A STUDY IN INTERREGIONAL STRUCTURAL ANALYSIS

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CONTENTS

I. INTRODUCTION

The following study is intended to discuss several aspects of an interregional structural model in order to find a theoretical base for empirical studies in this field.

For exposition purposes we start in Chapter 2 the discussion of a very complex model based on multiregional input-output approach. The advantage of this procedure lies (a) in the deep insights we can get into both the structure in interregional trade network and the intraregional structure of particular countries and (b) in the facto that we are able to subsume some of the existing models e.g. CMEA [9] etc. as a special case of our model resulting from various aggregation procedures and some additional assumptions. Empirical applications of such a model require very detailed data, which are at present not available. Nevertheless, this model remains our theoretical base for further analysis.

The next chapter is devoted to the analysis of the dual model. Here we show also some results which could be found by sensitivity analysis. The obvious complex level of the model is a major obstacle to use it for practical purposes. Therefore in the next chapter we move towards a more realistic model.

As a special problem we consider finally the influences on export and import determination in the context of our model.

2. A MULTIREGIONAL INPUT-OUTPUT MODEL

Probably the deepest insights into the structure of the interregional trade network one can get by means of a highly disaggregated input-output-type of model for n regions.

Output / Input			Region 1				Region 2				..	Region n			FD	GDP
			Sectors				Sectors					Sectors				
			1	2	...	m	1	2	...	m	..	1	...	m		
Region 1	Sectors	1	x^{11}_{11}	x^{11}_{12}	...	x^{11}_{1m}	x^{12}_{11}	x^{12}_{12}	...	x^{12}_{1m}	..	x^{1n}_{11}	...	.	Y^1_1	Q^1_1
		2	x^{11}_{21}	.	...	.	.	.	...	.	..	.	...	.	.	.
		:	:			:	:			:	::	:		:	:	:
		m	x^{11}_{m1}	.	...	x^{11}_{mm}	.	.	...	x^{12}_{mm}	..	.	...	.	Y^1_m	Q^1_m
Region 2	Sectors	1	x^{21}_{11}	x^{21}_{12}	...	.	.	.	...	.	..	.	...	.	Y^2_1	Q^2_1
		2	.	.	...	.	.	.	...	.	..	.	...	.	.	.
		:	:			:	:			:	::	:		:	:	:
		m	.	.	...	x^{21}_{mm}	.	.	...	.	..	.	...	.	Y^2_m	Q^2_m
.		.	.	.	...	.	.	.	...	.	..	.	...	.	.	.
.		.	.	.	...	.	.	.	...	.	..	.	...	.	.	.
Region n	Sectors	1	.	.	...	.	.	.	...	.	..	.	...	.	.	.
		:	:			:	:			:	::	:		:	:	:
		m	.	.	...	.	.	.	...	.	..	.	...	.	.	.

Where x_{kl}^{ij} represents the flow of the <u>quantity</u> x_{kl}^{ij} of commodity k from region i to industry l in region j, [1]
and let Q_l^j be the gross domestic product of l-th industry in region j, Y_l^{ij} represents the final demand of region j for commodity l produced in region i.

Under the assumption of fixed technology coefficients and constant returns to scale [2] we construct coefficients

$$a_{kl}^{ij} = \frac{x_{kl}^{ij}}{Q_l^j} \qquad\qquad a_{kl}^{ij} \geqq 0$$

indicating the quantity of commodity k of region i which is needed in order to produce one unit of gross product of commodity l in region j.

A formal description of the model can be given as follows:

(1) $AQ + Y = Q$

where

$$A = \begin{bmatrix} a_{11}^{11} & \cdots & a_{11}^{1j} & \cdots & a_{1m}^{1n} \\ \vdots & & \vdots & & \vdots \\ a_{k1}^{i1} & \cdots & a_{kl}^{ij} & \cdots & a_{km}^{in} \\ \vdots & & \vdots & & \vdots \\ a_{m1}^{n1} & \cdots & a_{ml}^{nj} & \cdots & a_{mm}^{nn} \end{bmatrix} = \begin{bmatrix} A^{11} & A^{12} & & \cdots & A^{1n} \\ \vdots & \vdots & & & \vdots \\ A^{i1} & A^{ii} & A^{ij} & \cdots & A^{in} \\ \vdots & \vdots & & & \vdots \\ A^{n1} & A^{n2} & & \cdots & A^{nn} \end{bmatrix}$$

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1) We assume here that each industry produces only one good (i.e. no joint production).

