# THE ROLE OF ENVY IN ULTIMATUM GAMES

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Abstract:

The behaviour of subjects in ultimatum bargaining experiments is very different from that predicted by standard theory. These "anomalies" are frequently explained by fairness considerations. In this paper we consider the possibility that the subjects are simply envious. We derive the implications of envy for the behaviour in ultimatum games and it will be shown that envy is a potential explanation for the most important experimental "anomalies". This points toward the need to set up experiments which allow to discriminate between fairness- and envy-motivated behaviour.

## I. Introduction

The last decade has witnessed substantial progress in the theoretical analysis of bargaining situations. Among the most prominent bargaining models has been the 2-player alternating offers model of sequential bargaining as developed by Stahl (1972), Rubinstein (1981) and Binmore (1987). In this type of model one player, say player 1, starts with a proposal on how to divide a cake of size c. Then player 2 responds by acccepting or rejecting the proposal. If he accepts the game is over and both players receive the proposed shares. In case of rejection it is player 2 who makes a proposal in the next bargaining round. After a renewed rejection person 1 makes a proposal again and so on. In the last round of a finite horizon sequential bargaining game the proposing player has the power to pose an ultimatum because the responding player's options are restricted to " YES" or "NO", i.e. he cannot make a further counterproposal. Moreover, if he rejects both players get nothing. Therefore the finite horizon version of an alternating offers game is often termed as ultimatum game.

Under the usual assumptions of game theory one can solve ultimatum games by backward induction to get the subgame perfect equilibrium. Take for example the one-period game and assume - in accordance with the standard theory - that both players exhibit non-interdependent preferences, i.e. every players is only interested in *his* monetary or physical payoff. Since in case of rejection player 2 receives nothing he is prepared to accept any proposal which provides him with *any* positive share of the cake, regardless of how small this positive share is. In addition he has no reason to reject a proposal which gives him nothing. Player 1 anticipates this and will thus demand the whole cake for herself. The subgame perfect equilibrium is, therefore, a demand of c by 1 which will be accepted by 2.[2]

Unfortunately, this prediction has not been confirmed by experimental studies of one-period ultimatum games. In the experiment conducted by Güth, Schmittberger and Schwarze (GSS 1982) each subject played the game twice. The proposing players demanded significantly less than 100% of the cake. The mean demand was 63% in the first and 68% in the second game - the demands deviate significantly from the result of standard theory in direction of an equal share. Moreover, contrary to the predictions of standard theory a non-negligible number of *positive* offers has been rejected.

Similar evidence is proposed by Kahneman, Knetsch and Thaler (KKT 1986) and Forsythe, Horowitz, Savin and Sefton (FHSS 1988).

In a two-period ultimatum game player 2 has the right to propose a division of the cake in the second period. Since the last period is like a one-period game standard predicts that he demands the whole cake for himself which will be accepted by person 1. Both person have discount factors $\delta_1$ and $\delta_2$ respectively ($\delta_1$, $\delta_2 \in [0,1]$) that reflect the impatience of the players. Therefore the player 2's first period utility of such

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[2]Notice that the acceptance of "zero" by 2 is a best reply because rejecting does not make him better off.

a second period agreement is just c times his discount factor $\delta_2$. Therefore 2 will reject every first period proposal that gives him less than $\delta_2 c$. When player 1 makes her proposal in the first period she takes this into account and offers 2 an amount of y such that $y=\delta_2$, to give 2 no reason for rejecting the proposal. Therefore, in a subgame perfect equilibrium $y=\delta_2$ is offered to 2 in the first period which will be accepted immediately. Standard theory predicts that player 2's part of the cake is $\delta_2$ and player 1's part is $1-\delta_2$. In case that 1 offers less than $\delta_2 c$, 2 rejects but in equilibrium an offer of less than $\delta_2 c$ will not occur. It is also worthwhile to notice that $\delta_1$ has no impact at all on the equilibrium.

The experimental evidence of Güth, Tietz (GT 1988) and Ochs, Roth (OR 1989) is again not very favourable for the predictions of the standard theory, to say the least. The demands deviate significantly from the standard theory prediction in direction of an equal share. In the GT-experiment with a discount factor of $\delta_2 = 0.1$ the mean first period demand was 71% of the cake and the mean *accepted* demand was 69%, whereas standard theory predicts an demand of 90%. With $\delta_2 = 0.9$, when standard theory predicts an demand of 10% the mean demand was 57% and the mean accepted demand was 48%. The same deviation in direction of an equal share can be seen from the results of the OR-experiment with an discount factor $\delta_2=0.6$.
On the other hand standard theory predicts the results quite well for the OR-experiment with $\delta_2=0.4$. In our view this is caused by another feature that can be seen from the experimental results -

the first mover advantage. In a situation with a high $\delta_2$, when the deviation in direction of an equal share is positive for player 1, this deviation is greater than the deviation in a situation with a low $\delta_2$ when the deviation is positive for person 2. To put it differently: The tendency for an equal share is stronger when this tendency is in favour of player 1 than when this tendency is in favour of player 2. Therefore the first mover advantage reduces the deviation when $\delta_2$ is lower than 0.5, it causes results more close to that predicted by standard theory.[3]

This first mover advantage can also be seen from the results of the GT experiment. On the average person 1 demanded more than the half of the cake even in the case if person 1 was in a very bad strategic position, i.e. for $\delta_2$=0.9. Furthermore if we look at the accepted demands with $\delta$=0.9 the deviation in direction of equal share is 48-10=38%, whereas with $\delta$=0.1 this deviation is only 90-69=21% - the tendency for an equal share is stronger when it is positive for player 1 (when $\delta_2$=0.1) than it is when the

[3]Experimental data more favourable for the standard theory are reported by Binmore et.al. (1985) and - for the two period experiment - also by Neeling et.el. (1988). In our view this is caused at least partly by the first mover advantage that weakens the tendency for an equal share in situations with a discount factor of 0.25 as it were the case in both of these experiments. Furthermore Binmore et. al. have been severly critized by Thaler (1988) and FHSS (1988) because the instructions given to the subjects included the rhetorical question, "How do we want you to play?", followed by the statement ,"YOU WILL DO US A FAVOUR IF YOU SIMPLY MAXIMIZE YOUR WINNINGS". Whereas in the other experiments the subjects were volunteers, and the experiments were conducted under conditions of strict anonymity, Neeling et. al. required participation by the students in a class, and their instructions included the sentence: "You will be discussing the theory this experiment is designed to test in class." Therefore the whole situation in this experiment resembles more an exam than a bargaining situation.

tendency is positive for player 2 (when $\delta_2=0.1$).

