Business Formation and Aggregate Investment

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Abstract

The paper proposes an intertemporal equilibrium model with monopolistic competition and free entry to explain the nexus between business formation and medium run growth. An investment externality is identified that results in underaccumulation of capital in the decentralized market equilibrium and, thus, creates an investment multiplier. Some form of investment promotion is called for. The paper compares the effectiveness of policies to promote business formation with a general investment subsidy.

Das Papier entwickelt ein intertemporales Gleichgewichtsmodell mit monopolistischer Konkurrenz und freiem Marktzutritt und erklärt den Zusammenhang zwischen Firmengründungen und mittelfristigem Wachstum. Es wird eine Externalität in den Investitionsentscheidungen festgestellt, die Unterakkumulation von Kapital im dezentralisierten Marktgleichgewicht hervorruft und einen Investitionsmultiplikator erzeugt. Die Externalität macht Investitionsförderung notwendig. Das Papier vergleicht die relative Wirksamkeit einer allgemeinen Investitionsprämie mit einem Steueranreiz zur Förderung von Firmengründungen.

1 Introduction

Many governments run programs which are intended to encourage business formation and, thus, to foster growth and enhance efficiency. Such policy initiatives presume a kind of market failure with private entrepreneurs being too hesitant in starting new businesses. In addition, the lack of entrepreneurial stamina would create unfavorable conditions for growth. This paper investigates the economic rationale of such policies in an intertemporal economy with monopolistic competition and free entry. Indeed, the competitive distortion is seen to create an investment insufficiency in the market equilibrium. Hence, some form of investment promotion is called for. We consider policies that are specifically designed to promote investment and encourage business formation. Are policy makers right after all in presuming that incentives for increased business formation will favorably affect the conditions for medium run growth and enhance economic efficiency? Specifically, we compare a fixed cost subsidy that makes market entry more profitable, with the effectiveness of a general investment subsidy.

Recent literature on New Keynesian Economics found that general equilibrium models with imperfect competition can generate multipliers much like in old Keynesian models of macroeconomic income determination [Blanchard and Kyotaki (1987), Cooper and John (1988), Mankiw (1988), Startz (1989), Heijdra and v.d.Ploeg (1994) and Benassy (1994) for an incomplete list and Silvestre (1993), Dixon and Rankin (1994) and Matsuyama (1994) for surveys]. Typically, these papers rely on a static framework where the multipliers result from the effect of monopoly profits on aggregate income and vice versa. This paper combines demand and supply of differentiated goods under monopolistic competition [Dixit and Stiglitz (1977)] with an intertemporal model of capital accumulation and shows that an *investment multiplier* is at work. The framework gives rise to increasing returns due to specialization in the sense of Romer (1987). Free entry and exit keeps profits to zero but results in a product diversity effect. When new firms enter the market, they introduce new products and services and contribute to progressive division and specialization of industrial production. An increasing range of differentiated varieties raises the productivity of intermediates in forming composite final goods. As a result, consumption and investment price indices fall which further enhances private demand. Specifically, investment picks up, boosts future income and creates markets for new products. An expanded product range, in turn, reinforces the incentives to invest. The paper rigorously shows how a fiscal stimulus initiates a cumulative process between investment and the supply of new products. Eventually, due to diminishing returns to capital, the process fades which makes the multiplier finite. The investment multiplier opens a new

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channel for magnifying policy effects in the spirit of the New Keynesian Macroeconomics literature.

The multiplier rests on an investment externality: private agents fail to recognize that their marginal projects, even though unprofitable at first sight, would boost aggregate activity and attract new firms. The increase in product diversity would cut investment costs and make the investments worthwhile after all. This shortsightedness translates into insufficient aggregate investment in the decentralized market equilibrium. The underaccumulation of capital was anticipated by Romer (1987) and Kiyotaki (1988), but this paper gives a particularly simple characterization and derives an intuitive formula for the multiplier. Romer (1987) actually concentrates on the case where the model gives rise to endogenous growth which is found here to be a knife edge case corresponding to a very specific constellation of parameters. The R&D based endogenous growth literature also integrates monopolistic market structure with an intertemporal equilibrium framework and allows for an ongoing supply of new products [see Grossman and Helpman (1991)]. The mechanics are, of course, completely different. This paper concentrates on the case with diminishing returns to capital and elaborates on the New Keynesian interpretation with multiplier effects that rank so prominently in recent real business cycle models. Kiyotaki (1988) studies a two period model with monopolistic competition and investment. Since it keeps the number of firms fixed it is obviously not suitable for an analysis of the nexus between net business formation and macroeconomic activity.

Recent empirical research focussed on the interaction between market power, business formation, product diversification and general macroeconomic activity. Hall (1988), Domowitz, Hubbard and Petersen (1988) and Morrison (1990), for example, estimated large markups of price over marginal costs in many U.S. industries and found market power to be a pervasive phenomenon. The real business cycle literature recognized the quantitative importance of imperfect competition in macroeconomic fluctuations. Adopting an imperfectly competitive intertemporal equilibrium approach improves on the fit between theoretical simulation models and empirical observations [see Hornstein (1993) and Rotemberg and Woodford (1992), for example]. Davis and Haltiwanger (1990) provide evidence that a large part of macroeconomic fluctuations is associated with business failures and startups. Chatterjee and Cooper (1993) report a contemporaneous correlation of .54 between detrended real GNP and net business formation. Jovanovic (1993) finds that product diversification increases during upturns. Only recently, the real business cycle literature adopted dynamic macroeconomic equilibrium models that allow for imperfect competition as well as business formation and endogenous product diversification [e.g. Devereux, Head and Lapham (1993) and Chatterjee and Cooper (1993). See also

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Keuschnigg and Kohler (1994) for a dynamic computable general equilibrium model along these lines]. This paper offers an analytical approach and highlights in an intuitive way the kind of propagation and magnification of exogenous shocks that is at work in these models. The empirical evidence also provides a compelling reason for policy analysis based on models with imperfect competition, free entry and an endogenously determined range of differentiated products. This paper examines the role of investment promotion and compares the relative effectiveness of policies to encourage business formation with a general investment tax credit.

To organize the presentation of the main results, the paper is devided in four sections. Section 2 presents the model and characterizes the market failure by comparing to the social optimum. Section 3 proceeds with a discussion of the long-run investment multiplier and compares the effects from an investment tax credit with those from a fixed cost subsidy. In addition, it checks some quantitative implications on the basis of empirically relevant parameter values. Section 4 summarizes the essential results of the paper.

2   The Model

2.1   Monopolistic Production

Love for diversity creates demand for a whole range of closely substitutable differentiated varieties. Assuming homothetic preferences, demand may be thought of as demand for a composite good representing a basket of different varieties. The composition of this bundle derives from expenditure minimization which yields an exact price index giving the cost per unit of the composite good,

$$P = \min_{x_j} \left\{ \int_0^n p_j x_j dj \quad s.t. \quad \left[\int_0^n x_j^{\frac{1}{\beta}} dj \right]^\beta \geq 1 \right\}, \qquad \beta = \frac{\sigma}{\sigma - 1} > 1. \tag{1}$$

Varieties are easily substituted against each other with an elasticity equal to $\sigma = \frac{\beta}{\beta - 1} > 1$. Demand for the composite good at a quantity $D = C + I$ stems from consumption and investment and creates derived demand for individual varieties $x_j = (P/p_j)^\sigma D$. We will assume that producers of individual varieties are identical. Accordingly, we will consider only the symmetric case where $D = n^\beta x$. Overall spending then amounts to $PD = npx$ with

$$P = \left[\int_0^n p_j^{1/(1-\beta)} dj \right]^{1-\beta} = n^{1-\beta} p. \tag{2}$$

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Each variety is produced by a single producer who is a monopolist on his market. His market power is limited though since varieties are close substitutes in demand. With perfect competition on factor markets, producers take factor prices as given. All producers have access to the same production technology which is assumed linearly homogeneous in capital and labor. To operate a factory, however, producers also need to hire factors for fixed overhead purposes. Such fixed costs give rise to increasing returns to scale that are internal to the firm. As a matter of simplification, we assume that overhead operations and production of variable output employ an identical technology. Denote factory output by x and the scale of overhead operations by $\bar{x}$. Given factor prices, cost per unit of output is

$$\phi(w_K, w_L) = \min_{k,l} \{w_K k + w_L l \quad s.t. \quad F(k,l) \geq 1\}. \tag{3}$$

Unit factor demands are given by $\phi_{w_L} = l$ and $\phi_{w_K} = k$. Optimal factor demand equates the marginal value products to factor prices, $w_i = \phi F_i(k,l)$. Individual workshops may rent any amount of capital at a rental rate w_K, and may employ labor at a wage w_L. Thus, capital and labor may be redeployed across workshops without any frictions. Total costs add up to $\Phi(x) = [x + (1-\tau)\bar{x}]\phi$. We allow for a fixed cost subsidy at rate τ. Of course, average costs are declining in output. Henceforth, we normalize prices to $\phi = 1$.