2) I.e. no externalities.

$$Q = \begin{bmatrix} Q^1 \\ Q^2 \\ \vdots \\ Q^n \end{bmatrix} \qquad Q^i = \begin{bmatrix} Q^i_1 \\ Q^i_2 \\ \vdots \\ Q^i_m \end{bmatrix}$$

$$Y = \begin{bmatrix} Y^{11} + Y^{12} + \dots + Y^{1n} \\ Y^{21} + Y^{22} + \dots + Y^{2n} \\ \vdots \qquad \vdots \qquad\qquad \vdots \\ Y^{n1} + Y^{n1} + \dots + Y^{nn} \end{bmatrix} \qquad E = \begin{bmatrix} 1 & \dots & 0 & \dots & 0 \\ 0 & \dots & 1 & \dots & 0 \\ 0 & \dots & 0 & \dots & 1 \end{bmatrix}$$

One should note that A^{ii} for all i is the usual intraregional input-output model for region i.

In order to make this model completely clear we construct the following simplified example where the number of regions n = 2 and the number of industries in both regions is also m = 2.

We have then the following relationships:

$$A = \left[\begin{array}{cc|cc} a^{11}_{11} & a^{11}_{12} & a^{12}_{11} & a^{12}_{12} \\ a^{11}_{21} & a^{11}_{22} & a^{12}_{21} & a^{12}_{22} \\ \hline a^{21}_{11} & a^{21}_{12} & a^{22}_{11} & a^{22}_{12} \\ a^{21}_{21} & a^{21}_{22} & a^{22}_{21} & a^{22}_{22} \end{array}\right] = \left[\begin{array}{c|c} A^{11} & A^{12} \\ \hline A^{21} & A^{22} \end{array}\right]$$

$$Q = \begin{bmatrix} Q_1^1 \\ Q_2^1 \\ Q_1^2 \\ Q_2^2 \end{bmatrix} = \begin{bmatrix} Q^1 \\ \\ Q^2 \end{bmatrix} \qquad Y = \begin{bmatrix} Y_1^{11} + Y_1^{12} \\ Y_2^{11} + Y_2^{12} \\ Y_1^{21} + Y_1^{22} \\ Y_2^{21} + Y_2^{22} \end{bmatrix} = \begin{bmatrix} Y^{11} + Y^{12} \\ \\ Y^{21} + Y^{22} \end{bmatrix}$$

$$\begin{bmatrix} A^{11} & A^{12} \\ A^{21} & A^{22} \end{bmatrix} \begin{bmatrix} Q^1 \\ Q^2 \end{bmatrix} + \begin{bmatrix} Y^{11} \\ Y^{21} \end{bmatrix} + \begin{bmatrix} Y^{12} \\ Y^{22} \end{bmatrix} = \begin{bmatrix} Q^1 \\ Q^2 \end{bmatrix}$$

$$A^{11}Q^1 + A^{12}Q^2 + Y^{11} + Y^{12} = Q^1$$
$$A^{21}Q^1 + A^{22}Q^2 + Y^{21} + Y^{22} = Q^2$$

$$AQ + Y = Q$$

$$(E-A)Q = Y$$

$$Q = [E-A]^{-1}Y$$

If we have e.g. forecasts of final demand, we are able to predict all interregional trade flows which are necessary to satisfy the estimated final demand. The matrix $[E-A]^{-1}$ gives us all direct and indirect requirements for the several commodities of the different regions.

It can be shown that a change in final demand for only one commodity in one region makes changes of GDP of all industries in all regions necessary.[1)]

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1) We do not want to go into deeper detail, see e.g. [13] etc.

The model described above is constructed under following assumptions:

(a) No technological substitution possibilities:
There exists only one technique to produce a certain commodity in each region. I.e. there is no choice between several techniques.

(b) No commodity substitution possibilities:
Furthermore technology assumptions are such that the same commodity inputs of different regions are needed in fixed proportions in order to produce a commodity in a certain region.

(c) The model excludes also joint production and externalities, which might sometimes be very important in reality.

There are no data available to compute such a big matrix because few countries have such detailed data.[1)]
Even if we are able to construct such a matrix, it will be unmanageable for political purposes. In order to be the basis for political decisions we can look at a more aggregate structure.

