NONLINEAR DYNAMICS OF SPOT AND FORWARD EXCHANGE RATES:

An Application of a Seminonparametric Estimation Procedure

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Abstract

This paper applies the seminonparametric nonlinear impulse response analysis proposed by Gallant, Rossi and Tauchen to weekly spot and forward Swiss franc/US dollar exchange rate. Five empirical regularities are found: (i) symmetric mean and volatility reaction pattern of the spot rate to pure spot shocks; (ii) symmetric mean, but asymmetric volatility responses of the forward rate to pure forward premium shocks; (iii) weak feedback from the forward to the spot rates; (iv) a forward premium shock triggers off a four-week cyclical impulse response, which is transmitted to a similar response cycle of the spot rate; (v) the volatility responses are neither monotone nor highly persistent as reported by numerous applications of ARCH models. Our finding offers a strong empirical support to the exchange rate model suggested recently by McCallum in which the monetary policy authorities systematically manage interest rate differentials so as to resist changes in exchange rates but also to smooth interest-rate movements.

KEYWORDS: Seminonparametric procedure, nonlinear impulse response, exchange rate dynamics.

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1. Introduction

It is a well documented fact that high frequency exchange rate changes have almost no autocorrelation, but strongly nonlinear dependencies are characteristic of short-term exchange dynamics. This finding is based on applications of the nonparametric BDS test to exchange rate data [for instance Hsieh (1989), Papell/Sayers (1990), Kugler/Lenz (1993)] as well as the estimation of specific non-linear models to exchange rate data. In particular, numerous ARCH and GARCH applications, surveyed by Bollerslev, Chou and Kroner (1992), account for the apparent dependence of conditional variances. In addition, there exist some studies detecting non-linear dependence in the conditional means of exchange rate changes as for instance, the application of non-parametric regressions methods by Diebold and Nason (1990) or the estimation of Self Exciting Threshold Autoregressive (SETAR) models by Kräger and Kugler (1993). These specific non-linear models are, however, not entirely satisfactory as some residual non-linear dependence and leptokurtic standardized residuals are detected. Moreover, parameter stability in terms of GARCH and SETAR models seems to be a problem [Kräger/Kugler (1993)] and the work of Diebold and Nason (1990) indicates that accounting for non-linear dependence in the conditional mean does not much improve ex ante forecasting over the random walk performance.

This state of affairs calls for modelling short run exchange rate dynamics in a more general non-linear framework. In fact, there are two recent publications of this kind. First we have to mention the paper by Gallant, Hsieh and Tauchen (1990) which applies the semi-nonparametric technique of Gallant and Tauchen (1989) to the British pound/US dollar exchange rate to estimate the univariate conditional density. Second, Canova (1993) successfully uses a Bayesian time varying coefficient model for weekly exchange rate data for prediction purposes. Canova´s specification encompasses many widely applied non-linear dynamic models.

Besides all these apparant evidence of non-linear dependence, there is another widely recognized fact about the foreign exchange market, namely the rejection of the unbiasedness hypothesis for the forward rate as predictor of the spot rate[1]. This finding is often attributed to a highly variable risk premium (Fama, 1984) or to systematic forecasting error (Froot and Thaler, 1990). However, a most recent paper by McCallum (1994) succesfully explains the rejection of the unbiasedness hypothesis with monthly data by interest rate and exchange rate

We wish to thank George Tauchen and Harald Zhang for their help and sharing their program. We are specially grateful to George Tauchen who through e-mail correspondence gave us extensive support. All remaining shortcomings are ours.

1) Compare Bekaert/Hodrick (1993) for a recent and very careful study and references of earlier work.

smoothing by central bank in a rational expectations framework. The corresponding model has some interesting implications, which are useful for the interpretation of bivariate time series analysis of the spot and forward exchange rate.

The aim of this paper is to extend by applying the seminonparametric non-linear dynamic modelling approach developed by Gallant, Rossi and Tauchen (1993) in a bivariate framework to the spot and forward exchange rate. We consider weekly spot and forward data for the Swiss franc/US dollar over the period 1977-91, where the forward contract period is one week, too. By this approach we provide a non-linear impulse response analysis of first and second order conditional moments. The remaining part of this paper is organized as follows: Section 2 contains the results and interpretion of a test of the unbiasedness hypothesis for the forward rate taking into account the conditional heteroscedasticity of exchange rate data. Section 3 provides a brief introduction to the SNP approach and its empirical results. Our conclusions are presented in section 4.

2. The Unbiasedness of the Forward Exchange Rates: A Preliminary Exploration

In this study we use a weekly data set for spot and forward rates of the Swiss france against the US dollar with one week maturity. Union Bank of Switzerland kindly provided a corresponding data set ranging from the second week of July 1977 until the third week of November 1991[2]. The data are bid rates of Wednesday taken between 9:15 and 9:30 a.m. Zurich time. Wednesday was selected in order to minimize the number of bank holidays and to avoid the "weekend effect"[3].

Before turning to the application of the SNP approach to this data set, we present some results of a standard analysis of our data. Our framework of analysis is a test of the unbiasedness hypothesis for the forward rate (f_t) as a predictor of the future spot rate (s_{t+1}) which takes into account the nonstationarity and conditional heteroscedasticity of exchange rate data. The basic equation is

$$s_{t+1} = \alpha + \beta f_t + \varepsilon_{t+1}$$

and the hypothesis of interest is $H_0 : \beta = 1$. In order to account for the non-stationarity of s_t we subtract s_t from both sides of the basic equation and get

$$\Delta s_{t+1} = \alpha + \beta (f_t - s_t) + \varepsilon_{t+1}$$

under H_0. Further, we assume that ε_{t+1} follows a normal GARCH (1,1) model. The estimation of this equation with our 750 weekly observations brought the following results (asymptotic t values in parantheses):

$$\Delta s_{t+1} = \underset{(-1.37)}{-0.23} - \underset{(-1.60)}{1.75}(f_t - s_t) + \hat{\varepsilon}_{t+1},$$

$$\varepsilon_{t+1} \sim N(0, h_{t+1}),$$

$$h_{t+1} = \underset{(3.56)}{0.79} + \underset{(5.24)}{0.16}\varepsilon_t^2 + \underset{(19.0)}{0.77} h_t,$$

2) More information on this data set are given by Lampietti (1993), who constructed it.

3) If Wednesday was a holiday next day figures are used, but the forward rate was corrected by day to day foreign exchange swap data in order to set a forward contract maturing the following Wednesday.

$$Q_{\varepsilon}(12) = 17.6 \qquad Q_{\varepsilon\varepsilon}(12) = 3.46 \qquad BDS(m=2) = -1.43$$
$$K = 3.46 \qquad S = -0.12$$

Q_{ε} and $Q_{\varepsilon\varepsilon}$ are the Ljung-Box statistic for the standardized residuals and residuals squared, respectively. BDS is the BDS test statistic with embeding dimension 2 and a distance of one standard deviation, whereas K and S are the residual coefficient of kurtosis and skewness, respectively.

The results correspond to the well known characteristic of foreign exchange rate data. First, the change of the spot rate is nearly nonautocorrelated, but clearly dependent. Second, the forward rate is indicated to be a biased predictor of the spot rate: The estimate of β has the wrong negative sign and the hyothesis $\beta = 1$ has to be rejected at usual significance levels. Third, a highly persistent GARCH effect is found: The sum of the two GARCH parameter estimates is 0.93. Fourth, the GARCH model may, according to the BDS test, let some residual dependence[4)] and the standardized residuals clearly exhibit excess-kurtosis.

The most disturbing aspect of the results reported above is the rejection of the unbiasedness hypothesis. Two recent studies attempt to explain this frequent finding of seemly forward foreign exchange market inefficiency by retaining the rational expectations hypothesis without relying on a highly variable risk premium. The paper of Bekaert and Hodrick (1993) shows that the rejection of the unbiasedness hypothesis cannot be attributed to measurement problems, omitted variables or regime shifts. However, the more recent paper of McCallum (1994) presents a very intersting argument concerning the effects of interest smoothing and reactions of the central bank to spot rate changes. At least to us this explanation is most promising for understanding the dynamic interrelationship of spot and forward rates. As the structural model introduced by McCallum may be useful for the interpretation of some of our results, we present it briefly. The interest rate differential, which is equal to the term premium under covered interest rate parity, is denoted by $fp_t = f_t - s_t$. The first equation assumes that the domestic central bank resists depreciation and smoothes interest rate movements:

$$fp_t = \lambda(s_t - s_{t-1}) + \gamma fp_{t-1} + \zeta_t,$$

where $\lambda > 0$, $0 < \gamma < 1$ and ζ_t is a white noise error term representing interest rate differential shocks. This equation is combined with uncovered interest rate parity

$$s_{t+1}^e - s_t = fp_t - \xi_t,$$

4) The asymptotic standard normal distribution of the BDS statistic is no longer valid if it is applied to GARCH residuals. True critical values are considerably smaller than the normal ones (Brock/Hsie/Le Baron 1991).

where ξ_t represents the risk premium which is assumed to follow an AR(1) process

$$\xi_t = \rho\xi_{t-1} + u_t \quad \text{with } |\rho| < 1.$$

The following solution is obtained under rational expectations:

$$fp_t = \frac{\lambda}{\lambda+\gamma}\xi_t = \rho fp_{t-1} + \frac{\lambda}{\lambda+\gamma}u_t,$$
$$\Delta s_t = \frac{\rho-\gamma}{\lambda} fp_{t-1} - \frac{1}{\lambda}\zeta_t + \frac{1}{\lambda+\gamma-\rho}u_t.$$

This solution has two very interesting implications: First, it may easily explain a negative and sizeable β coefficient in the regression

$$\Delta s_{t+1} = \alpha + \beta(f_t - s_t),$$

with λ sufficiently small and γ close to one as well as a smaller value of ρ. Second it implies that the innovation of the term premium can be interpreted as risk premium shock. Moreover, it means that the forward premium and the spot rate change have a recursive contemporaneous and Granger causality structure, which allows to identify uncorrelated interest rate and risk premium shocks in a VAR or more general non-linear multivariate time series model[5].

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5) This property seems to be valid for higher order AR processes for ξ_t. The solution of the model for the AR(2) case is available from the authors on request.

3. SNP Estimation and Nonlinear Impulse Response

We apply the SNP estimator suggested by Gallant and Nychka (1987) and Gallant and Tauchen (1989, 1992) to estimate directly the bivariate one-step ahead conditional density of spot rate and forward premium. Denote the bivariate series $y_t = (\Delta s_t, fp_t)$, where Δs_t is log differences of the spot rates and fp_t is log of the forward premium. The basic idea is to approximate the conditional density by multiplying a normal density by a polynomial expansion. After the joint density is estimated, the nonlinear impulse response technique developed by Gallant, Rossi and Tauchen (1993) is then applied to exploit the dynamics of these two variables.

