SPACE-TIME SOCIOLOGY

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Forschungsbericht/
Research-Memorandum Nr. 313
January 1993

ABSTRACT

Those are two contributions to spacetime sociology. The first is on space and the second on time. Both are part of a more comprehensive work which has been based on an extended epistemology of concepts.
As soon as you investigate into the origins of culture, you come upon the survival formulas of our paleolithic ancestors. Among those symbols there is a concept of orientation that can be followed forward until in our times or the times of Descartes respectively. This concept, in its exact mathematical form, possesses a symmetry which is isomorphic with the symmetry of classical logic or what Boole denoted the *laws of thought*.
The second contribution is on time and is similar to the first, but develops the idea by beginning at the other end of history, that is, it is begun in the present state of the art which is connected with nonlinearity and self-organizing systems. It is shown by the aid of a rather general scenario of a two-dimensional Feigenbaum attractor how an exact concept of linear time can be drived as an internal system parameter from the exchange of stability in such a chaotic system of population dynamics.

ZUSAMMENFASSUNG

Der vorliegende Entwurf zur *Raumzeit-Soziologie* kennt zwei Hauptthemen: das eine davon handelt vom *Raum* - und das andere von der *Zeit*. Beide stellen jeweils Teile eines größeren Forschungsvorhabens dar, welches sich als *Erkenntnistheorie von Begriffs-Bildungen* charakterisieren ließe.
Folgt man nämlich zu den Ursprüngen von menschlichen Umgangsformen und Kommunikationen, so wird man unweigerlich zu den Kulten und Zeremonien unserer paläolithischen Vorfahren geführt. Im Rahmen dieser frühesten Symboliken erlangt, so die zentrale Behauptung, ein Ordnungsprinzip eine hervorragende Bedeutung, nämlich jenes der *Orientierung*, dessen prominenter Stellenwert bis in Moderne, bis in die Gegenwart oder auch bis in die Zeit von René Descartes verfolgt werden könnte. Es läßt sich nun zeigen, daß dieser Begriff der *Orientierung* in seiner exakten mathematischen Form eine Symmetrie besitzt, welche isomorph zum klassischen Begriff der *Logik* ausfällt, welche von George Boole entwickelt worden ist.
Der zweite substantielle Beitrag ist dem Problem der *Zeit* gewidmet, das allerdings vom anderen Endpunkt der Entwicklung aus, nicht von seinen frühesten Anfängen, sondern von seinen gegenwärtigen Analyseformen angegangen wird, d.h. vom Kontext *nicht-linearer* und *selbstorganisierender* Systeme aus. Es wird mit Hilfe eines allgemeinen Szenarios mit einem zweidimensionalen *Feigenbaum*-Attraktor gezeigt, wie in einem chaotischen populationsdynamischen System ein präziser Begriff einer *linearen Zeit* als Systemparameter vom Austausch der Stabilität abgeleitet werden kann.

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ROOTTALK

Those are two contributions to spacetime sociology. The first is on space and the second on time. Both are part of a more comprehensive work which has been based on an extended epistemology of concepts.

The conception of social space as is used in nowaday sociology goes back to the thought of Gustave Flaubert who designed a social experiment that he denoted *sentimental education*. In *sentimental education* each actor is allowed to realize his own survival formula within the restricted social space of the dominant class he is in. Pierre Bourdieu has developed that concept so far that he could go beyond the traditional separation between subject and object. In his approach, social space appears as a matrix of group locations. Thus political action can be derived from relations between classes rather than from the nature of each class per se. This holds for the social space of a Berber speaking people just as well as for the social location of the state, the agency that has the power of legitimate naming and coding. But there is some difference between civilization and people without writing. Looking at the spatial arrangement of symbols and power in a settlement of some Sioux tribe of North America, the Bororo of South America or the Kabyles of North Africa etc. you find original patterns of orientation that cannot be found that easily in a modern city. But they belong to very old morphogenetic patterns. In a way, we must differ between the space of society and the sociology of space. Only the second can improve our understanding of such original structures and their relation to the genesis of social orientation.

As soon as you investigate into the origins of culture, you come upon the survival formulas of our paleolithic ancestors. Our very first experiment of sentimental education became established around the symbols of stone age worship. Among those symbols there is a concept of orientation that can be followed foreward until in our times or the times of Descartes respectively. This concept, in its exact mathematical form, possesses a symmetry which is isomorphic with the symmetry of classical logic or what Boole denoted the *laws of thought*. This is no arbitrary transposition from space to logic. But it is a matter of hardware. There is something in our brains that allows us to realize spatial symmetries of coordinates on the screen just as well as transitions among logical propositions. It seems that men have learned to think by playing around with objects in space, and this playing around was accompanied by the engravings: stone age ideograms. So before there was any exact concept of measurable space - the *exact* is in the sense of Körner - we had an exact concept of orientation, and this is still active in the (re)construction of social space. So there is logic from space.

The second contribution is on time and is similar to the first, but develops the idea by beginning at the other end of history, that is, it is begun in the present state of the art which is connected with nonlinearity and self-organizing systems. It is similar to the space approach in as much as time is considered as a *derived quantity*. Time is sociologically constructed too. Just as space, so time is no synthetic a priory condition to the mind. But both are constructed. It is shown by the aid of a rather general scenario of a two-dimensional Feigenbaum attractor how an exact concept of linear time can be drived as an internal system parameter from the exchange of stability in such a chaotic system of population dynamics.

Next it is investigated into the relation between time and migration by the aid of a *masterequation* as has been proposed by Wolfgang Weidlich and Günter Haag[1] and which has been tested by Karl H. Müller and others at the *Institute for Advanced Studies* in Vienna.[2] Though this approach is straightforward, there arise considerable problems of aggregation in social space and general questions of methodology. It is hoped that by this present work some hints concerning the future procedure are given.

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1 See e.g. W. Weidlich, G. Haag (1988)(eds.), *Interregional Migration. Dynamic Theory and Comparative Analysis.* Berlin *et al.*

2 See K.H. Müller, K. Pichelmann (1990)(eds.), *Modell zur Analyse des österreichischen Beschäftigungssystems. Prognose- und Szenarieninstrument der branchenspezifischen Beschäftigungsentwicklung.* Wien and K.H. Müller, L. Lassnigg (1992)(eds.), *Langfristige Szenarienanalyse des österreichischen Bildungssystems,* 2 vol. Vienna.

1. TRACING BACK TO THE ROOT

THE CONCEPT OF SPACE

As a mathematician one is using symbols. As a physicist one is dealing with space. But being a sociologist, one cannot take concepts for granted, but one has to ask about their sociology, how they were put together by social action. Because thought is a sociological process.

1.0. Cultural Origin of Orientation in Prehistory

Unfortunately, in the case of space, that process seems to have taken place about 200000 years before our time. So the reconstruction is rather arduous. It doesn't really matter how far exactly we have to look back. What matters is the basic structure we find there, whether 20000 years later or earlier. The job of exact dating we leave to the prehistorians and the paleonthologists. The very first concepts were symbols; spheres, strokes, circles, triangles, partitioned spheres, quartered circles and others. Among those there was a concept of orientation. This concept has evolved until in our times. Not that it has evolved continuously or regionally in any smooth manner. But it experienced several transitions beyond instability and regained its stability, thereby changing its shape many times. It was really subject to metamorphosis, often exposed to occurrences, visions and vague ideas, especially in the beginning. Today the concept of space is even applied in sociology. So we actually have both now, a sociology of space and a space of sociology, that is, social space in the sense of Bourdieu. I am convinced that, despite of Bourdieu's attempt to bring the idea down to clarity, we cannot really obtain a relevant concept of social space without reconstructing to some extent the sociology of space. If we don't do so, my intuition tells me, the concept of social space will unnecessarily subordinate to the approaches of mathematics and physics.

To understand the meaning of a spatial partition of a house or a village in an ethnic community, whether the Berber house or the men's house of Bororo or some Sioux village, you have to understand the basic pattern of the partition and its symmetry properties, how those relate to social action and so on. To figure out the cultural origin of that pattern you have to go back to the roots of conceptualization. When we investigate into a subject like *The Berber House or the World Reversed*[3] we are indeed confronted with a stone age acquisition of the mind, though we may not mind that in any case. But when we want to learn about the nature of human orientation we have to mind it. On the other hand, if we use a coordinate-system today and carry out some plot on a piece of paper, we are also carrying out a partitioning that is based on a very old structure of thought. To understand its original meaning, again we have to go back to the roots. We don't have to go back at any case, of course. But if we do, we are immediately obtaining some insight into the basic structure and socio-psychological function of the space-concept. Then we need not mechanically base our sociological concept on that of physics. Rather we find out the reason why the methods of Descartes, euclidean space, topology and so on, proved so powerful. May be that we can also understand a little better why that concept experiences such a dramatic change in our days. Since discontinuity and fractal dimension have taken the form of exact concepts of mathematics, some originally very small part of topology seems to transform the whole subject matter. We got to understand that, because it can tell us a lot about time and evolution.

Now some of us may still think of space and time in terms of Newton, Einstein or, say, Kant. But those are neither absolute, nor true or right, but they are but attempts to find exact concepts for space and time within a given cultural context. Some of the basic hypothesis may have been freely invented, as Einstein claimed, others may have been carefully thought out by pure reason, as Kant liked to say. But essentially they were products of the mind in a given cultural setting with a given scientific basis to rely on, with given beliefs and habits and so forth. Therefore many facts and questions that are important to us today have not been seen. Take for instance Kant. When he discovered by mere reason that space and time were synthetic a priori conditions to perception, he virtually made a statement about the evolution of the brain, possibly without being aware of it. I am not shure about that, but even if he was aware of it, he didn't yet have the tools to prove it. There wasn't too much to be known, then, about realization of time in neuronal networks, the appearence of self-organizing images in synergetic processes and the like. Being a sociologist today I cannot, of course, go into the question either. But I have to be aware, at least, of what goes on in modern fields of science like synergetics, neurophysiology and so on, if I do not want to entirely fall out of time. To speak about sociological subject matters such as history of culture and development of concepts is indeed one of my duties. Therefore let me consider those two denotations *synthetic* and *a priori* and ponder over their meaning sociologically: the *synthetic* refers to how the concept was put together by social action. That action is not really separate from neurophysiological evolution. And the *a*

3 P. Bourdieu (1976), "Das Haus oder die verkehrte Welt", in: ders. (1976), *Entwurf einer Theorie der Praxis*. Frankfurt am Main, 48 - 65.

priori means that the concept was culturally prior to all the other concepts to which it was a basis. That means that neither perception, nor cognition work without the arrangement of that concept. The arrangement of that concept is, however, altogether a physical, bioenergetic, neurophysiological and sociological matter. That's the *down-to-earth-translation* of *synthetic a priori*. I do not claim that I am telling you the truth, or that anybody should pray up to my approach. But I simply have no other way than to work out very carefully a consistent image of my own cognitive experience in relation to social science and science in general. That image tells me that the concept of space formed part of the sociological basis from where education and culture took off. The very first concepts of space were, of course not of the Euclidean type, as we use today, but they were concepts of orientation. Because orientation was the thing to be minded.

1.1. Bifurcation of Original Concepts

It is now an accepted fact of science that the origins of primitive concepts of tools as well as of the original ideas of orientation cannot be found in history, but have to be located in stone age cultures. It is not easy to obtain a correct image of these developments unless one liberates one's own mind from the many prejudices of positivism, evolutionism and Darwinism whether old-fashioned or modern. Those ideas as were prepared by german idealism, Goethe, Spencer and other representants of the *Zeitgeist* led to a radical shrinking of sociological theory in terms of a primitive belief: Due to the intelligence of natural selection the present day civilization has a very high spiritual value because it has often successfully passed the examinations whereas that which is old has not. So there was the simple formula: the older a society, the more primitive it is. Consequently the coloured paintings of stone age caves could not really be understood, since how could a primitive being create something so beautiful. The engravings of ideograms were altogether passed by. However, as those findings increased in number, their prehistoric origin was accepted in 1902. But they were not taken as expressions of cultural or religious activity, of course, but rather as some kind of random games, spiritual repetitions and so forth, so that the explanation could be reconciled with the prevailing notions of primitivity and evolution. The primitive stone age men somehow pencilled ahead with a musty head and incidentally left something beautiful which confused them. Adama van Scheltema gave such a sketch in *Die geistige Wiederholung*. But physiology and even sociology blew the same horn. Physiologists confirmed the idea of a retarded state of thought capable of projecting some kind of internalized visual images onto the outer. The sociologist Baege loyal to the psychological idea of imaginative association explained their truth to nature by a lack of internal associations so that these couldn't mutually block each other up. But it was not found out then why man set out on a dangerous journey into dark, deep-seated caves and grottoes. That journey confirmed the distance between light and dark, between outer and inner. Probably you travelled not only from light to dark, but also from warm to cold. Thus the contrast between living space and caves was conditioned. Paintings and engravings were bound to the places of worship. If you just took out a beautiful painting for your collection you essentially missed the context. But as the engravings, the ideograms, were bound to religious rites and worship, they had a symbolic meaning. So the caves were places of instruction. In Langenscheidt's Taschenwörterbuch of 1929 (which is exactly the publication year of Baege's *Sociologie des Denkens*) the *Kultusminister* appears as a *Minister of Public Worship and Instruction*. It seems, somehow the house (being a dictionary) doesn't loose a thing. Neither the sociological mainstream derived from Auguste Comte, nor the anthropological directions appealing to Frobenius were able to account for the correct relation between stone age arts and hunting. The first invented the imagination of man pressed by hunger, reveling in their representations of meat. The others imagined their belief in magic power, ritual practise of hunting magic. All these theories began with the axiom of an unintelligent man. There was some kind of reaction to this evolutionist attitude represented best by the writings of Grabbe, Büchner, Lenau, Heine, Nietsche and Spengler. They prefered the idea of a sequence of life-cycles of cultures, and foretold the end of the occident. This image wasn't so bad, because as there was the inevitable improvement of culture by selection, it did not disturb one too much if cultures were occasionally extinguished. My intuition tells me that the evolutionary approach to time is always accompanied by some uncritical, positive attitude towards the here and now. The here and now appears as the only vessel of history which is always a bit misunderstood. Therefore one has to stir a bit deeper towards the origin of evolution in the present structure of things. This might be dissipative structures, self-organizing biochemistry and all the rest of it. As all those people like Spencer, Spengler and the other bunch of loners are found in the citation index of Ilya Prigogine, one of the inventors of non-equilibrium thermodynamics - for him virtually everything seems to take place in transitions beyond equilibrium - we may ask ourselves about the role of selection today, the kind of concepts in use and so on. Well, today there is still the belief in things like evolutionary fitness, a quantity that describes the formation of stable clusters of species like a *deus ex machina*, just that there seems to be some chance that fitness is at least partly described by a cooperation function.[4] Such a parameter is always playing the role of an outside agent, not more and not less than what is called a *control-parameter*, and therefore it is not of much use in sociology today. But even in the days of Frobenius scientists saw the drawbacks of the *Kulturkreislehre*, and especially the ethnologists of the *Wiener Schule* strove hard for a historical view of prehistory. The scientists around Hoernes asserted that prehistory was denoted *Urgeschichte*. So *Urgeschichte* became part of history.

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4 A. Dress (1987), "Populationsdynamik auf Sequenzräumen", in: B. Küppers (1987)(ed.), *Ordnung aus dem Chaos*. München, 123.

Only from that time onward there could be developed some unprejudiced approach to ancient symbols. Especially in anthropology there was worked out an image where the spiritual development towards logical thought was no longer bound to economic development. Arambourg, Vallois and others explained that paleolithic men must have been highly intelligent.[5] Even biologists began to prove that for a human being without culture there was no chance of development. The fundamental difference between biological and sociological facts was now the following: the body was governed by laws of biology. But the mind brought forth the ideas that had to be handed over from each generation to the next. And each generation would add to the tradional stock of ideas something new. So there was a coexistence of tradition and development. But selection didn't play an important role in cultural evolution. But, step by step, mankind developed both culture and history.[6]

Leroir-Gourhan worked on some new approach to the understanding of paleolitic engravings by stressing their symbolic meaning.[7] But he somehow reduced the whole thing to vertility symbols which always form only part of religion as Hentze has pointed out. In his view religion is based on a whole, on what he denoted a world view that formed the centre of thought. So for Hentze there existed some sort of cultural ego that is to be preserved during the generation sequence. With him many others, as for instance Marie E. P. König, say that the evolution of religious thought begins with a primary world image or world view. Especially Marie König who investigated into the paleolitic material of the old caves and grottoes found expressed in engraved symbols and images the orientation of stone age society in space and time.[8]

She mentioned the viewpoint of Johann Jakob Bachofen that historical development includes both tradition of old and creation of new ideas. Original concepts have been more general and diffuse, but new thoughts brought into the old greater accuracy. Therefore, in the course of history, thought didn't become different, but more accurate. It didn't change its nature, say, from dull to conscious, but became increasingly complex. The same holds for the concepts of space and time. Those were preserved throughout history and developed into ideas of higher and higher complexity. But the core idea, the original concept of orientation, was always kept invariant. König writes:

> Das Beibehalten älterer Begriffe, die Strukturteile vergangener Geistesschichten waren, ermöglichte ihr Nochvorhandensein in viel späterer Zeit. C.G. Jung bezeichnete sie als *Archetypen*. Als Beispiel führte er aus dem Bereich der Zahlenwelt die Drei und die Vier an. Er hebt hervor,

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5 See e.g. C. Arambourg (1943), *La Gense de l'Humanité*. Paris and H.V. Vallois, M. Boule (1964), *Fossile Menschen. Grundlinien menschlicher Stammesgeschichte*. Baden-Baden.

6 See G. Siegmund (1966), *Die seelisch-geistige Sonderstellung des Menschen. Handbuch der Urgeschichte*, Bd.1. Bern.

7 See e.g. A. Leroi-Gourhan (1966), *Les Religions de la Prehistoire*. Paris.

8 M.E.P. König (1973), *Am Anfang der Kultur*. Wien.

> daß sie Maßtäbe gesetzt und das pattern of behavior beim Aufbau unserer Kultur gebildet hätten.[9]

In this same connexion Ronald D. Laing, Jean Piaget and others speak of (psycho)genetic structures of experience, morphogenetic structures of perception. This signifies the position of genetic structuralism.The original idea of space is a morphogenetic concept of orientation on which all the later concepts are based. Even the modern idea of sociological space is and must be based on it. There is no other way.

During the last cultural stage of the old stone age, the Acheulen, there were two different types of objects which unfolded from the original basis of thought. The first class contained multipurpose tools, the second a general class of cultural objects with little complexity of design. These varieties did not, of course, unfold from a dull mind. But both spread out of fundamental experiences. As for the making of tools, there was the basic experience of work and procedures, and there was an understanding of the relation between cause and effect. So out of the basic experience of capability there came the multipurpose tools. Some scientists say that *capability took shape*, others speak of *stone conceptions*, Arnold Gehlen for instance, or Richard Pittioni takes them to represent *expressions of the thought process*. They all try in their way to appreciate the effort made by our ancestors to give shape to the first comprehensions.

What strikes me is the belief that these first abstract conceptions were indeed first denotations. Stone age men may have given simple names to every little thing they worked with. Language may have been in some kind of experimental or chaotic state. That would not contradict the idea of first general classes of concepts. What if they were capable, as people without writing, of a denotation system with maximum complexity, but minimum or no abstraction at all? They may have had unstable, phonetic denotations for the trees in their environment without denoting the equivalences within one and the same species. When we take the example of the infant, as Marie König does, we can indeed say the infant means by *Wau Wau* a variety of animals with four legs and larger than a mouse, but not only dogs. But later *Wau Wau* is restricted to the species of dogs. So at first there is an inexact concept for a large variety of animals and later that concept becomes more exact. But with stone age men it might have been a little bit different. They may have had *wau waus*, *tzu tzus*, *tsu tsis* and *flabls* and all kinds of things, before they worked out a general denotation that remained somehow collectively stable. Yet that system may have been uncomparable with infant talk in that it had a much higher complexity. But today we simply do not know.

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9 Ibidem, 24.

What is left for us, however, is a collection of objects which make us aware of their general meaning. Therefore, whichever story we invent today, there is an important thing to be kept in mind, namely that there was a basic encounter of man with world where man learned to experience himself as an actor capable to give shape to the outward objects. The making of tools and instruments had to develop along the lines of the intrinsic logic of work and procedures. This logic, as Böhm has pointed out[10], led to the stepwise unfolding of a typology.

For the present purpose I need not explain all those events in greater detail. But I wish to stress that from the original idea of a tool with little complexity in itself, which allowed however for multiple applications, there spread out a variety of tools with increasing complexity and specialization.

There is a second general class of objects originating in a different domain of thought. That is, whereas in working the actor experienced himself as powerful in relation to nature, in worship his basic experience was different. Here he felt himself as exposed to the powers of nature, gravity, winds, temperature and so forth. In the *work relation* man appeared as an actor exercising control over outward events. In that relation nature and environment appeared as events rather than as actors. But in his *religious relation* nature appeared as a most powerful actor, and man was the event. Thus the roles in the exchange system man-nature were commuted. The inner was commuted with the outer. As Marie König put it, the content of thought of the primary world image was its internal power. To this feeling of power man felt himself exposed and from it he derived feelings of dependence, of connectedness and obligation. The commutation of outward with inward power signified a topological relation that led to the making of cultural objects rather than tools. Cultural objects can be realized by the fact that they do not appear as tools.

To put it modestly: among the first objects of worship formed by hand, there are spheroids made of stone or earth. So they denote a general equivalence class of *round objects* which show a minimum complexity of form. The eldest such spheroids were found in Europe, some in the Aisne-valley, France, others in Achenheim, Germany.

