COOPERATION UNDER UNCERTAINTY

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Abstract

In this paper we analyze with game–theoretic tools economic situations where two players know that in a world of certainty cooperation would make both players better off compared to a situation of non–cooperation, i.e. the sum of payoffs in case of non–cooperation is strictly less than the surplus emerging from cooperation. In case of complete information cooperation will always occur, but as we show, in case of incomplete information non–cooperation may be an equilibrium outcome – despite of gains from trade. We characterize in a simple bargaining framework the two pooling and the three separating equilibria in terms of prior probabilities. Furthermore we characterize implied rent payments and the influence of bargaining power on the division of the surplus.

Zusammenfassung

In dieser Arbeit untersuchen wir spieltheoretisch ökonomische Situationen in denen zwei Spieler wissen, daß unter Sicherheit Kooperation beide Spieler besserstellen würde – verglichen mit Nicht–Kooperation. D.h., die Summe der Auszahlungen im Falle der Nicht–Kooperation ist strikt kleiner als der Surplus der aus der Kooperation entsteht. Bei vollständiger Information wird es daher immer zu Kooperation kommen. Jedoch kann, wie wir zeigen werden, in einer Situation mit unvollständiger Information Nicht–Kooperation ein Gleichgewicht sein, obwohl es immer "gains from trade" gibt. Wir charakterisieren in einem einfachen Verhandlungsmodell die beiden pooling und die drei separierenden Gleichgewichte als Funktion der a priori–Wahrscheinlichkeiten, sowie die implizierten Rentenzahlungen und den Einfluß von Verhandlungsmacht auf die Aufteilung des Kooperationssurplus.

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1 Introduction

A lot of economic interactions involve bargaining in which players have the possibility of concluding a mutually beneficial agreement in presence of a conflict of interest. How agreements are reached and how the bargaining surplus is divided is the central issue of bargaining.

In this paper we analyze with the help of a non–cooperative game–theoretic bargaining model economic situations where two players know that in a world of certainty cooperation would make both players better off. If they would cooperate they could find a division of the gains from cooperation which would make each of them better off compared to a situation of non-cooperation. Put differently, the sum of the payoffs they get in case of non–cooperation is less than the profit resulting from cooperation – there are gains from trade in any case. "Cooperation" can be understood fairly general. Intuition suggests that in bargaining over the division of the cooperation surplus each player only agrees on the cooperation if he gets at least what is available for him in the case of non–cooperation. This payoff is the players' "outside option" payoff or, equivalently, his reservation payoff or opportunity cost. Outside option payoffs may be independent or depend on the type of the opponent. However, it is a lesson from bargaining theory that

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the outcome depends quite heavily on the specification of the bargaining process. As for example **Shaked and Sutton** [1984], **Sutton** [1986] and **Osborne and Rubinstein** [1990] have shown, the possibility of an outside option will in general influence the division of the bargaining surplus.

The purpose of this paper is to analyze in a framework of incomplete information the influence of the outside option payoffs, identified as bargaining power, on the division of the surplus from cooperation and to characterize the outcome in terms of bargaining power and prior probabilities. To tackle this problem a simple two–stage game is analyzed where in the first stage a player makes a "take–it–or–leave–it"–offer and in the second stage the other player accepts or rejects the offer. In case of an acceptance cooperation occurs and the players get their agreed upon shares of the surplus. Non–agreement implies that each player gets his outside option payoff. For simplicity, we assume that there are only two possible outside option payoffs (i.e. two possible types) for each player: the payoff may be "high" or "low". These payoffs and the probabilities with which they occur are assumed to be common knowledge.

If there are gains from trade and if there is complete information, every game–theoretic solution concept of such a bargaining will have cooperation as an equilibrium outcome. Furthermore, the responder in such an ultimatum game will get just his reservation payoff. However, if the bargaining parties only have incomplete information about the reservation payoff of the opponent, the situation may be different. We show that cooperation under uncertainty does not always happen – contrary to cooperation under certainty. With incomplete information non–agreement may be an equilibrium outcome (despite of gains from trade) and the bargaining outcome may therefore be inefficient. Furthermore, under uncertainty the equilibrium payoffs may include a rent for the responder, i.e. an overpayment relative to his reservation payoff. The equilibria of the game are characterized in terms of the prior probabilities. Bargaining power, identified by the size of the outside option payoff, influences the probability intervals where an equilibrium exists.

The paper is organized as follows: After the description of the model and the assumptions in the next Section, Section 3 presents some examples which can be discussed within our framework. Section 4 states and discusses the results comprising sequential pooling and separating equilibria. A short discussion of the influence of bargaining power is also contained in this section. Section 5 gives a summary. All proofs are delegated to the Appendix.

2 The model

There are two players who know that in a given economic environment cooperation would make them strictly better off as compared to non–cooperation. The players are of two possible types, i.e. they may be "weak" or "strong". Whether they are weak or strong is determined by nature. The meaning of weak and strong depends on the specific economic situation which has to be analyzed, i.e. the type depends on the outside options (the payoff a player gets in a situation of non-cooperation). Hence, we will characterize the types by their outside option payoffs. The structure of the game is as follows: Nature decides the types of the players and each player learns its type but not the type of the other player. The combination of types may determine the possible outside option payoffs in a situation of non- cooperation. The players now have to decide whether to cooperate or not. Cooperation yields a known payoff which has to be divided. One player makes a proposal which can be accepted or rejected by the other player. If the other player accepts, cooperation starts and the players get their respective agreed-upon division of the cooperation payoff. If they disagree, bargaining ends. In the latter case they receive their outside option payoffs. After this informal description we will give a more formal one.

First, nature decides with probability p_i that player i, $i = 1, 2$ is "weak", ($p_i =$ prob(i = weak)). With probability $(1 - p_i)$ player i is "strong". Each player learns his type but not the type of the other player. A player is labelled i_w if he is weak and i_s if he is strong. This description implies that there are four possible combinations of types: both players may be weak (1_w, 2_w), both players may be strong (1_s, 2_s) and one player may be weak and the other strong ((1_s, 2_w) or (1_w, 2_s)). At the next stage (the bargaining stage) we consider a very simple bargaining situation, where the players bargain over the division of the cooperation surplus Π_m. One player, labelled player 1, demands x, $0 \leq x \leq \Pi_m$. If this demand is accepted by player 2 cooperation will start and each player gets his respective profit share of the cooperation surplus. If player 2 refuses the proposal, cooperation does not take place and the players get their outside option payoffs. All payoffs are measured in von Neumann-Morgenstern utility. The extensive form of this game is given in Fig. 1.

Fig. 1

Notice that the proposal of player 1 is labelled x_t, $t = w, s$. Player 2 (in pure strategies) can respond by accepting ($R_t = 1$, $t = w, s$) or rejecting ($R_t = 0$, $t = w, s$) the proposal. The strategy set of player 1 is the interval $[0, \Pi_m]$ and player 2's strategy set is $\{0, 1\}$. The strategy set of the whole game therefore is $[0, \Pi_m] \times \{0, 1\}$. If the proposal is accepted player 1 gets x_w or x_s and player 2 the rest of the cooperation surplus Π_m, i.e. $(\Pi_m - x_w)$ or $(\Pi_m - x_s)$, respectively. If the proposal is not accepted the outside option payoffs result. As described above, four combinations of types are possible. Hence, four possible outside option payoffs for each player can result. These outside option payoffs are labelled Π^i_o, Π^i_l, Π^i_h and Π^i_c, $i = 1, 2$. Concerning these outside option payoffs the following notational convention will take place: If two strong players are matched $(1_s, 2_s)$ their outside option payoffs are denoted Π^i_c, $i = 1, 2$. If two weak players are matched $(1_w, 2_w)$, their outside option payoffs are denoted Π^i_o, $i = 1, 2$. In the cases where a weak player i and a strong player j are matched $(i_w, j_s, i \neq j)$ the payoffs are denoted Π^i_l and Π^j_h, $i \neq j$. The payoff of a weak player i *matched with a strong player* j is always denoted Π^i_l and the payoff of a strong player i *matched with a weak player* j is Π^i_h. Next we will make some assumptions about these outside option payoffs:

Assumption 1 *(i)* $\Pi^i_h \geq \Pi^i_c > \Pi^i_l$; *(ii)* $\Pi^i_h > \Pi^i_o \geq \Pi^i_l$, $i = 1, 2$.

Assumption 1 (i) says that a strong player i, matched with a weak player j, gets strictly more than a weak player i who is matched with a strong player j. The payoff of a strong player i, matched with a weak player j, Π^i_h, is greater than or equal to player i's payoff if matched with a strong player j, (Π^i_c). Furthermore, a strong player i, matched with a weak player j, gets more than a weak player i, matched with a weak player j: $\Pi^i_h > \Pi^i_o$. The payoff of a weak player i, matched with a weak player j, Π^i_o, is greater than or equal to the payoff of a weak player

i matched with a strong player j, (Π^i_l). This is (ii). Notice that our definition of types allows for the possibility that the outside option payoffs depend only on the player's own type ($\Pi^i_h = \Pi^i_c$; $\Pi^i_o = \Pi^i_l$). In this case, our definition of the types simply says that a strong player has a strictly greater outside payoff than a weak player (i.e. $\Pi^i_h = \Pi^i_c > \Pi^i_o = \Pi^i_l$). Furthermore, if the outside option payoff depends on the type of the other player, we allow for the possibility that a strong player i, who is matched with a strong player j, has a lower outside payoff than a weak player i who is matched with a weak player j, (i.e. ($\Pi^i_c \leq \Pi^i_o$). We also do not rule out the converse. Our definition of the types only requires that the payoff a weak player i can get if matched with a strong player j, (i.e. Π^i_l), is the lowest possible payoff for player i and. Conversely, the payoff a strong player i can get if matched with a weak player j, (i.e. Π^i_h), is the highest possible payoff player i can get. The next assumption is the "gains from trade" assumption which assures that cooperation is possible for all matching of types (i.e. bargaining over cooperation surplus always makes sense):

Assumption 2 $\Pi_m > \max\,(\Pi^1_c + \Pi^2_c;\;\; \Pi^1_h + \Pi^2_l;\;\; \Pi^1_l + \Pi^2_h;\;\; \Pi^1_o + \Pi^2_o)$.

3 Examples

This section discusses some examples which may illustrate our model.

EXAMPLE 1: A widespread model in the Industrial Organization literature is the duopoly model. Two firms interact either by quantity or price competition. In the standard model of oligopoly, the Cournot–model, firms interact by choosing quantities which are, under some conditions on the profit function, strategic substitutes.[1] This implies that the profit of a monopolist is always at least as high as the sum of the duopolists' profits. With complete information this creates a strong incentive for duopolists to merge and to become a monopolist. Incomplete information about the payoff of the other firm may change this incentive substantially. Consider for example a case where each firm has only some conjectures about the costs or the capacity of the other firm. If non– agreement in the bargaining over the division of the monopoly profit occurs firms play a Cournot–game

[1]See **Tirole** [1988] or **Geanakoplos, Bulow and Klemperer** [1985].

under incomplete information. For illustrative purposes assume that a homogenous goods' market is characterized by a linear inverse demand and that the firms produce at constant marginal costs. However, assume further that the firms may have capacity constraints which are less than the Cournot–Nash–quantity. A weak firm then corresponds to a capacity-constrained firm and a strong firm is unconstrained. If after non–agreement in a subsequent duopoly game two unconstrained firms are matched, each firm gets the unconstrained Cournot–Nash profit of Π_c^i. If two weak firms are matched each gets Π_o^i. A matching of a weak firm with a strong firm yields the weak firm Π_l^i and the strong one Π_h^i. Then one can show that the Cournot–profits indeed satisfy Assumption 1 (see **Gächter and Kirchsteiger** [1991]). The bargaining in our model may in this case be interpreted as a takeover offer.