Another feature found in the OR-experiment is the influence of the discount factor of person 1 $\delta_1$ in two period games that is not predicted by standard theory. For a given $\delta_2$ the mean first period demand of person 1 is decreasing when $\delta_1$ is increased.

As in the one period game the experimental data of the two period game reveal a considerable number of rejected offers. For example, in the OR-experiment about 15% of the first period offers were not accepted and even 45% of the last period offers have been rejected. In the GT-experiments the share of rejected offers is even larger. When $\delta_2=0.1$, 19% of the first period offers were not accepted and when $\delta_2=0.9$ the share of rejected offers raise to 62% (!).

Another important result concerns the counteroffers of person 2 after a rejection. Contrary to the predictions of the standard theory they have often been *lower* than the rejected offers. According to the standard theory a rational player 2 will only reject a first period offer if he plans to demand in the second period more than he was offered in the first period by player 1. But in the experiment of OR, 81% of the counteroffers were disadvantageous for player 2. In the experiment of GT disadvantageous counteroffers were forbidden. If someone made such a, let us say, irrational counteroffer, then it was counted as a disagreement and both player got automatically nothing. Even in this case 35% of the counteroffers were disadvantageous.

To summarize, experimental results deviate in the following way from the predictions of standard theory:

1) The offers differ systematically from those predicted by standard theory. This deviation is always in the direction of equal share.
2) A first mover advantage can be observed.
3) In the two period games the discount factor of player 1 plays a role although it should not. An increase of $\delta_1$ decreases the first period demand of person 1.
4) There is a remarkable amount of nonagreement, in the first period as well as in the last.
5) Most of the counteroffers are disadvantageous relative to the rejected offers.

It will be shown in this paper that envy can explain these experimental results of the one- and two period ultimatum games. The paper is organized in the following way. In the next section the interpretations of the experimental results are discussed. In section III and IV models of ultimatum games are presented that show the impact of envy on the one- and the two period ultimatum games. In section V we analyse the bargaining behaviour if the importance of envy varies with the income and in the last section the conclusions are drawn.

## II. Interpretations of the Experimental Results

GT (1990) offer an interpretation of the results that is the least favourable for the standard theory. They conclude that "observed bargaining behavior clearly contradicts the most obvious rationality requirements of game theory and also of economic theory. ... Experimentally observed ... behavior reveals how considerations of distributive justice seriously destroy the prospects of exploiting strategic power."[4]

In our view this conclusion does not necessarily follow from the experimental evidence cited in the preceding section. To "explain" the results 1) - 5) above one is neither forced to abandon the usual rationality assumptions nor it is necessary to assume that people are motivated by distributive justice. In the following sections we try to argue that the deviation from the predictions of the standard theory can be accounted for by rational behaviour of utility maximizing agents whose preferences exhibit some envy.

Our approach is closer to the interpretation of OR (1988). Because of the high amount of disadvantageous counteroffers in their experiment, OR came to the conclusion: "Monetary payoffs do not capture the utility of the bargainers. There are nonmonetary arguments in the bargainers utility function."[5] OR did not explicitly analyse the impact of such "uncontrolled elements" of the utility function. Rather they assumed a minimum acceptance threshold that takes the form of a minimum percentage of the cake. An offer that gives a person less than this minimum threshold is regarded as insultingly low and the disutility of accepting such an offer is greater than the utility of a monetary gain. OR called this uncontrolled element of the utility function that they use as motivation for their minimum threshold assumption "fairness". But in our view this expression is

[4]Güth, Tietz 1990, p. 446

[5]Ochs, Roth 1989, p 365

misleading, because according to their interpretation of the experimental results people are not concerned about every deviation from a fair share. They are only concerned if this deviation is disadvantageous for themselves.

Although we think that fairness motivation plays an important role in the explanation of data we doubt that they can explain the behaviour fully for the following reasons. First of all it seems difficult to reconcile the first mover advantage with fairness considerations. Is it fair that the proposing players in almost all cases demand more than the half of the cake? Why should it be fair that the player who starts proposing actually gains relative to the prediction of the standard theory when she is in a bad position than she actually looses relative to the standard predictions when she is in a good position? GT (1988) use the behavioural theory of distributive justice, also called "equity theory", as explanation for the demands and the actual agreements. According to this theory the relation of investments (or efforts) and rewards should be equal for all persons of the relevant group. To explain the first mover advantage GT suggest that the subjects regard the investments of those playing the role of player 1 being higher than the investments of those playing the role of person 2. But in their experiment as well as in the OR-experiment the roles of player 1 and player 2 respectively were randomly assigned to the subjects. In our view it is quite unplausible that the subjects regard a randomly chosen player 1 being entitled to get more because of her "higher investment" or "effort".

Furthermore the analysis of FHSS (1988) indicate that fairness preferences of player 1 cannot fully account for the data of the ultimatum games. These authors tested the hypothesis that the distribution in 1-period ultimatum and dictator games coincide. In a dictator game player 2 has no options at all, that is player 1 has dictatorial power to divide a given amount of money. The choice of the dictator can be interpreted as reflecting truly her concerns for a fair distribution. Thus, if "players give away money only for a desire to be fair, the distribution of proposals in dictator and ultimatum games with equal pies would be identical. This clearly did not happen in our games with equal pay, where the fairness hypothesis is clearly rejected"[6], because the dictators offered considerable less money to player 2 than the proposers in ultimatum games. One can of course argue that this behaviour of player 1 reflects at least *some* degree of fairness. The obvious counterargument is, however, that player 1 anticipates that 2 is envious and that she concedes a positive share to reduce the probability of disagreement. The proposal data of player 1 are, therefore, compatible with purely selfish behaviour.

