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Arno Riedl

Institut für Höhere Studien
Institute for Advanced Studies
Vienna

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Author(s):
Arno Riedl

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Josefstädter Straße 39, A-1080 Wien
E-Mail: office@ihs.ac.at
Web: www.ihs.ac.at

Institut für Höhere Studien (IHS), Wien
Institute for Advanced Studies, Vienna

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Arno Riedl
Institute for Advanced Studies
Department of Economics
Stumpergasse 56, A-1060 Vienna
Phone: +43/1/599 91-155
Fax: +43/1/599 91-163
e-mail: riedl@ihssv.wsr.ac.at

Institut für Höhere Studien (IHS), Wien
Institute for Advanced Studies, Vienna

Abstract

The opportunity to bargain often causes costs for at least one party in many economic situations, e.g. wage negotiations, joint ventures or interfirm cooperation. This paper studies such situations. A "strong" and a "weak " player have to agree how to divide the produced surplus. The "weak" player has to bear *opting in* costs. We characterize the set of subgame perfect equilibria. It is shown (i) that raising the costs of the weak party may strictly lower the payoff of the strong party, (ii) that for some cost levels the only equilibrium is inefficient, (iii) that if the players are sufficiently patient the outcomes of the "zero-cost model" and the "vanishing costs" version of our model do not coincide, and (iv) that in general multiplicity of equilibria arises.

Zusammenfassung

In vielen ökonomischen Situationen, wie z.B. Lohnverhandlungen, Joint Ventures oder anderen Formen der zwischenbetrieblichen Kooperation, hat zumindest eine der involvierten Parteien Verhandlungskosten zu tragen. Diese Arbeit analysiert solche Situationen. Ein "starker" und ein "schwacher" Spieler müssen ein Abkommen darüber treffen wie der von ihnen produzierte Surplus aufgeteilt werden soll. Der schwache Spieler ist derjenige welcher *Eintritts*kosten zu tragen hat. Die Menge der teilspielperfekten Gleichgewichte wird charakterisiert. Weiters wird gezeigt: (i) Eine Erhöhung der Eintrittskosten des schwachen Spielers bringt nicht immer einen Vorteil für den starken Spieler; (ii) für bestimmte Kostenniveaus ist das eindeutige Gleichgewicht ineffizient; (iii) Die Outcomes des "Null-Kosten" Modells und des präsentierten Modells mit verschwindenden Kosten stimmen nicht überein wenn die Spieler hinreichend geduldig sind; (iv) im allgemeinen existieren keine eindeutigen Gleichgewichte.

Keywords

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Comments:

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1 Introduction

The opportunity to bargain over some given surplus often causes costs for at least one party in many economic and political situations. Consider for example a situation where a union and a firm have to come to an agreement about future wages. The union may decide to act completely passively, accepting any wage offered by the firm. This would be an ultimatum game situation. On the other hand the union may decide to play an active part and force the firm to bargain about the wage. Such an enforcement will in general not be cost-less. The costs which the union and/or its members have to bear can be of various kinds. The simplest kind of costs for a union one can think of are just the membership fees which have to be paid by the workers. Furthermore, if a union wants to force the firm to bargain about the wage, it has to convince the firm that the workers are supporting the unions policy. Then the union has to bear costs because it, for example, has to organize a workers meeting to demonstrate its power. In this case the costs can be monetary ones, like a rent for a meeting room or/and compensation payments for the workers who show up at the meeting, or non-monetary ones, like foregone leisure time for the workers attending the meeting. A further example of costs to be borne by the union is the salary that has to be paid to the representative who is doing the negotiating with the firm. Since, in general, such costs are increasing with the duration of the bargaining process, as long as no agreement is reached, the union has to decide from time to time if it wants to continue the negotiations or to stop it and to accept an ultimatum offer made by the firm.

The possibility of such *opting in* costs can occur not only in union versus firm bargaining situations. Suppose, for example, the following situation: two partners are planning to run a production joint venture. They have to bargain over the potential surplus which can be extracted by a jointly run production-line, before the joint venture is actually set up. One partner owns the know-how of an efficient production process, whereas the other possesses the hardware. The latter may also have some know-how of how to run the production, but this may be an older technology and therefore less efficient. This is typically the case in production joint ventures between firms from Eastern Europe and firms from some Western Industrialized Country. There exists a possible efficiency gain if the two firms decide to cooperate in such a situation compared to the situation where they don't. Suppose furthermore that the firm who possesses the know-how can also sell this knowledge on a market. If the two firms want to cooperate, they have to agree and write a contract on how the produced surplus will be divided between them. As long as they are negotiating no production can take place. This can be due to the fact that the implementation of the technology is an irreversible process. The know-how possessing firm therefore will only agree to implement it if it can be sure that the gain from the know-how sale on the market is not higher than the gain from cooperation. While the bargaining process takes place the surplus is shrinking due to discounting. Additionally, the foregone gains of alternative utilization of the technology for this firm will be higher the longer the negotiations lasts. These foregone gains as well as the costs mentioned in the union/firm example can be regarded as *opting in* costs for one of the involved parties.

Several questions arise. What is the effect of *opting in* costs on equilibrium payoffs? Will these costs, as one would suggest at first sight, always work in favor of the party which doesn't have the cost? Is there a possibility of inefficient outcomes in the sense that the potential gains from bargaining will not be exploited? What is the relationship between the "zero-cost" model and a model with opting in costs approaching 0? Will the former always be the limit case of the latter one? In this paper we are trying to find answers to these questions. We model the economic situations mentioned above as a "Rubinstein-like" bargaining game to which we add the new feature that the weak party has to bear costs if she decides to opt into the bargaining process. This paper therefore is in line with the stream of contributions on noncooperative bargaining initiated by Rubinstein's (1982) seminal paper as well as the work of Ståhl (1972). The studies on outside options (see e.g. Shaked and Sutton (1984), Shaked (1987)) and money burning (Avery and Zemsky (1994)) are most similar to ours. Hendon *et al.* (1994) introduced switching costs for one player in addition to the usual discounting. They showed that if these costs are sufficiently large the subgame perfect equilibrium in a market with one seller and two buyers is always unique. These switching costs are somewhat similar to the *opting in* costs introduced in this paper. In our model, however, uniqueness is ensured for sufficiently large *and* sufficiently small opting in costs. We also show that there always exists a non-empty interval of such costs where multiplicity of equilibria will arise. Furthermore, the possibility of inefficient outcomes occurs which are not due to delay. We also find that (i) an increase of the opting in costs will not always work in favor of the stronger player and (ii) that under some circumstances it is strictly worse for the strong player if the costs of the weaker one increase. Hence, if a strong player has the option to choose between two weak players, he will possibly prefer to bargain with the relatively stronger one. Or if a firm has the possibility to influence the opting in costs of the union, it will not always try to raise these costs. Comparing what our model for costs strictly larger than 0 predicts, with the "zero-cost" model, we see that for almost completely patient players ($\delta \to 1$) the equilibrium payoffs of our model with vanishing opting in costs and the "zero-cost" model do not coincide. Furthermore, we also find that for reasonable alternative payoffs "outside", the unique equilibrium payoff of the "zero-cost" model is not even contained in the set of equilibrium payoffs of our "vanishing-costs" model. Hence for high discount factors the "zero-cost" model is not the limit case of our model with costs approaching 0.

The remainder of the paper is organized as follows. In section 2 the formal model is presented and in section 3 the results of the bargaining game are derived and presented. In section 4 we draw some conclusions.

2 The Model

Let us consider a situation where two parties have the possibility to extract a surplus from cooperation if, and only if, they are able to settle on an agreement how to divide these cooperative gains. At the beginning of each (potential) bargaining round one of the players has to decide if she wants to continue the negotiations. The decision to continue causes some effort costs. If she decides not to continue, the status quo remains. This gives both players some exogenous given "alternative" or "outside" payoffs. The parties face the following situation: They have to play a "Rubinstein-like" bargaining game. The game exhibits some asymmetry insofar as the bargaining position of one player, subsequently called w, is weakened compared to the other player's in two respects. Firstly, she has to implement some costly effort to have the opportunity to participate in the bargaining part of the game and secondly, her alternative payoff, called a, is smaller then the alternative payoff b of the other player, whom we shall call s. To get possible gains from cooperation, we assume that $a+b$ is smaller than the potential surplus. This surplus can be divided between the two parties if bargaining actually takes place. Without loss of generality we normalize this surplus to unity. Hence, the potential efficiency gains are given by $1-(a+b)$. We now want to define the game more formally.

The Bargaining Game

The game is played by two parties labeled s and w who bargain over the partition of a potential surplus with a size normalized to 1. The bargaining process takes place over time; the discrete time periods are of length 1 and are denoted by t, $t \in \{0, 1, 2, \ldots\}$. In each period one of the parties is selected randomly (with probability $\frac{1}{2}$ and independently of previous selections) to propose an offer[1] $\mathbf{x} = (x, y) \in [0,1] \times [0,1]$ with $x + y = 1$ where x (resp. y) is player s's (resp. player w's) share of the surplus. The other party immediately responds by accepting the offer, or rejecting it. If the offer is accepted an agreement is reached and the play ends with the agreed upon partition of the surplus. If it is rejected, the play will proceed into the next period.

The "Inside Option"

To get the opportunity to bargain with player s, player w has to decide if she wants to *opt in*, at the beginning of each period. If she decides to do so, she has

[1]Since we are interested in investigating the influence of asymmetries with respect to alternative points and opting in costs, we want to get rid of the so called "first proposer advantage". The proposed random selection procedure is the best and easiest way to do this.

to choose the action $e_t = 1$. By doing this player w immediately incurs costs of $c \geq 0$. Bargaining starts immediately after such a decision, with the selection of the proposer by nature. If player w withdraws from bargaining with the other player, the play ends with a payoff of b for player s and a payoff of a for player w.

The order of events in each period t thus is: At the beginning of the period party w has to decide upon opting in or not by choosing an $e_t \in \{0, 1\}$. If she decides to choose $e_t = 0$ the play ends without bargaining taking place. Otherwise a bargaining round will begin. A chance move then determines the identity of the proposer who then makes an offer to which the other party responds with acceptance or rejection. Acceptance terminates the game. Upon rejection the play proceeds into the next period $t + 1$.

insert Figure 1 here

Preferences and Costs

The parties are assumed to be risk neutral expected utility maximizers and they discount future income (as well as future costs) with a common discount factor $\delta \in]0, 1[$. This means that if they agree on a partition of the surplus of say (x, y) in period t, then player s gets the payoff $u = \delta^t x$, and player w's payoff is given by $v = \delta^t y - \sum_{i=0}^{t} \delta^i c$. If player w decides not to opt in in period t, after having chosen $e_i = 1$ in periods $i = 0, \ldots, t-1$, the strong player gets $u = \delta^t b$ whereas the weaker one gets $v = \delta^t a - \sum_{i=0}^{t-1} \delta^i c$, with $\sum_{i=0}^{-1} \delta^i \equiv 0$.

Strategies and Solution Concept

At each time period t there are tree consecutive instances where one or the other party might have to make a decision. These decisions are: (1) Player w has to choose between $e_t = 1$ and $e_t = 0$; (2) What proposal to make (if player w has chosen $e_t = 1$ and the player was selected to be the proposer); (3) Whether or not to accept the rivals proposal (if the player was selected not to propose). A strategy of a player in this extensive form game is a sequence of decision rules which specify the players action at every node of the game tree at which it is the player's turn to move, conditional on the history up to that node. We will denote a strategy of player s with σ and a strategy of player w with ω.

Since this is a game of perfect and complete information the natural solution concept to be employed is that of subgame-perfect equilibrium, developed by Selten[1965, 1975].

3 Results

This section is partitioned into three parts. First we define two constants which will play an important role in the analysis of the game. Both constants are related to particular levels of opting in costs. In section 3.1 some preliminary results concerning the existence and (non)uniqueness of subgame-perfect equilibria are presented. The main results of our analysis are stated in section 3.2.

Throughout the paper we will assume that $0 \leq a < b < 1$ and $a + b < 1$ holds. These assumptions reflect basic features of the situation we want to analyze. Namely, that one party, player w, is weaker than player s, and that there exist potential efficiency gains from bargaining. We now define the two constants mentioned in the preceding paragraph.

