IHS Macroeconomic Model ATMOD 0.6

Technical Appendix

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Non-technical summary

This technical documentation describes the macroeconomic model of the Austrian economy developed by a team of researchers from the Institute for Advanced Studies in Vienna as a part of a broader research agenda studying the effects of exogenous shocks on the Austrian economy. Parts of this technical documentation closely resemble the model descriptions available in previous studies, see Koch and Molnárová (2020) and Koch et al. (2019).

ATMOD is a state-of-the-art multi-industry New Keynesian dynamic stochastic general equilibrium (DSGE) model of a small open economy within the Euro Area. The model is calibrated such that it resembles the economic environment in Austria to the highest possible degree. It features the detailed production structure at the level of 74 individual industries, heterogeneous households, and an extensive government sector which interacts with the rest of the economy.

DSGE models attempt to explain the macroeconomic phenomena based on the microeconomic principles of optimizing agents and general equilibrium theory. In this way they avoid the Lucas critique (Lucas 1976), and are therefore suitable for policy analysis. The agents in DSGE models form expectations about the future rationally and act according to these expectations. This approach restricts the model structure and makes modelling more demanding in terms of matching empirical evidence and computational complexity. On the other hand, medium-size DSGE models are well equipped to explain the economic phenomena in an intuitive and tractable manner. The economic relationships identified by general equilibrium models can be directly related to the microeconomic behaviour of the agents, thus making the mechanisms behind the model outcomes transparent. This property is especially useful for policy analysis, where understanding the mechanisms driving the model outcomes is essential for the credibility of the analysis.

The model economy consists of domestic (Austrian) households, firms, government, and the rest of the world. The agents trade goods, production factors and financial assets. Austria is modelled as a small open economy within a monetary union. The main components of the model include:

- Households, differentiated into credit constrained and non-credit constrained type. This enables the model to approximately capture the heterogeneous reactions of households to changes in the economic environment.

- Production firms, differentiated into 74 industries that are connected through an input-output network. The industry structure of intermediate inputs, consumption, and investments corresponds to Austria.

- Government (public sector), conducting demand and supply side oriented economic policies. The set of policy instruments available to the government is modelled in a detailed way, including:

  - Tax system that resembles the most important taxes and social security contributions.

  - Public expenditures including government consumption, various types of public investment, and transfers.

- International trade of goods and financial assets with the rest of the world.

- Monetary policy, set by an (external) monetary authority.

Section 1 introduces the model. The calibration procedure including the data sources is described in section 2. In section 3 we discuss specific issues regarding the particular model application.

Contents

1 Model

In this chapter we describe the model of the Austrian economy used for the simulations of the macroeconomic effects of exogenous shocks. We build a multi-industry New Keynesian dynamic stochastic general equilibrium (DSGE) model of a small open economy within the Euro Area. The model is calibrated such that it resembles the economic environment in Austria to the highest possible degree.

DSGE models attempt to explain the macroeconomic phenomena based on the microeconomic principles of optimizing agents and general equilibrium theory. In this way they avoid the Lucas critique (Lucas 1976), and are therefore suitable for policy analysis. The agents in DSGE models form expectations about the future rationally and act according to these expectations. This approach restricts the model structure and makes modelling more demanding in terms of matching empirical evidence and computational complexity. On the other hand, medium-size DSGE models are well equipped to explain the economic phenomena in an intuitive and tractable manner. The economic relationships identified by general equilibrium models can be directly related to the microeconomic behaviour of the agents, thus making the mechanisms behind the model outcomes transparent. This property is especially useful for policy analysis, where understanding the mechanisms driving the model outcomes is often more important than the quantitative precision of the models.

The model was developed by a team of researchers from the Institute for Advanced Studies as a part of a broader research agenda studying the effects of exogenous shocks on the Austrian economy. Therefore, parts of the model description closely resemble the model descriptions available in previous studies, see Koch and Molnárová (2020) and Koch et al. (2019).

1.1 Model structure

The model economy consists of domestic (Austrian) households, firms, government, and the rest of the world. The agents trade goods, production factors and financial assets. Austria is modelled as a small open economy within a monetary union. Figure 1 depicts the model environment in a simplified manner. The main components of the model include:

- Households, differentiated into credit constrained and non-credit constrained type.
- Production firms, differentiated into 74 industries that are connected through an input-output network. The industry structure of intermediate inputs, consumption, investment, exports, and imports corresponds to Austria.
- Government (public sector), conducting demand and supply side oriented economic policies. The set of policy instruments available to the government is modelled in a detailed way, including:
  - Tax system that resembles the most important taxes and social security contributions.
  - Public expenditures including government consumption, various types of public investment, and transfers.
- International trade of goods and financial assets with the rest of the world.
- Monetary policy, set by an (external) monetary authority.

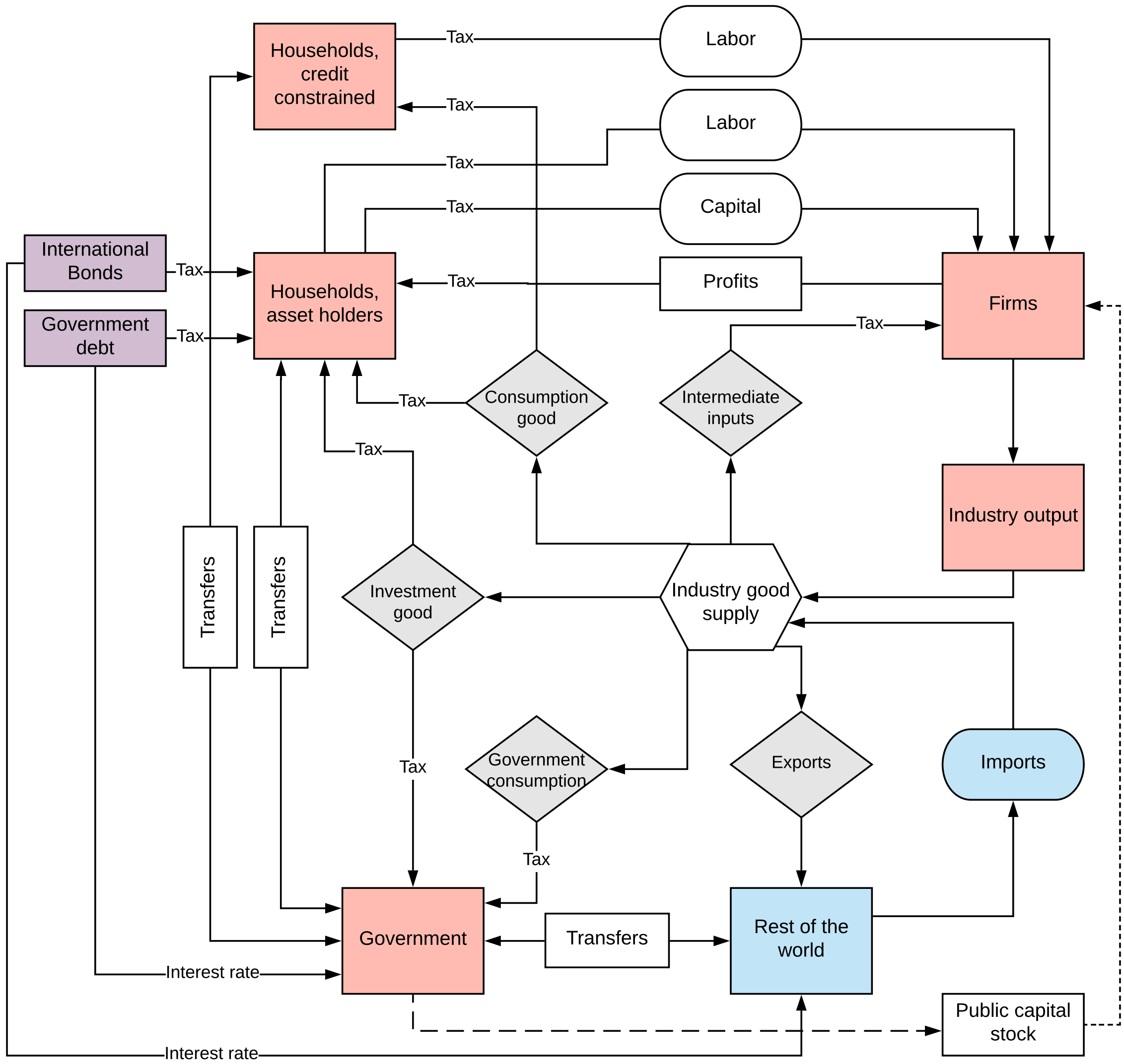

Figure 1: Model structure. Domestic agents (red), rest of the world (blue), uses of industry goods (grey), and financial assets (purple). Full lines represent financial transfers, which in most cases happen in exchange for goods, production factors, or assets. Dotted line symbolizes an influence on the economic environment without a financial compensation.

The model described in this chapter represents the stationary version of the economy, abstracting from deterministic economic growth and trend inflation. This representation of the model is equivalent to the representation in terms of growing variables, however, the interpretation of the variables and some parameters may differ.

1.1.1 Households

The economy is populated by a continuum of infinitely-lived households represented by a unit interval. Measure ω^K of the households is credit constrained, where $0 \leq \omega^K \leq 1$. We refer to the credit constrained households as *Keynesian*. The remaining households are not credit constrained and we refer to them as *Ricardian*. This specification with two types of households with different marginal propensities to consume allows us to study the effects of exogenous shocks on heterogeneous households in a simplified way, see Galí et al. (2007) and Debortoli and Galí (2017). All households provide labor input, earn wages, consume goods, pay taxes, and receive transfers from the government. Moreover, the Ricardian households also save resources in the form of risk-free international bonds, own capital stock, and receive firm profits.

Household preferences are modelled in line with the majority of the contemporary macroeconomic literature. However, the utility function differs from the standard functional forms because of the industry structure of our model. Particularly, in the economy consisting of multiple industries, the households must decide not only *how much* labor they want to supply, but also *in which industries* they prefer to work. The utility function reflects both dimensions of the labor supply decision. The objective of the households is to maximize their expected utility,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t^S - \frac{\left(N_t^S\right)^{1+1/\eta}}{1+1/\eta} \right], \tag{1}$$

where $S \in \{K, R\}$ denotes Keynesian, resp. Ricardian type of household. In period t, the household consumes C_t^S units of the final *consumption good*. Parameter $\beta \in (0,1)$ denotes the discount rate and $\eta > 0$ is the elasticity of total labor supply N_t^S. We model the labor supply decision of households in line with Horvath (2000) and Bouakez et al. (2014). The households supply differentiated labor input (hours) $l_{i,t}^S$ into each of the I industries, denoted by $i = 1 \dots I$. Total labor input is given as

$$N_t^S = \left(\sum_{i=1}^{I} \nu_i^{N,S} {l_{i,t}^S}^{\frac{\sigma_N+1}{\sigma_N}} \right)^{\frac{\sigma_N}{\sigma_N+1}}. \tag{2}$$

Thus, household utility is decreasing in number of hours worked in each industry and households prefer to distribute the labor input across industries according to exogenously given weights. The weight $\nu_i^{N,S}$ determines the relative amount of labor input that the household wishes to supply into industry i. The elasticity parameter $\sigma_N > 0$ determines how strongly the weights affect the realised labor input allocation across industries. For σ_N approaching infinity, labor inputs in various industries are perfect substitutes as far as the household is concerned. For $\sigma_N < \infty$, households prefer to diversify their labor input, thus the labor input is not perfectly mobile across industries. This specification allows for industry-specific conditions, e.g. wages, while maintaining the two types of representative households.