1) If we have only intraregional input-output and trade share data available we are able to construct the matrix under the assumption that

$$a_{ij}^{lk} = a_{ij}^{kk} = a_{ij}^{k}$$

see Chenery & Clark [6].

3. THE DUAL MULTIREGIONAL-MULTISECTOR MODEL

We consider now the dual problem of our model. We use additional notation:

p_i^k price of commodity i in region k.

v_i^k value added per unit (i.e. wages, profits, etc. per unit) in sector i of region k.

Since we also want to discuss effects of tax-rate changes we include the following:

$d_i^{k,l}$ tax rate, imposed by region k on quantity sold of commodity i to region l, independent into which sector they go;
if $d_i^{kl} > 1$ we think of a tax,

if $d_i^{kl} < 1$ we conceive it as a subsidy;

where $k, l = 1, \ldots, n$
$i = 1, \ldots, m.$

Using d_i^{kl} we define the following price system

$$p_j^l = \sum_{k=1}^{n} \sum_{i=1}^{m} p_i^k \, d_i^{kl} \, a_{ij}^{kl} + v_j^l \qquad \text{with } j = 1, \ldots, m; \quad k, l = 1, \ldots, n$$

or

$$(1) \quad p = \tilde{A}p + v$$

where

$$p = \begin{bmatrix} p^1 \\ \vdots \\ p^k \\ \vdots \\ p^n \end{bmatrix} \quad \text{and } p^k = \begin{bmatrix} p_1^k \\ \vdots \\ p_i^k \\ \vdots \\ p_m^k \end{bmatrix}, \quad v = \begin{bmatrix} v^1 \\ \vdots \\ v^k \\ \vdots \\ v^n \end{bmatrix} \quad v^k = \begin{bmatrix} v_1^k \\ \vdots \\ v_i^k \\ \vdots \\ v_m^k \end{bmatrix}$$

and

$$\tilde{A} = \begin{bmatrix} A^{11} & A^{21}\hat{D}^{21} & \dots & A^{n1}\hat{D}^{n1} \\ A^{12}\hat{D}^{12} & A^{22} & \dots & A^{n2}\hat{D}^{n2} \\ \vdots & \vdots & & \vdots \\ A^{1n}\hat{D}^{1n} & A^{2n}\hat{D}^{2n} & \dots & A^{nn} \end{bmatrix}$$

with

$$\hat{D}^{kl} = \begin{bmatrix} d_1^{kl} & & & & \\ & \cdot & & & \\ & & d_i^{kl} & & \\ & & & \cdot & \\ & & & & d_m^{kl} \end{bmatrix} \qquad \begin{matrix} k,l = 1,\dots,n \\ k \neq l \end{matrix}$$

Note that if k=l we can neglect the $\hat{D}$ matrix because we are assuming that the tax or subsidy-structure within one region is already included in the coefficients.

(2) $$v = (E-\tilde{A})p$$

and if det $E-\tilde{A} \neq 0$

(3) $$p = (E-\tilde{A})^{-1}v$$

With the help of these formulated models we are able to analyse the following problems:

1. Influences of a change in value added;
2. Influecences of technological changes - and
3. Effects of changes in the tax rate on prices.

Without loss of generality we consider for simplicity of the presentation two sectors in our further discussion. One should note that all results can easily be generalized.

(1) Effects of changes in value added.

We assume that v is given exogenously. In this case we have three possibilities:

a) Suppose a single element of the v^k vector of a certain region changes.

For example

$$\Delta v' = (1, 0, 0, 0)$$

Using equation (3) the change in prices is determined by the elements of the first column of the matrix $(E-\tilde{A})^{-1}$. One can prove that the main diagonal elements of the $(E-\tilde{A})^{-1}$ matrix are greater or equal one and all others are smaller than one.[1)] Formulated as a theorem: The above change in v leads to a change of all prices in the same direction which is more than proportional for price p_1^1 and less than proportional for all other prices.

b) Now we consider a change in the value added vector for one region.

For example

$$\Delta v = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

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1) We know $(E-\tilde{A})^{-1} = E+\tilde{A}+\tilde{A}^2+\ldots$. This series is convergent iff $\tilde{A}$ is productive (see Gale [8]), i.e. that the sum of entries in all rows or columns is smaller than one. Productivity of $\tilde{A}$ ensures also nonnegativity of $(E-\tilde{A})^{-1}$. See also Metzler's Theorem in Morishima [14].