Definition 1

$$(i) \quad c_0 \;:=\; \tfrac{1}{2}(1 - \delta b - a(2-\delta)), \quad (ii) \quad c_1 \;:=\; (1-2a)\tfrac{1-\delta}{2-\delta}.$$

Both constants specify particular levels of opting in costs. At first glance they seem to be completely arbitrary. To get some intuition we will look at them for discount factors approaching 1 (0 resp.). For $\delta \to 1$ we get that c_0 approaches $\frac{1}{2}[1-(a+b)]$ and c_1 approaches the value 0. In this case c_0 "is" half of the potential gains from cooperation. For $\delta \to 0$ both values are approaching $\frac{1-2a}{2}$. Because of our assumptions on a and b we have that $a \in [0, \frac{1}{2}[$. Using this information we can deduce that in case of vanishing patience the above defined cost levels are somewhere between 0 and $\frac{1}{2}$, i.e. between zero and half of the surplus which the parties can divide if they cooperate. c_0 as well as c_1 are strictly decreasing in the discount factor. For intermediate levels of δ they therefore lie in between their limit values.

Our assumptions on a and b ensure that the intervals $[0, c_0[$ and $[0, c_1[$ are non-empty. That $c_1 > 0$ is obvious because $a < \frac{1}{2}$ holds. To see that $c_0 > 0$ assume to the contrary that $0 \geq c_0$. Together with $a < b$ and $a + b < 1$ this leads to the contradiction $1 < \delta$. To investigate how the parameters δ, a and b interfere with c_0 and c_1 observe that $c_1 \leq c_0$ is equivalent to $a \leq \delta^{-1}(1-(2-\delta)b)$. In Figures 2(a) and 2(b) we have drawn this relationship for two different values of the discount factor δ in the (a,b)-plane.

insert Figure 2(a) and 2(b) here

The shaded region is the one where $c_1 < c_0$ holds. The plain one shows the (a,b) configurations where the reverse inequality, $c_1 > c_0$, holds. Fig. 2(a) shows that for relatively high discount factors, e.g. 0.9, the region of (a,b) values for which $c_1 \leq c_0$ holds is relatively large, compared to the region where the reverse inequality is valid. Even for small discount factors (Fig. 2(b)) the area where with $c_1 < c_0$ is always larger than the area with $c_1 > c_0$. If we think of a bargaining process in "real world" one could argue that it is natural to assume that the more "realistic" case will be that one of rather "high" discount factors. However, we will analyze the case $c_1 \leq c_0$ as well as $c_1 > c_0$.

3.1 Preliminaries: Existence and (Non)Uniqueness of Subgame-Perfect Equilibria

Our first result, stated in Lemma 1, is just a variation of Rubinstein's (1982) well known "Existence-Theorem". The difference to the classical result applies to the equilibrium payoffs only, and is due to the fact that we have introduced opting in costs.

Lemma 1 EXISTENCE OF A SUBGAME PERFECT EQUILIBRIUM FOR SUFFICIENTLY SMALL OPTING IN COSTS

If $c \in [0, c_1]$ then the strategy combination (σ_1^*, ω_1^*) described in Table I forms a subgame perfect equilibrium.

Table I: Equilibrium strategies for $c \in [0, c_1]$

		S_1
Player s	propose	$\overline{y}^s = \frac{1}{2}\delta - \frac{1}{2}\delta\frac{2-\delta}{1-\delta}c$
	accept	$y \leq \overline{y}^w$
Player w	e	1
	demand	$\overline{y}^w = \frac{1}{2}(2-\delta) - \frac{1}{2}\frac{\delta^2}{1-\delta}c$
	accept	$y \geq \overline{y}^s$
Transition		*absorbing*

(For a discussion of this method of representing an equilibrium see Osborne and Rubinstein (1990).)

Proof: To see that this pair of strategies form a subgame-perfect equilibrium (SPE) it suffices to check for one-shot-deviations, only (see Fudenberg and Tirole (1991), and Harris (1986)).

(A) Player s:

(α) To propose more than $\overline{y}^s$ is obviously worse than proposing $\overline{y}^s$.

(β) To propose less than $\overline{y}^s$ gives an expected payoff of $\delta\frac{1}{2}(1 - \overline{y}^s + 1 - \overline{y}^w) = \delta(\frac{1}{2} + \frac{1}{2}\frac{\delta}{1-\delta}c)$ whereas following the proposed equilibrium strategy gives $1 - \overline{y}^s$ for sure. It is easy to show that $1 - \overline{y}^s \geq \delta\frac{1}{2}(1 - \overline{y}^s + 1 - \overline{y}^w)$ is equivalent to $c \geq -\frac{1-\delta}{\delta}$ and therefore the inequality holds for all $c \geq 0$.

(γ) If he doesn't accept the proposal $1 - \overline{y}^w$, he will again get an expected payoff of $\delta\frac{1}{2}(1 - \overline{y}^s + 1 - \overline{y}^w)$ which is exactly $1 - \overline{y}^w$.

These arguments show that player s cannot do better in using any other strategy then σ_1^*, given that player w uses ω_1^*.

(B) Player w:

(α) To demand less than $\overline{y}^w$ is obviously not better than demanding exactly $\overline{y}^w$.

(β) If player w demands more than $\overline{y}^w$, player s will reject and player w can expect to get $\delta\frac{1}{2}(\overline{y}^s + \overline{y}^w) - \delta c$. It is easy to show that this is not larger than $\overline{y}^w$ for all $c \geq 0$.

(γ) If she rejects the proposal $\overline{y}^s$ made by player s, she will again get $\delta\frac{1}{2}(\overline{y}^s + \overline{y}^w) - \delta c$ which is exactly $\overline{y}^s$.

(δ) If player w chooses $e = 1$, she will get $\frac{1}{2}(\overline{y}^s + \overline{y}^w) - c$. Hence we have to show that $\frac{1}{2}(\overline{y}^s + \overline{y}^w) - c \geq a$ holds. After some calculations one can show that the above inequality is equivalent to $c \leq c_1$. Since we have assumed $c \in [0, c_1]$ this holds true and therefore this kind of deviation makes player w not better off.

These arguments show that player w cannot do better in using any other strategy then ω_1^* given that player s uses σ_1^*. Together with the observation in (A) this shows that the strategy combination (σ_1^*, ω_1^*) is indeed a subgame perfect equilibrium, given our assumptions.

□

As can be seen from Table I, the equilibrium strategies are such that player s always proposes $\overline{y}^s$ and accepts any offer not greater $\overline{y}^w$. Player w always opts in, always proposes $\overline{y}^w$, and accepts any offer not smaller $\overline{y}^s$. Hence, the equilibrium strategies are stationary as in the classical noncooperative bargaining model.

Proposition 1 below states that if the opting in costs are small enough, then the SPE described in Lemma 1 is unique. The intuition behind is similar to that of a bargaining game with small enough outside option values (see e.g. Shaked (1987)). The costs only have an effect on the agreed upon payoffs, but not on the opt in decision of the weak player.

Proposition 1 UNIQUENESS OF THE SUBGAME PERFECT EQUILIBRIUM FOR SUFFICIENTLY SMALL OPTING IN COSTS

If $c \in [0, \min\{c_0, c_1\}[$ then (σ_1^*, ω_1^*) from Lemma 1 is the unique subgame perfect equilibrium and the unique expected equilibrium payoff pair (u_1^*, v_1^*) is given by

$$(\frac{1}{2} + \frac{1}{2}\frac{\delta}{1-\delta}c, \frac{1}{2} - \frac{1}{2}\frac{2-\delta}{1-\delta}c)$$

.

Proof: Existence follows from Lemma 1. The players preferences satisfy all assumptions stated in Rubinstein (1982), and Osborne and Rubinstein (1990). In particular the "Increasing loss to delay"-assumption is satisfied. Therefore the above equilibrium is unique if $e_t = 1\ \forall t$. Hence, it remains to show that in each SPE after every history it is optimal for player w to opt in, i.e. to choose $e_t = 1$ instead of $e_t = 0$. If she chooses $e_t = 0$ she will get a immediately. Therefore we have to show that in any SPE of the bargaining situation (BS) she can expect to get strictly more than $a + c$. Denote by m_w^s (m_w^w resp.) the infimum of all accepted SPE-shares of the potential surplus for player w in a BS where player s (player w resp.) makes the offer. And $m_w := \frac{1}{2}(m_w^s + m_w^w)$ as the infimum of the expected accepted SPE-shares of the surplus for player w immediately before nature decides about the proposer. With this notation in mind we have to show that $m_w > a + c$. In any SPE player s has to propose at least $\max\{\delta a, \delta m_w - \delta c\}$ because otherwise player w is better off in rejecting the offer and waiting until the next period. In any BS on the other hand player s cannot expect to get more than $\max\{\delta b, \delta(1 - m_w)\}$. Therefore he will accept any offer $1 - y > \delta \max\{b, 1 - m_w\} \Leftrightarrow y < 1 - \delta \max\{b, 1 - m_w\}$. Hence, $\exists \epsilon > 0$ s.th. $y + \epsilon$ is still accepted by player s. But since player w is better off then such a y cannot be an equilibrium offer. We get

$$\begin{aligned} m_w^s &\geq \delta \max\{a, m_w - c\} \\ m_w^w &\geq 1 - \delta \max\{b, 1 - m_w\} \end{aligned}$$

We want to show that $a < m_w - c$. Assume to the contrary that $a \geq m_w - c$. Hence $m_w^s \geq \delta a$ and $m_w^w \geq 1 - \delta \max\{b, 1 - m_w\}$. If $1 - m_w \geq b$ then $m_w^w \geq 1 - \delta(1 - m_w) \Rightarrow m_w \geq \frac{1}{2}(\delta a + 1 - \delta + \delta m_w) \Leftrightarrow m_w \geq \frac{1-\delta+\delta a}{2-\delta}$. Together with $a + c \geq m_w$ this implies $(2 - \delta)(a + c) \geq 1 - \delta + \delta a$ which is equivalent to $c \geq (1 - 2a)\frac{1-\delta}{2-\delta} = c_1$. But this is a contradiction to $c < \min\{c_0, c_1\}$. If $1 - m_w < b$ then $m_w^w \geq 1 - \delta b$ and $m_w \geq \frac{1}{2}(\delta a + 1 - \delta b)$. Together with $a + c \geq m_w$ this implies that $a + c \geq \frac{1}{2}(\delta a + 1 - \delta b)$. But the last inequality is equivalent to $c \geq \frac{1}{2}(1 - \delta b - a(2 - \delta)) = c_0$ what again is a contradiction to our assumption $c < \min\{c_0, c_1\}$. Therefore $a < m_w - c$ indeed holds and thus it is always better for player w to opt in. Hence, the SPE (σ_1^*, ω_1^*) is unique if $c < \min\{c_0, c_1\}$.

The corresponding expected equilibrium payoff pair (u_1^*, v_1^*) is easily calculated, using $\overline{y}^s$, $\overline{y}^w$ of table 1 and $v_1 = \frac{1}{2}(y^s + y^w) - c$ as well as $u = \frac{1}{2}(1 - y^s + 1 - y^w)$.

□

In this equilibrium player w will always opt in because the opting in costs are small enough to ensure this behavior. The play ends with an immediate agreement in the first period with the expected equilibrium-shares

$$(y_s^*, y_w^*) = (\frac{1}{2} + \frac{1}{2}\frac{\delta}{1-\delta}c, \frac{1}{2} - \frac{1}{2}\frac{\delta}{1-\delta}c).$$

The alternative points (b, a) determine only upper bounds for the opting in costs for which the above equilibrium exists and is unique. They play no direct role in the determination of the equilibrium payoffs. This makes the situation similar to that one of a bargaining game with outside-options where the payoffs "outside" the bargaining game are too small to have an impact on the negotiated upon equilibrium shares. The additional costs which player w has to bear work like some additional impatience. It is therefore not surprising that, as it is easily seen, the payoff for the stronger player is increasing and the payoff for the weaker player is decreasing in c. Furthermore, the equilibrium-share of player w (s resp.) is smaller (larger resp.) then in the "Rubinstein"-game. For $c = 0$ we get the solution $(\frac{1}{2}, \frac{1}{2})$ of the "traditional" bargaining game. Next we'll analyze the other "extreme", namely the case of very high opting in costs.

Intuition tells us that for extremely large opting in costs the incentive to opt in will be destroyed. If the weak player decides to opt in she expects to get some compensation for her costs. But the "pie" is bounded and therefore the strong player cannot credibly make such compensation promises if the costs are very large. This leads to an equilibrium behavior of player s where he implements a strategy which prevents the weak player from opting in. These reasonings are described by Lemma 2 and Proposition 2 more formally.