The consumption good serves as the numeraire and all prices are expressed relative to its price before tax, P_t. The budget constraint of the Ricardian household is then formulated as

$$\begin{aligned}&\left(\sum_{i=1}^{I}(1-\tau_{i,t}^{l,R})(1-\tau_{i,t}^{s,R})w_{i,t}^{R}l_{i,t}^{R}\right)+\left(\sum_{j=1}^{I^K}\sum_{i=1}^{I}\left[(1-\tau_t^k)r_{i,t}^{k,j}+\tau_t^k\delta^j\right]\frac{k_{i,t-1}^{j}}{1-\omega^K}\right)+\\&+\left[R_t^B-\tau_t^B(R_t^B-1)\right]\frac{B_{t-1}}{1-\omega^K}+(1-\tau_t^k)\frac{T_t}{1-\omega^K}+\frac{LST_t^R}{1-\omega^K}-\frac{ResT_t}{1-\omega^K}=\\&=(1+\tau_t^C)C_t^R+\frac{1}{1-\omega^K}\left(\sum_{j=1}^{I^K}(1+\tau_t^{X,j})P_t^{X,j}X_t^{R,j}\right)+\frac{B_t}{1-\omega^K}.\end{aligned}\tag{3}$$

Variable $w_{i,t}^S$ for $S \in \{R, K\}$ represents gross real industry-specific wage per unit of labor input $l_{i,t}^S$. $k_{i,t}^j$ is private capital stock of type $j = 1, \dots I^K$ used for production in industry i, which is determined at the end of period t. Gross real return on capital $r_{i,t}^{k,j}$ and depreciation rate δ^j also depend on the type of capital. $P_t^{X,j}$ is the relative price of investment good of type j compared to the consumption good, $X_t^{R,j}$ is gross private investment (of Ricardian households) into type j capital. Ricardian households can save in the form of internationally traded one-period risk-free bonds B_t, which yield gross real return R_t^B. T_t are aggregate firm profits. The households have to pay tax rates $\tau_{i,t}^{l,S}$, $\tau_{i,t}^{s,S}$, τ_t^k, τ_t^B, τ_t^C, and $\tau_t^{X,j}$, denoting the labor income tax, social insurance contributions, taxes on capital asset income, interest on private bonds, consumption, and investment good, respectively. Moreover, they pay residual lump sum taxes $ResT_t$ and receive lump-sum transfers from the government, LST_t^S. All taxes are potentially subject to policy shocks and thus time-dependent. Variables τ_t^C and $\tau_t^{X,j}$ denote tax rates paid for bundles of industry goods described below.

The Keynesian households are excluded from all asset markets and thus cannot transfer resources over time. Their budget constraint is given by

$$\left(\sum_{i=1}^{I}(1-\tau_{i,t}^{l,K})(1-\tau_{i,t}^{s,K})w_{i,t}^{K}l_{i,t}^{K}\right)+LST_t^K/\omega^K=(1+\tau_t^C)C_t^K.\tag{4}$$

1.1.2 Consumption good

The households consume a bundle of differentiated *industry goods* produced by a variety of industries at home and abroad,

$$C_t^S=\left(\sum_{i=1}^{I}v_{i,t}{}^{\frac{1}{\sigma_C}}c_{i,t}^S{}^{\frac{\sigma_C-1}{\sigma_C}}\right)^{\frac{\sigma_C}{\sigma_C-1}},\tag{5}$$

where $S \in \{R, K\}$, $c_{i,t}^S$ is the amount of industry i good that is used for consumption by the household of type S and $v_{i,t}$ is the weight of good i in the consumption basket. Parameter $\sigma_C > 0$ represents the elasticity with which the households substitute between industry goods. The weights $v_{i,t}$ are subject to exogenous shocks to relative industry demand for goods of different industries. In steady state, the weights are calibrated to match the composition of household consumption expenditures from the input-output tables.

The nominal price of the consumption basket before tax can be expressed as

$$P_t C_t^S = \sum_{i=1}^{I} p_{i,t}^{NOM} c_{i,t}^S, \tag{6}$$

where $p_{i,t}^{NOM}$ is the nominal price of industry good i. Given industry-level consumption tax rates $\tau_{i,t}^c$, we can define the aggregate consumption tax rate that satisfies[1]

$$(1 + \tau_t^C) P_t C_t^S = \sum_{i=1}^{I} (1 + \tau_{i,t}^c) p_{i,t}^{NOM} c_{i,t}^S. \tag{7}$$

Using relative prices, 7 can be expressed in real terms as

$$(1 + \tau_t^C) C_t^S = \sum_{i=1}^{I} (1 + \tau_{i,t}^c) p_{i,t} c_{i,t}^S, \tag{8}$$

where $p_{i,t} = p_{i,t}^{NOM}/P_t$. The optimal demand for industry i good is iso-elastic, given by

$$c_{i,t}^S = \upsilon_{i,t} \left(\frac{1 + \tau_{i,t}^c}{1 + \tau_t^C} p_{i,t} \right)^{-\sigma_C} C_t^S. \tag{9}$$

It follows from equations 8 and 9 that relative prices $p_{i,t}$ satisfy

$$1 = \left(\sum_{i=1}^{I} \upsilon_{i,t} \left(\frac{1 + \tau_{i,t}^c}{1 + \tau_t^C} p_{i,t} \right)^{1-\sigma_C} \right)^{\frac{1}{1-\sigma_C}}. \tag{10}$$

We assume that industry-specific consumption tax rates follow

$$\tau_{i,t}^c = \tau_{i,ss}^c d_t^{\tau^C} d_{i,t}^{\tau^c}, \tag{11}$$

where $d_t^{\tau^C}$ is aggregate and $d^{\tau_{i,t}^c}$ industry-specific shock to tax rate.

1.1.3 Industry output

Each domestic industry consists of a continuum of monopolistically competitive firms represented by the unit interval. The differentiated *firm goods* aggregate to industry output according to

$$y_{i,t} = \left(\int_0^1 y_{ki,t}^{\frac{\sigma_I - 1}{\sigma_I}} dk \right)^{\frac{\sigma_I}{\sigma_I - 1}}, \tag{12}$$

where $y_{ki,t}$ denotes the output produced by an individual firm k in industry i and $\sigma_I > 0$ is the elasticity of substitution between the firm goods.[2] The industry output has five potential

[1] Since the composition of baskets for Keynesian and Ricardian households is the same, there is a unique price P_t and tax rate τ_t^C.

[2] Elasticity of substitution between firm goods $\sigma_{I,i}$ and all the other within-industry elasticities of substitution can in general be industry-depending. This includes the following elasticities: $\sigma_{y,i}$, $\sigma_{M,i}$, $\sigma_{MH,i}$, $\sigma_{X,i}$, $\sigma_{l,i}$, $\sigma_{k,i}$ $\upsilon_{A,i}$, $\upsilon_{E,i}$. In order to make the formulas readable, we omit the industry index i for all elasticities.

purposes. First, it constitutes the final *consumption good.* Second, it serves as *intermediate input* in the production of domestic firm goods. Third, it is used in the production of *investment goods* that build the domestic capital stock. Fourth, it is used for *government consumption.* Fifth, it is *exported.* The industries differ in the extent to which their output is used for each of these purposes.

The aggregator 12 implies iso-elastic demand for goods of firm k,

$$y_{ki,t} = \left(\frac{p_{ki,t}}{p^H_{i,t}}\right)^{-\sigma_I} y_{i,t}, \tag{13}$$

where $p_{ki,t}$ is the price set by firm k in industry i. The price of the domestically produced industry good $p^H_{i,t}$, defined by $p^H_{i,t} y_{i,t} = \int_0^1 p_{ki,t} y_{ki,t}\, dk$, can be expressed as

$$p^H_{i,t} = \left(\int_0^1 {p_{ki,t}}^{1-\sigma_I} dk\right)^{\frac{1}{\sigma_I - 1}}. \tag{14}$$

1.1.4 Firms

Monopolistically competitive domestic firms produce goods combining capital input, labor, and intermediate inputs. Firms maximize the expected discounted value of their future profits. In order to produce, the firms in industry i must pay the fixed cost $\Phi_i \geq 0$. All firms in an industry have access to the same technology and are ex ante the same. Therefore, in order to simplify the formulas, we omit the firm indices in the following firm-level equations. The gross output of a firm in industry i follows

$$y_{i,t} = J_t Z_t z_{i,t} \left[{\mu_{i,KL}}^{\frac{1}{\sigma_y}} \left(\left(d^k_{i,t} k_{i,t}\right)^{\alpha^y_i} (l_{i,t})^{1-\alpha^y_i} \right)^{\frac{\sigma_y - 1}{\sigma_y}} + {\mu_{i,M}}^{\frac{1}{\sigma_y}} {M_{i,t}}^{\frac{\sigma_y - 1}{\sigma_y}} \right]^{\frac{\sigma_y}{\sigma_y - 1}} - \Phi_i, \tag{15}$$

where Z_t is exogenous aggregate technology that affects all industries. In contrast, $z_{i,t}$ is industry-specific technology that only affects firms in industry i. Variable J_t denotes the level of public infrastructure, which depends on the stock of public capital K^G_{t-1},

$$J_t = \left(\frac{K^G_{t-1}}{K^G_{ss}}\right)^{\gamma_J},$$

where K^G_{ss} is the steady state value of the stock of public capital and parameter $\gamma_J > 0$ drives the sensitivity of firm output to changes in public infrastructure.

Labor and capital are combined into a composite production factor $KL_{i,t}$ using the Cobb-Douglas aggregator

$$KL_{i,t} = \left(d^k_{i,t} k_{i,t}\right)^{\alpha^y_i} (l_{i,t})^{1-\alpha^y_i}, \tag{16}$$

where α^y_i is the capital share and $d^k_{i,t}$ is capital productivity shock. The elasticity of substitution between the capital-labor composite $KL_{i,t}$ and the intermediate inputs bundle $M_{i,t}$ is $\sigma_y > 0$. Parameters $\mu_{i,KL}$ and $\mu_{i,M}$ are the appropriate weights.

Firms differentiate between labor input from Keynesian and Ricardian households and combine them into labor input composite

$$l_{i,t} = \left((1-\omega^K)(l^R_{i,t})^{\frac{\sigma_l - 1}{\sigma_l}} + \omega^K (a^K_i)^{\frac{1}{\sigma_l}} (l^K_{i,t})^{\frac{\sigma_l - 1}{\sigma_l}} \right)^{\frac{\sigma_l}{\sigma_l - 1}}, \tag{17}$$

where $\sigma_l > 0$ is the elasticity of substitution between types of labor input and $(a_i^K)^{\frac{1}{\sigma_l}}$ is the relative productivity of Keynesian workers.

Capital input composite $k_{i,t}$ is produced from I^K differentiated capital types with constant returns to scale technology

$$k_{i,t} = \left(\sum_{j=1}^{I^K} \chi^{j\,\frac{1}{\sigma_k}} k_{i,t}^{j\;\frac{\sigma_k - 1}{\sigma_k}} \right)^{\frac{\sigma_k}{\sigma_k - 1}}, \tag{18}$$

where $k_{i,t}^j$ denotes the capital input of type j that a firm in industry i uses to produce its output. Parameter $\sigma_k > 0$ is the elasticity of substitution between the capital types. Weights $\chi^j \geq 0$ determine the relative importance of capital inputs of different types. We calibrate the parameters χ^j to reflect the average capital composition implied by the input-output tables.

