Stock Markets Efficiency and Volatility Tests: A Survey

Chien-Te HSU

Forschungsbericht /
Research Memorandum No. 330
July 1993

Institute for Advanced Studies
Department of Economics
Stumpergasse 56
A-1060 Vienna, Austria
Tel: (1)59991-156
Fax: (1)59991-163
E-mail: chiente@ihssv.wsr.ac.at

If y_t is a martingale, its first difference x_t is a fair game with

$$E(y_{t+1} - y_t|I_t) = E(x_{t+1}|I_t) = 0. \tag{2}$$

In the context of stock market, the unpredictability of stock returns is equivalent to the rate of return r_t less a constant γ being a fair game, i.e.

$$E(r_{t+1} - \gamma|I_t) = E(\frac{P_{t+1} + D_{t+1} - P_t}{P_t} - \gamma|I_t) = 0, \tag{3}$$

where P_t is the stock prices, D_t the real dividends on the stock and γ a constant in this first-order stochastic difference equation.

If returns of a stock is a fair game, the stock prices P_t is given by

$$P_t = \beta E(P_{t+1} + D_{t+1}|I_t), \tag{4}$$

where $\beta = 1/(1+\gamma)$.

Equation (4) says that the stock price today equals the sum of the expected future price and dividends, discounted back to the present at a rate β. Substituting recursively for P_{t+1}, P_{t+2}, etc. into (4) and using the law of iterated expectations, gives

$$P_t = E(\sum_{i=1}^{n} \beta^i D_{t+i} + \beta^n P_{t+n}|I_t). \tag{5}$$

Suppose the transversality condition

$$lim_{n\to\infty}\beta^n E(P_{t+n}|I_t) = 0 \tag{6}$$

holds, and so as to rule out speculative bubbles[1], then

$$P_t = E(\sum_{i=1}^{\infty} \beta^i D_{t+i}|I_t). \tag{7}$$

Therefore, Samuelson's result shows that the fair game model (3) plus rational expectation imply that stock prices equal the sum of the expected

[1]see section 3.2 for bubbles.

present value of future dividends. And the present value model (7) forms the testable efficient market hypothesis which most of the volatility tests rely on.

2 First Generation Volatility Tests

Until the mid-1970s, stock market efficiency was tested using so-called returns tests. Since unforecastability of returns implies non-correlations between future returns and current information, one can simply regress returns on some variables considered as possible explanatory variables, such as past returns or other macroeconomic variables. Nonzero correlations suggest that stock markets are not informationally efficient. Most of the evidence accumulated prior to 1975 inferred that stock markets are efficient.

It was then realized that the same models which imply that returns should be unforecastable also imply that stock prices should have volatility which is low relative to the volatility of dividends. Both LeRoy and Porter (1981) and Shiller (1981) exploited this line and derived so-called variance-bound tests.

Define the discounted sum of future dividends as "ex-post rational prices" [2] of a stock, equation (7) can be written as

$$P_t = E(P_t^*|I_t), \tag{8}$$

with

$$P_t^* = \sum_{i=1}^{\infty} \beta^i D_{t+i}. \tag{9}$$

So the stock prices is the conditional expectation of its "ex-post rational prices". Since the variance of conditional expectation of a random variable is less than or equal to the variance of the random variable itself, we have

$$\frac{Var(P_t)}{Var(P_t^*)} <= 1.$$

This is the simplest form of so-called "variance-bound" inequality. LeRoy/

[2] Shiller (1981).

Porter and Shiller used different techniques to calculate this ratio and found this inequality extremely violated [3].

2.1 LeRoy and Porter's Orthogonality Tests

Define e_t as the unexpected one-period return (or forecast error) such that

$$e_t = P_t + D_t - E(P_t + D_t | I_{t-1}). \tag{10}$$

Then the stock prices can be written as

$$P_t = \beta(P_{t+1} + D_{t+1} - e_{t+1}). \tag{11}$$

Substituting P_{t+1}, P_{t+2}, etc. recursively into (11) and using equation (8) results in

$$P_t^* = P_t + \sum_{i=1}^{\infty} \beta^i e_{t+i}. \tag{12}$$

If e_t is stationary, $Var(e_t)$ is constant. Since rational expectation implies that P_t and e_{t+i} are uncorrelated, we get

$$Var(P_t^*) = Var(P_t) + Var(\sum_{i=1}^{\infty} \beta^i e_{t+i}). \tag{13}$$

Based on equation (13), the orthogonality test consists of evaluating the null hypothesis

$$H_0 : Var(P_t^*) = Var(P_t) + \frac{\beta^2 Var(e)}{1 - \beta^2}$$

against the alternative

$$H_1 : Var(P_t^*) < Var(P_t) + \frac{\beta^2 Var(e)}{1 - \beta^2}.$$

[3] The results are illustrated in the first two rows of table 1.

Since P_t^* is unobservable, LeRoy and Porter used a linear autoregressive model for dividends and estimate β as the reciprocal of 1 plus the average rate of return on stock. The estimates were then applied directly into equation (9) to estimate $Var(P_t^*)$. The null hypothesis was rejected, although the confidence interval for the null is extremely large.

