity structure of trade. However, we may write a factor-content of trade version of the $H-O$ theorem, irrespective of the number of input factors, and instead of a multiple ordering of products there is a unique ordering of trade-factor-intensities.

There is very little empirical support for an exact linear relationship between trade flows and factor supplies as predicted by the $H - O$ theory. This may be partly explained by the fact that usually the empirical studies are based on the quantity version of the $H - O - V$ theory, hence on the unrealistic assumption of factor price equalisation across countries. Based on Vanek's generalisation, Maskus (1985), Bowen, Leamer and Sveikauskas (1987), and Brecher and Choudhri (1988) have performed tests of the factor-content version of the $H - O$ theory, using independent measures of trade, factor intensities, and factor endowments. They found that the $H - O$ theorem departs significantly from its exact quantitative predictions, and this is not surprising given the extraordinary assumptions of the model. However, the consensus seems to be that factor endowments exert a positive and linear influence on the factor-content of trade flows, while they hardly constitute the only important explanation of trade patterns. Recent debates cast suspicion about the usefulness of the regression interpretation of the $H - O$ theory. As Leamer (1992) observes, many regressions tend to have an unclear theoretical foundation. Maskus (1991) notices that little can be said about the empirical determinants of the patterns of trade in a rigorous way.

A second doubt about the relevance of the $H - O$ theory is determined by the growing trade between developed countries with similar factor endowments, especially intra-industry (two-way) trade. An increased tendency not towards inter- industry specialisation, but rather to stimulating intra- industry trade is noticed in the context of European economic integration. The most usual explanations of intra- industry trade are product heterogeneity within aggregates and border and seasonal trade. In addition, starting with

Grubel and Lloyd (1975) a large evidence supportive of intra-industry trade was provided, suggesting that product differentiation and economies of scale rather are the appropriate explanation of this two-way trade. Meanwhile, interesting contributions by Krugman (1979), Dixit and Norman (1980), Ethier (1979, 1982), Helpman and Krugman (1985), offer theoretical models that refine the $H-O$ model by allowing for economies of scale, product differentiation, and departures from perfect competition. The outcome is a more generalised $H-O$ theorem that preserves the factor- endowment basis for inter-industry trade, while extending the theory to allow for intra-industry trade and explain it.

Based on the $H-O$ theory, this paper presents theoretical models for explaining a country's commodity pattern of net trade in a multi-factor, multi-good, multi-country setting. By contrast with previous existing studies, these models allow for cross-country technological and factor price differences, increasing returns to scale and product differentiation. They also allow for intermediate production, but preserve most of the strong assumptions of the $H-O$ theory such as internationally immobile input factors, identical and homothetic preferences in consumption, free trade, no transportation costs, and factor market and world commodity clearing. The presence of economies of scale and product differentiation makes the assumptions of the models slightly more realistic, while the standard model is more general. The relationship that determines optimal unit factor input requirements is now modified. The model with internal economies of scale and product differentiation allows us to write separate equations for exports and imports of the differentiated products. Finally, a model with international capital mobility is developed. Based on these theoretical models, I try to address the issue of proper empirical estimation of the $H-O-V$ equations. I propose a set of proper cross- industry regression equations to be used for explaining the countries' pattern of comparative advantage, and I reformulate the rank hypotheses that allow for direct tests of the $H-O-V$ theory.

The quantity version of the $H-O-V$ equation is based on the assumption of internationally identical technologies, in that factor returns and unit factor requirements are equal across countries, assumptions which are difficult to reconcile with the real world. By contrast, a value version of the $H-O-V$ model is derived without considering factor price equalisation across countries. A value version uses factor cost shares rather than input factor coefficients, which are parametric, hence independent of factor prices under $Cobb-Douglas$ technology. The value version of the $H-O-V$ equation written for a particular country is:

$$\Theta^{TOT}t^v = W\bar{v} - s\sum_j W^j\bar{v}^j, \qquad (1)$$

where t^v is the country's vector of net exports, in value terms, Θ^{TOT} is the matrix of total (direct plus indirect) factor cost shares, W is a diagonal matrix of factor returns, $\bar{v}$ is the vector of factor endowments, s is the ratio between the country's domestic absorption and world income, and a superscript j denotes countries. Equation (1) predicts that the total factor content of trade in value terms ($\Theta^{TOT}t^v$) is a linear function of national ($W\bar{v}$) and world factor endowments ($\sum_j W^j\bar{v}^j$) defined in value terms.

Even though one may have data on all the variables in equation (1), the partial nature of the factor- proportions theory should be acknowledged by always adding an error term. The ideal situation would be to have a complete regression model that combines, in a rigorous way, the factor-proportions theory with other determinants of trade, such as increasing returns to scale and product differentiation. Then, the empirical analysis may suggest, for example, that the more complete model does nest that suggested by factor-proportions theory, as a special case. Based on the theoretical models developed in the paper and following Bowden (1983), a proper form of the estimating equations for the cross-industry regression analysis is derived. The basic $H-O-V$ equation in its value version is translated into a proper estimation equation to be further used in a cross-industry regression framework:

$$t_i^{vA} = \alpha_0 + \sum_h \alpha_h \theta_{hi}^{TOT}(w_h\bar{v}_h - \frac{y}{y^w}\sum_j w_h^j\bar{v}_h^j) + \mu_i, \qquad (2)$$

where t_i^{vA} represents net exports in industry i, in value terms and adjusted for the country's trade imbalance (hence the superscript vA), $\bar{v}_h$ and w_h are the country's endowment and return for the input factor h, θ_{hi}^{TOT} is the total cost share of factor h per dollar value of commodity i, y and y^w are the country and world income, μ an error term, and α_h are the regression coefficients. Running cross-industry regressions using data on net exports (as dependent variable) and factor cost shares (as independent variables), the relationship between the commodity trade pattern and factor- intensities is predicted for each country. Using a two-stage estimation approach (see Balassa (1979)), the estimation coefficients obtained in the first estimation-step are then externally validated by being regressed on factor endowments in a cross-country framework. The interactions implied by equation (2) should be understood in the following way: if a factor h is intensively used in the production of good i, hence θ_{hi}^{TOT} is relatively large, and if the country is relatively well endowed with factor h, hence $(w_h \bar{v}_h - \frac{y}{y^w} \sum_j w_h^j \bar{v}_h^j)$ is positive, this combination favours the production and exports of good i.

Based on the above equation and the models developed in the present paper, proper estimation equations may be derived in the case of imperfect competition, increasing returns to scale, and product differentiation. The economies of scale and the product differentiation variables enter the explanatory part of the regression in an interactive term together with factor cost shares, and not as a separate independent variable. Usually, existing empirical studies add increasing returns to scale and product differentiation on the list of explanatory variables, but without providing any theoretical justification.

The paper is organised as follows. Sections 2.1 and 2.2 provide a theoretical framework for deriving the factor- content $H-O-V$ equation, the quantity and the value versions, in the presence of perfect competition and intermediate production. A modification of the model that takes into account non-neutral differences in the technological parameters across countries is proposed in Section 2.3. Section 2.4 derives the proper net trade equations in the presence of increasing returns to scale, either external or internal to the firms. I show that the resulting equations are identical to equation (1), except for the expression of Θ^{TOT}. In the latter case, separate factor-content equations for exports and imports of differentiated products are derived. Section 2.5, based on the argument that capital is mobile across countries, develops a model that relaxes the assumption of internationally immobile capital. Section 3 presents ranking hypotheses derived from the equations obtained so far that allow for direct tests of the $H-O-V$ theory. Based on the theoretical models developed in the previous sections, and following Bowden (1983), Section 4 derives the proper estimating equations for the cross- industry regression analysis. The basic $H-O-V$ equation in its value version is translated into a proper estimation equation to be further used in a cross-industry regression framework. According to the equations derived in Section 2 for the increasing returns to scale and product differentiation models, the estimation equations are then modified in order to check whether countries' commodity pattern of trade is better explained when we allow for these qualifications.

2 Generalisations of the *Heckscher – Ohlin – Vanek* Model

In a two-country, two-factor, two-commodity trade model, the $H-O$ factor-proportions theory provides an unambiguous explanation of the international trade pattern: each country will export the good that uses its relatively abundant factor intensively in production. In the multi-factor, multi-commodity, multi-country case, when the number of goods exceeds that of factors, the precise commodity pattern of production and trade is indeterminate. However, the factor-content of trade is still determinate, when redefining trade as an implicit flow of factors embodied in commodities. Therefore, a weaker version of the $H-O$ theorem may be proved, that involves the factor- content of trade rather than its commodity composition.

2.1 The Quantity Version of the $H-O-V$ Model

Vanek (1968) derives a proposition based on the standard assumptions of the $H-O$ theory, according to which a country's net exports of factor services reflect the country's relative ranking in terms of factor endowments. The following equation relates a country's factor-content of net trade to the differences between the country's resource endowment and that of the world:

$$Tt = \bar{v} - s\bar{v}^w, \tag{3}$$

where[1] T is an $m*n$ matrix (m factors and n industries) of total (direct and indirect) factor input coefficients for the country, with its elements indicating the total amount of each input factor required to produce one unit of final demand in each industry, t is country's $n*1$ vector of net exports of commodities, $\bar{v}$ is country's $m*1$ vector of factor endowments, $\bar{v}^w$ is an $m*1$ vector of world factor endowments, s is the ratio between country's domestic absorption and world income, $s=(y-b)/y^w$, where y is GNP and b the trade balance of the country, and y^w is world GNP.