3.1 SNP Estimators

The SNP is a semi-nonparametric density estimator based on a Hermite series expansion. To illustrate[6], suppose the multivariate process y_t with dimension M is strictly stationary with its conditional distribution given the entire past depends only on a finit number L of lagged values of y_t. Denote the one-step ahead conditional density of y_t as $f(y_{t+1}|x_t)$, where $x_t = (y'_{t+1-L}, y'_{t+2-L}, \ldots, y'_t)'$, which is a vector of lengh ML. Given the past of y_t, one can determine the conditional density $f(y_{t+1}|x_t)$ by minimizing

$$-\frac{1}{n}\sum_{t=1}^{n} \ln f(y_t|x_{t-1}) - \frac{1}{n}\int f(y,x_0)dy.$$

The second term above is negligible if the sample size n is large, so that the knowledge of the joint densiy $f(y_t, x_{t-1})$ is not required. Whereas $f(y|x)$ is time invariant under the assumption of strict stationarity, the assumption of Markovian structure of $f(y|x)$ allows any sort of conditional heterogeneity of the process y_t. The method proposed approximates $f(y|x)$ by a truncated Hermite series expansion. $f(y|x)$ can be consistently estimated if the number of terms in the expansion is an increasing function of the sample size. The Hermite polynomial has the form

$$f(y|x,\theta) \propto [p(z,x)]^2 \phi(y{:}\mu_x, \Sigma_x),$$

where z is the centered and scaled random variable corresponding to y_t with

$$z = R_x^{-1}(y - \mu_x)$$

6) The rest of this section follows Gallant, Hsie and Tauchen (1989) closely.

and $\phi(y:\mu_x,\Sigma_x)$ denotes the density of the M dimensional Gaussian process. The VAR nature of the leading term of the expansion is specified such that

$$\mu_x = a_0 + Ax_{t-1}^u,$$

where $x_t^u = (y'_{t+1-L_u}, y'_{t+2-L_u}, \ldots, y'_t)'$ denotes a vector of past value of y_t with length $L_u \le L$. a_0 is a M by 1 and A is a M by $M \times L$ coefficient matrices. Further, the ARCH-like characteristic of the leading term is specified in the way such that $\Sigma_x = R_x R_x'$, where

$$vech(R_x) = b_0 + B\left|x_{t-1}^{*,r}\right|$$

with $x_t^{*,r} = (y'_{t+1-L_r}, y'_{t+2-L_r}, \ldots, y'_t)'$ and $L_r \le L$. b_0 is a $(M+1)$ by 1 and B a $(M+1)\times ML$ dimensional coefficient matrices. $x_{t-1}^{*,r}$ denotes the centered and rescaled x_{t-1}^r. The constant of proportionality is $\dfrac{1}{\int [p(z,x)]^2 \phi(s)ds}$.

While the classical ARCH by Engle (1982) has the variance-covariance matrix of y_t depending on squared lagged residuals, the SNP suggested conditional heteroscedasticity is more akin to that of Nelson´s (1990) EGARCH model.

The multivariate Gaussian ARCH density $\phi(y:\mu_x,\Sigma_x)$ is multiplied by a multivariate polynomial $p(z, x^p)$ with degree K_z which has the form

$$p(z,x^p) = \sum_{|\alpha|=0}^{K_z}\left(\sum_{|\beta|=0}^{K_x} a_{\alpha\beta}(x^p)^\beta\right)z^\alpha$$

where $x^p = (y'_{1-L_p}, y'_{2-L_p}, \ldots, y'_0)'$ with length $L_p \le L$, $\alpha = (\alpha_1,\alpha_2,\ldots,\alpha_M)'$ and $\beta = (\beta_1,\beta_2,\ldots,\beta_{ML})'$ are vectors with integer elements, and

$$|\alpha| = \sum_{i=1}^{M}|\alpha_i|, \qquad |\beta| = \sum_{i=1}^{ML}|\beta_i|,$$

$$z^\alpha = \prod_{i=1}^{M}(z_i)^{\alpha_i}, \qquad (x^p)^\beta = \prod_{i=1}^{ML_p}(x_i^p)^{\beta_i}.$$

For example, in our application y_t is the bivariate process of spot rate and forward premium . Suppose $x_t^p = (y'_t)'$, then

$$p(z,x^p) = \left[\sum_{i=0}^{K_z} \sum_{j=0}^{K_z} a_{ij}(x^p) z_1{}^i z_2{}^j \right]$$

with

$$a_{ij}(x) = \sum_{l=1}^{K_x} \sum_{k=1}^{K_x} a_{ijkl} x_1^l x_2^k .$$

where x_1 and x_2 is the first and second element of x^p, respectively. So the coefficients of the polynomial $p(z,x^p)$ is themselves polynomials of x_t^p of degree K_x. The leading term in the approximation for the conditional density $f(y|x)$ resembles the widely used linear ARCH model for financial data. While the shape of the conditional density departuring from a Gaussian ARCH is controlled by K_z, K_x controlls conditional nonlinearities. If both K_z and K_x are equal to zero, y_t is a Gaussian ARCH process. If K_z is positive and K_x is zero, then the polynomial $p(z,x) = P(z)$, i.e. the coefficients are not dependent on the history so that y_t is non-Gaussian ARCH with homogeneous innovations.

SNP Fitting of the Conditional Density

Following the same model expansion strategy as in Tauchen, Zhang and Liu (1993), we use the Hannan-Quinn model selection criterion to pin down the best-fit model. The penalty on the rich parameterizations of Hannan and Quinn criterion is higher than the Akaike information criterion, but lower than the Schwarz criterion. The estimation results are presented in Table 1. The two additional parameters I_z and I_x are the degrees of supression in the interactions of the variable in the polynomial $p(z,x)$. From Table 1, the Hannan-Quinn preferred model is 4914210 with the number of parameters p_θ = 68, which implies a saturation ratio of 22.06 observations per parameter. The model 4914210 has a lag length of 4 in the VAR part (L_u = 4) and a larger lag length of 9 in the ARCH part (L_r = 9), which is a typical characteristic of financial data. Further heterogeneity beyond ARCH is incorporated via L_p = 1 and K_x = 1. A highly non-Gaussian error structure is reflected in the polynomial of degree 4 in z.(K_z = 4). I_z = 2 implies that in the polynomial $p(z,x) = \left[\sum_{i=0}^{4} \sum_{j=0}^{4} \alpha_{ij}(x) z_1{}^i z_2{}^j \right]$, the intersections of orders higher than 2 are supressed, i.e. the quartic interactions, (1,3), (2,2), (3,1) and the cubic interations (2,1) and (1,2) of z_1 and z_2 are excluded in the estimation. Hence, the estimated one-step ahead conditional density of $y_t = (\Delta s_t, fp_t)$ is essentially an ARCH model with a non-normal error density.

To sum up, we find that the most favorite model selected by Hannan-Quinn to capture the dynamics of the weekly Swiss franc/US dollar exchange rate is a non-linear process with heterogeneous innovations. Further, it has a quartic in the error density with coefficients depend linearly on one lag of each variable. And we use the model selected by Hannan-Quinn, i.e. the model 4914210, for the subsequent impulse response analysis.

3.2 Conditional moment profiles

In VAR models, impulse response analysis investigates the effect of small change in the "innovation" of the process on the system. In non-linear models, taking the perturbation of the conditioning arguments in the conditional density as the "shocks", the dynamics of the process with respect to some movement in the system can be studied by means of computing multistep ahead conditional expectations of the first and second moment, i.e. the conditional profiles of the process. As described in Gallant, Rossi and Tauchen (1993), the conditional moment profile of a strict stationary process is the multistep ahead forecast of the conditional moment. Under the assumption that the conditional density of the underlying process depends on at most L lags, the J-step conditional mean profile given initial condition x^0 is

$$\hat{y}_j(x^0) = \mathrm{E}(y_{t+j}|x_t = x^0) = \hat{y}_j^0 ,$$

for $j = 1,\ldots,J$, where x includes the contemporaneous and lagged value of y. If x^0 is changed by

$$x^+ = x^0 + \delta \text{ or}$$

$$x^- = x^0 - \delta$$

for some realistic value δ in the arguments of the conditional density, the J-step conditional mean profile becomes

$$\hat{y}_j(x^+) = \mathrm{E}(y_{t+j}|x_t = x^+) = \hat{y}_j^+$$

for $x^+ = x + \delta$ and

$$\hat{y}_j(x^-) = \mathrm{E}(y_{t+j}|x_t = x^-) = \hat{y}_j^-$$

for $x^- = x - \delta$, $j = 1,\ldots,J$. Accordingly, the positive and negative impulse response of J-step conditional mean are

$$\{\hat{y}_j(x^+) - \hat{y}_j(x^0)\}_{j=1}^{J}$$

and

$$\{\hat{y}_j(x^-) - \hat{y}_j(x^0)\}_{j=1}^{J},$$

respectively. These two terms provide a natural measurement to study the effect of the "shock" δ on the conditional mean of the system.

Analogous to the conditional mean, one can measure the effects of perturbing conditional arguments on the J-step ahead conditional variance. Since

$$\begin{aligned}\hat{V}_j(x^0) &= Var(y_{t+j}|x_t = x^0)\\ &= E\{[y_{t+j} - E(y_{t+j}|x_t = x_0)]\times[y_{t+j} - E(y_{t+j}|x_t = x_0)]' |x_t = x_0\}\end{aligned}$$

for $j = 1,\dots.,J$. Similarly,

$$\hat{V}_j(x^+) = Var(y_{t+j}|x_t = x^+) \text{ and}$$

$$\hat{V}_j(x^-) = Var(y_{t+j}|x_t = x^-),$$

for $j = 1,\dots.,J$, we get the positive and negative impulse responses of perturbations δ on the volatility which are

$$\{\hat{V}_j(x^+) - \hat{V}_j(x^0)\}_{j=1}^{J}$$

and

$$\{\hat{V}_j(x^-) - \hat{V}_j(x^0)\}_{j=1}^{J},$$

respectively.

Bivariate Spot-Forward Dynamics

In the following, we apply this methods described above to investigate the multi-step ahead spot and forward exchange rate dynamics. Being aware of the contemporaneous correlation structure between the two variables [pointed out by Gallant, Rossi and Tauchen (1993)], we investigate the effects of four types of shocks which are designed by inspection of the scatter (Fig. 1) to generate different combinations of some typical and realistic perturbations:

A shock:

$$\delta y_1^{A+} = 1.00\sigma_{\Delta S} \qquad \delta y_2^{A+} = 0.00$$
$$\delta y_1^{A-} = -1.00\sigma_{\Delta S} \qquad \delta y_2^{A-} = 0.00$$

B shock:

$$\delta y_1^{B+} = 0.00 \qquad \delta y_2^{B+} = 1.00\sigma_{fp}$$
$$\delta y_1^{B-} = 0.00 \qquad \delta y_2^{B-} = 1.00\sigma_{fp}$$

C shock:

$$\delta y_1^{C+} = 1.00\sigma_{\Delta S} \qquad \delta y_2^{C+} = 1.00\sigma_{fp}$$
$$\delta y_1^{C-} = -1.00\sigma_{\Delta S} \qquad \delta y_2^{C-} = 1.00\sigma_{fp}$$

D shock:

$$\delta y_1^{D+} = 1.00\sigma_{\Delta S} \qquad \delta y_2^{D+} = -1.00\sigma_{fp}$$
$$\delta y_1^{D-} = -1.00\sigma_{\Delta S} \qquad \delta y_2^{D-} = -1.00\sigma_{fp}$$

where $\sigma_{\Delta S}$ and σ_{fp} are the sample standard deviation of the spot changes and forward premium, repectively.

