# A Sequential Variance Ratio Test Based on the Closure Test Principle

Raimund ALT

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Institute for Advanced Studies
Department of Mathematical Methods
and Computer Sciences
Stumpergasse 56
A-1060 Vienna, Austria
Tel: (0222) 59 9 91-159
Fax: (0222) 597 06 35
e-mail: alt@ihssv.wsr.ac.at

## 1. Introduction

It is well known that the overall size of a sequence of tests can become quite large, if each single test is carried out at level $\alpha$. This is due to the possible occurrence of multiple type I errors when testing the individual null hypotheses. Situations of this kind can often arise, for example, in empirical applications of the variance ratio test proposed by Lo and MacKinlay (1988, 1989), where typically the use of multiple variance ratio estimates is employed. In a recent article Chow and Denning (1993) have provided a multiple variance ratio (MVR) test consisting of a sequence of single variance ratio tests where the size of each test is corrected in such a way that the overall test size is controlled by some prespecified level $\alpha$. Chow and Denning (1993) have also examined the size and power of the MVR test via simulation experiments and the reported results indicated quite a good performance of the MVR test.

In this paper we propose a sequential variance ratio (SVR) test which is based on the so-called 'closure test principle'. This principle was introduced by Marcus, Peritz and Gabriel (1976) and further developed by Holm (1979) and Sonnemann (1982). It facilitates the construction of multiple level $\alpha$ tests provided that a level $\alpha$ test is available for each single null hypothesis. By a multiple level $\alpha$ test we mean a test procedure which has the property that the probability of committing any type I error is always less or equal than $\alpha$ for any combination of true null hypotheses. Since the publication of the paper by Marcus, Peritz and Gabriel (1976) the application of the closure test principle has led to the improvement of many classical test procedures which results from the fact that 'closure tests' or 'closed test procedures' are as least as powerful (and in general more powerful) than their classical counterparts. The key features of the SVR test can be summarized as follows: (i) its construction is based on the closure test principle; (ii) it controls the multiple level $\alpha$; (iii) it is uniformly more powerful than the MVR test; (iv) it is easy to apply since one has only to compare pairwise a set of ordered p-values with a set of prespecified significance levels.

Closed test procedures seem to be almost unknown in econometrics. In particular, the surveys on multiple hypotheses testing given by Miller (1981) and Savin (1984), which are sometimes recommended in the econometric literature, did not mention them. Exceptions appear to be Krämer and Sonnberger (1986), who described the Holm procedure [see Holm (1979)], and Neusser (1991), who applied the closure test principle within a cointegration context. Due to the development of the theory of closed test procedures the above mentioned surveys are now somewhat out of date. More recent surveys are provided by Hochberg and Tamhane (1987) and Bauer, Hommel and Sonnemann (1988).

The paper starts with a description of the MVR test proposed by Chow and Denning (1993). Section 3 provides some theory on closed test procedures, in particular the multiple level $\alpha$ concept and the closure test principle. In Section 4 we will describe the construction of the SVR test and we will also demonstrate the key features of this test procedure. Section 5 contains a short summary and some concluding remarks.

## 2. The MVR test proposed by Chow and Denning

Let us consider the process $X_t$ which is defined by the recursion

$$X_t = \mu + X_{t-1} + \varepsilon_t \tag{1}$$

where $\mu$ is an arbitrary drift parameter. To examine the null hypothesis that $X_t$ is a random walk with homoskedastic increments[1], i. e.

$$H_0\colon \varepsilon_t \text{ i.i.d. } N(0,\sigma_0^2)$$

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[1] For the sake of simplicity we will confine ourselves to the case of testing the random walk hypothesis with homoskedastic increments. The presentation given in the following sections immediately carries over to the case of testing the random walk hypothesis with heteroskedastic increments, which is also covered by Chow and Denning (1993).