Notice that this model with its assumptions also allows for cooperation of firms which are not necessarily engaged in the same market, i.e. it is also suited for analyzing vertical integration or joint ventures.[2] In this case the outside option payoffs can be assumed to be independent.

EXAMPLE 2: One of the first examples of using an outside option in a bargaining was in the context of wage bargaining (**Shaked and Sutton** [1984]). As already discussed in our model the rejection of a demand can also be viewed as an outside option player 2 has which leaves both players with their outside option or reservation payoffs. Consider a wage bargaining between a firm and a representative worker whom the firm intends to employ. It is quite realistic to assume that the outside option payoff of the worker is the wage he can earn elsewhere or the unemployment benefit he or she could get or just his reservation wage. The outside option payoff of the firm is the profit the firm gets without the additional worker. It is even more realistic to assume that the firm does not know exactly the outside opportunities of the worker and that the worker has only some conjectures about his contribution to the profit of the firm. Furthermore one can assume that the outside options of the worker and the firm, respectively, are independent, i.e. $\Pi_h^i = \Pi_c^i$ and $\Pi_l^i = \Pi_o^i$, $i = 1, 2$. As already discussed, in this

[2]For more on models of cooperative behavior in industrial markets see **Jacquemin and Slade** [1989]. Models of vertical integration are discussed in **Tirole** [1988] and for more on joint ventures as a special legal form of a cooperative agreement see **Mariti and Smiley** [1983].

case Assumption 1 reduces to $\Pi_h^i > \Pi_l^i$ and Assumption 2 becomes (after a slight rearrangement) $\Pi_m - \Pi_h^1 > \Pi_l^2$. Let us identify player 1 with the firm and player 2 with the worker. Then Assumption 2 says that the minimal net profit of the firm resulting from the employment of a strong-type worker exceeds the highest possible reservation wage the firm must pay. If this inequality were violated the firm would not be interested in employing an additional worker if this worker were a strong type. The bargaining stage depicts price–taking behavior of the worker. If the worker agrees on the proposed wage he is employed and gets his agreed–upon wage and the firm gets their profit share.

4 The results

In this section the main results, i.e. the solutions of this model are given. Since our game does not have proper subgames, our solution concept is the sequential equilibrium concept by **Kreps and Wilson** [1982]. A sequential equilibrium consists of a sequentially rational strategy vector (x^*, R^*) and a system of beliefs μ^*. Strategies have to be such that each player behaves optimally in each of his information sets given the beliefs. The beliefs are derived by Bayes' rule and must be consistent with the equilibrium strategies. We derive two classes of equilibria, sequential pooling and sequential separating equilibria. A pooling equilibrium is an equilibrium where both types of player 1 play the same strategy. In our game this means that player 2 cannot infer the type of player 1 if he observes his proposal. In a separating equilibrium player 2 learns the type of player 1 after his proposal. We restrict ourselves to pure strategies. In deriving the results we proceed as follows: First the properties of equilibrium behavior (Lemma 1) are characterized after deriving the conditions for best responses. Proposition 1 summarizes this subsection by stating the possible candidates for equilibrium. Lemma 2 gives a further characterization of some equilibrium candidates. In the next two subsections the pooling and separating equilibria, respectively, are derived, depending on the probabilities p_i.

4.1 Properties of equilibrium behavior

Let $x = (x_w, x_s)$ be a pure strategy of player 1, where x_t, $t = w, s$, denotes the demand of a weak or a strong player 1, respectively. A strategy of player 2 is given by $R = (R_w(x), R_s(x))$, where $R_t(x) = 1$, $t = w, s$ if a type of player 2 accepts the demand x. Otherwise $R_t(x) = 0$.

The beliefs of player 2 are given by $\mu_t(x)$, $t = w, s$ which denote the probability of being matched with a weak player 1, given player 1's demand x. Let $b = (b_w(x), b_s(x))$ be a (possibly mixed) strategy of player 1 where $b_t(x)$, $t = w, s$ denotes the probability that type t of player 1 demands x. Then from the Bayesian rule follows:

$$\mu_w(x) = \mu_s(x) = \mu(x) = \frac{p_1 b_w(x)}{p_1 b_w(x) + (1 - p_1) b_s(x)} \quad \forall x : b_w(x) \text{ or } b_s(x) > 0, \tag{4.1}$$

This implies that the beliefs of player 2 are independent of his type for all x which are demanded with positive probability. Furthermore, consistency of beliefs (as a requirement for sequential equilibria) implies that the beliefs of player 2 are independent for all x including those which are played by both types of player 1 with zero probability.[3] Therefore the beliefs of player 2 are independent of his own type, i.e. $\mu_s(x) = \mu_w(x) = \mu(x)$ for all x.

The expected payoff of a weak-type player 2 is given by (notice that in the following we denote the *expected* payoff by Φ_{it}, ($i = 1, 2$, $t = w, s$); the *outside option* payoffs are denoted as described above):

$$\begin{aligned} \Phi_{2w}(R_w(x), \mu(x)) &= \Pi_m - x \quad \forall x : R_w(x) = 1 \\ \Phi_{2w}(R_w(x), \mu(x)) &= \mu(x)\Pi_o^2 + (1 - \mu(x))\Pi_l^2 \quad \forall x : R_w(x) = 0 \end{aligned} \tag{4.2}$$

$R_w(x)$ being a best response requires:

$$R_w(x) = \begin{cases} 1 & \forall x : \Pi_m - x > \mu(x)\Pi_o^2 + (1 - \mu(x))\Pi_l^2 \\ 0 & \forall x : \Pi_m - x < \mu(x)\Pi_o^2 + (1 - \mu(x))\Pi_l^2 \\ r_w \in \{0, 1\} & \forall x : \Pi_m - x = \mu(x)\Pi_o^2 + (1 - \mu(x))\Pi_l^2 \end{cases} \tag{4.3}$$

The expected payoff of the strong type of player 2 is given by

$$\begin{aligned} \Phi_{2s}(R_s(x), \mu(x)) &= \Pi_m - x \quad \forall x : R_s(x) = 1 \\ \Phi_{2s}(R_s(x), \mu(x)) &= \mu(x)\Pi_h^2 + (1 - \mu(x))\Pi_c^2 \quad \forall x : R_s(x) = 0 \end{aligned} \tag{4.4}$$

[3]For a discussion of this issue see **Fudenberg and Tirole** [1991], p.331 and 338.

$R_s(x)$ is a best response if:

$$R_s(x) = \begin{cases} 1 & \forall x : \Pi_m - x > \mu(x)\Pi_h^2 + (1-\mu(x))\Pi_c^2 \\ 0 & \forall x : \Pi_m - x < \mu(x)\Pi_h^2 + (1-\mu(x))\Pi_c^2 \\ r_s \in \{0,1\} & \forall x : \Pi_m - x = \mu(x)\Pi_h^2 + (1-\mu(x))\Pi_c^2 \end{cases} \tag{4.5}$$

Define a set $\Omega^w = \{x : R_w(x) = 1\}$ and similarly $\Omega^s = \{x : R_s(x) = 1\}$. Let X^w denote the highest demand which player 2_w does not reject and similarly X^s the highest demand which player 2_s does not reject. In order to guarantee that X^w and X^s exist we assume that $r_w = r_s = 1$.[4] Otherwise player 1 would have to maximize over open sets and no equilibrium would exist. From (4.3) and (4.5),

$$\begin{aligned} \Pi_m - \Pi_o^2 &\leq X^w \leq \Pi_m - \Pi_l^2 \\ \Pi_m - \Pi_h^2 &\leq X^s \leq \Pi_m - \Pi_c^2 \end{aligned} \tag{4.6}$$

Then, because of Assumption 1

$$\begin{aligned} \Pi_l^2 < \Pi_c^2,\quad \Pi_o^2 < \Pi_h^2 &\implies X^s \leq X^w \\ &\implies \Omega^s \subseteq \Omega^w \end{aligned} \tag{4.7}$$

The relationship $0 < X^s \leq X^w < \Pi_m$ will be central in all following arguments.

The expected payoff of a weak player 1_w given his strategy x_w is:

$$\Phi_{1w}(x_w) = \begin{cases} x_w & \text{if } x_w \in \Omega^s \\ p_2 x_w + (1-p_2)\Pi_l^1 & \text{if } x_w \in \Omega^w \text{ and } x_w \notin \Omega^s \\ p_2\Pi_o^1 + (1-p_2)\Pi_l^1 & \text{if } x_w \notin \Omega^w \end{cases} \tag{4.8}$$

This is the expected payoff of 1_w who will accept every demand $x_w \in \Omega^w$. If $x_w \in \Omega^w$ but $x_w \notin \Omega^s$ player 2 will accept if he is weak which happens with probability p_2 and reject if he is strong. A rejection gives player 1_w his reservation payoff Π_l^1. If $x_w \notin \Omega^w$ both types of player 2 will reject and the expected payoff of a weak player 1 therefore is $p_2\Pi_o^1 + (1-p_2)\Pi_l^1$.

With similar arguments one gets the expected payoff of player 1_s:

$$\Phi_{1s}(x_s) = \begin{cases} x_s & \text{if } x_s \in \Omega^s \\ p_2 x_s + (1-p_2)\Pi_c^1 & \text{if } x_s \in \Omega^w \text{ and } x_s \notin \Omega^s \\ p_2\Pi_h^1 + (1-p_2)\Pi_c^1 & \text{if } x_s \notin \Omega^w \end{cases} \tag{4.9}$$

[4] This assumption is also needed in Proposition 1.

The next step in deriving our results is, with the help of (4.1) – (4.9), to find strategies which cannot be an equilibrium behavior. This will lead us to a proposition about the candidates for equilibria in this game. All proofs of the following lemmas and propositions are stated in the Appendix.

Lemma 1 *(i) Let $x_t \in \Omega^s$, $x_t < X^s$, $t = w, s$, be a strategy of player 1_t. Then this strategy is strictly dominated by another strategy $x'_t = X^s$.*
(ii) Let x_t, $t = w, s$, be a strategy of player 1_t with $x_t \in \Omega^w$ but $x_t \notin \Omega^s$ and $x_t < X^w$. Then x_t is strictly dominated by another strategy $x'_t = X^w$.
(iii) Let x_w be a strategy of player 1_w with $x_w \notin \Omega^w$. Then x_w is strictly dominated by another strategy $x'_w = X^w$.

(4.7) implies that both types of player 2 will reject every demand $x_e \notin \Omega^w$. Let us denote these "excessive" demands as x_e. For all types of player 1 and all types of player 2 all these x_e are payoff–equivalent, because they are always rejected by player 2. Therefore it is enough to consider only a typical element $x_e \notin \Omega^w$. The following proposition makes a statement about the candidates for equilibria:

Proposition 1 *(i) Given the best responses of player 2 (see (4.3) and (4.5)), the equilibrium demands of player 1 have to be ($x_w = X^s$) or ($x_w = X^w$) if player 1 is weak and ($x_s = X^s$) or ($x_s = X^w$) or ($x_s = x_e$) if player 1 is a strong type.*
(ii) Demands such that the weak type of player 1 demands $x_w = X^w$ and the strong type of player 1 demands $x_s = X^s$ cannot be equilibrium demands.