Hence the conditional arguments in the one-step ahead conditional density are set to be

$$x^+ = (y'_{-L+1}, y'_{-L+2,}, \ldots, y'_0) + (0, 0, \ldots.\delta y^{+'})$$

$$x^- = (y'_{-L+1}, y'_{-L+2,}, \ldots, y'_0) + (0, 0, \ldots.\delta y^{-'})$$

$$x^0 = (y'_{-L+1}, y'_{-L+2,}, \ldots, y'_0).$$

In this design, ***A*** shock reflects a pure spot movement up or down by one standard deviation ($\sigma_{\Delta S}$ = 1.83134), whereas ***B*** shock reflects pure forward premium movement up or down by one standard deviation (σ_{fp} = 0.093242). ***C*** shock combines a spot movement together with a positive forward premium shock, whereas ***D*** shock combines a spot movement with a negative forward premium shock.

We now turn to the results of our impulse response analysis. Figure 4 - 7 contains the results for the pure spot rate shock (type ***A***). The reaction pattern of the conditional mean of the spot rate displayed in Figure 4 is symmetric about the baseline and heavily damped. Moreover, the conditional mean of the forward premium (Figure 5) is hardly affected by the type ***A*** shock. This result is consistent with the model of McCallum outlined in section 2: Our type ***A*** shock corresponds to a interest rate differential shock, which effects only s_t but has no effects on fp_t. The same result is obtained for the conditional variance of the forward premium (Figure 7). For the conditional variance of the spot rate we observe in Figure 7 a longer lasting effect which seems to be the same for a positive and negative disturbances. However, the adjustment to the baseline is clearly different from that implied by the ARCH exchange rate model estimates reported in the literature: There is no monotone reaction for conditional variance and the persistence is not very high in the sense that the conditional variance path reaches the baseline after 20 weeks.

The impulse response of the forward premium to its own shock (type ***B***) displayed in Figure 9 is characterized by a symmetric four week cycle about the base line, which is transmitted to the spot rate (Figure 8). This finding is also implied by McCallum´s model: A shock to the forward premium u_t, which is interpreted as a risk premium shock, has dynamic influence on the spot rate. Figure 10 indicates that the spot rate volatility is hardly affected by the forward premium shock, whereas the latter variable is characterized by an asymmetric volatility response to its own shock: The negative ***B*** shock has a clearly stronger impact on the forward premium volatility than the positive ***B*** shock in Figure 11.

Finally for the combined ***C*** and ***D*** shock in Figures 14, 15, 18 and 19, compared with 6, 7, 10 and 11, it shows that volatility impulse responses are dominated by their own shocks. In addition, the mean impulse responses displayed in Figure 12, 13, 16 and 17 can be approximately obtained by adding the impulse responses of the corresponding ***A*** and ***B*** type shocks. Therefore, the system we analyse behaves almost like a linear one with this respect: The point occurance of the shocks does not seem to influence their impact. Of course, this results suggest the use of a linear model for the conditional mean of spot and forward exchange rates.

3.3 Confidence Bands

Our analysis of the multistep ahead dynamics of the weekly spot and forward rates above uncovers two main interesting characteristics. First, symmetric and non-monotone volatility response of the spot rate to an interest rate shock (***A*** shock) but asymmetric and cyclical volatility response of the forward premium to a risk premium shock (***B*** shock) are revealed. Second, weak feedback from the forward premium to the spot rate suggests that the forward rate Granger-causes the spot rate.

In order to take into account the sampling variation in the estimation of $\hat{f}(y|x)$, we compute the confidence bands for these two preceding key findings. The bands were constructed using the bootstrap procedure described in Gallant, Rossi and Tauchen (1993) by refitting 300 simulated data sets from B4914210 model[7]. If the band include the null profile (a horizontal line through zero in our application), the effect of the impulse is insignificant.

Figure 20 and 21 shows 95% confidence bands about the estimates

$$\{ \hat{V}_{\Delta s,j}(x_A^+) - \hat{V}_{\Delta s,j}(x_A^-) \}_{j=1}^{30}$$

and

$$\{ \hat{V}_{fp,j}(x_B^+) - \hat{V}_{fp,j}(x_B^-) \}_{j=1}^{30}$$

respectively. If the population volatility function is symmetric, then the differences should be insignificant. Figure 20 shows that except for the first week, one can not reject the null hypothesis of symmetric response of the spot volatility to interest rate differential shocks. Strong evidence is also given in Figure 21 with regard to the asymmetric response of the forward volatility to risk premium shocks: The effects to negative shocks is larger than that to positive shocks.

Figure 22 and 23 show 95% confidence bands about the estimates:

$$\{ \hat{fp}_j(x_B^+) - \hat{fp}_j(x^0) \}_{j=1}^{30}$$

and

[7]) The computation proceeds as follows: First, 300 data sets with the same length are generated from the estimated conditional density $\hat{f}(y|x)$ using the original initial conditions. Second, the conditional density is reestimated from each simulated data set. Then the conditional moment profiles are computed from it. A 95% (or 90%) sup-norm confidence band is an ε - band around the profile from $\hat{f}(y|x)$ that is just wide enough to contain 95% (or 90%) of the 300 simulated profiles.

$$\{ \hat{fp}_j(x_B^-) - \hat{fp}_j(x^0) \}_{j=1}^{30}$$

repectively, in which the cyclical response patter of the forward premium to risk premium shocks are supported. 95% confidence bands of the estimates

$$\{ \hat{V}_{fp,j}(x_B^+) - \hat{V}_{fp,j}(x^0) \}_{j=1}^{30}$$

and

$$\{ \hat{V}_{fp,j}(x_B^-) - \hat{V}_{fp,j}(x^0) \}_{j=1}^{30}$$

are given in Figure 24 and 25, respectively. For the impulse response to negative risk premium shock the evidence of statstical significance is stronger than that to positive shock.

Finally, Figure 26 and 27 show the confidence estimates of spot response to effects of risk premium shock relative to baseline, i.e.

$$\{ \Delta\hat{S}_j(x_B^+) - \Delta\hat{S}_j(x^0) \}_{j=1}^{30}$$

and

$$\{ \Delta\hat{S}_j(x_B^-) - \Delta\hat{S}_j(x^0) \}_{j=1}^{30}.$$

The point estimates of deviations relative to baseline exclude or slidely include the null profile in the first few weeks, which indicate statistical significance at the 95% level.

4. Conclusion

The application of the seminonparametric nonlinear impulse response analysis proposed by Gallant, Rossi and Tauchen (1993) to weekly spot and forward Swiss franc/US dollar exchange rate provides the following results. In general we found a symmetric impulse response of the spot rate to positive and negative shocks, both in mean and volatility. The response of the forward rate to a pure forward shock is symmetric in mean but asymmetric volatility: forward volatility reacts stronger to its own negative shocks. Moreover, the combination of both shocks does not seem to have an influence on the dynamic effects of the spot and forward rate. Thus, the nonlinear system analyzed behaves similar to a linear system with these respects. A spot rate shock only affects its own conditional mean for two periods with clearly highest current impact. A forward premium shock triggers off a four week cycle of the forward premium conditional mean, which is transmitted to a similar response cycle of the spot rate. This result is consistent with the model of interest and exchange rate smoothing developed recently by McCallum in order to explain the rejection of the unbiasedness hypothesis of the forward rate. According to this model we interprete the pure spot shock as a interest rate differential shock, whereas a pure forward premium shock mainly reflects a innovation of the risk premium. With respect to the volatility impulse response we found that the conditional variances of the two series are mainly influenced by their own shocks. In addition, the volatility responses are neither monotone nor highly persistent as reported by numerous application of ARCH models to exchange rate data.

5. References

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Table 1. Bivariate SNP Estimation.

L_u	L_r	L_p	K_z	I_z	K_x	I_x	p_θ	s_n	*Hannan-Quinn*
0	0	1	0	0	0	0	5	2.84590	2.85321
1	0	1	0	0	0	0	9	2.63232	2.65503
2	0	1	0	0	0	0	13	2.53970	2.57250
2	3	1	0	0	0	0	19	2.49473	2.54267
2	3	1	4	4	0	0	27	2.26740	2.33553
2	3	1	4	3	0	0	27	2.25971	2.32783
2	3	1	4	2	0	0	28	2.25933	2.32999
3	0	1	0	0	0	0	17	2.50526	2.54816
4	0	1	0	0	0	0	21	2.34231	2.39530
4	9	1	4	2	1	0	68	1.87097	2.04255
4	4	1	0	0	0	0	29	2.28050	2.35367
4	9	1	0	0	0	0	39	2.17402	2.27243
4	9	1	4	4	0	0	47	1.92811	2.04670
4	9	1	4	2	0	0	48	1.92809	2.04921
4	**9**	**1**	**4**	**2**	**1**	**0**	**68**	**1.87097**	**2.04255**
5	0	1	0	0	0	0	25	2.30167	2.36475
5	1	1	0	0	0	0	27	2.27817	2.34630
5	2	1	0	0	0	0	29	2.26781	2.34099
5	3	1	0	0	0	0	31	2.25666	2.33488
5	4	1	0	0	0	0	33	2.23541	2.31868
5	5	1	0	0	0	0	35	2.20543	2.29374
5	5	1	1	0	0	0	37	2.19938	2.29274
5	5	1	1	1	0	0	37	2.19938	2.29274
5	5	1	2	2	0	0	39	2.15364	2.25205
5	5	1	3	3	0	0	41	2.15209	2.25554
5	5	1	2	2	1	0	49	2.12021	2.24385
5	5	1	2	1	1	0	49	2.12018	2.24382
5	5	1	2	0	1	0	52	2.11782	2.24903
5	5	1	2	2	2	0	64	2.08021	2.24170
5	5	1	2	1	2	0	64	2.08021	2.24170
5	5	1	2	0	2	0	70	2.07943	2.25606
5	5	2	2	0	1	0	64	2.10153	2.26302
5	5	2	2	0	2	0	124	2.02169	2.33457
5	5	1	3	2	1	0	55	2.08309	2.22187
5	5	1	3	2	1	0	55	2.08349	2.22227
5	5	1	3	1	1	0	58	2.08145	2.22780
5	5	1	3	2	1	0	55	2.08310	2.22187
5	5	1	2	0	1	0	52	2.11782	2.24903
5	6	1	0	0	0	0	37	2.19874	2.29210
5	7	1	0	0	0	0	39	2.19200	2.29040
5	8	1	0	0	0	0	41	2.18466	2.28811
5	9	1	0	0	0	0	43	2.14598	2.25448
5	9	1	1	1	0	0	45	2.14198	2.25552
5	9	1	2	2	0	0	47	2.11086	2.22945
5	9	1	3	3	0	0	49	2.10602	2.22966
5	9	1	4	4	0	0	51	1.92596	2.05464
5	9	1	4	3	0	0	51	1.92596	2.05464
5	9	1	4	2	0	0	52	1.92597	2.05718
5	9	1	4	1	0	0	54	1.92584	2.06210
5	9	1	4	2	1	0	72	1.86797	2.04964
5	9	1	4	1	1	0	78	1.86482	2.06163
5	9	1	4	0	0	0	57	1.92123	2.06505
5	9	1	4	0	1	0	87	1.85709	2.07661
5	9	1	4	0	2	0	132	1.80355	2.13662
5	9	1	4	0	2	0	132	1.80356	2.13663
6	0	1	0	0	0	0	29	2.29588	2.36905
7	0	1	0	0	0	0	33	2.29072	2.37398

Figure 1. Scatter plot of spot increase and forward premium.

