# The Vanishing Savings Motive

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# 1 Introduction

Relying on models of a small open economy populated by overlapping generations (OLG) with life-time uncertainty, some authors [e.g. Bovenberg (1991, 1992) and Matsuyama (1987)] recently found that non-monotonic adjustment of the current account is well possible. However, such dynamic phenomena still lack a convincing explanation. While Bovenberg noted this possibility and also generated interesting numerical cases, he did not provide much of an explanation regarding the source of such dynamics. When analyzing the effects of an oil price increase, Matsuyama did not interpret the non-monotonicity in the stock of domestic savings but argued that overshooting in the current account is caused by a portfolio substitution effect. Contrary to this contention, the present note argues that all of the interesting dynamics is due to the response of savings instead. Slowly falling (increasing) wages create transitory life-cycle savings (dis-)incentives which vanish in the long-run when wages become stationary again. Such a transitory life-cycle savings component comes on top of a base component created by the permanently operating long-run savings incentives, and it may easily give rise to non-monotonic adjustment of assets. Assets may even move first in a direction that is opposite to the implied long-run changes.

The vanishing savings motive is most clearly seen in a small open economy framework with an internationally determined interest rate. When the production sector adjusts gradually to some exogenous shocks, then wage income also changes gradually during a transitional period creating an additional life-cycle type savings component. Given that shocks are permanent and the interest rate is fixed, intertemporal relative prices remain unaffected and no intertemporal substitution is involved. With intuition coined by the analysis of representative agent (RA) models or two period lived OLG models with first period labor income, most economists would probably expect monotonic adjustment of the stock of savings. These alternative intertemporal models cannot capture the effects of slowly changing wage profiles on life-cycle savings decicions either because the adjustment of savings requires only one period, or because consumption is based on permanent income without any life-cycle considerations at all. Hence, the vanishing savings motive is a unique feature of OLG models with an extended life-cycle earnings profile.

The insight is relevant in many applications such as fiscal or commercial policy, terms of trade changes, price changes of imported raw materials, technology shocks to mention just a few. One easily imagines that production adjusts slowly to permanent shocks of such kind, and that intertemporal relative prices remain unaffected in a small open economy. To emphasize our point, we just take a gradual change in wages as given, and analyze the implications for savings. Choosing a largely identical notation, the presentation of the model is short since it is explained in all necessary detail in Blanchard (1985) and Blanchard and Fischer (1989) [see also Weil (1989) and Buiter (1988) for variations and a synthesis].

# 2 Overlapping Generations with Life-time Uncertainty

New agents continuously enter the economy. They are disconnected to preexisting dynasties and start their life with zero financial wealth. While the life span of each individual is uncertain, mortality risk cancels out in the aggregate and the fraction of the population dying at each date is deterministic. Since agents differ in their age and in the amount of previously accumulated wealth, they need to be carefully distinguished by their date of birth. Given a stream of wage income and initial assets, agents are assumed to maximize expected utility of consumption over their remaining life-time,

$$\begin{aligned} \max \quad U_{s,t} &= \int_t^\infty u(c_{s,z}) e^{-(\theta+p)(z-t)} dz, \\ s.t. \quad \dot{v}_{s,z} &= (r+p) v_{s,z} + y_{s,z} - c_{s,z}. \end{aligned} \tag{1}$$

Notation: $c_{s,z}$ consumption, $y_{s,z}$ labor income and $v_{s,z}$ assets of an agent born at date $s$ as of time $z$, $r$ constant interest rate, $p$ mortality rate and fair insurance premium, $r+p$ effective interest, $\theta$ subjective discount rate, $\theta+p$ risk adjusted discount rate. Wage income may accrue according to an earnings profile declining at a rate $\alpha$ with age in order to mimic the life-cycle motive of savings for retirement. Hence, generation specific wage income $y_{s,t}$ is tied to economy wide averages according to $y_{s,t} = Y_t a e^{-\alpha(t-s)}$. The coefficient $a = (p+\alpha)/p$ makes cohort specific wages sum up to aggregate wages.