The attempt to rationalize the experimental data by assuming interdependent preferences has been strongly critized by GT: "We strictly reject the idea to include results of analysing a social decision problem into the utility function of the interacting agents. Utility functions are an instrument of describing individual characteristics needed to define a social decision problem. Furthermore, all our experiences from ultimatum bargaining indicate that subjects do not maximize but are guided

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[6]Forsythe et.al. 1988, p. 23

by some conflicting behavioral norms. The utility approach necessarily neglects the dynamic structure of the intellectual process which subjects apply to derive their decision behavior."[7]

This quotation suggests that GT prefer an explanation of the evidence that is based on norm-guided behaviour rather than one that relies on a modification of preferences. In our view, however, this approach still leaves many questions unanswered. In particular, one needs a clear definition of a norm. Is a norm a behavioural regularity that is sustained by constraints which are external to the individual? In a recent paper Holländer (1990) explains the emergence of norms by the positive sentiments received by those who abide by the norm from their friends or other group members. But this argument is not applicable in the context of ultimatum games with anonymos players because the conformity with the norm cannot be observed. Or put it differently: The behavioural regularity is caused by an internalized norm.

In our view the concept of internalized norms demands that the assumption of a unitary self is replaced in favour of the multiple self assumption. If one allows for two selves with divergent objectives which both have a psychic impact on one another it becomes possible to capture the "dynamic structure of the intellectual process which subjects apply to derive their decision behaviour." Unfortunately, we do not know of a convincing model of that type. We do not see that the equity theory used by GT is better able to explain the "dynamic

[7]Güth, Tietz 1990, p 440

structure of the intellectual process" than the concept of preferences. In addition equity theory cannot capture the impact of conflicting norms on behaviour.
Thus, although we are in principle not hostile to the idea that we should model the dynamic interaction of different selves to analyze the impacts of internalized norms we are at the same time not persuaded by the solution offered by GT. Moreover, it may turn out that a more complicated model will in the end yield the same predictions as the conventual approach with a suitable augmented objective function. And finally, as it will turn out that our more moderate envy-approach is capable of explaining all of the above mentioned empirical regularities it derives some justification from the principle of parsimony.

## III. The One Period Envy Model

A cake c consisting of money should be divided.

$$c = x + y \qquad (3.1)$$

with x the amount of money player 1 gets and y the amount of money person 2 gets.
The preferences are represented by the continuous utility functions:

$$\begin{aligned} &\textit{Utility of person } 1: u = u\,(x,y) \quad u_x > 0 \quad u_y < 0 \\ &\textit{Utility of person } 2: v = v\,(y,x) \quad v_y > 0 \quad v_x < 0 \end{aligned} \qquad (3.2)$$

The more person 2 gets, the less is the utility of person 1, and the more person 1 gets, the less is the utility of person 2 - both players are envious.

Furthermore, it is assumed:

$$u_x(m,m) > -u_y(m,m)$$
$$v_y(m,m) > -v_x(m,m) \tag{3.3}$$
$$\text{for all } m > 0$$

This assumption means that both persons are not too envious - both care more about their own gain than about the gain of the other person. It implies:

$$u(m,m) > u(0,0)$$
$$v(m,m) > v(0,0) \tag{3.4}$$
$$\text{for all } m > 0$$

Person 1 makes the offer and she must give player 2 an amount of money $y^*$ that makes player 2 indifferent between acceptance and nonacceptance.

$$v(0,0) = v(y^*,x) = v(y^*,c-y^*) \tag{3.5}$$

Notice that because of (3.2):

$$\frac{dv(y,c-y)}{dy} = v_y - v_x > 0$$
$$v(0,c) < v(0,0) < v(c,0) \tag{3.6}$$

Therefore

$$\exists!\ y^*:\ 0 < y^*,\ v(0,0) = v(y^*, c - y^*) \qquad (3.7)$$

The equilibrium offer is greater than zero, whereas zero is the result predicted by standard theory.
Furthermore equal share gives person 2 a greater utility than disagreement because of (3.4). Therefore:

$$v(0,0) < v(m,m) \quad \Rightarrow \quad y^* < \frac{c}{2} \qquad (3.8)$$

The upper bound of the amount of envy, i.e. assumption (3.3) is sufficient to guarantee the possibility of an agreement. Look at the utility of person 1 from an offer $y^*$ that is acceptable for person 2. Because of (3.8):

$$u(c - y^*, y^*) > u(\frac{c}{2}, \frac{c}{2}) > u(0,0) \qquad (3.9)$$

An offer of $y^*$ gives subject 1 a greater utility than disagreement.
If the assumptions about the amount of envy (3.2) do not hold, i.e if the subjects are extraordinarily envious, than an agreement acceptable for person 2 would give person 1 less utility than disagreement. In this case person 1 would make an offer that is unacceptable for person 2 - disagreement would be the only equilibrium.
Such extraordinary envy is one possible reason for the disagreement found in the experiments. Another, more plausible reason is that the amount of envy is private information. Even

without having analyzed such a game with incomplete information it is reasonable to assume that person 1 will underestimate the envy of person 2 with a positive probability. In this case person 1 offers person 2 too little - disagreement occurs. Therefore both results of the one period ultimatum game experiments, positive offers and disagreement, can be explained by this one period envy model.

## IV. The Two Period Envy Model

As already explained in the two period ultimatum game experiments a deviation in direction of equal share, an influence of the discount factor of person 1, disagreement and disadvantageous counteroffers have been found. All of this features can be explained with envy. Furthermore, if this envy is variable - in a sense that will become clear in the next section - a first mover advantage is implied by the envy model.