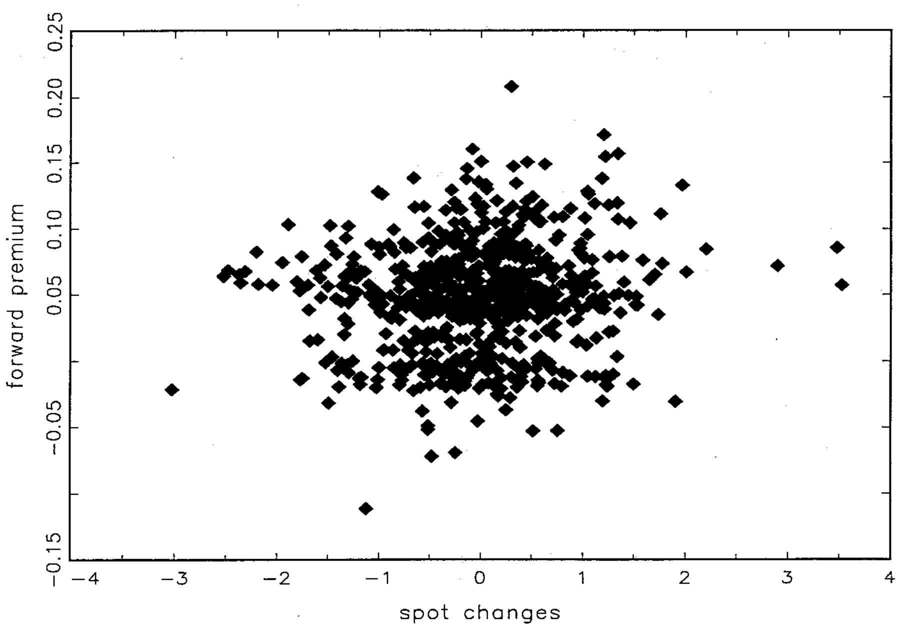


Figure 2. Bivariate conditional density, 4914210 fit.

(all lags are set at the unconditional mean of the data)

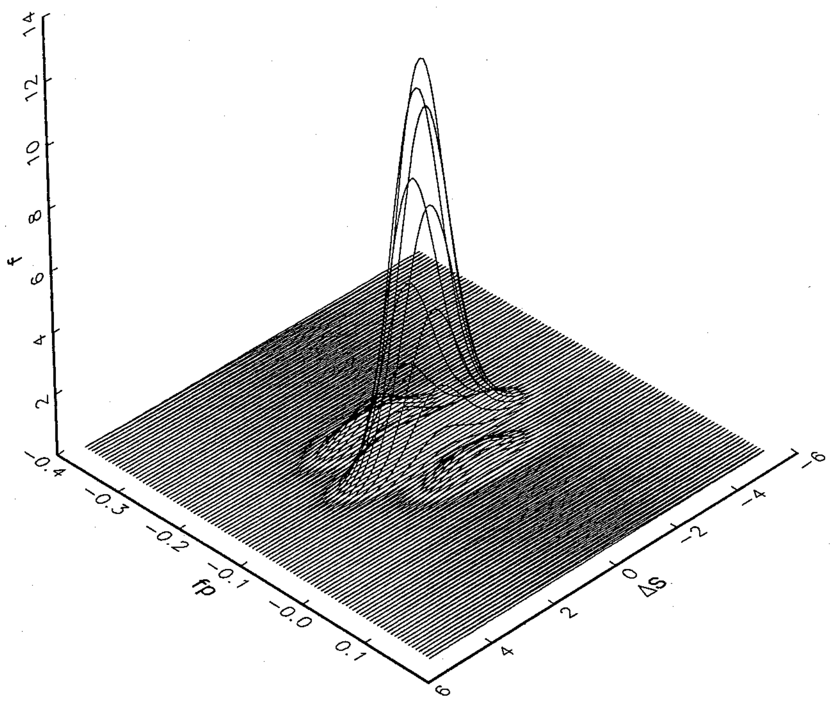
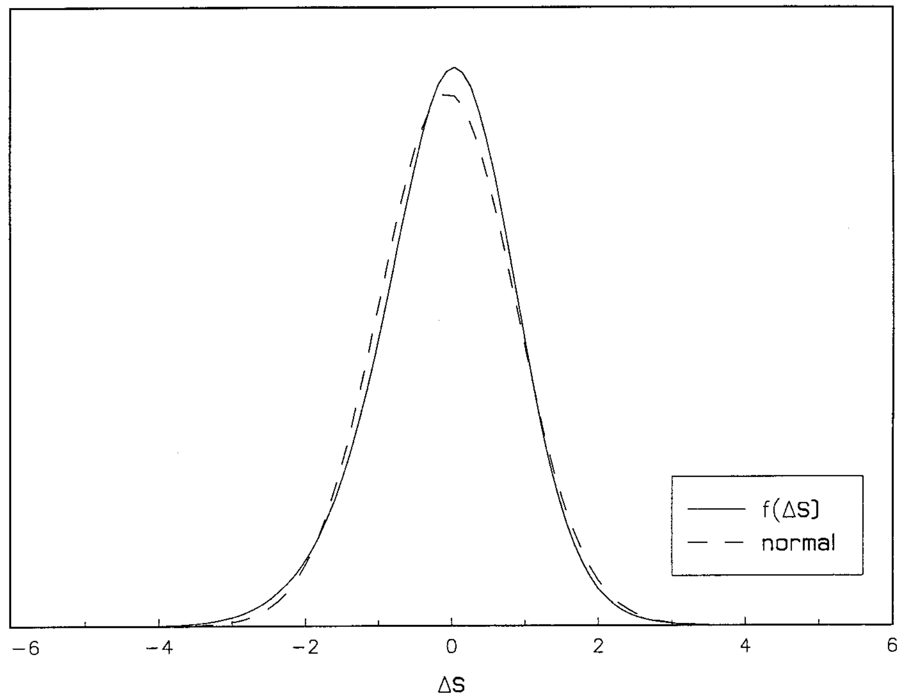
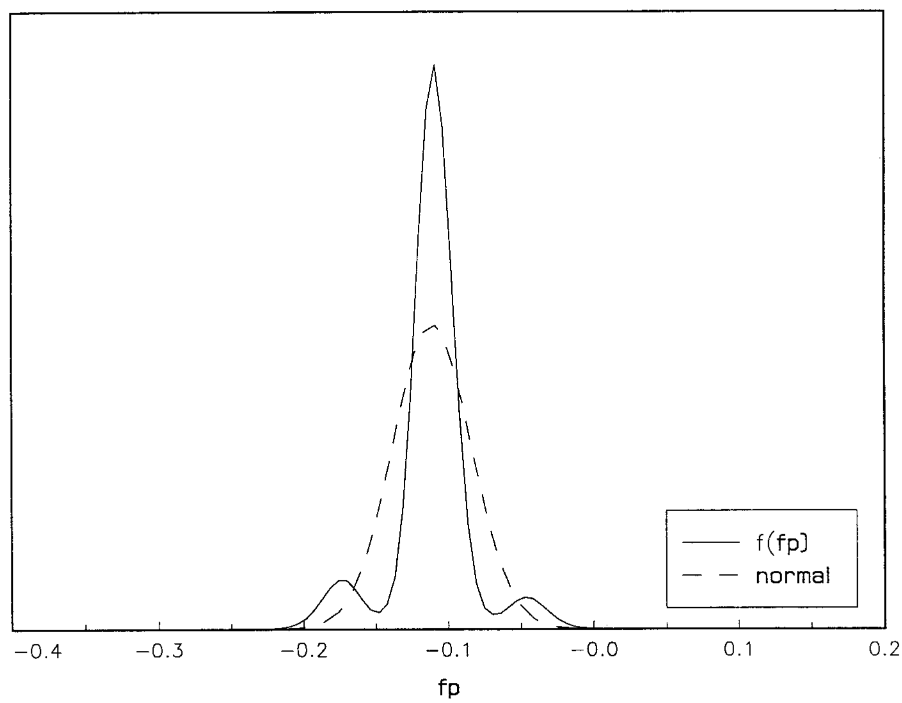


Figure 3. Marginal densities, 4914210 fit.



(a) Marginal density of ΔS and normal with the same mean and variance.



(b) Marignal density of *fp* and normal with the same mean and variance.

Figure 4. Impulse response of expected spot increase to ***A*** type shock

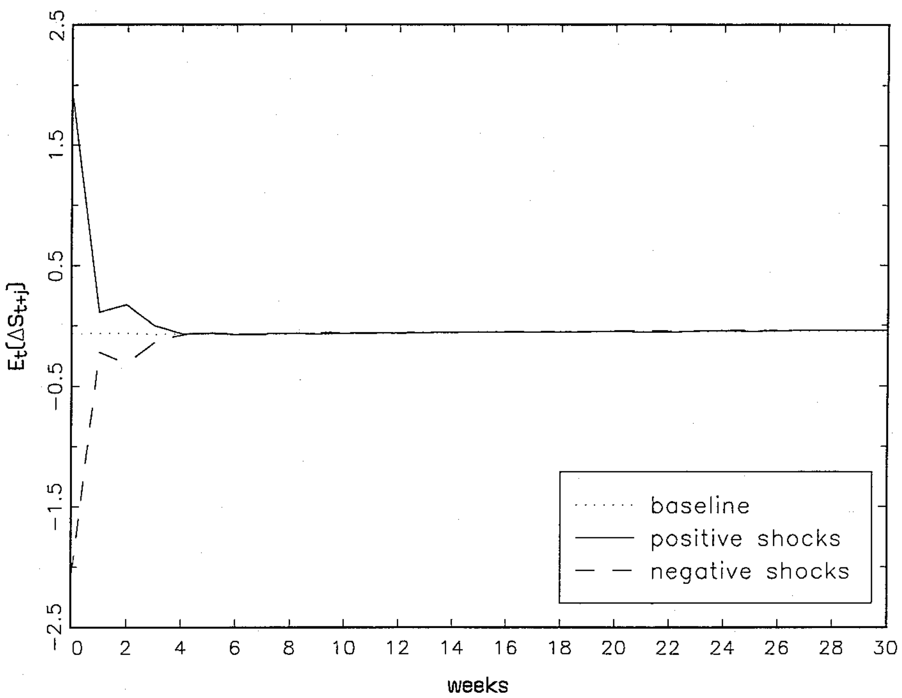


Figure 5. Impulse response of expected forward premium to ***A*** type shock.

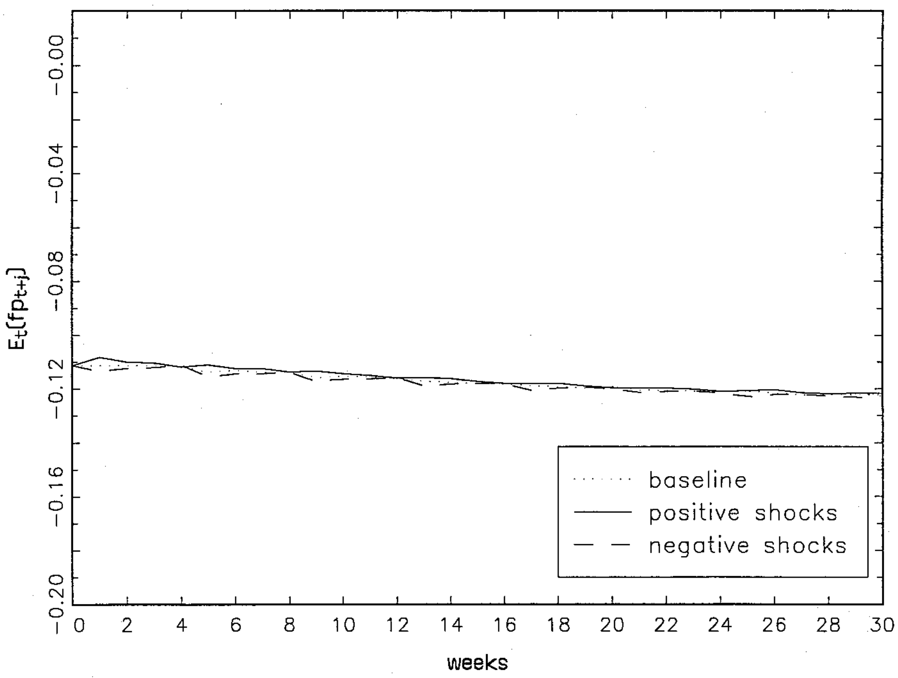


Figure 6. Impulse response of spot volatility to *A* type shock.