In a well diversified economy, the market for an individual variety is small if compared to the size of aggregate demand. Consequently, individual producers have a negligible influence on the overall price index P and on the state of aggregate demand, and they *perceive* their own price elasticity of demand to be $\sigma = -px'(p)/x$. Given this estimate, producers set prices to maximize profits $\pi = px - \Phi(x)$. In exploiting market power, they find it optimal to choose a price in excess of marginal costs. Since unit costs are normalized to unity, markup pricing determines a price equal to

$$p = \beta. \tag{4}$$

Profits attract new producers and each one offers his own differentiated brand. We assume that free entry and exit continues until all profit opportunities are exhausted.[1] Hence, in equilibrium, price must equal average cost, $p = \Phi(x)/x$. Using the markup equation, the zero profit condition determines the scale of each firm

$$x = (1-\tau)\bar{x}/(\beta - 1). \tag{5}$$

Firm output depends on taste and technology parameters. In addition, a fixed cost subsidy allows firms to break even at a smaller output level.

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[1] The appendix shows that profits indeed decline with the number of firms making firm entry a stable.

Absent any barriers to entry, the number of producers and, consequently, the number of available varieties respond to changes in the economic environment. Specifically, the number of firms hinges on the size of individual firm outputs and the size of the aggregate economy which, in turn, depends on factor endowments. Given a labor force $L = 1$ and an aggregate capital stock K, full employment requires

$$K = nk(x + \bar{x}), \qquad 1 = nl(x + \bar{x}). \tag{6}$$

Multiplying endowments with factor prices, one obtains aggregate income Y at factor cost. It is equal to total costs gross of the fixed cost subsidy [use the price normalization $w_L l + w_K k = 1$],

$$Y = w_L + w_K K = n(x + \bar{x}). \tag{7}$$

Competitive pricing on factor markets relates factor prices to their marginal products, $w_i = F_i(k, l)$. By linear homogeneity of the production technology, marginal products remain unchanged when inputs are scaled by a common factor $n(x + \bar{x})$. Using (6), we have $F_i(k, l) = F_i(K, 1)$, and we may now reconcile the monopolistic production model with the usual notation in neoclassical growth models:

$$Y = f(K), \qquad w_K = f'(K), \qquad w_L = f(K) - Kf'(K), \tag{8}$$

where $f(K)$ is the production function in intensive form since K coincides with the capital labor ratio in face of unitary labor endowment.

¿From (7), the number of firms hinges on aggregate factor income and the scale of each firm which is fixed by (5). Aggregate income depends on factor endowments and the production function as in (8). Combining the two conditions, one arrives at

$$n = f(K)/(x + \bar{x}). \tag{9}$$

2.2 Consumption and Investment

Now we integrate monopolistic production into a Ramsey type growth model with intertemporal determination of savings and investment. Given Fisher's separation theorem, we may consider separately the savings and investment decisions. As consumers, agents receive a stream of capital and labor income and choose an optimal intertemporal consumption allocation. As investors, agents derive revenues from renting capital stocks to individual producers and spend on investment such as to optimally accumulate capital stocks. As producers, they organize production of varieties.

Agents love variety and consume a range of differentiated products at each date. The basket of varieties forms a composite good C which agents consume at an optimal rate in order to maximize life-time utility. The desired consumption flow C is achieved by accumulating financial wealth A subject to a given initial stock of assets and a flow of disposable wage income $w_L - T$. The government collects a lump-sum tax T to finance the subsidies to business.

$$\max_C \int_0^\infty u(C_s)e^{-\rho s}ds \quad s.t. \quad \dot{A} = iA + w_L - T - PC, \quad A_0 > 0. \tag{10}$$

The usual Euler equation determines the optimal growth rate of consumption,

$$\dot{C}/C = \gamma(r - \rho), \qquad r = i - \dot{P}/P, \tag{11}$$

where i is the market interest rate and r the consumption based real interest rate which is compared with the subjective rate of discount ρ. Furthermore, $\gamma = -\frac{u'(C)}{Cu''(C)}$ denotes the intertemporal elasticity of substitution which is equal to the inverse of the elasticity of marginal utility.

The investor earns revenues by renting out the existing capital stock to individual producers at a competitive rental rate w_K. She spends $(1 - z)PI$ on the composite good to build up the capital stock and prepare future production and revenues. Capital depreciates at a rate δ. To enhance the incentives to invest, the government may subsidize part of the investment outlays at rate z. The investor maximizes the present value of dividends $\chi = w_K K - (1 - z)PI$:

$$V = \max_I \int_0^\infty \chi_s e^{-\int_0^s i_u du}ds \quad s.t. \quad \dot{K} = I - \delta K, \quad K_0 > 0. \tag{12}$$

Optimality requires that the total rate of return on capital consisting of a dividend and a capital gains rate must match the market rate of interest. Given that the investment subsidy is levied at a constant rate, the optimality condition is

$$r = i - \frac{\dot{P}}{P} = \frac{w_K}{(1 - z)P} - \delta. \tag{13}$$

Differentiation of (12) gives the no-arbitrage condition $iV = \chi + \dot{V}$.

2.3 Intertemporal Market Equilibrium

We now tie together savings and investment to determine the equilibrium dynamics of the imperfectly competitive economy. To close the model, one needs to add the government

budget constraint $T = zPI + \tau n\bar{x}$. Substituting the capital market equilibrium condition $A = V$ into the savings equation in (10) and using the no-arbitrage condition on equity wealth as well as the fiscal constraint gives the aggregate income expenditure identity, $P(C + I) = Y - \tau n\bar{x} = Y[\frac{\Phi(x)}{x+\bar{x}}]$. The second equality substitutes the solution for n given in (9). Henceforth, we use B to denote the ratio of private to social average costs: $B = \frac{\Phi(x)}{x+\bar{x}} = \frac{\Phi(x)/x}{(x+\bar{x})/x}$. In equilibrium, (9) relates the number of firms to the size of the capital stock. Consequently, the price index P given in (2) depends on the capital stock, and the real interest rate r noted in (13) as well. Define a real rental rate of capital $\tilde{w}_K = w_K/P$ and real income $\tilde{Y} = Y/P$. Both depend on the capital stock. Substituting the stock flow relationship in (12) into the income expenditure identity, one obtains the aggregate law of motion for capital stocks,

$$
\begin{array}{rll}
(a) & \dot{K} & = \tilde{Y}(K)B - \delta K - C, \\
(b) & \dot{C} & = \gamma C[r(K) - \rho].
\end{array}
\tag{14}
$$

The second equilibrium condition comes from the Euler equation governing consumption growth with the consumption based real interest rate determined by the optimality condition for investment in (13). Provided that the real income function $\tilde{Y}$ is increasing and concave, and that the real rental rate $\tilde{w}_K$ is diminishing in capital, one may draw the same type of phase diagram as in the standard Ramsey growth model with perfect competition in order to qualitatively describe the dynamics of intertemporal equilibrium.

