Modelling Intertemporal General Equilibrium: Appendix

by

Christian KEUSCHNIGG*

Wilhelm KOHLER*

Forschungsbericht /

Research Memorandum No. 304

July 1992

Institute for Advanced Studies
Department of Economics
Stumpergasse 56
A–1060 Vienna, Austria

Phone: (1) 59991–147
Fax: (1) 59991–163
E–mail: chkeu@ihssv.wsr.ac.at

University of Innsbruck
Department of Economics
Adolf Pichler Platz 6
A–6020 Innsbruck, Austria

Phone: (512) 507–2757
E–mail: Wilhelm.Kohler@uibk.ac.at

*We gratefully acknowledge financial support by the Austrian National Bank under grant no. 3327. Keuschnigg is also grateful for financial support received from Deutsche Forschungsgemeinschaft, Sonderforschungsbereich 303 at the University of Bonn.

Abstract

This extended appendix refers to our paper "Modelling Intertemporal General Equilibrium: An Application to Austrian Commercial Policy". We present a detailed analysis of the household sector consisting of overlapping generations with lifetime uncertainty. Furthermore, we prove Walras' Law and discuss the computation of stationary solutions of the computable equilibrium model. We report on the adjustments to the raw data of the Austrian economy which are necessary to yield a microconsistent data base suitable for calibration of an intertemporal equilibrium model. This appendix also includes additional calibration results and documents data as well as parameter values not reported in the main text. Finally, we provide a list of variable definitions to facilitate the reading of the model.

Zusammenfassung

Dieser Anhang bezieht sich auf unser Papier "Modelling Intertemporal General Equilibrium: An Application to Austrian Commercial Policy". Es wird im Detail der Haushaltssektor dargestellt, welcher aus überlappenden Generationen mit Lebensunsicherheit besteht. Außerdem beweisen wir Walras' Gesetz und erörtern die Berechnung von stationären Lösungen des numerischen Gleichgewichtsmodells. Wir schildern die Anpassungen in den Rohdaten der Österreichischen Wirtschaft, welche notwendig sind, um eine mikrokonsistente Datenbasis zu erhalten, auf die das intertemporale Gleichgewichtsmodell kalibriert werden kann. Der Appendix gibt zusätzliche Kalibrierungsergebnisse wider und dokumentiert Daten und Parameterwerte, welche im Haupttext nicht enthalten sind. Schließlich erstellen wir eine vollständige Variablenliste, um die Nachvollziehbarkeit des Modells zu erleichtern.

Contents

1 Introduction

This extended appendix refers to our paper "Modelling Intertemporal General Equilibrium: An Application to Austrian Commercial Policy". Section 2 presents, in greater detail than the paper itself, the overlapping generations structure of the model. Section 3 demostrates Walras' Law for the full model and proceeds in discussing the computation of stationary solutions. Section 4 contains additonal parameter values and data which were not reported in the main text. To facilitate the reading of the model, section 5 offers an exhaustive listing of variable definitions.

2 Overlapping Generations With Lifetime Uncertainty

The household sector consists of overlapping generations with lifetime uncertainty. The basic analytical model was pioneered by Blanchard (1985) who assumed identical birth and death rates and, therefore, a constant population. Frenkel and Razin (1987) developed a version in discrete time. Weil (1989) extended the model to the case of a zero death rate. In Weil's model, new infinitely lived families enter the economy at each instant but they are disconnected to existing generations. Buiter (1988) synthesized the two types of overlapping generations models. Our model follows Buiter (1988) except that it is in discrete time and additionally features endogenous labor supply.

2.1 Survival Probabilities and Demographics

We will consistently implement the notion of an end of period equilibrium as discussed by Turnovsky (1977) and Buiter (1980). Given wealth (or the values of stocks) at the beginning of the period, an agent plans the actions (flows) during that period to build up the desired stocks at the end of the period. This is also relevant for dating our demographic variables. Individuals face uncertain lifetimes, and their probabilities of dying or surviving are modelled according to table 1 which compares, for convenience, the discrete and continuous time cases. We assume that an agent borne at the end of period $t-1$ chooses a consumption flow $c_{t-1,t}$ or some other action during period t (or between points in time $t-1$ and t) and faces a probability θ of dying thereafter. Consequently, agents survive with probability $1-\theta$ to the next period.

We additionally assume that age cohorts are populated by a large number of identical individuals with independent risks of life. By the law of large numbers, the constant individual probability of dying θ is identical to the fráction of the cohort which dies at each date. Hence, total population evolves deterministically over time according to

$$N_t = (n+\theta)N_{t-1} + (1-\theta)N_{t-1} = (1+n)N_{t-1}. \tag{1}$$

At date $t-1$, a fraction $(n+\theta)$ of the existing population N_{t-1} is born with $(n+\theta)$ denoting the gross birth rate. Of the existing population, a fraction θ dies at the end of the period and a fraction

Table 1: Individual Probabilities

	discrete	continuous	interpretation
$f(a)$	$F(a) - F(a-1)$ $= \theta(1-\theta)^{a-1}$	$F'(a)$ $= \theta e^{-\theta a}$	probability of dying at age a
$F(a)$	$\sum_{s=1}^{a} f(s)$ $= 1 - (1-\theta)^a$	$\int_0^a f(s)ds$ $= 1 - e^{-\theta a}$	probability of dying at age a or younger
$1 - F(a)$	$(1-\theta)^a$	$e^{-\theta a}$	probability of dying at an age older than a
θ	$\frac{f(a)}{1-F(a-1)}$	$\frac{f(a)}{1-F(a)}$	instantaneous (conditional) probability of dying at age a
$1/\theta$	$\sum_{a=1}^{\infty} af(a)$	$\int_0^{\infty} af(a)da$	expected lifetime

In the discrete-time case we define $f(0) = 0$ as each agent lives at least one period. It is easily checked that $F(0) = 0$ and $F(\infty) = 1$.

$(1-\theta)$ survives until the next period. Hence, the net growth rate of the population is n. Repeatedly substituting out the term $(1-\theta)N_{t-1}$ gives the age distribution of the population at date t,

$$N_t = \sum_{a=1}^{\infty}(n+\theta)N_{t-a}(1-\theta)^{a-1}. \tag{2}$$

The term $(n+\theta)N_{t-a}$ indicates the birth weight of age cohort a born at date $t-a$. Of this cohort, a fraction $(1-\theta)^{a-1}$ survives until date t. Hence, (2) sums over the sizes, as of the end of period t, of all age cohorts born at the end of period $t-1$ or earlier. As a check for consistency, substitute $N_t = (1+n)^a N_{t-a}$ to obtain (2) as an identity. This gives the population weights of the different age cohorts at date t which are time autonomous,

$$\omega_a = \left(\frac{n+\theta}{1+n}\right)\left(\frac{1-\theta}{1+n}\right)^{a-1} = \omega_1\left(\frac{1-\theta}{1+n}\right)^{a-1}, \quad \sum_{a=1}^{\infty}\omega_a = 1. \tag{3}$$

Quite intuitively, the weight of young cohorts in the population increases with the net birth rate n. The above considerations on the demographics of such an OLG economy give us a simple rule for obtaining aggregate figures from cohort-specific variables. We have to think of any aggregate variable as the weighted sum of the corresponding age-specific variables for all cohorts born at the beginning of period $t-1$ or earlier, using ω_a as weights.

2.2 Individual Optimization

The model allows for two exogenous trend components: productivity increase and population growth. We consider the detrended economy. Due to the OLG structure of the model, however, one cannot simultaneously detrend all variables from both growth factors in a single step. Instead, we first express all variables as per efficiency unit (which increases at a rate x) when dealing with individual optimization within a cohort. Subsequently we take care of population growth when dealing with aggregation.

Agents are assumed to maximize expected utility of their lifetime consumption subject to an accumulation constraint for financial wealth. Expected utility, as of time t, of an individual belonging to cohort a is

$$EU_{t-a} = \sum_{T=t}^{\infty} \theta(1-\theta)^{T-t} \times \left[\sum_{s=t}^{T} (1+\rho)^{-(s-t)} u(X_s C_{t-a,s}, 1 - L^s_{t-a,s}) \right]. \tag{4}$$

The square bracket gives lifetime utility if life lasts T periods. The first multiplicative term gives the probability of that event. The sum of these products is expected lifetime utility. Intraperiod felicity is a function of consumption and leisure. In a steady state, individual consumption grows at the rate of wage increasing technological progress. Hence, one may think of consumption XC as being the product of a trend component X and a stationary component C which will, henceforth, be called 'consumption per efficiency unit.' Denoting labor supply by L^s, leisure is simply the time not spent at work, $1-L^s$, out of a unitary time endowment. The indices indicate that $C_{t-a,s}$, for example, is consumption per efficiency unit at date s of an individual born at date[1] $t-a$.