2.2 Shiller's Variance-Bounds Tests

Shiller's (1981) variance-bounds tests are based on the inequality

$$H_0 : Var(P_t) - Var(P_t^*) <= 0.$$

To solve the problem of unobservability of ex-post rational prices, Shiller used an observable version of it which is generated as follows. Under the assumption that the transversality condition

$$lim_{T\to\infty}\beta^T P_{t+T}^* = 0$$

holds, the ex-post rational price (9) is one of the solutions (the stable solution) to the first-order difference equation

$$P_t^* = \beta(P_{t+1}^* + D_{t+1}). \tag{14}$$

Shiller simply replaced P_t^* by the solution above that satisfies the terminal condition

$$P_T^* = \frac{1}{T}\sum_{t=1}^{T} P_t, \tag{15}$$

and compared the sample variance of P_t^* and P_t. To correct for trend, he divided the variables by a constant growth rate. Using a century-long data, Shiller found that the variance inequality is extremely violated, although no significance tests were reported.

2.3 Econometric Flaws of First-Generation Volatility Tests: Flavin and Kleidon's critics

Both Shiller and LeRoy/Porter found extreme excess volatility in stock prices. The statistical significance was however unclear. It was shortly after found out that these variance-bounds tests were subject to some econometric problems. Flavin (1983) focused on Shiller's tests and demonstrated that small-sample problems lead to bias against acceptance of market efficiency. She pointed out that both P_t and P_t^* are positively autocorrelated, where P_t^* is more strongly autocorrelated than P_t. This implies that

(i) the standard deviations as the sample counterparts of $Var(P_t^*)$ and $Var(P_t)$ are biased estimators of true parameters and

(ii) variance of P_t^* is estimated with a greater downward bias than that of P_t.

Hence, if the present-value model is rejected when the statistic $\hat{V}(P^*) - \hat{V}(P) < 0$, rejecting the null when it is true will occur with high probability.

Flavin also criticized the implementation of Shiller's ex-post rational prices P_t^*[4]. As described above, Shiller generates an observable version of P_t^* by using equation (14) recursively and imposing the terminal condition $P_T^* = \frac{1}{T}\sum_{t=1}^{T} P_t$. Flavin pointed out that for each t, this implementation gives a biased estimate conditional on the realization of the sample. An unbiased proxy for P_t^*, denoted as $P_{t|T}^*$, can be constructed as the solution to the recursion (14) and imposing the terminal condition $P_{T|T}^* = P_T$[5]. First, $P_{T|T}^* = E(P_T^*|I_T)$ by equation (8), i.e. $P_{T|T}^*$ is an unbiased estimator of P_T^* conditional on the information set I_T. Second, if $P_{t|T}^*$ is an unbiased estimator of P_t^* conditional on the realization of the whole sample for any date $t <= T$, $P_{t-1|T}^*$ is also an unbiased estimator of P_{t-1}^*. Since

$$P_{t-1|T}^* = \beta(D_t + P_{t|T}^*) = \beta[D_t + E(P_t^*|I_T)] = E(P_{t-1}^*|I_T), \qquad (16)$$

where the last equality follows from (14). Hence using this recursive induction it is proved that at each date t in the sample, $P_{t|T}^* = E(P_t^*|I_T)$.

The second problem of the first generation volatility tests lies on the assumption of stationarity of P_t and D_t. If the variables have unit roots, then the variances are functions of time. Any sample variance is not a good esti-

[4]This criticism doesn't apply to LeRoy/Porter's orthogonality tests, since they don't use an observable proxy for P_t^*.

[5]The proof is taken over from Gills and LeRoy (1991)

mator of the corresponding population variance. Kleidon (1986) ran Monte Carlo studies in which the present-value relation holds by construction and using a geometric random walk model for dividends[6], i.e.

$$ln(D_{t+1}) = ln(D_t) + \epsilon_{t+1}, \tag{17}$$

where ϵ_t is distributed independently as normal with mean μ and variance σ^2 and $D_0 = 1$. Further, $\mu + \sigma^2/2$ is assumed to be zero, so that $E(D_{t+i}|D_t) = D_t$, which means D_t is a martingale. Suppose that agents are fully informed of the stochastic process which drives dividends and they form their expectation based on this information set. Prices are then generated by

$$P_t = \sum_{i=1}^{\infty} \beta^i E(D_{t+i}|D_t, D_{t-1}, ...). \tag{18}$$

If dividends follow the process as in equation (17), the implied price is then

$$P_t = \frac{\beta}{1-\beta} D_t. \tag{19}$$

Under the assumption that dividends have an expected growth rate of zero, P_t is a martingale as well. Kleidon defined a statistic θ as

$$\theta = \frac{\sum_{t=1}^{T} (P_t^*)^2}{T} - \frac{\sum_{t=1}^{T} P_t^2}{T}, \tag{20}$$

then $E(\theta) > 0$ if the present value model is true. However, conducted on simulated data, Kleidon found out that, following Shiller's detrending procedure, the inequality $E(\theta) > 0$ was violated at a frequency of about 90 percent. He argued that first generation volatility tests of the variance inequality relation depend on the assumption of stationarity of the underlying series. The detrending procedure Shiller and LeRoy-Porter applied to the series produce a stationary series only if the original series is trend stationary. As a consequence, violation of a stationary assumption leads to bias against acceptance of efficiency and the interpretation of the test results as rejection of present-value model for stocks is not warranted.