In Untertürkheim, Germany, there were excavated 40 pebles carefully hewn to equal diameter. Because of their precise form they arose considerable interest among scientists. So spheroids represented the ideal form to an undifferentiate idea of *inner* as in relation to *outer*. Often they were found in the ceremonial centres of caves. Sometimes they appeared to have been replaced by skulls. Consider relations -

[10] J. Böhm (1962), *Interpretation und historischer Wert archäologischer Quellen*. Atti I. Rom.

man under celestial globe
skull in cave
spheroid in grotto
man in cave
jaspis in eye sockets
spheroid in spheroid
man looking up to cave walls/or looking down to spheroid

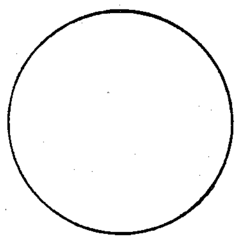
which all signify the same sort of a topological relation. When you enter a cave, which is only possible at a definite location, you look at the back of a skull showing the shape of a spheroid - König says, we can still convince ourselves of this today. Jaspers has already pointed out that the relation of inner to outer sphere can be understood as the subject-object relation. I think this is no speculation, but it can actually be derived from the whole reconstruction of the worship. I shall, however, not repeate that here. Rather I want to stress the relation between spheroid being in cave or spheroid as included in a larger one. That is, if you take a spheroid of, say, 10 cm diameter, you can hew it until it has shriveled to a diameter of, say, 3 cm. So you make the experience that in every sphere there is another. This is in analogy to the concept of topological neighbourhood. There is a seqence of spheric neighbourhoods with monotonously decreasing diameter. Such sequence of spheroids is analogous to the structure of the Ptolemäan world image. Over a time period of at least one hundred thousand years, skulls and stone balls have been found at ponds, spring-waters and other places of sacrifice. Those deep-seated places indicate the existence of sacrificial rites and worship of the powers of the nether world. Deep in the inner of the hyena cave Leroi-Gourhan found a dozen limeballs of the Moustrien period. The place of worship seems to have been separate from the dwelling-places. There are numerous examples of places where men have watched the spring or the regression and dry up of water, and where they signified that mysterious event by round objects such as skulls and spheroids.

1.2 Original Partition of Space

- and Metaphoric Transpositions

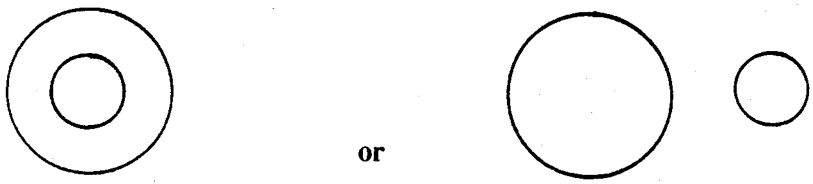
The original undifferentiated symbol of space was represented by skulls and spheroids made of stone, lime or earth. It was a binary partition into inner and outer because a ball separates an interior from an exterior region. By hewing the stone further, a spheroid could be turned into a smaller spheroid. By adding lime to a ball its size could be increased. So there was some idea of order in the size of spheroids and one of inclusion and exclusion. A ball or stone could be placed in an eye-socket, a skull could be placed in a cave or sinked in a pond. All those signify a relation of inclusion. Therefore we can identify the first archetypic symbol of orientation by modern means of graphic representation in the plane (fig. 1).

Figure 1: Archetypic Symbol



Together with the first relation man experienced, which is a relation of inclusion or exclusion as separation.

Figure 2: Original Topological Partition

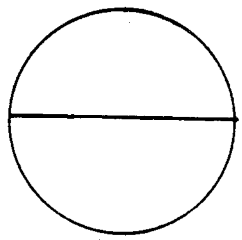


Celestial globe, cave, skull, limeball can all be considered as metaphoric transpositions of that first symbol. Cave under celestial globe, spheroid pebble in cave, stone in skull are representants of the topological relation of inclusion as spheroid included in larger spheroid. Skull next to pebble, pebble on pebble means exclusion.

1.2.1. Original Linear Partitions

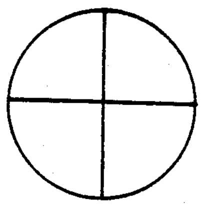
In those days two further partions which were linear, to put it in mathematical terms, have been known: there was a partition of space as power into a below and an above. There was the visible world on this side of the living, the world above, world of light and so on, and there was the world beyond, the other world of the invisible, of the dead. This partition into above and below, here and beyond, light and dark, warm and cold and so on, was at the same time a partition of the set of human skulls into living and dead. All those differences were obvious and analogous. But there existed a second partition of space that could have been derived from the immediate experience: the partition by a line running from east to west. The trajectory of the sun partitiones the sky into a southern and a northern hemisphere. The dead burried in east-west-direction divided the grave into two parts and so on. This can best be represented by halfed spheroids or disks which seem to have existed in the Moustrien. We chose to represent such partition by the figure 3.

Figure 3: Linear Partition



Such a partition could indeed also be represented by engraving a circle on a limeball. But I am not shure if such things were found. There is also a way to represent both partitions, above/below together with south/west in one object only. But here, in the old and middle paleolitic periods, the situation is somewhat unclear. Such a double partition could have been engraved into stone in many ways. It could have appeared hidden in a binary partition: e.g. as a diameter on a disk of stone or as a diametric partition of a bowl or as a equatorial channel on a pebble. Take the halfed stone bowl: in some indefinite way it can give rise to a perception of both partitions. Considering only the bowl, you find out the bowl alone can give you the impression of a below and an above, of a here and a beyond. So the division line in the middle can be taken to represent the O-W-axis. But at the same time that line also can represent the partition below/above. So there is some rest of ambiguity in the perception. But such rest may have been the cause to further developments. Whatever may have happened exactly, if there can be any exactness in the reconstruction of that event, there must have been considerable thought and progress until the stone disk or pebble or bowl were partitioned by an orthogonal linecross. It meant that the cosmic spheroid was no longer thought as a unity, but was arranged along an axis or straight line. The important idea was however not that of axis, but that of orientation. The additional engraving of the N-S-partition line on a hard disk of the stone age was a rather complex act of thought. It cannot be interpreted as a random event. The appearance of this *archetype* or morphogenetic structure of perception must be datet sometime in the Mousterien. A surprising stone document of that time has been found by L. Vertes 1964 in a Travertin-settlement next to the city of Tata, Hungary. It is a Nummulites perforatus with a diameter of 2,1 cm. It has been slightly rounded and polished, and there were engraved two almost orthogonal lines crossing in the middle of the disk. This figure represents a world image of orientation that can be traced foreward until to the times of Descartes. It belongs to the image of Kopernikus rather than Ptolemäus. What I intend to suggest by this statement is the idea that the temporal order of the basic structures of our world views can repeat troughout history. That is, just as the nummulites of Tata or the linecross in the cave of Rochers te Potets succede in time the limeballs and pebbles of the Acheulen, so the world views based on the ideas of Descartes and Euclidus follow the ancient images of Ptolemäus and others. The original idea of orientation is thus represented by the following figure.

Figure 4: Original Orientation Concept



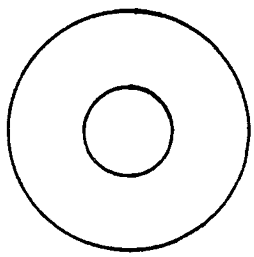
So we have obtained two stone age archetypes of orientation one of which seems to succede the other. One can be connected with the image of Ptolemäus, cosmos made up of celestial spheroids, even earth made of bowls and so on, the other is a relative of the coordinate-system. Also the first can be considered as a wave-image, whereas the second means location of particles. The first leads to the basic concept of neighbourhood in topology, the second to euclidean geometry, coordinates and all the rest of it. Topology is more general than euclidean geometry, indeed, and its roots in cultural history are deeper. That may be chance, but may be it is not.

1.2.2. Metaphoric Transpositions

To understand a symbol in sociological terms, one must be able to figure out its location in both social space and time. The original concept of space as orientation can virtually be located in a perception space that is at the same time physical and sociological. The space of objects cannot really be separated off the space of social locations. That's most interesting to discover because it leads to some surprising self-reference of the concept of space. Namely, there is the following partition of the original concept and social space:

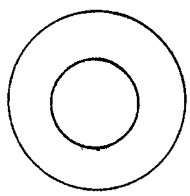
The original symbol of orientation, representing the partition of inner as in relation to outer -

Figure 5: Partition of Inner to Outer

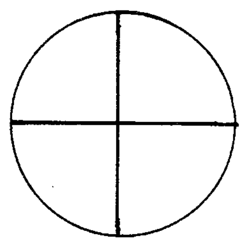


is found in places of worship and sacrifice. They are found in deep-seated caves apart from settlements and dwellings. Therefore those places of discovery can be entered into the later concept of the Mousterien where they must be placed in the lower half of the disk representing the below or the nether world. The dwellings have to be placed above the caves. On its evolutionary path from figure 1 to figure 4 -

Figure 6: The Evolutionary Path of the Orientation Symbol



to



the symbol moves upward, from deep-seated caves to the surface. Therefore this moving upward is a moving foreward in time, and thus relative to the sun a progression from east to west. While all that happened, groups of paleolitic men moved down *to the roots of mind*, down to the caves, many times, and they returned to the surface many times, and they moved from the outer into the inner and reverse.

It is possible to represent those events within the context of the old symbols. To represent the temporal path of the symbol within the symbol refers to the above mentioned case of self-reference:

the above
area of the here

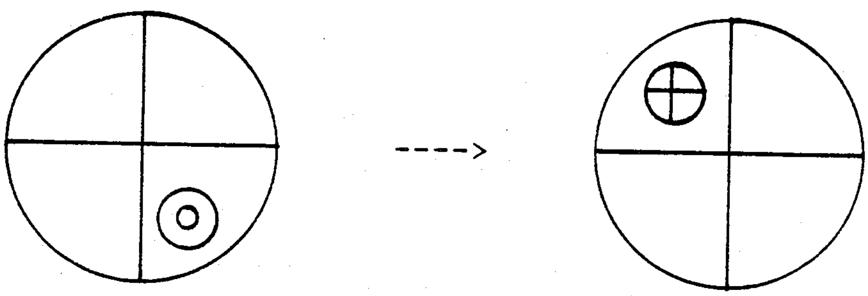
settlements, social region

---

deep seated caves, religious region
area of the beyond
the below

So the original symbol is located in the lower half

Figure 7: The Upward Movement of the Original Symbol



with settlements above caves of worship. In the course of history the original symbol develops and moves upward to the social region. (Today it is not even separated from households. There are coordinate-systems in every PC). Along this path there is arranged a synchronization of motion along different directions.

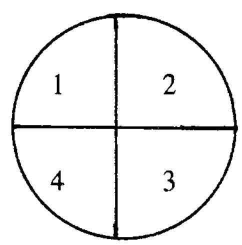
The whole arrangement of metaphoric transpositions of this motion is as follows: to move from the subconscious to the conscious, from the dark to the light, from the beyond to the here, means to move foreward, to move upward, from east to west, and from the inner to the surface. It also means, as M. König has pointed out, to move from an undifferentiated experience of power to a spatially and temporally differentiated experience of power. There is also involved a move on the dimension of fear and so on. There exist many more transpositions from that basic course of experience. But we shall not need all of them. There is also a direct correspondence of that motion with the flow of bioenergy in the human body. This can best be demonstrated by the aid of the old teaching of chinese acupuncture. We shall do this in a special section.

Next we shall investigate into the symmetry-properties of the quartered circle, that is, an experience of symmetry that has been made and internalized at least a hundred thousand years before our time. This basic symmetry of the original concept of space, we shall find out thereafter, is also the symmetry of classical logic. This is no accident, but it has to do with the organization of our brain and of language.

1.3 Symmetries of the Original Concept of Orientation

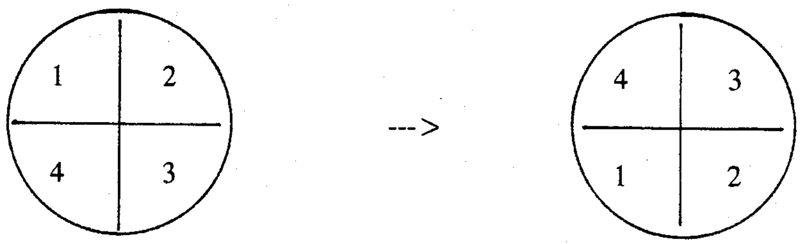
This is a twodimensional representation of the idea as was put into material form by the stone age nummulites of Tata, Hungary:

Figure 8: Spatial Orientation Symbol



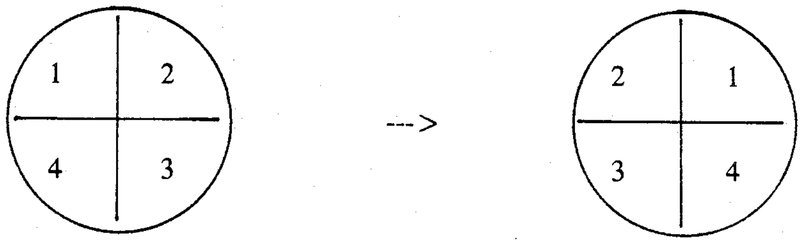
It signifies a concept of orientation in space. It is possible to carry out several rotations of that object without changing its location and orientation. First we can turn it around the horizontal axis A, so that the upper half is exchanged with the lower. Thereby 1 is exchanged with 4, and 2 with 3. This rotation will be considered as an element of motion and denoted G_1 :

$$G_1 = C_2'' = \Gamma_2$$



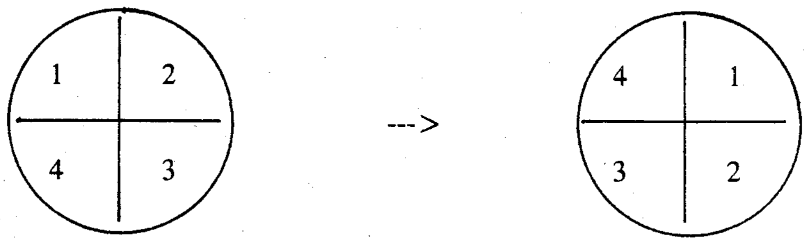
Now the order of numbers in clockwise direction has been altered, but the figure is the same, its orientation is the same and its centre has the same location. We briefly say that the *orientation stayed invariant*. But we may just as well carry out a rotation around the vertical axis B, which gives us G_2:

$$G_2 = C_2' = \Gamma_1$$



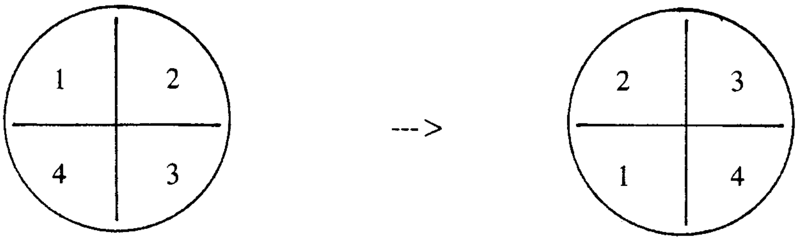
In rotations G_1 and G_2 the figure is turned out of the plane. But it is of course possible to rotate it without leaving the plane. Rotation by $\pi/2$ in clockwise direction gives

$$G_3 = S_4^3 = \pi_3$$



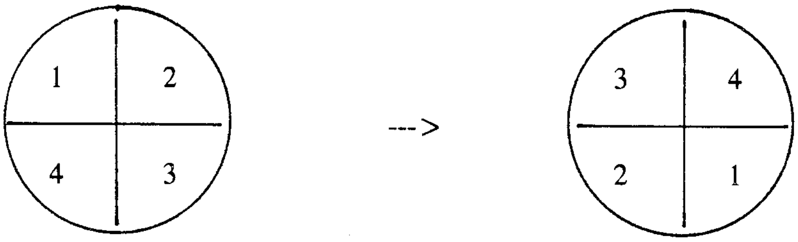
Rotation in counter-clockwise direction gives the fourth operation G_4 :

$$G_4 = S_4 = \pi_1$$



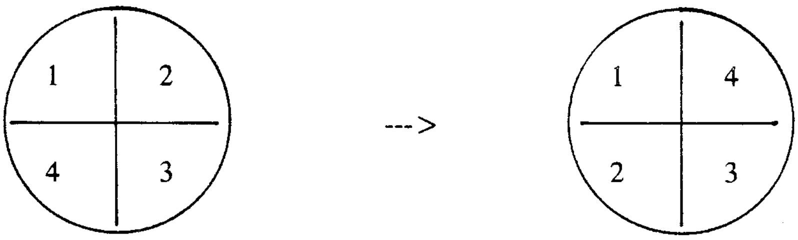
Rotation in the plane by π either in clockwise or in counter-clockwise direction give S_4^2 or G_5 :

$$G_5 = S_4^2 = \pi_2$$

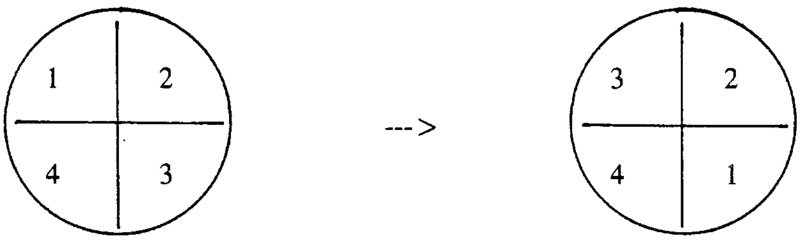


Now there are still two possibilities left if we introduce two diagonal rotation axis σ_d' and σ_d''. Rotation around the first gives G_6 -

$$G_6 = \sigma_d' = \sigma_1$$

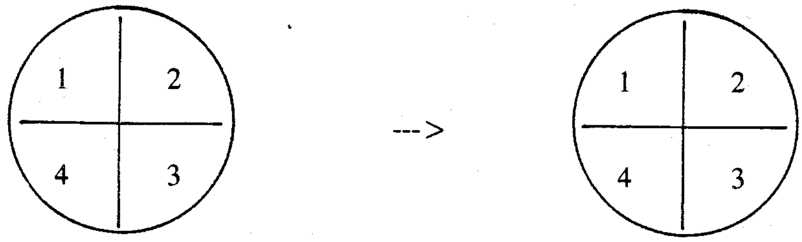


and $G_7 = \sigma_d'' = \sigma_2$



There is a last possibility which simply consists in leaving the whole arrangement unaltered. This is called the unit operation or unity of the algebra:

$$G_8 = E = C_2^2 = \sigma_d'^2 = S_4^4 \text{ etc.}$$

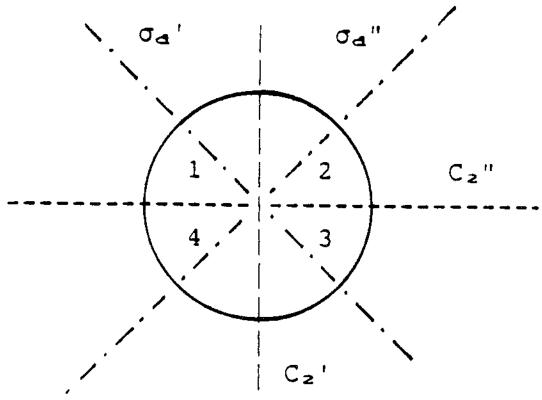


Let us now summarize those operations. We have obtained eight elementary rotations, E included, that leave both location and orientation of the linecross invariant. This is not only a mathematical operation, but it is an actual experience that can be made by turning, say, a wooden cross in one's hands. You find out by experimentation that certain things can be done that leave the orientation unchanged. But that means that cosmos and social space can be arranged in any of the eight fundamental ways without changing the essential structure of the arrangement.

After having found so many engravings of quartered circles and linecrosses in stone age caves, we can be shure that the orientation pattern together with its symmetry properties form an integral part of the human mind. This structure has actually been acquired during a long period of learning in worship and ritual sacrifice. It is now a morphogenetic structure of the human brain and the bodies bioenergetic system. But let's turn back to the algebra.

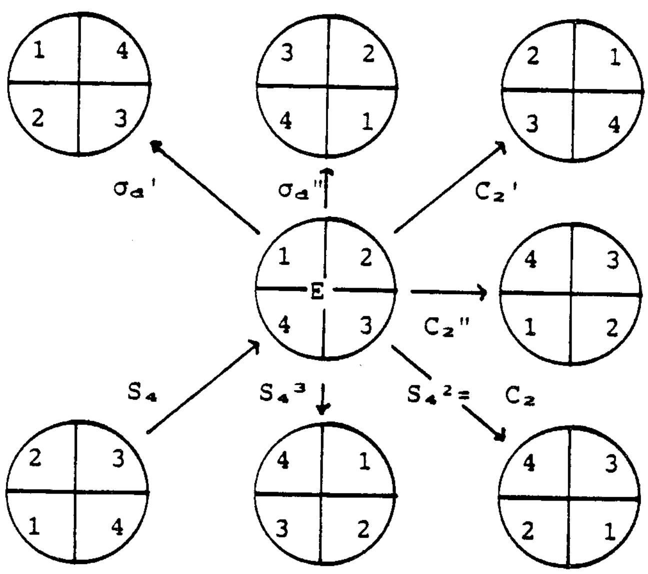
There appear three kinds of symbols, first the G with indices from 1 to 8, then there is a set consisting of C's, S's and e's, the so called *Schönfließ-symbols*. Arthur Schönfließ (1853-1928) was a German mathematician in Königsberg who established the theory of cristal structure. The Schönfließ-symbols are used by physicists and chemists because they are based on a classification of spatial symmetryoperations and rotation axis. For instance C_2 can be regarded as a rotation of the x-y-plane by π around the y-axis, σ_d' as a rotation by the same angle around a diagonal in the x-y-plane, but more precisely as a mirror reflection of a three-dimensional figure on a vertical plane through the diagonal x=y and so on. A schematic minimum plot to rotations in space takes the form of figure 9.