Lemma 2 EXISTENCE OF A SUBGAME PERFECT EQUILIBRIUM FOR SUFFICIENTLY LARGE OPTING IN COSTS

If $c \in [c_0, +\infty[$ then the strategy combination (σ_2^*, ω_2^*) described in Table II form a subgame perfect equilibrium.

Table II: Equilibrium strategies for $c \in [c_0, +\infty[$

		S_1
Player s	propose	$\overline{y}^s = \delta a$
	accept	$y \leq \overline{y}^w$
Player w	e	0
	demand	$\overline{y}^w = 1 - \delta b$
	accept	$y \geq \overline{y}^s$
Transition		*absorbing*

Proof: To see that this pair of strategies form a SPE we have to check for one-shot-deviations, again.

(A) Player s:

(α) To propose more than $\overline{y}^s$ is obviously worse than proposing $\overline{y}^s$.

(β) To propose less than $\overline{y}^s$ gives an expected payoff of δb. But following σ_2^* gives $1 - \delta a$, which is larger than δb since $1 > a + b$.

(γ) If he doesn't accept the proposal $1 - \overline{y}^w$ he will again get an expected payoff of δb which is exactly the same as he would get in following the proposed equilibrium strategy.

(B) Player w:

(α) To demand less than $\overline{y}^w$ is obviously not better than demanding exactly $\overline{y}^w$.

(β) If player w demands more than $\overline{y}^w$ player s will reject and player w can expect to get δa. Since $1 > a + b$ we have $\overline{y}^w = 1 - \delta b > \delta a$.

(γ) If she rejects the proposal $\overline{y}^s = \delta a$ made by player s she will get δa, and the deviation is not profitable.

(δ) If player w chooses $e = 1$ as ω_2^* prescribes, she will get a. Deviating gives the expected payoff $\frac{1}{2}(\overline{y}^s + \overline{y}^w) - c = \frac{1}{2}(1 - \delta b + \delta a) - c$. Suppose that the deviation is profitable, i.e. $a < \frac{1}{2}(1 - \delta b - \delta a) - c$. It is easily seen that this inequality is equivalent to $c < \frac{1}{2}(1 - \delta b - a(2 - \delta)) = c_0$ which contradicts $c \in [c_0, +\infty[$.

These arguments show that the strategy combination (σ_2^*, ω_2^*) is indeed a subgame perfect equilibrium, given our assumptions.

□

As for small opting in costs we are able to state a uniqueness result for sufficiently large opting in costs, too.

Proposition 2 UNIQUENESS OF THE SUBGAME PERFECT EQUILIBRIUM FOR SUFFICIENTLY LARGE OPTING IN COSTS

If $c \in]\max\{c_0, c_1\}, +\infty[$ then (σ_2^*, ω_2^*) stated in Lemma 2 is the unique subgame perfect equilibrium and the unique equilibrium payoff pair (u_2^*, v_2^*) is given by

$$(b, a).$$

Proof: Existence is ensured by Lemma 2. If $e_t = 0\, \forall t$ then the above equilibrium is obviously unique. It remains to show that in each SPE after every history it is indeed always optimal for player w to choose $e = 0$. Denote by M_w^s (M_w^w resp.) the supremum of all accepted SPE-shares of the surplus for player w in a BS where player s (player w resp.) makes the offer. Define $M_w := \frac{1}{2}(M_w^s + M_w^w)$ as the supremum of the expected accepted SPE-shares for player w immediately before nature decides about the proposer. Given these definitions we have to show that $a > M_w - c$. With arguments similar to those in the proof of Proposition 1 we get that

$$\begin{aligned} M_w^s &\leq \delta \max\{a, M_w - c\} \\ M_w^w &\leq 1 - \delta \min\{b, 1 - M_w\} \end{aligned}$$

and from table 2 we know furthermore that

$$M_w \geq \frac{1}{2} - \frac{1}{2}\delta(b - a)$$

must hold.

Suppose to the contrary that $a \leq M_w - c$. This implies

$$\begin{aligned} M_w^s &\leq \delta M_w - \delta c \\ M_w^w &\leq 1 - \delta \min\{b, 1 - M_w\} \end{aligned}$$

If $b \leq 1 - M_w$ then $M_w^w \leq 1 - \delta b$ which in turn implies $M_w \leq \frac{1}{2}(\delta M_w - \delta c + 1 - \delta b)$. This is equivalent to

$$M_w \leq \frac{1 - \delta b}{2 - \delta} - \frac{\delta}{2 - \delta} c.$$

Together with $M_w \geq \frac{1}{2} - \frac{1}{2}\delta(b - a)$ this implies $1 - \delta b - \delta c \geq (2 - \delta)\frac{1}{2}(1 - \delta(b - a)) \Leftrightarrow c \leq \frac{1}{2}(1 - \delta b - a(2 - \delta)) = c_0$. But this contradicts $c > \max\{c_0, c_1\}$.

If, on the other hand, $b > 1 - M_w$ then $M_w^w \leq 1 - \delta(1 - M_w)$, and therefore $M_w \leq \frac{1}{2}(\delta M_w - \delta c + 1 - \delta + \delta M_w)$ which is equivalent to

$$M_w \leq \frac{1}{2} - \frac{1}{2}\frac{\delta}{1 - \delta} c.$$

Together with $a + c \leq M_w$ this implies $a + c \leq \frac{1}{2} - \frac{1}{2}\frac{\delta}{1-\delta} c \Leftrightarrow c \leq (1 - 2a)\frac{1-\delta}{2-\delta} = c_1$ which again contradicts $c > \max\{c_0, c_1\}$. Hence we must have $a > M_w - c$ as desired.

□

The equilibrium strategies described in Table II are stationary. Player w never opts in, always demands $1 - \delta b$ and accepts any offer not smaller than δa. The strong player always proposes δa and accepts any offer which gives him at least δb. The weak player never opts in because the opting in costs are too large compared to the possible equilibrium shares of the surplus she can ensure for herself. Similar as in Proposition 1 the alternative points determine - via c_0 and c_1 - bounds on the costs for which the above equilibrium exists (is unique resp.). But in contrast to the equilibrium in Proposition 1 the equilibrium payoffs are now completely determined by these points b and a, since if the weak player chooses $e_0 = 0$ both parties get their alternative payoffs.

So far we have characterized the equilibria for the extreme cases of very small and very large opting in costs. We turn now to the case where the opting in costs are in some intermediate range, i.e. $c \in [\min\{c_0, c_1\}, \max\{c_0, c_1\}]$. The following Lemmata 3-4 show that in this case multiplicity of subgame-perfect equilibria arise. Propositions 3-4 state that not only the equilibria are not unique, but that they also lead to multiplicity of equilibrium-payoffs. Lemma 3 and Proposition 3 are concerned with the case $c_1 \leq c_0$ whereas Lemma 4 and Proposition 4 are concerned with the case $c_1 > c_0$.

Lemma 3 EXISTENCE OF SUBGAME PERFECT EQUILIBRIA FOR "INTERMEDIATE" OPTING-IN COSTS (PART 1)

If $c_1 \leq c_0$ *and* $c \in [c_1, c_0]$ *then the strategy combinations* (σ_3^*, ω_3^*) *described in Table III form subgame perfect equilibria.*

Table III: Equilibrium strategies for $c \in [c_1, c_0]$

		S_1	S_2	S_3	S_4
Player s	propose	$\overline{y}^s \in V^s$	$\underline{V}^s$	$\overline{V}^s$	$\underline{V}^s$
	accept	$y \leq \overline{y}^w$	$y \leq \underline{V}^w$	$y \leq \overline{V}^w$	$y \leq \overline{V}^w$
Player w	e	1	1	1	0
	demand	$\overline{y}^w \in V^w$	$\underline{V}^w$	$\overline{V}^w$	$\overline{V}^w$
	accept	$y \geq \overline{y}^s$	$y \geq \underline{V}^s$	$y \geq \overline{V}^s$	$y \geq \underline{V}^s$
Transition		if s rejects $y \leq \overline{y}^w \to S_4$	if s rejects $y \leq \underline{V}^w \to S_4$	if s rejects $y \leq \overline{V}^w \to S_4$	if w chooses $e = 1 \to S_2$
		if s proposes $y < \overline{y}^s \to S_3$			
		if w rejects $y \geq \overline{y}^s \to S_2$			
		if w demands $y > \overline{y}^w \to S_2$			

Where

$$V^s := [\underline{V}^s, \overline{V}^s], \quad \underline{V}^s := \delta a, \quad \overline{V}^s := \delta \frac{1-\delta b}{2-\delta} - \frac{2\delta}{2-\delta} c,$$
$$V^w := [\underline{V}^w, \overline{V}^w], \quad \underline{V}^w := a(2-\delta) + 2c, \quad \overline{V}^w := 1 - \delta b.$$

(In the "transition-row" of the above table the phrase "if $A \to S_i$" in the column S_j means that immediately after the event A has occurred the state changes from S_j to S_i.)

Proof: see Appendix.

To support the whole ranges V^s and V^w of stated in Table III as proposals accepted in equilibrium extreme punishment in case of deviation is needed. If one of the players deviates from the proposed equilibrium strategies the state will change such that in the future the worst possible equilibrium for the deviant is played. Suppose, for instance, that in state S_1 player s proposes less than prescribed. Then player w will change her behavior in such a way that the opponent gets less or at most equal to that payoff he would have been able to ensure to himself by following the proposed strategy. If player w deviates the punishment by player s works in an analogous way. As Table III indicates, in the case of intermediate opting in costs, the equilibrium path depends on both the initial state and the chosen proposals. In states $S_1 - S_3$ the weak player will opt in and the play ends in the first bargaining round ($t = 0$) with acceptance of one of the equilibrium offers shown in Table III. If the initial state is given by S_4 the weak player will not opt in and the play ends in the period zero without bargaining taking place. In this case both parties get their alternative payoffs, namely b and a. The expected equilibrium payoffs sustainable by (σ_3^*, ω_3^*) are easily calculated, using the intervals V^s and V^w. They are stated in the following Proposition.

Proposition 3 *If $c_1 \leq c_0$ and $c \in [c_1, c_0]$ then the expected equilibrium payoffs (u_3^*, v_3^*) sustainable by the strategy combination (σ_3^*, ω_3^*) from Lemma 3 are given by*

$$(U_3^* \cup \{b\}, V_3^*)$$

where the first entry are the payoffs for player s, the second entry are the payoffs for player w and

$$\begin{array}{rcl} U_3^* & := & [\frac{1-\delta(1-b)}{2-\delta} + \frac{\delta}{2-\delta}c, 1 - a - c], \\ V_3^* & := & [a, \frac{1-\delta b}{2-\delta} - \frac{2}{2-\delta}c]. \end{array}$$

For the case $c_0 < c_1$ similar results hold. The following Lemma follows straightforwardly from Lemmata 1 and 2.

Lemma 4 EXISTENCE OF SUBGAME PERFECT EQUILIBRIA FOR "INTERMEDIATE" OPTING IN COSTS (PART 2)

If $c_0 < c_1$ and $c \in [c_0, c_1]$ then there exists a subgame perfect equilibrium.

Proposition 4 shows that also in the case of $c_0 < c_1$ a whole range of payoffs is sustainable as a subgame-perfect equilibrium.