Intertemporal optimization yields the Euler equation governing generational consumption profiles, $\dot{c}_{s,t}/c_{s,t} = (r-\theta)/\sigma$, where the inverse of $\sigma$ indicates the intertemporal elasticity of substitution.[1] Whenever the interest rate exceeds the subjective rate of time preference, agents want consumption to increase with age. Hence, this condition determines – together with the life-cycle pattern of income – whether households save or dissave. When wages remain constant, the life-cycle income pattern declines with rate $\alpha$, and we may describe the permanent savings incentives by the parameter $\xi \equiv \alpha + (r-\theta)/\sigma$. For example, when agents are rather impatient and desire consumption to decline at a rate equal to $\alpha$, then new agents just consume current income and save nothing. If this parameter is positive, then consumption is tilted towards the future relative to an income stream declining at rate $\alpha$, and new agents save early in life to provide for old age consumption. Alternatively, one may view it as income being tilted towards the present.[2]

Agents allocate a fraction of their discounted life-time income to current consumption, $c = m(v+h)$, where $h$ is human wealth.[3] Given a fixed interest rate, the marginal propensity to consume out of wealth is constant and equal to $m = r + p - (r-\theta)/\sigma$. Since there is individual but no aggregate uncertainty, individual risks of life can be fully insured. The macroeconomic aggregates which are denoted by capital letters, obtain from summing over all generations. Consumption and asset accumulation by all generations taken together is described by [compare Blanchard and Fischer (1989), p.122, for $\sigma = 1$]

$$\begin{bmatrix} \dot{C}_t \\ \dot{V}_t \end{bmatrix} = \begin{bmatrix} \xi & -m(p+\alpha) \\ -1 & r \end{bmatrix} \begin{bmatrix} C_t \\ V_t \end{bmatrix} + \begin{bmatrix} 0 \\ Y_t \end{bmatrix}. \tag{2}$$

When wages are stationary, the levels of assets and consumption ultimately depend on the amount of wage income $Y$. Use $C = m(V+H)$ and get the stationary solution

$$\bar{C} = \frac{-m(p+\alpha)}{|Z|}\bar{Y}, \qquad \bar{V} = \frac{-\xi}{|Z|}\bar{Y}, \qquad \frac{\bar{V}}{V+\bar{H}} = \frac{\xi}{p+\alpha}. \tag{3}$$

The model makes sense only if the marginal propensity $m$ to consume out of life-time income is positive, and if stationary consumption is positively related to income $Y$. These restrictions guarantee a

[1] In case of a fixed interest rate, the RA model requires $r = \theta$ for existence of a stationary state.

[2] For a vivid discussion of the consumption tilting and other savings motives, see Frenkel and Razin (1987).

[3] Blanchard (1985) derives in detail the individual as well as the aggregate solutions. In his notation, $m = \Delta^{-1}$.

negative determinant $|Z| = r\xi - m(p+\alpha) < 0$ of the coefficients matrix.[4] Thus, saddle point stability of the dynamical system obtains with the two roots satisfying $\eta < 0 < \bar{\eta}$. The parameter $\xi$ captures the savings incentives. While individual savings incentives do not diminish with the level of financial wealth, aggregate accumulation grinds to a halt since the size of cohorts melts away with age. Hence, the savings parameter also fixes aggregate assets and, thus, the long-run composition of total wealth.

# 3 The Vanishing Savings Motive

In many applications, wage income will only gradually approach its long-run stationary level. For example, if capital is sticky because of installation costs, capital accumulates slowly and wages accordingly change only gradually as the production sector of a small open economy (with a fixed interest rate) adjusts to technology shocks, terms of trade changes, tariffs, oil price increases, capital taxation and so on. Suppose the adjustment speed of the production sector is equal to some $\mu < 0$. Hence, the change in wage income may be approximated by $Y_t = \bar{Y} + (Y_0 - \bar{Y})e^{\mu t}$. The remainder of the paper deals with the adjustment of aggregate assets as well as consumption to such an income path. As a first step, one needs to pin down the level of initial consumption that is consistent with bounded long-run solutions. From the aggregate consumption function,

$$C_0 = m(H_0 + V_0), \qquad H_0 = \int_0^\infty Y_t e^{-(r+p+\alpha)t} dt = [Y_0 - \bar{Y}]\frac{1}{r+p+\alpha-\mu} + \bar{Y}\frac{1}{r+p+\alpha}. \tag{4}$$