The real income function $\tilde{Y}$ reflects the external scale economies that arise in the macroeconomy when new firms enter the market and introduce new specialized products. An expansion of the resource base directly raises factor income Y by (8). It also makes room for more firms which introduce new varieties and contribute to increasing specialization and division of labor in industrial production. The productivity of specialized inputs in forming the composite good rises which squeezes the price index and gives a further boost to real income. Hence, the aggregate real income function is increasing returns to scale with respect to an expansion of factor endowments. In recognizing this interdependence one obtains a reduced form for real income and, similarly for the real rental rate of capital. In order to break even, workshops need to choose sufficiently large production runs. In particular, by the zero profit condition (5), output x is independent of the capital stock. The same holds for the output price in (4) while the number of firms depends on the size of the economy and, thus, on capital. Define $A \equiv (x + \bar{x})^{1-\beta}$ and relate the price index in (2) to the capital stock by inserting n from (9): $P = (p/A)f(K)^{1-\beta}$. Using the markup pricing condition (4), real income and the real rental rate of capital are

$$
\tilde{Y}(K) = \frac{A}{\beta}f(K)^\beta, \qquad \tilde{Y}'(K) = \beta\tilde{w}_K, \qquad \tilde{w}_K = \frac{A}{\beta}f(K)^{\beta-1}f'(K).
\tag{15}
$$

Diminishing private returns to capital imply that real income is increasing and concave in capital. Compute the elasticity of the real rental rate with respect to capital [capital's income share is $\alpha = \frac{K f'(K)}{f(K)}$, the elasticity of substitution in production is $\sigma^k = -\frac{(1-\alpha)f'(K)}{K f''(K)}$, and the hat notation indicates a percentage change]

$$\begin{aligned} \widehat{w}_K &= \eta \hat{K} + \hat{A}, & \eta &= -\frac{1-\alpha}{\sigma^k}(1 - \epsilon) < 0, \\ \epsilon &= (\beta - 1)\frac{\alpha \sigma^k}{1-\alpha}, & 0 &\leq \epsilon < 1. \end{aligned} \tag{16}$$

With perfect competition, varieties are perfect substitutes in demand ($\sigma \to \infty$) which leaves no room for market power and a positive markup: $\epsilon = 0$. As soon as producers are able to differentiate, they obtain market power and impose markup pricing. The number of firms starts to depend on the size of the economy as measured by the value of factor endowments or GDP at factor cost. Clearly, capital accumulation increases the number of firms and, thus, the range of differentiated products. An increased product variety in turn makes the composite capital good effectively cheaper and reinforces the incentives to invest. A cumulative process results that introduces increasing returns on the macro level even though technology is linear homogeneous at the firm level. As shown in (15), firm level technology f is wrapped into a convex function with elasticity $\beta > 1$. Hence, monopolistic competition makes the macroeconomic real income function less concave in capital. Specifically, monopolistic competition boosts the returns to capital. The condition $\epsilon < 1$ ensures that, at the macro level, the real marginal product, or the real rental rate of capital, remains diminishing. For example, in case of a CD production function with a unitary elasticity of substitution and a capital share of one third, the critical value of the markup factor to violate the condition for diminishing returns would be as high as $\beta = 3$. This is too high to be a realistic value, especially for macroeconomic averages.

The external economies created by new firms entering the market translate into increasing returns to scale on the macro level. These may, in principle, be strong enough to offset the decreasing returns to capital in the individual production functions and to give rise to sustained 'growth based on increasing returns due to specialization' [Romer (1987)]. In a sense, the model provides a microfoundation for Ak type endogenous growth models [see Rebelo (1991), King and Rebelo (1990)] where production is linear in the accumulating factor. In the limiting case where the elasticity of variety substitution attains its lower critical value [$\sigma = 1.5$ in the example], the aggregate real income function is indeed linear in capital.[2] We argue, however, that this limiting case is rather implausible in the

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[2]In case of a CD production function, $f(K) = K^\alpha$, the lower bound would satisfy $\sigma = 1/(1 - \alpha)$ or $\beta = 1/\alpha$ and the real income function would be $\tilde{Y} = \alpha A K$.

light of empirical evidence on basic parameters. Unlike Romer (1987) we emphasize the more realistic intermediate case which is in the spirit of New Keynesian economics with a new twist: capital accumulation expands product variety which reinforces the incentives to invest and, thereby, creates an investment multiplier.

2.4 Social Optimum

Given imperfect competition, the market equilibrium is bound to be non-optimal in some way.[3] In which way is private decision making distorted? To answer this question, we compare the selfish market outcome with the allocation that a benevolent planner would choose on behalf of the community. Such a comparison will then identify the nature of the externalities that are associated with private decisions. More precisely, we consider a second best social optimum that takes as given the markup pricing behavior of firms once they are in business.[4] Such an optimum can then be made viable as a decentralized market equilibrium by an appropriate choice of Pigouvian taxes or subsidies.[5] Given the aggregator function $D(n,x) = [\int_0^n x_j^{1/\beta} dj]^\beta$, the social planner's problem is

$$\max \int_0^\infty u(C_s)e^{-\rho s} ds \quad s.t. \quad \dot{K} \leq D(n,x) - \delta K - C, \quad f(K) \geq \int_0^n (x_j + \bar{x}) dj. \quad (17)$$

The second resource constraint is actually a short-cut that relies on linear homogeneity of the production technology and on symmetry in the production of individual varieties [see the discussion of (6) and (8)]. It says that overhead operations and variable output of all firms taken together must not exceed the total capacity for value added production. Again, we may normalize to unity the multiplier of the second constraint which reflects the resource cost of marginal value added. The first constraint requires that the total use of the final good for investment and consumption purposes must not exceed its supply. The planner controls consumption and regulates the number of firms, taking as given markup pricing and output reactions of private firms once they are in the market. Appendix A shows how output responds to an increase in the number of competitors, see (A.3).

The social planning approach dichotomizes into a static problem of choosing the optimal number of firms and an intertemporal problem of optimal consumption and

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[3]In the laissez faire equilibrium, $\tau = z = 0$ and $\Phi(x) = x + \bar{x}$ and $B = 1$ in (14).

[4]This follows Mankiw and Whinston (1986) who analyze optimal firm entry in a static partial equilibrium model, and conforms to the notion of a constrained optimum in Dixit and Stiglitz (1977).

[5]By way of contrast, the first best optimum could not be decentralized since the planner would set price equal to marginal cost and, thus, leave firms without any revenues to cover fixed overhead costs. In the absence of lump-sum transfers, firms would make losses.

investment. Note the derivatives of the aggregator function for the symmetric case: $D_n = \beta x n^{\beta-1}$, $D_{xj} = n^{\beta-1}$ and $D_x = nD_{xj} = n^\beta$. With this notation, the first order condition for optimal firm numbers is

$$\lambda D_n - [x + \bar{x}] + [\lambda D_x - n]\frac{\partial x}{\partial n} = 0, \tag{18}$$

where λ is the multiplier of the dynamic resource constraint. Private agents take the planner's marginal willingness to pay for an additional variety as their (inverse) demand function: $\lambda D_{xj} = \lambda(D/x_j)^{1/\sigma} = p$. From the viewpoint of a single producer, this demand function has a perceived price elasticity equal to σ from which they calculate their markup. Using $\lambda D_x = n\lambda D_{xj} = np$ and expanding by px,

$$[px - (x + \bar{x})] + [\lambda D_n - px] + n[p - 1]\frac{\partial x}{\partial n} = 0. \tag{19}$$

The first square bracket captures the profits of an individual producer. The second is the consumer surplus effect that results from an expansion of variety. The last represents a profit destruction effect that occurs when incumbent firms reduce their sales in response to new producers. The variety effect may be written as $px[\frac{D_n}{xD_{xj}} - 1] = px(\beta - 1)$. Due to (4), the profit destruction effect amounts to $px(\frac{\beta-1}{\beta})(\frac{n}{x}\frac{dx}{dn}) = px(1 - \beta)$ since the elasticity of firm output in response to new entrants is $-\beta$ [see (A.3)]. Hence, in the zero profit equilibrium, the optimality condition of the social planner is identically fulfilled since the variety and profit destruction effects exactly cancel. Accepting markup pricing, the zero profit condition gives $x = \frac{\bar{x}}{\beta-1}$, as in the laissez faire equilibrium. Consequently, the market equilibrium under monopolistic competition and free entry determines in (9) – for any *given* capital stock – the socially optimal number of firms $n^* = n(K)$.