Notice that Proposition 1 implies that r_w, $r_s = 1$, because otherwise equilibrium demands X^w, X^s would be the greatest elements of open sets, which do not exist (player 1 would have to optimize over an open set which is impossible). From Proposition 1 follows that there are five candidates for equilibria: first, we have two pooling equilibria – both types of player 1 demand X^s (see Proposition 2) and both demand X^w (Proposition 3). Propositions 4 – 6 prove the separating ones – player 1_w demands X^s and player 1_s demands X^w (Proposition 4); 1_w demands X^s and 1_s x_e (Proposition 5) and 1_w demands X^w and 1_s demands x_e (Proposition 6). Notice that two equilibrium candidates involve "excessive" demands x_e which will, in equilibrium, be rejected by both types of player 2. These demands are characterized in the following Lemma 2.

Lemma 2 *(i) If a weak player 1_w demands X^s and a strong player 1_s demands $x_e \notin \Omega^w$, the consistency of beliefs of a weak player 2_w implies that $x_e > \Pi_m - \Pi_l^2$. (ii) If $x_w = X^w$ and if $x_s = x_e$, the consistency of beliefs of player 2 implies that $x_e > \Pi_m - \Pi_l^2$.*

The content of Lemma 2 is that in those two equilibria in which the strong type of player 1 demands so much that both types of player 2 reject (see Proposition 1 and 5 and 6, resp.), this "excessive" demand must be greater than $\Pi_m - \Pi_l^2$. The next two subsections prove the equilibrium candidates stated in Proposition 1.

4.2 Pooling Equilibria

For analyzing the equilibria we have to define some conditions about the parameters p_1 and p_2 which have to be satisfied for existence of the equilibrium. As it will be seen during the proofs these conditions are necessary and sufficient to guarantee that the expected profits of player 1 are indeed maximized if he uses the proposed equilibrium strategies. For the existence of the first pooling equilibrium, Conditions 1 and 2 have to be met.

CONDITION 1: $\Pi_m - p_1\Pi_h^2 - (1-p_1)\Pi_c^2 - p_2\Pi_h^1 - (1-p_2)\Pi_c^1 \geq 0$

Notice that the lefthand-side of Condition 1 is decreasing in p_1 as well as in p_2 because $\Pi_h^2 \geq \Pi_c^2$ and $\Pi_h^1 \geq \Pi_c^1$ by Assumption 1. Notice further that for $p_1 = p_2 = 0$, Condition 1 holds with strict inequality because $\Pi_m - \Pi_c^2 - \Pi_c^1 > 0$ by Assumption 2. This implies that Condition 1 holds for low prior probabilities p_1, p_2 by continuity.

CONDITION 2: $\Pi_m - p_1\Pi_h^2 - (1-p_1)\Pi_c^2 - p_2[\Pi_m - p_1\Pi_o^2 - (1-p_1)\Pi_l^2] - (1-p_2)\Pi_c^1 \geq 0$

Notice that for $p_1 = p_2 = 0$ Condition 2 holds with strict inequality. This implies that there is a parameter region where Conditions 1 and 2 together hold.

Proposition 2 *If p_1 and p_2 are such that Conditions 1 and 2 hold, the following strategy-combination (x^*, R^*) and the beliefs $\mu^*(x)$ are a sequential pooling equilibrium:*

$$x^* = (x_w^*, x_s^*) \quad with \quad x_w^* = x_s^* = \Pi_m - p_1\Pi_h^2 - (1-p_1)\Pi_c^2 \tag{4.10}$$

$$
\begin{aligned}
R^* &= (R_w^*, R_s^*) \quad with \\
R_w^* &= \begin{cases} 1 & \forall x : \Pi_m - x \geq p_1 \Pi_o^2 + (1-p_1)\Pi_l^2 \\ 0 & \forall x : \Pi_m - x < p_1 \Pi_o^2 + (1-p_1)\Pi_l^2 \end{cases} \\
R_s^* &= \begin{cases} 1 & \forall x : \Pi_m - x \geq p_1 \Pi_h^2 + (1-p_1)\Pi_c^2 \\ 0 & \forall x : \Pi_m - x < p_1 \Pi_h^2 + (1-p_1)\Pi_c^2 \end{cases} \qquad (4.11) \\
\mu^*(x) &= p_1
\end{aligned}
$$

This pooling equilibrium is an equilibrium in which both types of player 1 demand exactly $x_s^* = x_w^* = X^s = \Pi_m - p_1\Pi_h^2 - (1-p_1)\Pi_c^2$. This implies that both types of player 2 get a share of $p_1\Pi_h^2 + (1-p_1)\Pi_c^2$ and will therefore agree – cooperation occurs for all kinds of possible matchings of types. Furthermore, the weak type of player 2 gets a rent, i.e. an overpayment relative to his expected reservation payoff $p_1\Pi_o^2 + (1-p_1)\Pi_l^2$, whereas a strong player 2 gets just his expected outside option payoff. The reason for this is that in this equilibrium the probability that player 2 is weak is relatively low and that is why player 1 accepts that a weak player 2 gets an overpayment. No type of player 1 risks a rejection. Therefore for high probabilities that player 2 is strong cooperation will always occur.

As it is proved in the Appendix, Condition 1 and 2 are indeed necessary and sufficient for this equilibrium. Condition 1 guarantees that the equilibrium demand X^s gives a strong player 1 a greater expected payoff than an excessive demand x_e (see (6.2) in the Appendix). Condition 2 guarantees that X^s gives a strong player 1 a greater expected payoff than a demand of X^w (see (6.1) in the Appendix). Notice the logic of the argument: *if* the parameters (i.e. the prior probabilities) are such that Conditions 1 and 2 are satisfied *then* these conditions are necessary and sufficient for the proposed equilibrium behavior. Comparing Condition 1 and 2 shows that Condition 1 is binding if and only if

$$\Pi_h^1 \geq \Pi_m - p_1\Pi_o^2 - (1-p_1)\Pi_l^2 \iff p_1 \geq \frac{\Pi_m - \Pi_h^1 - \Pi_l^2}{\Pi_o^2 - \Pi_l^2} \qquad (4.12)$$

Notice that it is possible that the righthand side of (4.12) is greater than 1. In this case, only Condition 2 is binding for the whole parameter region. Notice further that the lefthand side of Condition 1 is decreasing in p_1 and p_2. An increase in p_1 decreases X^s (if p_1 increases, player 2 wants a greater share of the cooperation surplus) and an increase in p_2 increases the expected payoff of an excessive demand. This implies that if Condition 1 is binding, p_1 and p_2 have to be relatively small. For a p_1 such that Condition 2 is binding ((4.12) does not hold), p_2 has to be low, too. To see this, notice that for $p_2 = 1$ Condition 2 cannot hold irrespectively of p_1. Notice further that the lefthand side of Condition 2 is decreasing in p_2 if (4.12) does not hold, i.e. if the inequality in (4.12) is reversed[5]. Because p_2 is the probability that X^w will be accepted Condition 2 holds for a low p_2. The influence of p_1 on Condition 2 is ambiguous, because an increase of p_1 decreases X^w as well as X^s. The result of these considerations is summarized in Fig. 2. In Fig. 2(a) the payoffs are such that only Condition 2 is binding (i.e. the righthand side of (4.12) is greater than 1). In Fig. 2(b) the payoffs are such that Condition 1 is also binding for a certain parameter region. A strategy combination as in Proposition 2 is an equilibrium if (p_1, p_2) is in the shaded areas.

Fig. 2

Notice that Fig. 2 is just an example but it is "generic" in the sense that different levels of outside option payoffs produce different probability regions but qualitatively the same picture.

A last remark concerns uniqueness. Because of the consistency of beliefs and the best response of player 2, the equilibrium demand X^s (and therefore the payoffs) are unique. However, the equilibrium behavior of player 2 and especially the highest demand player 2_w just accepts (X^w) are not unique because it depends on the assumed perturbation b^ϵ.[6] This implies that the parameter interval of the pooling equilibrium where both types of player 1 demand X^s is not unique. Different beliefs of player 2 about the probability of a type of player 1 making a mistake lead to different parameter regions where X^s is an equilibrium demand. If, for example, player 2 believes that 1_s makes mistakes with a lower probability than 1_w, $\mu(x)$ increases for all $x \neq \Pi_m - p_1\Pi_h^2 - (1-p_1)\Pi_c^2$ which leads X^w to decrease. This weakens Condition 2 and x^* is an equilibrium demand in a

[5]$d\,L/d\,p_2 = -(\Pi_m - p_1\Pi_o^2 - (1-p_1)\Pi_l^1) + \Pi_c^1 < -\Pi_h^1 + \Pi_c^1 \leq 0$, where L denotes the lefthand side of Condition 2.

[6]Notice that in a sequential equilibrium it suffices to find *some* sequence of completely mixed strategies b^ϵ which support the equilibrium in question. There may be other sequences of mixed strategies which lead to the same outcome.

greater parameter region of p_2. On the other hand, x^* is an equilibrium in a smaller parameter region if player 2 believes that 1_s makes mistakes with a higher probability than 1_w.

To summarize, if p_1 and p_2 are small which means that both players are likely to be strong, even a strong player 2 accepts a relatively high demand, i.e. X^s is high. Furthermore, no type of player 1 risks a rejection. Hence, for high probabilities that the opponent is strong an agreement will always occur.

For the next pooling equilibrium, Conditions 3 and 4 have to be met.

CONDITION 3: $p_2[\Pi_m - p_1\Pi_o^2 - (1-p_1)\Pi_l^2] + (1-p_2)\Pi_c^1 - p_2\Pi_h^1 - (1-p_2)\Pi_c^1 \geq 0$ which is the same as $\Pi_m - p_1\Pi_o^2 - (1-p_1)\Pi_l^2 - \Pi_h^1 \geq 0$

CONDITION 4: $p_2[\Pi_m - p_1\Pi_o^2 - (1-p_1)\Pi_l^2] + (1-p_2)\Pi_l^1 - \Pi_m + p_1\Pi_h^2 + (1-p_1)\Pi_c^2 \geq 0$

Proposition 3 *If the prior probabilities p_1 and p_2 are such that Conditions 3 and 4 hold, the following strategy-combination (x^*, R^*) and the beliefs $\mu^*(x)$ are a sequential pooling equilibrium:*

$$x^* = (x_w^*, x_s^*) \quad \textit{with} \quad x_w^* = x_s^* = \Pi_m - p_1\Pi_o^2 - (1-p_1)\Pi_l^2 \tag{4.13}$$

$$\begin{aligned}
R^* &= (R_w^*, R_s^*) \quad \textit{with} \\
R_w^* &= \begin{cases} 1 & \forall x : \Pi_m - x \geq p_1\Pi_o^2 + (1-p_1)\Pi_l^2 \\ 0 & \forall x : \Pi_m - x < p_1\Pi_o^2 + (1-p_1)\Pi_l^2 \end{cases} \\
R_s^* &= \begin{cases} 1 & \forall x : \Pi_m - x \geq p_1\Pi_h^2 + (1-p_1)\Pi_c^2 \\ 0 & \forall x : \Pi_m - x < p_1\Pi_h^2 + (1-p_1)\Pi_c^2 \end{cases} \\
\mu^*(x) &= p_1
\end{aligned} \tag{4.14}$$

In this pooling equilibrium both types of player 1 demand $X^w = \Pi_m - p_1\Pi_o^2 - (1-p_1)\Pi_l^2$. This implies that cooperation will occur if player 2 is weak, whereas non–cooperation occurs if player 2 is strong, because the offered share of $p_1\Pi_o^2 - (1-p_1)\Pi_l^2$ is less than his expected reservation payoff and therefore it will be rejected. A weak player 2 gets just his expected outside option payoff. The possibility of cooperation does not depend on the type of player 1. The reason for this result is that in this equilibrium both types of player 1 risk a rejection because the probability that player 2 is strong is relatively low, although, according to Assumption 2, player 1 would have profited from agreement even with a strong-type player 2. Non-cooperation can be an equilibrium outcome even if there are gains from trade for both parties.