Figure 9: Rotations in Space



The S_4^n with any natural number n represent rotations in the x-y-plane, n=1 signifies rotation by $\pi/2$, n=2 by π, n=3 by $3\pi/2$, and n=4 by 2π, so that n=5 is equivalent to n=1 or $S_4^5 = S_4$ which means that the element S_4 is of the order four, is a rotation of period 4 or a *group generator* of the cyclic group of order 4: Z_4. In the spatial interpretation of the Schönfließ notation, S_4 can be interpreted as a rotation by $\pi/2$ in counter-clockwise direction around the vertical z-axis combined with a reflection at the x-y-plane. Since, in our case, the symbol is merely an idealized two-dimensional linecross, the S_4-operation is but a rotation by $\pi/2$ in counter-clockwise direction around the centre in the x-y-plane or around some imaginary z-axis. So there is a slight difference between the more simple representation by the group-elements G_i and its corresponding Schönfließ-notation. The first consists of rotations only, and the axis involved are x, y, diagonals and z. Whereas in Schönfließ-notation those rotations are built up by true rotations, mirror reflections and rotatory reflections that consist of both rotations around main axis and reflections at planes x-y, x-z and so forth. The whole image of operations is:

Figure 10: Symmetryoperations of the Group D_{2d}



The second notation is by Γ's, π's and σ's. This signifies the permutation representation. That is, any of our rotations of the quartered circle can be represented by a permutation of four objects, being the four quarters of the disk. We have to use the following permutations:

Table 1: Permutations

$$\Gamma_2 = C_2'' = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \qquad \Gamma_1 = C_2' = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{bmatrix}$$

$$\pi_3 = S_4^3 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{bmatrix} \qquad \pi_1 = S_4 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{bmatrix}$$

$$\pi_2 = S_4^2 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{bmatrix} \qquad \sigma_1 = \sigma_d' = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 3 & 2 \end{bmatrix}$$

$$\sigma_2 = \sigma_d'' = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{bmatrix} \qquad E = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

Each symmetryoperation possesses an inverse operation. For example the inverse to π_1 is

$$\pi_1^{-1} = \begin{bmatrix} 2 & 3 & 4 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix} \qquad \pi_3 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{bmatrix}$$

So we can construct a complete list of symmetries, their representations in terms of permutations, cycles or in Schönfließ notation together with their inverses.

Table 2: The Elements of the Group D_{2d}

symmetry	permutation	cycles	inverse symmetry	
E	e	(1)(2)(3)(4)	E	e
S_4	π_1	(1234)	S_4^3	π_3
S_4^2	π_2	(13)(24)	S_4^2	π_2
S_4^3	π_3	(1432)	S_4	π_1
C_2'	Γ_1	(12)(34)	C_2'	Γ_1
C_2''	Γ_2	(14)(23)	C_2''	Γ_2
σ_d'	σ_1	(1)(24)(3)	σ_d'	σ_1
σ_d''	σ_2	(13)(2)(4)	σ_d''	σ_2

The important property of these elements is that they can be associated or multiplied, i.e. their algebraic property. They form an algebraic group. The multiplication can be defined by a diachronic application of operators, for example the product $\pi_1 \pi_2$ equals:

$$\pi_1\pi_2 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{bmatrix} = \pi_3$$

In this way a complete group table of association can be provided:

Table 3: Group Table of Associations

e	π_1	π_2	π_3	Γ_1	Γ_2	σ_1	σ_2
π_1	π_2	π_3	e	σ_1	σ_2	Γ_1	Γ_2
π_2	π_3	e	π_1	Γ_2	Γ_1	σ_2	σ_1
π_3	e	π_1	π_2	σ_1	σ_2	Γ_2	Γ_1
Γ_1	σ_1	Γ_2	σ_2	e	π_2	π_1	π_3
Γ_2	σ_2	Γ_1	σ_1	π_2	e	π_3	π_1
σ_1	Γ_2	σ_2	Γ_1	π_3	π_1	e	π_2
σ_2	Γ_1	σ_1	Γ_2	π_1	π_3	π_2	e

Algebraic groups can be defined in an exact way. For instance any of the above elements can be associated according to the complete group-table. Then it can be observed that the following law of association holds:

A.(B.C) = (A.B).C where . denotes the association. Generally any non-empty set is a group, if there exists an operation of association . which allows for a definite attachment of one and only one element of G to the association A.B, with A and B elements of G. If G is a group, the following laws have to hold:

Association A.(B.C) = (A.B).C

Existence of *unity*:

There is an E in G such that for all A we have A.E = E.A = A

Existence of the *inverse* element:

For any A in G there is an A' such that A.A'= A'A = E.

There are groups where in addition to Association a law of commutation holds: for any A, B: A.B = B.A. Such a group is denoted *abelian* or a commutative group. The group of symmetries of the original concept of space as was derived above is no commutative group! For instance we have $\Gamma_1\sigma_2 \neq \sigma_2\Gamma_1$ There is something interesting in groups in relation to wholism. As far as I know this has already been stressed by many system theorists.[11] That is, in a group G any element A can be understood as both an operator and an operation. You apply an operator being a group-element to another operator and thereby carry out an operation. Each operator obtains its identity only within the context of the association table. By applying one single operation to the whole group, you just shift the elements according to the group-table, but finally obtain as the resulting set the original group. But there are still more interesting features with groups. For example any group may contain proper subsets that are groups themselves. Those subsets are denoted subgroups of G. Now according to a theorem by Lagrange the number of elements of a subgroup of a finite group G must be a divisor of the number of elements, or *order*, of G. This divisor is the *index* of the subgroup in relation to G. For instance the above symmetry of the original concept of space, $\mathbf{D}_{2d}$, is a proper

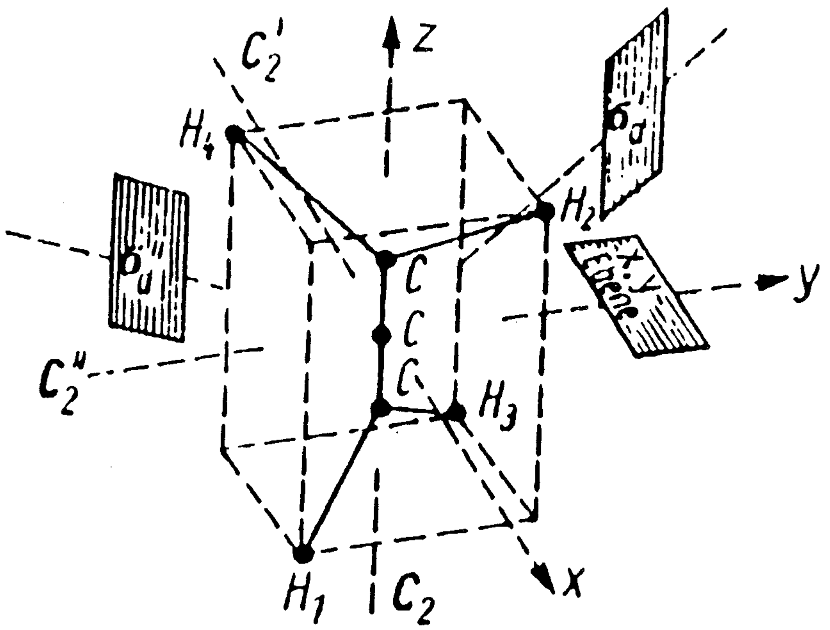
[11] See e.g. E. Laszlo (1971), "Systems and Structures - toward Bio-social Anthropology", in: *Theory and Decision* 2, 174-192.

subgroup of the complete permutation group of 4 elements S(4), the symmetric group of degree 4. This group S(4) contains as elements all possible permutations of four objects. Those are, however, not only 8, but 4 times 3 times 2 which is 24. Therefore we observe that $\mathbf{D}_{2d}$ is a proper subgroup of S(4) with index 3. In the following section we shall list all the essential mathematical properties of D_{2d} which will turn out relevant in social science.

1.4. Properties of $\mathbf{D}_{2d}$

The finite non commutative group $\mathbf{D}_{2d}$ of order 8 is called the symmetry of the two-sided double prism. It is isomorphic to $\mathbf{D}_4$, the spatial rotation-group of a square in space or dihedral-group. In agreement with the present representation, I shall prefer to call this group the *symmetry of the original concept of orientation* or *basic two-dimensional orientation group*. It can also be represented by the symmetries of the molecule of Allen (see figure 11, below):

Figure 11: Symmetry of the Allen-Molecule



DEFINITION 1: Centre of a Group

The complete subset of elements of a group G which commute with all elements of G form the centre of the group G.

The centre of the basic two-dimensional orientation group is the proper subgroup of index 4 denoted C_2. We have

$$\mathbf{C_2} = \{E, S_4^2\}.$$

DEFINITION 2: Cyclic Groups

A group which consists of all powers A^k where k is an integer is denoted *cyclic group generated by A*. A is called cyclic or primitive element of G. We write $G = <A>$.

Example:

$$\mathbf{S_4} = <S_4> = \{ S_4^o, S_4^1\ S_4^2, S_4^3 \}$$ is a cyclic subgroup of

the basic orientation group $\mathbf{D_{2d}}$.

All other cyclic subgroups of the basic orientation group are of the order 2. Those subgroups are

$$\mathbf{C_2} = \{E, C_2\} \qquad \mathbf{C_2}' = \{E, C_2'\} \qquad \mathbf{C_2}'' = \{E, C_2''\}$$

$$\Sigma' = \{E, \sigma_d'\} \qquad \Sigma'' = \{E, \sigma_d''\}$$

DEFINITION 3: Similarity Transformation and Conjugacy Classes

Consider elements A, B of the group G and the inverse element X^{-1} to X. B is said to be similar to A if B can be derived from A by a similarity transformation. We say that $A \approx B$ if there is an X in G such that $B = X^{-1}A.X$. The equivalence class of all elements similar to A is the class of similar or conjugate elements to A.

Example:

Consider the element S_4 and all group-elements of $\mathbf{D}_{2d}$. Using the group-table we obtain the conjugacy class of elements similar to representant S_4

Table 4: Similarity Transformations

X	X^{-1}	$X^{-1} S_4 X$
E	E	S_4
S_4	S_4^3	S_4
S_4^2	S_4^2	S_4
S_4^3	S_4	S_4
C_2'	C_2'	S_4^3
C_2''	C_2''	S_4^3
σ_d'	σ_d'	S_4^3
σ_d''	σ_d''	S_4^3

So we have $(X^{-1} S_4 X) = \{S_4, S_4^3\}$.

THEOREM G_1:

A group can be written as a union of disjoint classes of similar elements. When A, B, ... form a system of representants in G, we have G = (A) U (B) U (C) U ...; we then say that G decays into or is decomposed into conjugacy classes.

Example:

The basic orientation group $\mathbf{D}_{2d}$ decays into five conjugacy classes. The partition is the following:

$$\mathbf{D}_{2d} = \{E\} \cup \{S_4^2\} \cup \{S_4, S_4^3\} \cup \{C_2', C_2''\} \cup \{\sigma_d', \sigma_d''\} =$$

$$= [E] \cup [S_4^2] \cup [S_4] \cup [C_2'] \cup [\sigma_d']$$

SUBGROUPS OF ORDER 4 OF D_{2d}

The basic orientation-group contains 3 proper subgroups of order 4. Two of these are isomorphic with the Klein 4 group (Klein'sche Vierergruppe) $Z_2 \times Z_2$ and one is the cyclic group of order 4.

Thus, we have

$$\mathbf{D}_2 = \{E, C_2, C_2', C_2''\}$$
$$\mathbf{S}_4 = \{E, S_4, S_4^2, S_4^3\}$$
$$\mathbf{C}_{2d} = \{E, C_2, \sigma_d', \sigma_d''\}$$

INVARIANT SUBGROUPS

Definition: A subgroup I of G is an invariant subgroup of G, if for all X in G there is $I = X^{-1}\,I\,X$, that is I is invariant under all similarity-transformations.

Note that S_4 is an invariant subgroup of $\mathbf{D}_{2d}$, also the subgroups $\mathbf{D}_2$, $\mathbf{C}_{2d}$ and $\mathbf{C}_2$.

FACTORGROUPS

Definition: Let I be an invariant subgroup of G. Then the set of classes

$$F = (\ I,\ IA,\ IB,\ \ldots\ ) \text{ with } A,\ B,\ \ldots \text{ in } G$$

is a factorgroup or factor of G with respect to I.

Products are formed in the usual way; for instance we have (IA)(IB) = IIAB = IAB. I plays the role of the unity in the factorgroup, since I(IA) = IA and the element A-1N is the inverse of (NA). Consider the invariant $\mathbf{C}_2 = \{E, C_2\}$ of $\mathbf{D}_{2d}$. The elements of the factorgroup $\mathbf{D}_{2d}/C_2$ are C_2E, C_2S_4, $C_2C_2'$and $C_2\sigma_d'$. The products are formed according to the above rule. We obtain e.g. $(C_2S_4)(C_2C_2') = C_2S_4\sigma_d' = C_2\sigma_d''$, and therefore we find in the grouptable of $\mathbf{D}_{2d}/\mathbf{C}_2$ the association $S_4C_2' = \sigma_d'$ which is representative for the product $(C_2S_4)(C_2C_2') = C_2\sigma_d'$. The complete table of association of the factorgroup $\mathbf{D}_{2d}/\mathbf{C}_2$ reads follows:

Table 5: Group Table of the Factorgroup D_{2d}/C_2

	E	S_4	C_2'	σ_d'
E	E	S_4	C_2'	σ_d'
S_4	S_4	E	σ_d'	S_4
C_2'	C_2'	σ_d'	E	S_4
σ_d'	σ_d'	C_2'	S_4	E

This result is very important. It says that the structure of classical logic is a factor of the basic orientation group. We'll go into this in a while. At the basis of the cultural process there is a commutation of the original occurrence of space into a basic concept of orientation. This internalized structure of orientation has the exact mathematical form of a fundamental symmetry of orientation or equivalently a basic orientation group. Its form is that of a non-commutative algebraic group of the order 8 and can be found in the catalogue of crystal symmetries by Schönfließ. There it appears as the group $\mathbf{D_{2d}}$ of symmetries of the *two-sided doubleprism* or equivalently as $\mathbf{D_4}$ the spatial congruence group of the square. $\mathbf{D_{2d}}$ is an archetype of human orientation in both space and social space.

The original concept of space as $\mathbf{D_{2d}}$ has important algebraic properties with considerable consequences to social science, in particular to social organization in anthropology and sociology, but also in psychology. Those properties can be listed as follows:

$\mathbf{D_{2d}}$ possesses a centre consisting of two elements $\mathbf{D_2}=\{E, S_4^2\}$. The algebraic centre also represents a ritual centre of action in ethnology. It can determine upon central locations in social space, in particular in ethnic communities with a so called dualistic organization. These are organizations which are based on both concentric and diametral partitions of the society. In those societies logic and topologic go together. That is the logic of social exchange, the logic of rituals and ceremonies and the logic of myths are represented by topologic. Configurations of social space are mapped onto configurations of space. In such organizations the operator C_2 is a link between logic and topologic. $\mathbf{D_{2d}}$ possesses a cyclic subgroup of order 4, that is, Z_4. It is due to the existence of this subgroup that the basic orientation group, though of the order 8, is not isomorphic with the the sign-group of the chinese I Ging which is $\mathbf{Z_2}$ X $\mathbf{Z_2}$ X $\mathbf{Z_2}$. Moreover, there is a partition of the eight elements of $\mathbf{D_{2d}}$ into 5 equivalence classes. $\mathbf{D_{2d}}$ has 3 proper subgroups of order 4, two of them being of the Klein type. They are connected by the centre element C_2. This division is the deeper reason why some societies with a dualistic organization like the Bororo and several Sioux-tribes can be partitioned into three distinct social strata. The existence of this division can determine upon the location of a family in social space. This is somewhat contradictory because a division into three subgroups cannot easily brought into correspondence with the binary decomposition. Levi-Strauss has shown this and tried to overcome the paradox by a special construction which he denoted dualistic organization.[12] Indeed, $\mathbf{D}_{2d}$ can be generated by two operators of order 2, 8 is 2 times 2 times 2, no representant has the order three, but there occur only 1, 2 and 4 and so on. But it is due to the existence of the centre C_2 of order 2 connecting 3 subgroups of order 4 one of which is Z_4 which makes the difference to the I- Ging sign-group Z_2 X Z_2 X Z_2. This brings in what is called the *triadic point* in anthropology. The triadic point bridges the binary relation. Such triadic point is essential in the construction of $\mathbf{D}_{2d}$ from dualistic social organization. The last observation which is perhaps the most important at all is related to the organization of generative structures of thought as have been investigated by the researchers into genetic structuralism of intellectual development. Child psychology is a factor in the organization of social space. $\mathbf{D}_{2d}$ over C_2 is the factorgroup $\mathbf{D}_{2d}/\mathbf{C}_2$. This is isomorphic to the symmetry of binary logic or INCR. The acquisition of INCR at an age of about 11 is decisive for social location. We shall go into this topic in the next section. Note also that the original concept of space as $\mathbf{D}_{2d}$ actually gives rise to *archetypical numbers* as mentioned by C.G. Jung. But not only because of the geometric features of the quartered circle, but also by its algebraic structure. With a centre of order 2, 3 subgroups of order 4, 2 of them isomorphic with the symmetry of 2-valued logic, INCR, and a partition into 5 equivalence classes of operators, it gives a definite meaning to the natural numbers 2, 3, 4 and 5 in relation to orientation. All those properties of the basic concept of orientation as have been sketched on the previous two pages shall be explained in greater detail in the next two sections.[13] There is one line of development that reaches from the original concept of orientation unto the organization of ethnic communities, and another that explains the derivation of the *structure of thought* as logic from that same concept.

In order to be able to demonstrate these relations two things have to be done. First it must be shown that the exact concept of dualistic organization is isomorphic with D_{2d}. Next it must be shown that the symmetry of binary logic, as is brought in by de Morgan's laws, is a subsymmetry of D_{2d} .

[12] Claude Lévi-Strauss (1971), *Strukturale Anthropologie*. Frankfurt am Main, 148 - 180.

[13] Some of the ethnological data as derived from the ethnographic studies of Lévi-Strauss, Pater Albisetti von Steinen and Lowie can be found in B. Schuh (=Schmeikal) (1989), "Soziogene der Orientierung. Ikonen der Raumzeit", in: *Wisdom*, 3/4, 1 - 18.

2. SPACE AND LOGIC

Derivation of the Laws of Classical Logic

from the Original Concept of Orientation

The purpose of this section is to show that the genetic structure of cognition is a substructure of the genetic structure of orientation in space. It is no small substructure, however, since according to elementary algebra, its index relative to the orientation symmetry is 2. That is, orientation is roughly speaking twice as large as logic. To understand all that one has to go into some of the findings of genetic structuralism.

2.0. Two Theorems on Space and Logic (SLC)

The child psychologist Jean Piaget and other contributors to the genetic epistemology of structuralism used an algebraic form of logic which was very different from de Morgan's or Boole's *laws of thought*. They considered a metastructure of algebra that could itself be represented at the level of algebra, in particular as a symmetry or group. For Boolean logic this group is isomorphic with Z_2 X Z_2 and sometimes denoted the *semantic square*. As far as I can remember, none of my collegues at the logistics department, including myself, knew of that approach. Such lack of application generality was sometimes held against the inventors of genetic epistemology. But Piaget was indeed very right when he took INCR to be the basic structure of logic rather than Boolean algebra. This is, of course, no argument against Boolean algebra. It is just not the right way of representation when dealing with genetic structures of cognition.

Suppose you have an algebraic structure consisting of some fundamental set of elements or letters and various basic operations such as those of Boolean algebra. Now you may define second order operators acting on the basic ones, such as *inversion of binary operators*, e.g. $\cup$ is turned into $\cap$, *inversion of elements* such as p is turned over into $\neg p$. Then the meta-algebra derived in this way will have a certain symmetry. In particular, consider the operators

I	invariance of a Boolean term
N	negation of a Boolean term
C	inversion of the operators $\cap$ and $\cup$
R	inversion of elements (p turned into $\neg p$) in a Boolean term.

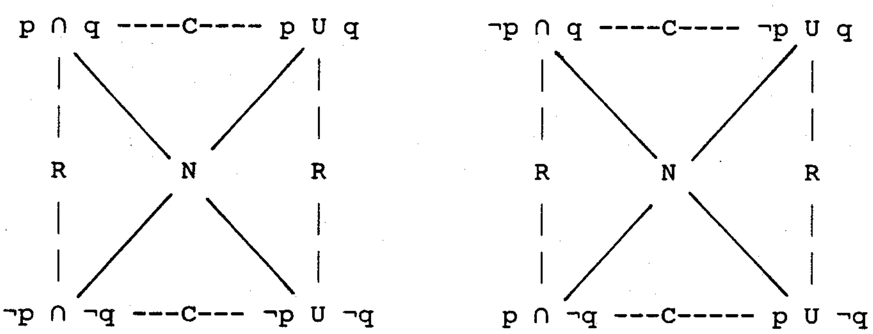
In french speaking art C denotes *correlative*, R means *reciprocity* and N involution as *negation.* I denotes identity. Next consider the complete set of well-formed Boolean terms of two letters. Those are

1. $p \cap q$ 2. $p \cup q$ 3. $\neg p \cap \neg q$ 4. $\neg p \cup \neg q$

5. $\neg p \cap q$ 6. $\neg p \cup q$ 7. $p \cap \neg q$ 8. $p \cup \neg q$.