Proposition 4 *If $c_0 < c_1$ and $c \in [c_0, c_1]$ then the strategy combination (σ_4^*, ω_4^*) described in Table IV below are subgame perfect equilibria. The expected payoffs (u_4^*, v_4^*) sustainable by (σ_4^*, ω_4^*) are given by*

$$(U_4^* \cup \{b\}, V_4^*)$$

where again the first entry are the payoffs for player s, the second entry are the payoffs for player w and

$$
\begin{aligned}
U_4^* &:= [\tfrac{1}{2} + \tfrac{1}{2}\tfrac{\delta}{1-\delta}c, 1 - a - c] \\
V_4^* &:= [a, \tfrac{1}{2} - \tfrac{1}{2}\tfrac{2-\delta}{1-\delta}c]
\end{aligned}
$$

Table IV: Equilibrium Strategies for $c \in [c_0, c_1]$

		S_1	S_2	S_3
Player s	propose	$\overline{y}^s \in V^s$	$\overline{V}^s$	δa
	accept	$y \leq \overline{y}^w$	$y \leq \overline{V}^w$	$y \leq 1 - \delta b$
Player w	e	1	1	0
	demand	$\overline{y}^w \in V^w$	$\overline{V}^w$	$1 - \delta b$
	accept	$y \geq \overline{y}^s$	$y \geq \overline{V}^s$	$y \geq \delta a$
Transition		if s proposes $y < \overline{y}^s \rightarrow S_2$	*absorbing*	*absorbing*
		if s rejects $y \leq \overline{y}^w \rightarrow S_2$		
		if w demands $y > \overline{y}^w \rightarrow S_3$		
		if w rejects $y \geq \overline{y}^s \rightarrow S_3$		

Where

$$
\begin{aligned}
V^s &:= [\underline{V}^s, \overline{V}^s], & \underline{V}^s &:= \delta a, & \overline{V}^s &:= \tfrac{1}{2}\delta - \tfrac{1}{2}\delta\tfrac{2-\delta}{1-\delta}c, \\
V^w &:= [\underline{V}^w, \overline{V}^w], & \underline{V}^w &:= a(2-\delta) + 2c, & \overline{V}^w &:= \tfrac{1}{2}(2-\delta) - \tfrac{1}{2}\tfrac{\delta^2}{1-\delta}c
\end{aligned}
$$

Proof: We only have to check for one-shot deviations in state S_1, because for states S_2 and S_3 optimality follows from the fact that they are the stationary equilibrium strategies of Tables 1 and 2.

(A) Player s:

(α) To propose more than $\overline{y}^s$ is obviously worse than proposing $\overline{y}^s$.

(β) If player s proposes less than $\overline{y}^s$ the state changes to S_2. The expected deviation payoff is therefore given by $\frac{1}{2}\delta(1 - \overline{V}^s + 1 - \overline{V}^w)$. Following the equilibrium strategy gives at least $1 - \overline{V}^s$.

Claim : $\overline{V}^w \geq \overline{V}^s$.
Since $\overline{V}^w \geq \overline{V}^s \Leftrightarrow 2 - \delta - \frac{\delta^2}{1-\delta}c \geq \delta - \delta\frac{2-\delta}{1-\delta}c \Leftrightarrow 1 - \delta \geq -\delta c$ which is true for all $c \geq 0$.

Suppose now that the deviation is profitable, i.e. $1 - \overline{V}^s < \frac{1}{2}\delta(1 - \overline{V}^s + 1 - \overline{V}^w) \leq \delta(1 - \overline{V}^s)$ which is a contradiction. The second inequality follows from the claim.

(γ) If he doesn't accept the proposal $\overline{y}^w$ the state again changes to S_2 and the expected deviation payoff is given by the discounted value of the expected equilibrium payoff u_1^* of Proposition 1, namely $\delta\frac{1}{2}+\delta\frac{1}{2}\frac{\delta}{1-\delta}c = \delta u_1^*$. Following the proposed equilibrium strategy gives at least $1-\overline{V}^w = \delta\frac{1}{2}+\delta\frac{1}{2}\frac{\delta}{1-\delta}c = \delta u_1^*$. Hence the deviation is not profitable.

(B) Player w:

(α) To demand less than $\overline{y}^w$ is obviously not better than demanding exactly $\overline{y}^w$.

(β) If player w demands more than $\overline{y}^w$ the state changes to S_3. Therefore she would get δa. Following the proposed strategy gives at least $\underline{V}^w = a(2-\delta)+2c$. Since $a(2-\delta)+2c \geq \delta a \Leftrightarrow c \geq -a(1-\delta)$ always holds for $c \geq 0$. The deviation is not profitable.

(γ) If she rejects the proposal $\overline{y}^s$ made by player s the state again changes to S_3. But following the proposed strategy gives at least $\underline{V}^s = \delta a$, the same she would get by deviating. Rejecting $\overline{y}^s$ is therefore not profitable.

(δ) If player w follows the proposed strategy and chooses $e = 1$ she will get at least the expected payoff $\frac{1}{2}(\underline{V}^s + \underline{V}^w) - c = \frac{1}{2}\delta a + \frac{1}{2}a(2-\delta) + c - c = a$. Which is the same as she would get if she deviates by choosing $e = 0$.

Hence the strategy combinations described by Table IV are indeed a SPE's. The expected payoff pairs (u_4^*, v_4^*) follow by simple calculations from the accepted equilibrium offers stated in Table IV and the fact that if the initial state is given by S_3 player s gets the alternative payoff b.

□

As in the case where $c_1 \leq c_0$ the equilibrium payoffs are again supported by equilibrium strategies where deviation from the proposed action is heavily punished. If, for example, player s deviates the state changes such that in the future always the worst possible equilibrium for player s is played. The punishment after a deviation of player w is similar. The "actual" outcome again depends on the initial states. In states S_1 and S_2 the weak player opts in and the play ends in the first bargaining round with an agreement on one of the shares given in Table IV. If the initial state is given by S_3 then the weak player stays out and the play ends without bargaining. The payoffs are then given by b for the strong player and a for the weak player.

3.2 Main Results

In this section we summarize the results obtained that far in Theorem 1 and show that the sustainable payoffs derived in the previous section give the full range of possible expected payoffs sustainable by a subgame-perfect equilibrium. Thereafter some important properties of the model are derived and limit case of vanishing impatience is analyzed. We also compare this limit case with the "zero-cost" model. First we sum up the results obtained that far (see Propositions 1-4):

Theorem 1 *The following expected payoffs can be attained by some subgame perfect equilibrium:*

Table V: Expected equilibrium payoffs

opting in costs	Player s	Player w
$c \in [0, \min\{c_0, c_1\}[$	$u_1^* = \frac{1}{2} + \frac{1}{2}\frac{\delta}{1-\delta}c$	$v_1^* = \frac{1}{2} - \frac{1}{2}\frac{2-\delta}{1-\delta}c$
$c \in [c_1, c_0]$ and $c_1 \leq c_0$	$u_3^* \in [\frac{1-\delta(1-b)}{2-\delta} + \frac{\delta}{2-\delta}c, 1-a-c] \cup \{b\}$	$v_3^* \in [a, \frac{1-\delta b}{2-\delta} - \frac{2}{2-\delta}c]$
$c \in [c_0, c_1]$ and $c_0 < c_1$	$u_4^* \in [\frac{1}{2} + \frac{1}{2}\frac{\delta}{1-\delta}c, 1-a-c] \cup \{b\}$	$v_4^* \in [a, \frac{1}{2} - \frac{1}{2}\frac{2-\delta}{1-\delta}c]$
$c \in]\max\{c_0, c_1\}, +\infty[$	$u_2^* = b$	$v_2^* = a$

In Propositions 1 and 2 we have already shown that for sufficiently small and sufficiently large opting in costs the payoffs sustainable by a subgame perfect equilibrium are unique for a given c. For the intermediate cases however we have only shown that the payoffs stated above are sustainable by *some* equilibrium strategies. We want to strengthen the statements for these cases now and show that the expected payoffs given in Propositions 3 and 4 in fact give the full range of expected equilibrium payoffs. We put our concentration on player w's payoffs, first. Remember that we have denoted the interval of player w's payoffs sustainable by (σ_3^*, ω_3^*) (resp. (σ_4^*, ω_4^*)) with V_3^* (resp. V_4^*).

Theorem 2 .

(i) If $c_1 \leq c_0$ and $c \in [c_1, c_0]$ then the set of expected payoffs to player w in subgame perfect equilibria is the interval V_3^*.

(ii) If $c_0 < c_1$ and $c \in [c_0, c_1]$ then the set of expected payoffs to player w in subgame perfect equilibria is the interval V_4^*.

Proof: See appendix.

Since, whenever player w decides to opt in, for every $u_3^* \in U_3^*$ the payoff for player s is uniquely determined by $v_3^* = 1 - (u_3^* - c)$ the payoffs for player s sustainable by a subgame perfect equilibrium in which player w opts in follow straightforwardly from Theorem 1 and Theorem 2. Thus, we get

Corollary 1 .
(i) If $c_1 \leq c_0$ and $c \in [c_1, c_0]$ then the set of expected payoffs to player s in subgame perfect equilibria where player w opts in is the interval U_3^*.

(ii) If $c_0 < c_1$ and $c \in [c_0, c_1]$ then the set of expected payoffs to player s in subgame perfect equilibria where player s opts in is the interval U_4^*.

insert Figure 3(a) and Figure 3(b) here

Figure 3(a) (resp. Figure 3(b)) describes the payoff for both players as a function of the opting costs c of the weak player. Figure 3(a) captures the case $c_1 \leq c_0$ and Figure 3(b) the case $c_0 < c_1$. Some facts are worth to be noted here. Firstly, it can be shown after some tedious calculations that, for $c \in [\min\{c_0, c_1\}, \max\{c_0, c_1\}]$, the payoff which player s can get if player w opts in is always greater than or equal to b as long as $c_1 \leq c_0$. If, however $c_0 < c_1$ holds the alternative payoff for player s will always be larger than any payoff he can get in an equilibrium where bargaining takes place. Secondly, player w never gets less than her alternative payoff a. Thirdly, multiplicity of equilibrium payoffs vanish if and only if $c_1 = c_0$. Fourthly, in both cases - for certain values of opting in costs - the possibility of inefficient outcomes which are not due to delay arises for certain values of opting in costs and fifthly, in the case of $c_1 \leq c_0$ the increase of the opting in cost of the weaker player may harm the strong in the sense that the equilibrium payoff player s can get is strictly smaller the higher the costs for player w are. We summarize these observations in the following Theorem.

Theorem 3 SOME IMPORTANT PROPERTIES OF THE MODEL

(i) **Multiplicity of equilibrium payoffs***:*

If $c \in [\min\{c_0, c_1\}, \max\{c_0, c_1\}]$ then, whenever $c_1 \neq c_0$ holds multiplicity of equilibrium payoffs arise.

(ii) **First period inefficiency***:*

If $c_1 \leq c_0$ and $c \in [c_1, c_0]$ then there always exists an equilibrium where player w decides not to opt in leading to the inefficient payoff pair (b, a).

If $c_1 < c_0$ then there exists a nonempty interval $[c_0, \hat{c}]$ such that if $c \in]c_0, \hat{c}[$ the unique equilibrium, where player w decides not to opt in, is inefficient.

If $c_0 < c_1$ and $c \in [c_0, c_1]$ then there always exists an equilibrium where player w decides to opt in leading to a joint payoff lower than $a + b$.

If $c_0 < c_1$ then there exists a nonempty interval $[\check{c}, c_0]$ such that if $c \in]\check{c}, c_0[$ the unique equilibrium, where player w decides to opt in is inefficient.

(iii) **Weakening the weak may harm the strong**:

If $c_1 < c_0$ then an increase of the opting in costs of player w may lower (resp. lowers for sure) the payoff of player s even if the efficient outcome occurs. I.e. the equilibrium payoffs of player s exhibit some kind of (strong) non-monotonicity in the sense that

$$\exists \underline{c} \text{ with } \underline{c} < c_1 \text{ and } \exists \overline{c} \text{ with } \overline{c} > c_1 \text{ s.th. } u_1^*(\underline{c}) > u_3^*(c)\ \forall c \in]\overline{c}, c_0[.$$

Proof: (i): This follows from Propositions 3 and 4. (ii): First and second statement: Observe that the joint payoff (net of costs) in the equilibria where player w opts in is given by $1-c$. This is larger than $a+b$ iff $c < 1-(a+b)$. Furthermore $c_0 < 1-(a+b) \Leftrightarrow c_1 < c_0$. Define $\hat{c} := 1-(a+b)$. From Lemma 3 and Proposition 3 we know that there always exists an equilibrium where the weak player decides not to opt in with the corresponding equilibrium payoff pair (b, a). For $c > c_0$ this equilibrium is unique (see Proposition 2). Hence for every $c \in [c_1, \hat{c}[$ the equilibrium where player w decides not to opt in is inefficient. The third and the fourth statement follow from (a) Lemma 1 and Lemma 2 (resp. Proposition 4), (b) the observation that the joint payoff (net of costs) in the equilibria where player w opts in is again $1-c$ which is smaller than $a+b$ iff $c > 1-(a+b)$ and (c) the fact that $c_0 > 1-(a+b) \Leftrightarrow c_1 > c_0$. If we define $\check{c} := 1-(a+b)$, now then we are done. In these cases the equilibria where the weak player decides to opt in are the inefficient once. (iii): Denote for any $c \in [c_1, c_0]$ the maximum of $U_3^*(c)$ (which is the set of the strong players expected equilibrium payoffs for the case $c \in [c_1, c_0]$) by $\overline{U}_3^*$. Observe that it is strictly decreasing in c and that $\overline{U}_3^*(c_1) = u_1^*(c_1)$. Since $u_1^*(c)$ is strictly increasing in c and both functions are continues in c the above statement holds.

