THE SAVING RATION IN A COMPUTER SIMULATION OF THE LIFE CYCLE MODEL*

Martin HUSZ

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Abstract

In her influential study with respect to the life cycle hypothesis, White (1978) applies computer simulation for an empirical analysis. She concludes that the pure life cycle model without bequests can account for, at best, 42 percent of observed personal saving. In the present paper, Harrod-neutral technological change is introduced to a comparable computer model. In contrast to White (1978) numerical computation of the steady state then reveals a plausible saving ratio. A second result reveals that the steady state values in a model of this class are extremely sensitive to parametrization. Unfortunately the exogenously given parameters have no sufficient empirical backing. This casts serious doubts on general refutations of the pure life cycle model based on computer simulations of the kind used by White (1978).

Kurzübersicht

In einer einflußreichen empirischen Studie kommt White (1978) mittels Computer-Simulationen zu dem Ergebnis, daß höchstens 42 Prozent des aggregierten Sparens durch die einfache Lebenszyklushypothese zu erklären sind. In der vorliegenden Arbeit wird bewiesen, daß bei Einführung von Harrod-neutralem technischen Fortschritt in ein vergleichbares Computermodell sehr wohl realistische Sparquoten generiert werden können. Ferner wird gezeigt, daß die Ergebnisse eines Computermodells dieser Art stark von der Parametrisierung abhängen. Nachdem diese Parameter empirisch gar nicht bis nicht ausreichend gesichert sind, müssen die Ergebnisse mit Vorsicht betrachtet werden, insbesondere wenn eine generelle Ablehnung der reinen Lebenszyklushypothese daraus gefolgert wird.

1. Introduction

Dynamic models based on the life cycle hypothesis are widely used in macroeconomic literature. Since Modigliani and Brumberg (1954) introduced the first formal life cycle model their ideas have been firmly established in economists' thoughts. The concept of a pure life cycle in its original form is not undisputed, though. In a frequently quoted study White (1978) applies computer simulation for an empirical analysis. She comes to the conclusion that the pure life cycle model with certain lifetime and without bequests can account for, at best, 42 percent of observed personal saving. Therefore she argues that simple life cycle models of the kind suggested by Ando and Modigliani (1963) must be rejected. This conclusion is not confirmed by Söderström (1982) who in his model allows for uncertainty of death and unplanned bequests to generate a realistic level of aggregate saving. Kotlikoff and Summers (1981), however, quote White's finding which is in line with their own results. In their study they argue that the life cycle model can explain only about a fifth of aggregate U.S. saving.

This has led to a lively and lasting debate. Both Modigliani (1988) and Kotlikoff (1988) published their opposite views in the Journal of Economic Perspectives. In his article Kotlikoff again emphasizes the importance of the study by White where old households do not show negative saving as predicted by the life cycle hypothesis. In his reply, among other points, Modigliani (1988) criticises computer simulations as a tool for settling empirical issues. He argues that the sensitivity of the outcome of the models with respect to the preference parameters is too large to allow for decisive conclusions. This motivates two questions which will be discussed in the present paper: Can a pure life cycle computer model generate net saving ratios high enough to explain empirical evidence? And how sensitive is this model to changes in its parametrization?

In the next section the theoretical model used for the computer simulations will be described. It is an overlapping generations model with perfect foresight including production and endogenous labor supply and is therefore more general than the model applied by White (1978). The reason for introducing production is to generate a general equilibrium with only technology and preference parameters being exogenously given. On the other hand bequests are still ruled out and with 55 periods households have the same lifetime. Though maybe not providing too much news to the discussion the paper might be seen as an useful exercise showing that by playing around with models of this kind results of a relatively wide range can be generated.

In section 3.1 the simulation mechanics will be illustrated.[2] The procedure is basically the same as described by White (1978) and used in other works on the subject. Some complications arise because of the endogenous labor supply in combination with productivity growth. This needs a more complex program, but the basic features of the model remain the same in principle. The results of the simulation will be given in section 3.2. The effects of variations of the parametrization on the saving ratio and other variables will be given in 7 tables. Each table is preceded by a short comment. Some concluding remarks in section 4 will recapitulate the most important points.

2. The Model

The general equilibrium model used here is closely related to the model developed by Auerbach and Kotlikoff (1987). It is a pure life cycle model with production and endogenous labor supply. Government activities are not included, however, and the utility function is simplified to allow for productivity growth. The introduction of Harrod neutral technical progress helps to generate higher saving ratios for plausible parameter settings.