Initial assets are predetermined. Aggregate human wealth is the present value of the flow of future wage income. As indicated by the two summands in (4), it is composed of a two parts. The first transitory component of human wealth may be positive or negative depending on whether short-run exceeds or falls short of long-run income. Its size depends on the adjustment speed $\mu$ and the excess of short-run over long-run income. The second permanent component is always positive. A separate appendix which is available upon request, derives the complete analytical solutions. Given the boundary conditions,

$$V_t = V_0 e^{\eta t} + \bar{V}(1 - e^{\eta t}) + (\xi - \mu)(\frac{Y_0 - \bar{Y}}{\bar{\eta} - \mu})(\frac{e^{\eta t} - e^{\mu t}}{\eta - \mu}). \tag{5}$$

Consider first the accumulation of assets in face of a constant stream of wage income and, consequently, constant human wealth. All the dynamics arises from the fact that the initial aggregate stock of assets is too low or too high. The last part of the solution in (5) becomes irrelevant as $Y_0 = \bar{Y}$. Hence, adjustment is monotonic. With constant human wealth and too low initial assets, the economy is initially populated by too many poor agents and only a few rich. Aggregate assets increase as all agents save. Later when a larger fraction of the population is rich, the accumulation of aggregate assets is brought to a halt since

[4] The effective discount rate for human wealth is $r+p+\alpha = \xi + m > 0$. Note furthermore that $r - m = (r-\theta)/\sigma - p < 0$ since the rate of extinction $p$ must exceed the growth rate of individual consumption $(r-\theta)/\sigma$ for aggregate consumption to be stationary in a steady state. The determinant may then be written as $|Z| = (\xi+m)(r-m) < 0$. It is readily verified that the characteristic polynomial $\psi(s) = |sI - Z|$ satisfies $\psi(\xi + r) = \psi(0) = |Z| < 0$, $\psi(r) = \psi(\xi) = -m(p+\alpha) < 0$ as well as $\psi(\xi + m) = \psi(r - m) = 0$. Hence, the unstable root is equal to the discount rate of human wealth, $\bar{\eta} = r + p + \alpha$, and the stable root is $\eta = (r-\theta)/\sigma - p$.

the rich die away at the rate of extinction $p$. Eventually, a stable distribution of rich and poor agents is obtained. With constant human wealth, consumption increases in line with aggregate assets until a stable composition of financial and human wealth is reached [see (3)]. Adjustment is monotonic.

Now consider the case where present income exceeds future income. The fact that wages decline gradually at rate $\mu$, however, creates a transitory life-cycle type savings motive which is a potentially powerful source of savings in the early phase of adjustment. This savings component vanishes in the long-run when wage profiles become flat again. The declining nature of wage income creates an augmented savings incentive $\xi - \mu$ during the transition. Hence, aggregate assets adjust excessively in the short-run which is captured by the last term in (5). It is easily verified that this transitory component starts out with zero and asymptotically approaches zero again. Since wage income declines, it is unambiguously positive during the transition, at least so if the augmented savings incentive $\xi - \mu$ is positive. The power of the transitory savings motive hinges both on the magnitude of the income gap $Y_t - \bar{Y}$, and on the rate $\mu$ with which it disappears. At the same time, $\mu$ captures the additional transitory life-cycle motive for savings. Agents living at some early date would see their life-cycle wage income decline at an augmented rate $\alpha + \mu$ which compares with the optimal growth rate of individual consumption $(r - \theta)/\sigma$ to determine the augmented savings incentive $\xi - \mu$. A high rate of wage adjustment indicates steep negatively sloped wage profiles which operate for a short time only and affect only those generations living very early in the transition period. Slow adjustment of wages implies moderately falling wage incomes over a longer time horizon. Hence, many generations living during the transition period are affected. As income adjustment is completed, the income gap $Y_t - \bar{Y}$ vanishes and new generations eventually miss any additional life-cycle savings incentive. The first two terms in (5) reflect the monotonic adjustment of aggregate assets that would obtain if agents saw a constant stream of wage income equal to $\bar{Y}$.