Applying the meta-operators N, C and R to those 8 terms, the following structure is obtained:

Figure 12: Operators of the INCR-Group



From this figure we can immediately see two things, the first being the fact that N essentially represents the laws of de Morgan, and the second that the fundamental set of dyadic relations in Boolean algebra decomposes into two disjoint subsets under the action of the INCR operators. That is, none of the operators I, N, C or R is capable of connecting the two subsets. But there is still a third thing to be realized: Associating the operators I, N, C, R by applying them diachronically, that is, one after the other, the algebra of I, N, C, R under such association turns out isomorphic with Z_2 X Z_2. Its group-table is

	I	N	C	R
I	I	N	C	R
N	N	I	R	C
C	C	R	I	N
R	R	C	N	I

which is indeed isomorphic with the Factor $\mathbf{D}_{2d}/\mathbf{C}_2$ of the basic orientation-group.[14] The fundamental set of dyadic relations of the Boolean algebra thus decomposes into two subsets with meta-structures of the form $\mathbf{D}_{2d}/\mathbf{C}_2$ which is Z_2 X Z_2. This group represents the symmetry of Boolean logic.

As Piaget and others have shown in experimental tests, the acquisition of the INCR group at a certain age as an internalized structure is fundamental for the development of intelligence. Therefore we can formulate the following theorem on the connection between space and logic (SLC):

SLC-THEOREM 1:

The symmetry of the Boolean laws of thought is contained in the symmetry of the original concept of space.

Or put in more mathematicas terms:

The symmetry INCR= $\mathbf{Z}_2$ X $\mathbf{Z}_2$ of classical logic is a proper subgroup with the index 2 of the basic two-dimensional orientation group $\mathbf{D}_{2d}$.

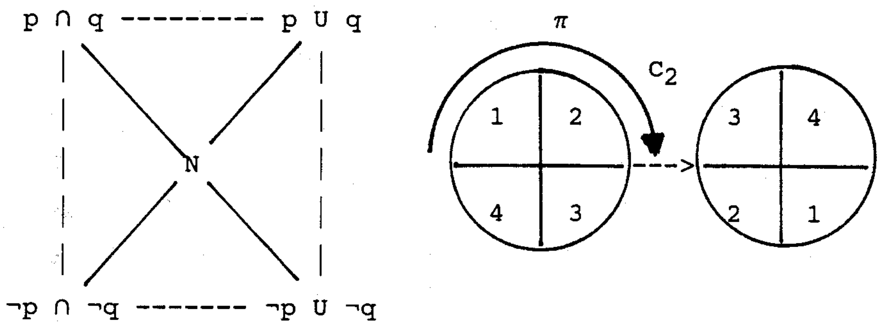
The SLC-theorem also says that logic is a part of topo-logic. The operative structure of cognition is virtually built on the basis of a genetic structure of orientation. It is based on an archetype. Therefore the group $\mathbf{D}_{2d}$ can be denoted a *topogenetic* structure whereas the INCR group is a psychogenetic structure of cognition. So we have found a topogenetic structure of the mind on which

[14] For comparison, see Table 5 on p. 34.

the psychogenetic structure of cognition is based. In other words, the laws of thought may have developed out of the original structure of orientation in space.

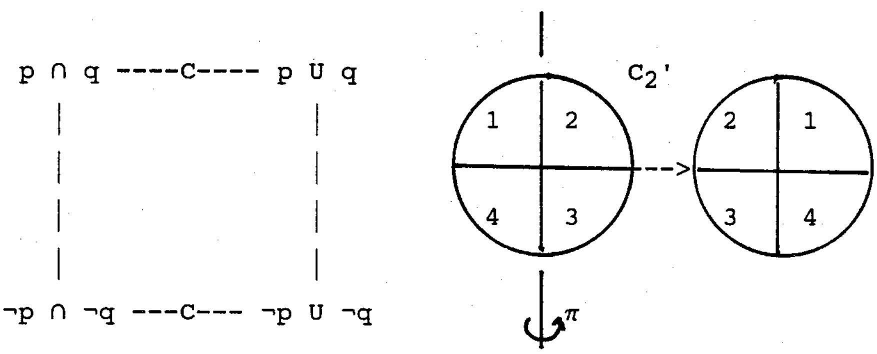
As the SLC-theorem says that the INCR is a proper subgroup of $\mathbf{D}_{2d}$, it would be interesting to know which of the subgroups $\mathbf{D}_2$ or $\mathbf{C}_{2d}$ represents best the INCR. That is, despite their being isomorphic with Z_2 X Z_2 we ask now which representation is better, is more natural, which one refers to details of representation that are more appropriate to describe the symmetry of the semantic square. Definitely, this is the subgroup $\mathbf{D}_2$. The spatial operations of $\mathbf{D}_2$ correspond directly with the logical operators I, N, C and R. Consider the figure

Figure 13: Operator C_2 of D_2 as Equal to N of INCR



By applying the operator C_2 to the semantic square as we would do with any ordinary square in space, corners on the main diagonals are exchanged. But this is actually resulting from the application of N. Similar figures can be discovered with C_2 ' and C_2 '':

Figure 14: Operator C_2' of D_2 as Equal to C of INCR:



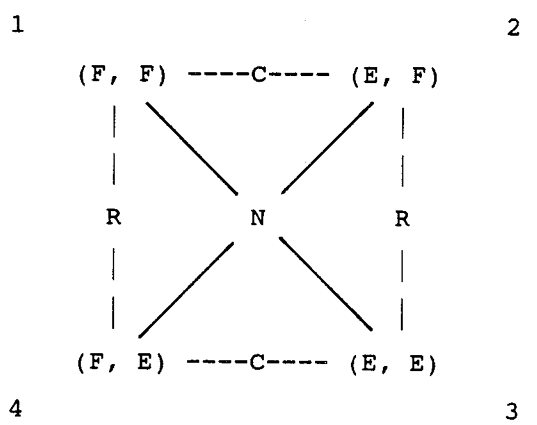
Since C_2' is a rotation of the square by π around the vertical axis, it represents the exchange of logical operators $\cap$ and U, that is, the C of INCR. Similarily the operator C_2'' of $\mathbf{D}_2$ is a rotation by π around the horizontal axis, it represents the logical operator R.

There are many representations of the INCR symmetry, some of which have been given by Jean Piaget. They are essentially related to the turning of two figures that have a front- and a backside, such as humans, animals, flowers and so on, in space. Thus C may signify the turning by π of the first figure, R the turning by π of the second figure and N the turning by π of both figures. Piaget has given the example of a board and a snail moving left or right and the operators are given by reversion of direction of motion. Or consider sociometry. Consider two friends A and B. Let (F, F) denote "A is a friend of mine" and "B is a friend of mine", and (E, E) "A is an enemy of mine" and "B is an enemy of mine". So there is an observer denoted *me* or *ego* and two relations between ego and A, and between ego and B. Let operators be defined as follows:

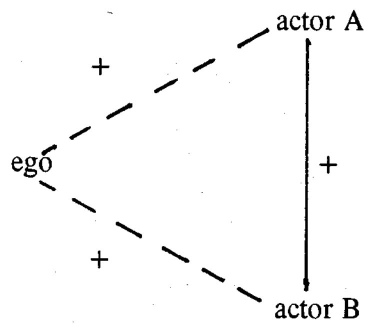
I	let friends of ego stay friends and foes stay foes
C	turn A's friendship into enmity and reverse
R	turn B's friendship into enmity and reverse
N	turn ego's friends into foes and reverse.

Then again you obtain the symmetry of the INCR group.

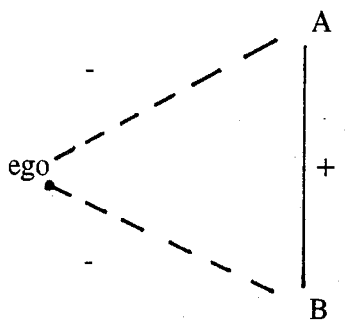
Figure 15: Symmetry of the INCR-Group



The corners in the graph have been numbered in order to bring out more clearly its relation to balance theory.[15] Considering *actual states of social affairs* in Fararo's sense and *ego* as the third actor, one obtains balanced graphs as signifying corners 1 and 3. Corner 1 signifies the relations



and Corner 3 means



According to the structure theorem by Cartwright and Harary[16] these structures are balanced, and the de Morgan type of operator N shifts between those balanced structures. On the other hand corners 2 and 4 refer to unbalanced states of the sociograph. That is, operators C and R effect the state of balance, while N leaves it invariant. However, products CR and RC again do not affect balance because they are equal to N.

Another example different from classical logic may be provided by Coleman's theory of social exchange.[17] Consider two actors and two events, their control matrix C_{ij} and interest matrix X_{ji} Furthermore, have regard to the possibilities of an event as being divisible or indivisible. In the first case control over the event may be shared by both actors, in the second only one at a time can control it. Now there exists the elementary structure of social exchange characterized by the following initial situation:

---

15 See T.J. Fararo (1989), *The Meaning of General Theoretical Sociology*. Cambridge, pp. 31 and pp. 281.

16 D. Cartwright, F. Harary (1965), *Structural Models: An Introduction to the Theory of Directed Graphs*. New York.

17 J.S. Coleman (1990), *Foundations of Social Theory*. Harvard University Press.

Table 6: Initial state of the exchange system

actor 1 controls event 1 entirely
actor 2 controls event 2 entirely
actor 1 is however not at all interested in event 1,
but directs all his interest onto event 2
actor 2 is not at all interested in event 2, but
analogously invests all interest in event 1.

In such a case the control matrix is given by

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and the interest matrix is

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

As can easily be verified by Coleman's equation, in such a case there will take place a complete exchange of control, and the equilibrium matrix will be

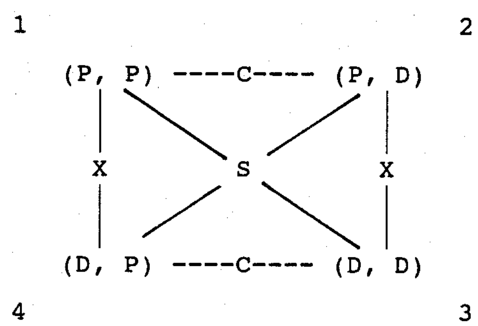
$$C^* = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

So in the end the one who is really interested will obtain total control over the event of interest. This is really the most elementary structure of social exchange. It can best be examplfied by playing children, say, Anna and Peter. Anna has an apple which she dislikes, and Peter has a ball he is not interested in. But Anna would like to have the ball, whereas Peter would like to eat Anna's apple. Therefore the best they can do is simply to exchange their commodities: Anna gives Peter her apple and gets his ball. The resulting equilibrium control structure will be denoted a partition of control over events in contrast to a distribution of control of events among more than one actors. Thus to every event, in the end, there belongs one and only one controlling actor, and therefore we obtain a homeomorphism of events onto actors. Events thus appear as indivisible commodities. But social reality is more complex and unsually there is a division of control. In the elementary configuration involving two actors and two events only, there may appear transitions from division of control among the two actors to partition of control. A quite anlogous statement holds for interests. There may be a partition of interest in that each actor is interested in one event only, e.g. in the one he controls. Or interest may be distributed over events. Now we may speak of microsociological exchange systems in the above sense as systems of individual belongings, if both control and interest are partitioned but not divided. Then the structure of control relations is a partition. The same holds for the structure of interests. As soon as either control or interest are divided, there arises confusion or conflict in the sense that it is no longer clear in such a system whether commodities are really personal belongings or not. Consider for instance Jill's car which arouses the interest of Jack and so on. Therefore we may define the *modality of division* as either distribution or partition of control among actors 1 and 2 or of interest among events 1 and 2. The operators are now defined as -

C	alters the modality of division of control
X	alters the modality of division of interest
S	changes modality of division of both
I	leaves modality unaltered

So we can ponder over the meaning of social processes where the modality of division of control is turned from monopolization by a single actor to distribution among two actors, and where the modality of division of interest is turned from fixation by a single event to division among two events. Those signify elementary happenings of communication and social exchange. The algebraic structure of possible associations of I, C, X and S is again that of $\mathbf{Z}_2 \ \mathrm{X}\ \mathbf{Z}_2$.

Figure 16: Algebraic Structure of Possible Associations



where

(P, P)	control and interest partitioned
(P, D)	control partitioned, interest distributed
(D, P)	control distributed, interest partitioned
(D, D)	both control and interest distributed.

To examplify the sociological meaning of the above graph we can invent a story of transactions and change that are carried out by Ann and Peter while we move from corner 1 over 2 and 3 back to corner 1 and then to 4:

(P, P) ...

Ann has an apple she would like to eat, and Peter has a ball and would like to play. So Ann is interested in her apple, but not in Peters ball, while Peter shows interest in playing with the ball, but it doesn't make any (utility-) difference for him whether he got Ann's apple or not. (thus interest is defined in terms of utility-difference). In this state there is some kind of order in the system. Things are in a way to where they belong. But:

As the two walk along it comes to Peters mind that he could convince Ann to play ball with him. Therefore he begins to tell her a story how good it would be to play with him and some others; if she would be interested to join in. After the game they could eventually drop into his home and get some cakes from his mother. It's of course not quite fair, but Ann listening to Peter becomes more and more convinced that this would be a good idea and says: oh, that's a nice idea, I'll join you. (At this point Peter has attained to arouse Ann's interest and to distribute it over the two events). So Peter would go on asking: "would you mind, Ann, giving me some bites of your apple in return, I'd like so much to bite your apple" (now Peter's interest is distributed too. So we have now a configuration of the type (P,D).

(P, D)

Ann says "of course" and gives Peter her apple, while Peter gives her the ball in return and comments: "you can already have the ball and play a little" (in this state both control and interest are *demonopolized* and distributed among both actors and events). Peter makes a first bite. It is still up to Ann to demand her apple. So both Ann and Peter got some control over the *declining* event *apple*.

(D, D)

↓

As Peter makes the first bites, lovely Ann says: "Peter you can eat my apple anyway, cause I shall get another when I get home". Thus it doesn't make a significant difference any longer whether she eats her apple or not. Also Peter enters a transitory state of losing interest in the ball. Thus both interest and control become concentrated again. Ann holds control over the ball while Peter has control over the apple.

(P, P)

↓

Ann has hoped that Peter would sometime stop eating her apple or at least leave her some bites. Therefore she runs away with the ball. That makes Peter aware of his interest in the ball. So he cries: "Stop Ann, give me back my ball!" But Ann says: "Okay, you can have it. But first give me back my apple!" At that instant interest is concentrated, but control is still distributed. Both have several resources to keep some control over the events apple and ball. So there is some sort of confusion in the system. Things are no longer where they belong to. ... and so on ad infinitum.

(D, P)

There are many things that can be learned from such a scenario. First we can observe that in microsociological situations, where transactions play a role, not only control but also interest can be exchanged. According to the rather complex situations of transactional analysis there may occur several moves during the exchange that are in analogy with the $\mathbf{Z}_2$ X $\mathbf{Z}_2$-operators. What is the surprising in such a situation is that action is somehow governed by a metastructure of classical logic although it does not follow that logic.

To show up the whole domain of applicability of the INCR group would be an inexhaustible undertaking. So let us only keep in mind, that Z_2 X Z_2 does not only represent the symmetry of Boole's *laws of thought*, but also the symmetry of a very general class of operations and actions. This symmetry being a factor of $\mathbf{D}_{2d}$ is part of the basic concept of orientation in space.

The SLC-THEOREM 2:

It asserts no less than that the symmetry of the original concept of space is indeed isomorphic with the symmetry of Boolean logic.

The SLC-theorem 2 is more severe than theorem 1. In this way the logic of thought is to be entirely derived from the archetype of orientation. This cannot immediately be seen from theorem 1 because the INCR group as was used by genetic structuralists has index 2 relative to the symmetry $\mathbf{D}_{2d}$ of orientation. Therefore one asks oneself if anything went wrong with the way logic was represented, or if there are some missing connectives between the elementary logical terms with two arguments. Both is true, in a way. First one gets the impression that the decomposition of logical terms into two distinct subsets by the action of INCR as is drawn on page 70 cannot quite be all. Consequently one begins to think if not some of the symmetry operators of D_{2d} can be translated in such ways that they connect the till now unconnected parts; since in thought those are not unconnected at all. As the first subgroup that could be identified with the INCR was C_2, the next step should consist in asking if it was possible to find out about the logical meaning of the subgroup $\mathbf{C}_{2d}$ and incorporate it into the calculation since it is also isomporphic with $\mathbf{Z}_2$ X $\mathbf{Z}_2$. The last step should be the incorporation of S_4. This, one has the feeling, must be the most difficult step since $\mathbf{S}_4$ is $\mathbf{Z}_4$ and one wonders what could be the role of a cyclic operator of order 4. But finally it should become clear why $\{E, C_2=S_4^2\}$ form the centre of the whole symmetry arrangement.

2.1. Incorporating $\mathbf{C}_{2d}$

The first step consists in the following task: consider $\mathbf{C}_{2d} = \{E, C_2, \sigma_d', \sigma_d''\}$ and construct $\sigma_d'$and σ_d'' such that the first semantic square with symmetric arguments $p \cap q$, pUq, $\neg p \cap \neg q$, $\neg pU\neg q$ can be connected with the second semantic square asymmetric in the arguments $\neg p \cap q$, $\neg pUq$, $p \cap \neg q$, $pU\neg q$, and take care that the definition is compatible with the group-table of $\mathbf{D}_{2d}$. Let I, N, C and R be defined as in the INCR-group as $I=E$, $N=C_2$, $C=C_2'$, and $R=C_2''$, C_2 representing the laws of de Morgan, C_2' exchange of operators $\cap$, U and R involution of arguments, p turning into $\neg p$, $\neg p$ into p and so on. It is clear that σ_d ' and σ_d " have to act asymmetric on the arguments.

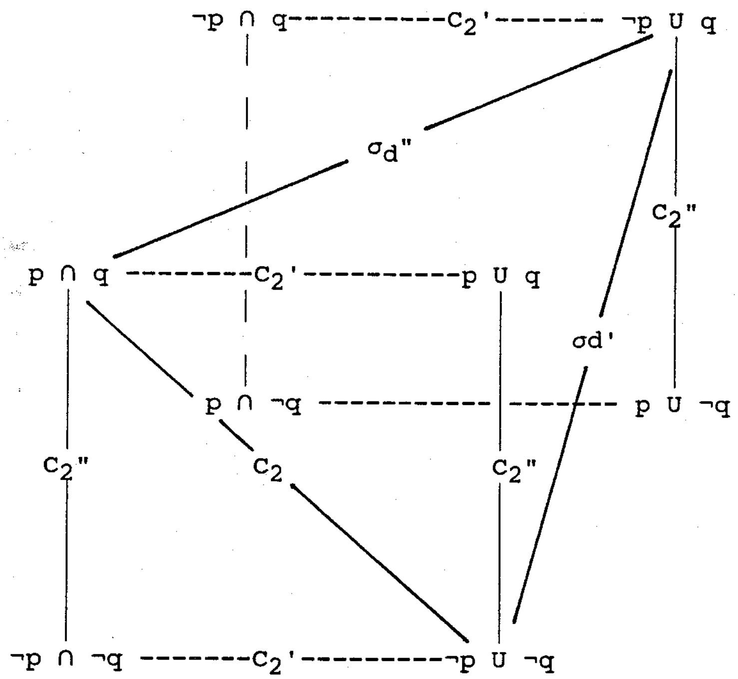
The following definition is compatible with the group-table of $\mathbf{D}_{2d}$:

σ_d' involute the second argument

σ_d" involute the first argument and exchange logical operators $\cap$, U.

Thus σ_d' turns $\neg q$ into q and q into $\neg q$ and so on. Let the two semantic squares form the front and the back of a cube. Then σ_d' acts *in the diagonals* of the left and right hand side planes of the cube, whereas σ_d" acts in the diagonals of the base and of the top plane. The figure looks as follows:

Figure 17: A Semantic Triangle



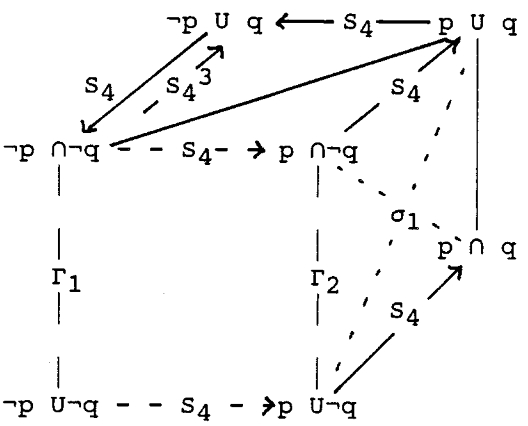
So we have now found one *semantic triangle* which connects the corners $p \cap q$, $\neg p$ U $\neg q$ and $\neg p$ U q. This plays an analogous role as the triangle formed by the sides C_2', C_2" and C_2 in the semantic square, and indicates that we have indeed a subsymmetry of the type $\mathbf{Z}_2$ X $\mathbf{Z}_2$ as is the case with INCR. There are indeed eight such triangles corresponding with the eight plane diagonals. They form two tetrahedrons that penetrate each other. The four edges on the front and backplane of the cube are C_2, the four edges on the left and right planes are σ_d' and the four edges on the base or top planes are σ_d".

Finally we have to find out about the role of S_4 and S_4^3 which is a little difficult. Obviously S_4 must somehow connect the front with the back of the *semantic cube*, it must be asymmetric in the arguments, and it can be related to the edges of the cube or to its spatial diagonals. Actually the following logistic definition is purposive:

> S_4... in symmetric terms involute first argument, in asymmetric terms involute the second and exchange logical operators $\cap$, U.
>
> S_4^3... in asymmetric terms involute first argument; in symmetric terms involute second argument and exchange operators $\cap$, U.