To summarize, the efficient markets hypothesis seemed to lose ground following the publication of Shiller's and LeRoy/Porter's volatility tests, both of

[6] Kleidon (1986), Proposition 4, pp964.

which found stock market volatility to be far greater than could be justified by the underlying present-value model. The papers of Flavin and Kleidon raised the doubt of the statistical validity of the initial volatility tests. This leaves open the question of whether these econometric flaws are so large as to explain the entire excess of $Var(P_t)$ over $Var(P_t^*)$.

Table 1. Volatility Tests (constant rate of return)

Author	Sample	V/V^*	P-Value of H_0
I. First generation (stationarity assumption):			
LeRoy/Porter	quarterly, 1955-73	16 - 148	.01 - .05
Shiller	annual, 1871-1979 1928-1979	31 - 176	n.a.
Kleidon	T = 100	25	.05 - .50
II. Second generation (a) unit arithmetic roots:			
Campbell/Shiller	annual, 1871-1985	1 - 67	.00 - .05
MRS	annual, 1871-1988	.27 - .4	.00 - .21
West	annual, 1871-1980	5 - 10	.00 - .01
(b) logarithmic unit roots:			
Campbell/Shiller	annual, 1871-1986 1926-1986	2 - 14	.00
LeRoy/Parke	annual, 1871-1988	.24 - 1.34	n.a.

Notes: In column (2) entry of "T = sample size" indicates a Monte Carlo study. In column (3) V/V^* denotes the ratio $\frac{Var(P_t)}{Var(P_t^*)}$.

3 Second Generation Volatility Tests

Nonstationarity does not invalidate the variance-bounds theorem. Equation (8), (9) imply the variance bounds inequality

$$Var(P_t) <= Var(P_t^*)$$

for each t whether or not dividends are stationary. It is the assumption that these variances are constant over time, which is adopted in econometric implementation of the variance-bounds test, that is violated if stationarity fails. Second-generation volatility tests explicitly allow for unit roots. In this section I will group the tests according whether the test is asymptotically valid under a unit arithmetic root (ΔP_t is stationary), or with a unit logarithmic root ($\Delta log(P_t)$ is stationary).

3.1 Unit Arithmetic Roots

3.1.1 West's Inequality

In his 1988a work, West showed a different approach to testing for excess volatility which is valid when dividends are generated by a linear process with a unit roots. His method has the extra advantage that it does not require a proxy for P_t^*, which avoids the problem of small sample bias. The idea of his test is following:

The constant-expected-return model supposes

$$P_t = \beta E(P_{t+1} + D_{t+1}|I_t).$$

Define ϵ_t as the forecast error with respect to I_t, then

$$\epsilon_{t+1} = P_{t+1} + D_{t+1} - E(P_{t+1} + D_{t+1}|I_t), \quad (21)$$

or equivalently

$$\epsilon_{t+1} = P_{t+1} + D_{t+1} - \beta^{-1}P_t. \quad (22)$$

Now suppose agents use a smaller information set H_t consisting of current and past dividends. The forecast error ϵ_{t+1}^H with respect to H_t is then

$$\epsilon_{t+1}^{H} = P_{t+1} + D_{t+1} - E(P_{t+1} + D_{t+1}|H_t), \tag{23}$$

or

$$\epsilon_{t+1}^{H} = P_{t+1} + D_{t+1} - \beta^{-1} P_t^H, \tag{24}$$

where

$$P_t^H = E(P_t^*|H_t) = \beta E(P_{t+1}^H + D_{t+1}|H_t), \tag{25}$$

i.e. P_t^H is the optimal forecast of P_t^* based on a subset of the full information.

Rearrange the equation above and impose the transversality condition $lim_{n\to\infty}\beta^n P_{t+n}^H = 0$, we get

$$P_t^H = P_t^* - \sum_{i=1}^{\infty} \beta^i \epsilon_{t+i}^H. \tag{26}$$

An analogous equation for P_t is

$$P_t = P_t^* - \sum_{i=1}^{\infty} \beta^i \epsilon_{t+i}. \tag{27}$$

Since the identity

$$P_t^* - P_t^H = (P_t^* - P_t) + (P_t - P_t^H) \tag{28}$$

is always true, we get the West inequality:

$$Var(\sum_{i=1}^{\infty} \beta^i \epsilon_{t+i}^H) >= Var(\sum_{i=1}^{\infty} \beta^i \epsilon_{t+i}). \tag{29}$$

If dividends have linear unit roots, for example $D_t = D_{t-1} + u_t$, the forecast errors are stationary. Their sample counterparts therefore provide consistent estimates of population values. West found that this variance inequality was strongly violated using simple autoregressive models for dividends. In other words, the autoregressive models forecast P_t^* so well that the variance in stock returns cannot be justified due to forecast errors.