□

insert Figure 4(a) and 4(b) here

In Figures 4(a) and 4(b) we depict the "first period inefficiency" statements of Theorem 3. In both figures the horizontal line shows the value of the sum of the alternative payoffs a and b. The negatively sloped line is given by $1-c$, the surplus (net of opting in costs) if the parties cooperate, i.e. if the weak player decides to opt in. We know that for all values of opting in costs between c_1 and c_0 a equilibrium exists where the weak player decides not to opt in. But as can be seen from Figure 4(a), if $c_1 \leq c_0$ the line $1-c$ is above the line $a+b$ for all $c \in [c_1, \hat{c}[$. For $c > c_0$ the only equilibrium has the feature that the weak player does not opt in. Hence, these equilibria are inefficient, in the sense that the potential gains from bargaining are wasted. If $c_0 < c_1$ (see Figure 4(b)) then for $c \in]\check{c}, c_1]$ the equilibria where player w opts in are inefficient since from an efficiency point of view it would be better if the weak player does not opt in. In Figure 4(b) this can be seen from the fact that the line $1 - c$ is always below the line $a + b$, for the appropriate values of c.

insert Figure 5 here

From Figure 5 one can see that for an appropriate $\underline{c}$ we can find a $\bar{c}$ such that the equilibrium payoffs for costs larger than $\bar{c}$ are all smaller then the unique equilibrium payoff at cost level $\underline{c}$. This is exactly what statement (iii) of Theorem 3 says. The reason why an increase of opting in costs does not always work in favor of the stronger party is that player s has to compensate player w if he wants to induce "opting-in behavior". If these costs exceed a certain value, namely c_1, the weakness of player w gets also features of a "threat", since as long as $c_1 < c_0$ holds the expected payoffs to player s in any equilibrium where player w opts in are larger than his outside alternative b. Therefore it is in his interest to give the weaker player an incentive to opt in. But since the surplus is of limited size he has to give up more of his share the higher the costs for player w are, lowering his own payoff, too. As can be seen from figure 3(a) there is also a discontinuity at c_0. At this level of opting in costs player s suddenly decides to stop the compensation payments although the net-size $1 - c$ of the surplus exceeds the sum $a + b$ of the alternative payoffs. So the question arises why player s will not continue to compensate player w up to the point where $1 - c = a + b$? The reason for the inefficiency which arises here is that the stronger player cannot commit himself to pay a compensation after the weaker player has opted in. The commitment is not possible because if he deviates player w has to punish player s. But any punishment contains the rejection of an offer and at costs higher than c_0 the punishment is too expensive and the threat of punishing the other player is not credible any more and therefore player s has always an incentive to deviate. The weak player anticipates this and therefore decides not to opt in. For the case $c_0 < c_1$ the inefficiency for $c \in]\check{c}, c_0[$ arises because player w has no incentive not to bargain with player s, since the stronger player has no possibility to threaten the weaker one. His only possibility would be to reject the offers made by player w. But this is not credible if he knows that player w will always opt in. And, on the other hand, player w will always opt since the costs are small enough.

We are turning now to the question: How will things change as impatience vanishes? First of all observe that for $\delta \to 1$ the cost level c_1 approaches 0 and $\lim_{\delta \to 1} c_0 = \frac{1}{2}(1 - (a + b))$. Hence the equilibrium stated in Lemma 1 vanishes, except for $c = 0$, and c_0 equals half of the possible gain from bargaining. Using this observation, Theorem 1, Theorem 2 and Corollary 1 we get

Corollary 2 EXPECTED EQUILIBRIUM PAYOFFS FOR VANISHING IMPATIENCE

Let $\delta \to 1$ *and denote* $\lim_{\delta\to 1} c_0 = \frac{1}{2}(1-(a+b))$ *by* $c_{0,\delta\to 1}$ *and* $\lim u_i^*$ $(\lim v_i^*)$ *by* $u_{i,\delta\to 1}^*$ $(v_{i,\delta\to 1}^*$ *resp.)* $(i = 2, 3)$, *then:*

(i) The following expected payoffs can be attained by some subgame perfect equilibrium:

Table VI: Expected equilibrium payoffs for vanishing impatience

opting in costs	player s	player w
$c \in]0, c_{0,\delta\to 1}]$	$u_{3,\delta\to 1}^* \in [b+c, 1-a-c] \cup \{b\}$	$v_{3,\delta\to 1}^* \in [a, 1-b-2c]$
$c \in]c_{0,\delta\to 1}, +\infty[$	$u_{2,\delta\to 1}^* = b$	$v_{2,\delta\to 1}^* = a$

(ii) If $c \in]0, c_{0,\delta\to 1}]$ *then the set of expected payoffs to player* w *in subgame perfect equilibria is the interval* $[a, 1-b-2c]$.

(iii) If $c \in]0, c_{0,\delta\to 1}]$ *then the set of expected payoffs to player* s *in subgame perfect equilibria where player* w *opts in is the interval* $[b+c, 1-a-c]$.

Notice that for $c = 0$ Proposition 1 applies and the equilibrium payoff pair is given by $(\frac{1}{2}, \frac{1}{2})$. For $c > 0$ the equilibrium payoffs are only determined by the alternative payoffs b, a and the opting in costs. In the model with vanishing impatience similar properties as in the general model hold. In particular, multiplicity of equilibrium payoffs do not vanish for any costs in the interval $]0, c_{0,\delta\to 1}]$. Hence even if the opting in costs are almost zero multiplicity arises. The possibility of first period inefficiency arises and the increase of opting costs may decrease the payoff of the stronger player, again. Let us summarize these observations.

Theorem 4 IMPORTANT PROPERTIES OF THE MODEL WITH VANISHING IMPATIENCE

(i) **Multiplicity of equilibrium payoffs***:*

Multiplicity of equilibrium payoffs arise for all $c \in]0, c_{0,\delta\to 1}]$.

(ii) **First period inefficiency**

If $c \in]0, c_{0,\delta\to 1}]$ *then there exists an equilibrium where player* w *decides not to opt in leading to the inefficient payoff pair* (b, a).

If $c \in [c_{0,\delta\to 1}, 1-(a+b)[$ *then the unique equilibrium leads to an inefficient outcome* (b, a).

(iii) **Weakening the weak may harm the strong***:*

If $c \in]0, c_{0,\delta\to 1}]$ an increase of the opting in cost of player w may lower the payoff of player s.

Proof: Similar to the proof of Theorem 3.

If we look at the model with zero opting in costs it is clear, from Proposition 1, that the expected equilibrium payoffs will be $(\frac{1}{2}, \frac{1}{2})$. This equilibrium payoff pair is supported by strategies that require player w always to opt in, and it is unique since a is strictly smaller than $\frac{1}{2}$. Of course this equilibrium will remain if impatience of the players vanish. Let us compare this with our findings stated in Corollary 2 for c approaching 0, now. Obviously there is a huge difference in the payoffs supported by subgame perfect equilibrium strategies. First of all the multiplicity does not vanish for vanishing opting in costs and even more surprising, for reasonable alternative payoffs for player s, namely for $b > \frac{1}{2}$, the equilibrium payoff pair of the "zero-cost" model is not even an element of the equilibrium payoff pairs of our model. We want to state this more formally now.

Theorem 5 THE "ZERO-COST" MODEL WITH VANISHING IMPATIENCE IS NOT THE LIMIT CASE OF THE PRESENTED MODEL WITH VANISHING IMPATIENCE

(i) In the presented model with vanishing impatience multiplicity does not vanish when the opting in costs approach zero. In particular, for $c \to 0$ the set of expected payoffs for player s (w resp.) is given by the interval $[b, 1-a]$ ($[a, 1-b]$ resp.).

(ii) For $b > \frac{1}{2}$ the unique equilibrium payoff pair of the "zero-cost" model is not contained in the set of equilibrium payoffs of the presented model with vanishing impatience and opting in costs approaching 0, i.e. $(\frac{1}{2}, \frac{1}{2}) \notin [b, 1-a] \times [a, 1-b] \forall b \in]\frac{1}{2}, 1]$.

Proof: The first statement follows from Corollary 2 and the second is obvious.

The statements of this Theorem are due to the fact that it is not equal if we take the limit of the opting in costs first and then the limit of the discount factors or if we do it the other way round. Since the opting in costs are discounted, the level of the discount factor plays only a role as long as the costs are non zero. With this in mind it is clear that the "zero-cost" model and our model with vanishing costs don't coincide.

4 Conclusions

We have examined a noncooperative bargaining model where one of the players has to bear so called *opting in* costs if she wants to participate in the bargaining process. Several results have been derived. Similar as in Shaked's (1987) costless opting out model and the "switching-cost" model of Hendon *et al.* (1994)

we get multiplicity of equilibria, and therefore possible inefficiency due to delay. However, unlike the results of other bargaining models, in our model the possibility of inefficiency which is not due to delay arises. We have shown that for a large range of opting in costs such equilibria exist, and that for some nontrivial interval of opting in costs the only equilibrium is one where the play ends in the first period with an inefficient outcome. We also got a result which we call "weakening the weak may harm the strong". What is meant by this phrase is that an increase of the opting in costs of the weaker player does not always work in favor of the stronger party which does not have opting in costs. In particular, for some cost levels *all* equilibrium payoffs for the stronger player are strictly decreasing with the opting in costs of the weaker player. In a union versus firm bargaining situation this result allows us to understand why firms are not always interested in raising the opting in costs of the union, if they are not strong enough to hinder the existence of a union or if it is illegal to prevent workers from organizing. This reflects the often observed fact of some sort of social partnership between a firm and a union.

We have also investigated the relationship between the "zero-cost" model and our model for costs approaching 0. The main conclusion here is that, for high discount factors, the "zero-cost" model is not the limit case of our model. For vanishing impatience the set of equilibrium payoffs of the "zero-cost" model and the "vanishing-cost" model do not coincide. Furthermore, for reasonable alternative payoffs for the stronger player (i.e. if the alternative payoff of the stronger player is larger than one half of the "pie"), the unique equilibrium payoff of the "zero-cost" model is not even contained in the set of equilibrium payoffs of our model with vanishing opting in costs.

Appendix

A Proof of Lemma 3:

Remember that the proposed equilibria (σ_3^*, ω_3^*) are described by Table III. For ease of exposition the table is given below, again.

Table III: Equilibrium strategies for $c \in [c_1, c_0]$

		S_1	S_2	S_3	S_4
Player s	propose	$\overline{y}^s \in V^s$	$\underline{V}^s$	$\overline{V}^s$	$\underline{V}^s$
	accept	$y \leq \overline{y}^w$	$y \leq \underline{V}^w$	$y \leq \overline{V}^w$	$y \leq \overline{V}^w$
Player w	e	1	1	1	0
	demand	$\overline{y}^w \in V^w$	$\underline{V}^w$	$\overline{V}^w$	$\overline{V}^w$
	accept	$y \geq \overline{y}^s$	$y \geq \underline{V}^s$	$y \geq \overline{V}^s$	$y \geq \underline{V}^s$
Transition		if s rejects $y \leq \overline{y}^w \to S_4$	if s rejects $y \leq \underline{V}^w \to S_4$	if s rejects $y \leq \overline{V}^w \to S_4$	if w chooses $e = 1 \to S_2$
		if s proposes $y < \overline{y}^s \to S_3$			
		if w rejects $y \geq \overline{y}^s \to S_2$			
		if w demands $y > \overline{y}^w \to S_2$			

Where

$$\begin{array}{lllllllll} V^s & := & [\underline{V}^s, \overline{V}^s], & \underline{V}^s & := & \delta a, & \overline{V}^s & := & \delta\frac{1-\delta b}{2-\delta} - \frac{2\delta}{2-\delta}c, \\ V^w & := & [\underline{V}^w, \overline{V}^w], & \underline{V}^w & := & a(2-\delta) + 2c, & \overline{V}^w & := & 1 - \delta b. \end{array}$$

To see that these pairs of strategies form SPE's we have to check for one-shot-deviations in each state. First we state a claim.