In general case, the intermediate input composite $M_{i,t}$ is constructed from industry goods using the two-layered constant returns to scale technology

$$M_{i,t} = \left(\alpha_i^{MHa\,\frac{1}{\sigma_M}} M_{i,t}^{H\;\frac{\sigma_M - 1}{\sigma_M}} + \sum_{j \in I^L} \alpha_{ji}^{\frac{1}{\sigma_M}} m_{ji,t}^{\frac{\sigma_M - 1}{\sigma_M}} \right)^{\frac{\sigma_M}{\sigma_M - 1}}, \tag{19}$$

$$M_{i,t}^H = \left(\sum_{j \in I^H} \alpha_{ji}^{MH\,\frac{1}{\sigma_{MH}}} m_{ji,t}^{\frac{\sigma_{MH} - 1}{\sigma_{MH}}} \right)^{\frac{\sigma_{MH}}{\sigma_{MH} - 1}}, \tag{20}$$

where $m_{ji,t}$ denotes the intermediate good from industry j that a firm in industry i uses to produce its output. I^H and I^L are two disjunct subsets of industries, denoting the group with higher, resp. lower substitutability. The parameters $\sigma_M > 0$ and $\sigma_{MH} > 0$ are the elasticities of substitution between intermediate inputs from different industries. The weight parameters are defined as

$$\alpha_i^{MHa} = \sum_{j \in I^H} \alpha_{ji}, \tag{21}$$

$$\alpha_{ji}^{MH} = \alpha_{ji} / \alpha_i^{MHa}, \tag{22}$$

and the weights $\alpha_{ji} \geq 0$ determine the relative importance of intermediate inputs from various industries. We calibrate the parameters α_{ji} to reflect the average composition from the input-output tables. Notice that in cases when either I^H or I^L is an empty set, that is whenever all intermediate inputs in industry i are subject to the same elasticity of substitution, expressions 19 and 20 reduce to a single CES aggregator

$$M_{i,t} = \left(\sum_{j=1}^{I} \alpha_{ji}^{\frac{1}{\sigma_M}} m_{ji,t}^{\frac{\sigma_M - 1}{\sigma_M}} \right)^{\frac{\sigma_M}{\sigma_M - 1}}. \tag{23}$$

The before-tax price indices $P_{i,t}^{MH}$ and $P_{i,t}^M$ are defined as

$$P_{i,t}^{MH} M_{i,t}^H = \sum_{j \in I^H} p_{j,t} m_{ji,t}, \tag{24}$$

$$P_{i,t}^{M} M_{i,t} = P_{i,t}^{MH} M_{i,t}^{H} + \sum_{j \in I^L} p_{j,t} m_{ji,t} = \sum_{j=1}^{I} p_{j,t} m_{ji,t}. \tag{25}$$

The product tax rates can in principle be industry-and-product specific, $\tau_{ji,t}^{m}$.[3] Industry tax rate index $\tau_{i,t}^{M}$ is defined as the weighted average of industry-level product taxes,

$$(1 + \tau_{i,t}^{M}) P_{i,t}^{M} M_{i,t} = \sum_{j=1}^{I} (1 + \tau_{ji,t}^{m}) p_{j,t} m_{ji,t}. \tag{26}$$

1.1.5 Price setting

The firms face standard Calvo-type rigidities when setting the prices. Since the firms in industry i are ex ante identical, they all choose the same optimal price $p_{i,t}^{*}$ conditional on adjusting the price in period t. Substituting into 14, the price of industry i goods evolves according to

$$p_{i,t}^{H} = \left[\theta_i \left(\frac{p_{i,t-1}^{H}}{\pi_t} \right)^{1-\sigma_I} + (1 - \theta_i) {p_{i,t}^{*}}^{1-\sigma_I} \right]^{\frac{1}{1-\sigma_I}}, \tag{27}$$

where $\pi_t = P_t / P_{t-1}$ denotes the price inflation and $\theta_i \in [0, 1]$ is the probability that a firm is not allowed to adjust the price in a given period.

In nominal terms, the price setting firm maximizes

$$\mathbb{E}_t \sum_{s=0}^{\infty} {\theta_i}^{s} Q_{t,t+s} \left[y_{i,t+s|t} \cdot p_{i,t}^{NOM} - \mathcal{C}_{i,t+s|t} \right], \tag{28}$$

where $p_{i,t}^{NOM}$ is the nominal price set at time t and $y_{i,t+s|t}$ is period $t+s$ demand for goods of a firm in industry i that was last setting its prices in period t. $Q_{t,t+s}$ is nominal discount factor between periods t and $t+s$, defined as

$$Q_{t,t+s} = \beta^{s} \frac{\lambda_{t+s}}{\lambda_t} \frac{P_t}{P_{t+s}}, \tag{29}$$

where λ_t denotes the marginal utility of consumption expenditures of the Ricardian household at period t. $\mathcal{C}_{i,t+s|t}$ are firm nominal costs of producing output $y_{i,t+s|t}$. Abstracting from the fixed costs, the firm production is constant returns to scale. Thus, we can express the nominal costs in terms of the real marginal costs $RMC_{i,t}$ as

$$\mathcal{C}_{i,t+s|t} = P_{t+s} RMC_{i,t+s} (y_{i,t+s|t} + \Phi_i).$$

1.1.6 Investment and Capital

Both public and private capital stocks of each type $j = 1, 2, ...I^K$ are built using specialized investment goods X_t^{j}. The formation of the investment good of type j follows

$$X_t^{j} = \left(\sum_{i=1}^{I} {\nu_i^{X,j}}^{\frac{1}{\sigma_X}} {x_{i,t}^{j}}^{\frac{\sigma_X - 1}{\sigma_X}} \right)^{\frac{\sigma_X}{\sigma_X - 1}},$$

[3] It seems reasonable to assume that $\tau_{ji,t}^{m}$ should be the same for all receiving industries i, although IOT do not support this assumption.

where $x_{i,t}^{j}$ is the amount of industry i good that is used for production of the investment good of type j, $\sigma_X > 0$ is the elasticity of substitution between industry goods and weight parameters $\nu_i^{X,j}$ drive the weight of industry i good in the investment good of type j. The corresponding (relative) price index before tax is denoted $P_t^{X,j}$. Investment is subject to product tax, which is in general industry-and-type specific, $\tau_{i,t}^{x,j}$.[4] Tax rate index corresponding to investment composite X_t^j is denoted $\tau_t^{X,j}$. Given the prices of industry goods, an optimizing investor chooses

$$x_{i,t}^{j} = \nu_i^{X,j} \left(\frac{(1+\tau_{i,t}^{x,j}) p_{i,t}}{(1+\tau_t^{X,j}) P_t^{X,j}} \right)^{-\sigma_X} X_t^j, \tag{30}$$

where the relative price of investment good j can be expressed as an aggregate of the relative prices of intermediate goods,

$$P_t^{X,j} = \left(\sum_{i=1}^{I} \nu_i^{X,j} \left(\frac{1+\tau_{i,t}^{x,j}}{1+\tau_t^{X,j}} p_{i,t} \right)^{1-\sigma_X} \right)^{\frac{1}{1-\sigma_X}}. \tag{31}$$

Denoting $x_{i,t}^{R,j}$ the investment good of type j used to build up capital stock in industry i, the total private investment is

$$X_t^{R,j} = \sum_{i=1}^{I} x_{i,t}^{R,j}. \tag{32}$$

Both private investment $X_t^{R,j}$ and public investment $X_t^{G,j}$ use the same investment good X_t^j, thus

$$X_t^j = X_t^{R,j} + X_t^{G,j}. \tag{33}$$

In the baseline specification of the model, we assume that capital stocks are industry-specific and installed capital cannot flexibly move between industries. The capital stock of each industry and type is subject to quadratic adjustment costs formulated as in Hayashi (1982).[5] The costs are quadratic in investment intensity $\iota_{i,t}^j$, defined as

$$x_{i,t}^{R,j} = \iota_{i,t}^j k_{i,t-1}^j. \tag{34}$$

The industry private capital stock of each type evolves according to

$$k_{i,t}^j = (1-\delta^j) k_{i,t-1}^j + \phi(\iota_{i,t}^j) k_{i,t-1}^j + \epsilon_t^{dKDj}. \tag{35}$$

Thus, for any investment intensity $\iota_{i,t}^j$, the part $\phi(\iota_{i,t}^j) k_{i,t-1}^j$ that is added to the capital stock is given by

$$\phi(\iota_{i,t}^j) = \iota_{i,t}^j - \frac{\kappa}{\delta^j} (\iota_{i,t}^j - \delta^j)^2,$$

where the depreciation rates δ^j vary across capital types. Cost parameter κ determines the size of the adjustment costs. Capital destruction shock is denoted ϵ_t^{dKDj}.

[4] It seems reasonable to assume that $\tau_{i,t}^{x,j}$ should be the same for all investment types j, although IOT do not support this assumption.

[5] This formulation of capital adjustment costs is equivalent to several other ways of introducing convex adjustment costs, cf. Wang and Wen (2010).

In an alternative specification of the model, we make the assumption that the aggregate capital stock of each type is rigid, but there are no frictions to capital mobility across industries. In such case, the households only face one investment decision for each type of capital. Thus, equations 34 and 35 become

$$X_t^{R,j} = \iota_t^j K_{t-1}^j, \tag{36}$$

and

$$K_t^j = (1-\delta^j)K_{t-1}^j + \phi(\iota_t^j)K_{t-1}^j + \epsilon_t^{dKDj}, \tag{37}$$

and equation 32 is redundant. All other equations hold with $r_{i,t}^{k,j} \equiv r_t^{k,j}$ and $\iota_{i,t}^j \equiv \iota_t^j$.

The public capital stocks $K_t^{G,j}$ evolve according to

$$K_t^{G,j} = (1-\delta^j)K_{t-1}^{G,j} + X_t^{G,j}, \tag{38}$$

where we assume the same depreciation rate as for the private capital and no capital adjustment costs for the public capital stock. The latter assumption is not important for the results, as the impact of public investment on aggregate productivity is determined through another free parameter, γ^J.

The total government capital stock is formed from the differentiated capital types with constant returns to scale technology

$$K_t^G = \left(\sum_{j=1}^{I^K} \chi^{j\,\frac{1}{\sigma_k}} K_t^{G,j\,\frac{\sigma_k-1}{\sigma_k}} \right)^{\frac{\sigma_k}{\sigma_k-1}}. \tag{39}$$

The elasticity of substitution between the capital types σ_k and weights χ^j are the same as for the private capital.

1.1.7 Government

The government collects taxes and spends resources on public consumption, investment, lump sum transfers to households and repaying the interest on its debt. The model is able to handle policy measures that directly affect government consumption, investment, transfers or tax rates in the form of exogenous policy shocks. Section 1.1.13 describes the shocks in detail.

Government consumption of good i follows[6]

$$c_{i,t}^G = c_{i,ss}^G + \frac{d_{i,t}^{cG}}{(1+\tau^{CG})p_{i,t}}, \tag{40}$$

where $c_{i,ss}^G$ is the steady state spending on good i, and $d_{i,t}^{CG}$ is the government consumption spending shock on goods of industry i. Notice that the shocks are expressed in terms of government spending on consumption before tax in current prices. It follows that the aggregate government spending on consumption before tax is

$$C_t^G = \sum_i p_{i,t} c_{i,t}^G. \tag{41}$$

[6]Dividing the government expenditure shocks by aggregate price level (P_t) leads to no difference in the linearised solution. Thus, we omit the term and assume no effect of the aggregate price level.