Recognizing that optimal firm numbers are related to the capital stock as noted in (9) and that variable output x is independent of capital, efficiency in intertemporal consumption choice requires

$$\dot{\lambda} - \rho\lambda = -\left\{f'(K) - \delta\lambda + [\lambda D_n - (x + \bar{x})]\frac{\partial n^*}{\partial K}\right\}. \tag{20}$$

Using the zero profit condition $px = x + \bar{x}$, the square bracket is recognized as the surplus or variety effect noted in (19) and is thus equal to $px(\beta - 1)$. Its power depends on how effective a higher capital stock is in expanding markets and attracting new firms. From (9), $\frac{\partial n^*}{\partial K} = \frac{f'(K)}{px}$. Hence, the social return to capital consists of the rental rate $w_K = f'(K)$ but is additionally boosted by the diversity effect of a higher capital stock, $(\beta - 1)f'(K)$. Private investors fail to recognize this externality. For a given capital stock, prices, outputs and firm numbers in the market equilibrium coincide with the solution

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of the social optimum problem. Hence, the shadow price of an additional unit of the composite good D is equal to the price index, $P = \lambda$. Dividing through by λ and noting the optimality condition $u'(C) = \lambda = P$ gives $\dot{C} = \gamma C[r^S(K) - \rho]$ where $r^S = \beta f'(K)/P - \delta$ denotes the social rate of return to capital.[6] This intertemporal efficiency condition is identical to (14b) except that the private rate of return r is replaced by the social rate r^S. Finally, substitute the solutions for n and x to relate the amount of the composite good to factor endowments: $D = n^\beta x = \tilde{Y}(K)$. Hence, the law of motion for capital is identical to the laissez faire version in (14a). The equilibrium conditions are easily analyzed in terms of a phase diagram such as figure 1. Recognizing $w_K = f'(K)$, the social return to capital exceeds the private return $r = \frac{w_K}{(1-z)P} - \delta$ by

$$r^S - r = (\beta - \frac{1}{1-z})\tilde{w}_K \quad \gtrless \quad 0 \qquad \Longleftrightarrow \qquad z \quad \lessgtr \quad \frac{\beta-1}{\beta} = \frac{1}{\sigma}. \qquad (21)$$

Private agents ignore the variety effect of additional investment. The externality could be offset by a sufficiently generous investment incentive. Without policy intervention, however, capital accumulation falls short of the socially optimal amount, see figure 1. The investment externality is also the source of the multiplier discussed in the next section. The results of this section are summarized in

Proposition 1 (Underaccumulation)
In the decentralized market equilibrium, the private return falls short of the social return to capital by $r^S - r = (\beta-1)\tilde{w}_K$. While firm entry is constrained optimal, private investment is too low. The optimal investment tax credit is $z^* = 1/\sigma$.

3 Investment Promotion and Welfare

3.1 The Investment Multiplier

Capital accumulation increases the size of the economy and creates markets for new diversified products. Exploiting profit opportunities, new firms enter the markets, introduce new brands and, thus, expand the product range. The availability of increasingly specialized varieties makes the composite capital good effectively cheaper which induces agents to invest even more. A cumulative process starts that reinforces investment incentives,

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[6]A shortcut to the social optimum problem would be to maximize the utility integral in (10) subject to the resource constraint (14a), yielding the same results.

magnifies the overall effects and, thus, creates increasing returns to scale on the macro level. This intuitive logic can be stated more rigorously. Log-linearization of (9) reveals how the number of firms changes with the capital stock, see (22a). Now find the implied change of the price index from (2), $\hat{P} = -(\beta - 1)\hat{n}$. In the long-run, the private return $r = \rho$ must remain constant to keep consumption stationary whence $\hat{w}_K - \hat{P} = -\hat{z}$ where $\hat{z} = dz/(1 - z)$. Substitute the changes in the price index and the rental rate, $\hat{w}_K = -\frac{1-\alpha}{\sigma^k}\hat{K}$, and remember the definition of ϵ in (16). Then the number of firms and the capital labor ratio respond according to

$$
\begin{array}{lll}
(a) & \hat{n} & = \alpha\hat{K}, \\
(b) & \hat{K} & = \frac{\sigma^k}{1-\alpha}\hat{z} + \frac{\epsilon}{\alpha}\hat{n}.
\end{array}
\tag{22}
$$

An investment tax credit (ITC) makes capital cheaper, increases its return and, thus, brings forth capital investments. For a given product range, the first round effect on the capital stock from (b), $\hat{K}_1 = \frac{\sigma^k}{1-\alpha}\hat{z}$, would drive down the return to a sustainable long-run rate. However, a larger capital stock raises aggregate economic activity and attracts more firms [$\hat{n} = \alpha\hat{K}_1$ by (a)]. New suppliers offer new products. With increasing specialization and division of labor, the average productivity of specialized inputs in forming the composite final good increases which effectively reduces its price index. Cheaper capital goods create more investment opportunities, giving rise to a second round effect [$\hat{K}_2 = \epsilon\hat{K}_1$ from (b)]. By the same reasons, this triggers a third round effect equal to $\hat{K}_3 = \epsilon^2\hat{K}_1$, and so on. Adding up, the overall expansion of capital stocks is

$$
\hat{K} = (1 + \epsilon + \epsilon^2 + \ldots)\hat{K}_1 = \frac{1}{1 - \epsilon}\frac{\sigma^k}{1 - \alpha}\hat{z}.
\tag{23}
$$

Alternatively, the total effect follows directly from the long-run restriction $\widehat{\widehat{w}}_K = \eta\hat{K} = -\hat{z}$ in (16), but (23) stresses the cumulative roundabout nature of the process. The *investment multiplier* is recognized in $(1 - \epsilon)^{-1}$. The discussion of (16) revealed that $\epsilon < 1$ is the condition for diminishing returns to capital at the macro level. In (23), the same condition is needed for the cumulative investment process to converge and the multiplier to be finite. In the borderline case of $\epsilon = 1$, the cumulative process would continue forever.[7] The paper focusses on the intermediate case in the spirit of New Keynesian economics which so far

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[7]The borderline case is viable only for a CD production function [$\tilde{Y} = \alpha AK$ as in fn.2] as otherwise the capital share α would change with accumulation and make $\epsilon = 1$ diverge from the borderline case. Given linearity in capital and $r > \rho$, the Euler equation in (14) pins down the growth rate $g = \dot{C}/C = \gamma[r - \rho]$. Differentiate the first equation in (14), substitute $\dot{K} = g\hat{K}$, and divide through by $\hat{K}$. Given the joint growth rate, we have $\dot{C}/\hat{K} = C/K$. Hence, consumption is tied to the capital stock according to $C/K = \alpha A - \delta - g$ and both grow at the same rate.

emphasized multiplier effects that arise in static models of monopolistic competition and endogenous labor supply. In a dynamic model, the increasing returns evolve via capital accumulation. While the multiplier on capital is long-run in nature and becomes effective only after some time, the magnifying effects on the growth rate of capital and, thus, on investment are felt immediately.