Claim A1: Define $\underline{V} := \frac{1}{2}(\underline{V}^s + \underline{V}^w)$ *and* $\overline{V} := \frac{1}{2}(\overline{V}^s + \overline{V}^w)$.

$$\begin{array}{llllllllll} (i) & \underline{V} & = & a + c & (iii) & \underline{V}^s & \leq & \underline{V} & \leq & \underline{V}^w \quad \forall c \in [c_1, c_0] \\ (ii) & \overline{V} & = & \frac{1-\delta b}{2-\delta} - \frac{\delta}{2-\delta}c & (iv) & \overline{V}^s & \leq & \overline{V} & \leq & \overline{V}^w \quad \forall c \in [c_1, c_0] \end{array}$$

$$(v) \quad \underline{V}^s \leq \overline{V}^s \quad \forall c \in [c_1, c_0]$$

The statements of Claim A1 are easily shown by simple but partly lengthy calculations.

(Aa) State S_1, Player s:

(α) To propose more than $\overline{y}^s$ is obviously worse than proposing $\overline{y}^s$.

(β) If he proposes less than $\overline{y}^s$ the state changes to S_3. Player w will reject and chooses $e = 1$. This gives an expected payoff of $\delta(1 - \overline{V})$. On the other hand if player s uses the proposed strategy he will get at least $1 - \overline{V}^s$. By Claim A1 (iv) $1 - \overline{V}^s \geq 1 - \overline{V} > \delta(1 - \overline{V})$. Hence, such a deviation is not profitable.

(γ) If he rejects the proposal $\overline{y}^w$ the state changes to S_4, where player w chooses $e = 0$. Therefore player s will get $\delta b = 1 - \overline{V}^w$ which is the payoff he will get at least if he doesn't deviate.

(Ab) State S_1, Player w:

(α) To demand less than $\overline{y}^w$ is obviously not better than demanding exactly $\overline{y}^w$.

(β) If player w demands more than $\overline{y}^w$ the state changes to S_2 and the expected payoff of this deviation will be $\delta\underline{V} - \delta c$. Following ω_3^* gives at least $\underline{V}^w \geq \underline{V} > \delta\underline{V} - \delta c$. (The first inequality follows by Claim A1 (iii)).

(γ) If she rejects the proposal $\overline{y}^s$ made by player s then the state will again change to S_2 with an expected payoff of $\delta\underline{V} - \delta c$. Following ω_3^* gives at least $\underline{V}^s$. But by definition of $\underline{V}^s$ and Claim A1 (i) we get $\underline{V}^s = \delta a \geq \delta(a+c) - \delta c = \delta\underline{V} - \delta c$.

(δ) If player w chooses $e = 1$ she will get at least $\underline{V} - c = a + c - c = a$, which is of course not smaller than a, the payoff which player w will get if she deviates in choosing $e = 0$.

(Ba) State S_2, Player s:

(α) To propose more than $\underline{V}^s$ is obviously worse than proposing $\underline{V}^s$.

(β) To propose less than $\underline{V}^s$ is also not profitable (see (Aa) (β)).

(γ) If he rejects the proposal $\underline{V}^w$ the state changes to S_4, where player w chooses $e = 0$. Therefore player s will get δb. Following σ_3^* gives $1 - \underline{V}^w$. But $1 - \underline{V}^w = 1 - a(2 - \delta) - 2c < \delta b \Leftrightarrow c > \frac{1}{2}(1 - \delta b - a(2 - \delta)) = c_0$, which contradicts our assumption $c \in [c_1, c_0]$. Therefore the deviation is not profitable.

(Bb) State S_2, Player w:

The optimality of player w's strategy is straightforward. (See (Ab) (α)-(β) and replace "at least" by "exactly").

(Ca) State S_3, Player s:

The optimality of player s's strategy follows from the arguments given in (A) (α)-(γ). (replace "at least" by "exactly", again).

(Cb) State S_3, Player w:

(α) To demand less than $\overline{V}^w$ is obviously not better than following ω_3^*.

(β) If player w demands more than $\overline{V}^w$ the state remains and the deviation-payoff is given by $\delta\overline{V} - \delta c$. Following ω_3^* gives $\overline{V}^w$. But Claim A1 (iv) tells us that $\overline{V}^w \geq \overline{V}$ which is in turn larger then $\delta\overline{V} - \delta c$.

(γ) If she rejects the proposal $\overline{V}^s$ player w will get again $\delta\overline{V} - \delta c$. Following ω_3^* gives $\overline{V}^s$. Hence we have to show that $\overline{V}^s \geq \delta\overline{V} - \delta c$, which turns out to be equivalent to $0 \geq 0$. Therefore this kind of deviation is not profitable for player w.

(δ) If player w chooses $e = 1$ she will get $\overline{V} - c$. If she deviates the payoff is given by a. Suppose that the deviation is profitable, i.e. $\overline{V} - c = \frac{1-\delta b}{2-\delta} - \frac{\delta}{2-\delta}c - c < a$ (The equality follows from Claim A1 (ii)). After some manipulations we get that this inequality is equivalent to $c > \frac{1}{2}(1 - \delta b - a(2-\delta)) = c_0$, which contradicts $c \in [c_1, c_0]$.

(Da) State S_4, Player s:

(α) To propose more than $\underline{V}^s$ is obviously not a profitable deviation.

(β) If he proposes less than $\underline{V}^s = \delta a$ the state remains and the deviation payoff for player s is given by δb. $\delta b \leq 1 - \delta a \Leftrightarrow \delta a + \delta b \leq 1$, which is true because of $a + b < 1$.

(γ) Rejecting $\overline{V}^w$ gives a payoff of δb which is exactly the same payoff player s could get if he follows σ_3^*. Therefore, the deviation is not profitable.

(Db) State S_4, Player w:

The optimality of player w's proposal and response is straightforward, because (by definition of $\underline{V}^s$ and Claim A1 (iv) - (v)) $\overline{V}^w \geq \overline{V}^s \geq \underline{V}_1 = \delta a$.

(δ) If player w deviates and chooses $e = 1$ the state will change to S_2. Therefore the deviation payoff is given by $\underline{V} - c = a + c - c = a$, which is the same as she would get following ω_3^*. Hence this kind of deviation is also not profitable.

□

B Proof of Theorem 2:

Part (i): We have to show that the set of expected payoffs to player w in subgame perfect equilibria is indeed given by the interval $V_3^* = [a, \frac{1-\delta b}{2-\delta} - \frac{2}{2-\delta}c]$. Since player w can always ensure a for herself by simply not opting in we only have to show that she cannot get more than $\frac{1-\delta b}{2-\delta} - \frac{2}{2-\delta}c$. For any expected payoff larger than a player w has to opt in. Hence if we define M_w, M_w^s, M_w^w (resp.) as in the proofs of Propositions 1 and 2 we have to show that $M_w \leq \frac{1-\delta b}{2-\delta} - \frac{2}{2-\delta}c + c = \frac{1-\delta b}{2-\delta} - \frac{\delta}{2-\delta}c$. From the proof of Proposition 2 we already know that

$$\begin{aligned} M_w^s &\leq \delta \max\{a, M_w - c\} \\ M_w^w &\leq 1 - \delta \min\{b, 1 - M_w\}. \end{aligned}$$

Claim A3: $a \leq M_w - c$.

Since on the one hand $m_w \leq a + c$ by Proposition 3 and on the other hand $m_w \geq a + c$ because otherwise player w would not opt in we get $a + c = m_w \leq M_w \Leftrightarrow a \leq M_w - c$

Claim A4: $b \leq 1 - M_w$.

Assuming that this is not true and using the definition of M_w as well as Claim A3 we get $M_w \leq \frac{1}{2}(\delta M_w - \delta c + 1 - \delta(1 - M_w)) \Leftrightarrow 1 - M_w \geq \frac{1}{2} + \frac{1}{2}\frac{\delta}{1-\delta}c$. Again using the assumption that $b > 1 - M_w$ we get $b > \frac{1}{2} + \frac{1}{2}\frac{\delta}{1-\delta}c \Leftrightarrow c < \frac{1-\delta}{\delta}(2b-1)$. This would contradict $c \in [c_1, c_0]$ if $\frac{1-\delta}{\delta}(2b-1) \leq c_1 = (1-2a)\frac{1-\delta}{2-\delta}$. After some manipulations we get indeed that this inequality is equivalent to $a \leq \frac{1-b(2-\delta)}{\delta}$ which is in turn equivalent to $c_1 \leq c_0$ and therefore true under our assumptions. Hence $b > 1 - M_w$ leads to a contradiction and we have indeed $b \leq 1 - M_w$.

With the help of claims A3 and A4 we get

$$\begin{aligned} M_w^s &\leq \delta M_w - \delta c \\ M_w^w &\leq 1 - \delta b \end{aligned}$$

These two inequalities together with the definition of M_w imply

$$M_w \leq \frac{1-\delta b}{2-\delta} - \frac{\delta}{2-\delta}c.$$

Part (ii): We have to show that the set of expected payoffs to player w in subgame perfect equilibria is indeed given by the interval V_4^*. From Proposition 4 we already know that

$$\begin{aligned} m_w &\leq a + c \\ M_w &\geq \tfrac{1}{2} - \tfrac{1}{2}\tfrac{\delta}{1-\delta}c \end{aligned}$$

As above player w cannot get less than a since she can always ensure this payoff to herself by simply not opting in. This again implies that whenever she decides to opt in she gets at least $a+c$. It remains to show that $M_w \leq \frac{1}{2} - \frac{1}{2}\frac{2-\delta}{1-\delta}c + c = \frac{1}{2} - \frac{1}{2}\frac{\delta}{1-\delta}c$ holds. As in the proof of part (i) the following inequalities

$$\begin{aligned} M_w^s &\leq \delta \max\{a, M_w - c\} \\ M_w^w &\leq 1 - \delta \min\{b, 1 - M_w\} \end{aligned}$$

and $a \leq M_w - c$ hold. We therefore get $M_w^s \leq \delta M_w - \delta c$, again. Suppose now that $b < 1 - M_w$. This implies that $M_w \leq \frac{1}{2}(\delta M_w - \delta c + 1 - \delta b) \Leftrightarrow M_w \leq \frac{1-\delta b}{2-\delta} - \frac{\delta}{2-\delta}c$. Together with $M_w \geq \frac{1}{2} - \frac{1}{2}\frac{\delta}{1-\delta}c$ this implies that $\frac{1}{2} - \frac{1}{2}\frac{\delta}{1-\delta}c \leq \frac{1-\delta b}{2-\delta} - \frac{\delta}{2-\delta}c$. After some manipulations this turns out to be equivalent to $(1-\delta)(2b-1) \leq \delta c$. Together with $\delta c \leq \delta c_1 = \delta(1-2a)\frac{1-\delta}{2-\delta}$ we get $(1-\delta)(2b-1) \leq \delta(1-2a)\frac{1-\delta}{2-\delta} \Leftrightarrow a \leq \frac{1}{\delta} - \frac{b(2-\delta)}{\delta} \Leftrightarrow c_1 \leq c_0$ which is a contradiction. Hence $b \geq 1 - M_w$ and

$$\begin{aligned} M_w^s &\leq \delta M_w - \delta c \\ M_w^s &\leq 1 - \delta + \delta M_w \end{aligned}$$

These inequalities together with the definition of M_w imply $M_w \leq \frac{1}{2}(2\delta M_w + 1 - \delta - \delta c \Leftrightarrow M_w \leq \frac{1}{2} - \frac{1}{2}\frac{\delta}{1-\delta}c$. $\square$