2.1. Households

2.1.1. The Life Cycle

The household sector consists of 55 overlapping generations $i=1,...,55$. Each household faces a fixed lifetime of 55 periods. Utility is derived from consumption and leisure. The elasticity α of utility u_{it} of household i in period t with respect to consumption c_{it} and the elasticity $(1-\alpha)$ with respect to leisure h_{it} are assumed to be constant, the elasticity of substitution being one:

$$u_{it} = c_{it}^{\alpha} h_{it}^{1-\alpha} \qquad 0 < \alpha < 1. \qquad (1)$$

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[2] The simulations were performed using a Gauss computer program written by the author and based on the equations presented in this paper. A program listing can be found in the appendix.

Relaxation of these strong assumptions in the context of the model is not easy. This will be shown in the discussion of the first order conditions. The lifetime utility function U_i of household i takes the following form:[3]

$$U_i = \frac{1}{1-1/\Gamma} \sum_{t=1}^{55} \frac{1}{(1+\delta)^{t-1}} u_{it}^{1-1/\Gamma}. \tag{2}$$

The parameter Γ equals the household's intertemporal elasticity of substitution between consumption in different years. The symbol δ denotes the rate of pure time preference used for discounting utility. As α, Γ and δ are constants, preferences do not change over time in the model. The model implies time separability which is another usual assumption. The household maximizes its lifetime utility function subject to its intertemporal budget constraint

$$\sum_{t=1}^{55} R_t \,(w_t\,(1-h_{it}) - c_{it}) \leq 0 \tag{3}$$

$$\text{with} \quad R_t = \prod_{s=2}^{t} (1+r_s)^{-1} \quad \text{and} \quad 0 \leq h_{it} \leq 1$$

where r_t is the one period interest rate and w_t is the wage rate in period t. The intertemporal budget constraint rules out the possibility of bequests. All life cycle income is consumed. The time endowment is normalized to one. Negative working time, i.e. $h_{it}>1$, is ruled out by assumption. Maximizing (2) subject to (3) leads to the first order conditions

$$c_{it} = \frac{\alpha}{1-\alpha} \; w_t \, h_{it} \tag{4}$$

$$c_{it} = \left[\frac{1+r_t}{1+\delta} \right]^{\Gamma} \left[\frac{w_t}{w_{t-1}} \right]^{(1-\alpha)(1-\Gamma)} c_{it-1} \tag{5}$$

$$h_{it} = \left[\frac{1+r_t}{1+\delta} \right]^{\Gamma} \left[\frac{w_t}{w_{t-1}} \right]^{(1-\alpha)(1-\Gamma)-1} h_{it-1}. \tag{6}$$

Note that in (4) consumption is a constant fraction of labor income $w_t(1-h_{it})$. It is this fact that allows the introduction of technological change. With the more general CES utility function used by Auerbach and Kotlikoff (1987) growth of labor efficiency

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[3] To keep indexing simple no special attention will be paid to generalisation. As only steady states are taken into consideration optimization over lifetime at the beginning of life is sufficient assuming rational expectations and perfect information.

would imply an exponentially increasing or decreasing trend in labor force participation. Not only would this lead to implausible results in the long run. It also would cause a problem in computer simulations because convergence is not guaranteed.[4]

Equation (5) says that consumption tends to increase if the interest rate r exceeds the rate of time preference δ. Furthermore consumption tends to grow with a rising wage rate if the intertemporal elasticity of substitution Γ is less than one. If on the other hand under the ceteris paribus assumption Γ is higher than one then the strong saving incentive would lead the households to consume less when wages rise.

Interpreting (6) it can be seen that the relation of r to δ has the same effect on leisure as it had on consumption. That does not apply to the wage effect. The wage rate can be regarded as the opportunity price for leisure. Leisure in the optimal allocation may be expected to decrease when the wage rate increases. Not regarding the interest effect an increasing wage rate corresponds to higher leisure chosen only if $(1-\alpha)(1-\Gamma)>1$. For $\alpha<1$ the condition only holds if $\Gamma<\alpha/(\alpha-1)$ which implies that Γ must be negative. This is not plausible which means that ceteris paribus leisure should in fact always decrease with rising wages. That is also valid for $\Gamma>1$. Consumption in this case is not substituted for leisure but for future consumption. Iteration of (5) leads to

$$c_{it} = \frac{G_t}{(1+\delta)^{\Gamma(t-1)}} \left[\frac{w_t}{w_1} \right]^{(1-\alpha)(1-\Gamma)} c_{i1} \qquad (7)$$

with $G_t = \prod_{s=2}^{t} (1+r_s)^{\Gamma}$

Substituting (4) and (7) for all h_{it} and c_{it} in (3) allows for determining c_{i1} by wage and interest rate only which are given to the household:

$$c_{i1} = \frac{\sum_{t=1}^{55} R_t w_t}{\sum_{t=1}^{55} R_t G_t (1+\delta)^{\Gamma(1-t)} \left[\frac{w_t}{w_1} \right]^{(1-\alpha)(1-\Gamma)}} . \qquad (8)$$

[4] Auerbach and Kotlikoff (1987,p.35) mention this problem in a footnote. They still prefer to model the preferences of households by a more general CES-function, thereby ruling out the possibility of technological change.