Figure 2 gives a lucid graphic demonstration of the long-run investment multiplier. We may draw (22) by plotting the capital stock horizontally and the product range vertically.[8] The line running through the origin is the free entry condition (22a) with a relatively flat slope equal to α. It captures the effect of increased market size associated with increased capital endowments on the number of firms. Condition (22b) captures the influence of the tax incentive and the capital goods price on capital investments. Upon inverting it, we obtain the other line which may be called the investment line. It has a negative intercept and a slope equal to α/ϵ which is steeper than the slope of the free entry line by virtue of the stability assumption $\epsilon < 1$. In the initial steady state, both lines would run through the origin. A permanent tax credit shifts the investment line to the east since for any given number of firms the tax incentive induces capital investments. The intersection of the investment line with the horizontal axis shows the first round effect on capital investments $\hat{K}_1$. These investments enlarge firm numbers in an amount that is seen by moving vertically to the free entry line. The productivity effect from firm entry and increased specialization reinforces the incentives to invest. Moving horizontally to the investment line, one obtains the implied capital investments. As the indirect effects become weaker in each round, this cumulative process eventually converges to determine the overall changes in capital stocks and firm numbers.

Proposition 2 (Investment Multiplier)
The externality in private investment decisions creates an investment multiplier. The size of the multiplier increases with the markup factor β, with capital's share in factor income α, and with the elasticity of factor substitution σ^k.

3.2   Investment Tax Credit or Fixed Cost Subsidy?

This section compares the effects of an investment tax credit with those of a fixed cost subsidy. The private economy's underaccumulation of capital creates a good and also

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[8]More precisely, the lines depict deviations from an initial steady state position which is associated with the origin. The figure is concerned with a long-run comparative statics exercise.

obvious reason for investment promotion. The investment tax credit reduces the effective price of the composite capital good and, thus, boosts the private incentives to invest. Could fixed cost subsidies achieve the same ends? In the real world, governments run various programs that are explicitly designed to encourage business formation. Fixed cost subsidies may be viewed as a rough approximation of such policies.

A fixed cost subsidy reduces overhead costs, allows all firms to break even at a smaller scale and encourages new producers to enter the market. The increased diversity in specialized inputs increases their productivity in forming the composite good, making it effectively cheaper. This cost reduction in the capital good reinforces the incentives to invest, possibly as much as an investment tax credit. A comparative dynamic exercise yields basic insights regarding the expected effects. To this end, appendix B log-linearizes the dynamic equilibrium conditions noted in (14) and arrives at

$$
\begin{bmatrix} \dot{\hat{K}} \\ \dot{\hat{C}} \end{bmatrix} = \begin{bmatrix} r^S & -\bar{c} \\ \gamma(\rho + \delta)\eta & 0 \end{bmatrix} \begin{bmatrix} \hat{K} \\ \hat{C} \end{bmatrix} + \begin{bmatrix} -(1-B)B\bar{y}\hat{\tau} \\ \gamma(\rho+\delta)(sB\hat{\tau} + \hat{z}) \end{bmatrix}. \tag{24}
$$

For more concise notation, the consumption and output to capital ratios are abbreviated by $\bar{c} = C/K$ and $\bar{y} = \tilde{Y}/K$. A share s of total cost is due to fixed costs, and $B = \frac{\Phi}{x+\bar{x}}$ denotes the ratio of private to social average cost. The social rate of return to capital, $r^S = B\tilde{Y}'(K) - \delta$, now slightly differs from the definition given in section 2.4 by the factor $B < 1$. We allow for a positive subsidy τ which alleviates the underaccumulation problem and reduces the gap between social and private returns in the initial equilibrium. The long-run effects of investment promotion obtain from the stationary version of (24). The second line yields the effects on capital investments which may then be substituted into the first line to obtain the long-run effects on consumption.

$$
\hat{K}_\infty = -\frac{1}{\eta}[sB\hat{\tau} + \hat{z}], \qquad \hat{C}_\infty = \frac{1}{\bar{c}}[r^S\hat{K}_\infty - (1-B)B\bar{y}\hat{\tau}]. \tag{25}
$$

Investment tax credits and fixed cost subsidies look quite similar in terms of their effects on capital investments if introduced in small magnitudes. For example, if a quarter of total costs were fixed [$s = .25$], a 20 % fixed cost subsidy would achieve the same effects on aggregate investment as a 5 % investment tax credit. The mechanism is, of course, quite different. When the government decides for an investment tax credit, it directly pays for part of the cost of the composite capital good and, thus, makes more investment profitable. By way of contrast, the government may want to encourage business formation by giving a fixed cost subsidy. Workshops now can afford to produce at a smaller scale and still break even which creates room for new producers. These offer new differentiated

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products and contribute to increasing division and specialization of industrial production. The average productivity of intermediate inputs in forming the composite capital good increases which makes it cheaper. Hence, aggregate investment is boosted just as with the investment tax credit.

If the fixed cost subsidy is introduced from a level of zero, the similarity between the two instruments continues to hold for consumption. Scale and variety were found to be optimal in the decentralized market equilibrium with zero profits, see section 2.4. A small subsidy would do no great harm. Even though the subsidy creates a distortion, the consequences are of second order magnitude only. The similarity breaks down, however, when a fixed cost subsidy was already given in the initial equilibrium. While an investment tax credit unambiguously increases long-run consumption, the fixed cost subsidy may or may not in this situation. With firms subsidized initially, the choice between variety and scale is already distorted which is reflected in a wedge $1 - B = \frac{\tau}{1-\tau}sB > 0$. Individual average cost falls below social average cost. An increase in the subsidy leads to further derationalization of industrial production. It increases the number of producers at the cost of a reduced scale of production. An increase in the subsidy now aggrevates an existing distortion resulting in first order efficiency losses. With the derationalization effect running counter to the capital accumulation effect, the overall increase in consumption becomes smaller and may turn out ambiguous in extreme cases.

A consistent welfare evaluation of the two rival policies needs to take account also of the transitional effects on consumption. With details given in the appendix, the two policies' impact on welfare is [see (B.6)]

$$\hat{U} = \frac{dU}{Cu'(C)} = -(1 - B)\frac{B\bar{y}}{\rho\bar{c}}\hat{\tau} - (r^S - \rho)\frac{\gamma(\rho + \delta)}{\Psi(\rho)}\left(\frac{1}{\rho} - \frac{1}{\bar{\zeta}}\right)[sB\hat{\tau} + \hat{z}]. \qquad (26)$$

As shown in appendix B, the unstable root exceeds the discount rate, $\bar{\zeta} > \rho$, and the characteristic polynomial satisfies $\Psi(\rho) < 0$. Hence, the sign of the second term depends on the sign of the wedges exclusively. It captures the welfare gains that result from the two policies' effect on capital accumulation. Baldwin (1992) showed that welfare from additional capital increases whenever the social return exceeds the private return, for whatever reasons. The present paper shows that in an intertemporal model with monopolistic competition, the economy underaccumulates capital from a social viewpoint. Hence, any shock, or any policy intervention, that enhances capital accumulation, creates dynamic welfare gains by alleviating the underaccumulation of capital. Several cases are interesting. First, if the economy starts from a laissez faire position ($\tau = z = 0$), then variety and scale are optimal ($B = 1$) but capital is underaccumulated. Since both

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instruments are promoting investment, welfare increases on account of induced capital accumulation, $\hat{U} = -(r^S - \rho)(\frac{1}{\rho} - \frac{1}{\zeta})\frac{\gamma(\rho+\delta)[s\hat{\tau}+\hat{z}]}{\Psi(\rho)}$. The power of this channel hinges on the size of the wedge between the social and the private rate of return or, equivalently, on how severe the underaccumulation of capital is in the initial situation: $r^S - \rho = (\beta-1)\tilde{w}_K > 0$. If both instruments are introduced with small rates, they can be made similar in terms of their welfare effects: $\hat{z} = s\hat{\tau}$. Second, the similarity in fact holds for any value of z, as long as $\tau = 0$ initially. Both instruments increase or decrease capital accumulation and welfare depending on whether the investment tax credit in place is too low or too high relative to the optimal one: $z \lessgtr z^* = \frac{1}{\sigma}$, see (21). If capital accumulation were optimal in the first place ($r^S = \rho$), further accumulation would not yield any increase in welfare.