In this case we conclude that the prices in region 1 will certainly increase over-proportional, since

$$p_1^1 = \sum_{j=1}^{2} b_{ij} > 1 \qquad \text{because } b_{ii} > 1 \ (i=1,2).$$

The prices in region 2 will also increase, but effects which are higher than in region one cannot be excluded.

c) Suppose all elements of the vector v are changed by the same amount.

Theorem:

If all elements of the v vector are changed by the same amount λ, then price changes are determined by the sum of the rows of the matrix $(E-\tilde{A})^{-1}$.

Proof :

From (3) follows $p = (E-\tilde{A})^{-1} \Delta v$.

$$\Delta v = \lambda . e \qquad \text{where } e = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \qquad \text{and } \lambda \text{ is a scalar.}$$

Then $\Delta p = \lambda(E-\tilde{A})^{-1} e$.

Denote:

$$(E-\tilde{A})^{-1} = \begin{bmatrix} r_{11} & \cdots & r_{1m} \\ r_{21} & \cdots & r_{2m} \\ \vdots & & \vdots \\ r_{m1} & \cdots & r_{mm} \end{bmatrix}$$

therefore

$$\Delta p = \lambda \begin{bmatrix} \sum_{j=1}^{m} r_{1j} \\ \sum_{j=1}^{m} r_{2j} \\ \vdots \\ \sum_{j=1}^{m} r_{mj} \end{bmatrix} \qquad \text{Q.E.D.}$$

Since we know that the sum of each row is greater than one we conclude that the change of the price vector goes into the same direction and with an increase higher than λ. Clearly, if all components of value added are changed proportionally, the prices will also do.

(2) Changes in the matrix A:

First we try to analyse the effect of a change [1] in one input coevvicient in a certain industry of one region on the prices of all industries in all regions. Then we are able to prove the following

Theorem:

If one entry of the input matrix A changes, the whole price vector will change also in the same direction and the price of that industry in that region for which the change occurs will change at the greatest rate.[2]

Proof:

We follow a method developed by B.W. Finkelstein[3]. The idea of this approach is the following:

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1) One can think e.g. a technical change.

2) The same theorem with different proof can be found in Morishima [15].

3) From Kossow and Dadajan [12].

(a) Construct a "difference matrix" B

(4) $B = A_1 - A_o$

where A_1 is the new matrix after the change and A_o is the original input coefficient matrix.

(b) Since we are interested only in the matrix $(E-A_1)^{-1}$; the problem is to find its relationship to the original $(E-A_o)^{-1}$ matrix which is known to us. Therefore we have to find a matrix Γ such that

(5) $(E-A_1)^{-1} = \Gamma(E-A_o)^{-1}$

using (4) we can write (5) as

$$(E-A_o-B)^{-1} = \Gamma(E-A_o)^{-1}$$

and after multiplication with $(E-A_o-B)$ we get

$$E = \Gamma(E-A_o)^{-1}(E-A_o-B)$$

and $E = \Gamma(E-A_o)^{-1} - \Gamma(E-A_o)^{-1}A_o - \Gamma(E-A_o)^{-1}B$

$$E = \Gamma(E-A_o)^{-1}(E-A_o) - \Gamma(E-A_o)^{-1}.B$$

$$E = \Gamma E-(E-A_o)^{-1}.B.$$

Therefore $\Gamma = \left[E-(E-A_o)^{-1}B\right]^{-1}$ (6)

(c) Now denote $(E-A_o)^{-1}.B = L,$ (7)

we get $\Gamma = \left[E-L\right]^{-1}$ (8)

According to the theorem only one element changes, say a_{11}^{12} (with no loss of generality). Then B looks as follows:

$$B = \begin{pmatrix} 0 & 0 & b & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Case I: b > 0[1)]

Denote $(E-A_o)^{-1} = R = \begin{pmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{pmatrix}$

Using (7) we find

$$L = RB = \begin{pmatrix} 0 & 0 & br_{11} & 0 \\ 0 & 0 & br_{21} & 0 \\ 0 & 0 & br_{31} & 0 \\ 0 & 0 & br_{41} & 0 \end{pmatrix}$$

We see that L has positive elements (if A_o is indecomposable) only in this column in which the change occurs.

Now if L is productive and non-negative we expand the C. Neumann series.

$$\Gamma = E + L + L^2 + L^3 + \dots + L^s + \dots$$

where $\Gamma \geqq 0$.