The requirement of a well-behaved stationary household equilibrium in the presence of constant wage growth implies restrictions for the felicity function. Specifically, the income and substitution effects of wage increases must exactly cancel to give a constant ratio of consumption to leisure as required in a steady state. At the same time, we want felicity to be homothetic to apply the principles of multistage budgeting, $u[v(XC, 1-L^s)]$. If this preference structure is to be compatible with the steady state restrictions, $u[.]$ must be strictly concave and isoelastic while $v(.)$ must be linearly homogeneous and display unitary elasticity of substitution. Hence, $v(.)$ must be Cobb-Douglas with constant expenditure shares. Denote the wage increasing productivity trend by $X_s = (1+x)^{s-t} X_t$ with the growth rate being a constant x. Given that $v(.)$ is Cobb-Douglas with an expenditure share α of consumption, and $u[.]$ is isoealstic, current felicity is

$$u[v(X_s C_{.s}, 1-L^s_s)] = \frac{[(X_s C_{.s})^{\alpha}(1-L^s_s)^{1-\alpha}]^{1-1/\gamma}}{1-1/\gamma} = X_t^{\alpha(1-1/\gamma)}[(1+x)^{\alpha(1-1/\gamma)}]^{s-t} u[v(C_{.s}, 1-L^s_s)]$$

where γ denotes the intertemporal elasticity of substitution in consumption. Now define the discount factor $\beta \equiv (1+x)^{\alpha(1-1/\gamma)}(1+\rho)^{-1}$. Furthermore, since $X_t^{\alpha(1-1/\gamma)}$ is a constant it may be taken out of the summation and ignored in the subsequent optimization exercise. Thus, we can write

$$\begin{aligned} EU_{t-a} &= \sum_{T=t}^{\infty} \theta(1-\theta)^{T-t} [\sum_{s=t}^{T} \beta^{s-t} u(v_{t-a,s})] \\ &= \theta \quad [u_t] \\ &+ \theta(1-\theta) \quad [u_t + \beta u_{t+1}] \\ &+ \theta(1-\theta)^2 \quad [u_t + \beta u_{t+1} + \beta^2 u_{t+2}] \\ &+ \theta(1-\theta)^3 \quad [u_t + \beta u_{t+1} + \beta^2 u_{t+2} + \beta^3 u_{t+3}] + \dots \end{aligned}$$

[1] Whenever there is no confusion, time indices are ignored. When it is not important to distinguish between different generations, we write $C_{t-a,s} = C_{.s}$ for short.

Summing the diagonals, expected utility is conveniently written as

$$EU_{t-a} = \sum_{s=t}^{\infty}[(1-\theta)\beta]^{s-t}u(v_{t-a,s}), \qquad \beta \equiv (1+x)^{\alpha(1-1/\gamma)}(1+\rho)^{-1}. \tag{5}$$

For a complete specification of the intertemporal optimization problem of each cohort, we need to characterize the accumulation of financial wealth. If an individual existing at date $t-1$ survives until date t, its financial wealth $A_{.t}$, measured per efficiency unit and in real terms, is

$$A_{.t} = (1+r_t+\pi_t)\frac{A_{.t-1}}{1+x} + \frac{y_{.t} - p_t^v v_{.t}}{\bar{p}_t}.$$

In the main text, $v = v(C, 1-L^s)$ may be viewed as an aggregate 'commodity' defined over leisure and a consumer good aggregate. Due to linear homogeneity, the least cost budget to obtain subutility v is $p^v v = p^c C + w(1-L^s)$ with exact price indices p^v and p^c for full consumption v and a commodity bundle C. Hence, the savings out of non-capital income is full disposable wage income $y_{.t}$ less expenditures for full consumption where $y_{.t}$ consists of wage income from total time endowment plus other components of non-capital income such as transfers. The real value of assets $A_{.t}$ is expressed in units of a commodity basket which is available at a price $\bar{p}_t$. The existing stock of assets earns a real interest r_t and an insurance premium π_t. The premium is paid by an insurance industry in exchange for the promise to leave all financial assets to the insurer in case of death[2]. Furthermore, savings out of non-capital income adds to financial wealth in case of survival. The insurers receive the assets including the interest of those who die, hence their revenues are $R_t = \theta(1+r_t)A_{.t-1}$. A fraction $(1-\theta)$ of the cohort survives and must be paid a premium π_t causing expenditure $E_t = \pi_t(1-\theta)A_{.t-1}$. Assuming perfect competition in the insurance industry, the fair premium is computed from the zero profit condition $R_t = E_t$ and is equal to $\pi_t = (1+r_t)\theta/(1-\theta)$. In case of survival, real financial wealth of the individual is

$$A_{.t} = \frac{1+r_t}{1-\theta}\frac{A_{.t-1}}{1+x} + \frac{y_{.t} - p_t^v v_{.t}}{\bar{p}_t}. \tag{6}$$

Agents face an effective interest factor $(1+r_t)/(1-\theta)$ which includes the risk premium π_t. The maximization of expected utility subject to the accumulation constraint and initial and terminal conditions $A_{.t-1} = A^0$ and $\lim_{T\to\infty} A_{.T} \geq 0$ is solved via the Lagrangean approach:

$$\mathcal{L}_t = \sum_{s=t}^{\infty}[(1-\theta)\beta]^{s-t}\left\{u(v_{.s}) + \mu_{.s}\left[\left(\frac{1+r_s}{1-\theta}\right)\left(\frac{A_{.s-1}}{1+x}\right) + \frac{y_{.s} - p_s^v v_{.s}}{\bar{p}_s} - A_{.s}\right]\right\}. \tag{7}$$

We have introduced the current value multiplier μ which is the shadow value of financial wealth. The first order necessary conditions for an (interior) optimum are

[2] If the consumer is in debt, he pays a premium to the insurer against the promise to bear all liabilities in case of death. This type of reverse life insurance provides the link between individual lifetime uncertainty and deterministic aggregate behavior. As pointed out by Blanchard and Fischer (1989), demand for such an insurance is motivated by the fact that an individual accumulating wealth would otherwise face the probability of dying without ever being able to use it.

$$
\begin{array}{llll}
(a) & u'(v_{.s}) & = & \mu_{.s}\frac{p^v_s}{\bar{p}_s}, \\
(b) & \mu_{.s} & = & \beta\mu_{.s+1}\left(\frac{1+r_{s+1}}{1+x}\right), \\
(c) & A_{.s} & = & \left(\frac{1+r_s}{1-\theta}\right)\frac{A_{.s-1}}{1+x}+\frac{y_{.s}-p^v_s v_{.s}}{\bar{p}_s}, \\
(d) & \lim_{T\to\infty} & & [(1-\theta)\beta]^{T-t}\mu_{.T}A_{.T}=0.
\end{array} \tag{8}
$$

Combining conditions (a) and (b) gives the Euler equation which shows how full consumption is allocated across periods to derive maximum utility,

$$
u'(v_{.s}) = \beta u'(v_{.s+1})\left[\left(\frac{1+r_{s+1}}{1+x}\right)\frac{p^v_s/\bar{p}_s}{p^v_{s+1}/\bar{p}_{s+1}}\right]. \tag{9}
$$

The left hand side gives the cost in terms of utility of foregoing one consumption unit in period s. Given the real interest rate on savings and price changes, the consumer can buy a quantity of consumption equal to the square bracket in period $s+1$ which yields a marginal utility gain $u'(v_{.s+1})$ per unit. Hence, the Euler equation compares the marginal utility cost with the discounted utility gain from transferring a unit of consumption from this to the next period. Note that it does not depend on the risk term θ. Repeated application of the Euler equation shows how the consumer trades off the gains and losses from shifting consumption between any arbitrary two periods.