Condition 3 guarantees that the expected payoff a strong player 1 gets if he demands X^w is greater than the expected payoff of an "excessive" demand x_e. Condition 4 guarantees that the expected payoff of a demand X^w of a weak player 1 is greater than his expected payoff when demanding X^s. To investigate the relation between Condition 3 and Condition 4 notice first that Condition 3 is only a constraint on p_1:

$$p_1 \leq \frac{\Pi_m - \Pi_l^2 - \Pi_h^1}{\Pi_o^2 - \Pi_l^2} \tag{4.15}$$

Notice that it is possible that Condition 3 is not binding at all, i.e. that the righthand side of (4.15) is greater than 1. If player 2 is strong, nonagreement occurs irrespectively whether player 1 demands X^w or x_e, because $X^w \geq X^s$, the highest demand a strong player just accepts. Therefore p_2 plays no role for Condition 3. But an increase of p_1 decreases X^w (if p_1 increases, player 2 wants a greater share of the cooperation surplus) which implies that a demand X^w becomes less profitable for player 1. Furthermore, if $p_2 = 0$, Condition 4 cannot hold irrespectively of the value of p_1. If $p_2 = 1$, Condition 4 holds irrespectively of the value of p_1. Furthermore, the lefthand side of Condition 4 is unambiguously increasing in p_2 if Condition 3 holds.[7] A rise in p_2 increases the possibility that demand X^w is actually accepted (because X^w is the highest demand a weak player 2 just accepts) and therefore increases the expected payoff from a demand X^w. However, a rise in p_1 decreases X^w and X^s and its effect on Condition 4 is therefore ambiguous.[8] The relationship between Condition 3 and 4 is summarized in Fig. 3(a) and 3(b):

Fig. 3

[7] $d\ L/d\ p_2 = \Pi_m - p_1\Pi_o^2 - (1-p_1)\Pi_l^2 - \Pi_l^1 > \Pi_m - p_1\Pi_o^2 - (1-p_1)\Pi_l^2 - \Pi_h^1 \geq 0$. L denotes the lefthand side of Condition 4 and the expression after the strict inequality is Condition 3.

[8] $d\ L/d\ p_1 = p_2(\Pi_l^2 - \Pi_o^2) + \Pi_h^2 - \Pi_c^2 \geq [\leq] 0$.

In Fig. 3(a) we show a situation where only Condition 4 is binding (i.e. the righthand side of (4.15) is greater than 1), where in Fig. 3(b) Condition 3 is also binding. Again notice that these pictures are just examples, although "generic" ones.

Concerning uniqueness, similar arguments to above apply. The stated equilibrium demands (and hence the payoffs) are unique. However, X^s, the amount a strong player just accepts, depends on the assumed perturbation b^ϵ and hence the parameter region within which the proposed equilibrium exists.

A strategy combination as stated in Proposition 3 is an equilibrium if p_2 is high enough that also a weak player 1 risks noncooperation (because the probability of noncooperation, $1 - p_2$ is low), but on the other hand p_1 is low enough that a demand acceptable for a weak player 1 (X^w) is high enough that a strong player 1 does not make an excessive demand.

Notice that for all conditions for pooling equilibria, (Conditions $1 - 4$), p_1 plays no role if the outside option of player 2 does not depend on player 1's type, i.e. if $\Pi_l^2 = \Pi_o^2$, $\Pi_c^2 = \Pi_h^2$. In this case, p_1 plays of course no role for the acceptance behavior of player 2. This situation is equivalent to a situation where player 2 already knows the type of player 1.

After this analysis of pooling equilibria we proceed in the next subsection by analyzing separating equilibria.

4.3 Separating Equilibria

The structure of the arguments used in this subsection is quite similar to those used in the analysis of pooling equilibria. As before, we proceed by stating conditions which have to be satisfied for the existence of a separating equilibrium. Conditions 5 and 6 are necessary and sufficient for the existence of the first separating equilibrium.

CONDITION 5: $\Pi_m - \Pi_h^2 - p_2(\Pi_m - \Pi_l^2) - (1 - p_2)\Pi_l^1 \geq 0$

Notice that the lefthand side of Condition 5 is decreasing in p_2. For $p_2 = 0$, Condition 5 holds, for $p_2 = 1$, it does not.

CONDITION 6: $p_2(\Pi_m - \Pi_l^2) + (1 - p_2)\Pi_c^1 - \Pi_m + \Pi_h^2 \geq 0$

The lefthand-side of Condition 6 is increasing in p_2 and, contrary to the former condition, Condition 6 holds for $p_2 = 1$. Furthermore, a comparison between these conditions reveals that Condition 6 holds with strict inequality if Condition 5 holds with equality. This implies that there exists an intermediate range of values of p_2 for which both, Condition 5 and 6 hold.

Proposition 4 *If p_2 is such that Conditions 5 and 6 hold, the following strategy-combination (x^*, R^*) and the beliefs $\mu^*(x)$ are a separating sequential equilibrium:*

$$x^* = (x_w^*, x_s^*) \quad with \quad x_w^* = \Pi_m - \Pi_h^2 \quad and \quad x_s^* = \Pi_m - \Pi_l^2 \tag{4.16}$$

$$\begin{aligned} R^* &= (R_w^*, R_s^*) \quad with \\ R_w^* &= \begin{cases} 1 & \forall x : \Pi_m - x \geq \Pi_o^2 \\ 0 & \forall x : \Pi_m - \Pi_o^2 < x < \Pi_m - \Pi_l^2 \\ 1 & if\ x : \Pi_m - x = \Pi_l^2 \\ 0 & otherwise \end{cases} \\ R_s^* &= \begin{cases} 1 & \forall x : \Pi_m - x \geq \Pi_h^2 \\ 0 & otherwise \end{cases} \end{aligned} \tag{4.17}$$

$$\mu^*(x) = \begin{cases} 0 & if\ x = \Pi_m - \Pi_l^2 \\ 1 & otherwise \end{cases} \tag{4.18}$$

This separating equilibrium is a case where a strong type of player 1 demands $x_s^* = X^w = \Pi_m - \Pi_l^2$ and a weak type $x_w^* = X^s = \Pi_m - \Pi_h^2$. A strong-type player 2 will accept the proposal of a weak- type player 1 because it gives him just Π_h^2, whereas the offer of a strong type would leave him with Π_l^2 which he will therefore reject. A weak player 2 gets a rent of $\Pi_h^2 - \Pi_l^2$ if the offer comes from a weak player 1; otherwise he gets just his reservation payoff.

Condition 5 guarantees that the expected payoff of a weak player 1 when demanding X^s is at least as high as when demanding X^w. Condition 6 ensures that player 1_s has no incentive to deviate from his strategy of demanding X^w (and risking a rejection by a strong-type player 2) because his expected payoff is at least as high as when demanding X^s or x_e. Because this is a separating equilibrium, the type of player 1 is revealed by his demand. Player 2 *knows* the type of player 1 ($\mu(x_s^*) = 0$ and $\mu(x_w^*) = 1$). This knowledge determines X^w and

X^s, respectively, and p_1 does not occur in the equilibrium demands. Therefore p_1 plays no role for the acceptance behavior of player 2.[9] The value of p_2, however, is important for the existence of this separating equilibrium. p_2 must be such that a strong type of player 1 has an incentive to risk a rejection of his offer (i.e. Condition 6 holds)[10] whereas a weak player 1 does not have this incentive (Condition 5 holds).

A last remark concerns the consistency requirement. With (4.16), X^w and X^s are independent of the assumed perturbations. However, for the consistency requirement of the beliefs it is necessary that player 2 assumes that the perturbations of a weak player 1 are much more likely than mistakes of a strong player. If not, i.e. if $\mu(x)$ is (sufficiently) below 1 for all $x : \Pi_m - \Pi_l^2 > x > \Pi_m - \Pi_h^2$, R_s^* would not be a best response anymore which implies that X^s would increase (see (4.5) and (4.6)). This in turn would contradict the requirement of consistent beliefs. Hence, the perturbations have to be such that $\mu(x) = 1$ for all $x : \Pi_m - \Pi_l^2 > x > \Pi_m - \Pi_h^2$. This equilibrium is not uniformly perfect (for a definition see **Güth** [1992]). As the proof of this equilibrium reveals, to be sequential all separating equilibria need asymmetric perturbations.

To summarize, in this equilibrium nonacceptance only occurs if both players are strong.

For the next separating equilibrium three conditions have to be met.

CONDITION 7: $p_2\Pi_h^1 + (1-p_2)\Pi_c^1 - p_2(\Pi_m - \Pi_o^2) + (1-p_2)\Pi_c^1 \geq 0$ which is the same as $\Pi_h^1 - \Pi_m + \Pi_o^2 \geq 0$

This condition cannot hold if the outside options do not depend on the type of the other player, i.e. if $\Pi_o^2 = \Pi_l^2$ and $\Pi_h^1 = \Pi_c^1$, because we have assumed that $\Pi_m > \Pi_h^1 + \Pi_l^2$.

CONDITION 8: $\Pi_m - \Pi_h^2 - p_2(\Pi_m - \Pi_o^2) - (1-p_2)\Pi_l^1 \geq 0$

[9]This is true for all separating equilibria just by definition of separating equilibria.

[10]If $\Pi_m \leq \Pi_h^2 + \Pi_c^1$, which can only be the case if the outside profits depend on the type of the other player, Condition 6 holds for any p_2. In this special case 1_s always has an incentive to risk a rejection even if he meets a strong player 2 with certainty ($p_2 = 0$). This is caused by the fact that X^s, the highest demand acceptable for 2_s, is very low ($\Pi_m - \Pi_h^2$) because 2_s believes that such a low demand is made by a weak player 1.

The lefthand side of Condition 8 decreases with p_2. For $p_2 = 0$, this condition holds; if $p_2 = 1$ it does not.

CONDITION 9: $p_2\Pi_h^1 + (1-p_2)\Pi_c^1 - \Pi_m + \Pi_h^2 \geq 0$

The lefthand side of Condition 9 increases with p_2. If Condition 7 holds, Condition 9 holds for $p_2 = 1$. For $p_2 = 0$, Condition 9 may also hold[11] (in this case Condition 9 is no longer a binding condition for the existence of the following equilibrium). Notice further that if Condition 7 holds and Condition 8 is satisfied with equality, Condition 9 also holds. This implies that there is an intermediate range of values of p_2 where both, Conditon 8 and 9 hold if Condition 7 holds.