In our case $L^s = b^s \begin{pmatrix} 0 & 0 & r_{11}r_{31}^{s-1} & 0 \\ 0 & 0 & r_{21}r_{31}^{s-1} & 0 \\ 0 & 0 & r_{31}r_{31}^{s-1} & 0 \\ 0 & 0 & r_{41}r_{31}^{s-1} & 0 \end{pmatrix} =$

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1) This case is economically rather strange. We refer to it only for the sake of completeness.

$$= b^s r_{31}^{s-1} \begin{pmatrix} 0 & 0 & r_{11} & 0 \\ 0 & 0 & r_{21} & 0 \\ 0 & 0 & r_{31} & 0 \\ 0 & 0 & r_{41} & 0 \end{pmatrix}$$

$$\text{Then } \sum_{s=1}^{\infty} L^s = \begin{pmatrix} 0 & 0 & r_{11} & 0 \\ 0 & 0 & r_{21} & 0 \\ 0 & 0 & r_{31} & 0 \\ 0 & 0 & r_{41} & 0 \end{pmatrix} \cdot \sum_{s=1}^{\infty} b^s r_{31}^{s-1} = \Gamma - E \qquad (9)$$

Now since we can write

$$p_1 - p_o = v'(E-A_1)^{-1} - v'(E-A_o)^{-1} = v'\Gamma(E-A_o)^{-1} - v'(E-A_o)^{-1} =$$

$$= v'(\Gamma-E)(E-A_o)^{-1}$$

From (9) we see that Γ-E has positive elements only in the column in which the change in A occurs. Denote

$$(\Gamma-E) = G = \begin{pmatrix} 0 & 0 & g_{13} & 0 \\ 0 & 0 & g_{23} & 0 \\ 0 & 0 & g_{33} & 0 \\ 0 & 0 & g_{43} & 0 \end{pmatrix}$$

then $v'G = (0,0, \sum_{i=1}^{4} v_k g_{i3}, 0)$

if we denote $\sum_{i=1}^{4} v_i g_{i3} = h_3$, which is positive.

$$\text{Then } p_1 - p_o = v'.G.R = (0,0,h_3,0) \begin{pmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{pmatrix} =$$

$$= h_3(r_{31}, r_{32}, r_{33}, r_{34}) .$$

Since we know that $r_{33} > 1$ and $r_{3i} < 1$ for $i = 1,2,4$ we see that the price of sector one in region two increases at the highest rate.

Case II: $b < 0$

This means that $A_o \geqq A_1$. Now $A_o \geqq A_1$ implies $(E-A_o)^{-1} - (E-A_1)^{-1}$ by application of C. Neumann series.

We now show that $(\Gamma-E) < 0$. Suppose that contrary $(\Gamma-E) \geqq 0$, then by multiplying both sides by $(E-A_o)^{-1} > 0$ (since A_o is indecomposable) we get

$$\Gamma(E-A_o) \geqq (E-A_o)^{-1} \quad \text{and}$$

$(E-A_1)^{-1} \geqq (E-A_o)^{-1}$ which contradicts that

$(E-A_1)^{-1} \leqq (E-A_o)^{-1} \Rightarrow \quad (\Gamma-E) = G < 0.$

We know that $\lim_{s\to\infty} L^s = 0$ although the series is now alternating in sign. 1)

Now $p_1-p_o = v'GR = g_3(r_{31}, r_{32}, r_{33}, r_{34})$

Now since $v' > 0$ and $G < 0$ therefore h_3 is negative, and since $r_{3j} > 0 \quad j=1,\ldots,4$ we conclude that $(p_1-p_o) < 0$ and since $r_{33} > 1$ the decrease of the third component of the price vector is the largest. Q.E.D.

(3) Effects of changes in tax rates

Without loss of generality suppose there is a change in the tax rate imposed on commodity 1 by region 2 for sales to region 1.

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1) Since $|L^s|$ is convergent.

The change in d_1^{21} leads to the following difference matrix

$$\tilde{B} = \tilde{A}_1 - \tilde{A}_o = \begin{pmatrix} 0 & 0 & b & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

We use the same approach as above in order to show the resulting effects on prices.