Lo and MacKinlay (1988) made use of the simple variance ratio relation

$$VR(q) \equiv q^{-1} \cdot Var(X_{t+q} - X_t)/Var(X_{t+1} - X_t) = 1 \tag{2}$$

Under $H_0$ this relation holds for any integer q greater than one (the case q=1 is trivial). Using a sample of nq + 1 observations $X_0,\ldots,X_{nq}$ Lo and MacKinlay (1988) derived the following test statistic

$$Z(q) \equiv (nq)^{1/2}\overline{M}_r(q)[2(2q-1)(q-1)/3q]^{-1/2} \tag{3}$$

where $\overline{M}_r(q) \equiv [\overline{\sigma}^2(q)/\overline{\sigma}^2(1)] - 1$ is the variance ratio estimate minus one, with

$$\overline{\sigma}^2(1) = (nq-1)^{-1} \sum_{k=1}^{nq} (X_k - X_{k-1} - \overline{X})^2 \tag{4a}$$

$$\overline{\sigma}^2(q) = s^{-1} \sum_{k=q}^{nq} (X_k - X_{k-1} - q\overline{X})^2 \tag{4b}$$

$$\overline{X} = (nq)^{-1} \sum_{k=1}^{nq} (X_k - X_{k-1}) = (nq)^{-1}(X_{nq} - X_0)$$

and $s = q(nq - q + 1)(1 - q/nq)$.

Lo and MacKinlay (1988) showed that Z(q) is (under $H_0$) asymptotically distributed as N(0,1). Testing $H_0$ by using the corresponding critical value of the standard normal distribution is straightforward.

In view of the fact that empirical research is often concerned with the examination of the hypothesis VR(q) = 1 for different values of q, Chow and Denning (1993, p. 386, foot-note 1) made the following remark: 'In testing multiple variance ratios, when the null hypothesis is $H_0$: VR(2) = VR(4) = ... = 1, one joint test statistic such as the F statistic may be appropriate. However, when $H_0$ is rejected further information concerning whether the individual variance ratios or all ratios are different from one is desirable. This can be tested simultaneously from a set of subhypotheses - $H_{01}$: VR(2) = 1, $H_{02}$: VR(4) = 1, $H_{03}$: VR(8) = 1, etc., by examining multiple Z test statistics.'

The examination of multiple Z test statistics, however, requires the construction of a suitable multiple comparison procedure in order to control the size of the overall test. To do this, we will first consider some basic probability inequalities which are of great importance in multiple comparison situations. Let $Z_1,\ldots,Z_m$ be standard normal variables with an arbitrary correlation matrix $\Omega$. If we set $\alpha' = \alpha/m$, then, due to the well known Bonferroni inequality, we get

$$P(\max_{1\leq i\leq m} |Z_i| \leq Z_{\alpha'/2}) = P(\bigcap_{i=1}^{m} \{|Z_i| \leq Z_{\alpha'/2}\}) \qquad (5)$$

$$\geq 1 - \sum_{i=1}^{m} P(|Z_i| \geq Z_{\alpha'/2})$$

$$= 1 - m\alpha'$$

$$= 1 - \alpha$$

where $Z_{\alpha'/2}$ is the upper $\alpha'/2$ point of the standard normal distribution. Sidak (1967) has proved the following inequality

$$P(\max_{1\leq i\leq m} |Z_i| \leq c) \geq \prod_{i=1}^{m} P(|Z_i| \leq c) \qquad (6)$$

which can be used to get a slight improvement over the Bonferroni inequality. If $c \equiv Z_{\alpha^+/2}$ is the upper $\alpha^+/2$ critical point of the standard normal distribution, where $\alpha^+ = 1 - (1-\alpha)^{1/m}$, then, specialising the above inequality, we get

$$P(\max_{1\leq i\leq m} |Z_i| \leq Z_{\alpha^+/2}) \geq \prod_{i=1}^{m} P(|Z_i| \leq Z_{\alpha^+/2}) \qquad (7)$$

$$= \prod_{i=1}^{m} (1-\alpha)^{1/m}$$

$$= 1 - \alpha$$

Since $\alpha/m \leq 1 - (1-\alpha)^{1/m}$, we have $Z_{\alpha^+/2} \leq Z_{\alpha'/2}$, which gives a slightly sharper bound. The following refinement of the Sidak inequality is due to Hochberg (1974):