The phase diagram of the savings system given in fig.1 provides a convenient graphic explanation. A rather flat $\dot{C} = 0$-line reflects weak permanent savings incentives. The initial position of the $\dot{V} = 0$ is nailed down by present wage income $Y_0$. As wage income is eroded, the $\dot{V}_0$-line shifts to the left with speed $\mu$ and eventually comes to a rest at a position determined by long-run wage income $\bar{Y}$. The saddle path SS would indicate the convergent path if wages were permanently equal to $\bar{Y}$. The curved line gives the convergent path which savings and spending actually follow in case of declining wages. As long as wage income is high, the dynamics of spending and savings is governed by the $\dot{C}$- and $\dot{V}_0$-schedules. With sufficiently large initial assets and continuity of the adjustment paths under perfect foresight, the starting position must lie in the north western region of the intersection of the $\dot{C}$ and $\dot{V}_0$-schedules where the dynamics indicate decreasing expenditures and increasing assets. If assets are rather low initially, the initial condition is placed to the south western region of the $\dot{C}$ and $\dot{V}_0$-schedules making both savings and spending increase in the initial phase. If wages decline fast, the $\dot{C}$ and $\dot{V}_\infty$-schedules soon begin to dominate the dynamics making assets as well as spending decline. If wages decline slowly instead, the $\dot{V}_0$-schedule dominates the dynamics for a longer period. Certainly assets must increase above the levels indicated by the $\dot{C}$-line in order to approach the saddle path SS indicating the direction of changes in spending and assets in the long-run when wages are low. These arguments clearly demonstrate that the source of overshooting in savings lies in gradually declining wages which continuously shift the $\dot{V}$-schedules in fig.1.

In economic terms, all of the generations living early in the transition period see their wages fall. Life-cycle considerations induce them to save a lot making aggregate financial wealth increase. Later on, wage profiles become flat again at a lower level and the temporary savings motive vanishes. Aggregate assets start to decline as those generations who became rich because of their previous savings die away and are replaced by young poor agents with diminished savings incentives. The decumulation of aggregate assets comes to a rest when the overall population approaches a stable distribution of rich and poor agents. The transitory savings motive can be easily isolated by considering the case of zero permanent savings incentives, $\xi = 0$. In this case, the $\dot{C}$-line in fig.1 coincides with the horizontal axis. If initial assets happen to be zero, then the transitory savings motive is the only source of savings. The phase diagram immediately reveals that aggregate assets start from zero and accumulate during an early phase of adjustment since initial agents start to save due to falling wages. Later on the population is quite wealthy but the size of the rich cohorts is constantly reduced by death and the population is continuously replaced by young poor agents who miss any savings incentives in face of flat wage profiles. In the long-run, when the population is completely replaced by poor agents who never experience any life-cycle type savings incentive, aggregate assets come to a rest at a level of zero. This shows that in an economy with a savings deficiency reflected in a small magnitude of the permanent savings incentive, the permanent savings component may be easily dwarfed by the magnitude of the transitory component giving an overshooting, non-monotonic adjustment of the stock of total savings.

# 4 Conclusions

This note demonstrated that the life-cycle properties of the overlapping generations model with life-time uncertainty may easily give rise to non-monotonic overshooting asset accumulation. Gradually changing wage income provides for a temporary life-cycle type savings motive. This may produce overshooting asset accumulation even though intertemporal price ratios remain invariant in a small open economy. Most economists probably find it rather surprising that adjustment should be non-monotonic if intertemporal substitution is excluded. The analysis of RA models or OLG models with two period life-cycles actually support the expectation of gradual and monotonic adjustment. In a sense, both models display only rudimentary savings dynamics when interest rates are fixed. The RA model implies complete consumption smoothing allowing for monotonic adjustment of net foreign assets only. When the two period OLG model misses old age wage incomes, it features a degenerate life-cycle that cannot capture the kind of transitory life-cycle motives emphasized in the present paper. Hence, it also allows for monotonic adjustment only. The OLG model with life-time uncertainty does include life-cycle savings considerations and, thus, is capable of generating a ray of rather surprising overshooting phenomena. For example, transitory savings may easily make the stock of assets move first in a direction opposite to the desired long-run change.