Construct again

$$\Gamma = (E-\tilde{L})^{-1} \quad \text{with } \tilde{L} = \tilde{R}.\tilde{B} \text{ and}$$
$$\tilde{R} = (E-\tilde{A}_o)^{-1}$$

Using the Neumann series:

$$\Gamma = (E-\tilde{L})^{-1} = E + \tilde{L} + \tilde{L}^2 + \ldots + \tilde{L}^s + \ldots$$

$$\text{where } \tilde{L}^s = (r_{31} + r_{32})^{s-1} \begin{bmatrix} 0 & 0 & (r_{11}+r_{12}) & 0 \\ 0 & 0 & (r_{21}+r_{22}) & 0 \\ 0 & 0 & (r_{31}+r_{32}) & 0 \\ 0 & 0 & (r_{41}+r_{42}) & 0 \end{bmatrix}$$

$$\sum_{s=1}^{\infty} L^s = \Gamma - E = \sum_{s=1}^{\infty} b^s (r_{31}+r_{32})^{s-1} \begin{bmatrix} 0 & 0 & r_{11}+r_{12} & 0 \\ 0 & 0 & r_{21}+r_{22} & 0 \\ 0 & 0 & r_{31}+r_{32} & 0 \\ 0 & 0 & r_{41}+r_{41} & 0 \end{bmatrix}$$

Now $\quad p_1 - p_o = v'\underbrace{(\Gamma-E)}_{G}\underbrace{(E-\tilde{A}_o)^{-1}}_{\tilde{R}}$

$$p_1 - p_o = v'G\tilde{R}$$

Therefore

$$p_1 - p_o = v'G.\tilde{R} = [0,0,h_3,0] \begin{bmatrix} \tilde{r}_{11} & & & \\ & \cdot & & \\ & & \cdot & \\ & & & \tilde{r}_{44} \end{bmatrix} =$$

$$= h_3 [\tilde{r}_{31}, \tilde{r}_{32}, \tilde{r}_{33}, \tilde{r}_{34}]$$

where

$$v'G = \left[0, 0, \sum_{i=1}^{4} v_i g_{i3}, 0\right]$$

with g_{i3} is an element of the G matrix

and $\sum_{i=1}^{4} v_i g_{i3} = h_3$.

The further analysis follows the same argument as above. We arrive then at the same result as before.
Interpreting this we have to note, that there is no influence of prices on demand for goods and also one has to be aware that the increase of a tax rate is conceived as a rise in the price of the selling region and therefore has its main impact on especially this price via the interrelatedness of the assumed economy.

4. TOWARDS A MORE REALISTIC MODEL

In order to make the model manageable for practical purposes, we need a higher degree of flexibility. Especially the assumptions (a) and (b) (mentioned in chapter 2) are too strong to be realistic. Therefore we have to weaken both and also to reduce the high level of complexity to a more manageable degree. We want to make in this chapter some general suggestions how to tackle this problems.

1. The necessity to weaken assumption (b) lies in the fact that several regions are competing on a common market with homogeneous goods. Therefore we should allow for substitution possibilities for homogeneous commodities produced in different regions, rather than to assume that the same commodity will be imported in fixed proportions from different regions. Now if we accept the fixed proportions assumption within each region we change the coefficients to coefficients a_{ij}^{l} i.e. in order to produce one unit of j in region l we need the quantity a_{ij}^{l} of commodity i. This quantity of commodity i can be delivered from inside the regional economy or from one or more regions outside. So we can write

$$a_{ij}^{l} = \sum_{k=1}^{m} a_{ij}^{kl}$$

The remaining question is to determine how much of this unit requirement should be bought from outside and from which regions a possible explanation can be given by means of a "gravity model".

E.G. $a_{ij}^{kl} = f(P_i, D^{kl}, PP^{kl}, T^{kl}, N^{kl}, \ldots)$

where

P_i	is a vector of prices of commodities i in all regions k;
D^{kl}	is the geographical distance between regions k and l,
PP^{kl}	stands for political preference between k and l;
T^{kl}	for tariff pressure in k and l;
N^{kl}	is an indicator of production capacity in region k,l.

In order to make such or a similar model operational, we have to specify (a) the functional form, because it is a priori not clear whether the function is e.g. multiplicative or additive or of some other form. (b) The specification and measurement of some variables may cause difficulties. Especially an important variable like political preference may create a lot of troubles.

2. Since there are not all data available to construct a multiregional input-output model we have to reduce it to a manageable degree. Therefore we have to handle a much more aggregated model.[1)]

In principle we find data which are already aggregated in four possible direction:

(a) Aggregation of inputs;[2)]
(b) Aggregation of o puts;
(c) Aggregation of importing regions into groups;
(d) Aggregation of exporting regions into groups.