Proposition 5 *If Conditions 7 - 9 hold, the following strategy-combination (x^*, R^*) and the beliefs $\mu^*(x)$ are a separating equilibrium:*

$$x^* = (x_s^*, x_w^*) \quad \text{with} \quad x_s^* = x_e > \Pi_m - \Pi_l^2 \quad \text{and} \quad x_w^* = \Pi_m - \Pi_h^2 \tag{4.19}$$

$$\begin{aligned} R^* &= (R_w^*, R_s^*) \quad \text{with} \\ R_w^* &= \begin{cases} 1 & \forall x : \Pi_m - x \geq \Pi_o^2 \\ 0 & \text{otherwise} \end{cases} \\ R_s^* &= \begin{cases} 1 & \forall x : \Pi_m - x \geq \Pi_h^2 \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{4.20}$$

$$\mu^*(x) = \begin{cases} 0 & \text{if } x = x_e \\ 1 & \text{otherwise} \end{cases} \tag{4.21}$$

This equilibrium is a case where player 1_s has to make an "excessive" demand to convince player 2 that he is a strong type (see (4.21) – in all other cases player 2 believes that player 1 is weak). This excessive demand leads to a rejection by both types of player 2. Contrary to the behavior of a strong type, the demand of a weak type of player 1, $x_w^* = X^s = \Pi_m - \Pi_h^2$ is "small enough" that both types of player 2 will accept – the strong player 2 gets just his reservation payoff and the weak type gets a rent. This is just what is expressed by Conditions 7 - 9 (see the proof in the Appendix); 7 and 9 being conditions on the optimality of an excessive demand. The former states that a strong type of player 1 has no

[11] We have not ruled out this by Assumption 2.

incentive to deviate to X^w, whereas the latter means that demanding X^s would result in a lower expected payoff. Condition 8 is necessary and sufficient for a weak players' behavior of not risking a rejection.

For this being an equilibrium, the probability p_2 that player 2 is weak, has to be small enough such that a weak player has no incentive to risk nonacceptance (i.e. Condition 8 must hold), and at the same time it has to be at least as high that a strong player 1 has no incentive to demand an x_s which is accepted by both types of player 2 (i.e. p_2 must be such that Condition 9 holds). Furthermore, player 2_w, facing an offer $x \leq \Pi_m - \Pi_l^2$, would have to believe that this offer was made by a weak type of player 1 (see (4.21)). To put it differently, player 1_s has to make an "excessive" demand to convince player 2 that he is strong. Condition 7 states that $\Pi_m - \Pi_o^2 \leq \Pi_h^1$, i.e. the highest acceptable demand for a weak player 2 under his assumption that such a demand comes from a weak player 1 ($X^w = \Pi_m - \Pi_o^2$) is less than the outside option payoff of a strong player 1 (matched with a weak player 2). Therefore a strong player 1 has no incentive to make an offer which is acceptable for a weak player 2, given this players' beliefs that such an offer is always made by the weak player 1.

The perturbed strategy b^ϵ and the resulting induced and consistent beliefs play a crucial role for this equilibrium because they determine the acceptance boundary X^w (see (4.3), (4.5) and (4.6)). If player 2 believes that both types of player 1 tremble with the same probability, X^w would increase and then player 1_s would not have any longer an incentive to make an excessive demand. To summarize, in this equilibrium an agreement will be reached (i.e. cooperation will occur) if player 1 is weak (irrespectively of the type of player 2). Otherwise the offer will be rejected. In this equilibrium there is a possibility that also a weak type of player 2 rejects the offer of a strong player 1.

For the third separating equilibrium we need Condition 10.

CONDITION 10: $p_2(\Pi_m - \Pi_0^2) + (1 - p_2)\Pi_l^1 - \Pi_m + \Pi_h^2 \geq 0$

Notice that Condition 10 is just the opposite of Condition 8 and the lefthand side of Condition 10 is increasing in p_2. For $p_2 = 0$ Condition 10 does not hold; if $p_2 = 1$, it is satisfied.

Proposition 6 *If Conditions 7 and 10 hold, the following strategy- combination* (x^*, R^*) *and the beliefs* $\mu^*(x)$ *are a separating equilibrium:*

$$x^* = (x_w^*, x_s^*) \quad with \quad x_s^* = x_e > \Pi_m - \Pi_l^2 \quad and \quad x_w^* = \Pi_m - \Pi_o^2 \tag{4.22}$$

$$\begin{aligned} R^* &= (R_w^*, R_s^*) \quad with \\ R_w^* &= \begin{cases} 1 & \forall x : \Pi_m - x \geq \Pi_o^2 \\ 0 & otherwise \end{cases} \\ R_s^* &= \begin{cases} 1 & \forall x : \Pi_m - x \geq \Pi_h^2 \\ 0 & otherwise \end{cases} \end{aligned} \tag{4.23}$$

$$\mu^*(x) = \begin{cases} 0 & if\ x = x_e \\ 1 & otherwise \end{cases} \tag{4.24}$$

This separating equilibrium shows the same behavior of a strong player 1 than in the former equilibrium and a different of a weak player 1. A strong player 1_s demands so much that both types of player 2 reject. Player 1_w, however, demands $x_w^* = X^w = \Pi_m - \Pi_o^2$ contrary to the former equilibrium where he demands $x_w^* = X^s = \Pi_m - \Pi_h$, i.e. in this equilibrium a weak player 1 demands more than in the former equilibrium. In this equilibrium also a weak type of player 1 risks a rejection, because a strong player 2 will reject the offered Π_o^2. A weak type gets just his outside option payoff.

For this being an equilibrium the probability p_2 must be such that Condition 10 holds, i.e. the expected payoff of demanding $X^w = \Pi_m - \Pi_o^2$ must be at least as high as the expected payoff of demanding X^s. In this case player 1_w has an incentive to risk a rejection by a strong player 2. If player 1_w demands X^w and if player 1_s demands something else, then X^w is independent of the off-equilibrium beliefs of player 2, i.e. independent of b^ϵ and the resulting beliefs $\mu^\epsilon(x), \epsilon \to 0$. X^w is in this case as low as possible. Because of this low X^w and because $\Pi_h^1 > \Pi_m - \Pi_l^2$, player 1_s has no incentive to demand X^w and his expected payoff is greater if he makes an "excessive" demand. b^ϵ is on the one hand important for the determination of X^s. If $\mu(x)$ is (sufficiently) below 1 for all $x : \Pi_m - \Pi_h^1 < x < \Pi_m - \Pi_o^2$, then player 2_s would accept an offer greater than $\Pi_m - \Pi_h^2$, which in turn would strengthen Condition 10 implying that the parameter section for which (x^*, R^*) of Proposition 6 being an equilibrium would be smaller. If, on the other hand, b^ϵ would be such that $\mu(x)$ is sufficiently below 1, for all $x : \Pi_m - \Pi_o^2 < x < \Pi_m - \Pi_l^2$, R_w^* would not be a best response anymore, which would imply that X^w increases. This in turn would contradict the requirement of consistent beliefs. Hence, for (x^*, R^*) being an equilibrium, the perturbations have to be such that $\mu(x) = 1$ for all $x : \Pi_m - \Pi_o^2 < x < \Pi_m - \Pi_l^2$.

In this equilibrium an agreement (and hence cooperation) will only occur if two weak players meet. Otherwise they will not agree and there will be no cooperation.

A comparison of Proposition 5 and 6 reveals that Condition 7 is crucial for these equilibria. If the outside option payoffs do only depend on the player's own type, Condition 7 cannot hold (see Assumption 1) - and the equilibria of Proposition 5 and 6, i.e. equilibria where player 1_s demands so much that this demand x_e will be rejected for sure, do not exist. In this case only one separating equilibrium exists, where player 1_s risks rejection (by demanding X^w), whereas player 1_w does not (he demands X^s). All separating equilibria do only exist if player 2 has some kind of asymmetric beliefs, i.e. if he believes that player 1_w "trembles" with a higher probability than player 1_s. Thus no uniformly perfect separating equilibria exist.

4.4 The influence of bargaining power

In this simple bargaining model bargaining power amounts to the level of the outside option payoff. Obviously, the higher a players' outside option payoff the higher his bargaining power. In deriving the equilibria, conditions as functions of the prior probabilities p_i were stated which have to be simultaneously satisfied for the existence of an equilibrium in question. The probability intervals which satisfy a certain set of conditions are determined by the levels of the outside options. Different outside option payoffs determine different probability intervals. Therefore Fig. 2 and 3 are just examples but "generic" ones. Bargaining power, i.e. the outside option payoffs, determines only the regions where a certain equilibrium exists. Which equilibrium actually prevails depends on the prior probabilities p_i.

In the following the influence of bargaining power is discussed in a framework where the reservation payoffs do not depend on the type of the opponent (see Example 2). This is a special case of our model which reduces the complexity of the game quite substantially and therefore allows to gain some insights about the influence of bargaining power which are harder to comprehend in the more general case.

If the outside options payoffs are independent only three kinds of equilibria exist: the two pooling ones and the separating equilibrium of Proposition 4. Performing the necessary calculations for the probability intervals of the equilibria shows the following: The pooling equilibrium of Proposition 2 exists if

$$p_2 \leq p_2^* = \frac{\Pi_m - \Pi_h^2 - \Pi_h^1}{\Pi_m - \Pi_l^2 - \Pi_h^1} \tag{4.25}$$

The pooling equilibrium of Proposition 3 exists if

$$p_2 \geq p_2^{**} = \frac{\Pi_m - \Pi_h^2 - \Pi_l^1}{\Pi_m - \Pi_l^2 - \Pi_l^1} \tag{4.26}$$

The separating equilibrium of Proposition 4 exists if p_2 lies between p_2^{**} (the lower bound) and p_2^* (the upper bound). This is summarized by Fig. 4:

Fig. 4

Notice that the probability regions for the equilibria overlap. If the probability that player 2 is weak is below p_2^{**} only the equilibrium described by Proposition 2 exists. Similarly, if the probability is above p_2^* only the equilibrium of Proposition 3 exists. The separating equilibrium lies between p_2^{**} and p_2^* but in this interval also the two pooling equilibria exist. Once again this picture is just an example but it is "generic" in the sense that different levels of outside option payoffs produce different probability regions but qualitatively the same picture. To be more concrete, with (4.25) and (4.26), a higher reservation payoff (i.e. a higher bargaining power) of the strong player 2, Π_h^2, shifts, ceteris paribus, both p_2^* and p_2^{**} downwards, i.e. the region where the equilibrium of Proposition 3 exists becomes larger whereas the region of Proposition 2 shrinks. The opposite happens if the reservation payoff of a weak-type player 2, Π_l^2, becomes higher: both p_2^{**} and p_2^* shift upward.[12] The same happens with higher outside option payoffs

[12]The greater the difference between the reservation payoffs of player 2's types, the higher the

of player 1. However, contrary to the former cases, a changed outside option payoff of one type of player 1 changes the interval where Proposition 4 holds. Notice once again that bargaining power, i.e. the reservation payoffs, determines only the regions where a certain equilibrium exists. Which equilibrium actually prevails depends on the prior probabilities p_i.