The next steps show how full consumption in any period depends on future lifetime income. Applying repetitively condition (c) yields

$$
\frac{A_{.t-1}}{1+x}\left(\frac{1+r_t}{1-\theta}\right) = \sum_{s=t}^{T}\frac{(p^v_s v_{.s}-y_{.s})}{\bar{p}_s}R_{t+1,s}+A_{.T}R_{t+1,T}, \quad R_{t,s}=\prod_{u=t}^{s}\frac{(1+x)(1-\theta)}{(1+r_u)}, \quad R_{t,t-1}\equiv 1. \tag{10}
$$

Similarly condition (b) gives $\mu_{.t} = [(1-\theta)\beta]^{T-t}R^{-1}_{t+1,T}\mu_{.T}$. Using this in the transversality condition (d), one obtains $\mu_{.t}\lim_{T\to\infty}A_{.T}R_{t+1,T} = 0$. Since for an interior solution the current shadow value of financial wealth $\mu_{.t}$ must be positive and finite [see (8a)], the discounted value of terminal financial wealth in (10) must vanish. Hence, the transversality condition restricts the consumer to satisfy the intertemporal budget constraint in real terms,

$$
\sum_{s=t}^{\infty}\frac{p^v_s}{\bar{p}_s}v_{.s}R_{t+1,s} = \mathcal{W}_{.t}, \qquad \mathcal{W}_{.t}\equiv\frac{1+r_t}{(1-\theta)(1+x)}[A_{.t-1}+H_{.t-1}], \qquad H_{.t}\equiv\sum_{s=t+1}^{\infty}\frac{y_{.s}}{\bar{p}_s}R_{t+1,s}. \tag{11}
$$

The present value of future spending on full consumption must not exceed total lifetime wealth which includes financial wealth plus expected human wealth $H_{.t}$. Note that real human wealth is defined ex current wage income. A closed form solution for current consumption is obtained by using the isoelastic functional form for felicity. Hence, the Euler equation relates full consumption in different periods by

$$
v_{.s} = \left\{\left(\frac{p^v_t/\bar{p}_t}{p^v_s/\bar{p}_s}\right)[(1-\theta)\beta]^{s-t}R^{-1}_{t+1,s}\right\}^{\gamma} v_{.t}. \tag{12}
$$

After substituting out future consumption in the intertemporal constraint, one can solve for present real expenditures[3]

$$\frac{p_t^v}{\bar{p}_t}v_{.t} = \Omega_t^{-1}\mathcal{W}_{.t}, \qquad \Omega_t \equiv \sum_{s=t}^{\infty}[(1-\theta)\beta]^{\gamma(s-t)}\left[\frac{p_s^v/\bar{p}_s}{p_t^v/\bar{p}_t}R_{t+1,s}\right]^{1-\gamma}. \tag{13}$$

The "consumption function" relates expenditures on present full consumption to lifetime wealth $\mathcal{W}_{.t}$. The factor Ω_t^{-1} is the marginal propensity to consume out of total wealth and depends on the price of present relative to future consumption. As a check for consistency, evaluate Ω for the logarithmic utility function $u(v) = \ln(v)$ with $\gamma = 1$. One obtains from (13) $\Omega^{-1} = [1-(1-\theta)\beta]$. Consumption is a constant fraction of lifetime wealth.

2.3 Aggregation

While individuals face a risk of life, the aggregate economy evolves deterministically due to the law of large numbers. Having derived individual decisions in the last section we now proceed with the aggregate economy. The following formula sums over all age cohorts to derive aggregate quantities and applies to consumption as well as other variables,

$$N_t v_t = \sum_{a=1}^{\infty}\left[(n+\theta)N_{t-a}(1-\theta)^{a-1}\right] \times v_{t-a,t}.$$

Note that the age specific variable $v_{t-a,t}$ relates to an individual of the respective cohort and is already detrended from productivity growth X_t. If we add consumption of all cohorts, the aggregate quantities contain a population growth trend N_t since the rate $n+\theta$ at which new cohorts enter the economy exceeds the rate θ at which existing cohorts are dying. To obtain **aggregate per efficiency unit** quantities v_t, we devide by N_t and obtain the following aggregation formula which was already mentioned above:

$$v_t = \sum_{a=1}^{\infty}\omega_a v_{t-a,t}, \qquad \omega_a = \left(\frac{n+\theta}{1+n}\right)\left(\frac{1-\theta}{1+n}\right)^{a-1}. \tag{14}$$

The population weights ω_a are defined in (3) and are time-autonomous implying that the age distribution of the population remains constant. The aggregate per capita quantities will be stationary in a steady state. Aggregation uses the fact that market prices are the same for all agents. Furthermore, we assume for simplicity of aggregation that disposable non-capital income $y_{t-a,t} = y_t$ is age-independent which requires specifically that government transfers to individuals are age-independent. Therefore we have by the definition of human wealth [see (11)] that aggregate human wealth per efficiency unit is equal to human wealth of each individual cohort, $H_{t-a,t} = H_t$. Human wealth is the same for all generations because they get the same disposable incomes $\{y_s\}_{s\geq t}$, have the same time horizon over the rest of their

[3]Knowing the current budget, demands for leisure and the consumption commodity can be obtained from separate static subproblems.

life and the same discount factors.[4] For similar reasons, the marginal propensity to consume out of total wealth Ω^{-1} is the same for all generations.

Applying the aggregation formula we can derive the aggregate per efficiency unit variables for the household sector as follows. Realizing that all agents face identical prices irrespective of their age, aggregate consumption is, by definition, given by

$$\frac{p_t^v}{\bar{p}_t} v_t = \Omega_t^{-1} \mathcal{W}_t. \tag{15}$$

We have omitted cohort indices to denote aggregate per efficiency unit variables. To obtain aggregate total wealth, use $(1+\bar{g}) \equiv (1+n)(1+x)$ and obtain from (11)

$$\mathcal{W}_{t-a,t} = \frac{1+r_t}{1+\bar{g}} \left[\frac{1+n}{1-\theta} A_{t-a,t-1} + \frac{1+n}{1-\theta} H_{t-a,t-1} \right]. \tag{16}$$

The aggregation of financial wealth yields

$$\begin{aligned} & \frac{1+n}{1-\theta} \sum_{a=1}^{\infty} \omega_a A_{t-a,t-1} \\ = & \frac{n+\theta}{1+n} \left\{ \frac{1+n}{1-\theta} A_{t-1,t-1} + A_{t-2,t-1} + \frac{1-\theta}{1+n} A_{t-3,t-1} + \dots \right\} \\ = & \sum_{a=1}^{\infty} \omega_a A_{t-1-a,t-1} = A_{t-1}. \end{aligned} \tag{17}$$

This procedure uses the fact that new generations are not connected to previous ones via bequests but start their lives with zero financial wealth: $A_{t-1,t-1} = 0$. Aggregate household behavior is then described, in addition to (15), by

$$\begin{aligned} (a) \quad & \mathcal{W}_t &&= \frac{1+r_t}{1+\bar{g}} \left[A_{t-1} + \frac{1+n}{1-\theta} H_{t-1} \right], \\ (b) \quad & A_t &&= \frac{1+r_t}{1+\bar{g}} A_{t-1} + \frac{y_t - p_t^v v_t}{\bar{p}_t}, \\ (c) \quad & H_{t-1} &&= \left(\frac{1+\bar{g}}{1+r_t} \right) \left(\frac{1-\theta}{1+n} \right) \left[\frac{y_t}{\bar{p}_t} + H_t \right], \\ (d) \quad & \Omega_{t-1} &&= 1 + (1-\theta)\beta^\gamma \left[\left(\frac{1+x}{1+r_t} \right) \frac{p_t^v / \bar{p}_t}{p_{t-1}^v / \bar{p}_{t-1}} \right]^{1-\gamma} \Omega_t. \end{aligned} \tag{18}$$

To obtain (18a), we used (17), (16) and $H_{t-a,t} = H_t$. The law of motion (18c) is obtained by writing in differences the definition of human wealth in (11). The law of motion of real assets (18b) stems from (6) and (17). While the evolution of aggregate per capita human wealth depends on the survival probability $(1-\theta)$, financial wealth in the aggregate does not. The intuitive explanation is that the insurance

[4] This is very much different from life cycle OLG models where agents face a fixed finite lifetime and death is certain to occur after T periods. In these models the marginal propensity to consume increases with age while human wealth declines with age. See Auerbach and Kotlikoff (1987) and Keuschnigg (1991).

premiums exchanged between the household and insurance sectors are essentially transfers that cancel in the aggregate. Finally, the inverse of the marginal propensity to consume must follow (18d) which is obtained by writing in differences the definition of Ω given in (13).