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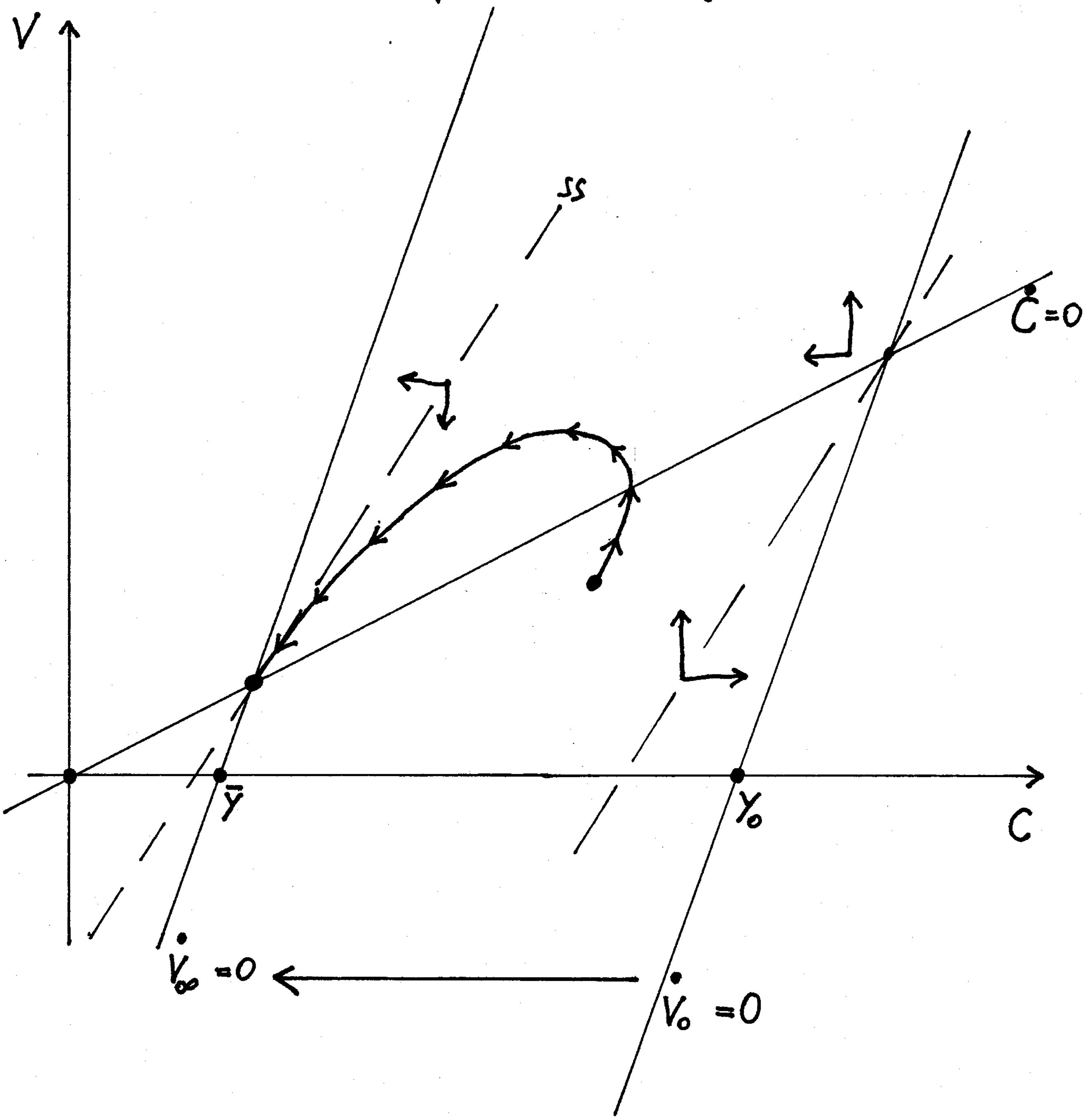
# Fig. 1: Aggregate Dynamics