Starting with consumption in the first period repeated use of equations (4)-(6) yields the values for consumption and leisure in the rest of the life-cycle. With consumption and leisure known saving can be determined as the difference between income and consumption. Income ensues from labor and from interest on assets:

$$s_{it} = w_t \, (1-h_{it}) + r_t \, a_{it} - c_{it} \tag{9}$$

Accumulating savings over the life cycle gives the stock of real assets. Similarily to (5) and (6) a difference equation for the assets a_{it} of household i in period t may be formulated:

$$a_{it+1} = (1+r_t) \, a_{it} + w_t \, (1-h_{it}) - c_{it}. \tag{10}$$

Assets in the next period equal assets from this period plus interest and saving. As bequests are not considered in this model an initial and an end condition have to be taken into account:

$$a_{i1} = 0 \quad \text{and} \quad a_{i56} = 0. \tag{10a}$$

2.1.2. Aggregation

Aggregate labor supply L_t^s is supposed to grow with a constant rate n each period. Aggregating the 55 generations in relation to the first period of the youngest household yields the following equation:

$$L_1^s = \sum_{i=1}^{55} \frac{1}{(1+n)^{i-1}} \, (1-h_{i1}). \tag{11}$$

Unfortunately the aggregation is not as simple as it might look at first sight. Labor supply depends endogenously on the wage rate, which changes with technological progress. For that reason it is necessary to compute the life cycle of each generation separately before being able to aggregate.[5] The same applies to aggregate consumption C, aggregate saving S and aggregate asset wealth A, where aggregation again in relation to the youngest household gives

$$C_1 = \sum_{i=1}^{55} \frac{1}{(1+n)^{i-1}} \, c_{i1} \tag{12}$$

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[5] By assuming inelastic labor supply this complication could be easily surmounted. It would then be sufficient to compute the life cycle of one representative household only. For aggregation the values of the household at different ages could be used.

$$S_1 = \sum_{i=1}^{55} \frac{1}{(1+n)^{i-1}} s_{i1} \tag{13}$$

$$A_1 = \sum_{i=1}^{55} \frac{1}{(1+n)^{i-1}} a_{i1} \tag{14}$$

respectively.

2.2. Firms

There is only one good produced in this economy. Like the utility function of the households the production function is assumed to be of the Cobb/Douglas-type. But unlike the previous case generalization would be relatively easy here. Still the special CES-function with an elasticity of substitution of one is used, mainly because empirical evidence suggests a value near to one.[6] Output Y_t therefore depends on capital K_t and labor L_t in the following form:

$$Y_t = A \left[\frac{K_t}{L_t \; (1+g)^{t-1}} \right]^{\sigma} L_t \; (1+g)^{t-1} . \tag{15}$$

A denotes a scaling constant while g represents the rate of Harrod neutral technological progress raising labor efficiency. Both are exogenously given. So is the parameter σ measuring the intensity of use of capital in production. The factor demand functions follow from profit maximization for given factor and output prices:

$$r_t = A \; \sigma \left[\frac{K_t}{L_t \; (1+g)^{t-1}} \right]^{\sigma-1} \tag{16}$$

$$w_t = A \; (1-\sigma) \left[\frac{K_t}{L_t \; (1+g)^{t-1}} \right]^{\sigma} (1+g)^{t-1} . \tag{17}$$

The adjustment of all factors is assumed to be costless.

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6 Auerbach and Kotlikoff (1987,p.52) quote work of Nerlove (1967) and Berndt and Christensen (1973); Although they use a CES-function for their analysis, their computer simulations, too, are performed with an elasticity of substitution of one.

2.3. Equilibrium Conditions

All prices are assumed to be market clearing. Moreover perfect foresight is assumed. Then in each period labor supply equals labor demand and the capital stock equals aggregate financial wealth:

Labor market: $L_{ts} = L_t$ (18)

Capital market: $A_t = K_t$. (19)

Furthermore saving must equal investment and output must be equal to aggregate consumption plus investment:

Goods market: $Y_t = C_t + I_t$. (20)

where $I_t = K_t - K_{t-1} = S_t$ (21)

A general equilibrium solution must be consistent not only with present but also with future prices. Using computer simulation a steady state can be calculated where this condition is satisfied. In the next chapter this procedure and the results are described.