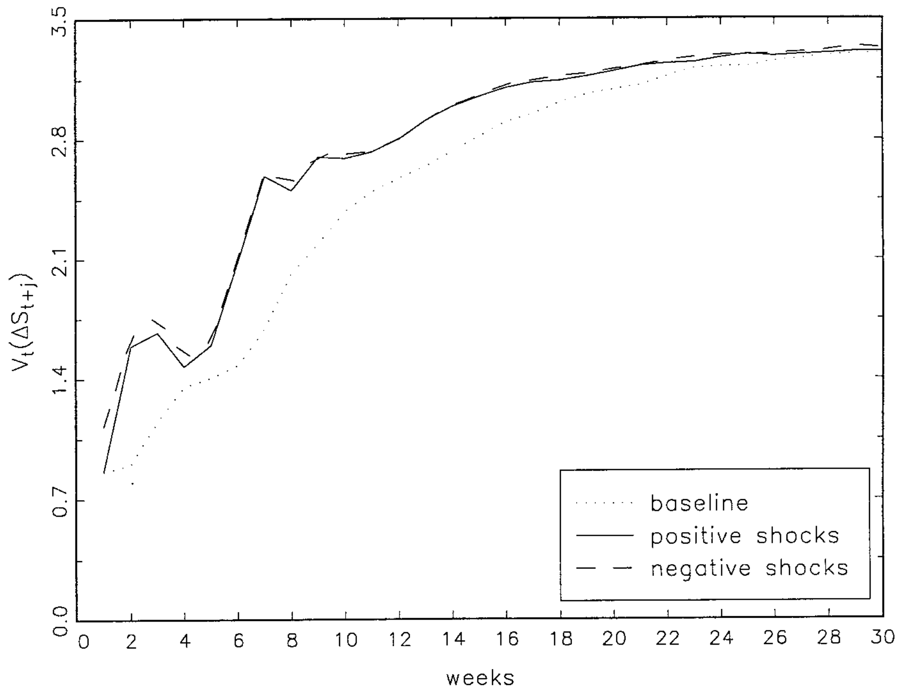


Figure 7. Impulse response of forward premium volatility to *A* type shock

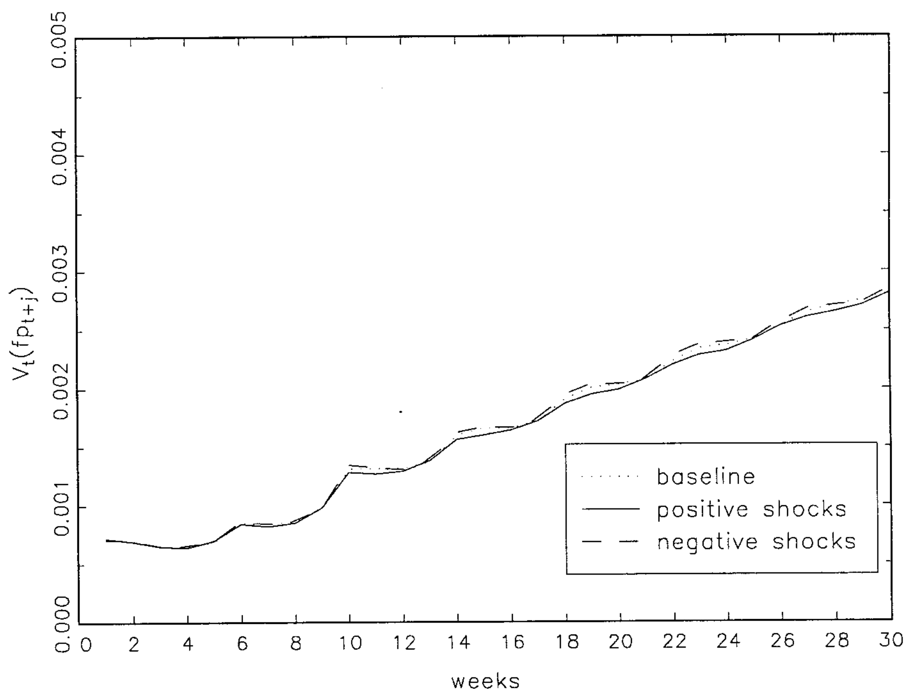


Figure 8. Impulse response of expected spot increase to ***B*** type shock.

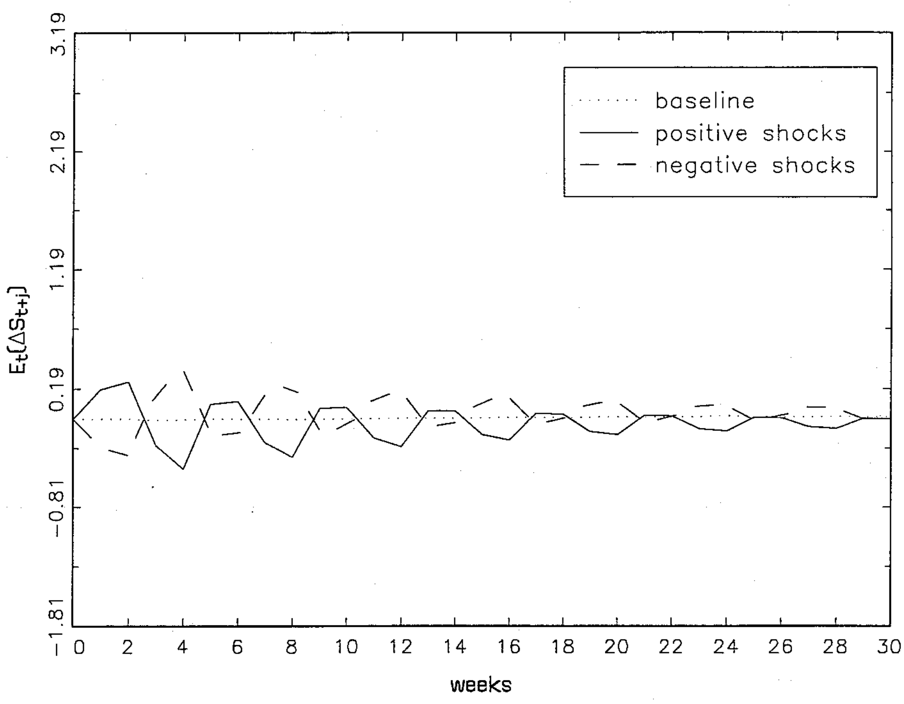


Figure 9. Impulse response of expected forward premium to ***B*** type shock.

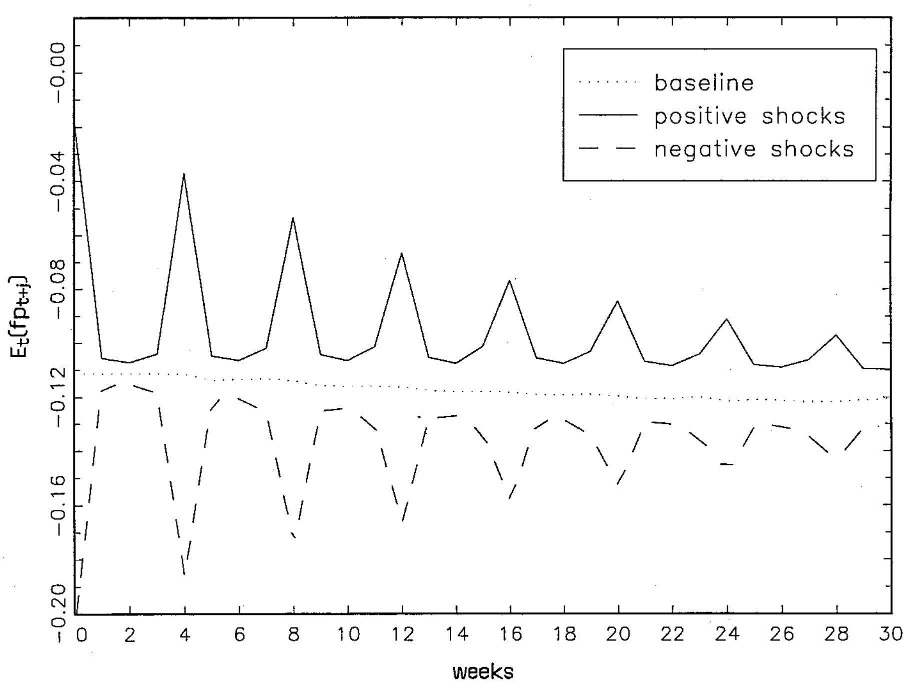


Figure 10. Impulse response of spot volatility to ***B*** type shock.

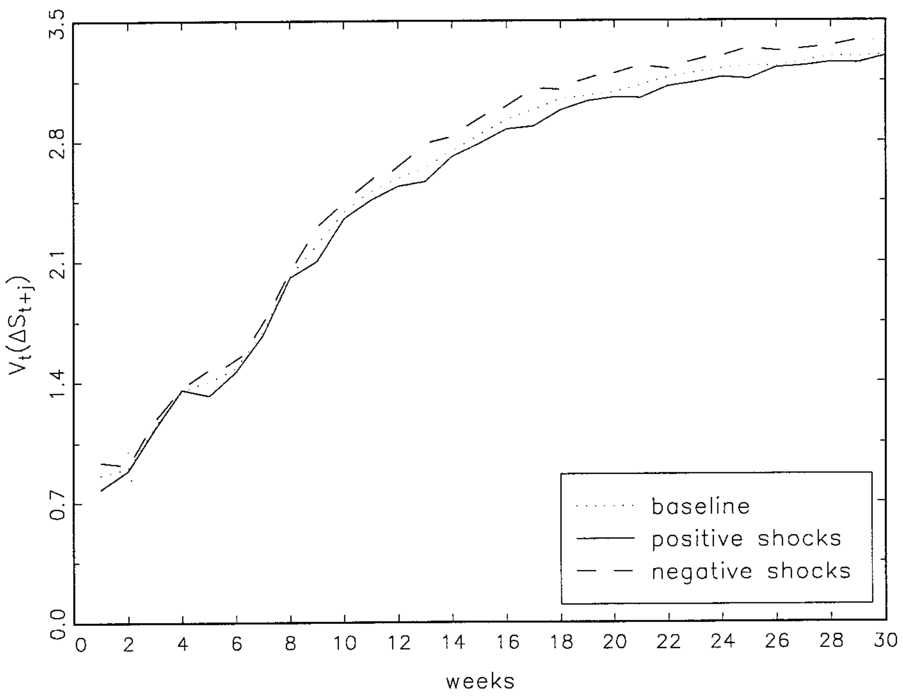


Figure 11. Impulse response of forward premium volatility to ***B*** type shock.