3.1.2 Mankiw, Romer and Shapiro: MRS Statistics

The MRS (1991) test recognized that in an efficient market the price of an asset must equal the discounted conditional expected payoff from holding the asset for k periods and reselling it. That is

$$P_t = E(P_t^{*k}|I_t), \tag{30}$$

where

$$P_t^{*k} = \sum_{i=1}^{k-1} \beta^i D_{t+i} + \beta^k P_{t+k}. \tag{31}$$

One thing to note is that there is different interpretation of P_t^* and P_t^{*k}: P_t^* is the fundamental value of stock, i.e. the sum of all discounted future cash-flow a stock posses, whereas P_t^{*k} is the perfect-foresight price for the strategy of buying a stock at time t and holding it for k periods.

Rational expectation implies that $P_t^{*k} - P_t$ should be uncorrrelated with any variable which is in the information set at time t. In particular, let P_t^o denotes some "naive forecast" of P_t^{*k} which is available at time t. Denote E_t for $E(.|I_t)$, then

$$E_t(P_t^{*k} - P_t)(P_t - P_t^o) = 0 \tag{32}$$

if equation (30) holds.

This in turn implies that

$$E_t[(P_t^{*k} - P_t^o)^2] = E_t[(P_t^{*k} - P_t)^2] + E_t[(P_t - P_t^o)^2]. \tag{33}$$

The same relation holds if the data is normalized by some variables available at time t. Especially, divide both side by P_t, from equation (33),

$$E_t[(\frac{P_t^{*k}}{P_t} - \frac{P_t^o}{P_t})^2] = E_t[(\frac{P_t^{*k}}{P_t} - 1)^2] + E_t[(1 - \frac{P_t^o}{P_t})^2]. \tag{34}$$

Such a transformation ensures stationarity so that the major criticisms of the treatment of trends of the first-generation volatility tests do not apply here.

Mankiw et al defined q_t as the corresponding sample realizations of equation (36),

$$q_t = (\frac{P_t^{*k}}{P_t} - \frac{P_t^o}{P_t})^2 - (\frac{P_t^{*k}}{P_t} - 1)^2 - (1 - \frac{P_t^o}{P_t})^2. \tag{35}$$

The null hypothesis from equation (35) is then $E_t(q_t) = 0$. By the law of iterated expectations, $E(q_t) = 0$, so the sample mean of q_t should be close to zero under the null hypothesis.

MRS used two different versions of naive price prediction. The first one is equal to the discounted value of the infinite stream of future dividends by assuming that real dividends never change from their most recently observed value, D_{t-1}. Thus

$$P_t^o = \frac{\beta}{1-\beta} D_{t-1}. \tag{36}$$

MRS also considered the other specification where a thirty-year moving average of dividends is capitalized:

$$P_t^o = \frac{\beta}{1-\beta} \sum_{i=1}^{30} D_{t-i}. \tag{37}$$

The conclusion of the MRS tests is that the rejections of the constant-return present-value model are only moderately significant. For most specifications, the rejection of the null is significant only at about the five or ten percent level. The rejections of market efficiency are moderately, but not overwhelmingly, statistically significant.

3.1.3 Campbell and Shiller: Linear VAR Approach

Campbell and Shiller (1987) found out that if the nonstationarity of the variables result from a unit arithmetic root, then the present value model expressed by equation (7) implies that P_t and D_t are cointegrated. Following Engle and Granger (1987), P_t and D_t are said to be cointegrated in order $1, 1$ (denoted as $CI(1,1)$) if there exists a scalar θ such that

$$S_t = P_t - \theta D_t \tag{38}$$

is stationary, where the cointegrating constant θ is given by

$$\theta = \frac{\beta}{1-\beta}.$$

Campbell and Shiller used a vector autoregression (VAR) to test a set of cross-equation restrictions implied by the present-value model. They used an information set H_t which contains current and lagged P_t and D_t. From equations (7) and (38), the present-value model can be rewritten as

$$S_t = E_t(\beta S_{t+1} + \theta \Delta D_{t+1}). \tag{39}$$

Using the law of iterated expectation, the present-value model implies

$$S_t = \theta \sum_{i=1}^{\infty} \beta^i E_t(\Delta D_{t+i}). \tag{40}$$

The spread S_t can be interpreted as the optimal forecast of weighted average of future changes in D_t. One can use the S_t and ΔD_t as stationary variables that summarize the bivariate history of P_t and D_t without loss of the information of cointegration relation on their levels.

Estimation of the VAR representation for ΔD_t and S_t is then:

$$\begin{pmatrix} \Delta D_t \\ S_t \end{pmatrix} = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} \begin{pmatrix} \Delta D_{t-1} \\ S_{t-1} \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix}, \tag{41}$$

where $\phi_{i,j}$ are lag polynomials[7], with $i, j = 1, 2$. Writing (41) more succinctly as

$$X_t = \Phi X_{t-1} + v_t, \tag{42}$$

Since X_t is included in the information set H_t, it follows that for all i,

$$E(X_{t+i}|H_t) = \Phi^i X_t. \tag{43}$$

Projecting equation (40) onto the information set H_t we have

[7] For simplicity reason the lag operator and the orders of ϕs are ignored. However, one should keep in mind that they are lag polynomials and their orders have to be estimated.