3. Computer Simulation

3.1. Simulation Procedure

A solution to the system of equations described in the previous chapter is obtained when all the variables reach their steady state values. These values are computed using a Gauss-Seidel algorithm. The algorithm begins with the assignment of starting values to the variables capital and labor. The capital labor ratio is then used to compute all the other endogenous variables. Using the computed values of these other variables a new value for the capital labor ratio is obtained. A combination of the old and new capital labor ratio serves as a new guess for the repetition of the procedure. The iteration goes on until all variables are consistent with all equations. The following sketch should help to explain the Gauss-Seidel method:

$$K_1/L_1 \rightarrow Y_1, w_t, r_t \rightarrow c_{i1} \rightarrow c_{it}, h_{it}, s_{it}, a_{it} \rightarrow C_1, L_1{}^S, S_1, A_1$$

In equations (15)-(17) an exogenous first guess for capital and labor is used to compute the output in the first period of the youngest generation as well as all the wage and interest rates for this period and the 54 periods before. Application of equation (8) then serves to compute the consumption values of each generation in its first period respectively. From this consumption and leisure in the other periods can be obtained by equations (4)-(6). Savings and assets follow from the use of equation (9) and (10). Aggregation as described in equations (11)-(14) then leads to aggregate consumption, leisure, saving and asset wealth.

By equating demand and supply in equations (18) and (19) new values for capital and labor are now available. The requested steady state is approached by summing up half of the old and half of the new value of the capital labor ratio and using this new guess to repeat the procedure. Equation (20) and (21) are used to control for convergence. The conditions (10a) that rule out the possibility of intergenerational transfers serve as an additional check.

The task of this paper is to analyze the effects of changes in the parameters of the model on the saving ratio. Table 1 contains the values of the standard parametrization to start with:

Table 1: Standard parametrization

Rate of labor efficiency growth	$g = 0.020$
Rate of population growth	$n = 0.010$
Intertemporal elasticity of substitution	$\Gamma = 0.250$
Pure rate of time preference	$\delta = 0.015$
Consumption preference parameter	$\alpha = 0.400$
Capital intensity parameter	$\sigma = 0.250$
Production function constant	$A = 0.839$

The choice of the production function constant A allows for the wage rate in the steady state without growth to be normalized to one. Similarily with the consumption preference parameter chosen exactly 8 hours a day are worked in the stationary case. Without growth of either population or labor productivity, however, the saving ratio must equal zero. With the growth rates of the base case described in the table above the model can explain a saving ratio of about 15 percent, which seems a good point to start. The other parameters are taken from Auerbach and Kotlikoff (1987) where in the face of the empirical evidence their choice is discussed in some detail.

Statements in the empirical literature normally predict parameters only within a wide range, though. Some parameters literally seem to be open for free choice. The consumption preference parameter α for instance is typically chosen in a way that allows for calibrating models to explain the features of interest. Evans (1984) in discussing the simulation study from White (1978) points out that the outcome of life cycle models can vary considerably with the choice of the elasticity of temporal substitution and the rate of time preference, too. The White-model does not contain endogenous labor or production. But as can be seen in the results of the present paper especially for Γ the argument still holds.

3.2. Results

In this chapter the effects of changes of exogenous variables on the steady state values of the economy are analyzed. It will be shown that the model described above can generate a realistic level of aggregate saving without taking uncertainty or bequests into consideration. Furthermore, it will then be possible to make statements about the sensitivity of the model to variations in growth rates, preference parameters and the rate of technical substitution. The base case results following from the standard parametrization are as follows:

saving ratio (S/Y)	0.152
capital labor ratio (K/L)	3.137
capital output ratio (K/Y) ...	2.810
wage rate (w)	0.837
interest rate (r)	0.089

With these results one of the questions put in the introduction of this paper can already be answered. The base case simulation generates a saving ratio of about 15 percent. Dean, Durand, Fallon and Hoeller (1990) show that of 21 OECD countries including the USA and Germany only 3 countries possess a higher net national saving ratio in 1981-88, namely Switzerland, Japan and Norway.[6] For Austria and many other countries the saving ratio has been considerably higher in the 60ies and 70ies. Of this time period, though, the net saving ratio in the USA has been highest with 10.6 percent in the 60ies and falling since. Hence a saving ratio of 15 percent given a growth rate of 3 percent overall seems not to be too unrealistic.