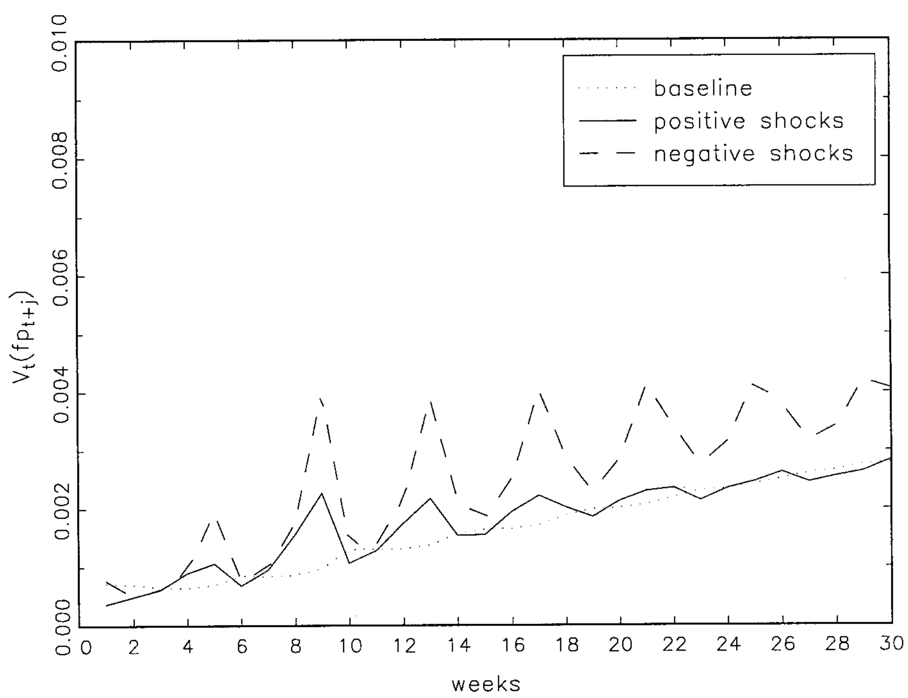


Figure 12. Impulse response of expected spot increase to ***C*** type shock.

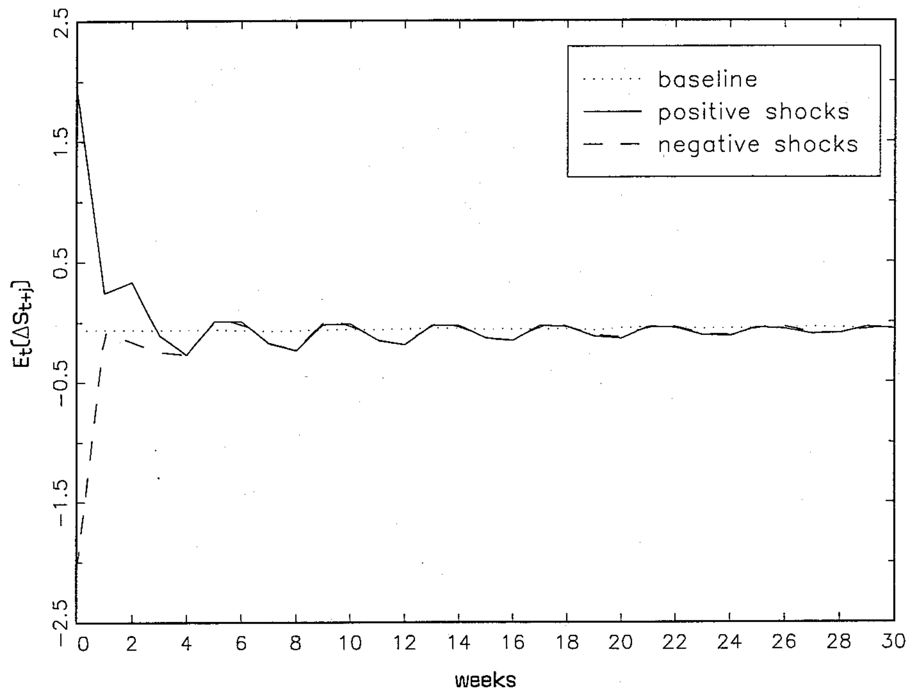


Figure 13. Impulse response of expected forward premium to ***C*** type shock.

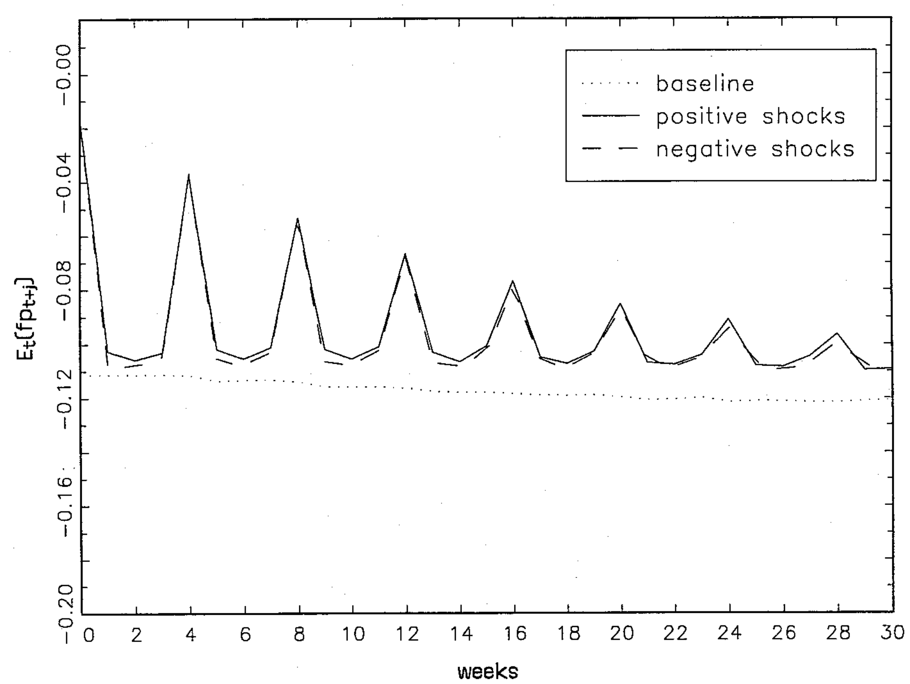


Figure 14. Impulse response of spot volatility to ***C*** type shock.

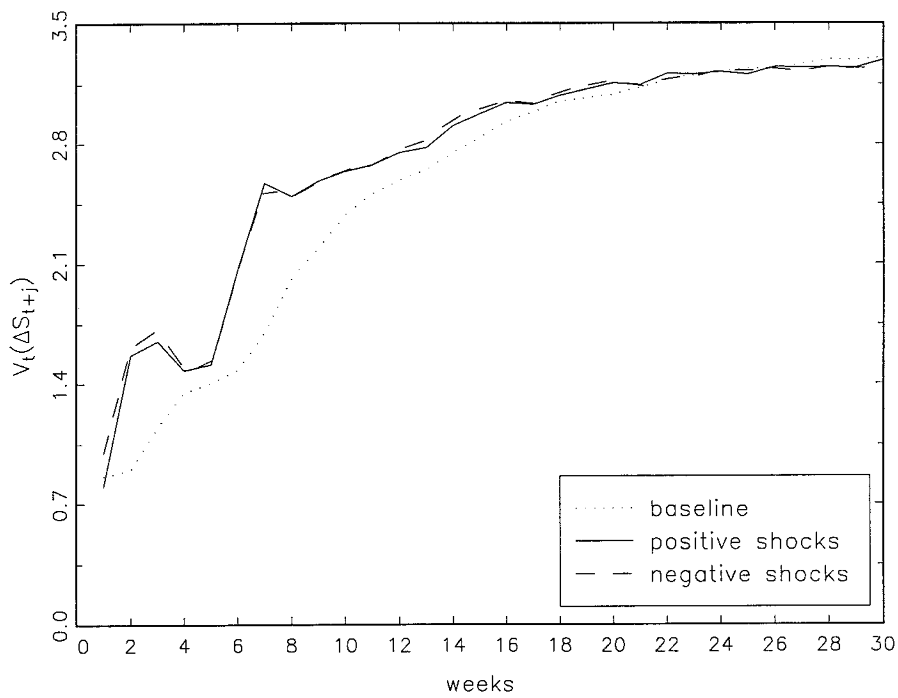


Figure 15. Impulse response of forward premium volatility to ***C*** type shock.

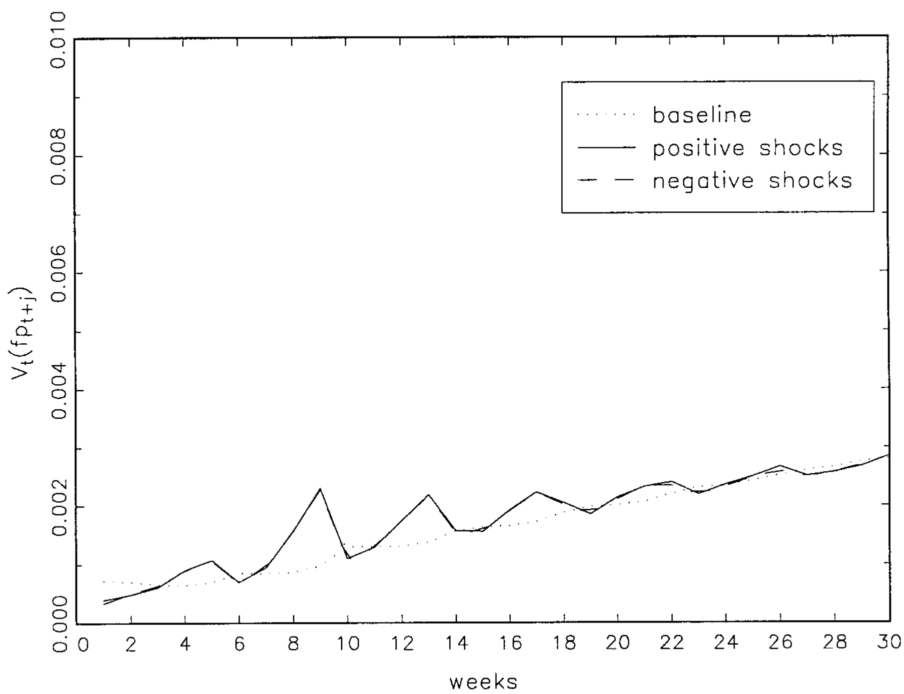


Figure 16. Impulse response of expected spot increase to ***D*** type shock.