**The Vanishing Savings Motive: Separate Appendix**
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Consider a dynamical system $\dot{X}_t = ZX_t + B_t$ such as given in (2) with $X_t = [C_t, V_t]'$ and $B_t = [0, Y_t]'$. This appendix derives explicit solutions for the dynamic variables contained in $X_t$. Since the absolute term depends on time, the standard solution formulas are not applicable any more. But one can solve via the method of Laplace transforms. Denoting the coefficients of the $Z$-matrix in (2) by $z_{11} = \xi$, $z_{12} = -m(p+\alpha)$, $z_{21} = -1$, and $z_{22} = r$, the Laplace transform of the system is

$$\psi(s)\begin{bmatrix} L_s[C_t] \\ L_s[V_t] \end{bmatrix} = \begin{bmatrix} (s-z_{22}) & z_{12} \\ z_{21} & (s-z_{11}) \end{bmatrix}\begin{bmatrix} C_0 \\ L_s[Y_t]+V_0 \end{bmatrix}. \tag{1}$$

Evaluating the transform at a rate $s = \bar{\eta} > 0$ equal to the positive eigenvalue makes the polynomial $\psi(\bar{\eta})$ zero. Upon the non-explosion condition, the l.h.s is zero which pins down the initial value of the jump variable. As the forcing variable $Y_t$ adjusts with the speed $\mu$, its Laplace transform is easily computed as

$$\begin{array}{llcl} (a) & Y_t & = & [Y_0 - Y_\infty]\, e^{\mu t} + Y_\infty, \\ (b) & L_s[Y_t] & = & [Y_0 - Y_\infty]\frac{1}{s-\mu} + Y_\infty \frac{1}{s}. \end{array} \tag{2}$$

The long-run solutions stated in (3) in the text may be more compactly written as

$$C_\infty = \frac{z_{12}}{\bar{\eta}\eta} Y_\infty, \qquad V_\infty = \frac{-z_{11}}{\bar{\eta}\eta} Y_\infty. \tag{3}$$

Evaluating (1) at rate $\bar{\eta}$ yields the restrictions

$$\begin{array}{llcl} (a) & (\bar{\eta} - z_{22})C_0 & = & -z_{12}\left\{L_{\bar{\eta}}[Y_t] + V_0\right\}, \\ (b) & -z_{21}C_0 & = & (\bar{\eta} - z_{11})\left\{L_{\bar{\eta}}[Y_t] + V_0\right\}. \end{array} \tag{4}$$

Note that the transform in (2b) is equal to aggregate human wealth at date 0 if evaluated at rate $s = \bar{\eta}$. Since the positive eigenvalue is the discount rate of human wealth, $\bar{\eta} = r+p+\alpha$, we have $-z_{12}/(\bar{\eta}-z_{22}) = m$. Furthermore, fn.3 in the main text shows $\xi + m = \bar{\eta}$ which also gives $(\bar{\eta}-z_{11})/-z_{21} = m$. Hence, both conditions given in (4) which are in fact equivalent, pin down initial consumption by $C_0 = m(H_0 + V_0)$.