2.4 Dynamics of Aggregate Consumption and Wealth

We now show the dynamic behavior of total real wealth as well as of aggregate consumption. From (18a,c) we have

$$\mathcal{W}_t = \frac{1+r_t}{1+\bar{g}} A_{t-1} + \frac{y_t}{\bar{p}_t} + H_t.$$

Denote real expenditures on total consumption by $M_t^v = p_t^v v_t / \bar{p}_t$. Using (18b) lagged by one period, substitute out A_{t-1} to obtain

$$\mathcal{W}_t = \frac{1+r_t}{1+\bar{g}} \left\{ \frac{1+r_{t-1}}{1+\bar{g}} A_{t-2} + \frac{y_{t-1}}{\bar{p}_{t-1}} + H_{t-1} - H_{t-1} - M_{t-1}^v \right\} + \frac{y_t}{\bar{p}_t} + H_t.$$

Now use the above definition of total wealth. Furthermore, use $M_t^v = \Omega_t^{-1} \mathcal{W}_t$ and eliminate H_{t-1} with (18c),

$$\mathcal{W}_t = \frac{1+r_t}{1+\bar{g}} (1 - \Omega_{t-1}^{-1}) \mathcal{W}_{t-1} + \frac{n+\theta}{1+n} (\frac{y_t}{\bar{p}_t} + H_t). \tag{19}$$

Note that the dynamic behavior of total wealth not only depends on its lagged value, but also on its composition between human and financial wealth. Multiplying both sides by Ω_t^{-1}, we obtain

$$M_t^v = \frac{1+r_t}{1+\bar{g}} \frac{\Omega_{t-1}}{\Omega_t} (1 - \Omega_{t-1}^{-1}) M_{t-1}^v + \Omega_t^{-1} \frac{n+\theta}{1+n} (\frac{y_t}{\bar{p}_t} + H_t). \tag{20}$$

Convergence of this difference equation requires that the coefficient of M_{t-1}^v be smaller than unity. The finiteness of individual planning horizons ($\theta > 0$) and the birth of new generations ($n > 0$) allow for steady state equilibria in real consumption spending. If the gross birth rate $n + \theta$ were zero, the intercept would vanish and the above difference equations could not determine steady state levels of total wealth and full consumption [see Frenkel and Razin (1987), chapter 10]. This case gives the model of a representative, infinitely lived agent where steady state general equilibrium requires that the coefficient of M_{t-1} be exactly unity and that the level of financial wealth is determined, in equilibrium, by the supply of assets in other sectors.

3 General Equilibrium

3.1 Walras' Law

The temporary equilibrium system of the model is given by (T.33) in the main text,[5] and it satisfies Walras' Law:

$$\sum_{i=1}^{n} p_i^h \zeta_i^C + w\zeta^L + \zeta^G + \zeta^K = 0. \tag{21}$$

For a proof, we first derive several useful relationships. The commodity market clearing condition (T.33a) implies

$$\sum_{i=1}^{n} p_i^h (C_i^h + G_i^h + \sum_{j=1}^{n} I_{ij}^h + E_i) = \sum_{i=1}^{n} Y_i (p_i^h - \sum_{j=1}^{n} a_{ji}^h p_j^h) \tag{22}$$

At the same time, since $(F_i - \Phi_i)/a_{0i} = Y_i$, the definition of the value added price $\tilde{p}_i$ (see (T.31) in main text) implies

$$\begin{aligned} &(a) \quad \tilde{p}_i (F_i - \Phi_i) = p_i^h Y_i - \sum_{j=1}^{n} \left[a_{ji}^h p_j^h (1 + t_{x,j}^{Q,h}) + a_{ji}^m p_j^m (1 + t_{x,j}^{Q,m} + t_{m,j}^{Q}) \right] Y_i, \\ &(b) \quad \sum_{i=1}^{n} Y_i (p_i^h - \sum_{j=1}^{n} a_{ji}^h p_j^h) = \sum_{i=1}^{n} \tilde{p}_i (F_i - \Phi_i) + T_x^{Q,h} + \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ji}^m p_j^m Y_i + T_x^{Q,m} + T_m^Q, \end{aligned} \tag{23}$$

where $T_x^{Q,h} \equiv \sum_i \sum_j a_{ji}^h p_j^h t_{x,j}^{Q,h} Y_i$, and analogously for $T_x^{Q,m}$ and T_m^Q.

Next, use the definition of commodity prices in (T.23) and of price indices in (T.24) of the main text to rewrite expenditure on the various components of final demand as

$$\begin{aligned} &(a) \quad p^c C = \sum_{i=1}^{n} p_i^h C_i^h + T_v^{c,h} + T_x^{c,h} + \sum_{i=1}^{n} p_i^m C_i^m + T_v^{c,m} + T_x^{c,m} + T_m^c, \\ &(b) \quad p^I I = \sum_{i=1}^{n} p_i^h \sum_{j=1}^{n} I_{ij}^h + T_v^{I,h} + T_x^{I,h} + \sum_{i=1}^{n} p_i^m \sum_{j=1}^{n} I_{ij}^m + T_v^{I,m} + T_x^{I,m} + T_m^I, \\ &(c) \quad p^G G = \sum_{i=1}^{n} p_i^h G_i^h + T_x^{G,h} + \sum_{i=1}^{n} p_i^m G_i^m + T_x^{G,m} + T_m^G. \end{aligned} \tag{24}$$

Here, $T_v^{c,h} \equiv \sum_i p_i^h C_i^h t_{v,i}^{c,h}$ stands for total value added taxes paid on private consumption of home produced goods. Analogous definitions apply to all other indirect tax liabilities (x indicates excise taxes and m import tariffs) and to the other categories of demand (I indicates investment demand and G government demand).

Net of tax dividends are given in (T.16) of the main text, for each sector i. Summing across sectors and using the wage definitions $w_i^g = (1 + t_{l,i})w$ in (T.17) and $w^n = (1 - t_y)(1 - t_s)w$ in (T.6) of the main text, dividends for the economy as a whole are

[5] We refer to equations in the main text by placing a T in front of the equation number.

$$\begin{aligned}
&(a)\quad \sum_{i=1}^{n}\chi_i = \sum_{i=1}^{n}(1-t_y)\tilde{p}_i\Big(F_i - \Phi_i - (1+t_{l,i})wL_i^d\Big) - (1-et_y)p^I I, \\
&(b)\quad \sum_{i=1}^{n}\chi_i = \sum_{i=1}^{n}\tilde{p}_i(F_i - \Phi_i) - \sum_{i=1}^{n} w^n L_i^d - p^I I - T_f, \\
&(c)\quad T_f \equiv t_y \sum_{i=1}^{n}\tilde{p}_i(F_i - \Phi_i) - et_y p^I I + (1-t_y)w\sum_{i=1}^{n}\left[t_s + t_{l,i}\right]L_i^d.
\end{aligned} \tag{25}$$

T_f denotes net tax revenue from factor income taxation. In deriving the expression for T_f, use was made of the fact that $w = w^n + w[t_s + t_y(1-t_s)]$. Note, however, that the definition of T_f does not yet take account of the income tax deduction $t_y d$. Therefore, overall tax revenues of the government amount to $T \equiv T_f + T_c - t_y d$ where tax revenues from all indirect taxes add up to

$$T_c \;\equiv \sum_{\substack{i=C,I \\ j=h,m}} T_v^{i,j} \;+ \sum_{\substack{i=C,I,G,Q \\ j=h,m}} T_x^{i,j} \;+ \sum_{i=C,I,G,Q} T_m^i. \tag{26}$$

Walras' Law is now proved by showing that capital market equilibrium $\zeta^K = 0$ is implied by market clearing in all other markets, $\zeta_i^C = \zeta^L = \zeta^G = 0$. To this end, substitute the definitions of the primary balances S^F, S^G and S^H (given after equation (T.32) in the main text) as well as the definition of total dividends (given in (25c) above) into the flow condition $\zeta^K = S^F + S^G - S^H - \sum_i \chi_i$ (see (T.34)). Use furthermore the definition of the budgets in (24) above and substitute (23) into the dividend equation (25b). The resulting expression can be rearranged, as in (21), in terms of the other excess demand functions (defined in (T.33a–c) of the main text). Q.E.D.