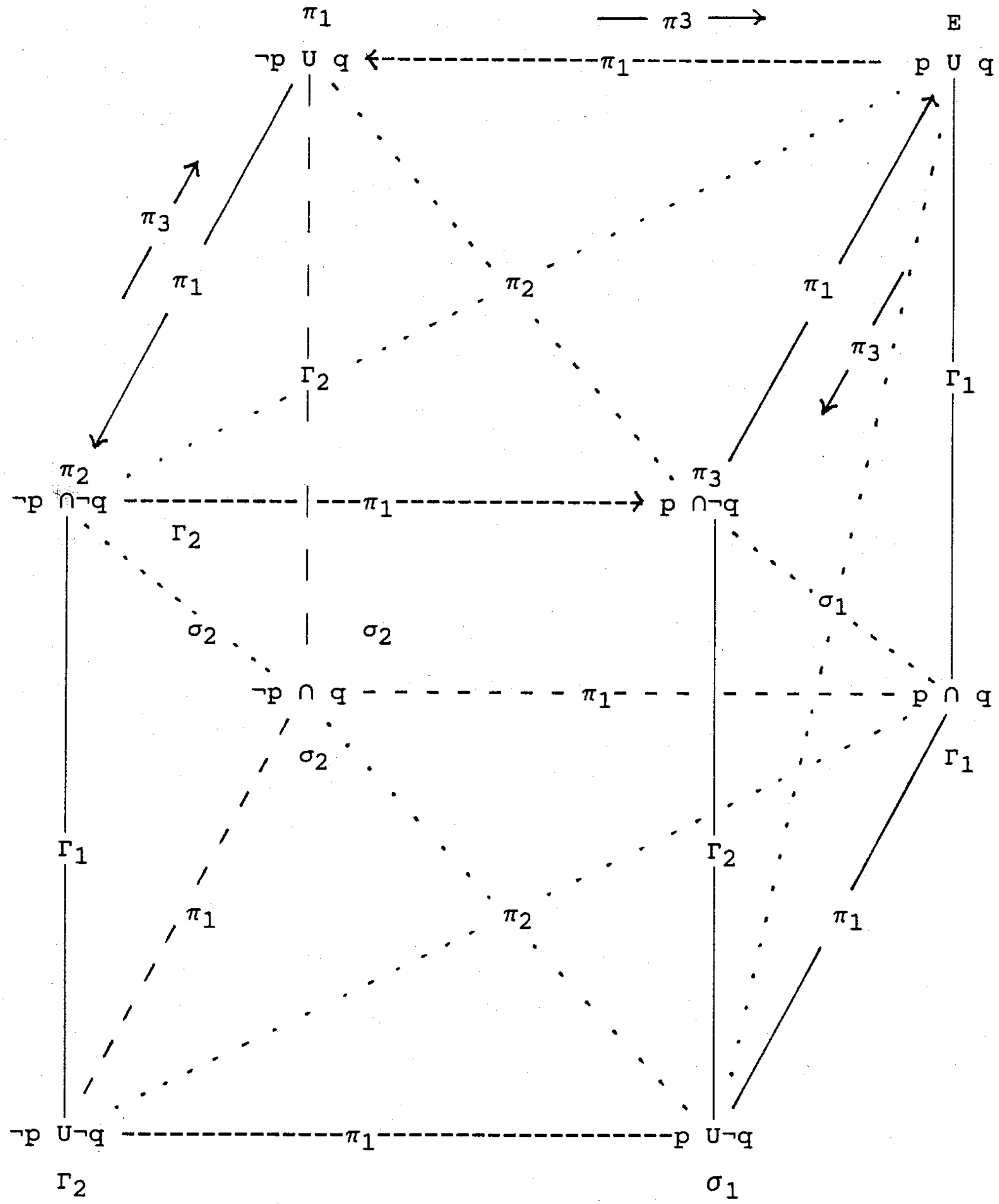
Thus both operators act in the edges connecting the front with the back plane as well as in the space diagonals, depending on symmetry of arguments. Each edge is split into two directed lines one of which is S_4 while the other is S_4^3 Therefore we obtain $S_4 S_4^3 = E$. In this way de Morgans rule is split into two components one of which leads e.g. from adjunction p U q to implication p --> q (which is $\neg$p U q), and from implication to the negation of adjunction $\neg$p $\cap$ $\neg$q.

Figure 18: Splitting of de Morgan's Rule



A sector of the whole algebra looks as follows:

Figure 19: Identification of the Operators of the Basic Orientation Group as Logical Operators



2.2. What happened?

Recall that we begun with the idea that genetic structuralism when identifying the operative structure of logic with the INCR group, has actually found the right vehicle to carry both classical logic and genetic structure. So we took, as is usual, the semantic square as a representative structure of the INCR. However, the fundamental logical terms in two arguments built up two such squares. Thus, in order to understand all possible transformations of all possible logical propositions formed by two statements p and q, we needed both semantic squares.

As soon as we considered both squares, all the other operators outside $\mathbf{D}_2$ obtained a definite logical meaning, although they represented symmetries of a subgroup of proper rotations in space, $\mathbf{SO}_+(3)$ (sometimes denoted $\mathbf{D}_3$). Each of the operators of the original concept of space performs a specific transformation within the space of logical propositions, and all of these operators actually exhaust the entire space of logical possibilities. Now, as we have seen that the orientation group $\mathbf{D}_{2d}$ contains 3 subgroups of order 4, two of which are INCR while the third is $\mathbf{Z}_4$ wouldn't it be interesting to find out about their logical meaning?

The situation is somewhat surprising. It turns out that it is better to think in terms of semantic triangles than in terms of semantic squares! How come? Well, the semantic square is built up by triangles anyway. Their sides are C_2' and C_2'' and centre element C_2. As we saw, the operator C_2 represents nothing other than de Morgan's laws, and the group C_2 is INCR. But there are still two other types of triangles that lead us through semantic space. The first is given by operators C_2, σ_d', σ_d'' which form the subgroup $\mathbf{C}_{2d}$.

And the second which is perhaps still more fascinating is $\mathbf{S}_4 = \{E, S_4, C_2\ S_4^3\}$. In every triangle there is thus a representative for a) de Morgan's laws and b) the centre of the orientation group. So each triangle is actually a decomposition of one of de Morgan's laws.

Consider paths $C_2C_2'C_2''$ or $C_2S_4S_4$ or $C_2S_4^3S_4^3$ or $C_2\sigma_d'\sigma_d''$ and so on. They all equal unity. Each will lead you around the edges of one type of triangle. In any case and in any triangle, C_2 is split into a product of two operators the first of which negates half of the truth while the second negates all of the truth. To get that point look at the following truth tables:

Table 7: Truth Tables

$S_4 \times S_4 = S_4^2 = S_4 \times S_4 = C_2$

-----> ----->   ----> ---->

p	q	¬p	¬q	p∩¬q	¬p∩¬q	¬pUq	¬p∩q	¬pU¬q	pU¬q
0	0	1	1	0 -	- 1	1	0-	--1	1
0	1	1	0	0	0--	--1	1	1--	-0
1	0	0	1	1--	--0--	--0	0-	--1	1
1	1	0	0	0	0--	--1	0	0--	-1

$\sigma_d'' \times \sigma_d' = C_2' \times C_2'' = C_2$

-----> ---->   -----> ---->

p	q	¬p	¬q	p∩q	¬pUq	¬pU¬q	¬pU¬q	¬p∩¬q	p∩q
0	0	1	1	0--	--1	1	1	1--	-0
0	1	1	0	0--	--1	1	1--	--0	0
1	0	0	1	0	0-	--1	1--	--0	0
1	1	0	0	1	1-	--0	0	0--	-1

Those are four examples of truth tables which belong to triangles $C_2S_4S_4$ (twice), $C_2\sigma_d''\sigma_d'$ and $C_2C_2'C_2''$ respectively. You always start with the left hand term and after two steps end up with the right hand term. The horizontal lines in the table indicate involutions of truth values by the action of the operators of $\mathbf{D_{2d}}$. You can easily figure out how the first operator actually negates half of the truth, while the second negates the rest. Thus de Morgan's rule of negation is always brought upon in two steps of negation. This is an invariant feature throughout the whole procedure. Now, all this looks rather complicated, but it is not. The whole algebra of D_{2d} as applied to logical propositions can actually be unfolded from two elementary operations; e.g. by C and σ_d'. C representing the C of the INCR equals C_2'. This operator exchanges the Boolean operators ∩, U. The other involutes the second argument.

For example you obtain:

$C_2' \times \sigma_d' = S_4$

p∩q ---> pUq ---> pU¬q

So you have obtained the action of S_4 from C and σ_d'. Provided you know the definition of S_4 then in the next step you can get C_2'', then C_2 next σ_d'' and finally S_4^3. Thus you can unfold the whole algebra from two very simple operations:

First exchange Boolean operators
Second involute second argument,

or in terms of spatial rotations of the quartered circle:

First rotate around C_2'-axis by π
Second rotate around diagonal σ_d' by π.

There are indeed other ways to define the generators of the symmetry. We may just as well start with S_4 and σ_d' which may possibly appear even more correct where the actual course of history is concerned:

SYMMETRY-GENERATORS:

Generator 1: In symmetric terms change first argument; in asymmetric terms change second argument and exchange Boolean operators. This is S_4.

Generator 2: Change second argument: σ_d'.

Whichever way you regard as best. In any case you can unfold from such generation principles the whole eightfold path of the original concept of orientation which is the root structure of thought. Thus the generators of the basic orientation symmetry are indeed generators of logic.

3. SOCIOLOGICAL TIME

Time as Exchange of Stability

In the following chapter sociological time will be based on our knowledge about self-organizing systems. Therefore nonlinearity, chaos and order, strange attractors, exchange of stability, commutation of diachronic, recursive processes into synchronic, stable patterns and similar quite formal things will play an important role. But at the same time we shall always rely on the basic epistemology, that is, occurrences, concepts and the role of cognition in the process of time. We shall observe how the coupling of two chaotic processes of population dynamics can bring about an internal time parameter, and we shall also work out how sociological time is to be derived within a synergetic migration system. By carrying out all the nessessary procedures, the logic of the whole thing shall come upon all by itself.

3.0. Basic Definitions

If we are highly aware of the process of theory building we shall see how a bundle of rather vague ideas is step by step transformed into inexact and thereafter into exact concepts. To demonstrate this let us procede very slowly.

Time is not motion. Let us be very aware of this. There may be motion and yet no change. Classical physics has not seen this. But in social science it can indeed be seen. To put it into loose and simple words at first: microsociologically, people may move around, yet there need not be any significant social change. So it's important at first to notice that time is not change, but change can be temporal. Not every motion is temporal, neither physical, nor psychological, nor sociological, but all three of them may be. This is to be kept in mind at first.

Next let us be aware, time can appear in many garments. You need time to acquire some technical skills, time to learn a language or time to build a theory. This is called acquisition time. You also need time to achieve some eductional level, time of achievment, time to travel from here to there, travel time, time, if you wish, of contemplation, time of no time experience. You need time to produce an electronic print or a scientific paper, production time, and the time you spend in the opera house, in the theatre, in the concert hall, at the pop festival and so on. But to listen to a concert doesn't really bring in time. Looked at it from the outside observer, your husband waiting at home for your return, you spend a certain amount of time in the concert, of course. But psychologically, sociologically, for the collectivity following the flow of sense-perception, the music, the applause, parfumes, the changing lights, the talk in the pause, there is no time at all. After all, that's what concerts are good for. You don't look at your watch when you listen to the music, unless it's bad music or, may be, you are a bad listener. The concert is a precise ticking over, a harmonious cooperation of musicians, a synergy of tones and acustic events in which the collective experience of time is annihilated.

You see, we had a take off with acquisition time, time to learn a language and so on, somehow related to effort, task, aim, stress and all the rest of it, and ended up with a state where there is nothing to be acquired, no tomorrow. You are just at ease with yourself and with the others. We are all listening to that wonderful music, to the applause , there's no stress, no aim, no effort, no look at the watch. The orchestra performance was taken by Lévi-Strauss and some linguists like Leach and indeed by many others as a means of instruction. The performance of mythical episodes, ceremonies, rituals, myths as a whole are all machineries to diminish or annihilate time. Therefore in society, let us be aware, actually two things are going on. One being the creation of time, the other being dissolution of time. Creation of time is bound to construction in the civilized sense, the building of a cognitive structure, such as learning a language, the structuring of society, establishing an educational system for instance. All those things that go on in your inner, which change your brain and at the same time are bound to a special design of society, and going on not only in you, but in many of us, in a whole social subsystem, group or acting collectivity; all those processes where there is a transition to cognitive order, either from chaos or from some previous order, represent temporal social processes. Those processes are diachronic developments, from one state to another, from e.g. being free of that cognitive structure, being a language, skills or whatever, to having acquired that structure, language, skills and so on. So time is essentially acquisition time, eventually production time, time of achievment and so forth, and at the basis of it all there is the learning, transfer of cognitive order, acquisition of certain memories, brain-structure and sociological competence. To acquire all that, a society is needed, subsystems of society with special functional units, schools, teachers, means of instruction, administration and all the rest of it. Once that whole system, brains included, is in a stable state, a stable synchronic sociological pattern is obtained. A traditional opera house may serve as some sort of example. In that thing there is motion too, yet this is synchronic. Because it's a stable structure, and it has settled the whole evolutionary affair. To put it briefly, for a synchronic social structure to occur, a transition beyond instability has to be made and completed. The evolutionary steps have to be finished, though the usual motion, people moving in and out and all the customary happenings, can go on as ever. Therefore we put foreward a first definition:

DEFINITION T_1:

Motion which involves development of cognitive and social structures, transitions to some new order of skills, competence and social space, is denoted evolutionary and diachronic. Such motion defines sociological time. Thus, sociological time is bound to collective observation, learning, cooperation and exchange of stability. It is the building up of order as an ongoing process.

DEFINITION T_2:

Motion that does not involve evolution of new cognitive and social structures, that does not include transitions beyond instability and leaves the social space unaltered, but does only rely on reproduction of the current system, like in a ritual or mythical performance, such motion does not define sociological time. Thus, motion beyond sociological time is bound to a stable, synchronic social pattern. In this case the *timeless* structure is already built up.

States as defined by definitions T_1 and T_2 are indeed extreme. Real social events will always somehow sail to and fro between time and no-time. But civilized societies as are prevailing at present are rather on the side of time. Civilization seems to be some sort of incarnation of an uncritical belief in evolution. Actually it might be better to let things rest for a while.

Thus, as was said above, time is acquisition time, achievment time, production time and so forth, and at the basis of it all there is some transfer of cognitive order, the acquisition of sociological competence and so on. After all, you must know where your place is in social space. That is what takes places at the individual and micrological level. Sociologically things are a little bit different. Consider the labour market. To reproduce, day after day, year after year, a stationary occupational structure, a certain educational system is required. And that educational system is very essentially the source of the diachronic procedures of society. Therefore, observing the cultural system as a whole we have to find out: one part of it, namely traditional art, established science etc., sinks time in performances, parties, presentations, ritual activities, and similar customary events, and a second part, the educational system, schools, colleges, universities and so on penetrates into our brains and gives them new skills, new structures, new memories. That latter subsystem is the essential source of sociological time. But there might be another, may be households, but I'm not shure of that yet. There might be something in households that is not yet seen, not only unemployees, indeed.

All that sounds somewhat unusual, may be, but it is not. In order to bring the point down to earth, let us ask a question: Is migration between differnt economic sectors, sociologically, time? What would you say? Foreget about the watch! Does migration bring on time? In present day synergetic approaches to migration and social mobility in general, often the following assumption about asymmetric contribution to transitionrates between different sectors is made[18]: the size of the symmetric contribution to transitionrates between different occupational sectors depends on the numbers of employees in those sectors and on the similarity of educational equipment with levels of qualification. That's quite common sense. Briefly, if two sectors are occupied with mainly different educational levels, migration between them is not probable. So, empirically you can show that intensity of motion, intersectoral migration in that case, is bound to a similarity of statistical moments of the distribution of educational levels. Therefore we have asked: does that motion represent sociological time? Is the watch ticking rapidly when migration is high? Not at all! Because, even when transitionrates are high, there is no sociological construction, no evolution, neither cognitive, nor sociologically taking place. Rather the contrary is the case! The complexity of the labour market is used to avoid evolution, in the sense of transition beyond cognitive instability, change of social location and so on. Also there is involved some kind of conservation of qualification. Loss of cognitive stability is avoided too. Loss of social status is avoided too, and so on. So we have to be very careful with time. Sociological time is actually bound to acquisition of cognitive structures, sociological competence and change of location in social space in the vertical sense. Action in stable (ritual) social structures does not bring on sociological time. But action in unstable, evolutionary structures creates the collective experience of time as foreward motion. When diachronic sociological processes constantly bring upon more or less stable synchronic structures in society, they are expected to create a stable social structure. That expectation is psychological time.

There were times, it is said[19], when population numbers in the world were fluctuating because of energy deficiency. Then mankind did not dispose of means to control energy supply and population-

[18] K. H. Müller (1990), "Langfristige Szenarienanalyse des österreichischen Beschäftigungssystems", in: ders., K. Pichelmann (1990)(eds.), *Modell zur Analyse des österreichischen Beschäftigungssystems. Prognose- und Szenarien-instrument der branchenspezifischen Beschäftigungsentwicklung*. Wien, 114.

[19] See e.g. C.M. Cipolla (1978), *The Economic History of World Population*. Harmondsworth.

numbers. There was no internal mechanism to stabilize a social structure over a long period of time. During these times the collective acquisition of knowledge in current use of symbols etc. must have been an extremely difficult sociological adventure. Now, when the internal mechanisms that allow for stabilization are not yet evolved, the whole outer structure, that is, population-numbers, may perform a transition into chaos, or there may be a to and fro between oscillations and chaotic motion with rare times of almost constant numbers. Probably such chaotic demographic systems have been coupled because of the fact that they had to use common ressources. So the question arises whether some sort of exchange of stability could take place in case of such couplings. It might also be interesting to ask if we can say anything about how people experienced time in those days. Actually we are able today to pin down such processes in an exact way, and we can learn from those simple examples, what is the logic of time in relation to self-organizing processes. There are both transitions between stability and chaos and exchange of fluctuations. Surprisingly, those systems contain hidden in their dynamics some sort of internal time parameters. These are however not to be confused with biological clocks. Rather they may be seen as parallel to the exchange of stability and instability.

3.1 Exact Requirements

Before we can go into coupled systems we have to discuss the now classic example of a logistic, nonlinear recursive function which involves chaotic motion. We shall not go into the whole mathematics of it because that would be much to difficult. But we can work out all the essential properties that are needed for moving a few steps foreward.

3.1.1. The Basic Model

The basic mechanism is given by the following functional f

(1) $$X_{n+1} = f(X_n) = A\,X_n\,(1 - X_n)$$

and describes the growth of a population from generation n to the next generation labelled n+1 by the constant control- or growth parameter A. Thus X_n denotes the population number of the n's generation, but is regarded as normalized between zero and one. Since it is a logistic equation of the saturation type, growth is being *saturated* by the term $(1-X_n)$. The larger the generation X_n, the smaller the saturating factor $(1-X_n)$. The term $X_n(1-X_n)$ has a maximum at $X_n=1/2$ where it takes the value 1/4. So X_{n+1} will not exceed unity unless A is larger than 4. Therefore A is considered bounded above by 4. The surprising feature of this iteration function is that it can disclose some rather vigilant motion of the population-number in case the generational growth is intensive. The reason is in the regulation mechanism provided by the binding factor $(1-X_n)$. A rapidly growing population also experiences rapid limitation by limited food- and energy supply. Now, what happens in detail?

As long as $A < 1$ the population will ultimately die out, and unless A exceeds 3 it will approach one definite stable value of $(A-1)/A$. This stable equilibrium value is given by the fixed point of the functional $f(x) = x$. When A gets larger than 3 there appears a stationary oscillation between two values. Therefore it is said that there is a bifurcation of f at $A=3$. The formerly stable, definite equilibrium value of X bifurcates into a stable 2-cycle. But that's not all. As we go on increasing the growth-parameter we observe more and more bifurcations. Beyond 3,45 there appears a bifurcation into stable 4-cycles, above about 3,547 there is a critical value for the inset of 8-cycles,, and at 3,566 stable 16-cycles come up. Beyond the critical value of 3,57 there is chaotic motion. Critical values of A to the inset of 2^n-cycles can be obtained by fixed points of higher orders of f(x). Beyond the inset of chaotic motion the iterated values of X take the shape of a random-walk. One cannot foretell where X will be going because that depends very sensibly on the initial values of both X and the control parameter.

In figure 20 (see page 59) there is a plot of X for $A=3{,}513$, that is, X moves within a 4-cycle. In figure 21 (see page 60) it is started off with $X=0{,}17$ with $A=3{,}8$. This leads to chaotic motion. Now it is possible to make a plot where X begins with some definite value and the control parameter is varied linearily such that the whole intervall between, say, unity and four is covered. In this case one obtains an image of a Feigenbaum scenario (figure 22, page 61). This image discloses how the complexity of the dynamic behaviour of the recursive system (1) increases with increasing growth parameter. Above $A=3{,}57$ chaotic behaviour may appear, but there are still regions of regular behavior built into the chaotic domain. For example there exists a stable 3-cycle which is a sufficient condition for any such system to have chaotic trajectories. So we can say there is a region of definite behaviour where A varies in a linear and monotonous fashion between zero and 3. In this intervall there is regular behavior, and X is either zero or moves in cycles of the order 2^n. But for values of A larger then 3 the motion of X is not definite, but either it is regular or irregular or both. So motion of X may be cyclic, it may show up with 5-cycles for instance, or it may be chaotic. Or it may be both chaotic and regular, depending on initial values of X. There appear regular patterns of motion after a history of more than 7000 iterations from the initial value of $X=0{,}17$ with $A=3{,}8$ (figure 21, page 60).