[1]Appendix A provides an extensive list of the symbols and definitions of variables and parameters used in the paper.

Equation (3) is based on many restrictive assumptions and it is obtained in the following way. In input-output formalism, gross output and final demand are related through the following relationship:

$$x^{fv} = (I - A^v)x^v, \tag{4}$$

where v is a value index, I is the $n * n$ identity matrix, A^v is an $n * n$ matrix, with elements indicating the value of output a particular industry must buy from each other industry to produce one dollar of its own product, x^v is an $n * 1$ vector of gross output in value terms produced by each industry, and x^{vf} is an $n * 1$ vector of final demand obtained from each industry.

The input-output tables report $(I - A^v)^{-1}$. Entries in each column of this matrix show the value of total output required by industry j from each other industry to produce one dollar of final demand of its own product.

The vector of final demand for a particular country[2] consists of net exports and final consumption, therefore the vector of net exports t equals the difference between output for final demand and consumption[3]:

$$t = (I - A)x - c, \tag{5}$$

where A is country's input-output matrix in quantitative terms[4]. I assume that A is the same for all the countries.

The assumption of identical and homothetic tastes results in the neutralisation of the consumption side of the model and it implies that each country consumes input factors in proportion to world factor endowments. It means that, in the absence of barriers to trade, all consumers face identical commodity prices, and they have identical and homothetic tastes. Hence, all consumers choose the same composition of the demand bundle (the

[2]The $H - O - V$ equation is derived for a particular country and no superscript or subscript is used to denote it.

[3]This a generally valid definition of net exports, irrespective of any technology assumption and whether intermediates or final goods, or both, are traded. It may be stated either in quantitative or value terms.

[4]The relationship between A and A^v is given by: $A^v = PAP^{-1}$, where P is an $n * n$ diagonal matrix of commodity prices.

assumption of identical consumption tastes across countries) and any country's consumption vector is a scaled down version of the world consumption vector (the assumption of homothetic tastes in consumption):

$$c = sc^w, \tag{6}$$

where c^w is the $n*1$ vector of world final consumption. Assuming identical matrices A for all countries, and using $x^{fw} = \sum_j (I - A)x^j$ and the world commodity market clearing condition $c^w = x^{fw}$, equation (5) becomes:

$$t = (I - A)(x - s\sum_j x^j), \tag{7}$$

or:

$$(I - A)^{-1}t = x - s\sum_j x^j \tag{8}$$

The consumption share, s, is determined by pre-multiplying equation (5) by the vector of prices p:

$$b = p^{'}t = p^{'}x^f - sp^{'}c^w. \tag{9}$$

The left-hand side of equation (9) represents the country's trade balance, the first term on the right-hand side represents country's GNP ($y = p^{'}x^f$), and $y^w = p^{'}c^w$ is world GNP. It follows that:

$$s = \frac{y - b}{y^w}. \tag{10}$$

We assume that firms are engaged in perfect competition and that the production function $F = F(v, x_q)$ is homogeneous of degree one in factor inputs v and input-output requirements x_q, and quasiconcave, where F, v, and x_q are $n * 1$ vectors. Therefore, we may write:

$$1 = F(\frac{v}{x}, \frac{x_q}{x}) = F(r, a), \quad (11)$$

where $F(r, a)$ is a unit isoquant, with r and a being the elements of matrices R and A, respectively. R is a $m * n$ country's direct factor input coefficients matrix, with its elements showing the amount of each direct factor input used to produce one unit of output within each industry.

We also assume that the production function is separable in its arguments (the factor inputs and the input-output requirements). The competitive firm chooses the direct unit factor requirements r and the input-output coefficients a such as to minimise unit production costs $\phi(w, p)$:

$$min_{r,a}(\phi(w,p) = A'p + R'w \;\; s.t. \;\; F(r, a) \geq 1). \quad (12)$$

Using the zero-profit condition in the presence of free trade:

$$p = (I - A^{'}(p))^{-1} R^{'} w, \quad (13)$$

and the first order conditions, we get the unit factor inputs $R(w)$ and the input-output requirements $A(p)$[5]. Given the CRS assumption, the cost-minimising input mix depends only on the relative factor returns and is independent of the level of output.

The factor market equilibrium conditions are given by:

$$R(w)x = \bar{v}, \quad (14)$$

where $\bar{v}$ is the factor supply vector. We assume full- employment of factors.

For the multi-factor, multi-industry, and multi-country model the general equilibrium set of equations is given by equations (13), (one zero-profit equation in matrix form for each country, hence nc in total), (14) (one factor-market equilibrium equation in matrix form for each country, hence mc equations in total), and the good market equilibrium

[5] Given the assumption of free trade, firms located in different countries choose identical input-output requirements. The assumption of different factor prices results in different unit factor inputs across countries.

conditions (one good market equilibrium condition in matrix form, hence n equations in total), where c denotes the number of countries in the model. Therefore there are $(nc + mc + n)$ equations in total and $(n + mc + nc - 1)$ unknowns: factor prices (m for each country, hence a total of mc), the quantities of goods (n for each country, hence a total of nc), and good prices ($n - 1$, given the assumption of free international trade and normalisation of the price in one industry). Therefore there is a determinate system with $(nc + mc + n)$ equations and $(n + mc + nc - 1)$ unknowns, one equation being redundant by Walras' Law[6].

Given the assumption of internationally identical technologies, $R^k(w^k) = R^j(w^j) = R$ for any two countries k and j (this implies that factor price equalisation holds, $w^k = w^j$), and multiplying equation (8) by R and using (14), we get:

$$R(I - A)^{-1}t = \bar{v} - s\sum_j \bar{v}^j. \tag{15}$$

The combination between the elements of any row h of matrix $R(w)$ and the elements of any column i of matrix $(I - A)^{-1}$ gives the total factor input requirements of factor h in industry i (that is, the total factor input h required for producing one unit of final demand in industry i). The matrix of total factor input requirements is given by the $m * n$ matrix T:

$$T = R(I - A)^{-1}. \tag{16}$$

The factor-content of trade Tt is a linear function of national and world factor endowments (see equation (3)):

$$Tt = \bar{v} - s\bar{v}^w.$$

The last equation represents the quantity version of the factor-content $Heckscher - Ohlin - Vanek$ model in the presence of factor price equalisation, free trade, identical

[6]The summation of the world commodity clearing conditions for the first $(n - 1)$ industries, in the presence of the assumptions of world income-expenditure equality and zero-profit conditions, gives the equilibrium condition for the n-th industry, which is therefore redundant.

and homothetic consumer tastes, perfect competition, constant returns to scale (CRS) production functions, and internationally identical technologies.

If matrix R were invertible, as in the case of $n = m$, net trade would be given by:

$$t = (I - A)R^{-1}(v - s\bar{v}^{w}). \tag{17}$$

Without factor price equalisation, there are country- specific factor input coefficients, this making necessary to use a different matrix $R(w)$ for each country. In addition, as Trefler (1993) observed, the net trade equation should be modified as to allow for the international differences in factor productivities. Trefler modified the quantity version of the $H - O - V$ model, in order to allow for factor augmenting international productivity differences. He defined a parameter π^{l} such as, if $\bar{v}^{l}$ is the endowment of factor l for a country, then $\bar{v}^{*l} = \pi^{l}\bar{v}^{l}$, where $\bar{v}^{*l}$ is the corresponding factor endowment measured in productivity-equivalent units. It follows that $w^{*l} = w^{l}/\pi^{l}$, where w^{*l} is the price per unit of $\bar{v}^{*l}$. Then, the $H - O - V$ equation (without intermediate production) becomes:

$$R^{*}(w^{*})t = \Pi\bar{v} - s\sum_{j}\Pi^{j}\bar{v}^{j}, \tag{18}$$

or:

$$R^{*}(w^{*})t = \bar{v}^{*} - s\sum_{j}\bar{v}^{*j}, \tag{19}$$

where $R^{*}(w^{*})$ is the country's technology matrix when its factors are measured in productivity equivalent units and computed using the US technology data, and Π is an $m * m$ diagonal matrix with elements π. He found that this modification of the $H - O - V$ model based on the assumption of equal technology with respect to productivity equivalent units explains much of the factor-content of trade and the cross- country variation in factor prices.

2.2 The Value Version of the $H - O - V$ Model

For the purpose of the present paper, a value form of the $H - O - V$ equation would be more useful. Usually observations on trade values, rather than quantity values, are available for empirical tests. Under $Cobb - Douglas$ production functions, the assumption of factor price equalisation can be relaxed without any of the above mentioned problems of the quantity version. A value version uses factor cost shares (instead of input factor coefficients), which are parametric, hence independent of factor prices under $Cobb - Douglas$ technology. Therefore, a value version of the $H - O - V$ equation holds independently of factor price equalisation, as long as we consider free trade. In addition, Trefler's (1993) qualification leaves the value form of the $H - O - V$ equation unchanged, therefore one would not need to correct it for the differences in factor productivities.