But let us first analyze the model with constant envy. In 2-period games the discount factors $\delta_1$, $\delta_2$ play the crucial role for the division of the cake ($\delta_i \in [0,1]$ for i=1,2). In theory $\delta_1$, $\delta_2$ reflect the intrinsic impatience of the players. This intrinsic impatience cannot be controlled in experiment. Furthermore intrinsic impatience can be neglected in the experiments because the second period follows immediately after the first period. To mimic the intrinsic impatience and to control for the discount factors in 2-period games the cake does not consist of real money but of experimental money, of "chips".

$$C = W_1 + W_2 \tag{4.1}$$

with $w_1$ and $w_2$ as the amount of chips the subjects receive in the case of agreement.

In case of a first period agreement the players get one unit of real money for one chip. In case of a second period agreement they only get $\delta_1$ and $\delta_2$ units of real money respectively for one chip. Therefore these $\delta_1$, $\delta_2$ used in the experiments are conversion factors. If the assumption of the standard model are correct, i.e. if the subjects have non-interdependent preferences the use of such conversion factors is an adequate way to mimic intrinsic impatience. But as we shall see conversion factors are no longer an adequate method to mimic impatience in experiments if the subjects have interdependent preferences (see footnote 9).

In our model the preferences are represented by the following utility functions:[8]

$$\begin{aligned} &\textit{Utility of person } 1: \quad u(x,y) = x - ay \\ &\textit{Utility of person } 2: \quad v(y,x) = y - bx \\ &\qquad a,b \in (0,1) \end{aligned} \tag{4.2}$$

x is the amount of real money person 1 gets, y the amount of real money 2 gets. a,b are constant envy parameters. If this parameter is greater than 0, the person is envious, if it is less than 0, he would be altruistic. However, for what follows it is assumed

[8]For mathematical convenience these utility functions are less general than those assumed in the one period case.

that a,b are greater than zero and less than one. This means that both are envious, but they care more about their own income than about the income of the other person. This is the same assumption as (3.3) in the one period case.

To calculate the subgame perfect equilibrium backward induction is used.

If agreement is reached in the second period, then:

$$y^2 = \delta_2 w_2^2$$
$$x^2 = \delta_1 (c - w_2^2) \tag{4.3}$$

In the second period person 2 makes the offer. Notice that u=v=0 holds if both get nothing. To make person 1 indifferent between accepting and rejecting the second period offer of person 2 must obey:

$$0 = x^{2*} - ay^{2*} = \delta_1 c - \delta_1 w_2^{2*} - a w_2^{2*} \delta_2 \tag{4.4}$$

with $w_2^{2*}$ as the amount of chips that person 2 gets from a second period agreement that is acceptable for person 1.[9]

Using (4.1), (4.3), (4.4) leads to:

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[9] If our assumptions about the preferences hold and if the subjects are envious, the use of conversion factors is no longer an adequate way to mimic intrinsic impatience. Also in the case of envious subjects real impatience plays no role for the second period agreement if the second period is already reached. It only plays a role for the first period agreement because person 2's discounted utility of a period-2-agreement is his reservation utility for the first-period offer. In contrast the conversion factors $\delta_1$ and $\delta_2$ also play a role for the second period equilibrium condition (see (4.4)). Therefore they are not adequate to control for intrinsic impatience if the participants of the experiments are envious.

$$w_2^{2*} = \frac{\delta_1 c}{\delta_1 + a\delta_2}$$

$$x^{2*} = \frac{a\delta_1\delta_2 c}{\delta_1 + a\delta_2} \tag{4.5}$$

$$y^{2*} = \frac{\delta_1\delta_2 c}{\delta_1 + a\delta_2}$$

By substituting in (4.2) the utility of person 2 from such an agreement in period 2 is given by:

$$V^{2*} = \frac{c\delta_1\delta_2\ (1-ab)}{\delta_1 + a\delta_2} \tag{4.6}$$

This utility is greater than zero, i.e. greater than the utility of disagreement if a,b are less than one. The upper bound for the envy parameters is sufficient to guarantee the possibility of a second period agreement.

If agreement is reached in the first period, then:

$$x^1 = c - w_2^1 = c - y^1$$
$$y^1 = w_2^1 \tag{4.7}$$

An accepted first period offer must give person 2 at least as much utility as he can obtain by rejecting the offer and making the counteroffer in period 2. Therefore a first period offer that is accepted by person 2 must obey:

$$y^{1^*} - bx^{1^*} = y^{2^*} - bx^{2^*} \tag{4.8}$$

Using (4.5), (4.7), (4.8) leads to:

$$y^{1^*} = \frac{c(\delta_1\delta_2 - ab\delta_1\delta_2 + b\delta_1 + ab\delta_2)}{(\delta_1 + a\delta_2)(1+b)} \tag{4.9}$$

$y^{1*}$ is the amount of money (and chips) that person 2 gets if agreement is reached in the first period.

Using (4.2), (4.7) and (4.9) the utility of person 1 from such an agreement is given by:

$$u^{1^*} = c\frac{\delta_1\ (1 - ab)\ (1 - \delta_2) + a\delta_2\ (1 - ab)\ (1 - \delta_1)}{(\delta_1 + a\delta_2)\ (1 + b)} \tag{4.10}$$

This utility is greater than zero, the utility of person one from a second period offer, if a,b are less than one - if the people are too envious, agreement is, like in the one period game, impossible.