3.2 Computation of Steady States

When computing a steady state, both expected and predetermined variables are unknown and must be determined endogenously. In addition to computing equilibrium in the spot market, one must ensure that the equations of motion are stationary. Therefore, we compute the extended "excess demand system" given in (27) where the price vector is $\vec{p} \equiv \{p_1^h, \ldots, p_n^h, w, z\}$. The first $n+2$ equations correspond to the temporary excess demand functions stated in (T.33a-c). The other functions must be included to guarantee stationarity of the corresponding equations of motion. Since in a steady state prices are time invariant, the expected variables of the household sector are obtained by directly evaluating the present value formulas. Moreover, we have $r = i^*$ since $\bar{p}$ remains constant in a steady state. Thus, in determining aggregate consumption we first use the stationary versions of (18c-d) to express H and Ω in terms of steady state prices. Insert then these expressions, alongside the iterated value of A, into the consumption function (15). As regards investment, we simply relate sectoral demands for the composite capital good to sectoral capital stocks via the stationary version of the capital accumulation equation (T.18). These computations lead to n excess commodity demand functions in (27a), and excess labor demand in (27b). The stationary excess demand function (27c) corresponds to (T.33c) and is discussed in the main text. It states that the government must generate a high enough primary surplus to keep its debt service stationary.

$$
\begin{aligned}
&(a) \quad \zeta_i^C(\vec{p},\vec{K},A) \equiv C_i^h + G_i^h + \sum_{j=1}^{n} I_{ij}^h + E_i + \sum_{j=1}^{n} a_{ij}^h Y_j - Y_i, \qquad i=1\dots n,\\
&(b) \quad \zeta^L(\vec{p},\vec{K},A) \equiv \sum_{j=1}^{n} L_j^d - L^s,\\
&(c) \quad \zeta^G(\vec{p},\vec{K},A) \equiv \frac{\bar{g}-i^*}{1+\bar{g}}\bar{D}^G - S^G,\\
&(d) \quad \zeta_i^V(\vec{p},\vec{K},A) \equiv \frac{\bar{g}-i^*}{1+\bar{g}}V_i + \frac{\chi_i}{\bar{p}}, \qquad i=1\dots n,\\
&(e) \quad \zeta^A(\vec{p},\vec{K},A) \equiv -\frac{\bar{g}-i^*}{1+\bar{g}}A + \frac{S^H}{\bar{p}},\\
&(f) \quad \zeta^F(\vec{p},\vec{K},A) \equiv \frac{\bar{g}-i^*}{1+\bar{g}}\left(\bar{p}A - \sum_{i=1}^{n}\bar{p}V_i - \bar{D}^G\right) - S^F.
\end{aligned}
\tag{27}
$$

In addition to spot market clearing, we have to impose steady state restrictions that keep household asset accumulation stationary, (27e), and that guarantee the no-arbitrage condition on stationary firm equity values, (27e). The former is obtained from the stationary version of (18b). The latter uses the stationary version of the no-arbitrage condition (T.14). According to Hayashi's theorem, firm values are computed by multiplying the iterated values capital stocks with the sectoral shadow values of capital which depend on steady state prices and parameters via the Euler equation (T.20b), again in stationary form. We obtain n "excess demands" $\zeta_i^V(\cdot)$, as in (27d). Note that both A and V_i are expressed in real terms. Finally, a similar stationarity restriction (27f) must hold with respect to the trade balance and net foreign assets D^F which are, of course, part of financial wealth: $\bar{p}A = \bar{p}\sum_i V_i + D^G + D^F$. The home economy must run a trade balance surplus large enough to service a stationary value of foreign indebtedness.

The arguments of the excess demand functions ζ are the equilibrating variables that we iterate upon when solving for a stationary equilibrium: $\vec{p} \equiv \{p_1^h,\dots,p_n^h,w,z\}$, $\vec{K} \equiv \{K_1,\dots,K_n\}$ and A. We have $2n+4$ equilibrium conditions and $2n+3$ variables. However, one of the equations is redundant. From (27c-f), we obtain

$$\bar{p}\zeta^A + \bar{p}\sum_i \zeta_i^V + \zeta^G + \zeta^F = \sum_i \chi_i + S^H - S^G - S^F. \tag{28}$$

As proved in the preceding subsection, this expression is zero by Walras' Law. Therefore, we drop the last of the steady state equilibrium conditions and solve for (27a-e) only. Exploiting the linearity properties of the model in a stationary state allows to reduce the solution space to dimension $n+3$ by calculating what we call 'commodity market clearing capital stocks' for any vector of prices, government transfers and financial wealth. Thus, we in effect iterate on these latter variables only when computing steady states.

4 Data Set and Calibration Results

4.1 Constructing Trade Elasticities

The parameters in question are the elasticity of substitution between imports and domestic goods and the price elasticity of export demand. We use five different sources of information on international trade elasticities: Harrison, Rutherford and Wooton (1991), Shiells, Deardorff and Stern (1986), Deardorff and Stern (1986), Lächler (1985), and Harris (1986). For the sake of easier concordance, we have initially worked with the 31 sector classification which is given in table 2, including the concordance to the 19 sector classification used in the paper.

Table 2: Sector Classification and Concordance: 31 Sectors

No.	Sector	Shorthand	Concordance
1	Agriculture a. Forestry	Agr/For	1
2	Mining	Min/Quar	2
3	Petroleum	Petrol	8
4	Stones, Clay a. Cement	Stones	9
5	Glass	Glass	9
6	Foodstuff	Foodst	3
7	Tobacco	Tobacc	3
8	Textiles	Text	4
9	Clothing	Cloth	4
10	Leather and Shoes	Leath/Sh	4
11	Chemicals (excl. Petroleum)	Chem	7
12	Iron a. Steel Production	Iron/St	10
13	Machinery, Steel and Metal Constr.	Machin	11
14	Casting	Cast	10
15	Non-ferrous Metals	Nonferr	10
16	Iron a. Metal Products, Precision Mech.	MetProd	11
17	Electric Machinery	ElMach	11
18	Vehicles Constr. a. Repair	Vehic	11
19	Sawing	Sawing	5
20	Wood Processing	WoodPr	5
21	Paper Production	Paper	6
22	Paper Processing	PaperPr	6
23	Construction	Constr	13
24	Energy and Water Supply	En/Wat	12
25	Commerce	Comm	14
26	Transport a. Communication	Transp	16
27	Banking a. Insurance	Bank/In	17
28	Hotels a. Restaurants	Hot/Cat	15
29	Other Services	Oth/Ser	18
30	Real Estate Services	RealEst	17
31	Public Services	Public	19

Take the import substitution elasticities first. Table 3 gives the elasticity values reported by these studies, whereby a zero entry in the table should be interpreted as no value being given for that sector in the respective source. The last column gives the elasticity values used for the present model which are obtained as follows. In view of the similarity between the Austrian and the German economies, the elasticities obtained by Lächler (his table 5) are preferred if they are statistically significant. In all other cases we take an unweighted average of the different values given by the various sources. For the services sectors (25 – 30) we arbitrarily use a value of 0.1 on the grounds that we would expect the elasticity to be "rather low" in these cases. The value for sector 31 is immaterial since there are no imports in this sector. Finally, for our computational model, the elasticities so obtained for the 31 sector classification are aggregated to 19 sectors using total imports as weights.

Table 3: Imported–Home Goods Substitution Elasticities (σ_i^m)

Sector		Harrison et al.	Shiells et al.	Deardorff & Stern	Lächler	Harris	Present
1	Agr/For	2.000	0.000	1.139	0.000	1.100	1.413
2	Min/Quar	0.500	0.000	0.000	0.053	1.100	0.551
3	Petrol	2.000	-0.340	2.359	0.112	1.100	1.046
4	Stones	1.300	2.110	2.784	-0.765	2.480	1.582
5	Glass	1.300	4.290	1.628	0.932	0.000	1.630
6	Foodst	0.500	0.460	1.133	0.797	1.280	0.797
7	Tobacc	0.000	-16.190	1.133	0.797	1.100	0.797
8	Text	2.000	2.580	1.147	1.361	1.100	1.361
9	Cloth	2.000	1.620	4.269	-0.465	4.070	2.299
10	Leath/Sh	6.800	3.150	1.810	0.998	1.710	0.998
11	Chem	2.000	9.850	2.612	1.065	2.200	1.065
12	Iron/St	0.500	3.050	1.446	2.841	2.010	2.841
13	Machin	0.500	3.340	1.022	-0.469	1.100	1.099
14	Cast	0.500	3.050	1.446	0.000	0.000	0.999
15	Nonferr	0.000	0.810	1.430	1.218	0.000	1.218
16	MetProd	2.000	1.540	3.674	4.911	4.220	4.911
17	ElMach	1.300	7.460	2.110	1.362	1.100	1.362
18	Vehic	0.000	2.010	3.585	0.818	4.840	0.818
19	Sawing	1.900	0.260	1.757	0.787	1.100	0.787
20	WoodPr	1.900	12.130	3.096	1.014	3.110	1.014
21	Paper	1.100	1.800	1.585	0.063	1.310	1.172
22	PaperPr	1.100	2.720	3.013	2.191	3.230	2.191
23	Constr	0.000	0.000	0.000	0.000	1.100	1.100
24	En/Wat	0.340	0.000	0.000	0.000	0.100	0.440
25	Comm	0.000	0.000	0.000	0.000	0.000	0.100
26	Transp	2.000	0.000	0.000	0.000	0.100	0.100
27	Bank/In	0.000	0.000	0.000	0.000	0.000	0.100
28	Hot/Cat	0.000	0.000	0.000	0.000	0.000	0.100
29	Oth/Ser	0.000	0.000	0.000	0.000	0.000	0.100
30	RealEst	0.000	0.000	0.000	0.000	0.000	0.100
31	Public	0.000	0.000	0.000	0.000	0.000	0.100