The consumption side of the model is identical to that described in the previous section, therefore equations (6), (9), (10) apply also here. On the production side, we assume a $Cobb - Douglas$ production function. As before, R is defined such as to minimise the unit production cost $\phi(w, p)$ and using the first-order and zero-profit conditions, we obtain:

$$WR(w) = \Theta P, \tag{20}$$

where Θ is an $m{*}n$ matrix of direct factor cost shares (the $Cobb{-}Douglas$ parameters), that is, each element of the $m * n$ matrix Θ represents the value of each factor per dollar value of gross output of each industry. W is an $m * m$ diagonal matrix of factor prices and P is an $n * n$ diagonal matrix of commodity prices. Given the assumption of full employment of input factors:

$$R(w)x = \bar{v}. \tag{21}$$

Premultiplying equation (21) by the matrix of factor prices W, we obtain:

$$WR(w)x = W\bar{v}. \tag{22}$$

The net trade is given by equation (15) and premultiplying equation (8) by WR and using equations (20) and (22), we obtain:

$$\Theta P(I - A)^{-1}t = W\bar{v} - s\sum_j WR(w)x^j. \tag{23}$$

Under $Cobb - Douglas$, $W^j R^j(w^j) = \Theta P$ for any country j, hence $\sum_j WR(w)x^j = \sum_j \Theta P x^j = \sum_j W^j R^j(w^j)x^j$ and, using equation (21), equation (23) becomes:

$$\Theta(I - A^v)^{-1}t^v = W\bar{v} - s\sum_j W^j \bar{v}^j, \tag{24}$$

where t^v is the net trade vector in value terms. Equation (24) represents the factor-content $H - O - V$ equation in its value version.

Using the notation: $\Theta(I - A^v)^{-1} = \Theta^{TOT}$, country's total (direct plus indirect) factor input cost shares $m * n$ matrix, equation (24) becomes:

$$\Theta^{TOT}t^v = W\bar{v} - s\sum_j W^j \bar{v}^j, \tag{25}$$

where $\Theta^{TOT}t^v = Wt_f$ and Wt_f is the vector of factor content of trade in value terms or the embodied trade in factor services. **The factor content of trade in value terms $\Theta^{TOT}t^v$ is a linear function of national and world factor endowments.**

In the case where the number of commodities is not equal to that of input factors, matrix Θ (and matrix Θ^{TOT}, respectively) is not invertible. In this case the pattern of production and trade can not be predicted, but the value of the factor-content of trade is determined. When applying Trefler's qualifications to equation (25) modified for no intermediate production, this leaves the value version unchanged. Equation (25), after applying Trefler's qualifications, becomes:

$$W^* R^*(w^*)t^v = W^* \bar{v}^* - s\sum_j W^* \bar{v}^{*j}, \tag{26}$$

which is equivalent to:

$$WR(w) = W\bar{v} - s\sum_j W^j \bar{v}^j, \tag{27}$$

or, using equation (20):

$$WR(w) = \Theta P,$$

$$\Theta(I - A^v)^{-1}t^v = W\bar{v} - s\sum_j W^j\bar{v}^j, \quad (28)$$

which is exactly equation (24). Hence, when using the value version of the $H - O - V$ model without factor prices equalisation, it is not necessary to correct for international differences in factor productivities.

2.3 The Value Version of the $H - O - V$ Model in the Presence of Non-Neutral Technological Differences

The model derived in this section is based on that developed previously, and it allows for non-neutral technological differences in the parameters Θ. The technological differences are non-neutral, because the adjustment of the direct factor cost shares is different for different factors. The idea of the model is based on the observation that, even after considering the differences in the input factor prices and in the unit factor requirements, the available data show important departures for some countries from Θ computed for a reference country, typically the US. The previous model can be quite easily transformed if we assume that matrix Θ is modified in an non-neutral way and differently across countries. We may write equation (20) for a particular country:

$$WR(w) = \Omega\Theta P, \quad (29)$$

where Ω is an $m * m$ diagonal matrix and $\Omega\Theta$ is the matrix of effective direct factor cost shares, after adjusting for non-neutral parametric technological differences. It follows that equations (23) and (24) written for a particular country become:

$$\Omega\Theta P(I - A)^{-1}t = W\bar{v} - s\sum_j WR(w)x^j, \quad (30)$$

or:

$$\Omega\Theta(I - A^v)^{-1}t^v = W\bar{v} - s\sum_j \Omega\Omega^{-1j}W^j\bar{v}^j, \quad (31)$$

where Θ is computed for the US. If we pre-multiply equation (31) by Ω^{-1}, we get:

$$\Theta(I - A^v)^{-1}t^v = W\Omega^{-1}\bar{v} - s\sum_j W^j\Omega^{-1j}\bar{v}^j, \qquad (32)$$

and if we use the notation $\Omega^{-1}\bar{v} = \bar{v}^*$ and $\Omega^{-1j}\bar{v}^j = \bar{v}^{*j}$, then:

$$\Theta(I - A^v)^{-1}t^v = W\bar{v}^* - s\sum_j W^j\bar{v}^{*j}. \qquad (33)$$

Equation (33) shows that, when using the US technology parameters, the factor-content of trade (in value terms) for any other country equals the vector of relative factor supply, after adjusting the factor endowment vectors for differences in productivity. This modification of the model is similar to Trefler's, except that here we do not have to adjust the factor returns as well, given that we used a value version of the $H-O-V$ model that already takes into account differences in factors return across countries. This is one of the most important advantages of the formulation of the $H-O-V$ model in value form as compared to the quantity version. The quantity version assumes that technology is identical across countries, in that *unit factor requirements* are the same, while the value version assumes that only *technology parameters* are identical, allowing for *different unit factor requirements* and *different factor returns* for different countries.

2.4 Modification of the $H-O-V$ Model in the Presence of Increasing Returns to Scale and Product Differentiation

The new theories of international trade allow for increasing returns to scale and product differentiation and thus introduce intra- industry trade. Ethier (1979) argues that economies of scale resulting from an increased division of labour rather than an increased plant size are international in scope rather than depending upon the size of the national market, as assumed in the traditional theory. Such international returns to scale are free from the presumption of indeterminacy and multiple equilibria characteristic to national returns to scale, and they provide a basis for a theory of intra-industry trade in intermediate goods between similar economies. The main argument of his paper is that in the modern world economy decreasing costs imply intra-industry trade in intermediate manufactures rather than 'arbitrary' patterns of industry specialisation. The more similar two countries are, the larger the volume of their bilateral trade in intermediate goods is.

Ethier (1982) shows that, in the framework of international scale economies, the factor-proportions theory is consistent with the factors explained above. He is building a two-good, two-factor model that explores the relationship between national and international returns to scale and the factor endowments theory of trade. His results show that international returns depend in an important way on the interaction between the two types of scale economies, national and international, and the basic theorems of the factor- proportions theory are robust in the presence of such scale economies. The national economies of scale involve considerations of the minimal plant size and they require production to be geographically concentrated. He assumes that final output in an industry is a function of components which are assembled to produce final goods. He models the economies of scale at the level of the firms producing components by the existence of a fixed cost. The international economies of scale depend upon the size of the market for finished manufactures, as an increase in the size of the market increases the equilibrium number of components, and they are external to the individual firms that assemble components into final goods. There are many competitive firms that see themselves as being subject to constant returns to scale. Therefore, there are constant returns to scale in the input level of the components, but increasing returns to scale in the number of components. In the framework of such a model he shows that the $H-O$ theorem (and other basic propositions derived from it) continues to hold and, in international equilibrium, each country necessarily exports the good intensive in its relatively abundant factor, if the two countries are not separated by a factor-intensity reversal. He shows that intra-industry trade, like the inter-industry trade, has a factor-endowment basis, and it is basically complementary to international factor mobility. Although the existence of product differentiation and internal economies of scale are essential to the theory, the degree of such phenomena does not need to be an essential determinant of the degree of intra-industry trade.

Markusen (1990) uses a simple two-good general equilibrium model for finding certain micro-economic determinants of external economies and for analysing the consequences of the external economies of scale for production and welfare.

Helpman and Krugman (1985) show that inter-industry trade is still explained by the relative factor endowments, when allowing for economies of scale, internal to the firm, and product differentiation.

Based on the value version of the $H-O-V$ equation, the present paper derives two models that allow for departures from the original factor-proportions theory: the first allows for increasing returns to scale external to the firm, but internal to the industry, and the second allows for increasing returns to scale modelled by the existence of a fixed cost at the level of the firm. All factor markets are competitive and in the competitive industries constant returns are assumed, so price must equal unit cost. By contrast with previous studies, the models are derived in a multi-country, multi-factor, and multi-good framework, and they allow for unit factor requirements and factor prices to differ across countries. Intermediate production is also considered.