5 Summary and Conclusions

In this paper we analyzed how uncertainty (incomplete information) influences the possibility of cooperation and how the cooperation surplus (the gains from trade) is divided if an agreement is reached. The model we used is quite general in the assumptions about the payoffs, i.e. it only defines what "weakness" and "strongness" in terms of outside option payoffs mean (where it is allowed that these payoffs depend on the type of the opponent: independency is a special case) and it assumes that there are always gains from trade. However, the model is quite restrictive concerning the "bargaining" stage which in fact is an ultimatum game – a "take-it-or-leave-it"-offer. In our opinion this does not pose a serious problem because all bargaining models are to some extent artificial and secondly, we are interested only in finite bargaining games and have therefore chosen the most tractable one – the one-stage bargaining. This allows us to exactly characterize the possible bargaining outcomes in terms of bargaining power which influences the likelihood of the occurrence of a certain equilibrium and this is what we intended to do in this paper. Furthermore, in economic applications (see the section on Examples) this stage has suitable interpretations.

Cooperation under uncertainty may result in an overpayment and in an inefficient outcome: an equilibrium may entail the possibility of a rejection despite of gains from trade. This inefficiency result is typical for bargaining under incomplete information (see e.g. **Chatterjee** [1985]). Our approach, however, allowed us to identify and to characterize these outcomes in terms of bargaining power.

possible rent paid to the weak-type player 2. A higher rent can be due to a higher reservation payoff of a strong type or a lower reservation payoff of the weak type. Both reasons shift p_2^* and p_2^{**} downwards.

6 Appendix

Proof of Lemma 1: (i) and (ii) follow directly from (4.8) and (4.9), respectively. The proof of (iii) is as follows: The expected payoff of player 1_w is $\Phi_{1w}(x_w) = p_2\Pi_o^1 + (1-p_2)\Pi_l^1$ if he demands x_w and $\Phi_{1w}(x'_w) = p_2X^w + (1-p_2)\Pi_l^1$ if he demands x'_w. Because of (4.6), $X^w \geq \Pi_m - \Pi_o^2$ and because $\Pi_m - \Pi_o^2 > \Pi_o^1$ (see Assumption 2), (iii) follows. □

Proof of Proposition 1: Part (i) follows straightforwardly from Lemma 1. (ii) $x = ((x_w = X^w), (x_s = X^s))$ being an equilibrium behavior would require:

$$\begin{aligned}
\Phi_{1w}(X^w) &= p_2X^w + (1-p_2)\Pi_l^1 \geq X^s = \Phi_{1w}(X^s) \\
\Phi_{1s}(X^w) &= p_2X^w + (1-p_2)\Pi_c^1 \leq X^s = \Phi_{1s}(X^s)
\end{aligned}$$

Because $\Pi_l^1 < \Pi_c^1$, these conditions lead to a contradiction, which proofs the proposition. □

Proof of Lemma 2: (i) Demands $x_w = X^s$, and $x_e \notin \Omega^w$ imply that the beliefs of player 2 become $\mu(x_e) = 0$. If $x_e \leq \Pi_m - \Pi_l^2$, this would, according to (4.3), imply that $R_w(x_e) = 1$ which further implies that $x_e \in \Omega^w$ (from the definition of Ω^w). But this contradicts the assumption that $x_e \notin \Omega^w$. The proof of (ii) works analogously. □

Proof of Proposition 2: Let $b^\epsilon(x)$ be a completely mixed strategy with

$$b_w^\epsilon(x) = b_s^\epsilon(x) = \begin{cases} 1 - \Pi_m\epsilon & \text{if } x = \Pi_m - p_1\Pi_h^2 - (1-p_1)\Pi_c^2 \\ \epsilon & \text{otherwise} \end{cases}$$

Then

$$\lim_{\epsilon \to 0} b^\epsilon(x) = x^*$$

The perturbed strategies induce consistent beliefs given by

$$\begin{aligned}
\mu^\epsilon(x \neq x_s^* = x_w^*) &= \frac{p_1\epsilon}{p_1\epsilon + (1-p_1)\epsilon} = p_1 \\
\mu^\epsilon(x = x_s^* = x_w^*) &= \frac{p_1(1-\Pi_m\epsilon)}{p_1(1-\Pi_m\epsilon) + (1-p_1)(1-\Pi_m\epsilon)} = p_1
\end{aligned}$$

Therefore consistent beliefs are given by:

$$\lim_{\epsilon \to 0} \mu^\epsilon(x) = \mu(x) = p_1 \quad \forall \ x$$

With these beliefs, the best response of player 2 is given by (4.11) (see (4.3) and (4.5)) and therefore $x^* = X^s$, i.e. both types of player 1 demand exactly X^s. From (4.11), X^w is given by $X^w = \Pi_m - p_1\Pi_o^2 - (1-p_1)\Pi_l^2$ and Lemma 1 tells us that $x^* = X^s$ being a best response of player 1 requires

$$\Phi_{1s}(X^s) \geq \Phi_{1s}(X^w) \tag{6.1}$$
$$\Phi_{1s}(X^s) \geq \Phi_{1s}(x_e) \tag{6.2}$$

Notice that the lefthand side of Condition 2 is equal to $\Phi_{1s}(X^s) - \Phi_{1s}(X^w)$ and the lefthand side of Condition 1 is equal to $\Phi_{1s}(X^s) - \Phi_{1s}(x_e)$. This implies that (6.1) and (6.2) are satisfied. Furthermore, it is required that

$$\Phi_{1w}(X^s) \geq \Phi_{1w}(X^w) \tag{6.3}$$

which is the same as

$$\Pi_m - p_1\Pi_h^2 - (1-p_1)\Pi_c^2 \geq p_2[\Pi_m - p_1\Pi_o^2 - (1-p_1)\Pi_l^2] + (1-p_2)\Pi_l^1$$

Because $\Pi_l^1 < \Pi_c^1$, Condition 2 implies (6.3). □

Proof of Proposition 3 (Sketch): The proof of this proposition is very similar to the proof of Proposition 3 and will therefore only be briefly stated. It can be shown that the beliefs $\mu(x) = p_1$ for all x are consistent. With these beliefs, the best response of player 2 is given by (4.14) (see (4.3) and (4.5)) and therefore $x^* = X^w$, i.e. both types of player 1 demand exactly X^w. From (4.13), X^s is given by $X^s = \Pi_m - p_1\Pi_h^2 - (1-p_1)\Pi_c^2$ and Lemma 1 tells us that it can never be a best response of player 1 to demand $X^s < x < X^w$ or less than X^s. Therefore $x^* = X^w$ being a best response of player 1 requires

$$\Phi_{1w}(X^w) \quad \geq \quad \Phi_{1w}(X^s) \tag{6.4}$$

This is the same as

$$p_2[\Pi_m - p_1\Pi_o^2 - (1-p_1)\Pi_l^2] + (1-p_2)\Pi_l^1 \geq \Pi_m - p_1\Pi_h^2 - (1-p_1)\Pi_c^2$$

Because of Condition 4, (6.4) is satisfied. Furthermore, the profit of a strong player when demanding X^w has to be at least as high as by demanding X^s, i.e.

$$\Phi_{1s}(X^w) \geq \Phi_{1s}(X^s)$$

which is

$$p_2[\Pi_m - p_1\Pi_o^2 - (1-p_1)\Pi_l^2] + (1-p_2)\Pi_c^1 \geq \Pi_m - p_1\Pi_h^2 - (1-p_1)\Pi_c^2$$

Because of (6.4) and $\Pi_c^2 > \Pi_l^2$, this is valid. For a best response it is also required that

$$\Phi_{1s}(X^w) \geq \Phi_{1s}(x_e)$$

or, equivalently

$$p_2[\Pi_m - p_1\Pi_o^2 - (1-p_1)\Pi_l^2] + (1-p_2)\Pi_c^1 \geq p_2\Pi_h^1 + (1-p_2)\Pi_c^1$$

This is just Condition 3. □

Proof of Proposition 4: The proof of this separating equilibrium is typical for all others and will therefore be stated in full length. Let $b^\epsilon(x)$ be a completely mixed strategy- combination:

$$b_w^\epsilon(x) \quad = \quad \begin{cases} 1 - \Pi_m\epsilon & \text{if } x = \Pi_m - \Pi_h^2 \\ \epsilon & \text{otherwise} \end{cases}$$

$$b_s^\epsilon(x) \quad = \quad \begin{cases} 1 - \Pi_m\epsilon^2 & \text{if } x = \Pi_m - \Pi_l^2 \\ \epsilon^2 & \text{otherwise} \end{cases}$$

Notice that

$$\lim_{\epsilon \to 0} b^\epsilon(x) = x^*$$

The perturbated strategies induce the following consistent beliefs:

$$\begin{aligned}
\mu^{\epsilon}(x \neq x_s^* \neq x_w^*) &= \frac{p_1\epsilon}{p_1\epsilon + (1-p_1)\epsilon^2} = \frac{p_1}{p_1 + (1-p_1)\epsilon} \longrightarrow_{\epsilon\to 0} 1 \\
\mu^{\epsilon}(x = x_s^* \neq x_w^*) &= \frac{p_1\epsilon}{p_1\epsilon + (1-p_1)(1-\Pi_m\epsilon^2)} \longrightarrow_{\epsilon\to 0} 0 \\
\mu^{\epsilon}(x = x_w^* \neq x_s^*) &= \frac{p_1(1-\Pi_m\epsilon)}{p_1(1-\Pi_m\epsilon) + (1-p_1)\epsilon^2)} \longrightarrow_{\epsilon\to 0} 1
\end{aligned}$$

Therefore consistent beliefs are given by

$$\lim_{\epsilon\to 0}\mu^{\epsilon}(x) = \mu^*(x) = \begin{cases} 0 & \text{if } x = \Pi_m - \Pi_l^2 \\ 1 & \text{otherwise} \end{cases}$$

Now we have to show that R^* is a best response of player 2 given his beliefs $\mu^*(x)$ (see (4.18). According to (4.2) and (4.4) we calculate the expected payoff of player 2 if he rejects, i.e. $\Phi_{2t}(R_t(x) = 0)$, $t = w, s$ and we then check whether this expected payoff is greater than the expected payoff if player 2 accepts the offer $\Pi_m - x$, i.e., for finding the optimal best response of player 2 we use (4.3) and (4.5) respectively. We first analyze the case where player 2 is a strong type:

$x : x \leq \Pi_m - \Pi_h^2 \Longrightarrow \mu(x) = 1 \Longrightarrow \Phi_{2s}(R_s(x) = 0) = \Pi_h^2$. Since $\Pi_h^2 \leq \Pi_m - x \Longrightarrow R_s^*(x) = 1$.

$x : \Pi_m - \Pi_h^2 < x < \Pi_m - \Pi_l^2 \Longrightarrow \mu(x) = 1 \Longrightarrow \Phi_{2s}(R_s(x) = 0) = \Pi_h^2$. Since $\Pi_h^2 > \Pi_m - x \Longrightarrow R_s^*(x) = 0$.

$x : x = \Pi_m - \Pi_l^2 \Longrightarrow \mu(x) = 0 \Longrightarrow \Phi_{2s}(R_s(x) = 0) = \Pi_c^2$. Since $\Pi_c^2 > \Pi_m - x \Longrightarrow R_s^*(x) = 0$.

$x : x > \Pi_m - \Pi_l^2 \Longrightarrow \mu(x) = 1 \Longrightarrow \Phi_{2s}(R_s(x) = 0) = \Pi_h^2$. Since $\Pi_h^2 > x \Longrightarrow R_s^*(x) = 0$.