The elasticity of substitution between imported and home goods is closely related to the price elasticity of import demand. Some authors use the convention of equating the negative of the substitution elasticity to the own price elasticity of import demand [see for instance Lächler (1985) and Harrison, Rutherford and Wooton (1991)]. Others, such as Shiells, Deardorff and Stern (1986) relate the elasticity of substitution to the cross price elasticity of demand for the imported good. We use these three sources of extraneous information for the price elasticities of demand for Austrian exports. We take the german values reported by Lächler to be representative for the price elasticities of demand for Austrian exports to Europe and the US elasticities reported by Shiells et al. to be representative for exports to non–European countries, and we use a weighted average of these two values for every sector, with regional shares for merchandise exports serving as weights. If any one of the two sources reports a missing or positive value, the corresponding value is taken from Harrison et al., and where no information is available at all we arbitrarily choose a value of 1.5. The respective elasticity values are given in table 4.

Table 4: Export Demand Elasticities (θ_i)

Sector		Harrison et al.	Shiells et al.	Lächler	Present
1	Agr/For	-2.000	0.000	0.000	-2.000
2	Min/Quar	-0.500	0.000	-0.053	-0.155
3	Petrol	-2.000	-0.790	-0.112	-0.135
4	Stones	-1.300	-1.370	0.765	-1.311
5	Glass	-1.300	-2.860	-0.932	-1.628
6	Foodst	-0.500	-0.700	-0.797	-0.769
7	Tobacc	0.000	-7.570	-0.797	-1.144
8	Text	-2.000	-1.410	-1.361	-1.374
9	Cloth	-2.000	-0.520	0.465	-1.761
10	Leath/Sh	-6.800	-2.010	-0.998	-1.333
11	Chem	-2.000	-6.820	-1.065	-2.741
12	Iron/St	-0.500	-2.280	-2.841	-2.756
13	Machin	-0.500	-0.880	0.469	-0.651
14	Cast	-0.500	-2.280	0.000	-0.927
15	Nonferr	0.000	-0.670	-1.218	-1.094
16	MetProd	-2.000	-0.940	-4.911	-3.760
17	ElMach	-1.300	-3.080	-1.362	-1.895
18	Vehic	0.000	-1.240	-0.818	-0.946
19	Sawing	-1.900	-1.320	-0.787	-0.854
20	WoodPr	-1.900	-9.560	-1.014	-2.861
21	Paper	-1.100	-1.800	-0.063	-0.464
22	PaperPr	-1.100	-1.460	-2.191	-2.023
23	Constr	0.000	0.000	0.000	-1.500
24	En/Wat	-0.340	0.000	0.000	-0.340
25	Comm	0.000	0.000	0.000	-1.500
26	Transp	-2.000	0.000	0.000	-1.500
27	Bank/In	0.000	0.000	0.000	-1.500
28	Hot/Cat	0.000	0.000	0.000	-1.500
29	Oth/Ser	0.000	0.000	0.000	-1.500
30	RealEst	0.000	0.000	0.000	-1.500
31	Public	0.000	0.000	0.000	-1.500

We are well aware of the fact that the above procedure of assigning parameter values to factor substitution and trade elasticities is problematical. Obtaining more reliable, authentic information for the Austrian economy to which the model is calibrated must indeed be one of the most important points on the agenda for future research. For now, the crucial question really seems to be whether using extraneous information as indicated above is any better than just using random numbers within a "meaningful" interval. Without being able to give any rigorous justification for our own judgement, we nevertheless feel that the answer to this question is affirmative.

4.2 Effective Indirect Tax Rates

We distinguish between three types of indirect taxes in our computational model: a general excise tax, a value added tax, and an import tariff. In the process of calibration, we compute effective indirect tax rates for all categories of demand as follows. We start with disaggregate information on indirect taxes paid on private consumption. This allows us to compute effective indirect tax rates on private consumption for all three types of taxes. Notice that these effective indirect tax rates differ between imports and home goods. Next we use these effective indirect tax rates to distribute across all sectors the known aggregate tax payments for other categories of final demand, as we did not have any disaggregate information on indirect taxes paid on non-consumption demand. These derived tax vectors are then used to calculate effective indirect tax rates for investment demand and government demand. Note again that there are different effective tax rates for imported and home goods.

Table 5: Tax Rates on Consumption

Sector		$t_{v,i}^{c,h}$	$t_{v,i}^{c,m}$	$t_{x,i}^{c,h}$	$t_{x,i}^{c,m}$	$t_{m,i}^{c}$
1	Agr/For	0.102	0.104	0.039	0.040	0.082
2	Min/Quar	0.088	0.102	0.000	0.000	0.072
3	Food	0.113	0.095	0.132	0.043	0.074
4	Tex/Clot	0.177	0.182	0.000	0.000	0.068
5	Wood	0.168	0.175	0.000	0.000	0.032
6	Paper	0.130	0.121	0.000	0.000	0.025
7	Chemic	0.132	0.140	0.000	0.000	0.046
8	Petrol	0.209	0.216	0.667	0.714	0.153
9	Nonferr	0.149	0.175	0.000	0.000	0.059
10	MetProd	0.054	0.145	0.000	0.000	0.010
11	MetProc	0.181	0.172	0.000	0.000	0.033
12	Energy	0.080	0.080	0.000	0.000	0.000
13	Constr	0.096	0.000	0.000	0.000	0.000
14	Trade	0.136	0.000	0.000	0.000	0.000
15	Hot/Cat	0.125	0.000	0.071	0.000	0.000
16	Trans	0.057	0.000	0.000	0.000	0.000
17	RealEst	0.071	0.000	0.021	0.000	0.000
18	OthSer	0.121	0.121	0.000	0.000	0.000
19	Public	0.067	0.000	0.000	0.000	0.000

Table 6: Tax Rates on Investment						
Sector		$t_{v,i}^{I,h}$	$t_{v,i}^{I,m}$	$t_{x,i}^{I,h}$	$t_{x,i}^{I,m}$	$t_{m,i}^{I}$
1	Agr/For	0.040	0.041	0.001	0.001	0.092
2	Min/Quar	0.040	0.000	0.000	0.000	0.000
3	Food	0.036	0.000	0.001	0.000	0.000
4	Tex/Clot	0.073	0.073	0.000	0.000	0.066
5	Wood	0.070	0.072	0.000	0.000	0.037
6	Paper	0.066	0.000	0.000	0.000	0.000
7	Chemic	0.055	0.056	0.000	0.000	0.053
8	Petrol	0.054	0.055	0.065	0.066	0.111
9	Nonferr	0.072	0.073	0.000	0.000	0.044
10	MetProd	0.051	0.037	0.000	0.000	0.032
11	MetProc	0.072	0.072	0.000	0.000	0.026
12	Energy	0.031	0.000	0.000	0.000	0.000
13	Constr	0.038	0.000	0.000	0.000	0.000
14	Trade	0.054	0.000	0.000	0.000	0.000
15	Hot/Cat	0.046	0.000	0.001	0.000	0.000
16	Trans	0.022	0.000	0.000	0.000	0.000
17	RealEst	0.000	0.000	0.000	0.000	0.000
18	OthSer	0.048	0.048	0.000	0.000	0.000
19	Public	0.026	0.000	0.000	0.000	0.000