The inverse operation on (1) yields the complete solutions for the adjustment paths. Using (2b) and (3), we first write the transform more conveniently as

$$\begin{array}{lcl} L_s[C_t] & = & \frac{1}{\psi(s)}\left\{C_0 s + \frac{z_{12}}{s}Y_\infty + \frac{z_{12}}{s-\mu}(Y_0 - Y_\infty) - z_{22}C_0 + z_{12}V_0\right\}, \\ L_s[V_t] & = & \frac{1}{\psi(s)}\left\{V_0 s - \frac{z_{11}}{s}Y_\infty + \left[\frac{s}{s-\mu} - \frac{z_{11}}{s-\mu}\right][Y_0 - Y_\infty] + z_{21}C_0 - z_{11}V_0 + Y_\infty\right\}. \end{array} \tag{5}$$

Note $\psi(s) = (s-\bar{\eta})(s-\eta)$. To find the reverse transform and the complete solutions for the transition, we need some further results to apply the formulas for Laplace operations. Specifically,

$$\begin{array}{llclcl} (a) & \frac{1}{\psi(s)} & = & \frac{1}{(s-\bar{\eta})(s-\eta)} & = & \frac{1}{(\bar{\eta}-\eta)}\left[\frac{1}{(s-\bar{\eta})} - \frac{1}{(s-\eta)}\right], \\ (b) & \frac{s}{(s-\mu)\psi(s)} & = & \frac{s}{(s-\mu)(s-\bar{\eta})(s-\eta)} & = & \frac{1}{(\bar{\eta}-\eta)}\left[\frac{s}{(s-\mu)(s-\bar{\eta})} - \frac{s}{(s-\mu)(s-\eta)}\right]. \end{array} \tag{6}$$

Show (6a) by the method of partial fractions. Since $\psi(s)$ has distinct roots, we have

$$\tfrac{1}{(s-\eta)(s-\bar\eta)} = \tfrac{C_1}{(s-\eta)} + \tfrac{C_2}{(s-\bar\eta)}.$$

Finding the common denominator on the r.h.s. and comparing the coefficients of the numerators on both sides yields $C_2 = -C_1$ and $C_1 = -1/(\bar\eta - \eta)$ which gives (6a). In (6b), we could have $\mu = \eta$ in principle. In this case of repeated roots, the following decomposition is useful:

$$\tfrac{s}{(s-\eta)^2} = \tfrac{C_1}{(s-\eta)^2} + \tfrac{C_2}{(s-\eta)}.$$

Comparing coefficients, we have $C_1 = \eta$ and $C_2 = 1$. We obtain the following inverse transforms:

$$\begin{aligned}
L^{-1}\left[\tfrac{1}{(s-\eta)(s-\mu)}\right] &= \tfrac{e^{\eta t}-e^{\mu t}}{\eta-\mu} = te^{\eta t} \quad \text{for} \quad \mu = \eta,\\
L^{-1}\left[\tfrac{s}{(s-\eta)(s-\mu)}\right] &= \tfrac{\eta e^{\eta t}-\mu e^{\mu t}}{\eta-\mu} = (1+\eta t)e^{\eta t} \quad \text{for} \quad \mu = \eta,\\
L^{-1}\left[\tfrac{1}{s\psi(s)}\right] &= \tfrac{1}{\bar\eta\eta} + \tfrac{e^{\bar\eta t}}{\bar\eta(\bar\eta-\eta)} - \tfrac{e^{\eta t}}{\eta(\bar\eta-\eta)},\\
L^{-1}\left[\tfrac{1}{(s-\mu)\psi(s)}\right] &= \tfrac{1}{(\bar\eta-\eta)}\left\{\tfrac{e^{\bar\eta t}-e^{\mu t}}{\bar\eta-\mu} - \tfrac{e^{\eta t}-e^{\mu t}}{\eta-\mu}\right\},\\
L^{-1}\left[\tfrac{s}{(s-\mu)\psi(s)}\right] &= \tfrac{1}{(\bar\eta-\eta)}\left\{\tfrac{\bar\eta e^{\bar\eta t}-\mu e^{\mu t}}{\bar\eta-\mu} - \tfrac{\eta e^{\eta t}-\mu e^{\mu t}}{\eta-\mu}\right\}.
\end{aligned}$$