2.4.1 Industry Increasing Returns to Scale, External to the Firm

The production side of the model allows the coexistence of n_1 industries that produce homogeneous products using constant returns to scale (CRS) technologies and n_2 industries with increasing returns to scale technologies (IRS), external to the firms, but internal to the industry. I assume that the economies of scale are the same world-wide for a particular industry. The firms are small enough to not perceive themselves as influencing the industry-wide economies of scale. Hence, returns to scale are *perceived* to be constant at the level of the firm, this being consistent with perfect competition. This type of external economies of scale output generated might be explained by the dissemination of production knowledge among firms belonging to an industry or by the fact that larger industry can support production of more varieties of intermediate inputs at lower costs and if the intermediate goods are tradable, the economies of scale are international.

We assume that the increasing returns to scale are present at the industry level in a multiplicative way, e.g.:

$$x_{ik} = (x_i)^{\epsilon_i} F_i(v_{ik}), \epsilon_i > 0, \tag{34}$$

where x_{ik} represents the gross output of firm k in industry i that depends parametrically on the gross output of industry i, x_i, ϵ_i is the external economies of scale in industry i, $F_i(v_{ik})$ is the CRS production function for a firm k in industry i, the same for all the firms within the industry, and v_{ik} is a vector of factor inputs employed by a firm k in industry i. Subscripts k and i refer to firms and industries, respectively. The first term on the right-hand side refers to the productivity effect, while the second refers to an index of factor input. The output in industry i is given by:

$$x_i = N_i x_{ik} = (x_i)^{\varepsilon_i} F_i(v_i), \tag{35}$$

where N_i is the number of firms in industry i and v_i is the vector of factor inputs employed by industry i[7]. As before, firms are choosing the unit factor inputs and input-output requirements in order to minimise their unit production costs. With increasing returns, unit factor requirements depend not only on factor returns, but also on the level of industry output. We assume that there are no cross-industry externalities and that technology is homothetic, therefore we may write:

$$R(w, x) = R(w) X^{-\varepsilon}, \tag{36}$$

where $X^{-\varepsilon}$ is an $n{*}n$ diagonal matrix, whose entries are $x_i^{-\varepsilon_i}$ and ε^i is zero for the first n_1 industries and close but larger than zero for the last n_2 industries. Since the firm considers the external effect independent of its actions, and since $F_i(v_{ik})$ is linear homogeneous in v_{ik}, the firm minimises its unit marginal cost, which is output independent. The first order conditions, together with the zero-profit condition:

$$p = (X^{\varepsilon} - A'(p))^{-1} R'(w) w, \tag{37}$$

imply that:

$$W R(w) X^{-\varepsilon} = \Theta P. \tag{38}$$

We recall that in the perfect competition model equation (38) is slightly different (see equation(20)):

$$W R(w) = \Theta P.$$

[7]With a *Cobb−Douglas* production function and only two input factors, namely capital K and labour L, $F_{ik}(v_{ik}) = L_{ik}^{\theta_{L_{ik}}} K_{ik}^{\theta_{k_{ik}}}$ and $x_i = N_i x_i^{\varepsilon_i} L_{ik}^{\theta_{L_{ik}}} K_{ik}^{\theta_{k_{ik}}} = x_i^{\varepsilon_i} L_i^{\theta_{L_i}} K_i^{\theta_{k_i}}$, where subscripts k and i denote firm and industry, respectively.

The factor market clearing condition is now:

$$R(w)X^{-\epsilon}x = \bar{v}, \tag{39}$$

and using the results from Section 2.2, in the presence of free trade, internationally identical technologies, and identical and homothetic consumer tastes, the $H-O-V$ equation for a particular country becomes:

$$\Theta(I - A^v)^{-1}t^v = W\bar{v} - s\sum_j W^j\bar{v}^j, \tag{40}$$

or

$$\Theta^{TOT}t^v = W\bar{v} - s\sum_j W^j\bar{v}. \tag{41}$$

Equation (41) is identical to equation (25), except that now $\Theta = WR(w)X^{-\epsilon}P^{-1}$, while in the perfect competition case, $\Theta = WR(w)P^{-1}$. The factor- content of trade is still a linear combination of domestic and world factor endowments.

This model allows us to formulate properly the regression equations, with economies of scale entering the explanatory part of the regression in an interactive term together with factor cost shares, rather than a separate independent variable.

When we consider the economies of scale to be international,

$$x_{ik} = (x_i^w)^{\epsilon_i}F_i(v_{ik}). \tag{42}$$

The $H-O-V$ equation is the same as in the case with national external increasing returns to scale, except that now $\Theta = WR(w)(X^w)^{-\epsilon}P^{-1}$, where a superscript w denotes the world.

2.4.2 Modification of the $H-O-V$ Model in the Presence of Increasing Returns to Scale and Product Differentiation

The new theories of international trade allow for increasing returns to scale and product differentiation and thus introduce intra- industry trade. Helpman and Krugman (1985) show that inter-industry trade is still explained by the relative factor endowments.

The derivation of the model is based on the two-sector model developed by Helpman and Krugman (1985). The production side allows for the coexistence of n_1 constant returns to scale (CRS) and n_2 increasing returns to scale (IRS) industries, with the economies of scale modelled by the existence of a fixed cost at the level of the firm. Increasing returns to scale at the level of the firm make the industry imperfectly competitive.

Each economy produces homogeneous and differentiated products. In addition to the homogeneous products, there are products that consumers like to consume in many varieties. Following Helpman and Krugman, I assume that preferences are represented by a two-level utility function:

$$U = U(u_1(.), u_2(.), ..., u_n(.)), \tag{43}$$

where $u_i(.)$ is the subutility function derived from the consumption of product i and U is the upper tier utility function.

I assume identical and homothetic preferences in consumption and a subutility function that rewards variety. Let $u_i = u_i(d_{i1}, d_{i2}, ...)$ be the subutility from consuming differentiated products, with $d_{i\omega}$ representing the quantity of variety ω of the differentiated product i that is being consumed. I assume that $u_i(d_{i1}, d_{i2}, ...)$ is the symmetrical constant elasticity of substitution function, given by:

$$u_i(d_{i1}, d_{i2}, ...) = (\sum_{\omega} d_{i\omega}^{\beta_i})^{1/\beta_i}, 0 < \beta_i < 1, \tag{44}$$

where $\beta_i = (1 - \frac{1}{\sigma_i})$ and $\sigma_i > 1$ is the constant elasticity of substitution between pairs of varieties of the differentiated products of industry i.

The symmetry assumption implies that consumers allocate the same expenditures $e_{i\omega} = e_i$ for any variety ω. The subutility $u_i(.)$ is larger, the larger the number of available varieties:

$$u_i(.) = n_i^{(1-\beta_i)/\beta_i} d_{i\omega}, \tag{45}$$

where n_i represents the number of varieties of product i.

In each industry producing differentiated products, each firm faces a demand curve with an elasticity perceived by the firms. The firm chooses the variety to produce and its price such as to maximise its profits. Given the assumption of a large number of firms, the elasticity faced by an individual producer may be approximated by the elasticity of substitution between any two varieties σ_i, and it determines the optimal mark-up for firms. The firm takes as given the prices of other firms within the industry and competes equally with any other firm. Assuming that each firm can costlessly differentiate its products, no variety will be produced by more than one firm, because no firm would like to share the market with other producers. I assume that firms producing different varieties of the same product use the same production functions. The economies of scale, internal to the firm, are modelled by the existence of a fixed cost $\phi_{i\omega}(w,p)\bar{x}$, hence total cost is given by $(\frac{\bar{x}_{i\omega}}{x_{i\omega}}+1)\phi_{i\omega}(w,p)x_{i\omega}$, where $\phi_{i\omega}(w,p)$ is the minimum unit variable cost for producing each variety ω of product i, w is a vector of factor prices, and p is the vector of prices. Hence, the firm problem is:

$$max_{p_{i\omega}}[p_{i\omega}x_{i\omega}-(x_{i\omega}+\bar{x}_{i\omega})\phi_{i\omega}(w,p)], \tag{46}$$

where $(\frac{\bar{x}_{i\omega}}{x_{i\omega}}+1)\phi_{i\omega}(w,p)$ represents the firm's average cost which is declining with output $x_{i\omega}$.

The firm's maximisation problem results in marking up price over marginal cost:

$$p_{i\omega}=\frac{\phi_{i\omega}(w,p)}{\beta_i}. \tag{47}$$

Assuming that all the firms in the differentiated product industries use the same technology, hence they have the same unit cost $\phi_{i\omega}(w,p)=\phi_i(w,p)$, all the firms producing different varieties of the same product have the same price for their varieties:

$$p_{i\omega}=p_i=\frac{\phi_i(w,p)}{\beta_i}. \tag{48}$$

Free entry and exit of firms implies a zero-profit condition in equilibrium. This implies further that the price of differentiated products equals average cost, $(\frac{\bar{x}_i}{x_i}+1)\phi_i(w,p)$ and determines output with zero profits:

$$x_{i\omega} = x_i = \frac{\bar{x}_i \beta_i}{(1-\beta_i)}, \qquad (49)$$

where $\bar{x}_{i\omega} = \bar{x}_i$, given the symmetry assumption. Therefore, firms in the IRS industries produce different varieties, in the same quantities and equally priced.