The analysis of these cases shows that the stated R_s^* is indeed a best response. The next step is to analyze the best response behavior of a weak player 2_w:

$x : x \leq \Pi_m - \Pi_o^2 \Longrightarrow \mu(x) = 1 \Longrightarrow \Phi_{2w}(R_w(x) = 0) = \Pi_o^2$. Since $\Pi_o^2 \leq \Pi_m - x \Longrightarrow R_w^*(x) = 1$.

$x : \Pi_m - \Pi_o^2 < x < \Pi_m - \Pi_l^2 \Longrightarrow \mu(x) = 1 \Longrightarrow \Phi_{2w}(R_w(x) = 0) = \Pi_o^2$. Since $\Pi_o^2 > \Pi_m - x \Longrightarrow R_w^*(x) = 0$.

$x : x = \Pi_m - \Pi_l^2 \Longrightarrow \mu(x) = 0 \Longrightarrow \Phi_{2w}(R_w(x) = 0) = \Pi_l^2$. Since $\Pi_l^2 \leq \Pi_m - x \Longrightarrow R_w^*(x) = 1$.

$x : x > \Pi_m - \Pi_l^2 \Longrightarrow \mu(x) = 1 \Longrightarrow \Phi_{2w}(R_w(x) = 0) = \Pi_o^2$. Since $\Pi_o^2 \geq \Pi_c^2 > \Pi_m - x \Longrightarrow R_w^*(x) = 0$.

Therefore, the stated R_w^* is indeed a best response of player 2_w.

Proposition 4 claims that $x_s^* = X^w$ and $x_w^* = X^s$, i.e. in this separating equilibrium a strong player 1 demands X^w and a weak player 1 demands X^s. We now have to show that this is a best response of player 1. The asserted behavior requires for player 1_w that

$$\Phi_{1w}(X^s) \geq \Phi_{1w}(X^w) \Longleftrightarrow \Pi_m - \Pi_h^2 - p_2(\Pi_m - \Pi_l^2) - (1-p_2)\Pi_l^1 \geq 0$$

Because of Condition 5, this is satisfied.

For player 1_s optimal behavior requires

$$\Phi_{1s}(X^w) \geq \Phi_{1s}(X^s) \Longleftrightarrow p_2(\Pi_m - \Pi_l^2) + (1-p_2)\Pi_c^1 - \Pi_m + \Pi_h^2 \geq 0$$

Because of Condition 6, this is satisfied.

A third condition for a best response in this equilibrium is that the expected payoff of a strong player 1 when demanding X_w is at least as high as when he demands x_e

$$\Phi_{1s}(X^w) \geq \Phi_{1s}(x_e)$$

or equivalently

$$p_2(\Pi_m - \Pi_l^2) + (1-p_2)\Pi_c^1 - p_2\Pi_h^1 - (1-p_2)\Pi_c^1 \geq 0 \Longleftrightarrow \Pi_m - \Pi_l^2 \geq \Pi_h^1$$

This is true by assumption. □

Proof of Proposition 5 (Sketch): This proof is very similar to the proof of Proposition 4 and is therefore only briefly stated. Similar arguments as in the proof of Proposition 4 show that beliefs given by

$$\mu^*(x) = \begin{cases} 0 & \text{if } x = x_e \\ 1 & \text{otherwise} \end{cases}$$

are indeed consistent. Furthermore, similarly to above it can be shown that R^* is a best response of player 2, given his beliefs $\mu(x)$ as in (4.21). According to Proposition 5 we consider the case where a strong type of player 1 demands $x_s = x_e$ and a weak type $x_w = X^s$. These demands being best responses require

$$\Phi_{1w}(X^s) \geq \Phi_{1w}(X^w) \Longleftrightarrow \Pi_m - \Pi_h^2 - p_2(\Pi_m - \Pi_o^2) - (1-p_2)\Pi_l^1 \geq 0$$

Because of Condition 8 this is satisfied. The optimality of the "excessive" demand x_e of player 1_s requires

$$\Phi_{1s}(x_e) \geq \Phi_{1s}(X^w) \Longleftrightarrow p_2\Pi_h^1 + (1-p_2)\Pi_c^1 \geq p_2(\Pi_m - \Pi_o^2) + (1-p_2)\Pi_c^1$$

Condition 7 implies that this is satisfied. The optimality of demand x_e further requires

$$\Phi_{1s}(x_e) \geq \Phi_{1s}(X^s) \Longleftrightarrow p_2\Pi_h^1 + (1-p_2)\Pi_c^1 \geq \Pi_m - \Pi_h^2$$

Because of Condition 9 this is satisfied. This analysis of the best response behavior of player 1 completes the proof. □

Proof of Proposition 6 (Sketch): The proof is very similar to the proof of Proposition 4 and is therefore only briefly stated. Similar arguments as in the proof of Proposition 4 show that beliefs given by

$$\mu^*(x) = \begin{cases} 0 & \text{if } x = x_e \\ 1 & \text{otherwise} \end{cases}$$

are indeed consistent. Similarly to above one can proof that the stated R^* is a best response to $\mu^*(x)$. What remains to be shown is that x^* is a best response to R^*. In (4.22) we consider a case where 1_s demands $x_s^* = x_e > X^w$ and 1_w demands $x_w^* = X^w$. These demands being best responses require

$$\Phi_{1s}(x_e) \geq \Phi_{1s}(X^w) \Longleftrightarrow$$

$$p_2\Pi_h^1 + (1-p_2)\Pi_c^1 - p_2(\Pi_m - \Pi_o^2) - (1-p_2)\Pi_c^1 \geq 0$$

Because of Condition 7 this is satisfied. Furthermore, for a best response it is required that

$$\Phi_{1w}(X^w) \geq \Phi_{1w}(X^s) \Longleftrightarrow p_2(\Pi_m - \Pi_o^2) + (1-p_2)\Pi_l^1 - \Pi_m + \Pi_h^2 \geq 0 \tag{6.5}$$

Because of Condition 10 this is also satisfied. The last requirement of the claimed strategy being a best response is

$$\Phi_{1s}(x_e) \geq \Phi_{1s}(X^s) \Longleftrightarrow p_2\Pi_h^1 + (1-p_2)\Pi_c^1 - \Pi_m + \Pi_h^2 \geq 0 \tag{6.6}$$

Because $\Pi_c^1 > \Pi_l^1$, and because $\Pi_h^1 \geq \Pi_m - \Pi_o^2$ (Condition 7), (6.5) implies (6.6). This proves that the asserted strategies are indeed best responses and hence Proposition 6. □

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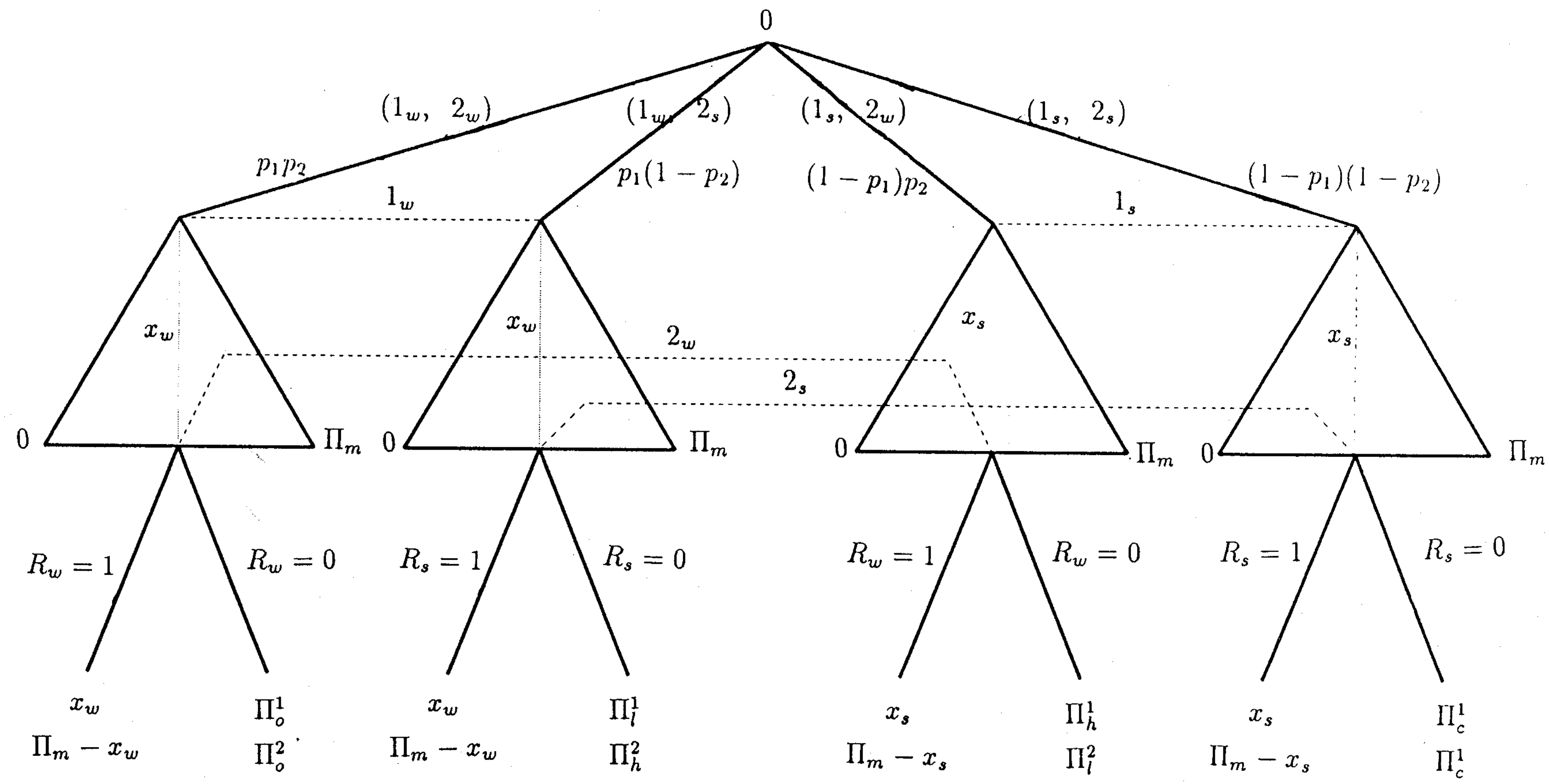
0
$(1_w,\ 2_w)$
$(1_w,\ 2_s)$
$(1_s,\ 2_w)$
$(1_s,\ 2_s)$
$p_1 p_2$
$p_1(1-p_2)$
$(1-p_1)p_2$
$(1-p_1)(1-p_2)$
1_w
1_s
x_w
x_w
x_s
x_s
2_w
2_s
0
Π_m
0
Π_m
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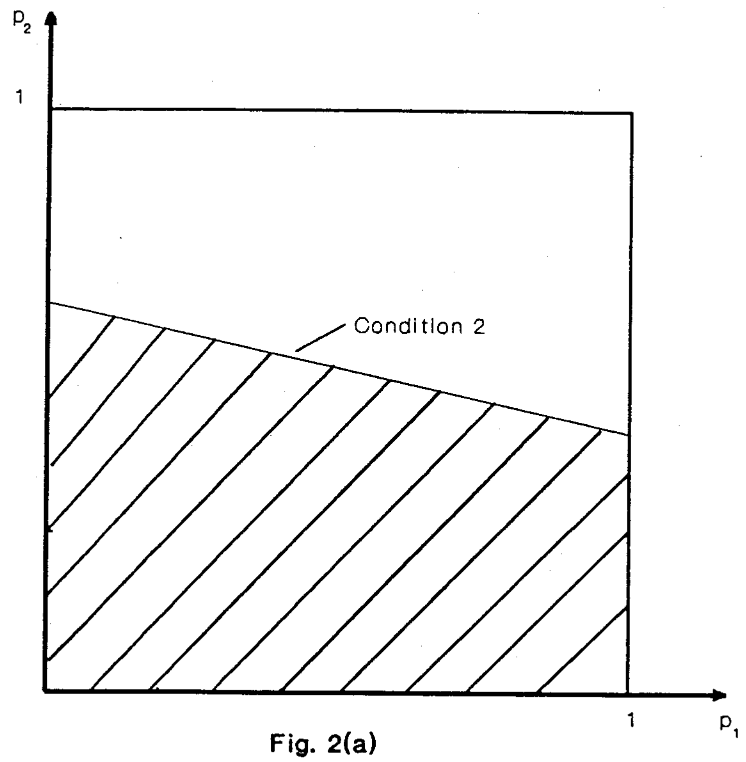
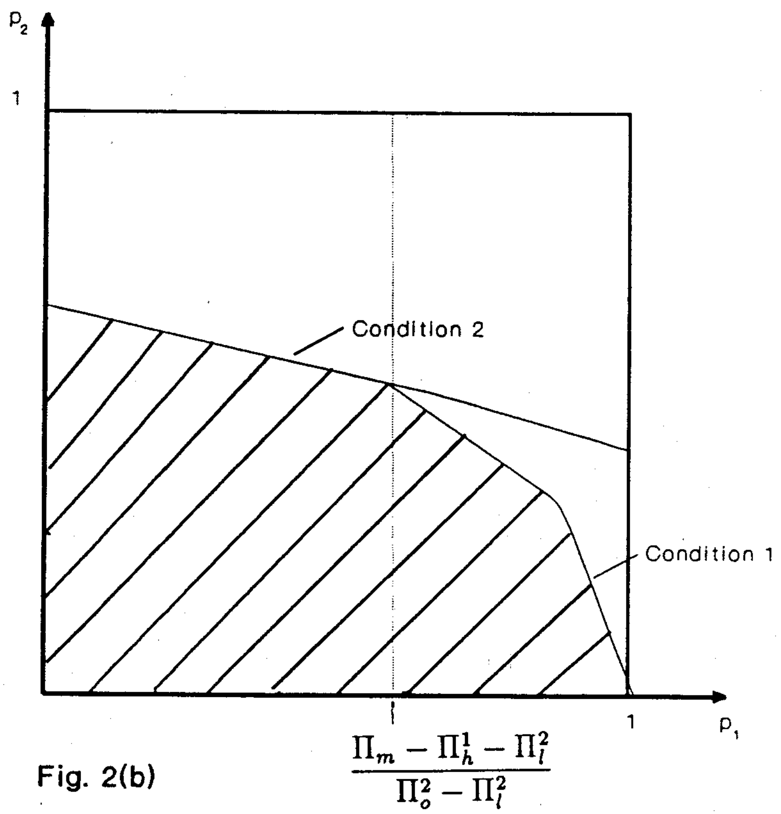
$R_s = 0$
x_w
$\Pi_m - x_w$
Π_o^1
Π_o^2
x_w
$\Pi_m - x_w$
Π_l^1
Π_h^2
x_s
$\Pi_m - x_s$
Π_h^1
Π_l^2
x_s
$\Pi_m - x_s$
Π_c^1
Π_c^1