$$P(\max_{1\leq i\leq m} |Z_i| \leq SMM(\alpha,m,N)) \geq 1-\alpha \qquad (8)$$

where SMM $(\alpha,m,N)$ is the upper $\alpha$ critical point of the Studentized Maximum Modulus distribution with m and N(sample size) degrees of freedom. If $N=\infty$, then the Hochberg inequality and the Sidak inequality are equivalent, which means that SMM $(\alpha,m,N) = Z_{\alpha^+/2}$. Chow and Denning (1993) used this result to construct the MVR test. They considered a set of different integers $q_1,\ldots,q_m$ (each greater than one) with the corresponding null hypotheses $H_{oi}$: VR $(q_i) = 1$, $i = 1,\ldots,m$. Since each test statistic $Z(q_i)$ is (under $H_{0i}$) asymptotically distributed as N(0,1), the critical region based on

$$\max_{1\leq i\leq m} |Z(q_i)| \geq Z_{\alpha^+/2} \qquad (9)$$

defines a test which controls (asymptotically) the overall level $\alpha$. The MVR test will be carried out by comparing each statistic $|Z(q_i)|$ with the critical value $Z_{\alpha^+/2}$ of the standard normal distribution.

## 3. Closed test procedures

This section provides some theory on closed test procedures which will be used in Section 4 to construct the SVR test. The theory presented here is quite general and can be applied to many situations in which multiple hypotheses are tested.

Let $\{H_{01},\ldots,H_{0m}\}$ be the set of interesting null hypotheses and let $\Phi=(\varphi_1,\ldots,\varphi_m)$ be a multiple test procedure where $\varphi_i$ is a test for the hypothesis $H_{0i}$, $i=1,\ldots,m$. The critical region of $\varphi_i$ will be denoted by $\{\varphi_i = 1\}$. A usual requirement for a multiple test procedure is the overall level $\alpha$ property

$$P(\bigcup_{i=1}^{m} \{\varphi_i = 1\} | H_{01},\ldots,H_{0m}) \leq \alpha, \qquad (10)$$

which means that the probability of rejecting at least one of the null hypotheses $H_{01},\ldots,H_{0m}$, given that all null hypotheses are true, should be less or equal than $\alpha$. Since the publication of the paper by Marcus, Peritz and Gabriel (1976) this requirement is more and more replaced by the multiple level $\alpha$ concept. The test procedure $\Phi = (\varphi_1,\ldots,\varphi_m)$ controls the multiple level $\alpha$, if for each subset $I \subset \{1,\ldots,m\}$ the probability of rejecting at least one of the null hypotheses $H_{0i}$, $i \in I$, given that all null hypotheses $H_{0i}$, $i \in I$, are true,[2] is less or equal than $\alpha$, or, to put it in another way

$$P(\bigcup_{i\in I} \{\varphi_i = 1\} | H_{0i}, i \in I) \leq \alpha \quad \text{for each } I \subset \{1,\ldots,m\}. \tag{11}$$

This concept is of course much stronger than the overall level $\alpha$ concept. It requires that the probability of committing any type I error should always be less or equal than $\alpha$ for each combination of true null hypotheses. It is obvious that a multiple level $\alpha$ test also controls the overall level $\alpha$, since $I = \{1,\ldots,m\}$ is the index set of the combination of all null hypotheses. The converse is of course in general not true. Using the multiple level $\alpha$ concept we will now formulate the 'closure test theorem'.

*Theorem 1. (Marcus, Peritz and Gabriel)*

Let $\{H_{01},\ldots,H_{0m}\}$ be a set of null hypotheses which is closed under intersection, i. e. $H_{0i} \cap H_{0j} \in \{H_{01},\ldots,H_{0m}\}$ for any two indices $i \neq j$. If $\varphi_i$ is a level $\alpha$ test for $H_{0i}$, $i=1,\ldots,m$, then the test procedure $\Psi = (\psi_1,\ldots\psi_m)$, where each component $\psi_i$ is defined by the test rule

$\psi_i$ rejects $H_{0i}$, if each subhypothesis $H_{0j} \subset H_{0i}$ is rejected by its level $\alpha$ test $\varphi_j$,

controls the multiple level $\alpha$.