Figure 20: Phase Diagram with Constant Control-Paramter (1000 Iterations)

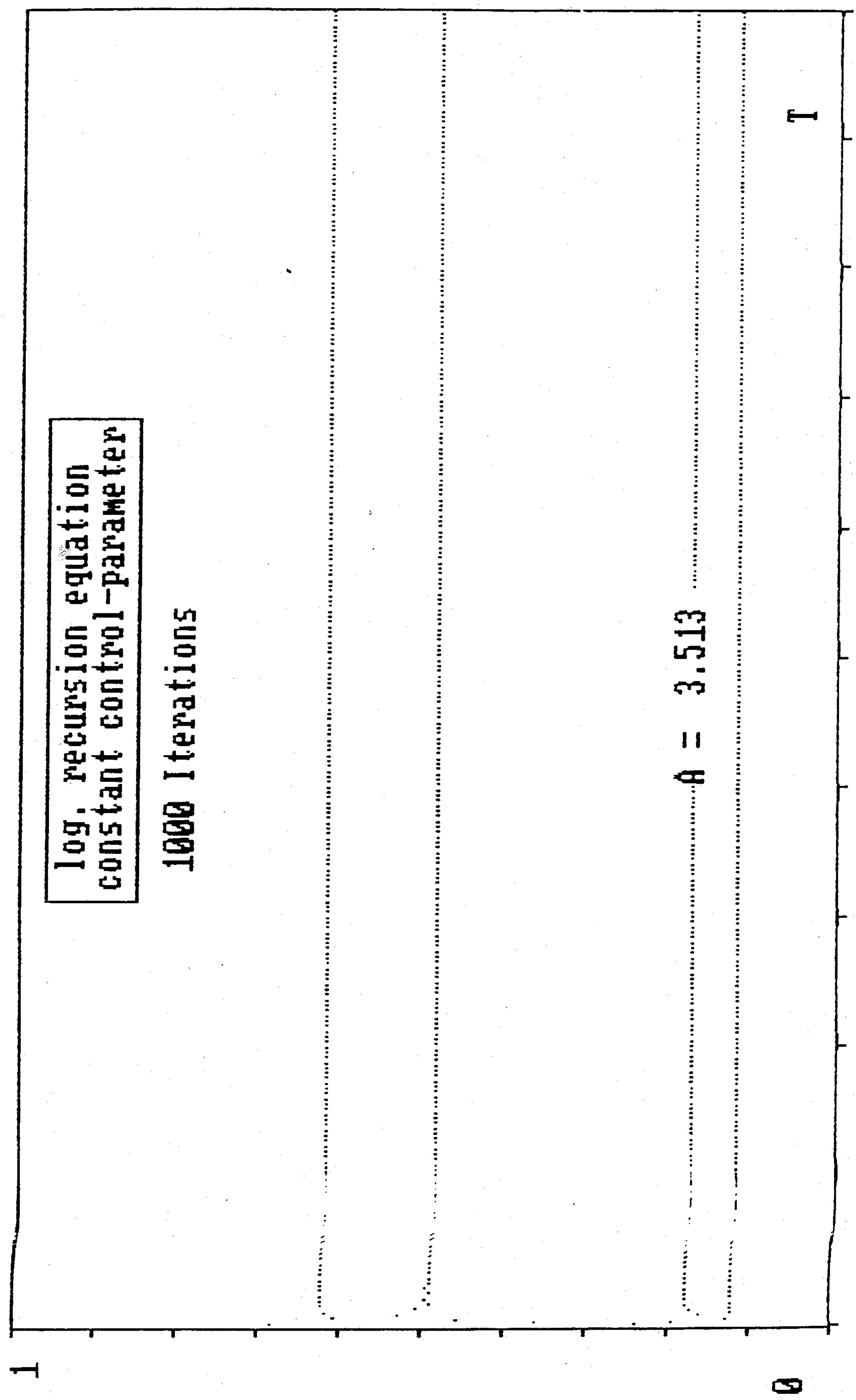


Figure 21: Phase Diagram with Constant Control-Paramter (10.000 Iterations)

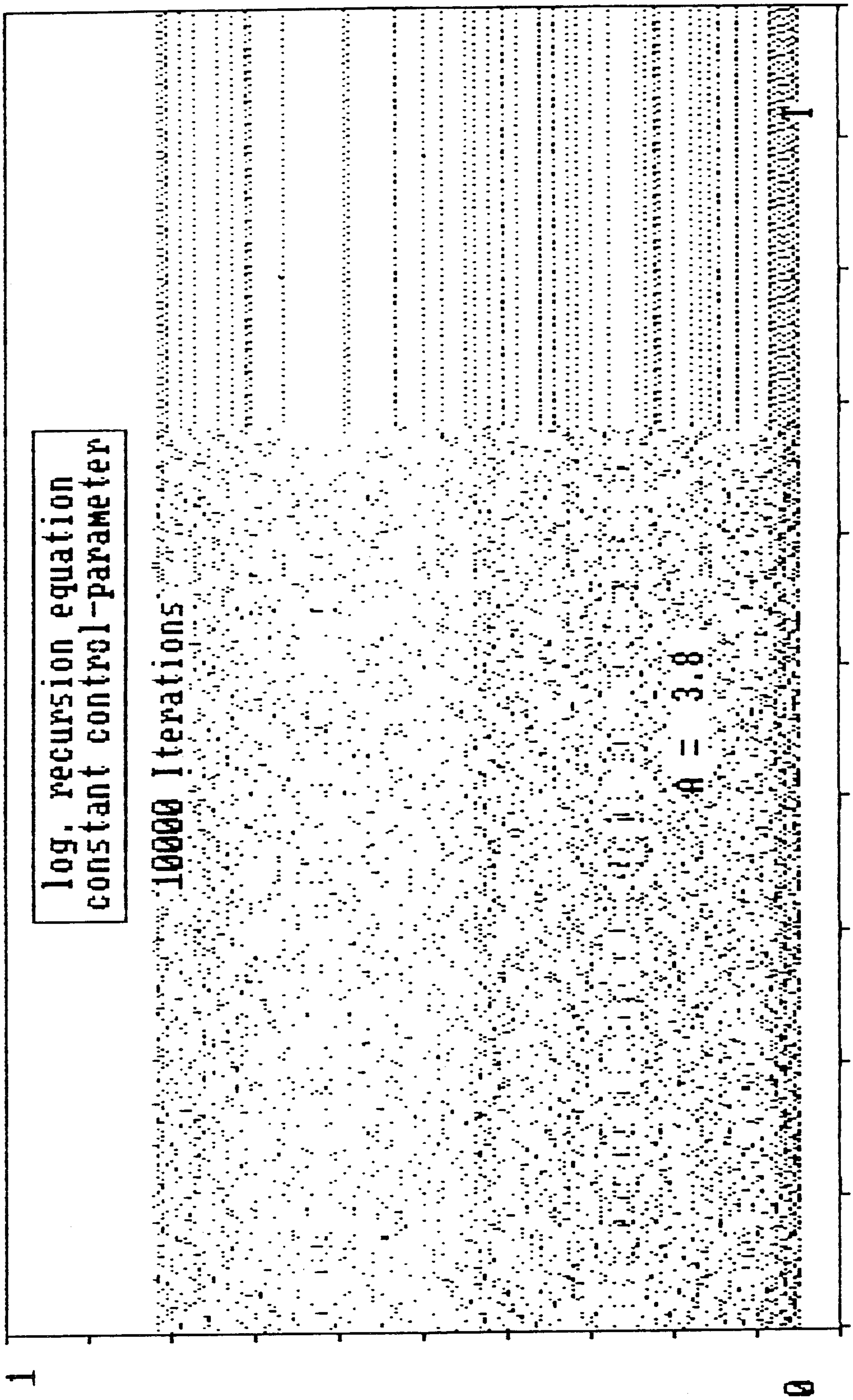
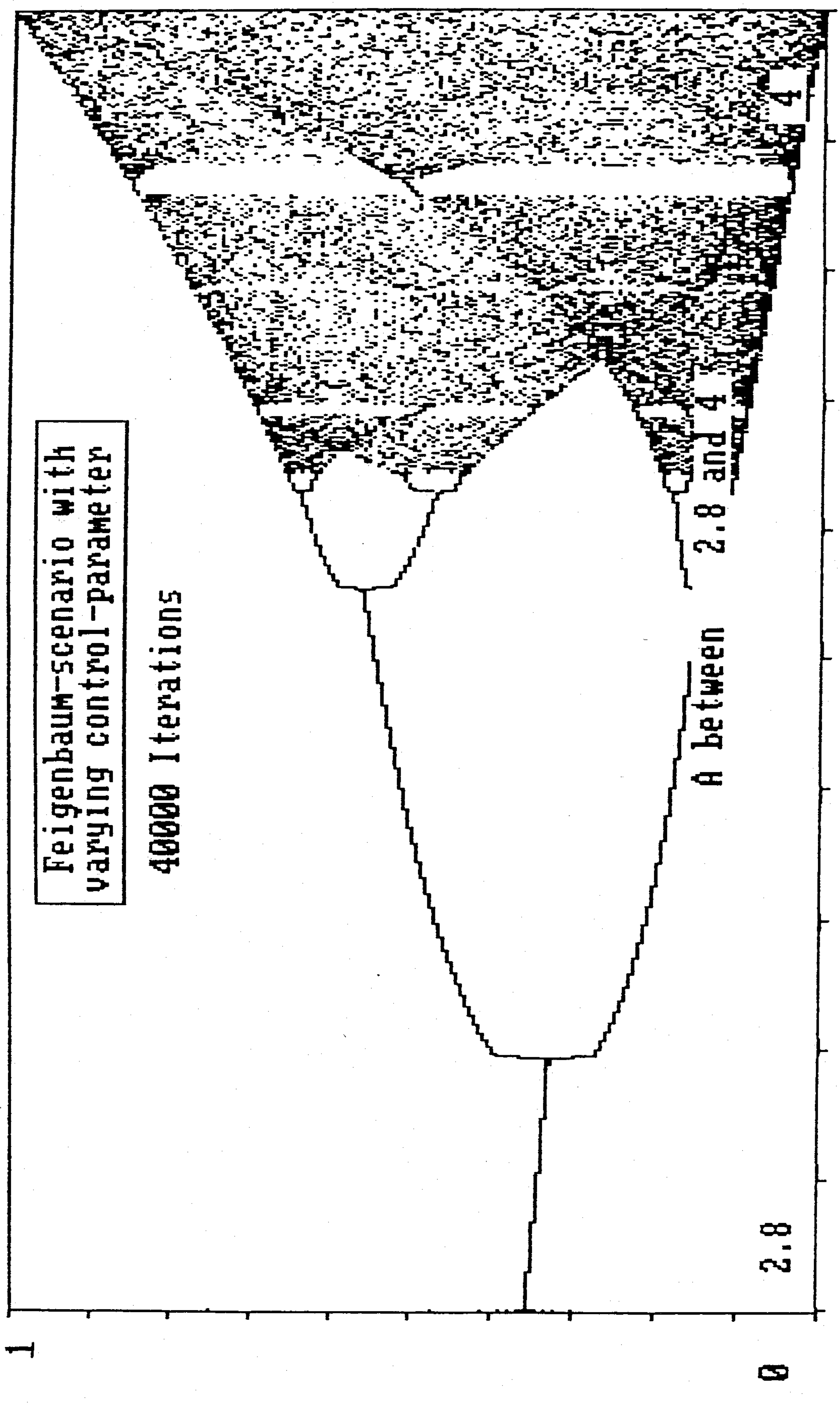


Figure 22: Feigenbaum-Scenario with Varying Control-Parameter



3.1.2. A Note on Exactness

Whether motion turns from irregular to regular within some given period of *time* does not only depend on the initial values of X and A, but also on the number of iterations you are ready to wait for such a transition. Therefore the customary predication of the control parameter as either chaotic or regular in the sense of a statement like "A is in the chaotic domain of the attractor" is inexact. You may base a predicate of the form *chaotic(A)* on Ljapunov stability. But even then, with given initial value X_o and iteration number N, the predicate Chaotic($A(X_o,N)$) can possess neutral canditates unless it is properly defined. Because the Ljapunow function, in whichever way you define it, may suddenly rise which means sudden gain of stability. Therefore it needs a lot of wise specification to formulate an exact concept of chaos. As far as I know, there has not been drawn any final decision on that issue. In the diachronic process, if you merely look at the sequence of numbers, a trajectory may appear rather chaotic, but when you look at the synchronic image, at the plot, you may discover various sorts of geometric regularities that appear to be built into the chaos. In other words, Ljapunov stability is not quite the desired measure of regularity because it is derived from the diachronic process. But the regularity appears in the synchronic pattern of the plot. That's not quite the same.

So we got to keep that in mind and be satisified with some intuitive notion of chaos and believe mathematicians that they have found several exact definitions, as is done in most textbooks. What can be said is that the motion of the system is regular unless A exceeds a critical value close to 3,57. While the control parameter is increased iteration by iteration, the complexity of trajectories also increases. Beyond a certain critical value there appear chaotic trajectories, and there exist various ways to define exact mathematical concepts to such chaotic motion. Then it can be shown that the sequence of the X_n cannot be distinguished from random walks or white noise though it is diachronically produced in a deterministic way. Thus equation (1) is said to represent the recursive function of a deterministic chaos. The totality of possible trajectories is denoted a strange attractor. It is made up by a discontinuous set or fractal. For the present we need not go any deeper into the mathematics of fractal dimension, and we also do not exploit the whole mathematical structure of the Feigenbaum scenario. But we recall that the totality of the Feigenbaum attractor shows a higher complexity than one single trajectory belonging to one definite constant value of the control parameter. When A is varied over the intervall (0, 4) by a computer, a subset of the whole variety of the attractor can be plotted or printed. This subset gives us some sort of representative visual image of its total shape.

3.2 Coupled Populations

Next let us consider two coupled population systems. In this case two populations X and Y are growing according to their control parameters A and B, and both are saturated by their own growth as well as that of the other. That can be interpreted such that X and Y are considered to grow on account of identical food- or energy supplies. The two-dimensional recursive system now takes the following form:

$$(2) \qquad X_{n+1} = A\, X_n(1-X_n)(1-Y_n)$$

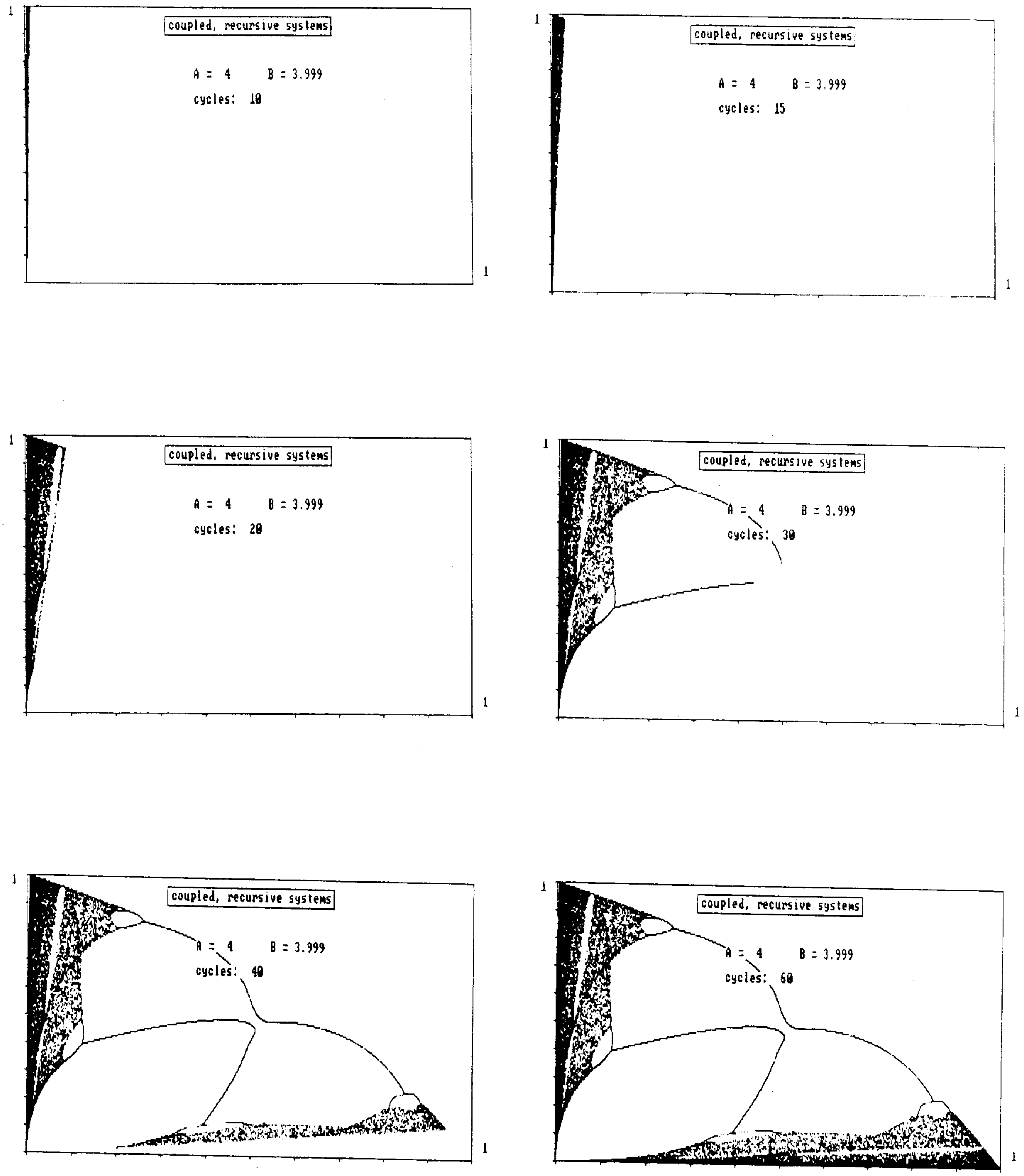
$$Y_{n+1} = B\, Y_n(1-Y_n)(1-X_n)$$

Variables X and Y are denoted phase variables. Their motion is plotted in X-Y-phasespace. Examples are given in figures 23 (see page 64) for different iteration numbers. These are blocked into *cycles* of 1000 iterations each. What can be seen from the figures? Don't they remind us of a Feigenbaum scenario? How come? After 60 cycles we have obtained something like a double-Feigenbaum attractor though we chose two constant values for A and B. Well, that's the essential lesson to be learned from that example. Two chaotic systems with definite parameters can span the whole attractor. Consider the cases where X is positive and Y vanishes or Y is positive and X vanishes. Then we are left with only one logistic recursion equation either for X or for Y, and both have a Feigenbaum scenario. Since we have chosen A and B to be equal or close to 4 we obtain for the decoupled case merely two chaotic trajectories. As soon as both X and Y begin with positive initial values, we obtain an unfolding of a double attractor. We have chosen all parameters and initial values in such a way that the whole attractor becomes visible. So we have not varied any of the control parameters, yet we obtain a very complex double Feigenbaum scenario. That's the first thing we observe.

3.2.1. A Note on Coupling and Unfolding

Coupling can give rise to an unfolding of an attractor. The second thing that can be observed concerns the size and order of control parameters. Note that we began with almost vanishing initial-value of X=0,001 and large Y=0,999 (the two need not add up to unity!), both control parameters were maximum, but B was a little bit smaller than A which was 4.

Figure 23: Coupled, Recursive Systems



As soon as the iteration begins what can be seen? It can be seen that Y, the large population begins with chaotic motion that covers the whole intervall between zero and unity. But X stays small and grows only very slowly and appearantly monotonous. By the time X becomes larger and larger whereas Y remains in a state of irregular motion, but the intervall covered by that motion becomes smaller. After some 25 cycles Y has left the chaotic domain and moves in 16-, 8-, and then in 4-cycles. By that time X has increased to the considerable size of about 0,2. In the least complex situation that arises after about 32 cycles both X and Y are very close to each other. There are of course values for A and B where the system makes a transition beyond even the last bifurcation. Then the population numbers move on a single line. In any case that situation is not stable. But the system walks further through all the previous bifurcations and irregular episodes, just that now the roles of X and Y are exchanged. As Y decreases X goes through a Feigenbaum history and finally becomes chaotic.

3.2.2. The Double Feigenbaum Scenario:

Exchange of Stability

So we sum it all up: In a coupled, two-dimensional, logistic iterative system of the saturation type there is an exchange of stability, an exchange of chaos and an exchange of order. Provided the first population begins with an almost vanishing initial value and the second with a large value, say, close to unity, that population will nevertheless ultimately die out while the first will take over and grow until it is forced to move through a chaotic scenario, if and only if its growth parameter is larger than that of the second population. In that case the system begins with *chaotic Y*, but ends up with *chaotic X*.

There is some quite natural interpretation to such a double Feigenbaum scenario. Intensive generational growth is so to say something dangerous in population dynamics. Since it causes demographic *turbulences*. Too many individuals may be forced to die. By coupling such a population with chaotic growth to some other, the irregularity can be reduced, in a way absorbed by the other and thereby exchanged. That is, since the other has some still more *dangerous* growth, the coupling will not really help, will not banish chaos from the scenario. There is a simple rule in such a system: the more chaotic population (more accurately the one with larger growth) has to carry the load. The other will die out. There is no real equilibrium that can be attained in such a system. But there is at first an exchange of stability but on the whole an exchange of chaos. If you consider one half of the history only, you find both populations in quite stable arrangements, but if you look at the whole, the one will ultimately die out while the other absorbes all the chaos. Now how about time? Is there anything that can be said about time? Anything different from the decoupled case?

3.3 Time and Exchange of Stability

For just a moment, consider time as given by increasing iteration number. From the figures it can be seen that the system in a double Feigenbaum scenario first runs through a Feigenbaum-attractor in reversed time and then in foreward-time. First it leaves the chaotic domain and becomes decreasingly complex, then there is a turning point in the middle of the scenario, then complexity increases, number of bifurcations increase until finally the chaotic domain is entered once again. Systemtime, whatever that might be exactly, is reversed in the middle of history. This somewhat reminds one of the rise and fall of an empire. The population of Y somehow begins in high life, large and chaotic, but by the time through the coupling to another population becomes less and less chaotic. In the middle of its history it has shriveled a little, but has gained considerable stability. That instant marks a point of culmination, and from that time on it gradually fades away forever.