Consider next a situation where some fixed cost subsidies are already in place to begin with. The first welfare term relates exclusively to the fixed cost subsidy and is negative. It is the present value of a static output loss that recurs in each period since the fixed cost subsidy exacerbates a preexisting distortion. By subsidizing fixed costs, the government creates misguided incentives for smaller production runs and excessive entry. While more variety strengthens the incentives to invest and generates welfare gains from induced capital formation in case of an insufficiently high ITC, it also means a deviation from socially optimal entry and scale of production with the effect of a static output loss. Hence, a fixed cost subsidy is less attractive in terms of its welfare effects. This inferiority reflects nothing more than Bhagwati's (1971) principle of targeting: a distortion is best countered by a tax instrument that directly acts on the relevant margin. Since policies that are intended to encourage business formation relate only indirectly to the investment externality, they are bound to be less attractive. By way of contrast, an investment tax credit directly boosts the private rate of return to capital which is perceived to be too low by private investors.

Proposition 3 (Inferiority of Fixed Cost Subsidy)

(a) Starting from a value of zero, $\tau = 0$, a small increase $\hat{\tau}$ of the fixed cost subsidy and an increase of the investment tax credit equal to $\hat{z} = s\hat{\tau}$ have equivalent effects on capital accumulation, consumption and welfare.

(b) With large increases, or if increased from an already positive level $\tau > 0$, the fixed cost subsidy becomes inferior. It creates, or exacerbates, a distortion with respect to scale and variety.

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3.3   How Important Is It?

In the decentralized market equilibrium under imperfect competition and free entry, private investors fail to take account of the full social returns to capital. Consequently, the market equilibrium results in insufficient capital accumulation. Investment promotion is called for. Furthermore, the externality creates a multiplier that magnifies the economy's response to exogenous shocks. Just how large are the multipliers? And how big should the investment tax credit be? Table 1 provides a perspective by assigning empirically relevant numbers to key parameters. The most important is the markup factor β. Recent industry studies found that market power is a pervasive economy wide phenomenon and provided estimates for markups ranging between 1.2 to 2 [see Hall (1988), Domowitz, Hubbard and Petersen (1988) and Morrison (1990)]. Table 1 combines these numbers with commonly used values for other basic parameters. The first two columns report values for capital's income share α and the markup factor β. The next three columns compute the long-run elasticities of the capital stock from (23). According to line 1, introducing a one percent ITC would raise the long-run capital stock by two percent in total. Given a unitary elasticity of factor substitution, the direct effect amounts to $\frac{1}{1-\alpha}$. With ϵ defined in (16), the multiplier is $\frac{1}{1-\epsilon}$ and the total effect is simply the product of columns 3 and 4. The sixth column uses (21) and reports the wedge between the private and social returns $r^S - \rho = (\beta - 1)\tilde{w}_K$ that obtains in the laissez faire equilibrium. The real interest rate equals ρ in the long-run which implies a real rental rate of capital equal to $\tilde{w}_K = \rho + \delta$. Finally, column 7 calculates the optimal investment subsidy from (21).

Taking a capital income share equal to $\alpha = \frac{1}{3}$ and a markup factor of 2, the multiplier is seen to double the long-run effect on the capital stock! While this is on the high side, column 4 shows that the multiplier rapidly increases with the markup factor. Taking more reasonable markups of 25 or 50 %, the investment multiplier will magnify the direct long-run effect on the capital stock by a further 10 to 30 %. Finally, the model implies quite large investment tax credits to achieve optimal capital accumulation. Given a 25 % markup of price over marginal cost, an ITC as high as 20 % is suggested to compensate for the externality and to encourage private investment.

4   Conclusions

Monopolistic competition and free entry create an investment externality that results in underaccumulation of capital in the market equilibrium. Private investors fail to recog-

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nize that the marginal investment projects, even though unprofitable at first sight, would attract new businesses that add to product diversity and contribute to increasing division and specialization of industrial production. As a result, the price of the composite capital good would fall and, thus, make the investment worthwhile after all. This shortsightedness creates an investment multiplier: some expansionary shock induces new investments that are decided on the basis of a given price index for capital goods. Now agents are "surprised" to learn that increased product diversity cuts investment costs and makes additional projects profitable. New investments attract additional firms giving rise to a second round of cost reductions, investment expansion and so on. Drawing on empirical estimates of key parameters, the multiplier was seen to be rather powerful and could possibly double the long-run effects on the capital stock.

With insufficient aggregate investment in the decentralized market equilibrium, some form of investment promoting policies are called for. Many governments run programs which are intended to encourage business formation and, thus, to foster growth and enhance efficiency. Such policy initiatives presume a kind of market failure with private entrepreneurs being too hesitant in starting new businesses. In addition, the lack of entrepreneurial stamina would create unfavorable conditions for medium run growth. This paper investigates the economic rationale of such policies within a framework of a growing economy with monopolistic competition and free entry. We find that policies aimed at increased business formation are, indeed, effective in generating aggregate investment. Increased variety raises the average productivity of intermediate inputs in building the capital stock. Hence, the capital becomes effectively cheaper which further strengthens the incentives to invest. However, policies for increased business formation provide only an indirect cure for the investment insufficiency. Increased variety comes at the cost of derationalization of production and results in the end in an inefficient tradeoff between variety and scale. Hence, the paper finds business formation policies to be second rate. At the root of the problem is an investment externality that makes the private rate of return fall short of the social rate. Hence, an investment subsidy that directly boosts the private return to capital is more appropriate than a policy that encourages business formation. Checking the empirical evidence on important parameters, an investment subsidy of 20 % and more seems advisable.

Appendix A: Profits and Firm Entry

For an analysis of the social optimum, one needs to know how outputs of incumbent firms change in response to an increase in the number of competitors. For the zero profit equilibrium to be viable, profits must be declining in the number of firms. Otherwise, the entry process would be unstable as ever more firms would be attracted by increasingly higher profits. For a consistent explanation of the static equilibrium in case of non-zero profits, the aggregate income expenditure identity must include aggregate profit income $\Pi = n\pi$ in addition to factor income $Y = f(K)$ which is given at any moment in time:

$$Y + \Pi = E = npx. \tag{A.1}$$

Consider a small perturbation of the zero profit equilibrium which is characterized by $\pi = 0$ or $px = x + \bar{x}$. Since the capital stock is predetermined at any moment in time, $\hat{E} = \hat{\Pi}$ where we compute the change in profits relative to aggregate spending: $\hat{\Pi} = \frac{d\Pi}{npx} = \frac{d\pi}{px} = \hat{\pi}$. Producers will always apply the markup pricing rule (4), hence $p = \beta$ and $\hat{p} = 0$. At the zero profit equilibrium, aggregate spending and profits respond as

$$\hat{E} = \hat{n} + \hat{x}, \qquad \hat{\pi} = [(\beta - 1)/\beta]\hat{x}. \tag{A.2}$$

The two conditions determine the change output levels when new competitors enter the market. The effects on monopoly profits and aggregate spending are immediate,

$$\hat{E} = \hat{\Pi} = \hat{\pi}, \qquad \hat{x} = -\beta\hat{n}, \qquad \hat{\pi} = (1 - \beta)\hat{n}. \tag{A.3}$$

Note that the usual income multiplier relating to monopoly profits is in operation. To see this, consider an exogenous increase in firm numbers $\hat{n}$. For a given level of spending, new firms steal business from incumbent producers causing sales to fall by $\hat{x} = -\hat{n}$, see (A.2). Profits and, in turn, aggregate income fall by $\hat{\pi} = -(\frac{\beta-1}{\beta})\hat{n} = \hat{E}_1$. By (A.2), depressed aggregate spending dictates further production cutbacks, $\hat{x} = \hat{E}_1$, which again squeezes profits and aggregate income by $\hat{\pi} = (\frac{\beta-1}{\beta})\hat{E}_1 = \hat{E}_2$. Another cycle of production cutbacks and profit destruction creates a third round effect on spending equal to $\hat{E}_3 = (\frac{\beta-1}{\beta})^2\hat{E}_1$. Eventually, these indirect demand effects die out. The total effect noted in (A.3) results from summing over all direct and indirect income effects, $\hat{E} = \sum_{i=1}^{\infty} \hat{E}_i = \beta\hat{E}_1 = (1 - \beta)\hat{n}$.