*Proof.* Let $I \subset \{1,\ldots,m\}$ be an arbitrary index set. We have to show that

$$P(\bigcup_{i\in I} \{\psi_i = 1\} | H_{0i}, i \in I) \leq \alpha$$

[2] We will assume here that the intersection $H_{01} \cap \ldots \cap H_{0m}$ is nonempty, otherwise the above definition should be modified in such a way that only those subsets $I \subset \{1,\ldots,m\}$ are considered, for which $H_{0I} = \bigcap_{i\in I} H_{0i}$ is nonempty.

where $\{\psi_i = 1\}$ denotes the critical region of the test $\psi_i$. Since $\{H_{01},...,H_{0m}\}$ is closed under intersection there exists an index j, for which $H_{0j} = \bigcap_{i \in I} H_{0i}$. Now, since $H_{0j} \subset H_{0i}$ for each $i \in I$, we have $\{\psi_i = 1\} \subset \{\psi_j = 1\}$ for each $i \in I$, which is an implication of the test rule given above. But then we have $\bigcup_{i \in I} \{\psi_i = 1\} \subset \{\psi_j = 1\}$, which implies

$$P(\bigcup_{i \in I} \{\psi_i = 1\} | H_{0i}, i \in I) \leq P(\psi_j = 1 | H_{0j}) \tag{12}$$

$$\leq P(\varphi_j = 1 | H_{0j})$$

$$\leq \alpha$$

where the last inequality uses the assumption, that $\varphi_j$ is a level $\alpha$ test for $H_{0j}$. This completes the proof.

The above theorem describes the construction of a multiple level $\alpha$ test which is based on two assumptions. The set $\{H_{01},...,H_{0m}\}$ should be closed under intersection and for each single null hypothesis $H_{0i}$ there should exist a level $\alpha$ test $\varphi_i$. Then, for each hypothesis $H_{0i}$, we can define another test $\psi_i$, which rejects $H_{0i}$ if and only if each subhypothesis $H_{0j} \subset H_{0i}$ is rejected by its level $\alpha$ test $\varphi_j$. This is the closure test principle and the resulting test procedure $\Psi = (\psi_1,...,\psi_m)$ is called 'closure test' or 'closed test procedure'. The assumption that $\{H_{01},...,H_{0m}\}$ is closed under intersection is actually not a restriction. If $\{H_{01},...,H_{0m}\}$ is not closed under intersection, it is always possible to add 'auxiliary hypotheses' of the form $H_{0I} = \bigcap_{i \in I} H_{0i}$ in order to get a closed set of null hypotheses. The only crucial point is to find suitable level $\alpha$ tests for the hypotheses $H_{0I}$, $I \subset \{1,...,m\}$.

## 4. The sequential variance ratio test

In the following theorem we will define the SVR test which turns out to be a shortcut version of a specific closed test procedure. Again we consider m null hypotheses $H_{01},\ldots,H_{0m}$, with $H_{0i}$: VR $(q_i) = 1$, $i=1,\ldots,m$, which correspond to m different integers $q_1,\ldots,q_m$ (each greater than one).

*Theorem 2.* For $i=1,\ldots,m$ let $Z_i \equiv Z(q_i)$ be a test statistic, which is (under $H_{0i}$) asymptotically distributed as N(0,1) and let $p_i = P(|Z_i| \geq |\tilde{z}_i| \,|H_{0i})$ be the p-value[3] belonging to the observed value $|\tilde{z}_i|$ of the test statistic $|Z_i|$. If $p_{(1)} \leq \ldots \leq p_{(m)}$ are the ordered p-values, where $H_{0(1)},\ldots,H_{0(m)}$ are ordered analogously, then the sequential procedure defined by the test rule

$$\text{Reject } H_{0(i)} \text{ if } \quad p_{(j)} \leq 1-(1-\alpha)^{1/(m-j+1)} \quad \text{for } j = 1,\ldots,i$$

controls asymptotically the multiple level $\alpha$.