Which level of aggregation seems to be appropriate depends mainly on the problems we want to solve. Also we have to note that we are now not any longer confronted with "quantities" but "values" since for aggregation procedures we need a common standard. In most of the countries prices are used as

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1) For these difficult problems of aggregation see e.g. Green [1o].

2) This kind of aggregation has been done in the next chapter in which the inputs in region j are lumped toghether irrespectively in which industry they go.

the common standard. Therefore we have not any longer a purely production oriented system but rather a value structure. One main problem is to separate the influences of quantity changes and price changes. We do not discuss this problem further in this paper.

3. Since we cannot expect constancy of a certain input-output structure over time expecially because we do not allow a choice of technique in our model and also because of influence of technical change, we have to overcome these regidities for forecasting purposes. Various procedures have been developed on this point.[1)]

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1) We want to mention just a few procedures:

(a) A well-known procedure is for example the RAS method. This method is described e.g. by M. Bacharack [1]. (For other methods see e.g. Carter-Bródy [3], [4], [5].)

(b) It can also be handled by means of some kind of a gravitational model to estimate the matrix coefficients from behavioral equations which takes into account demand and supply factors. (See [7]).

(c) A third method can be found in "LINK-procedures" (see [11]) which lies in the construction of a regression model with time trend and price effects to bet some kind of adaptation.

5. INTERREGIONAL TRADE FLOW ANALYSIS

Starting from our model discussed in chapter 2 we consider now the interregional connectedness of the whole economy with special regard to influences on export and import determination. We list the following additional symbols:

X_k^{ij} exports of sector k of region i to region j[1];
note that

$$\sum_k X_k^{ij} = X^{ij} = \text{total exports of region } i \text{ to region } j$$
$$= M^{ij} = \text{total imports of region } j \text{ from region } i$$
$$\sum_j X^{ij} = X^i = \text{total exports of region } i$$

$\bar{Y}^i$ final domestic demand of region i (i.e. final demand of region i less exports of region i)

W^i value added of region i

$\;^i$ input flow matrix of region i

A^i input coefficient matrix of region i

Consider the following structure where there are n regions and m sectors.

X^1	$X_1^{12} \cdot \cdot \cdot X_m^{12}$	...	$X_1^{1n} \cdot \cdot \cdot X_m^{14}$	$\bar{Y}^1$	Q^1
$X_1^{21} \cdot \cdot \cdot X_m^{21}$	X^2	...	$X_1^{2n} \cdot \cdot \cdot X_m^{2n}$	$\bar{Y}^2$	Q^2
$\vdots$	$\vdots$	$\vdots\vdots\vdots$	$\vdots$	$\vdots$	$\vdots$
$X_1^{41} \cdot \cdot \cdot X_1^{41}$	$X_1^{42} \cdot \cdot \cdot X_m^{42}$	...	X^n	$\bar{Y}^n$	Q^n
W^1	W^2	...	W^n		
Q^1	Q^2	...	Q^n		

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1) See footnote 2) on page 19.

In $\mathbb{X}^i$ one cannot distinguish between domestic and imported inputs.

Additionally to the input coefficients assume now the following:

(1) $m_k^{ij} = \frac{X_k^{ij}}{Q_k^j}$ $\quad i \neq j,\ k=1,\dots,m$
$\quad i,j = 1,\dots,n$

i.e. the proportion of imports of region j from sector k of region i to the gross product of sector k of region j.
Therefore in matrix notation (^ denotes a diagonal matrix)

(2) $\hat{X}^{ij} = \hat{m}^{ij}.\hat{Q}^j$

The equation system can be written as follows:

(3) $AQ + \bar{Y} = Q$ $\quad$ or $\bar{Y} = Q(E-A)$

where

$$A = \begin{bmatrix} A^1 & \hat{m}^{12} & \hat{m}^{13} & \dots & \hat{m}^{1n} \\ \hat{m}^{21} & A^2 & \hat{m}^{23} & \dots & \hat{m}^{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ \hat{m}^{n1} & \hat{m}^{n2} & \hat{m}^{n3} & \dots & A^n \end{bmatrix} \qquad \bar{Y} = \begin{bmatrix} \bar{Y}^1 \\ \bar{Y}^2 \\ \vdots \\ \bar{Y}^n \end{bmatrix} \qquad Q = \begin{bmatrix} Q^1 \\ Q^2 \\ \vdots \\ Q^n \end{bmatrix}$$