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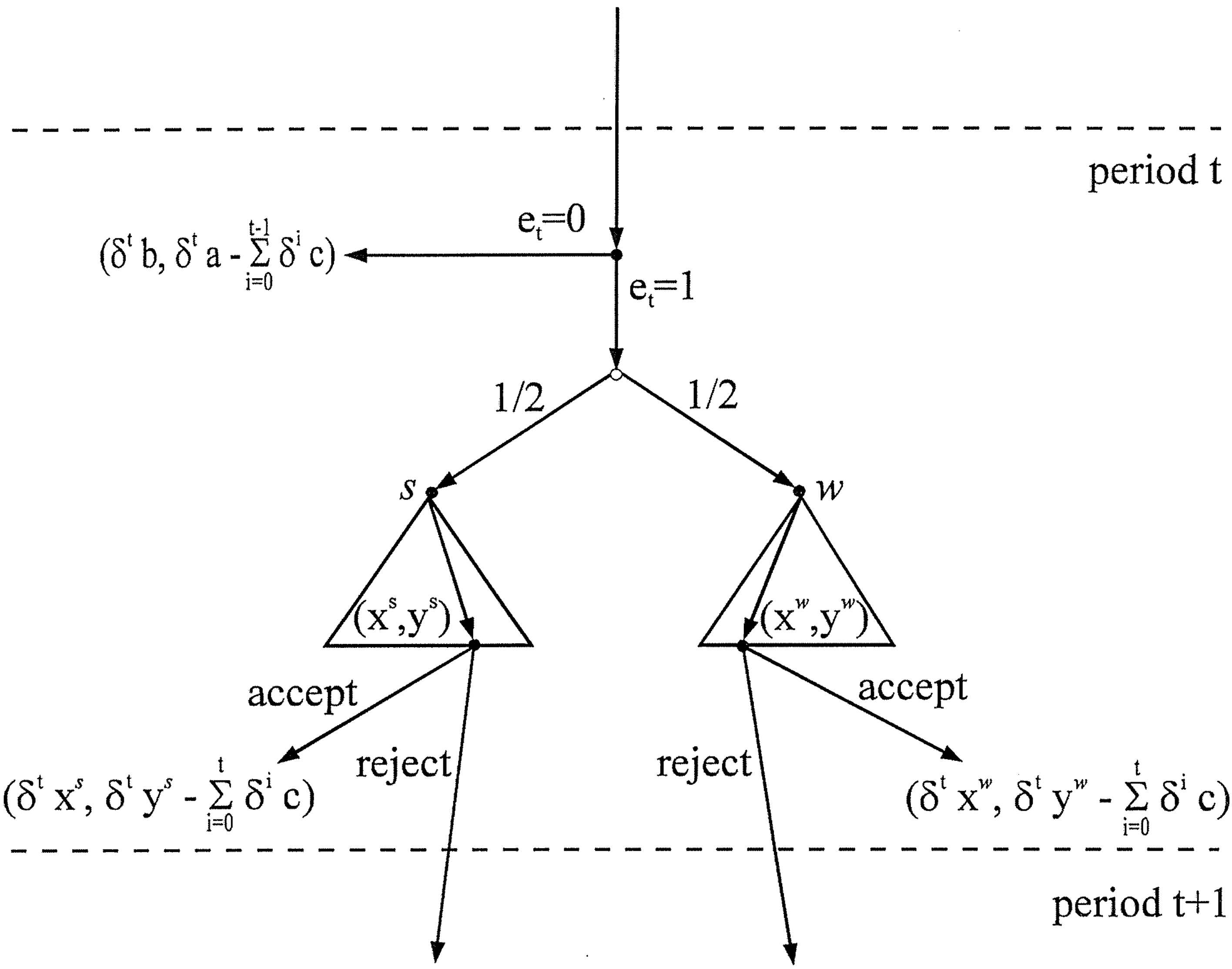