In steady state, expenditure shares of industry i goods in total government consumption are calibrated to match the data and we denote the shares by ν_i^{CG}. When discussing the *aggregate* shock to government consumption, we assume that the government is allocating its consumption such that the industry shares stay the same.

While government consumption does not directly influence the welfare of the households, government investment affects the aggregate productivity, therefore influencing their expected lifetime wealth. Government investment in type j capital follows

$$X_t^{G,j} = X_{ss}^{G,j} + \frac{d_{j,t}^{XG}}{(1+\tau_t^{X,j})P_t^{X,j}}, \tag{42}$$

where $X_{ss}^{G,j}$ is the steady state investment of type j, $d_{j,t}^{XG}$ is the corresponding government investment spending shock. When discussing the *aggregate* shock to government investment, we assume that the government is allocating its consumption such that the expenditure shares stay constant across investment types.[7]

The government makes lump sum transfers to Keynesian households (LST_t^K), Ricardian households (LST_t^R), and to the rest of the world (FT_t). All three transfer variables are subject to government spending shocks:

$$LST_t^K = LST_{ss}^K + d_t^{LST^K}, \tag{43}$$

$$LST_t^R = LST_{ss}^R + d_t^{LST^R}, \tag{44}$$

$$FT_t = FT_{ss} + d_t^{FT}. \tag{45}$$

The government can borrow or lend resources in the specialised bond market for a given gross real interest rate R_t^G. We assume that all government bonds are held by foreign investors. The budget constraint of the government can be expressed as

$$B_t^G = B_{t-1}^G R_t^G + (1+\tau^{CG})C_t^G + \left(\sum_{j=1}^{I^K}(1+\tau_t^{X,j})P_t^{X,j}X^{G,j}\right) + LST_t + FT_t - TaxRev_t, \tag{46}$$

where $TaxRev_t$ are the total tax revenues of the government and $LST_t = LST_t^K + LST_t^R$.

As the composition of investment good of each type is the same for government and private investment, the tax rates $\tau_t^{X,j}$ are the same as in the private sector. The government consumption, however, has composition that differs from the private consumption. Therefore, the government faces a different consumption tax rate τ^{CG}, identified as the average rate paid by the government in the steady state,

$$(1+\tau^{CG})C^G = \sum_{i=1}^{I}(1+\tau_i^{cg})p_i c_i^G. \tag{47}$$

[7] In this version of the model, government is not able to increase investment of a particular good i. The reason is that we assume the same composition of the private and public investment this assumption will be relaxed in the future, such that it only holds in the steady state.

Total tax revenues of the government are given by

$$
\begin{aligned}
TaxRev_t = &\sum_{i=1}^{I}\left[(\tau_{i,t}^{s,R}+\tau_{i,t}^{l,R}(1-\tau_{i,t}^{s,R}))(1-\omega^K)w_{i,t}^R l_{i,t}^R+(\tau_{i,t}^{s,K}+\tau_{i,t}^{l,K}(1-\tau_{i,t}^{s,K}))\omega^K w_{i,t}^K l_{i,t}^K\right]\\
&+\tau_t^C\left[(1-\omega^K)C_t^R+\omega^K C_t^K\right]+\tau^{CG}C_t^G+\sum_{j=1}^{I^K}\tau_t^{X,j}P_t^{X,j}X_t^j+\sum_{i=1}^{I}\tau_{i,t}^M P_{i,t}^M M_{i,t}\\
&+\tau_t^k\sum_{j=1}^{I^K}\sum_{i=1}^{I}(r_{i,t}^{k,j}-\delta^j)k_{i,t-1}^j+\tau_t^k T_t+\tau_t^B(R_t^B-1)B_{t-1}+ResT_t.
\end{aligned}
\tag{48}
$$

Social security contribution rate $\tau^{s,S}$ is adjusted in order to account for the effect of exogenous shocks on automatic stabilizers in the form of government transfers. As unemployment is not modelled separately in our model, the effective social insurance contributions tax also takes into account the average gain (resp. loss) in unemployment benefit payments.

In the benchmark calibration of the model, the labor income tax rates are only differentiated by household type and kept constant across industries. As common in the macroeconomic literature, time-dependent labor income tax rates are used as fiscal instrument that ensures the stability of the model, see e.g. meta-study of the European Central Bank (de Walque et al. 2015). The labor income tax rates adjust endogenously in reaction to the level of government debt, following

$$
\tau_{i,t}^{l,S}=\tau_{i,ss}^{l,S}+\rho^{\tau_l}(\tau_{i,t-1}^{l,S}-\tau_{i,ss}^{l,S})+(1-\rho^{\tau_l})\gamma^{\tau_l}\frac{B_{t-1}^G-B_{ss}^G}{VA_{ss}}. \tag{49}
$$

The parameters ρ^{τ_l} and γ^{τ_l} determine the speed and strength of the endogenous reaction to the level of government debt. Thus, they can be used to study the effect of various approaches to fiscal consolidation. Alternatively to the distortionary fiscal instrument (labor income tax), the fiscal consolidation in our model can be conducted using the non-distortionary lump sum tax $ResT_t$. Although the assumption that the fiscal consolidation does not distort the economic allocations is less realistic, it is often considered in the macroeconomic literature as a natural benchmark.

1.1.8 International trade

The domestic economy trades goods and financial assets with the rest of the world. Both home economy and the rest of the world are parts of a monetary union and share a common currency. Unlike the home economy, the rest of the world is big. Thus, the nominal price of the foreign consumption goods, $p_{i,t}^{FNOM}$, is not affected by the prices in the home economy and can be treated as exogenous. Expressed in terms of the numeraire home consumption good C_t, the price of the foreign good is

$$
p_{i,t}^F=p_{i,t}^{FNOM}/P_t. \tag{50}
$$

For most (but not all) of the model applications, we assume $p_{i,t}^{FNOM}$ is exogenously given as

$$
p_{i,t}^{FNOM}=p_{i,ss}^{FNOM}\cdot d_{i,t}^{pF}, \tag{51}
$$

where the shock process $d_{i,t}^{pF}$ is industry-specific shock to price of importing good i. Alternatively, we can model the aggregate import price shock d_t^{PF}.

The trade volumes between the home economy and the rest of the world depend on relative prices between home and foreign goods.

Exports We assume that the real exports from domestic industries to the rest of the world follow a reduced-form demand function

$$ex_{i,t} = ex_i^{ss}(p_{i,t}^H/p_{i,t}^F)^{-\upsilon_E} \cdot d_{i,t}^{EXP} \tag{52}$$

where $\upsilon_E > 0$ is the price elasticity of exports and $d_{i,t}^{EXP}$ is exogenous shock to industry exports. Alternatively, the export shocks can be aggregate, denoted d_t^{EXP}.

Imports Each of the industry goods is used in the home economy for one of the five purposes (private consumption, government consumption, investment, intermediate inputs, exports). For each of these purposes, both goods produced at home and abroad are required. The total amount of good i available for each of the uses, $g \in \{c^K, c^R, c^G, x, ex, m\}$, is a CES composite of domestic and foreign goods,

$$g_{i,t} = \left((1-\alpha_i^{it})^{\frac{1}{\upsilon_A}} g_{i,t}^{H\,\frac{\upsilon_A-1}{\upsilon_A}} + \alpha_i^{it\,\frac{1}{\upsilon_A}} g_{i,t}^{F\,\frac{\upsilon_A-1}{\upsilon_A}}\right)^{\frac{\upsilon_A}{\upsilon_A-1}}, \tag{53}$$

where α_i^{it} is a parameter that pins down the import intensity of the industry i. Parameter $\upsilon_A > 0$ is the Armington elasticity which measures the sensitivity of imports to relative prices. Given the price of home-produced good, $p_{i,t}^H$, and the price of foreign-produced good, $p_{i,t}^F$, both expressed in terms of domestic consumption good C_t, price index $p_{i,t}$ is defined as

$$p_{i,t}g_{i,t} = p_{i,t}^H g_{i,t}^H + p_{i,t}^F g_{i,t}^F. \tag{54}$$

The optimal demand for domestic and foreign goods follows

$$g_{i,t}^H = (1-\alpha_i^{it})\left(\frac{p_{i,t}^H}{p_{i,t}}\right)^{-\upsilon_A} g_{i,t}, \tag{55}$$

$$g_{i,t}^F = \alpha_i^{it}\left(\frac{p_{i,t}^F}{p_{i,t}}\right)^{-\upsilon_A} g_{i,t}, \tag{56}$$

where industry price (index) can be expressed as

$$p_{i,t} = \left((1-\alpha_i^{it})p_{i,t}^{H\,1-\upsilon_A} + \alpha_i^{it} p_{i,t}^{F\,1-\upsilon_A}\right)^{\frac{1}{1-\upsilon_A}}. \tag{57}$$

Since equation 56 holds for each use of industry goods separately, the total imports follow

$$im_{i,t} = \alpha_i^{it}\left(\frac{p_{i,t}^F}{p_{i,t}}\right)^{-\upsilon_A}(c_{i,t} + c_{i,t}^G + \sum_{j=1}^{I^K} x_{i,t}^j + \sum_{k=1}^{I} m_{ik,t} + ex_{i,t}), \tag{58}$$

where

$$c_{i,t} = \omega^K c_{i,t}^K + (1-\omega^K)c_{i,t}^R. \tag{59}$$

1.1.9 Market clearing

All markets clear in equilibrium. In particular, for each intermediate good i, the total production plus imports equals the amount of good i used for final consumption, government consumption, investment, intermediate inputs to production in all industries and exports,

$$y_{i,t} = c_{i,t} + c_{i,t}^G + \sum_{j=1}^{I^K} x_{ji,t} + \sum_{k=1}^{I} m_{ik,t} + ex_{i,t} - im_{i,t}. \tag{60}$$

The firm-level factor inputs clear the industry-level factor supply. Industry-level factor inputs clear the aggregate factor supply.

1.1.10 Net foreign assets

Within the domestic economy, only Ricardian households are able to trade the international (private) risk-free bonds. Because the Ricardian households are all the same, the demand for the bond will be either positive or negative for all of them. Therefore, they are only able to trade the bond with the rest of the world. It follows that the total domestic bond holdings must be equal to the net foreign asset position of the Ricardian households,

$$NFA_t^R = B_t. \tag{61}$$

The net foreign asset position of the whole domestic economy equals the holdings of private bonds minus the government debt held abroad,

$$NFA_t^H = B_t - B_t^G. \tag{62}$$

Combining budget constraints of the households and the government with the goods market clearing condition implies that changes in foreign asset positions equate net exports (see section 1.2.1 for detail).

$$NetExp_t = B_t - B_{t-1}R_t^B - (B_t^G - B_{t-1}^G R_t^G) + FT_t, \tag{63}$$

where the real value of net exports $NetExp_t$ is

$$NetExp_t = \sum_{i=1}^{I} (p_{i,t} ex_{i,t} - p_{i,t}^F im_{i,t}). \tag{64}$$

From 63 and 61 we can express the net foreign asset position of the Ricardian households as

$$NFA_t^R = NFA_{t-1}^R R_t^B + NetExp_t - B_{t-1}^G R_t^G + B_t^G - FT_t. \tag{65}$$

1.1.11 Monetary policy and financial markets

The monetary union shares a common monetary policy which does not react to conditions in the home economy. We assume that the monetary authority sets an exogenously given nominal interest rate

$$R_t = R^* \cdot d_t^M. \tag{66}$$

where d_t^M is monetary policy shock. We assume for simplicity that target rate R^* is chosen such that there exists a zero-inflation steady state for the home economy.