3.3.1. Derivation of Systemtime

Consider some chaotic trajectory from a one-dimensional, iterative system. You can cut its plot into strips each belonging to different periods of iteration time, exchange them in any manner, so that iteration time is mixed up, and look at the result. The muddle-plot will not be different from the original. Therefore in a chaotic plot you cannot reconstruct the order of time from the data alone. Intuitively way may say, a chaotic walk has no time. There need to be some sort of transition to order or at least away from order. Only then there may be a little hope to reconstruct some sort of systemtime from the phase plot. Considering figures 23 once again, if you fix a pointer at the origin in vertical position and then monotonously turn it to the right until the X-axis is reached, you essentially obtain the movement of the edge of the process while iterations increase. So there seems to exist something in the whole chaotic event that moves in some monotonous manner. The movement of a pointer fixed at the origin can, of course, best be described by the arctan(Y/X) up to some linear transformation. However, since motion is chaotic, you can only have something like an average angle of the pointer. That is, we have to consider a moving average of the arctan(Y/X). Carrying out the averaging within some fixed intervall of, say, 100 iterations and moving through the whole attractor, a smooth logistic curve is obtained (see fig. 24, page 68). It's nothing essentially different from a solution of the Verhulst equation. That's strange, isn't it? The fractal solution to the recursive equation contains an internal parameter that looks like a solution to the continuous problem! One is tempted to pin down a definition like the following:

A POSSIBLE DEFINITION:
The strange attractor of a two-dimensional Feigenbaum scenario is resulting from a fractal decay of continuous time.

But that's not the most important thing to ponder over at present. But what is more important is the logistic shape of the moving average arctan(Y/X). This tell's one that a log-transformation might be better than arctan. Therefore we try a moving average log(Y/X) up to some linear transformation, and the result is quite convincing: Packed into the strange attractor there is a linear parameter - derived, linear systemtime (see figure 25, page 69). As exchange of stability proceeds, linear systemtime goes foreward too. In the concrete case a linear transformation of the moving average ln(Y/X) was carried out depending on the initial value of that logarithm. Finally unity was added and the result was divided by 2. The reason for such manipulation was that the moving average ln(Y/X) becomes negative at some special moment. That moment is exactly the point of minimum complexity of motion, that is, in the middle of the scenario. Therefore if you consider absolute values instead, there is a fall and a rise of linear time which resembles exactly the concept of *relative time* experienced first as a gain of stability and then as a loss of stability. In the middle of history the scenario is actually reversed. So there exists a derived parameter that is capable to reflect such time-reversal (see figure 26, page 70).

3.3.2. An Existence Theorem

Let's now sum up our different approaches to derivation of time. The very first measure was obtained from the visual perception which told us in what way the process grows iteration by iteration. That led to the arctan-measure.

(3) $t = \alpha_0 \, \mu(\arctan(Y/X))$

Figure 24: Moving Average

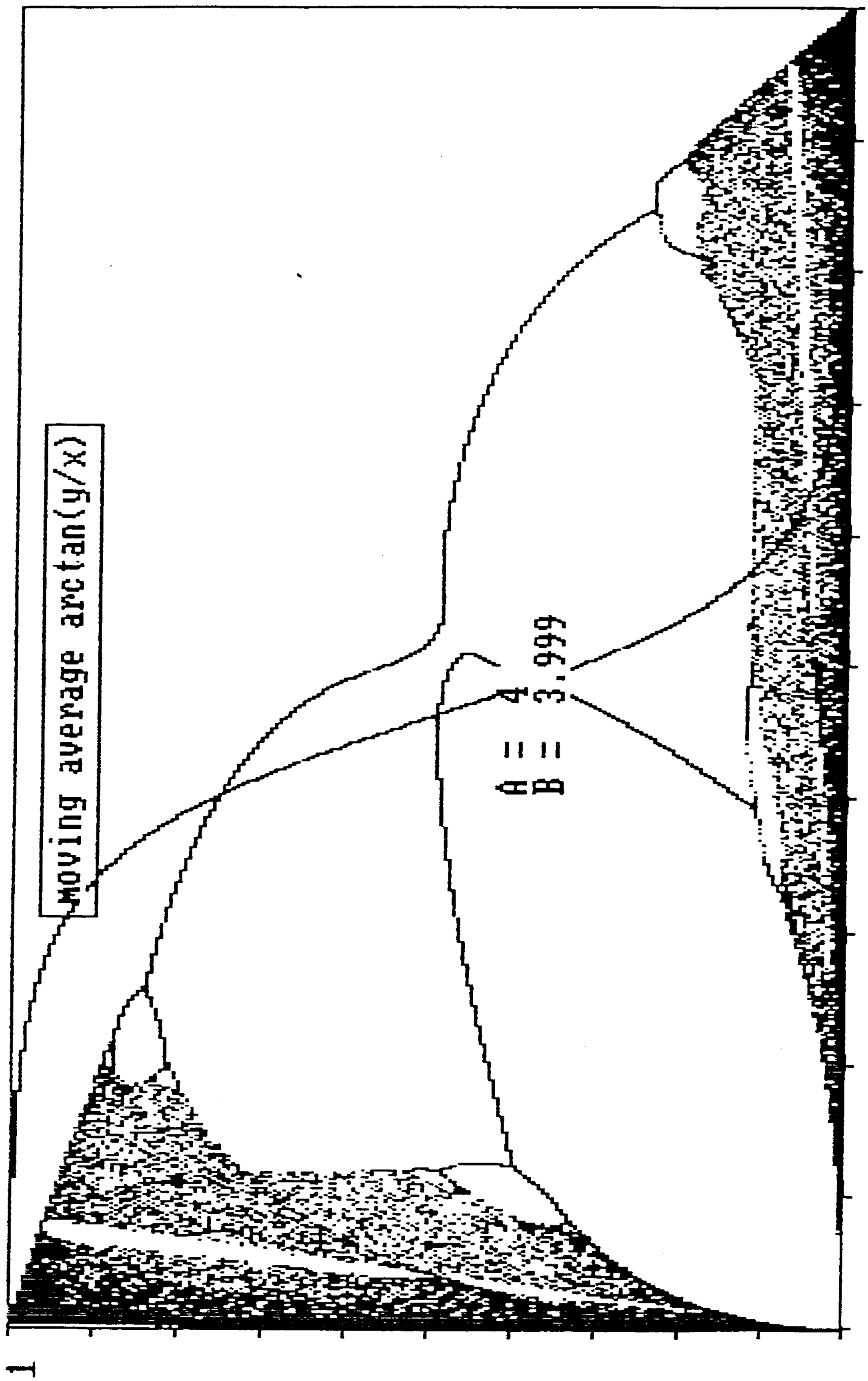


Figure 25: Emergent Linear Systemtime

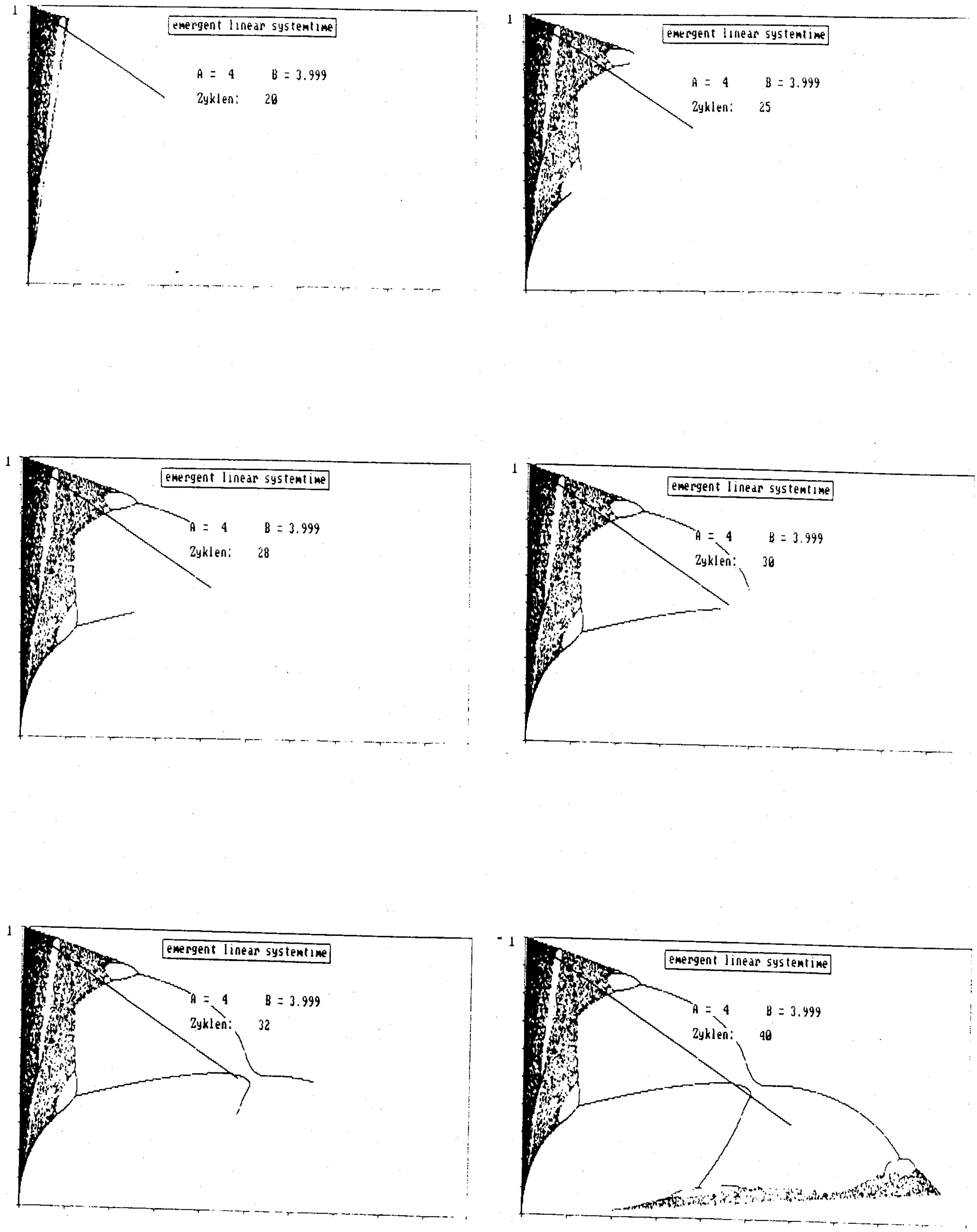
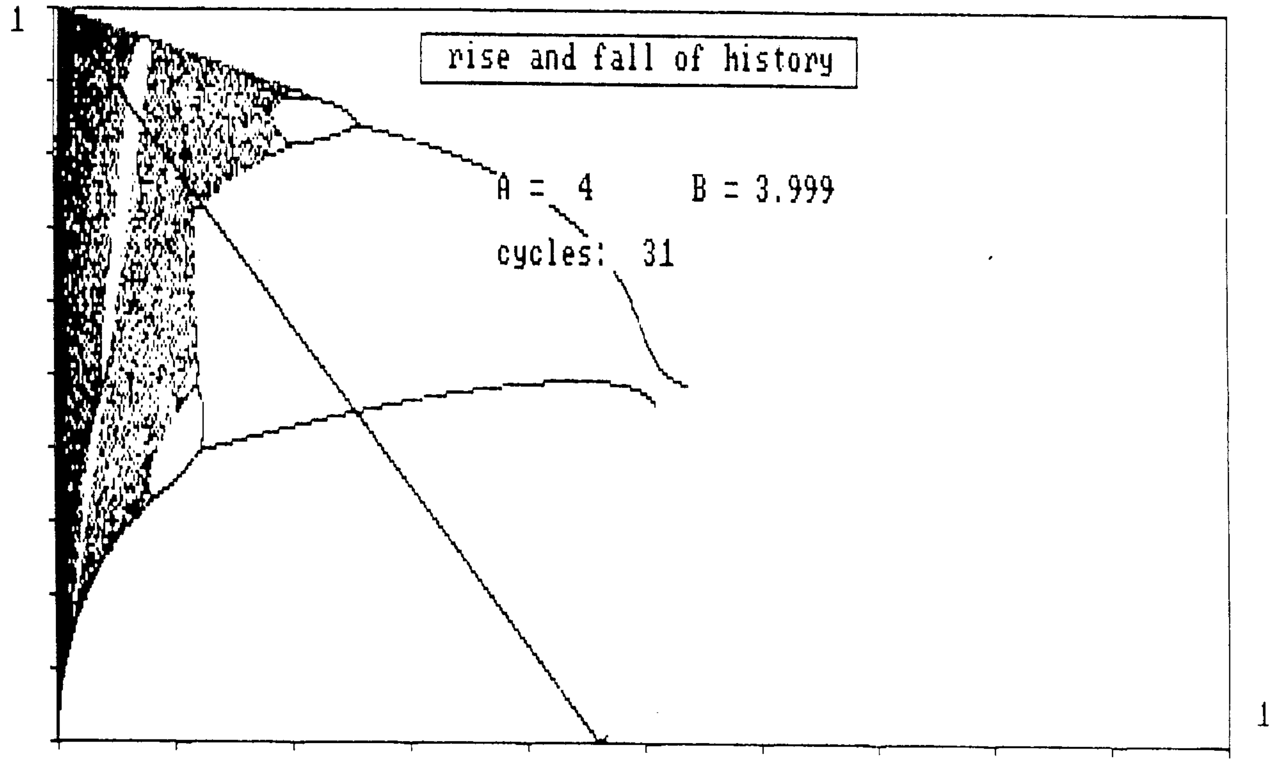


Figure 26: Rise and Fall of History



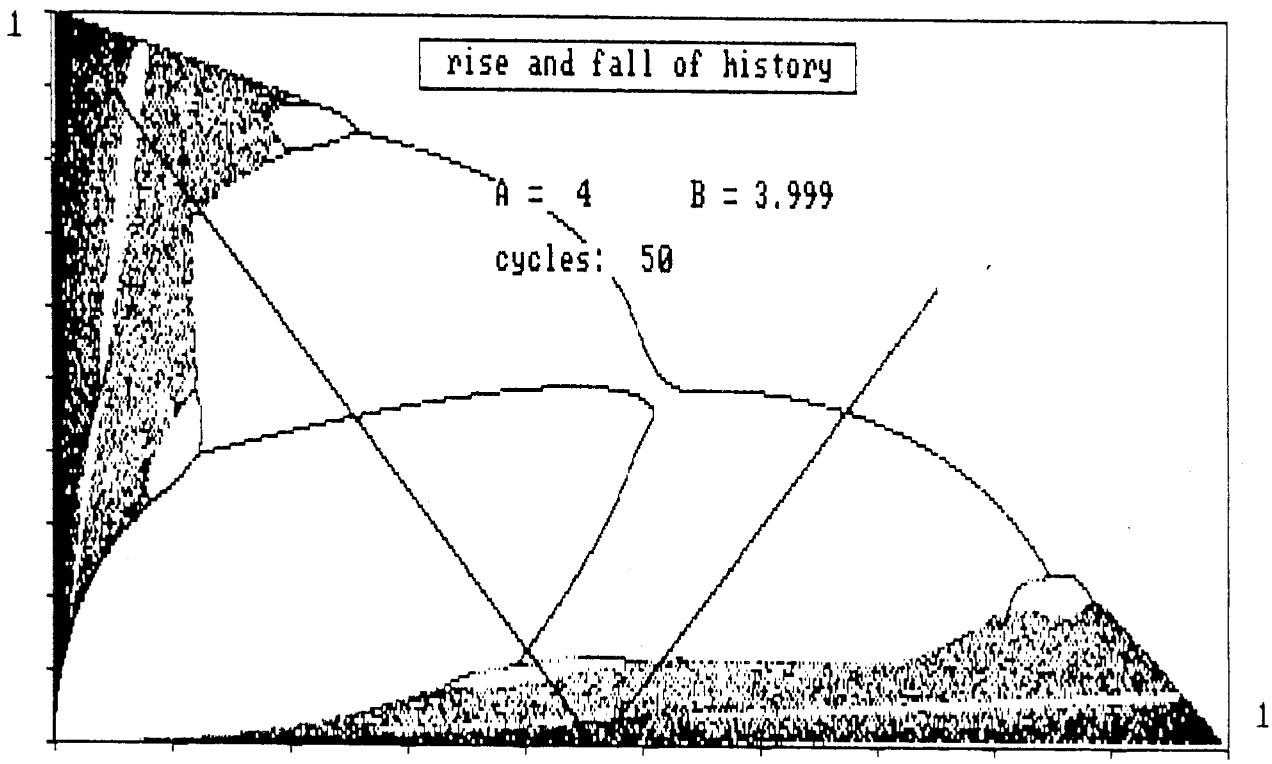


Figure 27: Four-Dimensional Systemtime

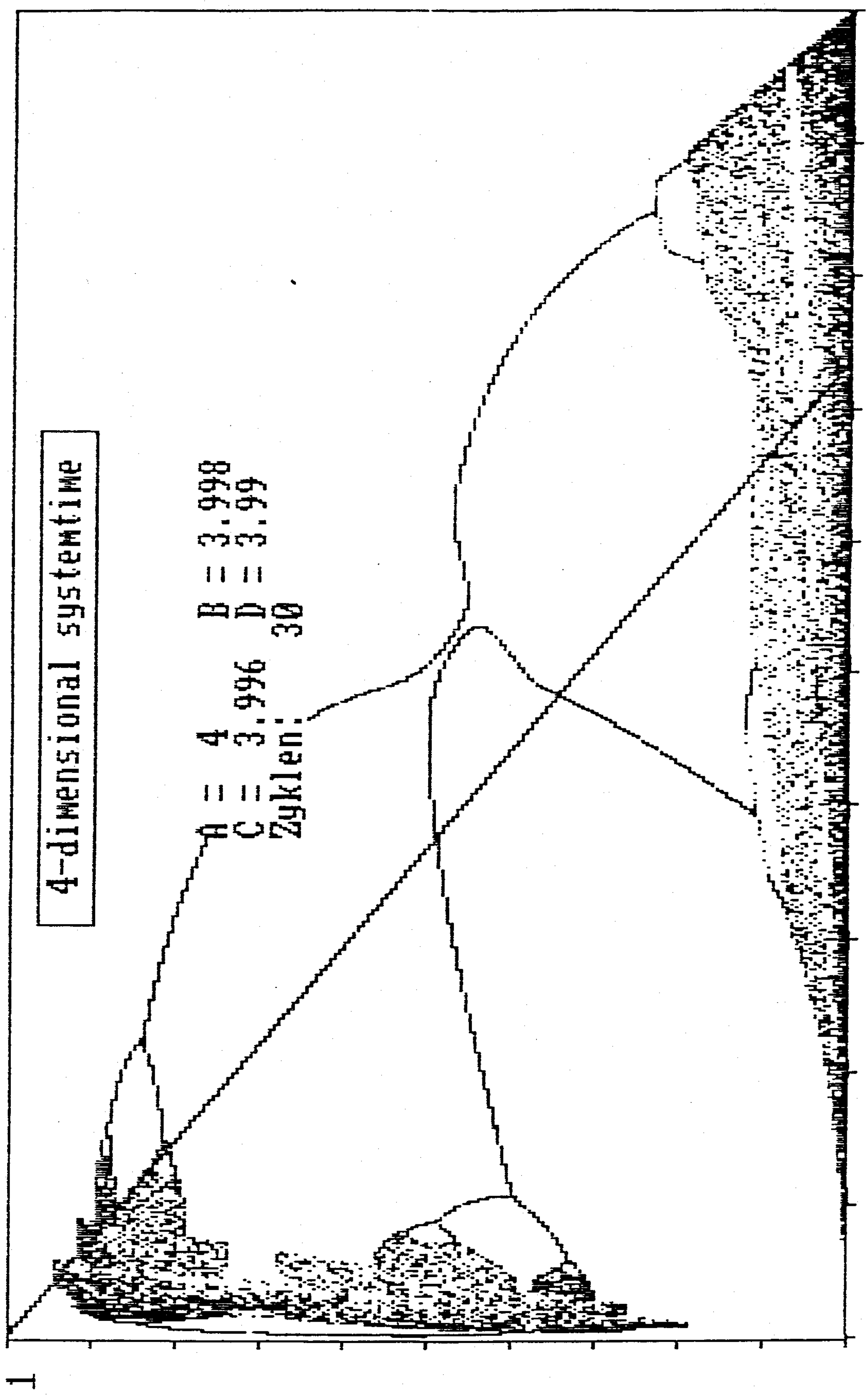
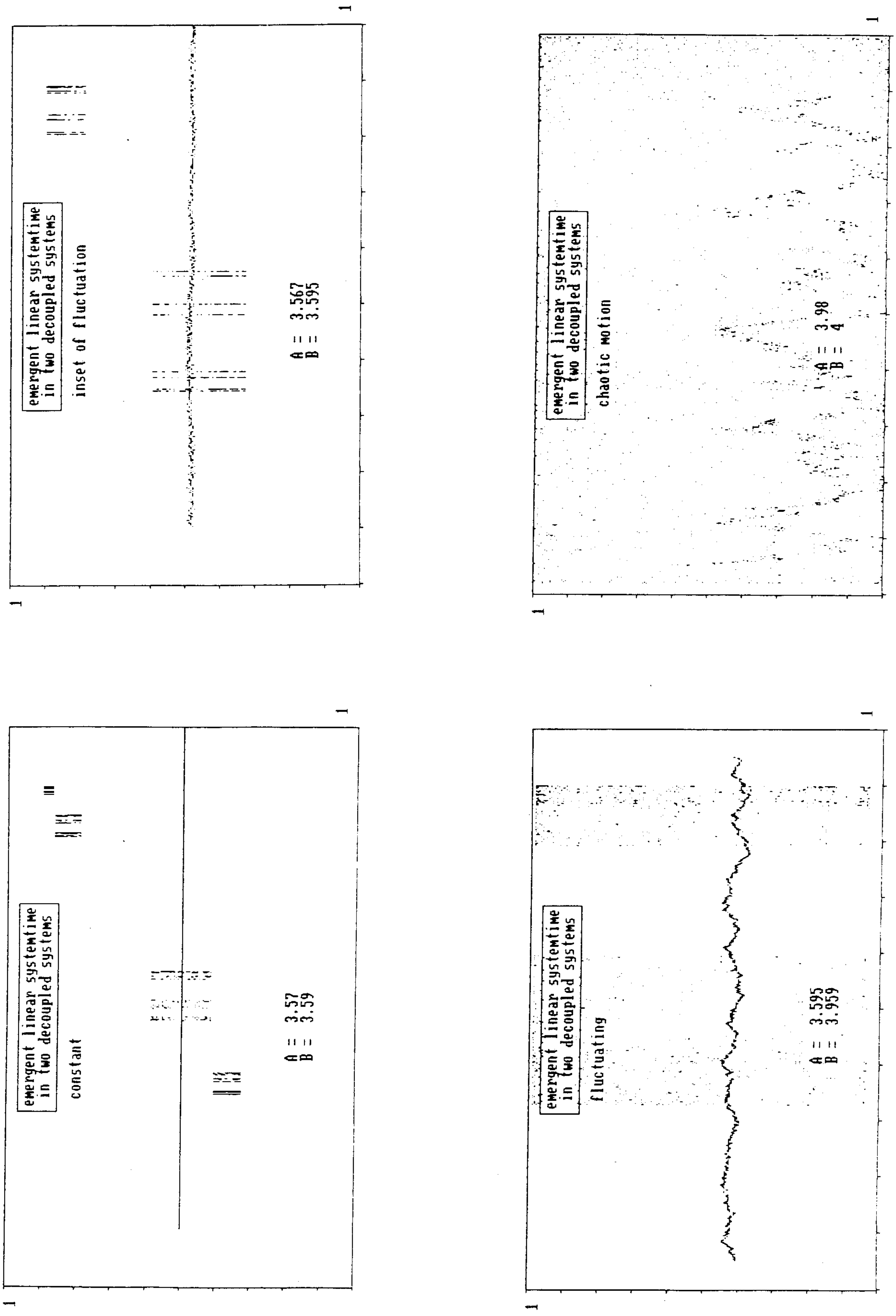


Figure 28: Emergent Linear Systemtime in Two Decoupled Systems



with $\alpha_o = 1/\arctan(Y_o/X_o)$ X_o, Y_o ... initial values and μ is the moving average over some fixed period, in the concrete cases of N=100 iterations.

$$(4) \qquad \mu(\arctan(Y/X);n,N) = (1/N) \sum_{i=n}^{n+N} \arctan(Y_i/X_i) = \mu(n)$$

So with fixed averaging period of N=100 iterations we obtain a moving average μ which is essentially varying with the left intervall limit n. Thus we have $\mu=\mu(n)$.