Relative to the cake size player 1 offers player 2:

$$d_2^{1^*} = \frac{y^{1^*}}{c} = \frac{\delta_1\delta_2 - ab\delta_1\delta_2 + b\delta_1 + ab\delta_2}{(\delta_1 + a\delta_2)\ (1 + b)} \tag{4.11}$$

The derivative of this relative offer $d_2^{1*}$ with respect to $\delta_2$ is given by:

$$\frac{\partial d_2^{1^*}}{\partial \delta_2} = \frac{\delta_1^2(1-ab)}{(\delta_1 + a\delta_2)^2(1+b)} > 0 \tag{4.12}$$

Now introduce a new variable k, defined by:

$$k := d_2^{1^*} - \delta_2 \tag{4.13}$$

Remember that without the envy term $d_2^{1*}$ should be equal $\delta_2$ (see section 1). This implies that k is the relative amount of money person 2 gets more than predicted by standard theory.
Using (4.11) leads to:

$$k = -\frac{a\delta_2(b(\delta_1+\delta_2-1)+\delta_2)+b\delta_1(\delta_2-1)}{(\delta_1+a\delta_2)(1+b)} \tag{4.14}$$

Notice that if a,b go to zero, than k is zero and we are back in the standard model.

$$\lim_{\substack{a\to 0\\ b\to 0}}(k) = 0 \tag{4.15}$$

Otherwise, regardless of the values of a and b and $\delta_1$, k is greater than zero for $\delta_2$=0 and less than zero for $\delta_2$=1.

$$\begin{aligned} k(\delta_2=0) &= \frac{b}{b+1} > 0 \\ k(\delta_2=1) &= -\frac{a(1+b\delta_1)}{(a+\delta_1)(b+1)} < 0 \end{aligned} \tag{4.16}$$

The derivative of k with respect to $\delta_2$ is always negative.

$$\frac{\partial k}{\partial \delta_2} = -\frac{a^2(\delta_2)^2(b+1)+a\delta_1(b(\delta_1+2\delta_2)+2\delta_2)+b(\delta_2)^2}{(a\delta_2+\delta_1)^2(b+1)} < 0 \quad (4.17)$$

k is monotonically decreasing in $\delta_2$, but because of (4.12) this derivative is greater than -1.

Furthermore, the second derivative with respect to $\delta_2$ is also negative.

$$\frac{\partial^2 k}{(\partial \delta_2)^2} = -\frac{2a\delta_1(1-ab)}{(a\delta_2+\delta_1)^3(b+1)} < 0 \qquad (4.18)$$

Therefore the graph of the function with respect to $\delta_2$ is:

(Insert figure 1)

As it can be seen from the figure, if $\delta_2$ is close to zero (in which case the game is approximately a one period game), k is positive, which means that the result is more close to equal share than in the model without envy. On the other hand, if $\delta_2$ is close to one, k is always negative, 2 gets less than in the model without envy - again a deviation in direction of equal share.

Furthermore, if the situation is good for player 2, i.e. if $\delta_2$ is high, then a further increase of $\delta_2$ leads to a relatively great further deviation in direction of equal share. Person 2 profits only a little by the increase of $\delta_2$. On the other hand, if $\delta_2$ is small, i.e. if the position of person 2 is bad, then a further decrease of $\delta_2$ leads to a relatively small further deviation in direction of equal share. In this case person 2 has a relatively small disadvantage of a further deterioration of his position.

The standard theory predicts that $\delta_1$ has no influence at all. But in the envy model the derivative of k with respect to $\delta_1$ is positive.

$$\frac{\partial k}{\partial \delta_1} = \frac{a(\delta_2)^2(1-ab)}{(a\delta_2+\delta_1)^2(b+1)} > 0 \qquad (4.19)$$

The more person 1 gets for a chip in the second period, the less chips person 2 must give him in the second period and the greater is the utility of person 2 of a second period agreement. An increase of player 2's second period utility, which is his reservation utility for the first period offer, improves of course 2's position for the first period agreemeent. This is just the same influence of $\delta_1$ that is reported from the OR experiment.

The derivative of k with respect to a is negative, with respect to b positive:

$$\begin{aligned} \frac{\partial k}{\partial a} &= -\frac{\delta_1\delta_2(b\delta_1+\delta_2)}{(a\delta_2+\delta_1)(b+1)} < 0 \\ \frac{\partial k}{\partial b} &= \frac{a\delta_2(1-\delta_1)+\delta_1(1-\delta_2)}{(a\delta_2+\delta_1)(1+b)^2} > 0 \end{aligned} \qquad (4.20)$$

The more envious person 1 is and the less envious person 2 is, the worse is the outcome for player 2.

In this model disagreement may occur if, as already mentioned, the persons are very envious. Another, more plausible reason is that the envy parameters are private information. The resulting counteroffer may be disadvantagous, if $\delta_1$ or $\delta_2$ are greater than zero. Using (4.5), (4.7), (4.8) to calculate $y^{1*}$ depending on $y^{2*}$ gives:

$$y^{1*} = c\left(\frac{b}{1+b} - \frac{b\delta_1}{1+b}\right) + y^{2*}\left(1+\frac{b\delta_1}{\delta_2(1+b)}\right) > y^{2*} \quad \Rightarrow \qquad (4.21)$$
$$\exists\, y^{1'}:\; y^{2*} < y^{1'} < y^{1*}$$

$y^{1*}$ is strictly greater than $y^{2*}$. Now, if player 1 slightly underestimates b, the envy parameter of person 2, and if person 1 therefore offers $y^{1'}$, which is less than $y^{1*}$, then the resulting counteroffer $y^{2*}$ is less than $y^{1'}$, the counteroffer is disadvantageous.

The reason for this possibility of disadvantageous counteroffers is that person 2 is content with less money in the second period than in the first because person 1 also gets less in the second period and therefore 2's envy is smaller in the second period.

As already explained, person 1 is in a good position if she is very envious. But to analyze the first mover advantage one to look at the situation of an equal of the envy parameters of both persons because it cannot be assumed that all the subjects that played the role of person 1 in the experiments were more envious than those playing the role of person 2.

Furthermore the first mover advantage has been also found in the GT-experiments where both persons had the same discount factors. Therefore we look at situations with $\delta_1=\delta_2=\delta$.