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Appendix B: Equilibrium Dynamics and Welfare

Local equilibrium dynamics is usefully characterized by taking a log-linear approximation of (14) at the economy's initial steady state (ISS) position. The characterization of comparative statics relies heavily on stationarity properties of the ISS such as $r = \rho$, see (11). By the zero profit condition (5), the cost share $s \equiv \frac{(1-\tau)\bar{x}}{\Phi}$ satisfies $(\beta-1)(1-s) = s$ or $\beta(1-s) = 1$. Thus, the share of fixed costs net of the subsidy is tied to the markup factor and, correspondingly, to the substitution elasticity by $s = 1/\sigma$. The zero profit condition now implies that a fixed cost subsidy of size $\hat{\tau} = \frac{d\tau}{1-\tau}$ causes derationalization equal to $\hat{x} = -\hat{\tau}$. The subsidy reduces total costs by $\hat{\Phi} = -\hat{\tau}$ and, consequently, allows firms to break even at a smaller scale. Derationalization of production affects the productivity parameters $A = (x+\bar{x})^{1-\beta}$ and $B = \frac{\Phi(x)}{x+\bar{x}}$. Notice that $\frac{x}{x+\bar{x}} = (1-s)B$, $\frac{\bar{x}}{x+\bar{x}} = sB/(1-\tau)$, and therefore $(1-B) = \frac{\tau}{1-\tau}sB$. The productivity terms change by $\hat{A} = (1-\beta)(1-s)B\hat{x} = sB\hat{\tau}$ and $\hat{B} = [(1-s)B - 1]\hat{\tau}$. Consequently, $\hat{A} + \hat{B} = (B-1)\hat{\tau}$. One derives

$$\frac{d(\tilde{Y}B)}{K} = B\tilde{Y}'(K)\hat{K} + \bar{y}B(\hat{A} + \hat{B}), \tag{B.1}$$

where $\bar{y} = \tilde{Y}/K$ denotes the output to capital ratio. Similarly, the consumption to capital ratio is abbreviated by $\bar{c} = C/K$. Using the short-hand $r^S = B\tilde{Y}'(K) - \delta$, the log-linearized form of the law of motion in (14a) is read from the first line of (24). Defining $\hat{C} = \frac{d\dot{C}}{C}$ and using the fact that $r = \rho$ holds in the ISS, the linearized form of the Euler equation is $\hat{C} = \gamma(\rho + \delta)[\hat{\tilde{\omega}}_K + \hat{z}]$. Now, one uses (16) to obtain the final form given given in the second line of (24).

A consistent welfare evaluation requires to consider the complete adjustment trajectory of the economy. In short-hand notation, (24) reads as $\dot{X} = ZX + G$ where X denotes the vector of the two dynamic variables, Z is the coefficients matrix and G is a vector with elements $g_1 = -(1-B)B\bar{y}\hat{\tau}$ and $g_2 = \gamma(\rho+\delta)[sB\hat{\tau} + \hat{z}]$. To check stability, one evaluates the characteristic polynomial $\Psi(\omega) = |\omega I - Z| = \omega^2 - r^S\omega + \bar{c}\gamma(\rho+\delta)\eta$ and finds that it satisfies $\Psi(r^S) = \Psi(0) = |Z| = \bar{c}\gamma(\rho+\delta)\eta < 0$. Hence, the eigenvalues $\{\zeta, \bar{\zeta}\}$ split into

$$\zeta < 0 < r^S < \tilde{\zeta}. \tag{B.2}$$

In the market equilibrium, $\rho < r^S < \bar{\zeta}$. Since the characteristic polynomial is quadratic, it would definitely satisfy $\Psi(\rho) < \Psi(r^S) = \Psi(0) < 0$. With investment incentives in the ISS, $\rho \gtrless r^S$, but we assume that $\Psi(\rho) < 0$ always.

The change in life-time welfare is the present value of all future changes in consumption discounted at rate ρ. It is most directly computed as the Laplace transform of the complete

solution for consumption. Taking the transform of (24), one obtains[9]

$$\Psi(\omega) \begin{bmatrix} L_\omega[\hat{K}_t] \\ L_\omega[\hat{C}_t] \end{bmatrix} = \begin{bmatrix} \omega & -\bar{c} \\ \gamma(\rho+\delta)\eta & (\omega-r^S) \end{bmatrix} \begin{bmatrix} g_1/\omega \\ g_2/\omega + \hat{C}_0 \end{bmatrix}. \tag{B.3}$$

Although the method is more general, we consider only the simplest case of a constant policy change whence $L_\omega[G] = G/\omega$. The instantaneous jump in consumption is determined by the requirement that the solution remain bounded. Evaluating the transform at a rate $\omega = \bar{\zeta} > 0$ equal to the positive eigenvalue makes the polynomial $\Psi(\bar{\zeta})$ identically zero. With bounded solutions, the l.h.s. is zero which fixes the instantaneous jump,

$$\hat{C}_0 = \frac{g_1}{\bar{c}} - \frac{g_2}{\bar{\zeta}}. \tag{B.4}$$

Log-linearizing the utility integral in (10) reveals that the welfare effect equals the transform of the change in the consumption flow at rate ρ. The second line in (B.3) gives

$$\frac{dU}{Cu'(C)} = L_\rho[\hat{C}_t] = \frac{1}{\Psi(\rho)} \left\{ \gamma(\rho+\delta)\eta\frac{g_1}{\rho} - (r^S - \rho)\left[\frac{g_2}{\rho} + \hat{C}_0\right] \right\}. \tag{B.5}$$

Substitute now (B.4) for the instantaneous jump in consumption, collect the terms that multiply with g_1, and use $\Psi(\rho) = \rho^2 - r^S\rho + \bar{c}\gamma(\rho+\delta)\eta < 0$. Then

$$L_\rho[\hat{C}_t] = \frac{g_1}{\rho\bar{c}} - (r^S - \rho)\frac{g_2}{\Psi(\rho)}\left[\frac{1}{\rho} - \frac{1}{\bar{\zeta}}\right]. \tag{B.6}$$

Substituting the g-coefficients gives (26) in the text.

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[9]The Laplace transform of X_t at rate $\omega > 0$ is defined as $L_\omega[X_t] = \int_0^\infty X_t e^{-\omega t} dt$. It may be checked that $L_\omega[\dot{X}_t] = \omega L_\omega[X_t] - X_0$. See Judd (1982) for applications in economics.