Table 7: Tax Rates on Government				
Sector		$t_{x,i}^{G,h}$	$t_{x,i}^{G,m}$	$t_{m,i}^{G}$
1	Agr/For	0.001	0.001	0.069
2	Min/Quar	0.000	0.000	0.062
3	Food	0.001	0.001	0.062
4	Tex/Clot	0.000	0.000	0.056
5	Wood	0.000	0.000	0.028
6	Paper	0.000	0.000	0.023
7	Chemic	0.000	0.000	0.040
8	Petrol	0.060	0.060	0.081
9	Nonferr	0.000	0.000	0.043
10	MetProd	0.000	0.000	0.024
11	MetProc	0.000	0.000	0.024
12	Energy	0.000	0.000	0.000
13	Constr	0.000	0.000	0.000
14	Trade	0.000	0.000	0.000
15	Hot/Cat	0.001	0.001	0.000
16	Trans	0.000	0.000	0.000
17	RealEst	0.031	0.000	0.000
18	OthSer	0.000	0.000	0.000
19	Public	0.000	0.000	0.000

As the value added tax is, in effect, a tax on consumption only, intermediate input use is exempt from this tax. Although investment demand is, in principle, not subject to the value added tax either, we nevertheless observe value added taxes on investment demand in the data. Some firms are exempt from value added taxes in the first place and, therefore, are not eligible for deducting the value added taxes paid on their investment purchases. As a result, our data set shows a significant amount of value added taxes on investment purchases. We treat this along the lines suggested above. For obvious reasons, the effective value added tax rates on investment are much lower than for consumption. Finally, government purchases are exempt from the value added tax since 1976.

Our data set contains a matrix of import duties on intermediate input use. We use it together with the input transactions to calculate effective import tariffs on intermediates. We have only very sparse information on excise taxes paid on intermediate input use. In particular, with the information at hand we were unable to calculate separate effective excise tax rates for different using sectors. Therefore, we had to assume that effective excise tax rates are equal for all using sectors, and we calculated these as suggested above using effective excise tax rates on private consumption.

4.3 National Accounts

Table 8.1: Production Account

Expenditure		Revenue	
Domestic intermediates	129.02	Domestic intermediates	129.02
Taxes on dom. int.	0.98	Private consumption demand	85.18
Imported intermediates	35.21	Government purchases	37.00
Taxes on imp. int.	1.18	Investment demand	44.43
Wage bill	100.00	Export demand	65.46
Ind. wage tax	19.13		
Capital income	75.58		
Total	361.10	Total	361.10

Table 8.2: Enterprise Accounts			
Primary Income and Expenditure			
Expenditure		Revenue	
Imported investment goods	10.45	Capital income	75.58
Taxes on imp. inv. goods	1.01		
Domestic investment goods	44.43		
Taxes on dom. inv. goods	2.12		
Profit tax	10.47		
Dividends [χ_i^0]	7.09		
Total	75.58	Total	75.58
Equity Related Flows: $V^0\bar{p}^0 = 374.71$(*)			
Expenditure		Revenue	
Return on equity [$rV\bar{p}^0$]	19.90	Dividends [χ^0]	7.09
		Capital gains [$\bar{g}V^0\bar{p}^0/(1+\bar{g})$]	12.81
Total	19.90	Total	19.90
(*) We simply write V^0 for $\sum_{i=1}^n V_i^0$.			

Table 8.3: Government Accounts			
Primary Income and Expenditure			
Expenditure		Revenue	
Domestic purchases	37.00	Indirect comm. taxes	26.61
Taxes on dom. purch.	0.04	Profit tax + income tax	27.14
Purchase of imports	2.00	Social security tax	11.99
Taxes on imp. purch.	0.07	Indirect wage tax	19.13
Transfers	39.99	Inc. tax on interest paid on gov. debt	-2.38
Primary government balance [$-S^{G,0}$]	3.39		
Total	82.49	Total	82.49
Debt Related Flows: $D^{G,0} = 179.10$			
Expenditure		Revenue	
Interest payments [$iD^{G,0}/(1+\bar{g})$]	11.89	Primary balance [$-S^{G,0}$]	3.39
		Tax on interest [$it_yD^{G,0}/(1+\bar{g})$]	2.38
		Net deficit [$\bar{g}D^{G,0}/(1+\bar{g})$]	6.12
Total	11.89	Total	11.89

Table 8.4: Foreign Sector Accounts			
Primary Income and Expenditure			
Expenditure		Revenue	
Exports	65.46	Private consumption imports	18.25
Ind. tax on exports	2.59	Investment imports	10.45
		Government imports	2.00
		Intermediate imports	35.21
		Foreign sector balance $[S^{F,0}]$	2.14
Total	68.05	Total	68.05
Foreign Debt Related Flows: $-D^{F,0} = 113.23$			
Expenditure		Revenue	
Foreign sector balance $[S^{F,0}]$	2.14	Interest payments $[-rD^{F,0}/(1+\bar{g})]$	6.01
Current account $[\bar{g}D^{F,0}/(1+\bar{g})]$	3.87		
Total	6.01	Total	6.01

Table 8.5: Household Accounts			
Primary Income and Expenditure			
Expenditure		Revenue	
Consumption of domestic goods	85.18	Wage income	100.00
Ind. tax on dom. cons.	14.45	Profit tax	10.47
Consumption of imported goods	18.25	Transfer income	39.99
Ind. tax on imp. cons.	4.19		
Social security payments	11.99		
Income tax + profit tax	27.14		
Tax on interest income on gov. debt	-2.38		
Primary household balance $[S^{H,0}]$	-8.34		
Total	150.47	Total	150.47
Wealth Related Flows: $A^0\bar{p}^0 = 440.57$			
Expenditure		Revenue	
Tax on interest $[it_yD^{G,0}/(1+\bar{g})]$	2.38	Interest payments $[iD^{G,0}/(1+\bar{g})]$	11.89
Interest paid on for. debt $[rD^{F,0}/(1+\bar{g})]$	6.01	Net return on equity $[rV^0\bar{p}^0/(1+\bar{g})]$	19.90
Primary balance $[S^{H,0}]$	8.34	New foreign debt $[\bar{g}D^{F,0}/(1+\bar{g})]$	3.87
Net deficit $[\bar{g}D^{G,0}/(1+\bar{g})]$	6.12		
Capital gains $[\bar{g}V^0\bar{p}^0/(1+\bar{g})]$	12.81		
Total	35.67	Total	35.67

5 List of Variables

Table 9: Summary Information on Variable and Parameter Notation	
Part 1: Variables	
Name	Description
C_i^h, C_i^m	Total consumption of home produced and imported commodity i, respectively.
c_i^h, c_i^m	Consumption of home produced and imported commodity i, respectively, per unit of C_i.
C_i	Total consumption of commodity aggregate i, composed of a home produced and an imported commodity.
c_i	Consumption of commodity aggregate i per unit of C.
C	Total aggregate commodity consumption.
c	Aggregate commodity consumption per unit of v.
v	"Full" consumption aggregate, composed of commodities and leisure.
h	Consumption of leisure per unit of v.
L^s	Total labor supply.
I_{ij}^h, I_{ij}^m	Investment demand for home produced and imported commodity i, respectively, by firms of sector j.
i_{ij}^h, i_{ij}^m	Investment demand for home produced and imported commodity i, respec-. tively, per unit of I_{ij}
I_{ij}	Investment demand by firms of sector j for the sector i commodity aggregate.
I_i	Investment demand of all sectors for the sector i commodity aggregate.
i_i	Investment demand for the sector i commodity aggregate per unit of I.
I	Total demand for the composite capital good.
I_j	Demand for the composite capital good by firms of sector j.
G_i^h, G_i^m	Government demand for home produced and imported commodity i, respectively.
g_i^h, g_i^m	Demand for home produced and imported commodity i, respectively, per unit of G_i.
G_i	Government demand for the commodity aggregate i.
g_i	Demand for the commodity aggregate i per unit of G.
G	Level of government procurement (exogenously determined at $\bar{G}$).
E_i	Foreign demand for home produced commodity i.
M_i	Total quantity of commodity i imports.
Y_i	Gross output of commodity i.
Q_{ij}^h, Q_{ij}^m	Total demand for home produced and imported commodity i, respectively, for intermediate input use by sector j.
q_{ij}^h, q_{ij}^m	Demand for home produced and imported commodity i, respectively, per unit of Q_{ij}.
Q_{ij}	Total demand for commodity aggregate i for intermediate input use by sector j.