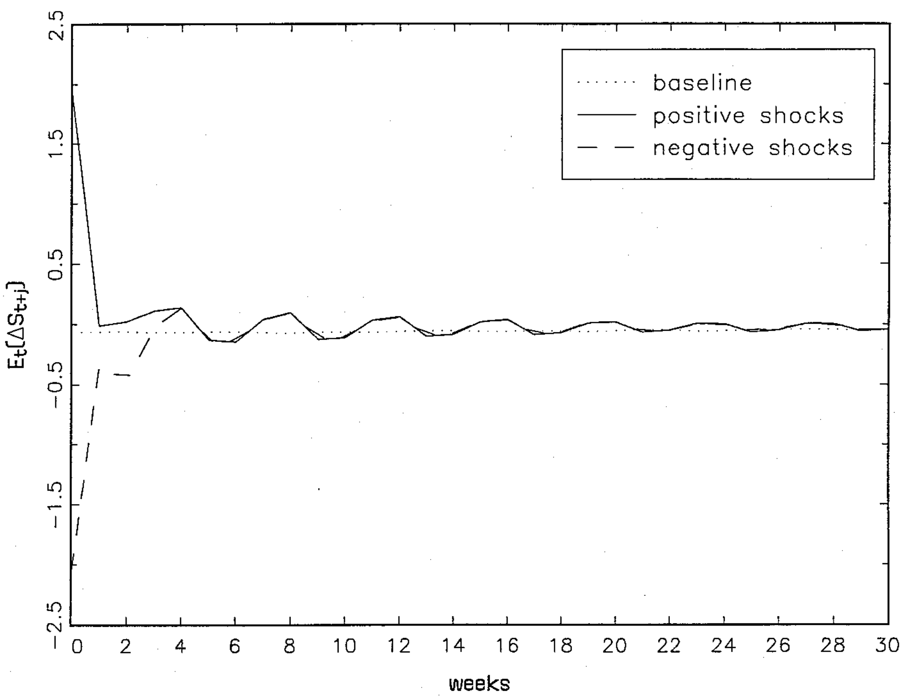


Figure 17. Impulse response of expected forward premium to ***D*** type shock.

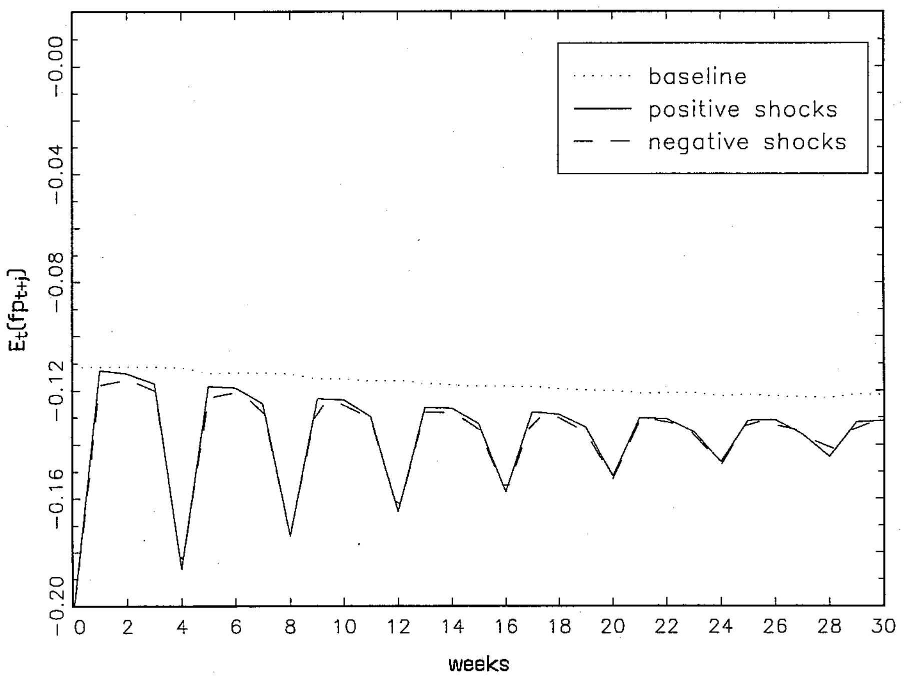


Figure 18. Impulse response of spot volatility to ***D*** type shock.

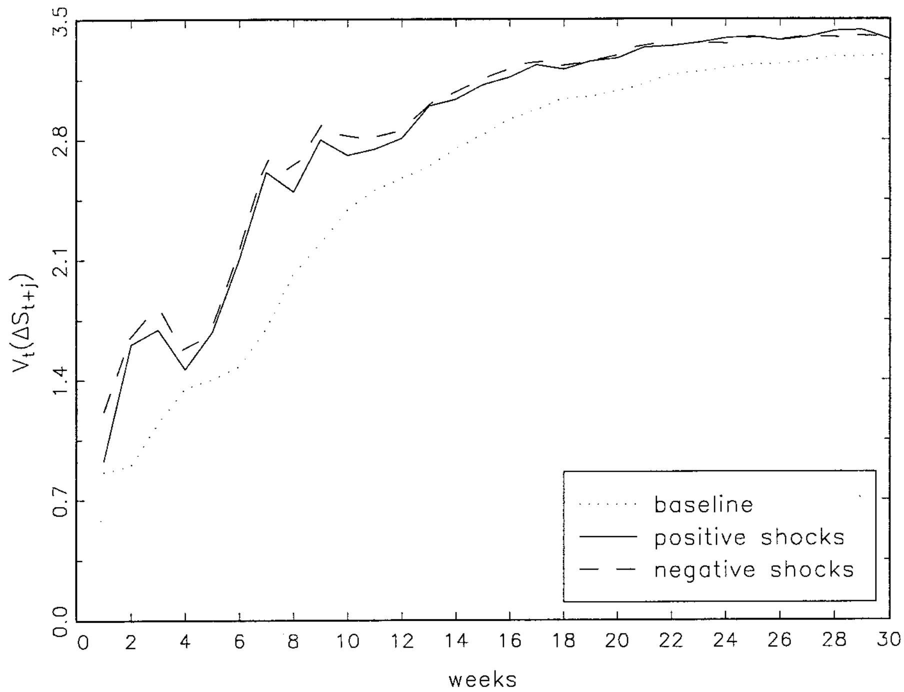


Figure 19. Impulse response of forward premium volatility to ***D*** type shock.

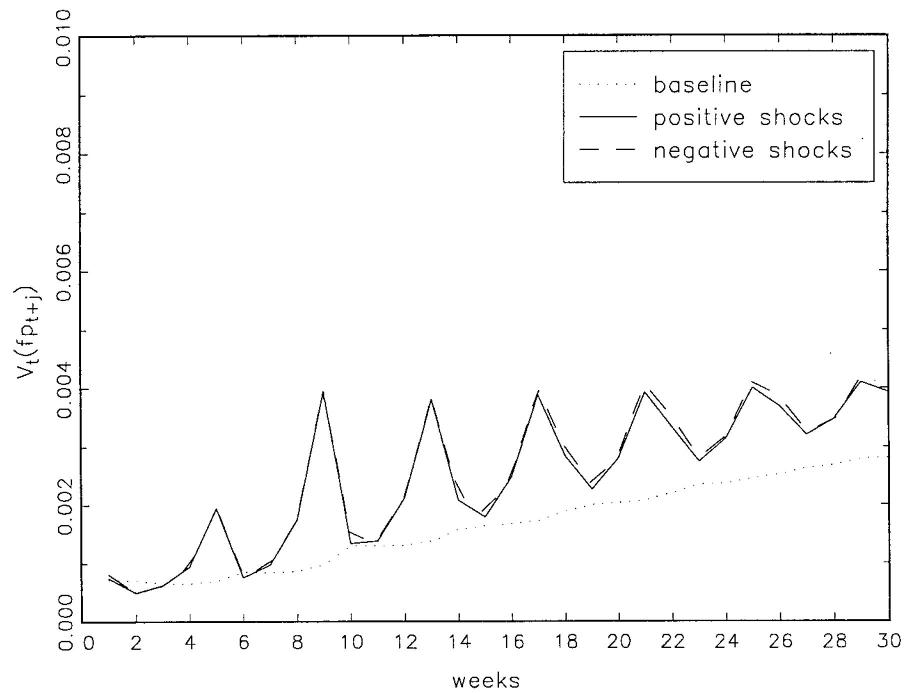


Figure 20. 95% confidence band for volatility response of spot rate to positive minus negative ***A*** type shock

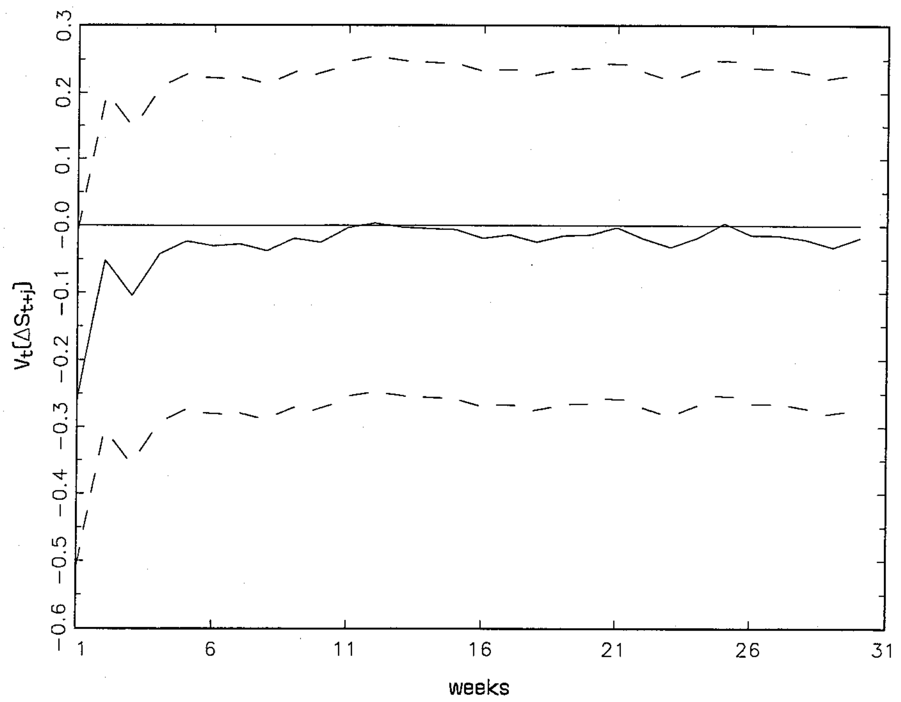


Figure 21. 95% confidence band for volatility response of forward premium to positive minus negative ***B*** type shock

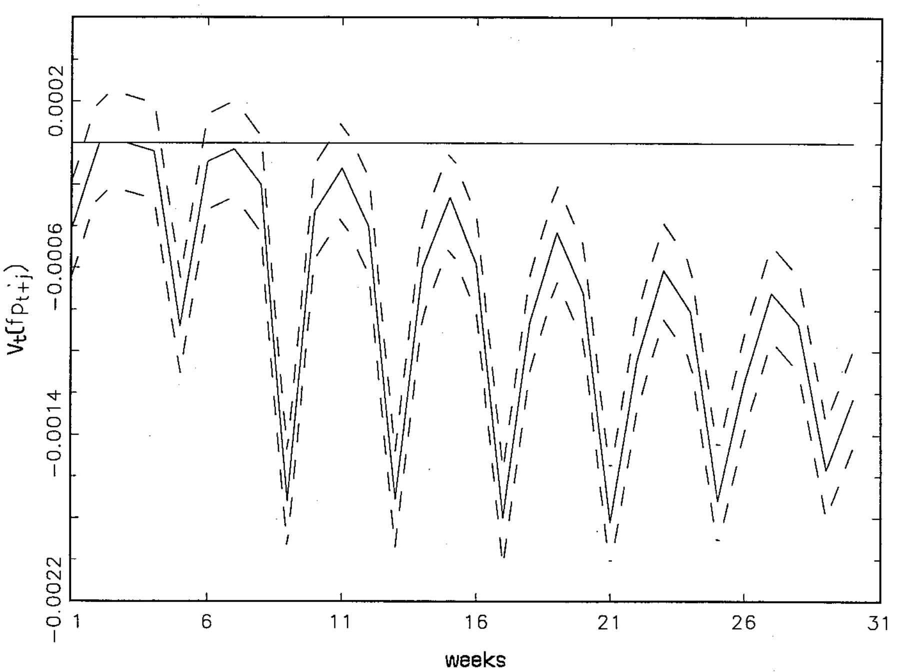


Figure 22. 95% confidence band for impulse response of forward premium to positive ***B*** type shock relative to baseline