Fig. 1



p_2
1
Condition 2
1
p_1

Fig. 2(a)



p_2
1
Condition 2
Condition 1
1
p_1
$\frac{\Pi_m - \Pi_h^1 - \Pi_l^2}{\Pi_o^2 - \Pi_l^2}$

Fig. 2(b)

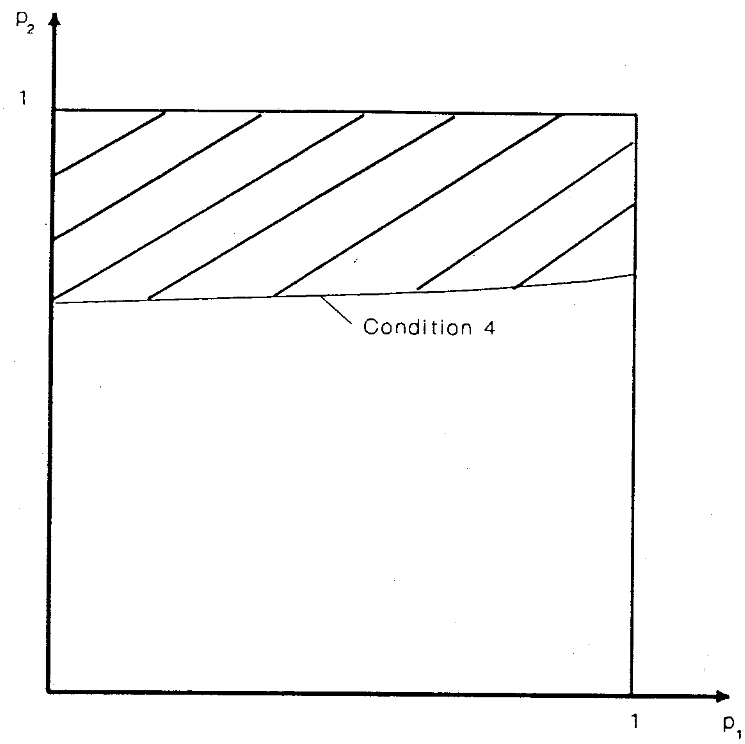


Fig. 3(a)

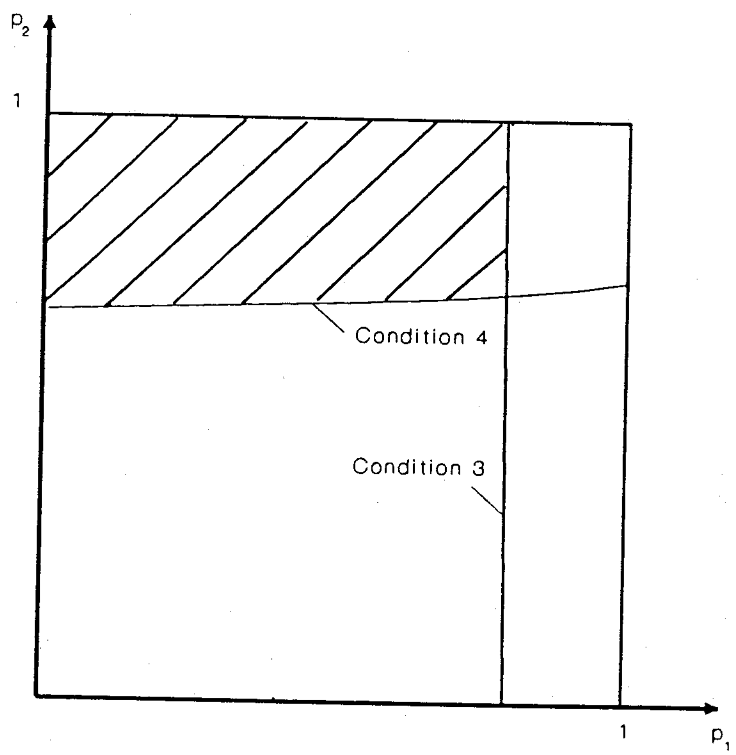
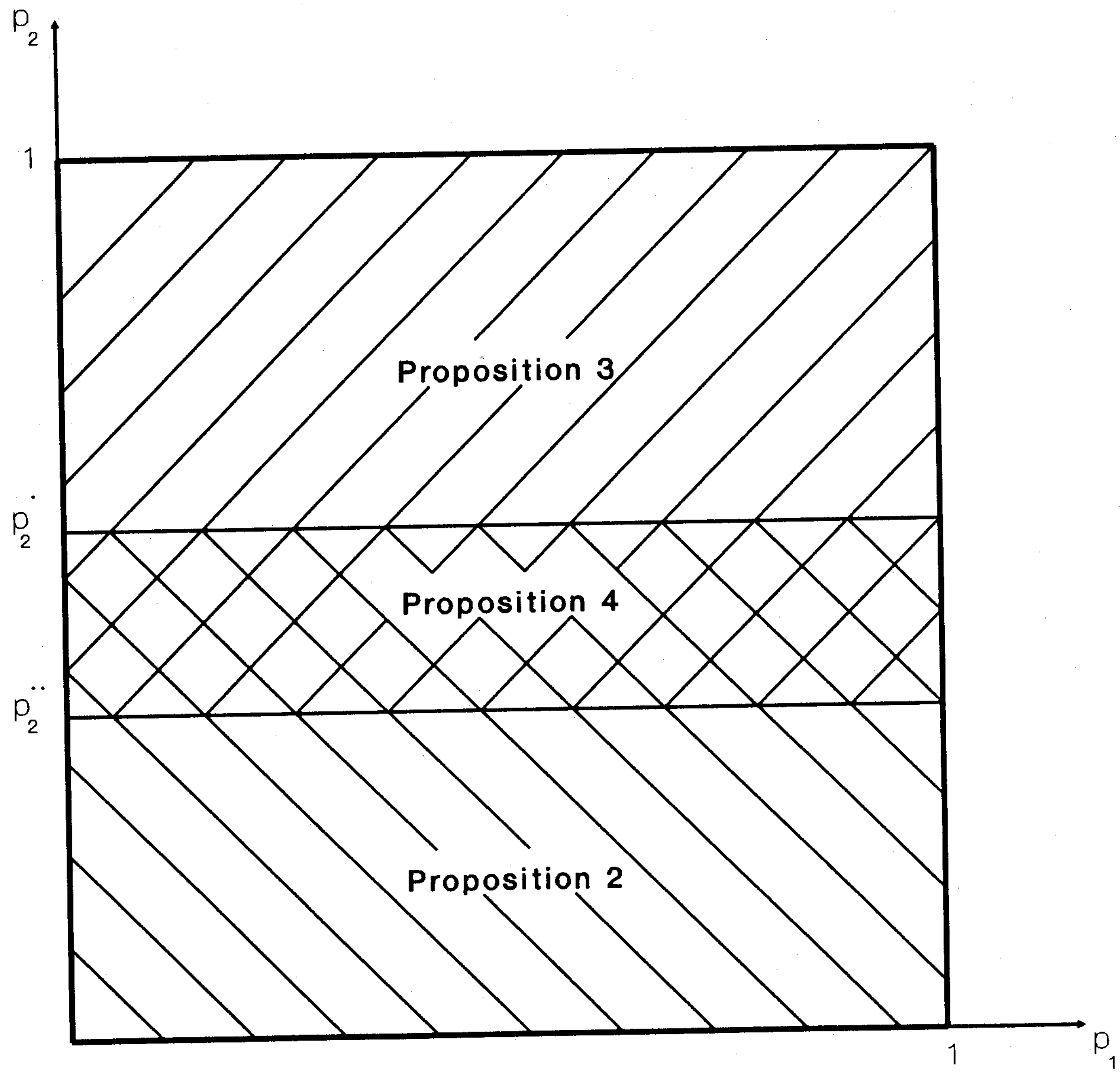


Fig. 3(b)



p_2
1
$\dot{p}_2$
$\ddot{p}_2$
Proposition 3
Proposition 4
Proposition 2
1
p_1

Fig. 4