In the CRS industries there are no fixed costs. Firms produce homogeneous products and the industries are perfectly competitive. I assume that all firms use internationally identical $Cobb-Douglas$ production functions and choose unit factor inputs and input-output requirements in order to minimise unit production costs. Given perfect competition and price taking behaviour in factor markets, unit factor requirements are given by:

$$WR(w) = \Theta \beta P, \qquad (50)$$

where Θ is the matrix of direct factor cost shares, with its elements being the $Cobb-Douglas$ parameters, and β is an $n*n$ diagonal matrix, with the first n_1 diagonal elements equal to 1 and the last n_2 equal to β_i. As previously in the perfect competition model (see Section 2.2) or the model with external economies of scale (see Section 2.4.1), by using the factor market equilibrium and assuming internationally identical production functions, we get the net trade equation written for a particular country. The factor market equilibrium condition is in this case:

$$R(w)\beta^{-1}x = \bar{v}. \qquad (51)$$

The net trade equation for a particular country becomes:

$$\Theta(I - A^v)^{-1}t^v = W\bar{v} - s\sum_j W^j \bar{v}^j, \qquad (52)$$

or:

$$\Theta^{TOT}t^v = W\bar{v} - s\sum_j W^j \bar{v}^j, \qquad (53)$$

where:

$$\Theta^{TOT} = \Theta(I - A^v)^{-1}, \qquad (54)$$

and $(I - A^v)^{-1}$ represents the total (direct and indirect) requirements matrix (with its elements showing the value of gross output required, directly and indirectly, from each industry per one dollar of delivery to final demand of each commodity).

Equation (53) is identical to equation (25), except that now $\Theta = WR(w)P^{-1}\beta^{-1}$, while in the perfect competition case, $\Theta = WR(w)P^{-1}$.

Every country produces some varieties of the differentiated products, but all countries consume all the varieties. As in the Helpman and Krugman (1985) or Dixit and Norman (1980) models, there are both comparative advantage (inter-industry) effects, and intra-industry effects. Therefore, intra-industry trade results, in contrast with the $H-O$ model, and it is possible to write separate equations for exports and imports.

Therefore, for the n_2 industries that produce differentiated products, we may now write separate equations for exports and imports. Each country will produce a certain number of varieties of each differentiated product and will consume a proportion of each of its own varieties equal to the ratio between its domestic absorption and world income, s, and will export the difference between its domestic produced and consumed varieties. Each country will consume, is addition to its own domestically produced varieties, a fraction s of the varieties produced elsewhere in the world. Therefore, exports and imports for the differentiated products are given by:

$$z = (1 - s)x^f \tag{55}$$

and

$$m = s(\sum_j x^{jf} - x^f), \tag{56}$$

where z is an $n_2 * 1$ vector of exports of the differentiated products, m is an $n_2 * 1$ vector of imports of the differentiated products, x^f is an $n_2 * 1$ vector of output for final demand of the differentiated products, and j denotes countries.

I follow now an approach similar to that used so far. One should notice that, even though we write export and import equations for the differentiated products only, these equations contain terms referring to all other perfect competitive industries, through the indirect factor requirements matrix.

Using equations (55) and (56), the equations that define the value of the factor-content of exports and imports for a particular country are:

$$\Theta(I - A^v)^{-1} z^v = \Theta^{TOT} z^v = (1 - s) W \bar{v} \tag{57}$$

and

$$\Theta(I - A^v)^{-1} m^v = \Theta^{TOT} m^v = s(\sum_j W^j \bar{v}^j - W \bar{v}), \tag{58}$$

where v is a value index, z^v is the value vector of exports of the differentiated products and m^v is the value vector of imports of the differentiated products. The left-hand side of equations (57) and (58) is identical with that of equation (52), except for equation (52) is written for net exports, while equations (57) and (58) are written for exports or imports.

We define intra-industry trade i for a country as the difference between total (exports and imports) trade and net exports:

$$i = (z + m) - |t|. \tag{59}$$

Using equations (57), (58), and (52), we may write an equation for the intra-industry trade i^v for the industries producing differentiated products:

$$\Theta(I - A^v)^{-1} i^v = 2s(\sum_j W^j \bar{v}^j - W \bar{v}) \tag{60}$$

for the industries where the country is a net exporter of differentiated products, and:

$$\Theta(I - A^v)^{-1} i^v = 2(1 - s) W \bar{v} \tag{61}$$

for the net importer industries. Equation (60) shows that, when a country is a net exporter of a differentiated product, the factor-content of intra-industry trade of this product is still explained by its relative factor abundance.

Equations (53), (57) and (58) allow us to formulate proper regression equations and estimate the model with economies of scale and product differentiation.

2.5 The $H-O-V$ Model with Internationally Mobile Capital

Wood (1994) noticed that, given the fact that capital is internationally mobile, it cannot influence the pattern of trade of goods, which is determined by the endowments of immobile factors only. The exclusion of capital from the input factors explaining the pattern of net trade might improve the results of the tests of the $H-O$ theory. He proposed a model in which the production factors are skilled and unskilled labour, and suggested that capital should be defined as finance, and not as capital goods.

Ethier and Svensson (1986) examined the theorems of international trade with factor mobility and found explicitly that comparative advantage could be applied to factor trade as well as to commodity trade. The model developed in this section is based on an idea suggested by Ethier and Svensson who developed a $H-O$ model with both mobile and immobile factors. In addition to the n industries, we consider an imaginary $n+1$-th industry, with capital its only input (unit capital requirement equal to one) and the service of capital its only output. Hence, capital (as an input factor) is internationally immobile, while its service is a traded good. All the other input factors are considered to be internationally immobile. We have now a model with $n+1$ industries and m factors, and the previous type of analysis may be applied. Capital, as an input factor is used in the production process of all industries, as it were an intermediate good. As a traded good, there is no intermediate usage of capital.

Equation (5) still applies for the first n industries:

$$t = (I-A)x - c.$$

In addition, there is an $(n+1)$-th equation for the $(n+1)$-th imaginary industry:

$$t^k = k - \sum_{i=1}^{n} r_i^k x_i, \tag{62}$$

where k is the country's capital endowment, r_i^k is the unit capital requirement in industry i, x_i is the output of industry i, and t^k is country's net export of capital. The second term on the right-hand side of equation (62) represents the total capital used in the domestic intermediate production.

As before, competitive firms chose unit factor inputs that minimise unit factor costs, taking as given the prices of goods and the price of capital. All firms face the same international prices for goods, given the assumption of free trade. The price of capital is determined either by the market, or by the government in some countries. Hence, the price of capital may differ from a country to another.

For the first $(m-1)$ input factors the factor market conditions are, as before, given by equation (21):

$$R(w)x = \bar{v},$$

and equation (62) applies for capital.

For the first $(m-1)$ factors, the net trade equation is given as before by equation (25):

$$\Theta^{TOT} t^v = W\bar{v} - s\sum_j W^j \bar{v}^j, \tag{63}$$

where $\Theta^{TOT} t^v = W t_f$ is the vector of factor- content of trade in value terms or the embodied trade in factor services.

The equation for capital is:

$$t_f^k = w_k(k - t^k) - s\sum_j [w_k^j k^j + (w_k - w_k^j) t^{k,j}] \tag{64}$$

or:

$$t_f^{k,A} = w_k(k - t^k) - \frac{y}{y^w}\sum_j [w_k^j k^j + (w_k - w_k^j) t^{k,j}], \tag{65}$$

or:

$$t_f^{k,A} = w_k(k - t^k) - \frac{y}{y^w}\sum_j [w_k^j k^j + w_k^j (\frac{w_k}{w_k^j} - 1) t^{k,j}], \tag{66}$$

where w_k is the price of capital (interest rate), $t_f^k = \Theta_k t^v$ is the country's value of the capital content of net trade (capital services embodied in net exports of goods), with Θ_k being column k of matrix Θ', and $t_f^{k,A}$ is the value of capital content of net trade if trade were balanced (for definition see Section 3). If we assume perfect mobility of capital, $w_k = w_k^j$, for any country j, and equation (66) becomes:

$$t_f^{k,A} = w_k(k - t^k) - \frac{y}{y^w} \sum_j w_k^j k^j. \tag{67}$$

This model may be estimated either in the perfect competition framework (see Section 2.2) or in the presence of increasing returns to scale and product differentiation (see Section 2.4).

3 Rank Propositions

Based on the value version of the $H-O-V$ model there are two important rank hypotheses to be tested. Remember that the $H-O-V$ equation in the value version written for a particular country is:

$$Wt_f = W\bar{v} - s \sum_j W^j \bar{v}^j, \tag{68}$$

or:

$$Wt_f^A = W\bar{v} - \frac{y}{y^w} \sum_j W^j \bar{v}^j, \tag{69}$$

where $t_f = W^{-1}\Theta^{TOT}t^v$ is the factor-content of trade and $t_f^A = t_f - (b/y^w)\sum_j W^{-1}W^j\bar{v}^j$ is the factor-content of trade if trade were balanced.