$$S_t = \theta \sum_{i=1}^{\infty} \beta^i E(\Delta D_{t+i} | H_t). \tag{44}$$

A set of restrictions can be obtained by using the "pick out" row arrays $g' = (0, 1)$ and $h' = (1, 0)$. Since

$$g' X_t = S_t$$

and

$$h' X_t = \Delta D_t,$$

multiplying equation (43) by h' we get

$$E(\Delta D_{t+i} | H_t) = h' E(X_{t+i} | H_t) = \Phi^i h' X_t. \tag{45}$$

Substituting equation (45) into the right-hand side of (44)

$$S_t = g' X_t = \theta \sum_{i=1}^{\infty} \beta^i \Phi^i h' X_t, \tag{46}$$

Equation (48) implies that[8]

$$g' = \theta \sum_{i=1}^{\infty} \beta^i \Phi^i h' = \theta h' \beta \Phi (I - \beta \Phi)^{-1}. \tag{47}$$

Multiplying both side of (49) by $(I - \beta\Phi)$,

$$g'(I - \beta\Phi) = \theta h' \beta \Phi. \tag{48}$$

Now a set of cross-equation restrictions is obtained. Writing it more explicitly,

$$\begin{pmatrix} 0, & 1 \end{pmatrix} \begin{pmatrix} 1 - \beta\phi_{11} & -\beta\phi_{12} \\ -\beta\phi_{21} & 1 - \beta\phi_{22} \end{pmatrix} = \theta \begin{pmatrix} 1, & 0 \end{pmatrix} \begin{pmatrix} \beta\phi_{11} & \beta\phi_{12} \\ \beta\phi_{21} & \beta\phi_{22} \end{pmatrix}. \tag{49}$$

[8] Under stationarity condition the sum $\sum_{i=1}^{\infty} \beta^i \Phi^i$ converges so that $(I - \beta\Phi)$ is invertable.

Solving these equations above, the matrix Φ is

$$\Phi = \begin{pmatrix} \phi_{11} & \phi_{12} \\ -\theta\phi_{11} & \frac{1}{\beta} - \theta\phi_{12} \end{pmatrix} \tag{50}$$

Using the VAR approach described above, Campbell and Shiller conducted a volatility test to see whether the movement of real stock prices P_t can be explained by the movement of future dividends by comparing the variance of "theoretical spread" S_t', derived from the unrestricted VAR estimation, and empirical spread. If the present- value model is true, the variance ratio $Var(S_t)/Var(S't)$ should be less or equal to 1.

Their test results from the comparison of $\widehat{Var(S_t)}$ and $\widehat{Var(S_t')}$ showed some evidence against the null hypothesis of $Var(S_t)/Var(S_t') = 1$. The point estimate of the variance ratio is 4.79 and 67.22 when $\hat{\theta} = 31.092$ and 12.195, respectively. Generally speaking, the present-value model is a poor fit for stocks, although it cannot be rejected significantly.

3.2 Unit Logarithmic Roots

3.2.1 Campbell and Shiller: Log Dividend/Price Ratio Model

Campbell and Shiller (1988a) used a model that is log-linear in prices, dividends and discount factors. They derived a log-linear approximation to the present-value model by first linearizing the log return $log(\frac{(P_{t+1}+D_{t+1}}{P_t})$ to yield the approximate log return:

$$\psi_t = \rho(p_{t+1} - p_t) + (1 - \rho)(d_t - p_t) + k, \tag{51}$$

where p_t is log prices per share, d_t is log dividend per share, ρ is the discount factor around which the return was linearized and k is a constant. The discount factor ρ equals $\frac{1}{1+r^*}$, where r^* is the average dividend-price ratio. So the approximated return ψ is just a weighted average of growth rate of price $(p_{t+1} - p_t)$ and of the log dividend-price ratio $(d_t - p_t)$ where most of the weight is put on the growth rate of price, plus a constant.

The log-linear version of efficient market hypothesis is then

$$E_t\psi_t = E_t r_t, \tag{52}$$

where r_t is a discount rate, and

$$p_t = E_t(p_t^*), \tag{53}$$

where p_t^* is given by

$$p_t^* = (1-\rho)\sum_{i=0}^{\infty}\rho^i d_{t+i} - \sum_{i=0}^{\infty}\rho^i r_{t+i} + \frac{k}{1-\rho}. \tag{54}$$

So the ex-post value p_t^* is equal to a weighted average of future log dividends minus a term proportional to a weighted average of expected future one-period interest rates.

Expressing the log dividend/price ratio as the rational expectation of the present value of future dividend growth rates and discount rates, the problem of unit roots is avoided. Using a log analogy VAR framework described in 3.2.1, they conducted various tests. The fundamental conclusion from their empirical work is that the variance of the approximate log return ψ_t should be less than it is, given the vector of information that includes price. Table 1 compares the standard deviation of ψ_t with that of ψ_t', the theoretical log return at time t predicted by the model. The latter is only a quarter as large.