Next we saw that this measure, though smooth and monotonous, was nonlinear. So we decided to use a loglinear transform. That led to:

$$(5) \qquad t = | 2\beta_o\mu(\ln(Y/X)) |$$

with $\beta_o=1/2\ln(Y_o/X_o)$ | and moving average

$$(6) \qquad \mu(\ln(Y/X);n,N) = (1/N) \sum_{i=n}^{n+N} \ln(Y_i/X_i)$$

This is the *history measure* where time is reversed in the middle of the scenario because the process of order is reversed. So there is a first run trough a Feigenbaum scenario somehow from the backward, that is, with decreasing irregularities and increasing order where the moving average logarithm is positive. And then there is a second run with reversed time where order is decaying and irregularities increase again, that is, the Feigenbaum scenario is run through in the other direction. Although this approach is appealing due to its capability to account for time-reversal, it can be critisized from another viewpoint. Time in this account is always seen from the standpoint of one population, either X or Y. Because during the first half of the process Y is chaotic and during the second it is X. So we may wish to look at the whole and obtain a linear, monotonous measure of time. This is the number

$$(7) \qquad t = \beta_o\mu(\ln(Y/X)) + 1/2$$

There are, of course, modifications by linear transformations possible, especially by improving β_o. But the essential result is now there:

EXISTENCE THEOREM:

A two-dimensional Feigenbaum attractor of a coupled population system contains an internal, linear time-parameter.

It would be interesting to find out if such a result can be generalized to coupled population systems of dimension larger than two. This is indeed the case.

3.4 Higher Dimensions

In principle, the derivation procedure of linear, derived systemtime can be carried out for Feigenbaum scenarios of arbitrary dimension. I have tested if the procedure works for dimension four and the result was good. The following interactive system was considered:

$$
\begin{aligned}
(8) \qquad X_{n+1} &= A\, X_n\, W_n \\
Y_{n+1} &= B\, Y_n\, W_n \\
Z_{n+1} &= C\, Z_n\, W_n \\
U_{n+1} &= D\, U_n\, W_n
\end{aligned}
$$

with the nonlinear term of order 4

$$W_n = (1-X_n)(1-Y_n)(1-Z_n)(1-U_n)$$

and all variables bounded below by zero, above by unity, all constants bounded below by zero and above by 4. This system has the advantage that it behaves somehow normal. It is a generalization to the Feigenbaum scenario, and does not leave its boundaries because it accounts for nonlinearities in an appropriate way. Its attractor is some amazing composition of Feigenbaum episodes.

Figure 27 (see page 71) shows solutions to a special problem. Their projection onto the X-Y-phasespace is plotted. The straight line is representing derived linear systemtime.

3.5 Decoupling and Decay of Time

It would be interesting to investigate how derived time reacts to decoupling. That can best be demonstrated by disregarding the interaction terms and consider the two separate logistic recursion equations

$$(9) \qquad \begin{aligned} X_{n+1} &= A\, X_n(1-X_n) \\ Y_{n+1} &= B\, Y_n(1-Y_n) \end{aligned}$$

only. Than it turns out that, in many cases, especially in the regular domain, derived time as computed by equation (7) stays constant. But even at the inset of chaos, e.g. for A=3,57 and B=3,59, t is constant. In other words, there is motion, but no time. Then there are cases where t begins to fluctuate or such where it oscillates irregularily and finally the measure may decompose and perform some chaotic walk. The following examples are shown in figures 28 (page 72):

case 1:	t is constant	... A=3,57 and B=3,59
case 2:	t begins to fluctuate	... A=3,567 and B=3,595
case 3:	t oscillates at random	... A=3,595 and B=3,939
case 4:	t moves chaotically	... A=3,98 and B=4

In all the figures you realize a plot of the phasespace superimposed with the measure of t. In case 4 the t-measure can hardly be distinguished from the chaotic motion of the phase-variables. In any case it is clear that t can no longer be considered as a linear and monotonous measure of time. Therefore it can be concluded that it is really the coupling between systems X and Y according to equation (2) which gives rise to what was denoted a measure of linear systemtime?[20]

3.6. Time in Migration

For the present purpose, I hope, I need not give you a new definition of social mobility. Because there are enough textbooks where such can be found. Also I shall not repeat in greater detail any of

20 At this point, a brief note on *loglinear models* seems appropriate: There is some methodological consequence to be taken from the above calculations. In some self-organizing systems the logarithms of odd-ratios of phase-variables represent internal time-parameters. In case the loglinear association of such measures turns out significant that may be due to some mechanism of synchronization.

the old ones. But I'd prefer to show you where the problem of derived, sociological time is to be located in migration models, in order that the theoretical procedure doesn't turn out as a mere illusion. Migration, as you know, is a constitutive topic in both regional science and social science in general. But despite its many interesting empirical and theoretical aspects, the whole process is embedded into a rather complex network of economic events as well as social and political action. Therefore it is a challenge to interdisciplinary cooperation and theoretical development. Furthermore, it is the only field that can be formalized to the standard of fully developed, dynamic models in terms of equations of motion.

3.6.1. The Masterequation - Approach

The central objective of migration theory is to understand the migration of individuals between different geographic regions and socioeconomic sectors. So we have demographic mobility as regional migration, occupational mobility not only as vertical mobility, but also as intersectoral migration and so on. Usually the whole population under investigation is decomposed into subpopulations P_i that are considered as internally homogenous, whatever that may mean, but different with respect to migratory behaviour. I said *whatever that may mean*, because often it is lacking exact sociological meaning. In the most simple cases the population configuration is defined by national identity only, or by regional residence only, or by occupational provenance only and so on. The change of configuration is described either by a stochastic differential equation or by some probability rate equation. Often the second is taken either because it is more simple or the statistical procedure not so expensive or the data is insufficient or any combination of those.

Now, consider some set S of subpopulations P_i belonging to occupational sectors i and the binary relation SxS. Both sets change by the time due to transitions from sector i into sector j and from j into i. Then we say that the probability rate equation describes the incoming probability flow or - transfer per unit time from the neighbouring sectors j into sector i and also an outgoing flow from i into sectors j. The most simple form of such *masterequation* is

$$(10) \qquad ds_i/dt = \sum_{j \neq i} s_j\, q_{ji} - \sum_{j \neq i} s_i\, q_{ij}$$

where the s_i are e.g. sizes of occupational sectors in terms of numbers of individuals in subpopulation i, and the q's denote transitional probabilities per unit time. The meaning of such probability flow equation is the following:

The size s_i of each occupational sector i changes by the time and that change is given by the time-derivative ds_i/dt. Now there is a positive contribution to that change and a negative. The positive is given by all the individuals who migrate from neighbouring sectors $j \neq i$ to sector i, that is, it is proportional to the transitional probability q_{ji} per unit time and to the sizes of sectors j. The more individuals there are in sector j, the more there can turn over to i depending on the value of q_{ji} and all the contributions of sectors $j \neq i$ are summed up. The negative contribution to change is given by those who leave sector i and turn over into neighbouring sectors. This is proportional to products s_i q_{ij}. These negative contributions to differential change also have to be summed up to give one negative saldo. Note that equation (10) is essentially one of Coleman's basic equations to the analysis of categorial data.

In order that equation (10) obtains a sociological meaning it must be filled with theoretical assumptions. Those theoretical assumptions can either be invented or carefully extracted from empirical findings. So the whole job of designing the right model consists in finding the right *decomposition* of the transitional probabilities. That signifies the essential analytical procedure. If that procedure is carried out in an illusory way, a possible mistake as will soon be clear, both sociological theory and the synergetic approach will come short of meaning. In the nonlinear synergetic approaches q_{ji} is regarded time dependent, and in a first analytic step is decomposed into a symmetric mobility factor f_{ij} and a push-pull factor $G_{ij}(t)$ accounting for asymmetric flows -

$$(11) \qquad q_{ij}(t) = v_o(t)\, f_{ij}\, G_{ij}(t)$$

where $v_o(t)$ is a global time dependent mobility which equally effects all sectors. The symmetric factor f_{ij} is said to

> include all effects which will either facilitate or impede a transition from i to j independent of any gain of utility in such a transition. This mobility will in particular depend on the *effective distance* between the regions i and j.[21]

But it is a rather serious question what an effective distance should be. This is of a special importance in any evolutionary approach. Because the partition on which the whole calculation is based should somehow represent an evolutionary partition which actually is the outcome of a bifurcation of the very process that is going to describe it. Did you get that point? We are likely to run into a problem of the Gödel type. Is it at all possible? Obviously every model should account for as

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[21] W. Weidlich, G. Haag (1988), *Interregional Migration. Dynamic Theory and Comparative Analysis*. Berlin *et al.*, 16.

much empirical facts as possible. Otherwise it is forced to be an epicycle-theory, you know, that phantasy by which planet trajectories have been described before Kopernikus and Kepler. Therefore, is it at all possible to design the model along the path of historic bifurcations, so that we base it all on the right partition? You see? We use such funny words as *population configuration*, *occupational sectors* and the like. But do those *configurations* account for the actual bifurcation process? Shouldn't we at first construct the right kind of lock, before we put the key in? Therefore we have to set up the following hypothesis before we can go any further:

PARTITION HYPOTHESIS:

In case that a masterequation for a nonlinear, self-organizing system of social mobility is designed, this equation, since it is a model of the factual, empirical process of bifurcation and organization, must be based on a partition or disaggregation of the data which is in accordance with the process itself.

That is, the process designed requires its own aggregation! On the other hand it must be clear that sociology also brings to light such aggregates or configurations of populations. That is, sociological understanding too requires its own way of partitioning the data. It is not realistic to set up the equation without investigating into both empirical and theoretical sociology.

Therefore, to design a model with regard to given statistical supply only, has not much value. It can at best show how the formal procedure works, and if it works at all. But it cannot lead to a realistic model of the system dynamics.

The symmetric factor also includes a measure of similarity of sectors. It is sometimes taken to be a term of the sort

(12) $$\text{similarity of sectors} = (s_i s_j)^{\alpha(t)}$$

Altogether f_{ij} often takes the form

(13) $$f_{ij} = (s_i s_j)^{\alpha(t)}\, e^{-\beta d_{ij}}$$

where d_{ij} is the *effective distance* and a constant. The push-pull factor $G_{ij}(t)$ describing the asymmetric effects is thought to represent attractiveness of a sector. It looks like

$$(14) \qquad G_{ij} = e^{(a_j(t)-a_i(t))}$$

where the a_i's denote attractiveness or *utility* of a sector. They are sometimes thought to saturate according to some Verhulst type of equation:

$$(15) \qquad a_i(t) = \sigma\, s_i^* - \tau\, s_i^{*2} + u_i$$

Here the s_i^* are due to a bandwagon effect and $-\tau\, s_i^{*2}$ is a saturation effect. They are obtained by some normalization of the form

$$(16) \qquad s_i^* = (s_i-\mu_s)/\mu_s$$

with μ_s mean sector size

$$(17) \qquad s_i^* = (s_i^2 - \mu_s^2)/\mu_s^2$$

The effect of (16) is that an attractive effect is switched on in the transitionrate q_{ji} and (17) causes that effect to saturate in some reasonable limits. The whole nonlinear decomposition of the transitionrates finally takes the form

$$(18) \qquad q_{ij}(t) = v_o(t)(s_i s_j)^{\alpha(t)}\, e^{-\beta d_{ij}}\, e^{(a_i-a_j)}$$

A model of the form (10), *birth- and death processes* included, with a decomposition (18) has actually been set up and tested by Karl H. Müller, Günter Haag, Martin Munz and Christian Weber[22] for the occupational system of Austria. Karl H. Müller considered six occupational sectors: agriculture, industry, service related to undertakings, service related to households, state, and households. The distances were defined in the following way:

$$(19) \qquad d_{ij} = (1/\sum_k T_k)\sum_k T_k \mid s_{ik}-s_{jk} \mid^{\sigma}$$

[22] See K.H. Müller, K. Pichelmann (1990)(eds.), *Modell zur Analyse des österreichischen Beschäftigungssystems. Prognose- und Szenarieninstrument der branchenspezifischen Beschäftigungsentwicklung*. Wien, 129 - 156.

Here T_k denotes the mean acquisition time of educational level k; k is one of four different education levels from elementary school to highschool; s_{ik} is the proportion of educational k-levels in occupational sector i and σ is necessary to carry out a weighing. It measures the importance of intersectoral education differences.

There was obtained a complete 6x6 migration matrix for the time period from Dec. 31 1985 to Dec. 31 1986. The set of control parameters was reduced considerably. Yet there could be carried out an estimation procedure, and the model turned out statistically significant. Now, despite the whole bundle of methodological problems challenged by such procedure, there is a first idea that can be obtained from the design of *effective distances* as given by equation (19) as also from the *bandwagon-effects* (15). Let's go into it.

There is no immediate *utility* connected with a symmetric move from occupational sector i to sector j. Also the contribution to the transitionrate $q_{ji}(t)$ of d_{ji} according to equations (18) and (19), is small when the mean acquisition time of educational level k is large. Furthermore, it is forced to decrease in case the fraction of high k-levels is large in sector i and small in sector j, and it is even smaller if the difference of educational levels is important for a transition to occur. I think that from this the meaning of the central statement to be made can easily be realized. In civilized life, progress and development are transported by the educational system. Due to the importance and lenght of acquisition time of high educational levels, transitions between occupational strata with very different educational demands are very unprobable. Those sectors are, in a way, separated by history, not only by social history, but also by the individual curricula.

Recall what has been said in definitions T_1 and T_2 at the beginning of this chapter. Where cognitive and social structures are developing, where social competence is acquired, where social space is created, there is evolution. (I do not say that this is and must be true forever and in any case. Times may change. But this is what is said, what can be heared and what people believe and prefer to any other idea of social location). So this allows for a definition of sociological time. As was said:

Migration is not time.

But time occurs where there are developing social strata with a maximum distance, in the sense of equation (19), to all other strata of society. Therefore the symmetric factor in the negative logarithm of transitional probabilities q_{ij} accounts for a component of systemtime. That is when there is a hierarchy of d_{ij} measures with a high mean and little entropy, then that can be taken as an indicator of an intensive evolutionary process in the sense of progress, cognitive development and hierarchization of social locations. But we are not yet ready. There remain a few, but very significant problems of methology and sociological theory. We shall deal with them next and give a list.

3.6.2. Critical Notes

NOTE 1: Neglecting Small Scale Self-Organization

Rough disaggregation neglects the synergetic reality of the underlying components. This is a special pity in a synergetic approach. Looking at the time-series of population numbers in the finer configurations of occupational mobility, one virtually sees by naked eye nonlinear oscillations. All those smaller time-series add up to the rough migration matrix under consideration.

NOTE 2: Linear Composition of Nonlinear Systems

There is the seemingly simple problem how a bundle of smaller nonlinear processes add up to some larger nonlinear process, eventually of the same type. Numerically and empirically configuration numbers can just be added to form the data of a larger aggregate.

NOTE 3: Partitioning

This topic has been already dealt with in the partition-hypothesis. The attractor of the masterequation under consideration should give rise to those categories, partitions, social strata etc. as you are using before you design and estimate the process. Does a requirement like that lead to a problem of exactness? Does it lead to undecidability?

NOTE 4: Emergence of the New

Problem 4 is even more serious. Synergetics, analysis of the evolutionary type and similar approaches to *life as a whole* are often carried away by some kind of blind belief in creativity, evolution, chaotic systems and the like.

But the most essential difference between the model and social reality is usually passed by or not seen or neglected. Consider the *emergence* of new professions like system operator, research-designer, mind-scientist, biodynamic masseuse, flower-therapist, the whole *configuration* of therapists, healers and so on, the scenario-writers, private researchers with desktop-Ventura-publication-equipment or whatever. Can you imagine that those professions will ever appear in your phasespace-plots as they do in reality? Definitely there is no chance for such creation to be brought forth by cognition alone. The best computer will not manage. Because we define our phasespace in advance, before the whole design- and estimation job can be carried out.

Within that space, representing occupational variables for example, the system is forced to move. The best that can happen then is that there appears some bifurcation in some subset of phase-variables. But then we don't know what kind of thing it is into which it bifurcates. So all we can say is, there is something going on, but we don't know what it is. That is the most, thought can do. But even then it might be that we don't really know what it is that bifurcates, because we have never understood where and how the whole creation process begins. But the reason is not only a matter of the intellect. It also depends on our own way of experience, on our own embedding into the process of civilization. Being thus captured by the process of time, how will you understand the other side of the coin which is no-time?

NOTE 5: Discrete Time

It makes a difference whether you take off with continuous time, as for instance is done with masterequation (10), or if you chose discrete, generation time. In the first case you may observe several bifurcations and equilibria. But in the second case you are approaching fractal space. If the discrete filter of the system you regard right is not identical with that of the continuous problem (10), but yet very similar, you have very good chances to obtain a strange attractor. Therefore it makes a big difference if you regard time as being either continuous or discrete. I've shown such a case at a congress on computer simulation in Nuremberg 1989.[23] It's a slight difference that makes the difference.

NOTE 6: On Risks and Local Maxima

Process (10) is not the right means to model evolution. It rather draws from the data and pencils local maxima, but does not discover new possibilities or take any decisive risks.

The best that can be done at present consists in figuring out the fundamental aggregates or configurations, as physicists use to say, in order not to entirely model away from social reality. If there is something like a set of evolutionary segments in society, then there should exist a criterion such as acquisition time of educational levels, or some other, by means of which the distances between segments can be maximized, so that intersectoral mobility almost vanishes. Those layers are then canditates for subsystems of a self-organizing process that is not identical with the *masterequation*.

NOTE 7: On Time

In the continuous *masterequation*-approach time does not follow naturally from the exchange of stability as in section (3.3), but it is based on an inuitive, empirical concept of space as distance between occupational divisions of the labor market.

An evolutionary approach should rather be based on generation of coded sequences of life-episodes that naturally incorporates systemtime.

[23] See B. Schmeikal (1989), "Sociology and Mathematics of Nonlinear Qualitative Social Systems", in: *Fourth International Nuremberg Symposion on Computersimulations in the Social Sciences.* (unpublished).

3.7. Critique of Evolution

There are some germans who say: "Manche Sache im Lebe ganget halt net und die ganget halt net."

That would mean that a peasant doesn't change his soil, a shoemaker will always remain a shoemaker, a farmer will never become a fashion designer, a professional violinist will not turn over to biochemistry, even a bassist will never be able to become a violinist, visual artists will not turn to acustic arts, the cooper will not work on strange attractors and the porter in our institute will never replace the professor.

This is the essence of the evolutionary, traditionalist approach to social life, and personally I dislike the idea because it is primitive.

We are, as we did before, experiencing a time of an unreflected belief in quality, selection, social status and all the rest of it. Since there is no real sociological competence, no sound qualification in action and activity, people need to work out very rigid codes and symbols that allow them to put each other into boxes more easily.

To disguise the spook, surfaces are polished.

Clearly that doesn't really work. It will always have to break down in the end.

But what surprises me is that mathematics reacts to that. Because it is a very fine instrument and has its own intelligence.

It has taken precautions that the blind cannot find the right model.

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