For a particular input factor h, equations (68) and (69) become:

$$w^h t_f^h = w^h \bar{v}^h - s \sum_j w^{jh} \bar{v}^{jh} \tag{70}$$

and:

$$w^h t_f^{hA} = w^h \bar{v}^h - \frac{y}{y^w} \sum_j w^{jh} \bar{v}^{jh}. \tag{71}$$

Equations (70) and (71) may be written:

$$(\frac{w^h t_f^h}{\sum_j w^{jh}\bar{v}^{jh}})/s = (\frac{w^h \underline{a} r v^h}{\sum_j w^{jh}\bar{v}^{jh}})/s - 1 \tag{72}$$

and:

$$(\frac{w^h t_f^{hA}}{\sum_j w^{jh}\bar{v}^{jh}})/(\frac{y}{y^w}) = (\frac{w^h \bar{v}^h}{\sum_j w^{jh}\bar{v}^{jh}})/(\frac{y}{y^w}) - 1, \tag{73}$$

for any country and any input factor h. Further, using: $\sum_j w^{jh}\bar{v}^{jh} = \bar{w}^{wh}\bar{v}^{wh}$, where by $\bar{w}^{wh}$ and $\bar{v}^{wh}$ we denote the world weighted average for the return on factor h and the world endowment of factor h, respectively, equations (72) and (73) become:

$$(\frac{w^h t_f^h}{\bar{w}^{wh}\bar{v}^{wh}})/s = (\frac{w^h \bar{v}^h}{\bar{w}^{wh}\bar{v}^{wh}})/s - 1 \tag{74}$$

and:

$$(\frac{w^h t_f^{hA}}{\bar{w}^{wh}\bar{v}^{wh}})/(\frac{y}{y^w}) = (\frac{w^h \bar{v}^h}{w^{wh}\bar{v}^{wh}})/(\frac{y}{y^w}) - 1. \tag{75}$$

Then, as Bowen, Leamer and Sveikauskas (1987) show, the sign of the net trade in factor services corrected for the trade imbalance will reveal the abundance of a factor, compared with other factors on average. However, their tests differ from those proposed here, as they develop them using the quantity rather the value version of the $H-O-V$ theory.

The derivation of the propositions follows Leamer (1980). He uses the $H-O-V$ model for a two-factor case to prove that a country is revealed to be relatively well endowed with capital (compared to labour) iif one of the following conditions hold:

$$K_x - K_m > 0, \;\; L_x - L_m < 0, \tag{76}$$

$$K_x - K_m > 0, \;\; L_x - L_m > 0, \;\; (K_x - K_m)/(L_x - L_m) > K_c/L_c, \tag{77}$$

$$K_x - K_m < 0, \;\; L_x - L_m < 0, \;\; (K_x - K_m)/(L_x - L_m) < K_c/L_c, \tag{78}$$

where K_x, L_x, K_m, L_m, K_c, and L_c are capital and labour incorporated in exports, imports, and consumption, respectively. The *Leontief Paradox* (Leontief (1953)) is based on the proposition that if $(K/L)_x < (K/L)_m$, the country is relatively better endowed with labour. But this is true only if the net exports of labour services have opposite sign with the net exports of capital services. When both are positive, as in Leontief's data, the correct comparison is between $(K/L)_t$ and $(K/L)_c$, where t denotes net exports. Hence, it can be shown that Leontief's data for 1947 satisfy the second condition, therefore the US is proved to be relatively well endowed with capital[8]. Leamer gives five corollaries, which outline necessary and sufficient conditions for trade to reveal the abundance of capital (as compared to labour), in a two-factor model of international trade.

However, Leamer' corollaries are derived based on the quantity version of the $H-O$ model, that assumes factor price equalisation across countries. Kohler (1991) examines the arbitrariness of empirical tests of rank order and sign propositions derived from the $H-O-V$ model of the factor- content of trade. He shows that, when using different rank and sign propositions derived from the $H-O$ model, there are certain conditions under which a given data set will support one hypothesis, while rejecting another. Based on the data set used by Bowen, Leamer, and Sveikauskas (1987), he checks the empirical relevance of this fact and he shows that the results of such rank and sign tests are not robust. However, his tests are based on the quantitative version of the $H-O-V$ model, that implies that factor price equalisation across countries holds.

This section proposes necessary and sufficient conditions for trade to reveal the relative abundance of a particular factor when compared to any other factor for a particular country, or the relative abundance of a country in comparison with another country for a certain factor. These propositions are derived using a value version of the $H-O-V$ model in a multi-factor, multi-good, multi-country setting, that allows for different factor returns and unit factor requirements across countries.

The first proposition refers to the ranking of any two input factors for a particular country, and may be formulated either in quantitative or in value terms. The second proposition refers to the ranking of any two countries for a particular input factor, stated either in quantitative or in value terms. Based on equations (74) and (75), there are four possibilities for deriving and stating each proposition.

[8]However, as Maskus (1985) shows, this result does not hold for 1958 and 1972.

Using equation (74) written for a particular country and two different input factors, h and l, we get the first proposition:

Proposition 1 a *If* :

$$\frac{w^h \bar{v}^h}{\bar{w}^{wh} \bar{v}^{wh}} / s <> \frac{w^l \bar{v}^l}{\bar{w}^{wl} \bar{v}^{wl}} / s, \tag{79}$$

then:

$$\frac{w^h t_f^h}{\bar{w}^{wh} \bar{v}^{wh}} / s <> \frac{w^l t_f^l}{\bar{w}^{wl} \bar{v}^{wl}} / s. \tag{80}$$

An alternative quantitative version of *Proposition 1 (a)* may be written as:

Proposition 1 b

If :

$$\frac{\bar{v}^h}{\bar{v}^{wh}} / s <> \frac{\bar{v}^l}{\bar{v}^{wl}} / s, \tag{81}$$

then:

$$\frac{t_f^h}{\bar{v}^{wh}} / s <> \frac{t_f^l}{\bar{v}^{wl}} / s + \left(\frac{\bar{w}^{wl}}{w^l} - \frac{\bar{w}^{wh}}{w^h}\right). \tag{82}$$

Using equation (75), *Proposition 1* may be written as:

Proposition 1 c

If :

$$\frac{w^h \bar{v}^h}{\bar{w}^{wh} \bar{v}^{wh}} / (y/y^w) <> \frac{w^l \bar{v}^l}{\bar{w}^{wl} \bar{v}^{wl}} / (y/y^w), \tag{83}$$

then:

$$\frac{w^h t_f^{hA}}{\bar{w}^{wh}\bar{v}^{wh}}/(y/y^w) <> \frac{w^l t_f^{lA}}{\bar{w}^{wl}\bar{v}^{wl}}/(y/y^w), \tag{84}$$

where $t_f^{hA}=t_f^h-\frac{b}{y^w}\sum_j \frac{w^{jh}\bar{v}^{jh}}{w^h}$ *and* $t_f^{lA}=t_f^l-\frac{b}{y^w}\sum_j \frac{w^{jl}\bar{v}^{jl}}{w^l}$ *are the factor* h *and* l *value content of trade if trade were balanced, and* b *is the country's trade balance.*

A quantitative version of *Proposition 1 (c)* may be formulated as:

Proposition 1 d

If :

$$\frac{\bar{v}^h}{\bar{v}^{wh}}/(y/y^w) <> \frac{\bar{v}^l}{\bar{v}^{wl}}/(y/y^w), \tag{85}$$

then:

$$\frac{t_f^{hA}}{\bar{v}^{wh}}/(y/y^w) <> \frac{t_f^{lA}}{\bar{v}^{wl}}/(y/y^w) + (\frac{\bar{w}^{wl}}{w^l} - \frac{\bar{w}^{wh}}{w^h}). \tag{86}$$

Given *Proposition 1* in its quantitative version (b or d), if input factors for a particular country are ranked according to their endowment ratios relative to the world, this would be reflected in the ranking of the factor- content (adjusted factor-content) of net exports of the factors relative to the world endowment thereof, *adjusted* for differences from the world (weighted average) factor returns and country's size. If factors are ranked according to the *value* of their endowment ratios relative to the world (*Proposition* 1a or 1c), this would be reflected in the ranking of the *values* of the factor-content (adjusted factor-content) of net exports, relative to the value of the world endowment. In this case, no adjustment is necessary for countries' size.