The saving profile over the life cycle is not too unplausible either. The youngest household starts with saving 26.4 percent of its income in the first period. Relative saving reaches a peak in period 22 when the household saves 29.3 of its income. Absolute saving arrives at its maximum in period 35, when still 26.1 percent of income are saved. Saving decreases from then on and the last 10 periods of its life cycle the youngest household's saving is negative.

The answer to the second question refering to the effects of changing preference and technology is summarized in 7 tables, which will now be discussed. In each table the bold numbers represent the base case solution.

3.2.1. Growth Rates

The effects of changing the rate of population growth on the base model solution can be seen in table 2. The effect on the saving ratio is considerable. Even if the rate of productivity growth were assumed to be zero, a growth rate of 2.5 percent would be sufficient to generate a saving ratio of more than 10 percent.[7] The steady state wage rate decreases with population growth, because there is more labor supply, whereas the steady state interest rate increases. As labor per capita decreases the capital labor ratio is falling.

Table 2: Effect of changes in the rate of population growth

n	S/Y	K/L	K/Y	L/day	wage	interest
0.000	0.129	3.187	2.844	8.752	0.840	0.088
0.010	**0.152**	**3.137**	**2.810**	**8.699**	**0.837**	**0.089**
0.020	0.173	3.071	2.766	8.626	0.833	0.090
0.030	0.193	2.991	2.712	8.531	0.827	0.092
0.040	0.211	2.898	2.648	8.412	0.820	0.094
0.050	0.227	2.795	2.579	8.268	0.813	0.097

[7] compare table 4.

As can be seen in table 3 the effect of productivity growth is even stronger than the effect of population growth. All variables of the model economy show strong reactions. A basic difference to the effect shown in table 1 lies in the adaptation of time worked. Productivity growth causes a rising wage rate over the life cycle of a single household. But in aggregate the steady state wage rate is lower when the productivity growth rate is higher and still households on aggregate work more. How can that be explained?

Comparison of the life cycle of different generations gives the following picture: With higher productivity growth wage rate and interest rate rise within the life cycle of one generation. But they increase more for younger generations than for the older ones. The youngest generation works more in earlier years and saves more for the future. In relation to the youngest generation older generations work less in their youth, but relatively more in their old age. The youngest generation accumulates more assets, older generations accumulate less. In aggregate steady state labor with productivity growth is higher, though little, while capital decreases considerably, when productivity growth is introduced. On the other hand, due to the additional saving of the young the saving ratio rises strongly. Table 3 shows the aggregate effects.

Table 3: Effect of changes in the rate of productivity growth

g	S/Y	K/L	K/Y	L/day	wage	interest
0.000	0.046	6.129	4.645	7.882	0.990	0.054
0.010	0.106	4.335	3.582	8.330	0.907	0.070
0.020	**0.152**	**3.137**	**2.810**	**8.699**	**0.837**	**0.089**
0.030	0.184	2.317	2.239	9.987	0.776	0.112
0.040	0.207	1.745	1.810	9.210	0.723	0.138
0.050	0.224	1.342	1.486	9.385	0.677	0.168

Table 4 is divided into 4 blocks. The first block consists only of one line showing the solution to a model without growth. Under perfect foresight no aggregate saving is generated in this case. The second block of table 4 gives the "pure" effect of the population growth rate. As population growth leaves productivity unchanged, wage rate and interest rate in steady state are the same for all generations in all periods. Therefore all generations allocate resources in the same way. This is not the case for the third block of table 4 where the "pure" effect of the productivity growth rate is shown. The reason for this has already been given above. In addition to the investigation of a change in merely one of the two growth rates their combined effect is analysed as well in block 4 of table 4. This also implies the base case solution. Table 4 does not show the effect of the growth rates on aggregate labor. It should be mentioned, however, that labor tends to increase when both growth rates are set equal which means that the effect of productivity growth tends to be stronger. Without any growth working time is normalized to 8 hours a day.