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Table 1: Investment Multipliers

(1) cap.s.	(2) markup	(3) direct	(4) mult.	(5) total	(6) dist.	(7) ITC
0.250	2.000	1.333	1.500	2.000	0.120	0.500
0.250	1.500	1.333	1.200	1.600	0.060	0.333
0.250	1.250	1.333	1.091	1.455	0.030	0.200
0.250	1.100	1.333	1.034	1.379	0.012	0.091
0.333	2.000	1.500	2.000	3.000	0.120	0.500
0.333	1.500	1.500	1.333	2.000	0.060	0.333
0.333	1.250	1.500	1.143	1.714	0.030	0.200
0.333	1.100	1.500	1.053	1.579	0.012	0.091

Columns (1) capital share α, (2) markup factor β, (3) direct effect, (4) multiplier effect, (5) total effect, (6) distortion $r^S - \rho$, (7) optimal investment tax credit z^*. Other basic parameters: $\rho = .04$, $\delta = .08$, $\sigma^k = 1$.
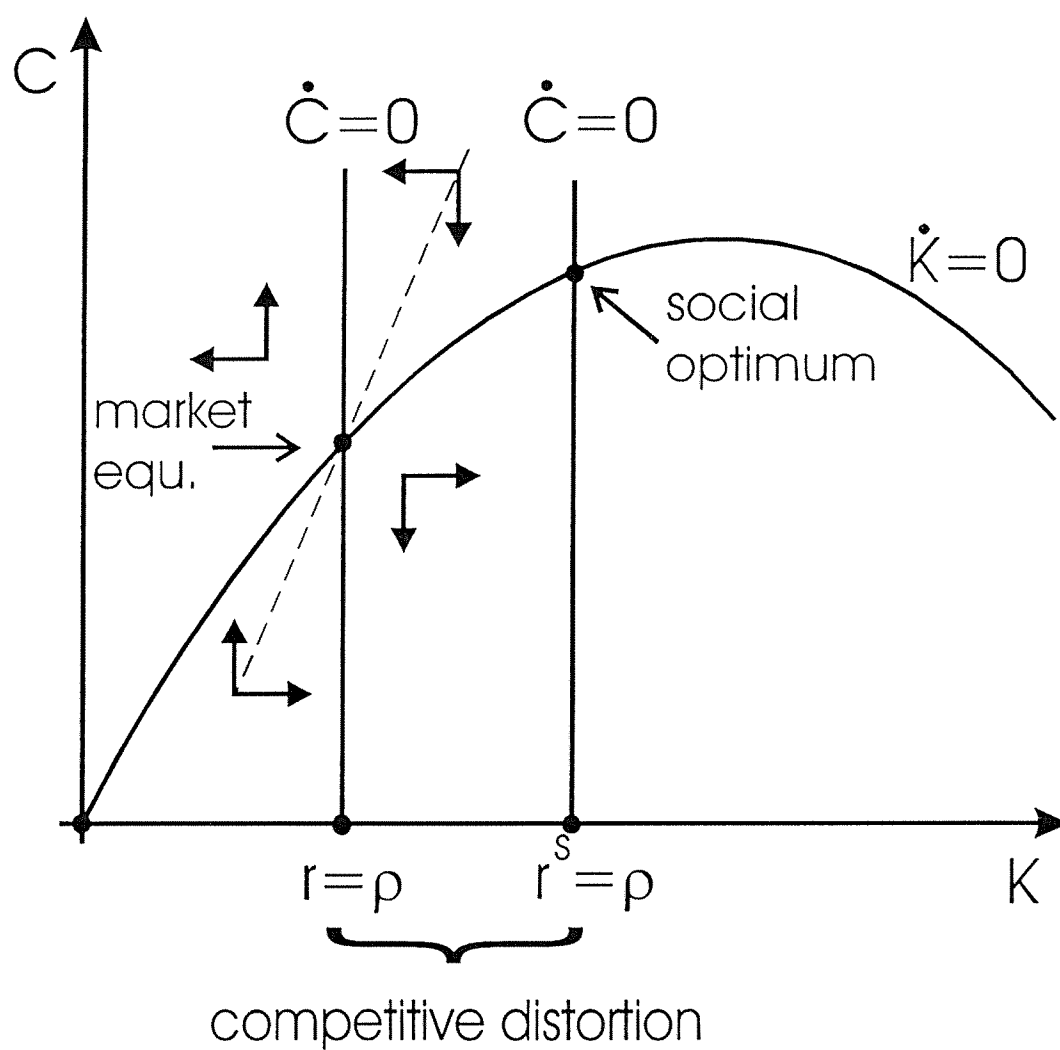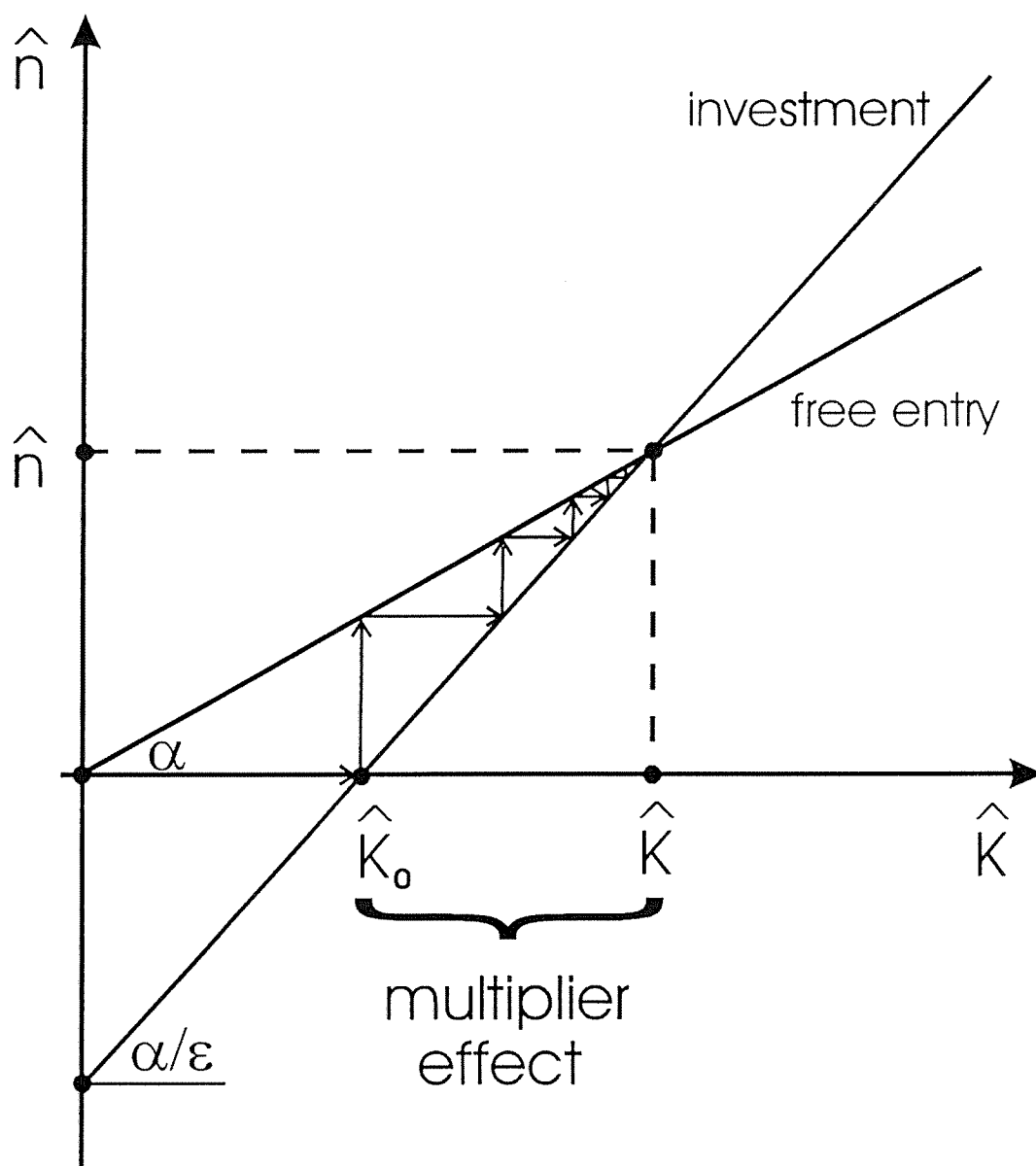
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Fig. 1: Underaccumulation

Fig. 2: Investment Multiplier

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