Therefore if $|E-A| \neq 0$ and if there exists an economic meaningful solution (i.e. the Hawkins-Simon-condition holds) we can write

$Q = (E-A)^{-1}.\bar{Y}$

Denote $(E-A)^{-1} = B$ $\begin{bmatrix} B^{11} & B^{12} & \dots & B^{1n} \\ \vdots & \vdots & & \vdots \\ B^{n1} & B^{n2} & \dots & B^{nn} \end{bmatrix}$

(4) $Q = B.\bar{Y}$

From this equation we can easily derive exports:

Since

$$Q^2 = B^{ii}\bar{Y}^i + \sum_{\substack{j \\ i \neq j}}^{n} B^{ij}\bar{Y}^j \qquad i = 1,\ldots,n$$

the exports of region i are given by

$$(5) \quad X^i = \sum_{\substack{j \\ i \neq j}}^{n} B^{ij}\bar{Y}^i \qquad i = 1,\ldots,n$$

showing that the exports of region i are now a function of the domestic final demand of all other region. Denote by B^* the matrix B where all main diagonal submatrices are replaced by Zero matrices

$$\text{i.e.:} \quad B^* = \begin{bmatrix} 0 & B^{12} & \ldots & B^{1n} \\ B^{21} & 0 & \ldots & B^{2n} \\ \vdots & \vdots & & \vdots \\ B^{n1} & B^{n2} & \ldots & 0 \end{bmatrix}$$

then equation (5) can be written

$$(6) \quad X = B^*\bar{Y} .$$

The elements of matrix B^* now indicate by what extent the exports of sector k of region i will change as a consequence of a unit change in the final domestic demand for sector l in region j, $(j \neq i)$. $(k,l = 1,\ldots,m)$.

Imports are derived as follows:

From definition

$$M^j = \sum_{i}^{n} M^{ij} = \sum_{i}^{n} \sum_{k}^{m} X_k^{ij} = \sum_{i}^{n} \sum_{k}^{m} Q_k^j \, m_k^{ij} \qquad (j=1,\ldots,n)$$

Using the matrices defined above we get

$$M^j = Q^j \sum_i^n \hat{m}^{ij} = Q^j \hat{m}^j \qquad \text{where } \hat{m}^j = \sum_i^n \hat{m}^{ij} \qquad (j=1,\ldots,n)$$

Now we can write

(7) $M = Q.\hat{m}$

where $m =)M^1, M^2, \ldots, M^n)$

$$Q = (Q^1, Q^2, \ldots, Q^n)$$

$$\hat{m} = \begin{pmatrix} \hat{m}^1 & & 0 \\ & \hat{m}^2 & \\ & & \ddots \\ 0 & & \hat{m}^n \end{pmatrix}$$

Since $Q = B.\bar{Y}$ or $Q = \bar{Y}.B^T$ with Q and $\bar{Y}$ row vectors and B^T the transposed B matrix, we get

(8) $M = \bar{Y}.B^T.\hat{m} = \bar{Y}.R$ where $R = B^T.\hat{m}$

We call the elements of R the total import-coefficients. We see that imports are a function of domestic demand of all regions and the elements of R show the effect of a unit change in domestic demand for sector k in region i on imports of sector l (k,l=1,...,m) of region j.

Finally we want to mention the relationship of this model to the usual trade share matrix approach.
The trade share matrix T is usually defined:

$T = \{t^{ij}\}$ where

(9) $t^{ij} = \dfrac{X^{ij}}{M^j} \qquad i,j = 1,\ldots,n$

Therefore, using (1) aggregated we have

$$X^{ij} = m^{ij}Q^{j} = t^{ij}M^{j}$$

$$m^{ij} = t^{ij}\left(\frac{M^{j}}{Q^{j}}\right) = t^{ij}.ß^{j} \qquad \text{denoting } \left(\frac{M^{j}}{Q^{j}}\right) \text{ by } ß^{j}$$

representing the import shares

and in general

$$T = \hat{m}^{ij}.\hat{ß}^{-1} .$$

Now for practical purposes we are confronted with some problems, already discussed in general in chapter 2. Because in this model demestic demand is given exogenously we need to specify behavioral equations for forecasting purposes. Similarly since we cannot hope that our matrix A remains constant over time, we need some kind of adaption in order to get better forecasts, e.g. RAS or modified methods which are mentioned in chapter 4.

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