Table 4: Combined effect of growth rate changes

g	n	S/Y	K/L	K/Y	wage	interest
0.000	0.000	0.000	6.387	4.790	1.000	0.052
0.000	0.010	0.046	6.129	4.645	0.990	0.054
0.000	0.020	0.090	5.847	4.483	0.978	0.056
0.000	0.030	0.129	5.547	4.309	0.965	0.058
0.000	0.040	0.165	5.238	4.128	0.951	0.060
0.000	0.050	0.197	4.929	3.944	0.937	0.063
0.010	0.000	0.074	4.453	3.655	0.914	0.068
0.020	0.000	0.129	3.187	2.844	0.840	0.088
0.030	0.000	0.168	2.333	2.251	0.777	0.111
0.040	0.000	0.197	1.745	1.810	0.723	0.138
0.050	0.000	0.217	1.333	1.479	0.676	0.169
0.010	0.010	0.106	4.335	3.582	0.907	0.070
0.020	**0.010**	**0.152**	**3.137**	**2.810**	**0.837**	**0.089**
0.020	0.020	0.173	3.071	2.766	0.833	0.090
0.030	0.010	0.184	2.317	2.239	0.776	0.112
0.030	0.020	0.198	2.289	2.219	0.774	0.113
0.030	0.030	0.211	2.250	2.191	0.770	0.114
0.040	0.010	0.207	1.745	1.810	0.723	0.138
0.040	0.020	0.216	1.738	1.805	0.722	0.139
0.040	0.030	0.224	1.722	1.792	0.721	0.139
0.050	0.010	0.223	1.342	1.486	0.677	0.168
0.050	0.020	0.229	1.345	1.489	0.677	0.168
0.050	0.030	0.234	1.343	1.487	0.677	0.168

3.2.2. Preference Parameters

Within the present design of the model a strong influence of all preference parameters on the steady state saving ratio could be expected. Surprisingly enough this is not the case as far as the rate of pure time preference is concerned. Yet the effect of the other two parameters is indeed as powerful as anticipated.

This applies especially for the consumption preference parameter α. Its choice is crucial for the outcome of the model. Yet there is no empirical evidence about its real value. From table 5 it becomes obvious, though, that it should lie somewhere close to 0.4. An increasing consumption preference makes consumption more valuable in respect to leisure. Therefore households increase labor supply. With given production technology the wage rate then decreases and the interest rate increases. An increasing interest rate means that households have to save less when young to secure income out of assets in their old age. For that reason the effect on the saving ratio is negative: The saving ratio tends to decrease when α increases, as shown in table 5.

Table 5: Effect of changes in the consumption preference parameter

α	S/Y	K/L	K/Y	L/day	wage	interest
0.200	0.232	4.828	3.883	4.534	0.932	0.064
0.300	0.182	3.814	3.254	6.578	0.879	0.077
0.400	**0.152**	**3.137**	**2.810**	**8.699**	**0.837**	**0.089**
0.500	0.130	2.648	2.475	10.917	0.802	0.101
0.600	0.114	2.277	2.210	13.248	0.772	0.113
0.800	0.092	1.750	1.814	18.307	0.724	0.138

As mentioned above the effect of rate of pure time preference δ on the saving ratio is surprisingly weak. It has the correct sign, though. It is quite obvious that the saving ratio should be lower when future consumption is subject to a higher rate of discounting. This relation is properly reflected in table 6.

Table 6: Effect of changes in the rate of pure time preference

δ	S/Y	K/L	K/Y	L/day	wage	interest
-0.015	0.168	5.063	4.024	8.712	0.944	0.062
0.000	0.158	3.960	3.347	8.700	0.887	0.075
0.015	**0.152**	**3.137**	**2.810**	**8.699**	**0.837**	**0.089**
0.030	0.147	2.527	2.390	8.704	0.793	0.104
0.045	0.144	2.074	2.061	8.712	0.755	0.121

The intertemporal rate of substitution Γ has maybe the most powerful effect on the saving ratio. It determines the relative change in the ratio of any two years' consumption with respect to a change in the relative prices. The easier present consumption is substituted for future consumption the higher the saving ratio should be. Therefore a high Γ in table 7 corresponds to a high saving ratio.

Table 7: Effect of changes in the intertemporal rate of substitution

Γ	S/Y	K/L	K/Y	L/day	wage	interest
0.125	0.086	1.640	1.728	8.328	0.712	0.145
0.250	**0.152**	**3.137**	**2.810**	**8.699**	**0.837**	**0.089**
0.500	0.245	5.183	4.096	9.279	0.949	0.061
1.000	0.368	7.564	5.438	10.199	1.043	0.046
2.000	0.523	9.952	6.681	11.714	1.117	0.037

3.2.3. Production Parameters

Due to the assumption of an Cobb-Douglas technology there are only two parameters of production: The scaling constant A and the capital intensity parameter σ. Changing the scaling constant influences only the wage rate and the capital labor ratio. Both variables are increased by increasing A. The other variables and especially the saving ratio remain unchanged:

A = 0.500 -▸ w = 0.420, K/L = 1.574
A = 0.700 -▸ w = 0.658, K/L = 2.465
A = 0.839 -▸ w = 0.887, K/L = 3.960
A = 1.000 -▸ w = 1.058, K/L = 3.966
A = 1.200 -▸ w = 1.350, K/L = 5.057

Changing the intensity in the use of capital σ necessarily has a big impact on the capital labor ratio and on both factor prices. It is also obvious that labor input should decrease when the capital intensity parameter σ increases. The effect of a change in σ on the saving ratio on the other hand is practically nil. In table 8 the whole range of empirically relevant σ-values is covered, but the saving ratio remains always in the vicinity of 15 percent.