Armed with these results, the inverse of (5) emerges as

$$\begin{aligned}
C_t &= \left\{z_{12}V_0 - z_{22}C_0\right\}\left[\tfrac{e^{\bar\eta t}-e^{\eta t}}{\bar\eta-\eta}\right] + z_{12}Y_\infty\left[\tfrac{1}{\eta\bar\eta} + \tfrac{e^{\bar\eta t}}{\bar\eta(\bar\eta-\eta)} - \tfrac{e^{\eta t}}{\eta(\bar\eta-\eta)}\right]\\
&\quad + C_0\left[\tfrac{\bar\eta e^{\bar\eta t}-\eta e^{\eta t}}{\bar\eta-\eta}\right] + (Y_0 - Y_\infty)\tfrac{z_{12}}{\bar\eta-\eta}\left[\tfrac{e^{\bar\eta t}-e^{\mu t}}{\bar\eta-\mu} - \tfrac{e^{\eta t}-e^{\mu t}}{\eta-\mu}\right],\\
V_t &= \left\{z_{21}C_0 - z_{11}V_0 + Y_\infty\right\}\left[\tfrac{e^{\bar\eta t}-e^{\eta t}}{\bar\eta-\eta}\right] - z_{11}Y_\infty\left[\tfrac{1}{\eta\bar\eta} + \tfrac{e^{\bar\eta t}}{\bar\eta(\bar\eta-\eta)} - \tfrac{e^{\eta t}}{\eta(\bar\eta-\eta)}\right]\\
&\quad + V_0\left[\tfrac{\bar\eta e^{\bar\eta t}-\eta e^{\eta t}}{\bar\eta-\eta}\right] + (Y_0 - Y_\infty)\tfrac{1}{\bar\eta-\eta}\left[\tfrac{(\bar\eta-z_{11})e^{\bar\eta t}-(\mu-z_{11})e^{\mu t}}{\bar\eta-\mu} - \tfrac{(\eta-z_{11})e^{\eta t}-(\mu-z_{11})e^{\mu t}}{\eta-\mu}\right].
\end{aligned} \tag{7}$$

By (2b) evaluated at $\bar\eta$ and (4), all the terms involving $e^{\bar\eta t}$ add up to zero. This is intuitive since (4) is implied by the condition that the solution is bounded. Hence, it must suppress the influence of the explosive eigenvalue $\bar\eta > 0$ on the solution trajectory. Collect the terms involving $e^{\eta t}$, use the long-run solutions given in(3), exploit once again the restriction (4) for simplification and obtain

$$\begin{aligned}
C_t &= C_0 e^{\eta t} + C_\infty(1-e^{\eta t}) + (\tfrac{Y_0-Y_\infty}{\bar\eta-\mu})(\tfrac{e^{\eta t}-e^{\mu t}}{\eta-\mu})\,(-z_{12}),\\
V_t &= V_0 e^{\eta t} + V_\infty(1-e^{\eta t}) + (\tfrac{Y_0-Y_\infty}{\bar\eta-\mu})(\tfrac{e^{\eta t}-e^{\mu t}}{\eta-\mu})(z_{11}-\mu).
\end{aligned} \tag{8}$$

This formula yields straightforward interpretations. The first two terms correspond to the standard solution formula for a constant change in wage income. The last term stems from the fact that wage income changes only gradually making short- and long-run income fall apart. It is easily verified that the last term starts out with zero and asymptotically approaches zero again. It is unambiguously positive during the transition in case that financial wealth was positive initially ($z_{11} - \mu > 0$). Adding this third term to the monotonic paths emerging from the first two terms may produce non-monotonicities in the overall dynamics of consumption and the stock of savings. The possibilities for overshooting or non-monotonic adjustment hinge on gradual adjustment of wage income. In case that wages adjust at a rate identical to $\bar\eta$, we substitute

$$\lim_{\mu\to\eta} \tfrac{e^{\eta t}-e^{\mu t}}{\eta-\mu} = \lim_{\mu\to\eta} te^{\mu t} = te^{\eta t}.$$

This term stems from the inverse transform of $\frac{1}{(s-\eta)(s-\mu)}$ in case of $\eta = \mu$ which gives $L^{-1}[\frac{1}{(s-\eta)^2}] = te^{\eta t}$.