3.2.2 LeRoy and Parke

LeRoy and Parke (1992) proposed correcting for the trend in stock prices by dividing prices by dividends. They assumed that dividends follow a geometric random walk process so that price/dividend ratio is stationary and conduct four types of volatility tests on this ratio, rather on price itself. The tests are classified via a two-by-two scheme. Firstly, bounds tests and orthogonality tests are constructed. Bounds tests determine the validity of the variance bounds inequality along the line of

$$Var(P_t) <= Var(P_t^*).$$

If the current dividend is in I_t, the present value model (8) implies that for each t,

$$\frac{P_t}{D_t} = E(\frac{P_t^*}{D_t}|I_t) \tag{55}$$

and

$$Var(\frac{P_t}{D_t}) <= Var(\frac{P_t^*}{D_t}). \tag{56}$$

Orthogonality tests, devised by LeRoy and Porter (section 1.2), take the form of an equality:

$$Var(P_t^*) = Var(P_t) + \frac{\beta^2 Var(P_{t+1} + D_{t+1} - \beta^{-1}(P_t)}{1 - \beta^2}. \tag{57}$$

The analog to (57) that is valid under log-linear dividend processes is [9]

$$Var(\frac{P_t^*}{D_t}) = Var(\frac{P_t}{D_t}) + \frac{\beta^2[Var(\frac{P_t}{D_t} + E(\frac{P_t}{D_t})^2]}{1 - \beta^2(\sigma^2 + \mu^2)} V(e_t), \tag{58}$$

where μ and σ are the mean and standard deviation, respectively, of the dividend growth rate. The interpretation of (58) is the same as that of (13) in section 1.2: in (60) the variance of the price-dividend ratio plus the variance of the rate of return equals the variance of the ex-post rational price-dividend ratio. The more information agents have, the higher is $Var(\frac{P_t}{D_t})$ and the lower is $Var(e_t)$.

LeRoy and Parke then conducted model-free tests and model-based tests to generate the ex-post rational prices: the model-free procedure uses $P_{t|T}^*$, the observable version of P_t^* obtained from the recursion (14) and the terminal condition $P_{T|T}^* = P_T$. Imposing the assumption that dividends follow a geometric random walk, $ln(D_{t+1}) = ln(D_t) + \epsilon_{t+1}$, the model-based procedure first estimates the discount factor and the mean and variance of the dividend growth rate, then enters these as arguments in the expression for the variance of ex-post rational prices.

LeRoy/Parke found that while the model-free bounds test rejects the present-value model and the model-based bounds test accepts it, both the model-free and model-based orthogonality tests reject the null-hypothesis significantly for any specification of agents' information.

3.3 Summary

The critics on first-generation volatility tests gave proponents of market efficiency reason to hope that the apparent evidence of excess volatility was

[9] See LeRoy/Parke pp.988 for a derivation.

entirely a consequence of flawed econometric procedures. The next round of volatility tests explicitly allow for unit roots, but they still tend to find some excess volatility. The second-generation volatility tests make it evident that, despite resolution of the statistical controversy surrounding the initial tests, the variance-bounds violations were here to stay.

4 Possible Explanations

The simple constant-return present-value model represented by equation (8) and (9) does not seem to be supported by the data. In this section, some suggested explanations for excess volatility of stock prices are reviewed. The first alternative is the relaxation of the fair game assumption of stock return. The second alternative is the relaxation of the transversality condition (6) imposed on stock prices. The third alternative, which is the most radical one, suggests that the stock prices are moved by fads.

4.1 Time-Varying Discount Rates

The first generation and most of the second generation volatility tests [10] impose a constant discount rate. A natural candidate to explain any excess price volatility is movements in expected returns. Prices will vary, even if dividends don't change, if the rate at which dividends are discounted varies over time. In response to this criticism, volatility tests have been generalized to include measures of time-varying discount rates. A generalized present-value model with time-varying discount rates (expected returns) is

$$P_t = E[\sum_{j=1}^{\infty}(\prod_{k=1}^{j} \beta_{t+k})D_{t+j}|I_t], \tag{59}$$

where β_{t+k} is the discount rate such that the Euler equation

$$1 = E(\beta_{t+1}R_{t+1}|I_t) \tag{60}$$

holds with

$$R_t = 1 + r_t = \frac{P_{t+1} + D_t}{P_t}. \tag{61}$$

[10] With exceptions of Campbell/ Shiller (1988a, b) and Mankiw/Romer/Shapiro (1991).

Hence, if β_t equals β, a constant, equation (59) specializes to the constant discount-rate model studied before, and equation (60) specifies that discounted returns $\beta_{t+1}R_{t+1}$ should be unforecastable.

However, the discount rate β_t is not directly observable, so one must use some model or proxy for the discount rates to conduct a test. Two volatility tests that allow for time-varying discount rates have been conducted: the first study by Campbell/Shiller (1988a,b) and Mankiw/Romer/Shapiro assumed that dividends were discounted by an interest rate plus a constant risk premium. Since real interest rates do not vary much, these interest-rate-based tests continue to reject the present-value model.