The internationally traded private bonds B_t yield real return

$$R_t^B = \frac{R_{t-1}}{\pi_t} \cdot D_{t-1} \cdot e^{-\gamma^{NFA}\left(\frac{NFA_{t-1}^R}{VA_{t-1}} - \frac{NFA_{ss}^R}{VA_{ss}}\right)}, \tag{67}$$

where the nominal interest rate R_t is set by the monetary authority and π_t is consumer price inflation. To ensure model stability, we assume that the international financial markets assign a (symmetric) risk premium to Austrian private bonds, which increases in net foreign private debt and is normalized to zero in the initial steady state, cf. Schuster (2019), Fenz et al. (2012).

Moreover, we introduce the exogenous disturbance term D_t which we interpret as the aggregate demand shock. The shock represents a wedge between the nominal interest rate set by the monetary authority and the interest rate available to the household. This type of demand shock is used in the recent DSGE literature, for example Smets and Wouters (2007), where it is referred to as the *risk premium shock*. The risk premium shock is one particular way of representing an aggregate demand shock. Demand-side shocks are typically characterized as disturbances that generate positive correlation between aggregate consumption, investment, output, employment, prices, and interest rates. The risk premium wedge shares these characteristics and is similar in effect to several other shocks standard in the literature, such as monetary policy shocks, discount factor shocks, financial intermediation shocks and others.

As an alternative to the risk premium shock, the model also features an alternative interest rate wedge which only changes the interest rate *perception* by the household, D_t^p. Shock to the perceived interest rate changes the intertemporal rate of substitution in the household Euler equation, but does not change the actual interest rate, see section 1.2 for detail. Another alternative would be the shock to household depreciation rate β (not implemented).

The government interest rate follows

$$R_t^G = R_{ss}^G \cdot e^{\gamma^{BG}\left(\frac{B_{t-1}^G}{VA_{t-1}} - \frac{B_{ss}^G}{VA_{ss}}\right)}, \tag{68}$$

where R_{ss}^G is the steady state real interest rate on government bonds. The international financial markets assign a risk premium to Austrian government bonds, which increases in government debt and is normalized to zero in the initial steady state. The elasticity of the interest rate on government bonds is determined by parameter γ^{BG}.

1.1.12 Wage rigidity

The hourly wages in the model are subject to a real rigidity, a property which is commonly featured in DSGE models in order to improve their empirical performance. The real wage rigidity channel on one hand stabilizes the private demand through the income effect and works against the stabilizing role of prices in general equilibrium on the other hand. We implement the wage rigidity in reduced form by introducing a wedge in the optimal labor supply decision of households outside of the steady state, see section 1.2 for detail.

1.1.13 Shocks

In principle, every variable or parameter in the model can be subject to an exogenous shock. Adding exogenous shocks with normal distribution and zero mean to the model does not alter the steady state and should not change the stability and determinacy properties. We denote

$d^x_{i,t}$ resp. d^X_t the exogenous shock process related to variable $x_{i,t}$, resp. X_t. This section summarizes the set of exogenous shocks currently featured in the model and briefly discusses their interpretation.

Besides the standard macroeconomic shocks that affect productivity and demand, the model features shocks that can be used to approximate the reaction of the Austrian economy to various external shocks, such as public health crisis, natural disasters, international trade shocks, and financial shocks. The second group are the policy shocks that are used to model various policy interventions. We formalize the fiscal policy intervention as an expansionary disturbance $d_t = \epsilon^d_t$ in period 1. Afterwards, the policy shock d_{t+s} follows autoregressive process. For autoregressive parameter equal to zero this includes one-period shocks. Moreover, each policy shock can be anticipated by the agents for up to A^P periods. Thus, the model is able to handle any given sequence of fiscal shocks specified in a given policy measure as a linear combination of anticipated one-period shocks. Notice that the size of the expenditure shocks is expressed in real terms.

Aggregate shocks

Aggregate exogenous shock processes:

- Shock processes Z_t, D_t, D^p_t, d^M_t, d^k_t, d^{EXP}_t, d^{PF}_t follow AR(1) process

$$\log(d_t) = \rho^d \log(d_{t-1}) + \epsilon^d_t, \qquad \epsilon^d_t \sim \mathcal{N}(0, \sigma^2_d), \tag{69}$$

  where ϵ^d_t is an unanticipated disturbance term for shocks process d and ρ^d is the corresponding persistence parameter.

- Exogenous capital destruction shock ϵ^{dKD}_t is i.i.d, $\epsilon^{dKD}_t \sim \mathcal{N}(0, \sigma^2_{dKD})$.

Aggregate policy shock processes:

- Policy shock to aggregate tax rate τ^C_t is modelled as AR(1) process

$$\log(d^{\tau^C}_t) = \rho^{d\tau} \log(d^{\tau^C}_{t-1}) + \epsilon^{d\tau^C}_t, \qquad \epsilon^{d\tau^C}_t \sim \mathcal{N}(0, \sigma^2_{d\tau^C}). \tag{70}$$

- Expenditure shocks d^{CG}_t, d^{AXG}_t, $d^{LST^R}_t$, $d^{LST^K}_t$, d^{FT}_t follow AR(1) processes

$$d_t = \rho^d d_{t-1} + \epsilon^d_t, \qquad \epsilon^d_t \sim \mathcal{N}(0, \sigma^2_d), \tag{71}$$

  where ϵ^d_t is an unanticipated expansionary disturbance term for shocks process d and ρ^d is the corresponding persistence parameter.

Shocks disaggregated at level of capital types

- Exogenous capital destruction shocks $\epsilon^{dKD,j}_t$ are i.i.d, $\epsilon^{dKD,j}_t \sim \mathcal{N}(0, \sigma^2_{dKD,j})$.

- Policy shocks to tax rates $\tau^{X,j}_t$ are modelled as AR(1) process

$$\log(d^{\tau^{X,j}}_t) = \rho^{d\tau} \log(d^{\tau^{X,j}}_{t-1}) + \epsilon^{d\tau^{X,j}}_t, \qquad \epsilon^{d\tau^{X,j}}_t \sim \mathcal{N}(0, \sigma^2_{d\tau^{X,j}}). \tag{72}$$

- Spending shocks $d^{XG,j}_t$ are modelled as

$$d^{XG,j}_t = \rho^{dXG} d^{XG,j}_{t-1} + \epsilon^{dXG,j}_t, \qquad \epsilon^{dXG,j}_t \sim \mathcal{N}(0, \sigma^2_{dXG,j}), \tag{73}$$

Industry shocks

- Industry technology follows AR(1) process

$$\log(z_{i,t}) = \rho^{dz}\log(z_{i,t-1}) + \epsilon^{z}_{i,t} \qquad \epsilon^{z}_{i,t} \sim \mathcal{N}(0, \sigma^2_{z,i}). \tag{74}$$

- The relative demand for industry good is normalized such that $v_{i,t} = 1$ in each period:

$$v_{i,t} = \tilde{v}_{i,t} / \sum_i \tilde{v}_{i,t}, \tag{75}$$

  where the exogenous process $\tilde{v}_{i,t}$ follows AR(1)

$$\log\left(\frac{\tilde{v}_{i,t}}{\nu_i}\right) = \rho^{dv}\log\left(\frac{\tilde{v}_{i,t-1}}{\nu_i}\right) + \epsilon^{v}_{i,t} \qquad \epsilon^{v}_{i,t} \sim \mathcal{N}(0, \sigma^2_{v,i}). \tag{76}$$

  Steady state values ν_i are calibrated from the industry cost shares of consumption good.

- Government consumption shocks $d^{cG}_{i,t}$ follow AR(1)

$$d^{cG}_{i,t} = \rho^{dcG} d^{cG}_{i,t-1} + \epsilon^{dcG}_{i,t}, \qquad \epsilon^{dcG}_{i,t} \sim \mathcal{N}(0, \sigma^2_{dcG,i}). \tag{77}$$

- Policy shocks to industry tax rate $d^{\tau^c}_{i,t}$ follow AR(1)

$$\log(d^{\tau^c}_{i,t}) = \rho^{d\tau}\log(d^{\tau^c}_{i,t-1}) + \epsilon^{\tau^c}_{i,t}, \qquad \epsilon^{d\tau^c}_{i,t} \sim \mathcal{N}(0, \sigma^2_{d\tau^c,i}). \tag{78}$$

- Import price shocks $d^{pF}_{i,t}$ follow AR(1)

$$\log(d^{pF}_{i,t}) = \rho^{dPF}\log(d^{pF}_{i,t-1}) + \epsilon^{dpF}_{i,t}, \qquad \epsilon^{dpF}_{i,t} \sim \mathcal{N}(0, \sigma^2_{dpF,i}). \tag{79}$$

1.1.14 Macroeconomic quantities and additional variables

Nominal industry-specific value added is defined as

$$va^{N}_{i,t} = P_t\left(p^{H}_{i,t} y_{i,t} - (1+\tau^{M}_{i,t})P^{M}_{i,t} M_{i,t}\right). \tag{80}$$

Real industry-specific value added is defined at constant prices as

$$va^{Re}_{i,t} = P_{ss}\left(p^{H}_{i,ss} y_{i,t} - (1+\tau^{M}_{i,ss})P^{M}_{i,ss} M_{i,t}\right). \tag{81}$$

Industry-specific value added should not be confused with the contribution of each industry good to aggregate value added (use side), which is defined as

$$va^{c}_{i,t} = P_t\left(p_{i,t}(c_{i,t} + c^{G}_{i,t} + \sum_{j=1}^{I^K} x^{j}_{i,t} + ex_{i,t}) - p^{F}_{i,t} im_{i,t} - \tau^{M}_{i,t} P^{M}_{i,t} M_{i,t}\right). \tag{82}$$

Nominal aggregate value added is defined as

$$VA^{N}_{t} = \sum_{i=1}^{I} va^{N}_{i,t} = \sum_{i=1}^{I} va^{c}_{i,t} = \tag{83}$$

$$= P_t \sum_{i=1}^{I}\left(C_t + C^{G}_{t} + \sum_{j=1}^{I^K} P^{X}_{j,t} X_{j,t} + NetExp_t - \sum_{i=1}^{I} \tau^{M}_{i,t} P^{M}_{i,t} . M_{i,t}\right). \tag{84}$$

Aggregate value added expressed in relative prices is

$$VA_t \quad = \quad VA_t^N / P_t, \tag{85}$$

which differs from *real* aggregate value added defined at constant prices as

$$VA_t^{Re} = \sum_{i=1}^{I} va_{i,t}^{Re}. \tag{86}$$

Nominal gross domestic product is defined as

$$GDP_t^N = P_t \left((1+\tau_t^C)C_t + (1+\tau^{CG})C_t^G + \sum_{j=1}^{I^K} (1+\tau_t^{X,j}) P_{j,t}^X X_{j,t} + NetExp_t \right). \tag{87}$$

Real gross domestic product is defined accordingly using constant prices (first order approximation)

$$GDP_t^{Re} = P_{ss} \left((1+\tau_{ss}^C)C_t + (1+\tau^{CG}) \sum_{i=1}^{I} p_{i,ss} c_{i,t}^G + \sum_{j=1}^{I^K} (1+\tau_{ss}^{X,j}) P_{j,ss}^X X_{j,t} + \sum_{i=1}^{I} \left(p_{i,ss} ex_{i,t} - p_{i,ss}^F im_{i,t} \right) \right). \tag{88}$$

Employment is defined as

$$EMP_t = \sum_{i=1}^{I} \frac{\omega^K l_{i,t}^K + (1-\omega^K) l_{i,t}^R}{HPE_i}, \tag{89}$$

where HPE_i is an industry-specific average ratio of additional hours per employed person. Alternatively, for short-run fluctuations we use the concept of *newly generated jobs* defined as

$$EMP_t^{NJ} = \Omega^{NJ} EMP_t, \tag{90}$$

where we assume that a fixed share Ω^{NJ} of fluctuations in hours is covered by new jobs, while the remaining share $1 - \Omega^{NJ}$ is absorbed by existing jobs.