Table 9: Summary Information on Variable and Parameter Notation	
Part 1: Variables, continued	
Name	Description
a^h_{ij}, a^m_{ij}	Input–output coefficient for home produced and imported input.
a_{ij}	Input–output coefficient for aggregate commodity inputs.
a_{0j}	Input–output coefficient for value added input.
F_j	Quantity of value added produced by firms of sector j.
L^d_j	Labor demand by firms of sector j.
$K_{j,-1}/(1+\bar{g})$	Capital stock of firms in sector j, expressed per efficiency unit of the current period.
Φ_j	Value added loss due to installation of physical capital in sector j.
p^h_i, p^m_i	Market price of the domestic and imported commodity i, respectively, net of all indirect taxes.
$p^{c,h}_i, p^{c,m}_i$	Price of the domestic and imported commodity i, respectively, paid by domestic consumers.
$p^{G,h}_i, p^{G,m}_i$	Price of the domestic and imported commodity i, respectively, paid by the government.
$p^{I,h}_i, p^{I,m}_i$	Price of the domestic and imported commodity i, respectively, paid by domestic investors.
$p^{Q,h}_{ij}, p^{Q,m}_{ij}$	Price of the domestic and imported commodity i, respectively, paid by intermediate input users of sector j.
p^c_i	Price index for the sector i commodity aggregate for private consumption.
p^G_i	Price index for the sector i commodity aggregate for government demand.
p^I_i	Price index for the sector i commodity aggregate for investment demand.
p^c	Aggregate price index for commodity consumption.
p^G	Price index for aggregate government procurement.
p^I	Price index for the composite capital good (investment demand).
p^Q_{ij}	Price index for the sector commodity aggregate used as input in sector j.
p^v	Price index for "full" consumption.
$\bar{p}$	Price index for a commodity basket in terms of which real asset positions are expressed.
M^v	"Full" consumption budget.
w	Market wage rate.
w^g	Wage rate gross of indirect tax on labor use.
w^n	Wage rate net of income tax and social security tax.
t_s	Social security tax rate.
t_y	General income tax rate.
$t_{l,j}$	Sector–specific indirect tax rate on labor use.
$t^c_{m,i}$	Effective tariff rate on private consumption of imported commodity i.
$t^G_{m,i}$	Effective tariff rate on investment demand for imported commodity i.
$t^I_{m,i}$	Effective tariff rate on investment demand for imported commodity i.
$t^Q_{m,ij}$	Effective tariff rate on intermediate input demand for commodity i by sector j for imported commodity i.

Table 9: Summary Information on Variable and Parameter Notation	
Part 1: Variables, continued	
Name	Description
$t_{v,i}^{c,h}, t_{v,i}^{c,m}$	Effective value added tax rate on private consumption of home produced and imported commodity i, respectively.
$t_{v,i}^{I,h}, t_{v,i}^{I,m}$	Effective value added tax rate on investment demand for home produced and imported commodity i, respectively.
$t_{x,i}^{c,h}, t_{x,i}^{c,m}$	Effective excise tax rate on private consumption of home produced and imported commodity i, respectively.
$t_{x,i}^{G,h}, t_{x,i}^{G,m}$	Effective excise tax rate on government demand for home produced and imported commodity i, respectively.
$t_{x,i}^{I,h}, t_{x,i}^{I,m}$	Effective excise tax rate on investment demand for home produced and imported commodity i, respectively.
$t_{x,ij}^{Q,h}, t_{x,ij}^{Q,m}$	Effective excise tax rate on intermediate input demand by sector j for home produced and imported commodity i, respectively.
y	Nominal non–wealth income (net labor income plus transfers).
z	Nominal lump–sum transfers from goverment to households.
d	Nominal tax deduction.
T	Total tax revenue, net of lump–sum tax deduction.
S^G	Primary government deficit net of interest payments, in nominal terms.
S^H	Household savings out of non–wealth income, in nominal terms.
S^F	Foreign trade surplus, in nominal terms.
A	Financial wealth, expressed in real terms.
r	Real interest rate of domestic financial wealth.
D^F	Net foreign assets owned by domestic households, expressed in nominal terms.
D^G	Government debt, expressed in nominal terms (exogenously determined at $\bar{D}^G$).
V_j	Total firm values in sector j, expressed in real terms.
$\mathcal{W}$	Total wealth, expressed in real terms.
H	Human wealth ex current income.
u	Momentary utility.
U	Lifetime utility.
Ω^{-1}	Marginal propensity to consume out of wealth.
χ_j	Dividends paid by sector j firms, expressed in nominal terms.
$\tilde{p}_j$	Price of the value added product of sector j.
q	Shadow value of the marginal unit of capital stock (Tobin's q).
uc_j	User cost of a unit of capital stock in sector j.
X	Labor efficiency factor.
N	Population size.

Table 9: Summary Information on Variable and Parameter Notation

Part 2: Parameters	
Name	Description
$\xi_i^{n,m}$	Share parameter of the sector i commodity aggregate for consumption ($n = c$), investment demand ($n = I$), and government procurement ($n = G$).
$\xi_{ij}^{Q,m}$	Share parameter of the sector i commodity aggregate as used by sector j for intermediate input.
σ_i^m	Elasticity of substitution between imported and home produced goods of sector i.
θ_i	Price elasticity of export demand.
κ_i^n	Share parameter of the overall commodity aggregate for consumption ($n = c$), investment demand ($n = I$), and government procurement ($n = G$).
ψ	Parameter of the installation cost function Φ.
ϕ_j	Scale parameter of the value added production function.
η_j	Labor share parameter of the value added production function.
μ_j	Elasticity of substitution between labor and capital in the value added production function.
e	Tax deduction rate for investment expenditure.
δ	Rate of decay of physical capital (identical for all sectors).
θ	Probability of dying between two periods.
γ	Intertemporal elasticity of substitution for "full" consumption.
π	Insurance premium received in case of survival.
α	Consumption share parameter in v.
β	Utility discount factor.
i^*	Real interest rate in terms of imported goods, net of the income tax rate.
i	Interest rate gross of income tax.
ρ	Subjective rate of time preference.
x	Rate of growth of labor productivity.
n	Rate of growth of the population.
ω_{t-a}, t	Population weight of generation $t - a$ as of period t.
$\bar{g}$	Rate of growth of the efficiency unit.

General remark: Variables appearing in sqare brackets in the figures above denote matrices, variables appearing with arrows denote vectors.

References

[1] Auerbach, Alan J. and Laurence J. Kotlikoff, 1987, Dynamic Fiscal Policy, Cambridge: Cambridge University Press.

[2] Blanchard, Olivier J., 1985, Debt, Deficits, and Finite Horizons, Journal of Political Economy 21, 223–247.

[3] Blanchard, Olivier J. and Fischer, S., 1989, Lectures on Macroeconomics, Cambridge: MIT Press.

[4] Buiter, Willem H., 1980, Walras' Law and All That: Budget Constraints and Balance Sheet Constraints in Period Models and Continuous Time Models, International Economic Review 21, 1–16.

[5] Buiter, Willem H., 1988, Death, Birth, Productivity Growth and Debt Neutrality, Economic Journal 98, 279–293.

[6] Deardorff, A.V. and Stern, R.M. (1986), *The Michigan Model of World Production and Trade*, Cambridge/Mass. - London: MIT Press.

[7] Frenkel, Jacob A. and Assaf Razin, 1987, Fiscal Policy and the World Economy, Cambridge: MIT Press.

[8] Harris, R.C. (1986), "Market Structure and Trade Liberalization: A General Equilibrium Assessment", in: Srinivasan, T.N. and Whalley, J. (eds.), *General Equilibrium Trade Policy Modeling*, Cambridge/Mass.: MIT-Press, 231–250.

[9] Harrison, G.W., Rutherford, T.F. and Wooton, I. (1991), "An Empirical Database for a General Equilibrium Model of the European Communities", *Empirical Economics*, 16, 95–120.

[10] Keuschnigg, Christian, 1991, The Transition to a Cash Flow Income Tax, Swiss Journal of Economics and Statistics 127, 113–140.

[11] Lächler, Ulrich (1985), "The Elasticity of Substitution between Imported and Domestically Produced Goods in Germany", *Weltwirtschaftliches Archiv*, 121, 74–96.

[12] Shiells, C.R., Deardorff, A.V. and Stern, R.M. (1986), "Estimates of the Elasticities of Substitution between Imports and Home Goods for the United States", *Weltwirtschaftliches Archiv*, 122, 497–519.

[13] Turnovsky, Stephen J., 1977, Macroeconomic Analysis and Stabilization Policy, Cambridge: Cambridge University Press.

[14] Weil, Philippe, 1989, Overlapping Families of Infinitely-Lived Agents, Journal of Public Economics 38, 183–198.