The second proposition refers to the ranking of any two countries for a particular input factor. Its derivation is based on equations (74) and (75), written for two different countries i and k, and a particular input factor l. There are four possibilities to state *Proposition 2.* The first derivation is based on equation (74):

Proposition 2 a

If :

$$\frac{w^{kl}\bar{v}^{kl}}{\bar{w}^{wl}\bar{v}^{wl}}/s^k <> \frac{w^{il}\bar{v}^{il}}{\bar{w}^{wl}\bar{v}^{wl}}/s^i, \tag{87}$$

then:

$$\frac{w^{kl}t_f^{kl}}{\bar{w}^{wl}\bar{v}^{wl}}/s^k <> \frac{w^{il}t_f^{il}}{\bar{w}^{wl}\bar{v}^{wl}}/s^i. \tag{88}$$

This may be written also in quantitative terms:

Proposition 2 b

If :

$$\frac{\bar{v}^{kl}}{\bar{v}^{wl}}/s^k <> \frac{\bar{v}^{il}}{\bar{v}^{wl}}/s^i, \tag{89}$$

then:

$$\frac{t_f^{kl}}{\bar{v}^{wl}}/s^k \quad <> \frac{t_f^{il}}{\bar{v}^{wl}}/s^i + (\frac{\bar{w}^{wl}}{w^{il}} - \frac{\bar{w}^{wl}}{w^{kl}}). \tag{90}$$

When using equation (75) we get:

Proposition 2 c

If :

$$\frac{w^{kl}\bar{v}^{kl}}{\bar{w}^{wl}\bar{v}^{wl}}/(y^k/y^w) <> \frac{w^{il}\bar{v}^{il}}{\bar{w}^{wl}\bar{v}^{wl}}/(y^i/y^w), \tag{91}$$

then :

$$\frac{w^{kl}t_f^{klA}}{\bar{w}^{wl}\bar{v}^{wl}}/(y^k/y^w) <> \frac{w^{il}t_f^{ilA}}{\bar{w}^{wl}\bar{v}^{wl}}/(y^i/y^w). \tag{92}$$

A last quantitative version of *Proposition 2* is:

Proposition 2 d

If :

$$\frac{\bar{v}^{kl}}{\bar{v}^{wl}}/(y^k/y^w) <> \frac{\bar{v}^{il}}{\bar{v}^{wl}}/(y^i/y^w), \tag{93}$$

then :

$$\frac{t_f^{klA}}{\bar{v}^{wl}}/(y^k/y^w) <> \frac{t_f^{ilA}}{\bar{v}^{wl}}/(y^i/y^w) + (\frac{\bar{w}^{wl}}{w^{il}} - \frac{\bar{w}^{wl}}{w^{kl}}). \tag{94}$$

Given *Proposition 2* in its quantitative version (*b* or *d*), if for a particular input factor countries are ranked according to their endowment ratios relative to the world after adjusting for countries' size, this would be reflected in the ranking of countries according to their factor-content (adjusted factor-content) of trade relative to the world endowment, *adjusted* for world differences in factor return and countries' size. If the countries are ranked according to the *value* of their endowment ratios relative to the world after adjusting for the countries' size (*Proposition* 2 *a* or *c*), this would be reflected in the ranking of the *values* of the factor-content (adjusted factor-content) of net exports, relative to the value of the world endowment and *adjusted* for differences in the countries' size.

The present formulation of these two ranking propositions differs from that by Leamer (1980) or Bowen, Leamer and Sveikauskas (1987). Their derivation is based on the quantitative version of the $H-O-V$ equation, hence does not allow for unit factor requirements and factor prices to differ across countries. By contrast, the present setting has the advantage of allowing for world differences in factor prices, hence allowing for differences in the unit factor requirements and factor returns across countries. Previous empirical studies (e.g. Brecher and Choudhri (1982), Maskus (1985), Bowen, Leamer and Sveikauskas (1987)) use the quantity version of the $H-O-V$ equation, which does not allow for the unit factor requirements to differ across countries, this being an important source of error in their results. They conclude that $H-O-V$ propositions that trade reveal factor abundance are not supported by data. Generally, data on the unit factor requirements for the US have been used and, given the important differences in this respect across countries, the results are incorrect.

We have formulated propositions according to which the ranking of the adjusted net exports of factor services (in value or quantitative terms adjusted for world differences in factor returns) should conform to the ranking of factors (in value or quantitative terms) by their abundance. In addition to rank tests, sign tests may also be reported, by computing the percentage of matches between the left-hand side and the right-hand side of the $H-O-V$ equation, either for factors within each country or for each factor across countries. *Propositions 1* and 2 may be modified to allow for increasing returns to scale, and product differentiation and internationally mobile capital. When relaxing the assumption of perfect competition for the commodity market, *Propositions 1* and 2 are perfectly preserved, except for the expression of the factor-content of trade, which now is computed based on a different matrix of factor cost shares. An appendix to this section gives the rank propositions for the model with internationally mobile capital (presented in Section 2.5).

Appendix

This appendix states the rank *Propositions 1* and 2 for the model with internationally mobile capital. Equations (71) and (73) written for capital become:

$$t_f^{k,A} = w_k(k - t^k) - \frac{y}{y^w}\sum_j [w_k^j k^j + (w_k - w_k^j)t^{k,j}] \tag{95}$$

and:

$$\frac{t_f^{k,A}}{\sum_j [w_k^j k^j + (w_k - w_k^j)t^{k,j}]}/(\frac{y}{y^w}) = \frac{w_k(k - t^k)}{\sum_j [w_k^j k^j + (w_k - w_k^j)t^{k,j}]}/(\frac{y}{y^w}) - 1. \tag{96}$$

Propositions 1 and 2 are modified in the following way. *Proposition 1* stated in value terms becomes:

Proposition 1 *If* :

$$\frac{w_k(k - t^k)}{\sum_j [w_k^j k^j + (w_k - w_k^j)t^{k,j}]}/s <> \frac{w^l \bar{v}^l}{\sum_j w^{jl}\bar{v}^{jl}}/s, \tag{97}$$

then :

$$\frac{t_f^k}{\sum_j [w_k^j k^j + (w_k - w_k^j) t^{k,j}]} / s <> \frac{t_f^l}{\sum_j w^{jl} \bar{v}^{jl}} / s, \tag{98}$$

where l is an input factor other than capital.

The second proposition becomes:

Proposition 2 *If :*

$$\frac{w_k^g k^g - t^{k,g}}{\sum_j [w_k^j k^j + (w_k^g - w_k^j) t^{k,j}]} / s^k <> \frac{w_k^i k^i - t^{k,i}}{\sum_j [w_k^j k^j + (w_k^i - w_k^j) t^{k,j}]} / s^i, \tag{99}$$

then :

$$\frac{t_f^{k,g}}{\sum_j [w_k^j k^j + (w_k^g - w_k^j) t^{k,j}]} / s^g <> \frac{t_f^{k,i}}{\sum_j [w_k^j k^j + (w_k^i - w_k^j) t^{k,j}]} / s^i, \tag{100}$$

where i and g are two countries.

4 From Theory to Empirics

We are interested in explaining countries' trade pattern in the framework of the factor-proportions theory, using a cross-industry regression analysis. This empirical approach uses measures of trade and factor intensities and from them infer the factor abundance vectors, in a cross-industry framework. If the estimated coefficient of a factor is positive, the country is implied to be abundant in that resource. Generally, the results of these type of studies are controversial, as different authors obtained completely different results. This might be explained, to some extent, by their lack of theoretical foundations. There is no accord on the precise form of the estimation equation, the definition of variables (dependent and independent), the estimation procedure (*OLS* or *GLS*, applied to bilateral or multilateral trade), etc.

Deardorff (1984) tries to formulate the theory with a parametrized representation of both the production and consumption sides of the model, in order to justify the usage of a regression type of framework. He assumes both the production functions and the preferences to be $Cobb - Douglas$, internationally identical, and he finds a relationship between autarky and free trade commodity prices, and factor endowments. By assuming $Cobb - Douglas$ production and utility functions, the autarky factor and good prices can be expressed in terms of observable factor cost shares and factor endowments, and therefore his results can be used in empirical analysis.

Harkness (1978, 1981) is among the few trade economists that tried to find a theoretical justification for using cross- commodity regressions for explaining a country's commodity trade pattern. He tries to characterise the link between commodity trade and factor-service trade, hence between commodity trade and factor-abundance in a form that maintains the spirit of the $H - O$ theorem for a two-factor world. He proposes a relationship between commodity net exports and factor intensities as:

$$t_i/x_i = \sum_h \hat{\beta}_h \theta_{hi} + \hat{\mu}_i, \tag{101}$$

where t_i and x_i are the net exports and output in industry i, θ_{hi} is the factor cost share of factor h in the production of good i, $\hat{\beta}$ is an OLS-computed partial regression coefficient, determined from the multiple regression of t_i/x_i on all factor cost shares, in a cross-industry framework, and $\hat{\mu}$ is an OLS-computed error term. The relationship described in equation (101) is a *descriptive*, and not a *structural* one, and is characterising the relationship between commodity net exports and factor-intensities which, on average, obtains in the general equilibrium. Hence, $\hat{\beta}$ is an OLS-computed descriptive statistic summarising the average, *partial* relationship between net exports and factor-intensity, and not a *structural parameter*, invariant across commodities. The computed OLS coefficient is, in fact, a proxy for the hypothetical direct coefficient that would be defined when factor intensities were mutually uncorrelated across commodities. His formulation of the $H - O$ theorem is as follows:

> *"Given that factor complementarities can be controlled through multiple regression analysis, the sign and rank order of $\hat{\beta}_h$ will duplicate those of the net indirect exports as a proportion of total domestic supply, and thereby, according to Vanek, those of the corresponding relative factor abundance."*

The very good fit he obtained suggests that omitted variables, if any, are highly collinear with factor intensities, their influence being captured by $\hat{\beta}$.