Remaining aggregate variables are defined bottom-up from the industry-level variables.

- Labor input
  Hours worked by a Keynesian household

  $$L_t^K = \sum_i l_{i,t}^K \tag{91}$$

  Hours worked by a Ricardian household

  $$L_t^R = \sum_i l_{i,t}^R \tag{92}$$

  Total hours

  $$L_t = \omega^K L_t^K + (1-\omega^K) L_t^R \tag{93}$$

- Private stock of type j capital

  $$K_t^j = \sum_i k_{i,t}^j \tag{94}$$

- Total private capital stock (approximate)

$$K_t = \sum_{j=1}^{IK} K_t^j \tag{95}$$

- Investment
  Total private investment

$$X_t^R = \sum_{j=1}^{IK} P_t^{X,j} X_t^{R,j} \tag{96}$$

  Total government investment

$$X_t^G = \sum_{j=1}^{IK} P_t^{X,j} X_t^{G,j} \tag{97}$$

  Total investment

$$X_t = \sum_{j=1}^{IK} P_t^{X,j} X_t^j \tag{98}$$

- Industry profits

$$prof_{i,t} = (p_{i,t}^H - RMC_{i,t})y_{i,t} - RMC_{i,t}\Phi_i \tag{99}$$

  Aggregate profits

$$T_t = \sum_{i=1}^{I} prof_{i,t} \tag{100}$$

- Consumption

$$C_t = \sum_i p_{i,t}c_{i,t} \tag{101}$$

  Real consumption at fixed prices

$$C_t^{Re} = P_{ss} \sum_i p_{i,ss}c_{i,t} \tag{102}$$

- Imports and exports

$$EX_t = \sum_{i=1}^{I} p_{i,t}ex_{i,t} \tag{103}$$

$$IM_t = \sum_{i=1}^{I} p_{i,t}^F im_{i,t} \tag{104}$$

- Real imports and exports at constant prices

$$EX_t^{Re} = P_{ss} \sum_{i=1}^{I} p_{i,ss}ex_{i,t} \tag{105}$$

$$IM_t^{Re} = P_{ss} \sum_{i=1}^{I} p_{i,ss}^F im_{i,t} \tag{106}$$

- Aggregate gross output

$$Y_t = \sum_{i=1}^{I} p_{i,t}^H y_{i,t} \tag{107}$$

Real aggregate gross output at constant prices

$$Y_t^{Re} = P_{ss} \sum_{i=1}^{I} p_{i,ss}^H y_{i,t} \tag{108}$$

- Average wages

$$W_t^K = \frac{\sum_i w_{i,t}^K l_{i,t}^K}{L_t^K} \tag{109}$$

$$W_t^R = \frac{\sum_i w_{i,t}^R l_{i,t}^R}{L_t^R} \tag{110}$$

$$W_t = \frac{\sum_i w_{i,t} l_{i,t}}{\sum_i l_{i,t}} \tag{111}$$

- Tobin's Q, denoted Q^T

$$Q_{i,t}^{T,j} = 1/\phi_I(\iota_{i,t}^j) \tag{112}$$

- Inflation in nominal price of investment good of type j

$$\pi_{t+1}^{X,j} = \frac{P_{t+1}^{X,j}}{P_t^{X,j}} \frac{P_{t+1}}{P_t} = \frac{P_{t+1}^{X,j}}{P_t^{X,j}} \pi_{t+1}. \tag{113}$$

1.1.15 Overview of model variables

Industry variables
Exogenous shock processes:
z_i, v_i, d_i^{cG},$d_i^{\tau^c}$, d_i^{ex}, d_i^{pF} ...
Other:
y, k, l, l^R, l^K, M, M^H, c, c^R, c^K, c^G, ex, im, p, P^M, P^{MH}, w, w^R, w^K, RMC, p^*, p^H, p^F, τ^c, $prof$, r^k , Ω, Ψ
Summary: $28 \times I$ variables + shock processes

Variables differentiated by capital type
Exogenous shock processes:
$d^{X,j}$,$\tau^{X,j}$, ...
Other:
K^j, $K^{G,j}$, X^j, $X^{R,j}$, $X^{G,j}$, $P^{X,j}$, , $\tau^{X,j}$, $\pi^{X,j}$
Summary: $8 \times I^K$ variables + shock processes

Aggregate variables
Exogenous shock processes:
Z, D, Dp, d^M, d^k, d^{EX}, d^{PF}, d^{τ^C}, d^{CG}, d^{AXG}, $d_t^{LST^K}$, $d_t^{LST^R}$, d_t^{FT}, ...
Government:
B^G, C^G, X^G, K^G, $TaxRev$, $\tau^{l,R}$, $\tau^{l,K}$, τ^C, LST^R, LST^K, FT, $ResT$
Aggregates:

K, L, L^K, L^R, X, X^R, C, C^R, C^K, VA, Y, EX, IM, $NetExp$, NFA, EMP, T
Prices:
R, RB, R^G, π, P, W, W^K, W^R, Q
Summary: 38 variables + shock processes

Two dimensional variables
Dimension $I \times I^K$: k^j, x^j, $x^{R,j}$, $r^{k,j}$, ι^j, $Q^{T,j}$
Dimension $I \times I$: $m_{i,j}$
Summary: $6 \times I \times I^K + I \times I$ variables.

Summary
Total number of variables necessary to solve the model (excl. shock processes):
$38 + 8 \times I^K + 28 \times I + 6 \times I \times I^K + I \times I$ variables
Variables k^j, x^j, $m_{i,j}$ are implemented as functional forms to reduce the dimensions.
Additional/auxiliary variables and shocks might be added in various versions of the model.

1.2 Model solution and equilibrium

1.2.1 First order conditions

This section sketches the derivation of the first order conditions following from the optimal behaviour of the agents. First order conditions describe the optimal choices and are important for determining the equilibrium allocations. They are also key in understanding and interpreting the model results.

Problem of Ricardian households Ricardian households maximize their objective function (1) with respect to the budget constraint (3), capital evolution equation (35), non-negativity constraints on $k^j_{i,t}$, $l^R_{i,t}$ and C^R_t and no-Ponzi conditions corresponding to assets B_t, $k^j_{i,t}$. The problem of the household is convex and leads to an interior solution, thus the non-negativity constraints are not binding. We solve the reduced problem of maximizing (1) with respect to (3) and (35) by differentiating the Lagrangian w.r.t. the control variables:

$$C^R_t: \quad \lambda_t(1+\tau^C_t) = U_C(C^R_t, N^R_t) \tag{114}$$

$$l^R_{i,t}: \quad -\lambda_t(1-\tau^{l,R}_{i,t})(1-\tau^{s,R}_{i,t})w^R_{i,t} = U_{l_i}(C^R_t, N^R_t) \tag{115}$$

$$B_t: \quad \lambda_t = \beta\,\mathbb{E}_t\,\lambda_{t+1}[R^B_{t+1} - \tau^B_{t+1}(R^B_{t+1}-1)] \tag{116}$$

$$k^j_{i,t}: \quad \nu^j_{i,t} = \beta\,\mathbb{E}_t\left[\nu^j_{i,t+1}(1-\delta^j+\phi(\iota^j_{i,t+1})) + \lambda_{t+1}\frac{(r^{k,j}_{i,t+1} - \tau^k_{t+1}(r^{k,j}_{i,t+1}-\delta^j)) - (1+\tau^{X,j}_{t+1})P^{X,j}_{t+1}\iota^j_{i,t+1}}{1-\omega^K}\right] \tag{117}$$

$$\iota^j_{i,t}: \quad \lambda_t\frac{(1+\tau^{X,j}_t)P^{X,j}_t}{1-\omega^K} = \nu^j_{i,t}\phi_\iota(\iota^j_{i,t}) \tag{118}$$

where λ_t and $\nu^j_{i,t}$ are Lagrange multipliers corresponding to constraints (3) and (35), respectively. U_C and U_{l_i} denote the derivatives of household objective function w.r.t. the corresponding variables.
Equations 114 and 115 together lead to the intratemporal condition

$$-w^R_{i,t}(1-\tau^{s,R}_{i,t})(1-\tau^{l,R}_{i,t})U_C(C^R_t, N^R_t) = (1+\tau^C_t)U_{l_i}(C^R_t, N^R_t). \tag{119}$$

Equations 114 and 116 together yield the household Euler equation

$$\frac{U_C(C_t^R, N_t^R)}{1+\tau_t^C} = \beta\, \mathbb{E}_t[R_{t+1}^B - \tau_{t+1}^B(R_{t+1}^B - 1)] \frac{U_C(C_{t+1}^R, N_{t+1}^R)}{1+\tau_{t+1}^C}. \tag{120}$$

We add the shock to households' perceived interest rate (wedge D_t^P) and substitute in the utility function to express the Euler equation as

$$\frac{C_{t+1}^R}{C_t^R} = \beta\, \mathbb{E}_t \left[(1-\tau_{t+1}^B) R_{t+1}^B D_t^p + \tau_{t+1}^B\right] \frac{1+\tau_t^C}{1+\tau_{t+1}^C}. \tag{121}$$

Using the standard definition of Tobin's Q (denoted $Q_i^{T,j}$), equation 118 can be rearranged as

$$\nu_{i,t}^j = \frac{(1+\tau_t^{X,j})P_t^{X,j}}{1-\omega^K} \frac{\lambda_t}{\phi_\iota(\iota_{i,t}^j)} = \frac{(1+\tau_t^{X,j})P_t^{X,j}}{1-\omega^K} \lambda_t Q_{i,t}^{T,j}, \tag{122}$$

and plugged into equation 117 to obtain the optimal investment conditions

$$Q_{i,t}^{T,j} = \mathbb{E}_t\, Q_{t,t+1} \pi_{t+1}^{X,j} \frac{1+\tau_{t+1}^{X,j}}{1+\tau_t^{X,j}} \left[\frac{r_{i,t+1}^{k,j} - \tau_{t+1}^k (r_{i,t+1}^{k,j} - \delta^j)}{(1+\tau_{t+1}^{X,j})P_{t+1}^{X,j}} - \iota_{i,t+1}^j + Q_{i,t+1}^{T,j}(1-\delta^j + \phi(\iota_{i,t+1}^j)) \right]. \tag{123}$$

Problem of Keynesian households Keynesian households maximize their objective (1) with respect to the budget constraint (4), and non-negativity constraints on $l_{i,t}^K$ and C_t^K. The problem of the household is convex and leads to an interior solution, thus the non-negativity constraints are not binging. Solving the reduced problem of maximizing (1) with respect to (4) leads to:

$$C_t^K: \quad \lambda_t^K(1+\tau_t^C) = U_C(C_t^K, N_t^K) \tag{124}$$

$$l_{it}^K: \quad -\lambda_t^K(1-\tau_{i,t}^{l,K})(1-\tau_{i,t}^{s,K}) w_{it}^K = U_{l_i}(C_t^K, N_t^K) \tag{125}$$

where λ_t^K is Lagrange multiplier corresponding to constraint (4). Equations 124 and 125 together lead to the intratemporal condition

$$-\, w_{i,t}^K(1-\tau_{i,t}^{l,K})(1-\tau_{i,t}^{s,K}) U_C(C_t^K, N_t^K) = (1+\tau_t^C) U_{l_i}(C_t^K, N_t^K). \tag{126}$$

Labor supply wedge: wage rigidity We implement the real wage rigidity in a reduced form by introducing a wedge in the optimal labor supply decisions 119, 126. The households supply labor according to the following equation:

$$-\, w_{i,t}^S = \left(\frac{(1+\tau_t^C) U_{l_i}(C_t^S, N_t^S)}{(1-\tau_{i,t}^{l,S})(1-\tau_{i,t}^{s,S}) U_C(C_t^S, N_t^S)} \right)^{(1-\omega)} \left(w_{i,ss}^S\right)^\omega, \tag{127}$$

for $S \in \{K, R\}$. Notice that equation 127 has the same steady state as equation 119 resp. 126 but implies a dampened reaction of wages compared to labor supply for any $0 < \omega \leq 1$. The case $\omega = 0$ corresponds to conditions 119 and 126.