Formally the first mover advantage can be defined by the following condition:

$$k(\delta=\delta^\alpha) < -k(\delta=\delta^\beta=1-\delta^\alpha)$$
$$\text{for } \delta^\alpha \in [0,\frac{1}{2}] \quad \text{for } a=b \tag{4.22}$$

The intuition behind this definition is that the amount of k should be greater for a great $\delta$ ($\delta^\beta$ situation - in this case the deviation from the standard result is positive for person 1) than for a low $\delta$ ($\delta^\alpha$ situation -in this case the deviation is positive for person 2) under the assumption that both subjects are equally envious - In a situation when the tendency of an equal share is positive for person 1 this tendency is stronger than in a situation when it is positive for person 2.

Using (4.14) and the relation $\delta^\beta=1-\delta^\alpha$ leads to:

$$k(\delta^\alpha) + k(\delta^\beta) = 0 \tag{4.23}$$

Therefore:

$$k(\delta^\alpha)=-k(\delta^\beta) \tag{4.24}$$

The amount of k is the same for $\delta=\delta^{\alpha}$ and for $\delta=\delta^{\beta}=1-\delta^{\alpha}$ for all $\delta^{\alpha}$ between zero and one half - the advantage arising from envy is just symmetric for both persons, the first mover advantage cannot be explained in this model.

In this model we have assumed that the envy parameters are constant. In the next section we will make another, more plausible assumption that will cause a first mover advantage.

## V. The Impact of Variable Envy Parameters

Until now we have assumed that the envy parameters are constant. But one can imagine that the importance of one's own monetary gain relative to the importance of the monetary payoffs of the others is increasing with one's own gain. For example, let us take a look at two games. In the first game a cake of 10 US$ has to be divided and person one offers 1 US$. In the second game 10 mill.US$ has to be divided and the offer is 1 mill. US$. It is quite likely that person 2 would rather reject the first than the second offer. Such a behaviour cannot be explained by the model of the previous chapter because the portion of the cake acceptable for person 2 is always the same for given discount factors (see (4.11)).

To capture the decreasing relative importance of envy, assume that the utility functions are given by:[10]

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[10]If x,y are interpreted as nominal values of money the "envy attitude", i.e. e(.) changes also when only the measure of money is changed. This implies that the preferences represented by such utility functions are not invariant against changes of the measure of money. Therefore x,y should be interpreted as real values of money that are invariant against any change of the measure of money. The "envy attitude" does not change with the

$$u(x,y) = x - e(x)y$$
$$v(y,x) = y - e(y)x \qquad (5.1)$$

As already explained we assume that both players have the same "envy attitude" when we analyse the first mover advantage. Therefore we use the same envy function e(.) for both players. This function is continous and has the properties:

$$e(0) < 1 \quad e' < 0 \quad e(m) > 0 \;\; for\ all\ m \geq 0 \qquad (5.2)$$

These assumptions imply that the envy parameter is decreasing with an increase of a person's income, but since e(.) is always greater than zero, both persons care at least a bit about the income of the other person.

We will consider a two period ultimatum game. To analyse the first mover advantage we look at the case where both persons have the same discount factor $\delta$.

In the second period, the equilibrium condition is given by:

$$0 = x^{2^*} - e(x^{2^*})y^{2^*} \qquad (5.3)$$

Using (5.5) leads to:

$$x^{2^*} = \frac{c\delta e(x^{2^*})}{1+e(x^{2^*})}$$
$$y^{2^*} = \frac{c\delta}{1+e(c\delta - y^{2^*})} \qquad (5.4)$$

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unit of measurement and only a change of the real gain from the game has an influence on the "envy attitude".

The right hand side of condition (5.3) is less than zero for $x^{2*}=0$ (except in the case when $\delta=0$) and greater than zero for $x^{2*}=\delta c$. Furthermore the right hand side of the equilibrium condition is strictly increasing with $x^{2*}$. Therefore $x^{2*}$ is strictly positive (except in the case when $\delta=0$) and the solution is unique.

Notice that if $\delta$ is greater than zero $x^{2*}$ is less and $y^{2*}$ is greater than $\delta c/2$ because with $x^{2*}=y^{2*}=\delta c/2$ the right hand side of the equilibrium condition is strictly positive. Therefore:

$$\begin{aligned} & x^{2*} < \frac{\delta c}{2} < y^{2*} \quad \text{for } \delta > 0 \\ & x^{2*} = y^{2*} = 0 \quad \text{for } \delta = 0 \end{aligned} \tag{5.5}$$

Person 2's utility from such an agreement is:

$$v^{2*} = y^{2*} - e(y^{2*})\,x^{2*} \geq 0 \tag{5.6}$$

Because of (5.5) this utility is greater or equal than zero, an agreement acceptable for person 1 is also acceptable for person 2.

The derivatives with respect to $\delta$ are:

$$\begin{aligned} & \frac{\partial x^{2*}}{\partial \delta} = x^{2*\prime} = \frac{ce(x^{2*})}{1-e'(x^{2*})\,x^{2*}+e(x^{2*})} > 0 \\ & \frac{\partial y^{2*}}{\partial \delta} = y^{2*\prime} = c\frac{1-e'(x^{2*})}{1-e'(x^{2*})\,y^{2*}+e(x^{2*})} > 0 \end{aligned} \tag{5.7}$$

Furthermore, because of (4.3), (5.2), (5.7) and the assumption $\delta_1=\delta_2=\delta$:

$$\begin{aligned} & x^{2*'}+y^{2*'} = c \\ & 0 < x^{2*'} < \frac{c}{2} \\ & \frac{c}{2} < y^{2*'} < c \end{aligned} \tag{5.8}$$

The equilibrium condition for the first period is given by:

$$y^{1*}-e(y^{1*})x^{1*} = y^{2*}-e(y^{2*})x^{2*} \tag{5.9}$$

For $y^{1*}=0$ the left hand side of the condition is less than the right hand side , for $y^{1*}=c$ it is greater . Therefore $y^{1*}$ is between zero and c. The solution for $y^{1*}$ is unique, because the left hand side of this condition is strictly increasing with $y^{1*}$. (4.7) and (5.9) leads to:

$$y^{1*} = e(y^{1*})(c-y^{1*})+y^{2*}-e(y^{2*})x^{2*} \tag{5.10}$$

Besides $y^{1*}$ is strictly greater than $y^{2*}$ (except when $\delta=1$).

$$\begin{aligned} & y^{1*} > y^{2*} \quad \text{for } \delta < 1 \\ & y^{1*} = y^{2*} \quad \text{for } \delta = 1 \end{aligned} \tag{5.11}$$

To see this, assume for a moment that $y^{1*}$ is less or equal $y^{2*}$. Then, because of (5.2), $e(y^{1*})$ would be greater or equal $e(y^{2*})$.