Table 8: Effect of changes in the capital intensity parameter

σ	S/Y	K/L	K/Y	L/day	wage	interest
0.150	0.142	1.710	1.881	9.446	0.773	0.080
0.250	**0.152**	**3.137**	**2.810**	**8.699**	**0.837**	**0.089**
0.350	0.155	5.494	3.608	7.822	0.990	0.097
0.450	0.154	10.405	4.324	6.812	1.324	0.104

4. Conclusion

Two questions were asked at the beginning of the paper: Can a pure life-cycle model explain observed saving ratios? And how sensitive is the result to changes in preference and technology parameters? Looking at the tables of the former section both questions can be answered to some extent. The base case simulation with plausible parameter setting generates a saving ratio of 15 percent. This seems reasonable. But answering the second question casts some doubts on the applicability of such a result. The proviso expressed by Modigliani (1988) seems to be highly justified. The results of the model hinge on exogenously given parameters which have no sufficient empirical backing. Calibrating the model is therefore ad hoc. And a change in parametrization can have important effects on the results.

As far as the saving ratio is concerned not all parameters have the same strong influence. Changes in the rate of pure time preference have a surpringly weak effect on the saving ratio. This is a major difference to the results of Evans (1984) or Jaeger and Keuschnigg (1988) which might be due to the introduction of leisure into the utility function. The consumption preference parameter used is also of great importance for the saving ratio. But this again proves that most of the models are not insensitive to generalization. An empirical analysis based on such a model must be read with much care. And this must be especially the case when a general refutation of the life cycle hypothesis follows from the study. This is in agreement with a more profound critique by Hurd (1990)[8] who only accepts proper use of panel data as reliable evidence for refuting the life cycle hypothesis.

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8 Hurd (1990); e.g. compare p.628.