Firm problem For given firm prices, the demand for products of the firm is determined. Firm k in industry i faces the problem of optimal choice of production inputs, such that the demand is satisfied.

$$\min_{k^j_{ki,t}, l^S_{ki,t}, m_{k,1..I,i,t}} (1-\omega^K) w^R_{i,t} l^R_{ki,t} + \omega^K w^K_{i,t} l^K_{ki,t} + \sum_{j=1}^{I^K} r^{k,j}_{i,t} k^j_{ki,t} + (1+\tau^M_{i,t}) \sum_{j=1}^{I} p_{j,t} m_{k,ji,t} \quad (128)$$

such that

$$y_{ki,t}(p_{ki,t}) + \Phi_i = F^i(Z_t, z_{i,t}, k_{ki,t}, l_{ki,t}, M_{ki,t}, K^G_{t-1}, d^k_{i,t}), \quad (129)$$

where F^i is the production function in industry i and input aggregates $l_{ki,t}$, $k_{ki,t}$, $M_{ki,t}$ are defined according to equations 17 - 19, respectively. We denote $w_{i,t}$, $r^k_{i,t}$, $P^M_{i,t}$ the corresponding price indexes, respectively. Due to the constant returns to scale technology, the price indexes of all production factors are the same for all firms in industry i, as they optimally choose the same composition of inputs.

Differentiating the Lagrangian w.r.t. $l_{ki,t}$, $k_{ki,t}$, $M_{ki,t}$ we obtain the standard conditions

$$\frac{w_{i,t}}{\lambda_{ki,t}} = \frac{\partial F^i}{\partial l_{ki,t}}, \quad (130)$$

$$\frac{r^k_{i,t}}{\lambda_{ki,t}} = \frac{\partial F^i}{\partial k_{ki,t}}, \quad (131)$$

$$(1+\tau^M_{i,t})\frac{P^M_{i,t}}{\lambda_{ki,t}} = \frac{\partial F^i}{\partial M_{ki,t}}, \quad (132)$$

where $\lambda_{ki,t}$ is the Lagrange multiplier associated with condition 129.

The wage index of labor used in industry i, $w_{i,t}$, follows from the constant returns to scale function of labor input l_i,

$$w_{i,t} = \left((1-\omega^K)(w^R_{i,t})^{1-\sigma_l} + a^K_i \omega^K (w^K_{i,t})^{1-\sigma_l}\right)^{\frac{1}{1-\sigma_l}}. \quad (133)$$

Firm demand for the two types of labor follows

$$l^R_{ki,t} = \left(\frac{w^R_{i,t}}{w_{i,t}}\right)^{-\sigma_l} l_{ki,t}, \quad (134)$$

$$l^K_{ki,t} = a^K_i \left(\frac{w^K_{i,t}}{w_{i,t}}\right)^{-\sigma_l} l_{ki,t}. \quad (135)$$

The price of capital good k_i follows from the production function of capital aggregate k_i

$$r^k_{i,t} = \left(\sum_{j=1}^{I^K} \chi^j \left(r^{k,j}_{i,t}\right)^{1-\sigma_k}\right)^{\frac{1}{1-\sigma_k}}. \quad (136)$$

Firm demand for the various types of capital follows

$$k^j_{ki,t} = \chi^j \left(\frac{r^{k,j}_{i,t}}{r^k_{i,t}}\right)^{-\sigma_k} k_{ki,t}. \quad (137)$$

The price index of intermediate goods used in industry i, $P_{i,t}^M$, follows from the production function of intermediate good M_i:

$$P_{i,t}^M = \left(\alpha_{ji}^{MHa} \left(P_{i,t}^{MH} \right)^{1-\sigma_M} + \sum_{j \in I^L} \alpha_{ji} p_{j,t}{}^{1-\sigma_M} \right)^{\frac{1}{1-\sigma_M}}, \tag{138}$$

$$P_{i,t}^{MH} = \left(\sum_{j \in I^H} \alpha_{ji}^{MH} p_{j,t}{}^{1-\sigma_{MH}} \right)^{\frac{1}{1-\sigma_{MH}}}. \tag{139}$$

Combining conditions 130 and 131 leads to

$$\frac{w_{i,t}}{r_{i,t}^k} = \frac{\partial F^i / \partial l_{ki,t}}{\partial F^i / \partial k_{ki,t}}, \tag{140}$$

and combining conditions 131 and 132 gives

$$\frac{w_{i,t}}{(1+\tau_{i,t}^M) P_{i,t}^M} = \frac{\partial F^i / \partial l_{ki,t}}{\partial F^i / \partial M_{ki,t}}. \tag{141}$$

Following a standard procedure, it is straightforward to derive the optimality condition

$$RMC_{i,t} = \frac{r_{i,t}^k}{\partial F^i / \partial k_{ki,t}}, \tag{142}$$

and analogous conditions for $l_{ki,t}$, $M_{ki,t}$, where the real marginal costs are are independent on the firm's decisions:

$$RMC_{i,t} = \frac{1}{J_t Z_t z_{i,t}} \left(\mu_{i,KL} \left(P_{i,t}^{KL} \right)^{1-\sigma_y} + \mu_{i,M} \left((1+\tau_{i,t}^M) P_{i,t}^M \right)^{1-\sigma_y} \right)^{\frac{1}{1-\sigma_y}}, \tag{143}$$

$$P_{i,t}^{KL} = \left(\frac{r_{i,t}^k}{d_{i,t}^k \cdot \alpha_i^y} \right)^{\alpha_i^y} \left(\frac{w_{i,t}}{1-\alpha_i^y} \right)^{1-\alpha_i^y} \tag{144}$$

Equation 142 thus implies that all firms operating in industry i choose inputs such that the partial derivatives $\partial F^i / \partial k_{ki,t}$ are the same across firms. Equations 140 to 142 thus hold at the industry level as well. Apart from the fixed costs, the production technology is constant returns to scale, therefore real costs are linear in output,

$$Costs(y_{ki,t} + \Phi_i) = RMC_{i,t} \times (y_{ki,t} + \Phi_i). \tag{145}$$

Plugging the production function 15 into 142 and reorganizing, we get

$$k_{ki,t} = \Theta_{i,t}^K (y_{ki,t} + \Phi_i), \tag{146}$$

$$l_{ki,t} = \Theta_{i,t}^L (y_{ki,t} + \Phi_i), \tag{147}$$

$$M_{ki,t} = \Theta_{i,t}^M (y_{ki,t} + \Phi_i) \tag{148}$$

where $\Theta_{i,t}^X = \Theta^X(J_t, Z_t, z_{i,t}, d_{i,t}^k, r_{i,t}^k, w_{i,t}, P_{i,t}^M, \tau_{i,t}^M, \sigma_y, \mu_{i,KL}, \mu_{i,M}, \alpha_i^y)$ depends on industry-level variables and parameters only.

Industry quantities We now derive the total demand for a particular good i and show it is independent of firm-specific variables up to the first order approximation. The industry demand can be expressed using the goods market clearing condition 60. We already expressed terms $c_{i,t}$, $c^G_{i,t}$, $x^j_{i,t}$, $ex_{i,t}$, $im_{i,t}$ independently on individual firm decisions, and prices are independent as well. We now complete by determining the demand for good i as intermediate input, $m_{ij,t}$.

Industry demand for labor follows from the market clearing condition

$$
\begin{aligned}
l_{i,t} &= \int_0^1 l_{ki,t}\, dk && (149)\\
&= \Theta^L_{i,t} \int_0^1 y_{ki,t} + \Phi_i\, dk && (150)\\
&= \Theta^L_{i,t}\Phi_i + \Theta^L_{i,t} y_{i,t} \int_0^1 \left(\frac{p_{ki,t}}{p^H_{i,t}}\right)^{-\sigma_I} dk, && (151)
\end{aligned}
$$

where constant $\Theta^L_{i,t}$ depends on industry-level prices and parameters only. We define the dispersion term

$$Disp_{i,t} = \int_0^1 \left(\frac{p_{ki,t}}{p^H_{i,t}}\right)^{-\sigma_I} dk. \tag{152}$$

A standard result from the New Keynesian literature shows that the dispersion term has only second-order effects around the zero-inflation steady state. It follows that

$$l_{i,t} \approxeq \Theta^L_{i,t}(y_{i,t} + \Phi_i), \tag{153}$$

up to the first order approximation. In line with 153, demand for intermediate good composite $M_{j,t}$ in industry j is

$$M_{j,t} \approxeq \Theta^M_{j,t}(y_{j,t} + \Phi_j). \tag{154}$$

An optimizing firm k in sector j chooses intermediate input from sector i according to

$$m_{k,ij,t} = \alpha_{ij} \left(\frac{1+\tau^m_{ij,t}}{1+\tau^M_{j,t}} \frac{p_{i,t}}{P^M_{j,t}}\right)^{-\sigma_M} M_{kj,t} \tag{155}$$

Total intermediate input i as an input into industry j production can be expressed as

$$
\begin{aligned}
m_{ij,t} &= \int_0^1 m_{k,ij,t}\, dk && (156)\\
&= \alpha_{ij} \left(\frac{1+\tau^m_{ij,t}}{1+\tau^M_{j,t}} \frac{p_{i,t}}{P^M_{j,t}}\right)^{-\sigma_M} \int_0^1 M_{kj,t}\, dk && (157)\\
&\approxeq \Gamma_{ij,t}(y_{j,t} + \Phi_j), && (158)
\end{aligned}
$$

where the parameter $\Gamma_{ij,t}$ is independent of the firm's actions,

$$\Gamma_{ij,t} = \alpha_{ij} \left(\frac{1+\tau^m_{ij,t}}{1+\tau^M_{j,t}} \frac{p_{i,t}}{P^M_{j,t}}\right)^{-\sigma_M} \Theta^M_{j,t}. \tag{159}$$

Thus, the demand for industry goods does not depend on the actions of individual firms up to the first order approximation, as none of the terms in goods market clearing condition 60 does.