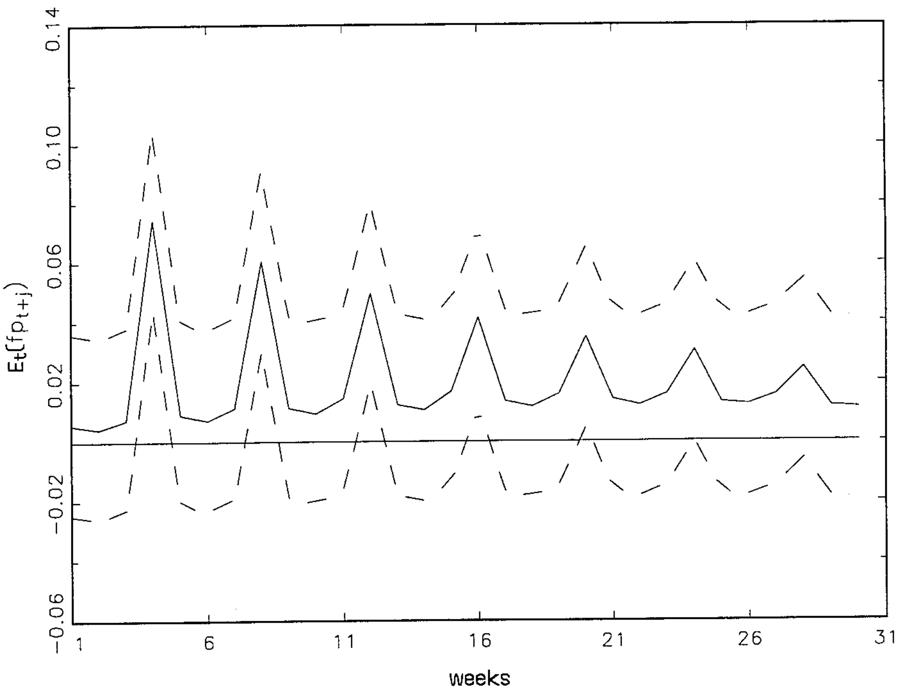


Figure 23. 95% confidence band for impulse response of forward premium to negative ***B*** type shock relative to baseline

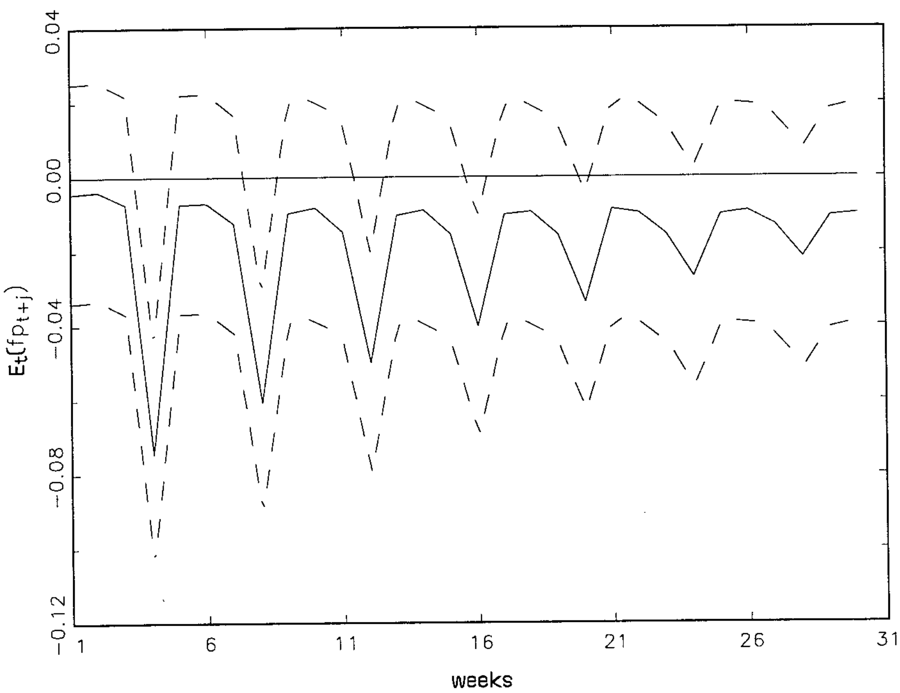


Figure 24. 95% confidence band for volatility response of forward premium to positive ***B*** type shock relative to baseline

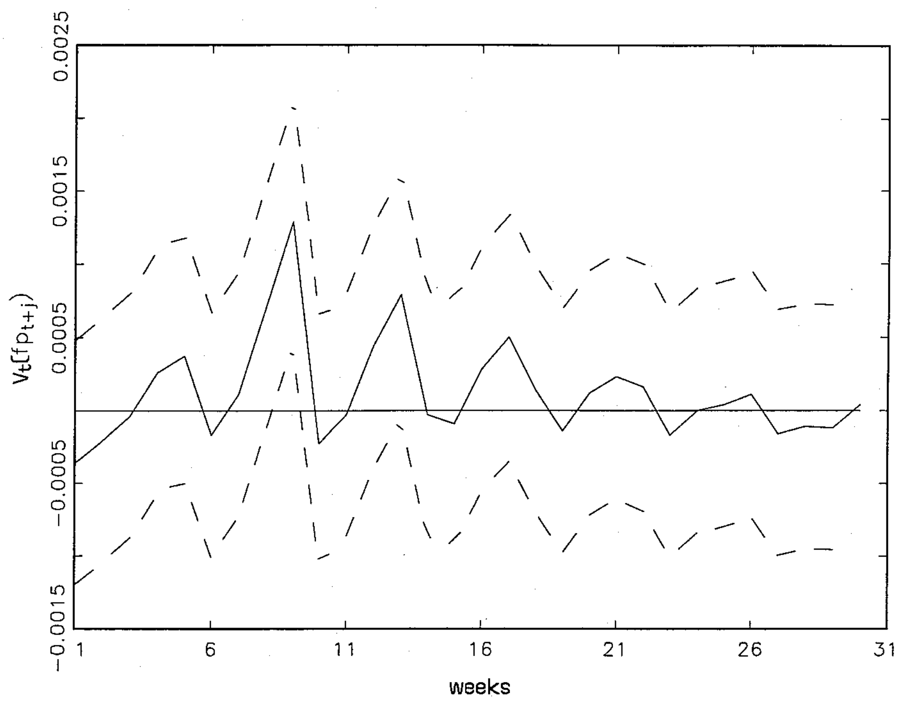


Figure 25. 95% confidence band for volatility response of forward premium to negative ***B*** type shock relative to baseline

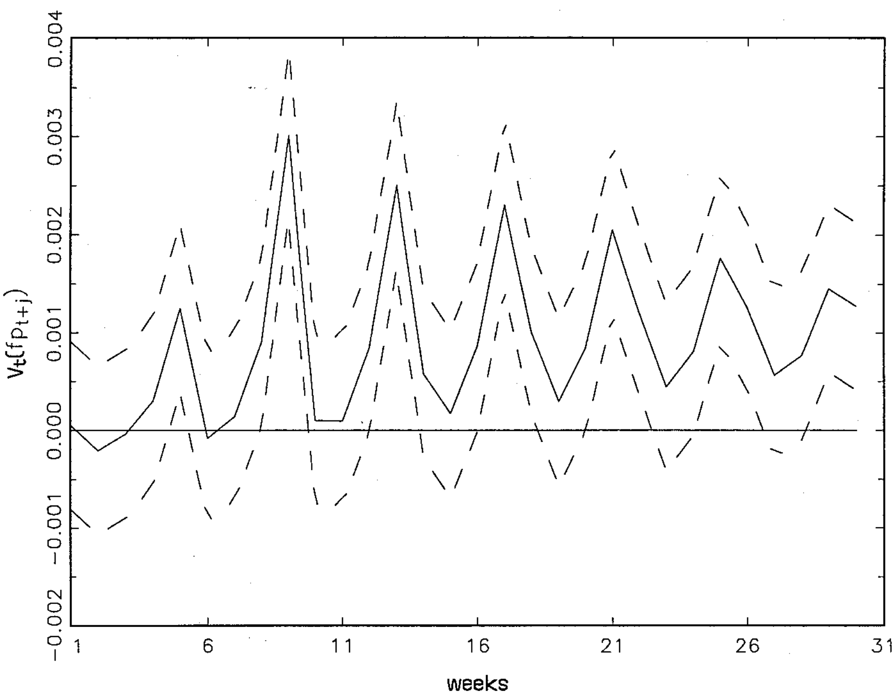


Figure 26. 95% confidence band for impulse response of spot rate to positive ***B*** type shock relative to baseline

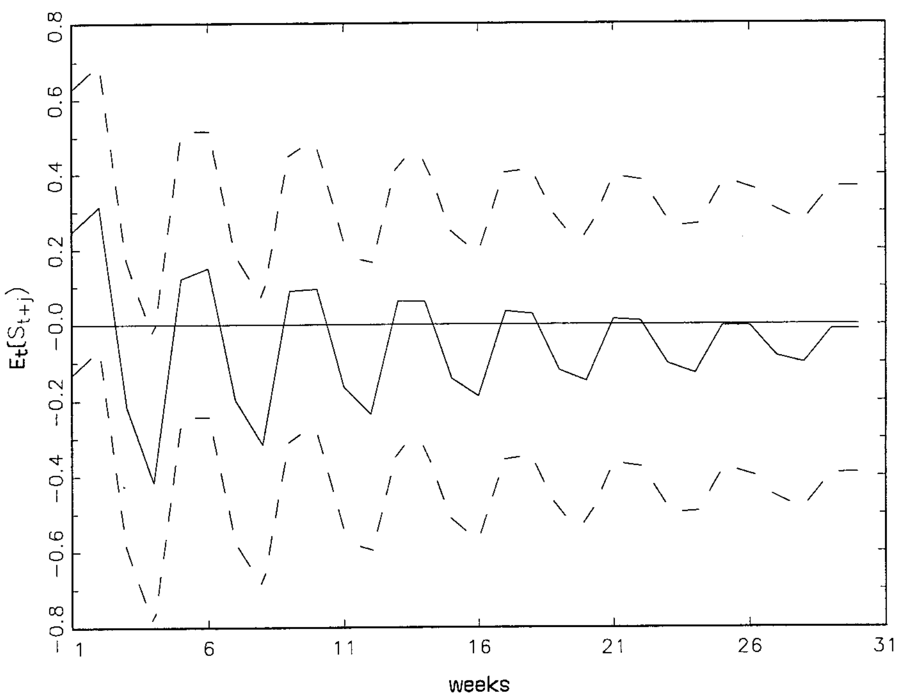


Figure 27. 95% confidence band for impulse response of forward premium to negative ***B*** type shock relative to baseline