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Appendix: The Computer Model

```
/***************************************************************************
**   OLG55.G              Martin Husz 1991/7    program software: GAUSS 2.1  **
**                                                                           **
**   Program computes the steady state of an model  with  55  overlapping   **
**   generations.  The  algorithm  follows  the  procedure  described  in   **
**   Auerbach and Kotlikoff (1987,p.47). Utility and production  function   **
**   are of the Cobb-Douglas type. The consumer problem is solved for all    **
**   55 generations separately. The disaggregated variables are organized    **
**   in 55x55-matrices or 55-vectors with the following order:              **
**                                                                           **
**        matrixrow:     Cross section over the generations                  **
**                       (e.g.2.row: 55 gen. in their respective 2.period)   **
**        matrixcolumn:  Life cycle of one generation                        **
**                       (e.g.2.col.: Development within previous gener.)    **
**        vektors:       Development within generation                       **
**                                                                           **
***************************************************************************/

"";
"OLG55.G: Life cycle simulation for 55 generations";
"************************************************";
"";
format /rd 12,7;
trace 0;

/*********************
**  Parametrization  **
*********************/

C1 = zeros(55,55);          /*  consumption   - matrix                       */
S1 = zeros(55,55);          /*  saving        - matrix                       */
H1 = zeros(55,55);          /*  leisure       - matrix                       */
L1 = zeros(55,55);          /*  labor         - matrix                       */
A1 = zeros(55,55);          /*  asset         - matrix                       */

n = 0.01;                   /*  rate of population growth                    */
g = 0.02;                   /*  rate of labor productivity growth            */
f = 0.4;                    /*  consumption preference parameter             */
d = 0.015;                  /*  rate of pure time preference                 */
m = 0.25;                   /*  intertemporal elasticity of substitution     */

Y  = 0;                     /*  aggregate output                             */
K  = 60;                    /*  aggregate capital                            */
L  = 20;                    /*  aggregate labor                              */
KL = zeros(55,1);           /*  capital labor ratio in efficiency units      */

a = 0.8387161;              /*  skaling constant (n,g=0,a=0.8387161->w=1)    */
q = 0.25;                   /*  capital intensity parameter                  */
w = zeros(55,55);           /*  wage rate     - matrix                       */
r = zeros(55,55);           /*  interest rate - matrix                       */

rr = ones(55,55);           /*  matrix of discount factors                   */
r1 = ones(55,55);           /*  matrix of cumulated interest rates           */
vv = ones(55,55);           /*  matrix for calculating H1[1]                 */
nn = ones(55,1);            /*  vektor of cum. population growth rates       */
gg = ones(55,1);            /*  vektor of cum. productivity growth rates     */
```

```
e5 = cumsumc(ones(55,1));
nn = (shiftr((cumprodc(nn+n))',1,1))';
gg = (shiftr((cumprodc(gg+g))',1,1))';
KL = K./(L.*gg);

/*****************************************
**  Iteration algorithm (Gauss/Seidel)  **
*****************************************/

t=1;
iter = 1;
imax = 100;
diff = 1;
do while diff >= 0.00000001;
   "Iter.=";;print /lz iter;;
   "K/L =";;K/L;;
   " cntr.=";;sumc(abs(a.*KL^q.*L.*gg - w[.,1].*L - r[.,1].*K));;
   " diff.=";;diff;

   /* Calculation of w and r over the life cycle of all 55 generations */

   t=1;
   do while t<=55;
      r[.,t] = a .* KL^(q-1) .* q .* (1+g)^((q-1)*(t-1));
      w[.,t] = a .* KL^q .* (1-q) .* gg .*(1+g)^((q-1)*(t-1));
      t=t+1;
   endo;

   /* Calculation of discount factors */

   t=2;
   do while t<=55;
      rr[t,.] = rr[t-1,.] .* (1./(1+r[t,.]));
      r1[t,.] = r1[t-1,.] .* (1+r[t,.])^m;
      vv[t,.] = rr[t,.] .* r1[t,.] .* (1+d)^((-m)*(t-1));
      t=t+1;
   endo;

   /* Calculation of 1.period values for all generations */

   C1[1,.] = f .* (sumc(rr .* w) ./ sumc(vv .* (w./w[1,.])^((1-m)*(1-f))
))';
   H1[1,.] = C1[1,.] .* ((1-f)/f) ./ w[1,.];
   L1[1,.] = 1 - H1[1,.];
   S1[1,.] = w[1,.].*L1[1,.] - C1[1,.];

   /* Calculation of the other life cycle values for all generations */

   t=2;
   do while t<=55;
      C1[t,.] = C1[t-1,.] .* ((1+r[t,.])./(1+d))^m
                .* (w[t,.]./w[t-1,.])^((1-m)*(1-f));
      H1[t,.] = C1[t,.] .* ((1-f)/f) ./ w[t,.];
      L1[t,.] = 1-H1[t,.];
      A1[t,.] = A1[t-1,.].*(1+r[t-1,.]) + w[t-1,.].*L1[t-1,.] - C1[t-1,.];
      S1[t,.] = w[t,.].*L1[t,.] + A1[t,.].*r[t,.] - C1[t,.];
      t=t+1;
   endo;
```

```
    H1=substute(H1,H1.>1,1);
    L1=substute(L1,L1.<0,0);

    /* Aggregation relative to first period of youngest generation */

    C = sumc(diag(C1)./nn);
    H = sumc(diag(H1)./nn);
    L = sumc(diag(L1)./nn);
    K = sumc(diag(A1)./nn);
    S = sumc(diag(S1)./nn);
    Y = a * KL[1]^q * L;

    /* New guess for capital labor ratio for next iteration step */

    KL   = 0.5.*KL + 0.5.*(K./(L.*gg));
    diff = abs(Y-C-S);
    iter = iter+1;
    if iter >= imax;
       break;
    endif;
 endo;

 /*************
 **  Output  **
 *************/

 output file=olg55.out on;

 "";
 "   Outputfile of program OLG55.G";
 "   ******************************";

 if iter>=imax;
    "   No convergence; difference of control variable:";;diff;
 else;
    "";
 endif;

 "   Determinants";
 "   ************";
 "   Productivity growth rate          g =";;g;
 "   Population growth rate            n =";;n;
 "   Rate of pure time preference      d =";;d;
 "   Intertemporal elast. of subst.    m =";;m;
 "   Consumption preference parameter  f =";;f;
 "   Capital intensity parameter       q =";;q;
 "   Skaling constant                  a =";;a;
 "";
 "   Steady state solution";
 "   *********************";
 "   Prices:     w =";;w[1,1];;"       r =";;r[1,1];
 "   Output:     Y =";;Y      ;
 "   Capital:    K =";;K      ;;"    K/Y =";;K/Y;;"     K/L =";;K/L;
 "   Labor:      L =";;L      ;;"  L/day =";;sumc(diag(L1))/55*24;
 "   Leisure:    H =";;H      ;;"  H/day =";;sumc(diag(H1))/55*24;
 "   Consumpt.:  C =";;C      ;;"    C/Y =";;C/Y;
 "   Saving:     S =";;S      ;;"    S/Y =";;S/Y;
output off;
```