A second study infers discount-rate variation from measures of real investment opportunities (marginal rates of substitution) in the economy. For example, the consumption-based asset-pricing model (Lucas (1978)) ties discount rates to aggregate consumption. In Lucas' model, if the representative consumer maximizes his expected life time utility

$$E\sum_{i=0}^{\infty}(1+\rho)^{-i}U(C_{t+i}), \tag{62}$$

where $U(C_t)$ is the agent's instantaneous utility function and ρ is his time preference coefficient. The equilibrium prices satisfy the stochastic Euler equation

$$P_tU'(C_t) = (1+\rho)^{-1}E_t[(P_{t+1}+D_{t+1})U'(C_{t+1})], \tag{63}$$

where $U'(C_t)$ and $U'(C_{t+1})$ are the marginal utilities. The interpretation of the Euler equation (63) is following: suppose an investor is considering selling one share of stock and consuming the proceeds. The utility gain is $P_tU'(C_t)$. The budget constraint implies a drop in consumption at $t+1$ of $P_{t+1}+D_{t+1}$. The RHS of equation (63) gives the expected utility cost of the decline in consumption, discounted back to t. At his optimum, the agent's utility gain at t must equal the expected utility loss at $t+1$. Under the assumption of risk neutrality, $U'(C_t) = U'(C_{t+1})$. So equation (63) reduces to the constant-return present-value model (4). Therefore, martingales generally would obtain only if agents are risk neutral.

Campbell/Shiller (1987, 1988a) constructed volatility tests with this consumption-based model with constant relative risk aversion, $U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}$. They found little theoretically plausible connection between stock prices and their measures of expected returns.

There are some problems associated with the test of the consumption-based asset-pricing model described above. For example, the validity of the parameterization of the utility function, or the assumption of the representative agent, etc. However, if one accepts the constant relative risk aversion utility function, then not much evidence supports that the consumption-based asset-pricing model properly explain excess volatility: greater risk aversion implies greater price volatility than in the present value model. But for the utility to produce violation of the variance bounds of the observed magnitude, the aggregate consumption series must be much more volatile than it actually is. The stylized fact is that the post World War II consumption series is very smooth, which implies an implausibly high estimated value of the coefficient of relative risk aversion.

4.2 Bubbles

Another possible explanation for excess volatility is that prices are disturbed by "rational bubbles". The stochastic difference equation such as (4) has multiplicity of solution. The solution (7) is unique provided that the transversality condition (6) holds. If the transversality condition doesn't hold, there is a continuum of solutions

$$P_t = E(\sum \beta^{i+1} D_{t+1} | I_t) + B_t \equiv P_t^f + B_t, \tag{64}$$

where B_t is any stochastic process that satisfies

$$E(B_t | I_{t-1}) = \beta^{-1} B_{t-1}, \tag{65}$$

or

$$B_t = \beta^{-1} B_{t-1} + b_t, \tag{66}$$

where b_t is the innovation in the bubble at time t, with $E(b_t | I_{t-1}) = 0$. P_t^f denotes the value of the prices that depends only on fundamentals. B_t is by definition a rational bubble, an extraneous event that affects stock prices because everyone expects it to do so. Often the idea of existence of bubbles is traced to Keynes's (1936) description of an equity market as an environment in which speculators anticipate "what average opinion expects average opinion to be," rather than focusing on things fundamental to the market.

A bubble thus represents a deviation of the stock prices from the value implied by market fundamentals. Bubbles suggest precisely the speculators' behavior efficient-markets critics had in mind: stock prices moves, without news about dividends, simply due to self-fulfilling prophecies of market participants. So the question here is, does the excess volatility of stock prices strongly suggest rational bubbles? The answer up to today is still ambiguous. Several reasons, however, make the bubble alternative less attractive. First, there are some volatility tests which implicitly allow bubbles under the null. That is, these tests do not actually impose the transversality condition (6). For example, the MRS tests (section 2.1.2) use the k−step-ahead price as the terminal price,

$$P_t = E_t(\sum_{i=1}^{k-1} \beta^i D_{t+i} + \beta^k P_{t+k}). \tag{67}$$

Hence bubbles cannot explain rejection of such volatility tests.

Second, from equation (66) we see that if bubbles exist, they must be expected to grow at the rate of $\beta^{-1} = 1 + \gamma$. Thus bubbles suggest that stock prices should grow at a more rapid rate than dividends[11]. Therefore, the dividend/price ratio should fall and capital gains should take an increasing large share of ex post returns. However, according to West (1988), for the Standard and Poor's data, 1871-1986, this is not the case. The mean ex post return in the first half of the sample, 1872-1928, is 8.6 percent, with a mean dividend/price ratio of 0.053; in the second half of the sample they are 8.3 percent and 0.051.

Third, it turns out that the transversality condition (6) imposes no testable restrictions in a finite sample. The seeming excess volatility in stock prices shows that price changes are not justified by the subsequent dividends in the sample, so price must move in response to changing expectations of some event not seen in the sample. In a bubble, that event is the limit of the discounted terminal price. But the bubble alternative cannot be distinguished from changing news about dividends beyond the sample (e.g. disasters, wars, etc.).

In sum, many authors (including Shiller, West) do not interpret volatility-test rejection as evidence for bubbles, for a variety of theoretical and empirical reasons.