Furthermore, because of (4.3) and (4.7), $x^{1*}$ would be unambigously greater than $x^{2*}$ (when $\delta < 1$). But by using the equilibrium condition (5.11) $y^{1*}$ would then be greater than $y^{2*}$ - a contradiction. Like in the previous model there is again the possibility of a disadvantageous counteroffer.

On the other hand, $y^{1*}$ is less than c, because with $y^{1*}=c$ the left hand side of the equilibrium condition is strictly greater than the right hand side.

Substitution of (4.7) and (5.4) in (5.9) and rearranging the term leads to:

$$x^{1*} = \frac{c-\delta c}{1+e(y^{1*})} + \frac{x^{2*}(1+e(y^{2*}))}{1+e(y^{1*})} \tag{5.12}$$

Because of (5.2), (5.11) and (5.12) $x^{1*}$ is greater than $x^{2*}$.

$$\begin{aligned} x^{1*} &> x^{2*} \quad for\ \delta < 1 \\ x^{1*} &= x^{2*} \quad for\ \delta = 1 \end{aligned} \tag{5.13}$$

If an agreement is possible in the first period the utility of person 1 from an first period agreement must be greater or equal than zero i.e. person 1's utility from an second period agreement.

$$x^{1*} - e(x^{1*})\,y^{1*} \geq 0 \tag{5.14}$$

Using (4.7) leads to the condition:

$$x^{1*} \geq \frac{e(x^{1*})\,c}{1+e(x^{1*})} \tag{5.15}$$

Calculating $x^{1*}$ by using (4.7), (5.4), (5.10) leads to:

$$x^{1*} = \frac{e(x^{2*})c}{1+e(x^{2*})} + \frac{c(1-\delta)}{(1+e(y^{1*}))(1+e(x^{2*}))} \tag{5.16}$$

Because of (5.12) and (5.16) condition (5.15) is satisfied - an offer acceptable for person 2 is also acceptable for person 1. Substituting (4.3) in (5.10) and calculating the derivative of $y^{1*}$ with respect to $\delta$ one gets:

$$\frac{\partial y^{1*}}{\partial \delta} = \frac{[y^{2*\prime}+e(y^{2*})y^{2*\prime}-e(y^{2*})c]+e'(y^{2*})y^{2*\prime}(y^{2*}-c\delta)}{1-e'(y^{1*})(c-y^{1*})+e(y^{1*})} > 0 \tag{5.17}$$

Because of (5.4) and (5.8) $y^{1*}$ is increasing with $\delta$.
To analyse the properties of this model we use again the variable k that measures the relative deviation of the results of our model from that of the standard model. Analogous with (4.13) of the previous section k is defined by:

$$k := \frac{y^{1*}}{c} - \delta \tag{5.18}$$

Analogous with (4.22) of the previous section we define the first mover advantage as:

$$k(\delta^{\alpha}) < -k(\delta^{\beta}) \quad with \ \delta^{\alpha}+\delta^{\beta} = 1$$
$$for\ all \ \delta^{\alpha} \in [0, \tfrac{1}{2}] \tag{5.19}$$

The amount of deviation is greater when $\delta$ is high (in this case the deviation is favourable for person 1) than when $\delta$ is low (in this case the deviation is favourable for person 2). Using (5.18) leads to:

$$\begin{aligned} k(\delta^{\alpha})+k(\delta^{\beta}) &< 0 \quad => \\ y^{1}(\delta^{\alpha})+y^{1}(\delta^{\beta}) &< c \end{aligned} \tag{5.20}$$

To calculate the first mover advantage compare two games - a game with a discount factor $\delta^{\alpha} \in [0,1/2]$, called the $\alpha$-game, and a game with a discount factor $\delta^{\beta}=(1-\delta^{\alpha})$, called the $\beta$-game.
The first period equilibrium conditions are given by (see 5.10):

$$\begin{aligned} \alpha\text{-}game\text{: } y^{1\alpha} &= y^{2\alpha}-e(y^{2\alpha})x^{2\alpha}+e(y^{1\alpha})x^{1\alpha} \\ \beta\text{-}game\text{: } y^{1\beta} &= y^{2\beta}-e(y^{2\beta})x^{2\beta}+e(y^{1\beta})x^{1\beta} \end{aligned} \tag{5.21}$$

We define:

$$\begin{aligned} &f:=e(x^{2\alpha}) \quad g:=e(y^{1\alpha}) \quad h:=e(y^{2\alpha}) \\ &j:=e(x^{2\beta}) \quad k:=e(y^{1\beta}) \quad l:=e(y^{2\beta}) \end{aligned} \tag{5.22}$$

Because of (5.7), (5.11) and (5.17):

$$\begin{aligned} x^{2\alpha} &\le x^{2\beta} \\ y^{2\alpha} &< y^{1\alpha} \\ y^{2\alpha} \le y^{2\beta} &\le y^{1\beta} \\ y^{1\alpha} &\le y^{1\beta} \end{aligned} \tag{5.23}$$

Furthermore $y^{1\alpha}$ is greater or equal $x^{2\beta}$. Because of (5.17) $y^{1\alpha}$ is lowest when $\delta^{\alpha}=0$ and because of (5.7) $x^{2\beta}$ is greatest when $\delta^{\beta}=1=1-\delta^{\alpha}$. In this case $y^{2\alpha}$ and $x^{2\alpha}$ are zero and the equilibrium conditions for $x^{2\beta}$ and $y^{1\alpha}$ are given by:

$$\begin{aligned} x^{2\beta}-e(x^{2\beta})(c-x^{2\beta}) &= 0 \\ y^{1\alpha}-e(y^{1\alpha})(c-y^{1\alpha}) &= 0 \end{aligned} \qquad (5.24)$$