An important contribution in explaining the link between the factor-proportions theory and the regression analysis is provided by Bowden (1983). Following Bowden (1983) and Kohler (1988) we show how, based on the theoretical models developed in Section 2, one may derive the proper estimation equations to be further used in a cross-industry empirical study. We found in Section 2.2 that a value version of the $H - O - V$ equation written for a particular country within the perfect competition framework is given by equation:

$$\Theta^{TOT} t^{v} = W\bar{v} - s\sum_{j} W^{j}\bar{v}^{j}. \tag{102}$$

There should be no debate about trade being treated as the dependent variable in a regression study, while factor endowments and factor intensities as exogenous. When the regression analysis is properly derived from the factor- proportions theory, it is agreed upon the proper choice of the trade variable as being net exports, rather than exports or imports, as many empirical studies in fact did. If in equation (102), we denote the right-hand side by an $m * 1$ vector g of relative factor endowments (in value terms), then it becomes:

$$\Theta^{TOT} t^{v} = g, \tag{103}$$

and if we premultiply equation (103) by the row vector g', we get:

$$g'\Theta^{TOT} t^{v} = g'g > 0. \tag{104}$$

Equation (104) is a restriction across commodities and factors that must hold in a post-trade equilibrium. It states that net commodity exports t_i must be higher, in some sense on average across all commodities, the higher the inner product $g'\theta_i$, where θ_i is column i of the (total) factor cost shares matrix Θ^{TOT}. However, trade theory does not allow for any specification of this relationship for each specific commodity.

Even though one may have data on all the variables in equation (102), the partial nature of the factor- proportions theory should be acknowledged by always adding an error term. The ideal situation would be to have a complete regression model that combines, in a rigorous way, the factor-proportions theory with other determinants of trade, such as increasing returns to scale and product differentiation. Then, the empirical analysis may suggest, for example, that the more complete model does nest that suggested by factor-proportions theory, as a special case.

This discussion suggests that in deriving the cross-industry regression equations, the next step should be an intuitive one and it involves an approximation. We may write $t_i^v = F(g'\theta_i)$ for each industry i or, as Bowden (1983) does:

$$t_i^v = \sum_{h=1}^{m} \alpha_h g_h \theta_{hi} + \mu_i, \tag{105}$$

where μ_i is an error term in industry i. In a cross- industry regression, the estimated coefficient may be interpreted as $\alpha_h g_h$, which may be then "externally validated" by actual observations of g_h in a second-stage estimation. The estimated coefficients obtained in a first- stage estimation are regressed on factor endowments in a cross-country framework (see Balassa (1979, 1986) and Balassa and Bauwens (1988)).

As Kohler (1988) noticed, a zero expectation of the error term implies an unspecified cross-commodity restriction on the parameters. As for the variance of the error term, in a cross-commodity regression, one should consider the problem of heteroskedasticity and hence, use GLS estimators. If we consider the existence of country-specific non-$H-O$ determinants, this may be modelled through intercepts in equation (105). However, as Bowen and Sveikauskas (1992) have shown, that inclusion of a constant term implies a specific trade imbalance correction. Following Bowen and Sveikauskas, the correct trade equation to estimate becomes:

$$t_i^v - b\frac{x_i^{wfv}}{y^w} = \alpha_0 + \sum_h \alpha_h \theta_{hi}^{TOT}(w_h\bar{v}_h - \frac{y}{y^w}\sum_j w_h^j \bar{v}_h^j) + \mu_i, \tag{106}$$

where x_i^{wfv} is a vector of world total production for final demand of good i, in value terms, and α_0 is a constant term measuring the level of net exports when domestic production is zero. The interactions implied by equation (106) should be understood in the following way: if a factor h is intensively used in the production of good i, hence θ_{hi}^{TOT} is relatively large, and if the country is relatively well endowed with factor h, hence $(w_h\bar{v}_h - \frac{y}{y^w}\sum_j w_h^j \bar{v}_h^j)$ is positive, this combination favours the production and exports of good i.

Based on equation (106) and the results in Section 2, one may explain the countries' commodity pattern of trade when allowing for increasing returns to scale and product differentiation. As already discussed in Section 2.4, the independent variables, hence the factor cost shares, are modified according to equations (38) and (50). Therefore, there are no additional explanatory variables, but rather new interactive ones. In the case where the increasing returns to scale are internal to the firms, separate estimation equations for the exports and imports of differentiated products may be derived.

5 Conclusions

Based on the $H-O-V$ model in its value version, this paper presents theoretical models for explaining a country's commodity pattern of trade in a multi-factor, multi-good, multi-country framework. By contrast with previous studies, these models allow for cross-country technological and factor price differences, and for departures from some of the assumptions of the original $H-O$ theory, such as increasing returns to scale and product differentiation, or internationally mobile capital. They also allow for intermediate production, while preserving most of the strong assumptions of the $H-O$ theory such as internationally immobile input factors (other than capital), identical and homothetic preferences in consumption, free trade, no transportation costs, and factor market and world commodity clearing. The presence of economies of scale and product differentiation and internationally mobile capital makes the assumptions of the models slightly more realistic, while the standard model is more general. The presence of product differentiation and economies of scale internal to the firm allows us to write separate factor-content equations for exports and imports of the differentiated products.

Based on these theoretical models, I reformulate the rank hypotheses that allow for direct tests of the $H-O-V$ theory. I also try to address properly the issue of cross-industry empirical estimation of the $H-O-V$ equations, especially when allowing for the commodity markets to be imperfectly competitive. I show that the economies of scale and product differentiation variables enter the explanatory part of the regression in an interactive term together with factor cost shares, rather than as additional explanatory variables.

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Appendix A

List of symbols and definitions of variables and parameters used in the paper:

a) **Superscripts:**

- i, j, g countries,
- w world,
- f final demand,
- h, l input factors,
- v value term,
- A adjusted,
- $'$ transpose of a matrix,
- TOT total.

b) **Subscripts:**

- k firm or capital,
- i industry,
- ω variety of a differentiated product,
- f factor-content,
- l labour,
- q intermediate.

c) **Matrices:**

- I $n * n$ identity matrix,

- A^v $n * n$ input-output matrix , with elements indicating the value of output a particular industry must buy from each other industry to produce one dollar of final demand its own product,
- R $m * n$ matrix of direct factor requirements, with elements r showing the quantity of each direct factor input required to produce a unit of gross output in a particular industry,
- T $m * n$ matrix of the total (direct and indirect) input factor requirements, with elements showing the total requirement of each factor input per unit of final demand in each industry,
- Θ $m * n$ matrix of direct factor cost shares (the $Cobb - Douglas$ parameters), with elements θ representing the value of each factor in a dollar value of gross output of each industry,
- W $m * m$ diagonal matrix of factor prices,
- P $n * n$ diagonal matrix of commodity prices,
- β $n * n$ diagonal matrix of mark-up prices,
- Ω $m*m$ diagonal matrix, with entries showing the non-neutral adjustment on direct factor cost shares.

d) **Vectors:**

- x $n * 1$ vector of gross output produced,
- c $n * 1$ vector of final consumption,
- t $n * 1$ vector of net trade,
- z $n_2 * 1$ vector of exports of the differentiated products,
- m $n_2 * 1$ vector of imports of the differentiated products,
- i $n_2 * 1$ vector of intra-industry trade in the differentiated products,
- d_i $n_1 * 1$ vector of world demand for the homogeneous products,

- $d_{i\omega}$ world demand for variety ω of the differentiated product i,
- $\bar{v}$ $m * 1$ vector of factor endowments,
- v $m * 1$ vector of factor demand,
- $\phi(w,p)$ $n * 1$ vector of unit production costs,
- w $m * 1$ vector of factor prices,
- $\bar{\rho}$ $n_2 * 1$ vector of fixed costs in the IRS industries,
- ε $n_2 * 1$ vector of the external economies of scale in $theIRS$ industries,
- σ $n_2 * 1$ vector of the constant elasticities of substitution between pairs of varieties of the same differentiated product,
- e $n * 1$ vector of expenditures,
- n $n_2 * 1$ vector of the number of varieties of the differentiated product,
- p $n * 1$ vector of commodity prices,
- α $n * 1$ vector of regression coefficients,
- μ $n * 1$ vector of error terms.

e) **Other symbols:**

- s ratio between a country's domestic absorption and world income,
- y GNP,
- b trade balance of a country,
- k capital endowment of a country,
- t^k net exports of capital of a country,
- $F(v)$ production function for a firm in an industry,
- $u_i(.)$ subutility function derived from the consumption of product i,

- U upper tier utility function,
- m number of input factors,
- n number of industries,
- c number of countries.

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