[11] More generally, there is a bubble if and only if the (log) price has a unit root not found in dividends (Cochrane (1991))

4.3 Nonstandard Models: Fads and Noise Traders

As mentioned in the beginning of this paper, efficient market hypothesis is actually a joint hypothesis of rational expectation plus martingale assumption. While most of the volatility tests still tend to reject the generalized present value model for stock prices, i.e. dropping out the martingale assumption, rational bubbles do not seem to provide a satisfactory explanation. For many authors (including for example LeRoy, Shiller, Summers, West etc.), the constant rejection of volatility tests suggests that it might be useful to consider some alternative to the efficient market hypothesis, not modifications of it. The fads model proposed by Shiller and Summers, argues that price movement are due to changes in opinion or psychology. Market participants may not be full rational and have rational expectations. In "fads" interpretations of the excess volatility, the stock price deviates from the present value of future dividends due to the presence of irrational traders in the market.

Empirical rejection of efficient market hypothesis suggests some predictability of stock returns. Evidence has been found that a number of variables, including price/dividend ratios and long-run corporate earnings[12] do in fact forecast small persistent changes in returns. Fama and French (1988) estimated directly the correlation between average returns over the interval from $t-T$ to t - denoted as $r_{t-T,t}$ - with $r_{t,t+T}$ for various values of T. They found a $U-$ shaped pattern: For T of one year the correlation is almost zero. For T on the order of three to five years about 35 percent of the variation of $r_{t,t+T}$ is explained by $r_{t-T,t}$. For T of ten years the correlation reverts to approximately zero. Shiller (1984) and Summers (1986) propose that instead of modeling stock prices as a martingale, one should consider assuming that price consists two components: a random walk plus a fad variable, where the latter is a slowly mean-reverting stationary process. West (1988b) gave an example of this kind, in which

$$log(D_t) = \mu + log(D_{t-1}) + \epsilon_t, \tag{68}$$

$$log(P_t) = \tau + log(D_t) + a_t, \tag{69}$$

$$a_t = \phi a_{t-1} + \nu_t, \tag{70}$$

[12] For example, Campbell and Shiller (1988b) found that earnings is a strong predictor of dividend growth even conditional on the current log price/dividend ratio.

where $|\phi| < 1$, ϵ_t and ν_t are uncorrelated white noises. Equation (68) says that dividends follow a logarithmic random walk, as suggested in Kleidon (1988) and LeRoy and Parke (1992). Equation (69) and (70) say that the mean log price/dividend ratio τ is perturbed by the AR(1) random variable a_t. According to Shiller and Summers, one can interpret a_t as a "fad", optimistic or pessimistic waves which influence investors expectations , that drives the stock price away from the fundamental value.

However, several studies have questioned Fama and French's conclusion. Lo and MacKinlay (1988) found that weekly and monthly stock returns have positive autocorrelation coefficients on the order of 30 percent. Kim, Nelson and Startz (1989) found evidence of mean reversion only in data sets that include the 1930s - for the post World War II period they found no evidence of negative return autocorrelation. The question whether excess volatility and mean reversion imply each other is still open.

Nonetheless, the core of the efficient market hypothesis - the assumption of rational expectation of all agents - seems not to be very plausible for many people. In a series of papers, De Long, Schleifer, Summers and Waldmann (1989. 1990a,b)[13] raised the question of rationality of all agents. An earlier argument for full rationality is that traders who act irrationally will lose wealth on average. So the irrational agents will disappear from the population in the long run. De Long et al, however, argued that if the irrational agents have unrealistically optimistic estimation of possible outcomes, they may be less risk averse than the rational agents. They developed a model in which some fraction of trading is done by naive (noise) traders, another fraction of trading is done by sophisticated (rational) investors. Risk is created by noise traders, which sophisticated investor must take into account. Since in a population of risk-averse agents the average return to risk takers exceed those to risk avoider, this implies that the risk takers as a whole do better than the risk avoiders. So "irrationality" may actually be rewarded in the aggregate[14] Thus, stock prices are more volatile than under the present value model in an equilibrium. This equilibrium is stable in the sense that noise traders do not necessarily die out in the long run. There is still little quantitative evidence in favor of fads as an explanation of stock price volatility, however[15].

[13] Denoted as DSSW in the following.

[14] Robert Kunst told me that if the noise traders can make more money in the stock market than the sophisticated ones, then they are not "irrational". In the DSSW model there is a wedge between utility maximization and return maximization. Rational trader is the one who maximizes his utility function, and the noise trader maximizes his expected return.

[15] See next chapter for noise traders models.

5 Conclusions

The efficient market hypothesis dressed in the literature is actuarially a complex hypothesis of rational expectation and martingale model for stock prices. The first-generation volatility tests from LeRoy/Porter and Shiller rejected the efficient market hypothesis. The test results were striking since these were one of the first empirical evidence against market efficiency. It was pointed out by Flavin and Kleidon that the initial volatility tests suffer some econometric flaws which make the test results unconvincing. The question raised hereafter is whether these econometric problems are severe enough to account for the seemly excess volatility. The next-round volatility tests emphasized on developing tests that have acceptable econometric properties under realistic dividend models. With the exception of MRS, the second-generation tests still found statistically significant excess volatility, however the order of magnitude is smaller.

Whether the constant rejection of the present value model is significant evidence against market efficiency is debatable. For some researchers, the evidence up to today is strong enough to be a signal for paradigm shift, in which the rational expectation assumption being replaced by a model in which fads or noise traders' misperception are the driving force behind day-to-day stock price movements. Other researchers, however, still expect that extending rational economic models can explain the price volatility. In sum, the debate still goes on.

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