*Proof.* Obviously the set $\{H_{0I}\colon H_{0I} = \bigcap_{i\in I} H_{0i},\ I \subset \{1,\ldots m,\}\}$ is closed under intersection. Let $I \subset \{1,\ldots,m\}$ be an arbitrary index set and $\alpha(I) = 1-(1-\alpha)^{1/|I|}$, where $|I|$ is the number of elements in I, and let $Z_{\alpha(I)/2}$ be the upper $\alpha(I)/2$ critical point of the standard normal distribution. Then the following statements hold asymptotically

$$P(\max_{i\in I}|Z_i| \geq Z_{\alpha(I)/2}\,|H_{0I}) \leq 1 - \prod_{i\in I} P(\,|Z_i| \leq Z_{\alpha(I)/2}\,|H_{0I}) \tag{13}$$

$$= 1 - \prod_{i\in I} \;(1-\alpha)^{1/|I|}$$

$$= \alpha$$

This provides a test for $H_{0I}$ which controls asymptotically the level $\alpha$. The application of the closure test principle results in a test procedure $\Psi = (\psi_I, I \subset \{1,\ldots,m\})$ which therefore controls asymptotically the multiple level $\alpha$. Now we will demonstrate that

[3] This value is calculated using the standard normal approximation.

both the closed test procedure $\Psi$ and the above sequential procedure lead to identical decisions concerning $H_{01},\ldots,H_{0m}$. To simplify the notation let us assume that the p-values $p_1,\ldots,p_m$ are already ordered increasingly and for the moment let us also assume that there are no ties, i. e. $p_1 < p_2 < \ldots < p_m$. Suppose that $\psi_i$ rejects $H_{0i}$, which means that the condition

$$\max_{i\in I}|\tilde{z}_i| \geq Z_{\alpha(I)/2}$$

is fulfilled for each subhypothesis $H_{0I}$, with $i\in I$. This implies the existence of an index $j\in I$ such that $|\tilde{z}_j| \geq Z_{\alpha(I)/2}$, or $p_j \leq 1-(1-\alpha)^{1/|I|}$. Choosing the index sets $\{1,\ldots,m\}$, $\{2,\ldots,m\},\ldots,\{i,\ldots,m\}$, we get $p_1 \leq 1-(1-\alpha)^{1/m}$, $p_2 \leq 1-(1-\alpha)^{1/(m-1)}$, … , $p_i \leq 1-(1-\alpha)^{1/(m-i+1)}$, which implies the rejection of $H_{0i}$ by the sequential procedure. Suppose now, that $H_{0i}$ is not rejected by $\psi_i$. Then there exists an index set I, with $i\in I$, such that

$$p_j \geq 1-(1-\alpha)^{1/|I|} \qquad \text{for all } j\in I.$$

If $|I| > m-i+1$, then there must be an index $j\in I$ with $j < i$. Let j be the smallest index such that $j\in I$ and $j < i$. Since $|I| \leq |\{j,\ldots,m\}| = m-j+1$ we have $p_j > 1-(1-\alpha)^{1/(m-j+1)}$ and therefore the sequential procedure cannot reject $H_{0i}$. The same is true for $|I| \leq m-i+1$ which is straightforward. The occurrence of ties does not cause any problems. Suppose $p_i = p_{i+1}$ and $p_i > 1-(1-\alpha)^{1/(m-i+1)}$, then neither $H_{0i}$ nor $H_{0,i+1}$ will be rejected. If $p_i \leq 1-(1-\alpha)^{1/(m-i+1)}$, then $H_{0i}$ will be rejected, provided $H_{01},\ldots H_{0,i-1}$ were already rejected. But then $H_{0,i+1}$ will be rejected as well, since $p_{i+1} = p_i \leq 1-(1-\alpha)^{1/(m-i+1)} < 1-(1-\alpha)^{1/(m-i)}$. Thus both test procedures lead to identical decisions concerning $H_{01},\ldots,H_{0m}$. Since the closed test procedure controls asymptotically the multiple level $\alpha$, the sequential procedure does also have this property. This proves the theorem.