In this extreme case the solution for both conditions are equal. Otherwise $y^{1\alpha}$ is greater than $x^{2\beta}$. Therefore and because of (5.23):

$$\begin{gathered} k \leq g \leq j \leq f \\ g \leq h \\ k < l \leq h \end{gathered} \qquad (5.25)$$

Using (5.4), (5.10), (5.20), (5.21) and (5.22) the condition for the first mover advantage becomes after some lengthy manipulations:

$$\begin{aligned} 0 < {} & fj[gl-gk-\delta(gl-hk)] + f[jl-gk-\delta(jl-hk)] \\ & + gj[l-k] + [1+f][j-g] + [jl-gk] + \delta j[fh-gl] \qquad (5.26) \\ & + \delta[fg-jk] + \delta[fh-jl] + \delta[f-j] + \delta[g-k] \end{aligned}$$

Because of (5.25) all the differences in the brackets are positive. Therefore condition (5.26) always holds - the model implies a first mover advantage.

The reason for this first mover advantage is intuitively quite obvious. Person 1's envy is important for the second period offer when the monetary cake size (and therefore the amount of money the persons can get) is relatively small compared with the first period, when person 2's envy plays the crucial role. Therefore person 1's envy is - in some sense - greater than person 2's envy, but not because person 1 is a more envious subject, but because her envy is important in a situation when all the subjects act more enviously.

Whether this model with a decreasing importance of envy gives an adequate description of the real behaviour of the people is of course an empirical question. In the experiments made until now only a very slight influence of the cake size - that is implied by this model - is reported.[11] But the cake size was always relatively small and therefore the results may be consistent with this model. Furthermore all persons (economists and noneconomists) with whom we spoke about these experiments first asked us for the cake size and then told us that their behaviour would have had depended on it. The very plausible example we made at the beginning of this chapter (comparison of 2 games, one with a big and one with a small cake) was made by almost everyone to explain the crucial role of the absolute amount of money for the possibility of rejecting an offer.

## VI. Conclusions

Our envy model shows the impacts of the consideration of envy and how envy can explain the evidence of the experiments, especially

[11]See GT 1987, p. 444

the deviation in direction of equal share, the importance of $\delta_1$, disagreement, disadvantagous counteroffers and the first mover advantage. But of course envy is only one possible reason for the experimental results.

Another reason may be the violation of the common knowledge assumption - the people simply do not understand the game properly. In this case people may apply a social norm like "equal division" just as a rule of thumb to the avoid difficult calculations for solving the problem. If such calculation problems have been causing the deviation from the results predicted by standard theory the results should tend to converge to the theoretical solution if the game is repeated. But when GT repeated their experiment with a discount factor of 0.1 the average offer differed even more from the theoretical offer than in the first game. Furthermore, one- and two-period ultimatum games are quite simpe to understand and the subjects in the GT- and the OR-experiment were students of economics, in the case of the GT-experiment even familiar with noncooperative game theory. Therefore we regard a lack of understanding as a quite unplausible reason for the experimental data.

As already mentioned one cannot distinguish properly between norm (fairness) and envy motivated behaviour with the experiments made until now. But we have no doubt that interpersonal comparison in the sense of envy plays an important role for the experimental results as well as for many "real world" situations. It remains to further investigation to conduct experiments that are able to distinguish properly between norm- and envy-motivated behaviour, to assess the importance of the specific factor and to analyze the impacts of envy and norms on the behaviour.

References

Binmore, K. (1987): "Perfect Equilibrium in Bargaining Models"; in: Binmore, K., Dasgupta, P.: "The Economics of Bargaining"; Blackwell, Oxford

Binmore, K.,Shaked, Sutton,J. (1985): "Testing Noncooperative Game Theory: A preliminary study"; American Economic Review, 75, 1178 - 1180

Forsythe, R., Horowitz, J., Savin, N., Sefton, M. (1988): "Replicability, Fairness and Pay with Simple Bargaining Games"; Working paper 88-30, University of Iowa, Iowa City

Güth, W., Schmittberger, R., Schwarze, B. (1982): "An Experimental Analysis of Ultimatum Bargaining"; Journal of Economic Behaviour and Decision, 3, 367 - 388

Güth, W., Tietz, R. (1988): "Ultimatum Bargaining for a Shrinking Cake: An Experimental Analysis"; mimeo

Güth, W., Tietz, R. (1990): "Ultimatum Bargaining Behaviour: A Survey and Comparison of Experimental Results"; Journal of Economic Psychology, 11, 417 - 449

Holländer, H. (1990): "A Social Exchange Approach to Voluntary Cooperation"; American Economic Review, 80, 1157 - 1167

Kahneman, D., Knetsch, J., Thaler R. (1986): "Fairness and the Assumptions of Economics"; Journal of Business, 59, 285 - 300

Neelin, J., Sonnenschein, H., Spiegel, M. (1988): "A Further Test of Noncooperative Bargaining Theory"; American Economic Review, 78, 824 - 836

Ochs, J., Roth, A. (1989): "An Experimental Study of Sequential Bargaining"; American Economic Review, 79, 355 - 384

Roth, A. (1988): "Laboratory Experimentation in Economics: A Methodological Overview"; Economic Journal, 98, 974 - 1031

Rubinstein, A. (1982): "Perfect Equilibrium in a Bargaining Model"; Econometrica, 50, 97 - 109

Stahl, I. (1972): "Bargaining Theory"; Economic Research Institute, Stockholm

Thaler, R. (1988): "Anomalies: Ultimatum Game"; Journal of Economic Perspectives, 2, 195 - 206

figure 1: k(delta2) for a=0,6; b=0,4; delta1=0,8