Figure 1: The structure of period t

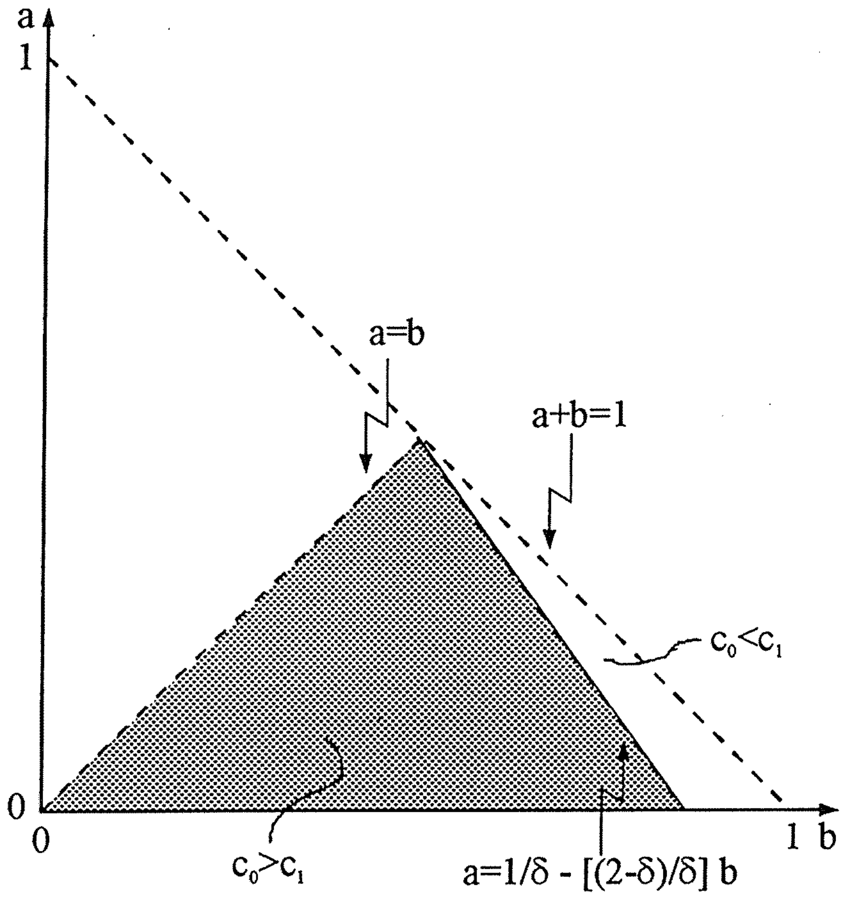

Figure 2 (a): δ=0.9

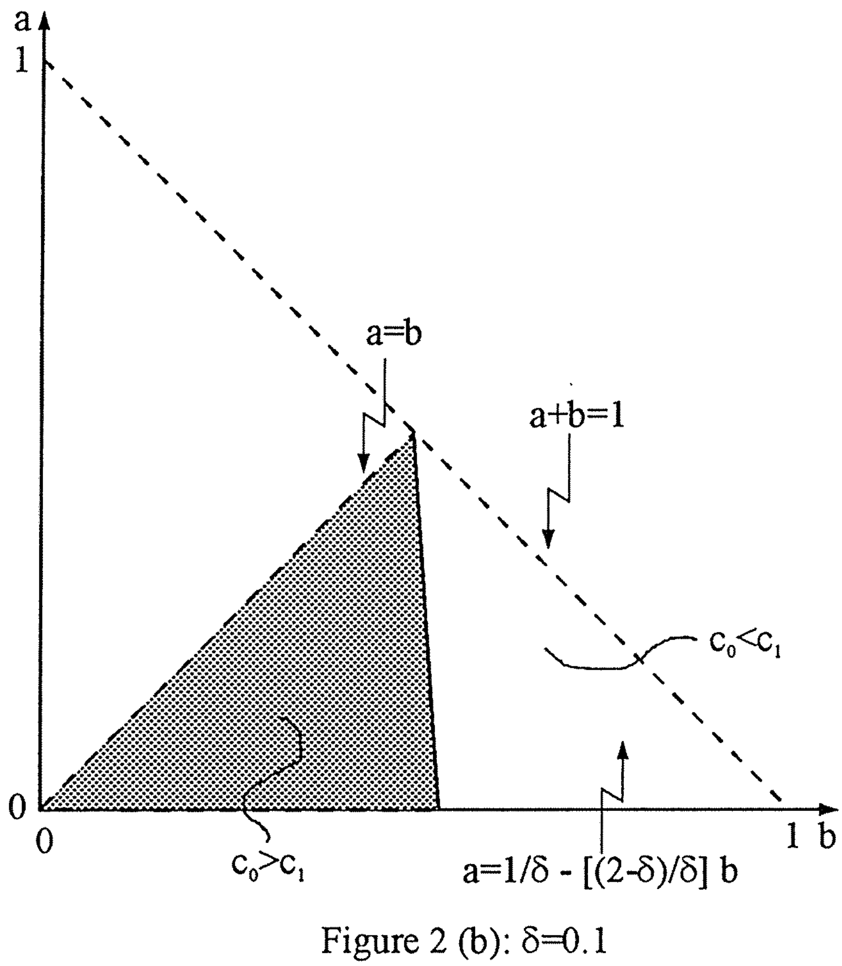

Figure 2 (b): δ=0.1

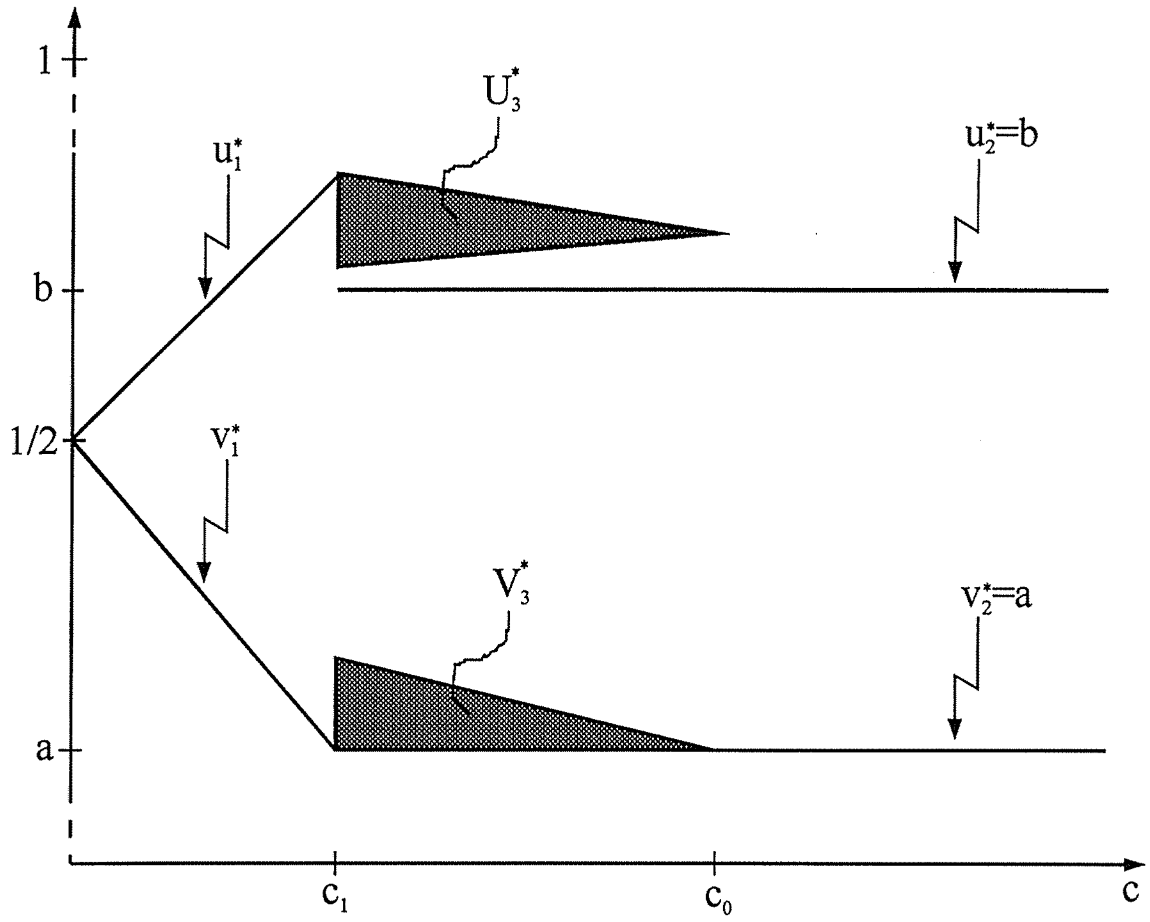

Figure 3 (a): Equilibrium payoffs for $c_1 < c_0$

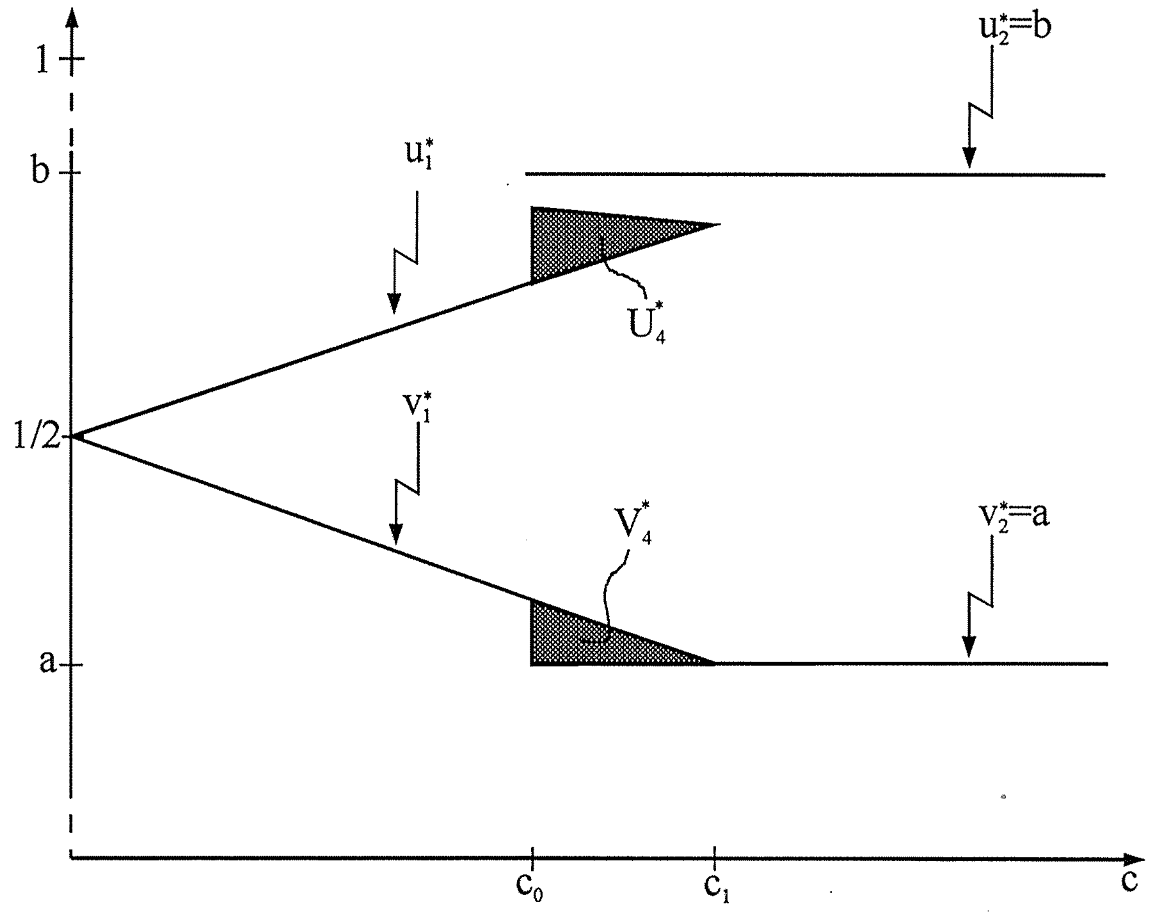

Figure 3 (b): Equilibrium payoffs for $c_1 > c_0$

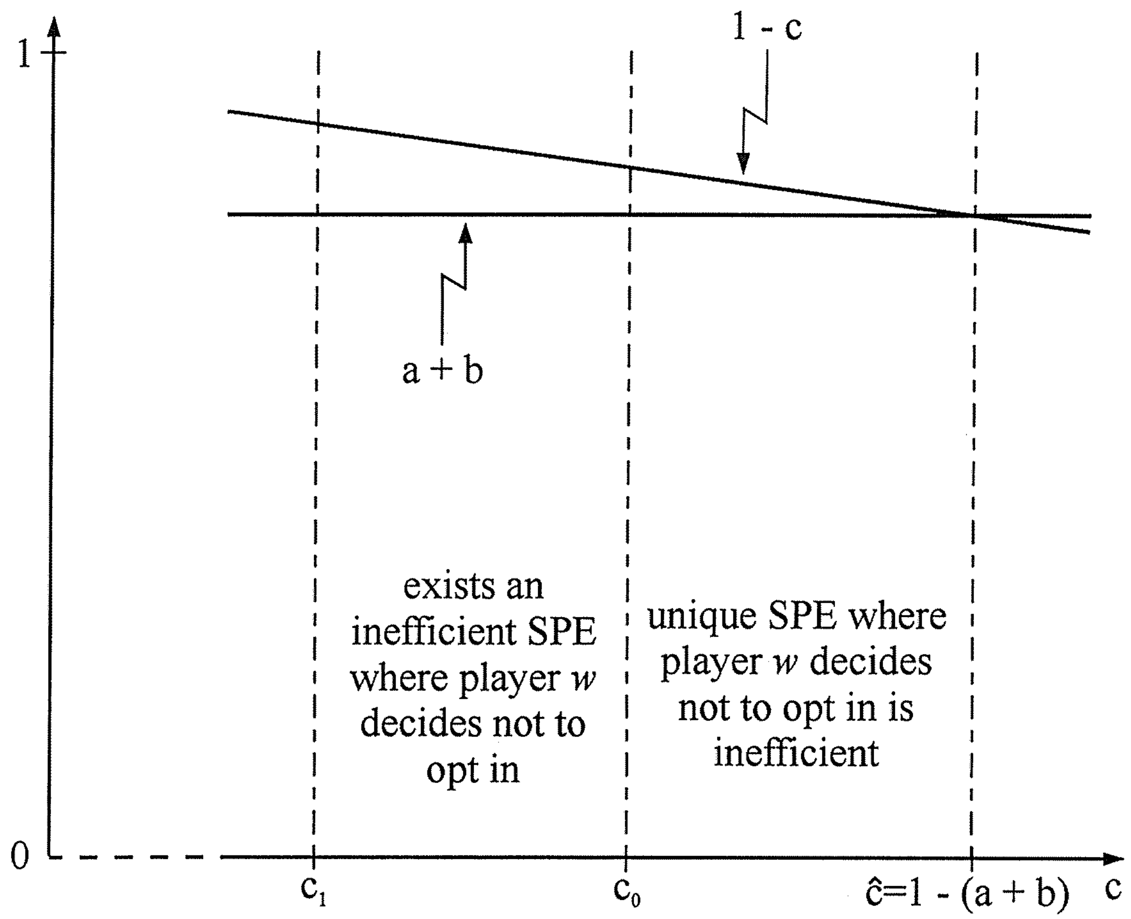

Figure 4 (a): Cost levels with inefficient outcomes for for $c_1 < c_0$

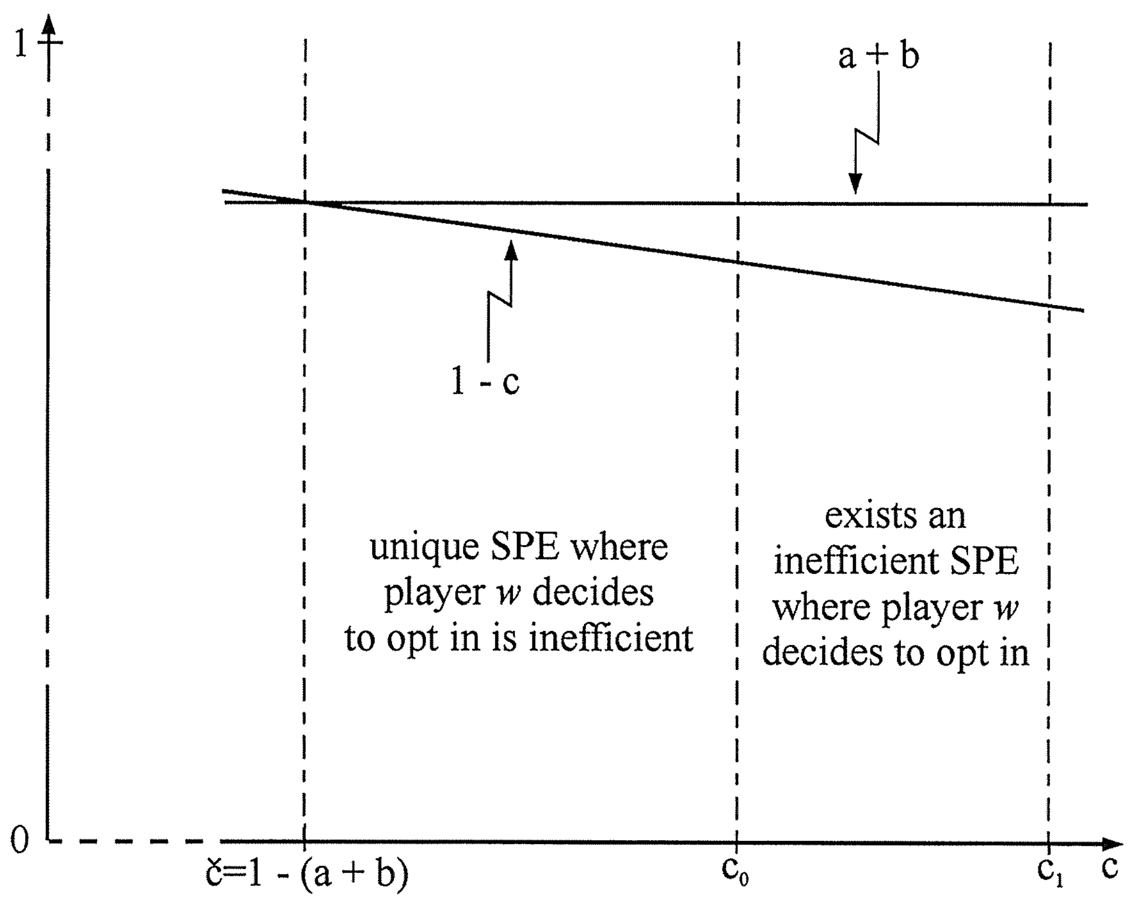

Figure 4 (b): Cost levels with inefficient outcomes for for $c_1 > c_0$

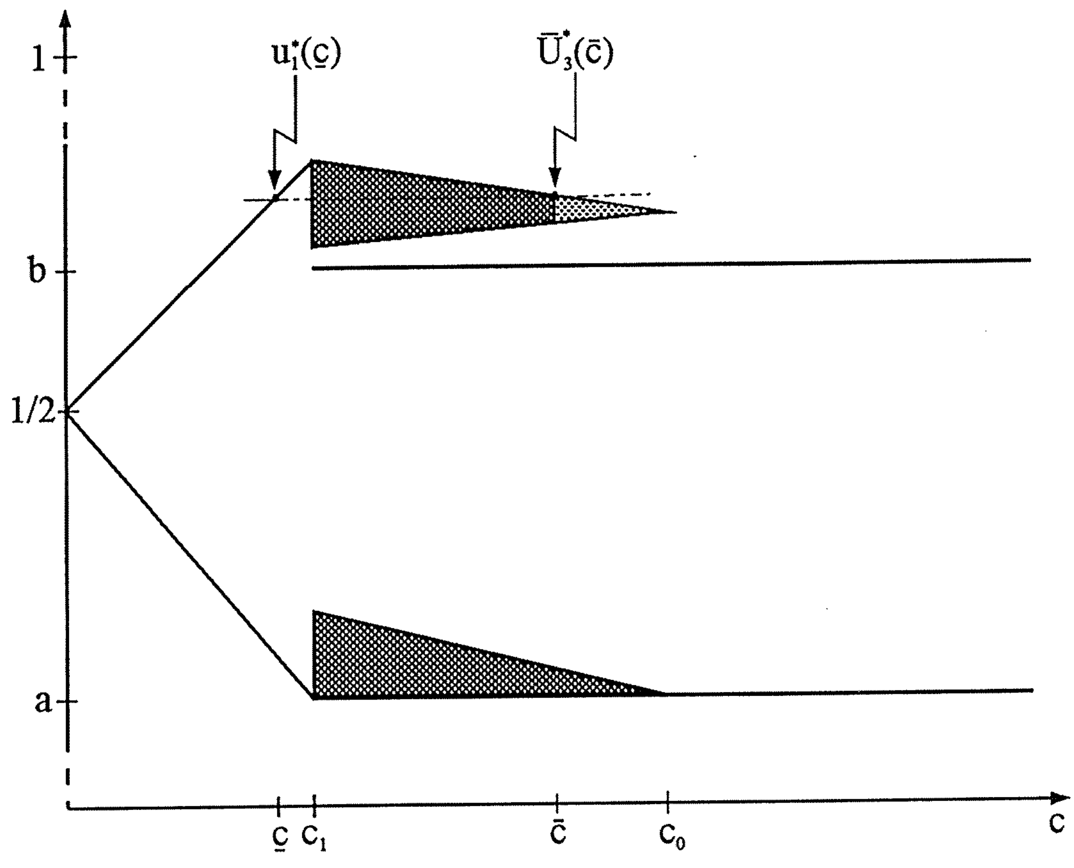

Figure 5: Weakening the weak may harm the strong

Institut für Höhere Studien
Institute for Advanced Studies
Stumpergasse 56
A-1060 Vienna
Austria

Phone: +43-1-599 91-145
Fax: +43-1-599 91-163