Let us describe again the SVR test using the following step-by-step algorithm:

Step 1: If $p_{(1)} > 1 - (1 - \alpha)^{1/m}$, no hypothesis will be rejected and the procedure stops. Otherwise $H_{0(1)}$ is rejected → Step 2.

Step 2: If $p_{(2)} > 1 - (1 - \alpha)^{1/(m-1)}$, no other hypothesis will be rejected and the procedure stops. Otherwise $H_{0(2)}$ is rejected → Step 3.

Step 3: If $p_{(3)} > 1 - (1 - \alpha)^{1/(m-2)}$, no other hypothesis will be rejected and the procedure stops. Otherwise $H_{0(3)}$ is rejected → Step 4

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Step m: If $p_{(m)} > 1 - (1 - \alpha) = \alpha$, $H_{0(m)}$ will not be rejected and the procedure stops. Otherwise $H_{0(m)}$ is rejected and the procedure stops too.

This procedure is very similar to the Holm procedure [see Holm (1979], which uses $\alpha/m$, $\alpha/(m-1)$, ... ,$\alpha$ instead of $1 - (1 - \alpha)^{1/m}$, $1 - (1 - \alpha)^{1/(m-1)}$, ... ,$\alpha$.

The prerequisites of the SVR test are rather simple. One has only to calculate the (asymptotic) p-values $p_1,\ldots,p_m$ of the corresponding test statistics and to compare pairwise the ordered values $p_{(i)}$ with the increasing levels $1 - (1 - \alpha)^{1/(m-i+1)}$. The difference between the SVR test and the MVR test is now straightforward. Basing the MVR test on the ordered p-values (which does not change any decision of the test) the MVR test uses the same significance bound $1 - (1 - \alpha)^{1/m}$ at each step of the procedure. But, with the exception of the first step, where the SVR test also uses the bound $1 - (1 - \alpha)^{1/m}$, the SVR test is able to reject more null hypotheses due to the increasing significance bounds of the following steps. In particular, each single null hypothesis which is rejected by the MVR test is also rejected by the SVR test. Therefore the SVR test is uniformly more powerful than the MVR test. But the SVR test still controls the multiple level $\alpha$. Incidentally, the last two statements also establish the multiple level $\alpha$

property of the MVR test.

The use of the levels $1 - (1 - \alpha)^{1/m}, \ldots, \alpha$ rather than the constant level $1 - (1 - \alpha)^{1/m}$ for the MVR test can lead to a nontrivial gain in power for the SVR test. For m=4 and $\alpha$=0.05, for example, the MVR test uses the level 0.0127 at each step, while the levels for the SVR test are 0.0127, 0.0169, 0.0253 and 0.05, respectively. Nevertheless the probability of committing any type I error is always controlled by $\alpha$=0.05.

Due to the above discussion the SVR test appears to be an attractive variant for testing multiple variance ratio hypotheses.

## 5. Summary and concluding remarks

This paper describes a sequential variance ratio test which is based on the closure test principle, introduced by Marcus, Peritz and Gabriel (1976). This test procedure is uniformly more powerful than the multiple variance ratio test, recently proposed by Chow and Denning (1993), while still controlling the probability of committing any type I error.

The closure test principle can be applied to many situations where the testing of multiple hypotheses has to be controlled by some prespecified level $\alpha$. It can be used to test linear hypotheses within regression models [see Alt (1990)], to test sets of nested hypotheses [see Bauer and Hackl (1987) or Alt (1988)] or to test for the rank of a matrix, just naming a few examples. Due to the attractive features of closed test procedures, especially their superiority compared with classical test procedures, the closure test principle offers a promising tool for further econometric applications.

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