Price setting The period $t+s$ demand for the product of a firm that has last updated its price in period t can be expressed using 13 as

$$y_{ki,t+s|t} = \left(\frac{p_{ki,t}^{NOM}}{p_{i,t+s}^{H,NOM}}\right)^{-\sigma_I} y_{i,t+s}, \tag{160}$$

where superscript NOM denotes the nominal prices. As we showed previously, $y_{i,t+s}$ does not depend on the decisions of firm k.
The price-setting problem of each firm is to maximize 28:

$$\max_{p_{ki,t},y_{ki,t},\cdot\cdot,y_{ki,\infty}} \mathbb{E}_t \sum_{s=0}^{\infty} {\theta_i}^s Q_{t,t+s} \left[y_{ki,t+s|t}(p_{ki,t}^{NOM} - NMC_{i,t+s}) - \Phi_i NMC_{i,t+s}\right]. \tag{161}$$

with respect to 160. Nominal marginal costs $NMC_{i,t}$ are defined as

$$NMC_{i,t} = P_t \cdot RMC_{i,t}. \tag{162}$$

Differentiating the Lagrangian we obtain

$$p_{ki,t}^{N} \quad : \quad \mathbb{E}_t \sum_{s=0}^{\infty} {\theta_i}^s Q_{t,t+s} y_{ki,t+s|t} - \sum_{s=0}^{\infty} \varrho_{t+s} \sigma_I \frac{(p_{ki,t}^{NOM})^{-\sigma_I - 1}}{(p_{i,t+s}^{H,NOM})^{-\sigma_I}} y_{i,t+s} = 0, \tag{163}$$

$$y_{ki,t+s|t} \quad : \quad {\theta_i}^s Q_{t,t+s}[p_{ki,t}^{NOM} - NMC_{i,t+s}] = -\varrho_{t+s}, \tag{164}$$

ϱ_t denoting the Lagrange multiplier corresponding to the constraint 160 at time t. Substituting 164 into 163 we get

$$\mathbb{E}_t \sum_{s=0}^{\infty} {\theta_i}^s Q_{t,t+s} \left[y_{ki,t+s|t} - [p_{ki,t}^{NOM} - NMC_{i,t+s}] \sigma_I \frac{(p_{ki,t}^{NOM})^{-\sigma_I - 1}}{(p_{i,t+s}^{H,NOM})^{-\sigma_I}} y_{i,t+s}\right] = 0 \tag{165}$$

$$\mathbb{E}_t \sum_{s=0}^{\infty} {\theta_i}^s Q_{t,t+s} y_{ki,t+s|t} \left[p_{ki,t}^{NOM} - \frac{\sigma_I}{\sigma_I - 1} NMC_{i,t+s}\right] = 0 \tag{166}$$

In a symmetric equilibrium, all firms within an industry i that set their prices in period t choose the same optimal price $p_{ki,t}^{NOM} = p_{i,t}^{NOM*}$. The term $\mu = \frac{\sigma_I}{\sigma_I - 1}$ expresses the gross price markup.

The numerical solution of the the model requires us to express the FOC 166 in terms of recursively defined sums. We can derive that the equation is equivalent to (in real prices)

$$\Omega_{i,t} p_{i,t}^* = \mu \Psi_{i,t} p_{i,t}^H, \tag{167}$$

where the recursive expressions for Ω and Ψ give

$$\Omega_{i,t} = y_{i,t} + \theta_i \, \mathbb{E}_t \left(\frac{p_{i,t+1}}{p_{i,t}} \pi_{t+1}\right)^{\sigma_I} Q_{t,t+1} \Omega_{i,t+1}, \tag{168}$$

$$\Psi_{i,t} = y_{i,t} \frac{RMC_{i,t}}{p_{i,t}} + \theta_i \, \mathbb{E}_t \left(\frac{p_{i,t+1}}{p_{i,t}} \pi_{t+1}\right)^{\sigma_I + 1}. \tag{169}$$

The derivation of the recursive solution is omitted from this documentation for space reasons.

Derivation of NFA equation 63 First, we combine equation 54 with the goods market clearing condition 60 and get that

$$p_i(c_{i,t} + c^G_{i,t} + \sum_{j=1}^{I^K} x^j_{i,t} + \sum_{k=1}^{I} m_{ik,t} + ex_{i,t}) = p^H_{i,t} y_{i,t} + p^F_{i,t} im_{i,t}. \tag{170}$$

Summing 170 across industries gives

$$Y_t = \sum_{i=1}^{I} p^H_{i,t} y_{i,t} = C_t + C^G_t + \sum_{j=1}^{I^K} P^{X,j}_t X^j_t + \sum_{i=1}^{I} P^M_{i,t} M_{i,t} + NetExp_t. \tag{171}$$

At the same time, industry-level production (by definition) gives

$$p^H_{i,t} y^H_{i,t} = prof_{i,t} + (1-\omega^K) w^R_{i,t} l^R_{i,t} + \omega^K w^K_{i,t} l^K_{i,t} + \sum_{j=1}^{I^K} r^{k,j}_{i,t} k^j_{i,t-1} + (1+\tau^M_{i,t}) \sum_{j=1}^{I} p_{j,t} m_{ji,t}. \tag{172}$$

Summing 172 across industries gives

$$Y_t = \sum_{i=1}^{I} p^H_{i,t} y_{i,t} = T_t + \sum_{i=1}^{I} ((1-\omega^K) w^R_{i,t} l^R_{i,t} + \omega^K w^K_{i,t} l^K_{i,t}) + \sum_{i=1}^{I} \sum_{j=1}^{I^K} r^{k,j}_{i,t} k^j_{i,t-1} + \sum_{i=1}^{I} (1+\tau^M_{i,t}) P^M_{i,t} M_{i,t} \tag{173}$$

Combining equations 171 with 173 we get

$$NetExp_t = \tag{174}$$

$$= T_t + \sum_{i=1}^{I} ((1-\omega^K) w^R_{i,t} l^R_{i,t} + \omega^K w^K_{i,t} l^K_{i,t}) + \sum_{i=1}^{I} \sum_{j=1}^{I^K} r^{k,j}_{i,t} k^j_{i,t-1} \tag{175}$$

$$+ \sum_{i=1}^{I} \tau^M_{i,t} P^M_{i,t} M_{i,t} - (C_t + C^G_t + \sum_{j=1}^{I^K} P^{X,j}_t X^j_t) \tag{176}$$

Summing up budget constraints 3, 4 and 46 we get

$$B_t - B_{t-1} R^B_t - (B^G_t - B^G_{t-1} R^G_t) + FT_t = \tag{177}$$

$$= T_t + \sum_{i=1}^{I} ((1-\omega^K) w^R_{i,t} l^R_{i,t} + \omega^K w^K_{i,t} l^K_{i,t}) + \sum_{i=1}^{I} \sum_{j=1}^{I^K} r^{k,j}_{i,t} k^j_{i,t-1} \tag{178}$$

$$+ \sum_{i=1}^{I} \tau^M_{i,t} P^M_{i,t} M_{i,t} - (C_t + C^G_t + \sum_{j=1}^{I^K} P^{X,j}_t X^j_t) \tag{179}$$

Finally, we obtain that net foreign asset position of the households as given by 63.

1.2.2 Equilibrium

The equilibrium allocation is determined by the following set of equations, which determine the variables listed in section 1.1.15:

Industry-level equations
Together $28 \times I$ equations + shock processes:

- n^I shock processes
- I consumption tax definition 11
- $2I$ intra-temporal FOCs from Ricardian and Keynesian household problem 127
- $2I$ conditions firm demand for labor input by household type 134, 135
- I equations for wage indexes of industry labor inputs 133
- I production functions 15
- $2I$ equations for price indexes of industry intermediate inputs 138, 139
- $3I$ FOCs from firm cost optimisation 140 - 142
- I second layer of intermediate input basket 20
- $3I$ recursive formulation of FOCs of firm price setting problem 167 - 169
- I industry home price evolution equations 27
- I industry foreign price equations 50
- I industry price equations 57
- $2I$ optimal demand for industry good in final consumption by type 9
- I total demand for industry good in final consumption 59
- I industry government consumption 40
- I good market clearing conditions 60
- I profit equations 99
- I industry exports 52
- I industry imports 58
- I industry costs of capital 136

Capital-type level equations
Together $8 \times I^K$ equations + shock processes:

- n^{I^K} shock processes
- I^K private investment 32
- I^K government investment 42
- I^K government capital evolution equations 38
- I^K investment good inflation 113
- I^K total investment definitions 33
- I^K relative price of investment goods 31
- I^K aggregate capital stock by type 94
- I^K investment product tax definitions

Aggregate equations
Together 38 equations + shock processes:

- n^A aggregate shock processes
- 1 aggregate price level
- 1 consumption price index 10
- 1 monetary policy interest rate rule 66

- 1 effective return on bonds 67
- 1 government bond interest rate 68
- 1 government bond evolution equation 46
- 1 aggregate government consumption 41
- 1 total tax revenue 48
- 1 aggregate consumption tax definition 8
- 2 tax instrument evolution equations 49
- 3 lump sum transfers 43, 44, 45
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- 2 aggregate total capital: private and government 95, 39
- 1 aggregate government investment definition 97
- 1 aggregate private investment definition 96
- 13 definitions VA, EMP, C, L, L^K, L^R, EX, IM, T, Y, W, W^K, W^R: 85, 89, 91 - 93, 100, 101, 103, 104, 107, 109 - 111,

Two-dimensional sets of equations
Together $6 \times I \times I^K + I \times I$ equations:

- $I \times I^K$ demand for goods for investment use 30
- $I \times I^K$ private investment definitions 34
- $I \times I^K$ private capital evolution equations 35
- $I \times I^K$ Tobin's Q 112
- $I \times I^K$ FOCs from Ricardian household problem 123
- $I \times I^K$ demand for capital types 137
- $I \times I$ intermediate input flows 155

1.2.3 Numerical solution

We solve the model by linearising the equations around the deterministic steady state. In the first step, we find the deterministic steady state. In the second step, the linear solution is computed in HetSol Toolkit developed by Michael Reiter.

2 Data and Calibration

2.1 Data sources

The main data source are the Austrian input-output tables (IOT), national accounts (VGR), national tax list, and EU Statistics on income and living conditions (EU-SILC).

Austrian input-output tables. We use the information from tables *28 - Input-output table at basic prices, domestic output and imports* and *27 - Employment (Products)* published by Statistics Austria. Table 27 provides information on employment and hours worked in each industry, separately for employed and self-employed persons. Table 28 provides information on the input-output structure of the economy which includes industry output, use of intermediate inputs and other production factors (make-side), product taxes, use of industry output differentiated by purpose (use-side) and imports.[8]

The information from the input-output tables is used to calibrate the steady state of the model economy. To minimize the effect of short term fluctuations, we use averages over the IOT information from years 2012 to 2017.

Other data sources. To characterise the two types of households we use the data from the 2016 EU Statistics on income and living conditions in Austria published by Statistics Austria, (StatAT 2017). The EU-SILC database gathers the information on income, government transfers, and other income-related statistics from 6,000 individual households representative of all 3.9 million households in Austria.

For public and private investment and unemployment benefit payments we use the information from the national accounts, tables 57 and D.62, as reported by the Austrian statistical office (StatAT 2018).

We also use the information on tax revenues from the National Tax List published by Statistics Austria, see table *Steuern und Sozialbeiträge in Österreich: Einzelsteuerliste / National Tax List*. The data set includes the information on annual tax revenues and social security contributions disaggregated by the type of tax. In line with the IOT we use the information from 2012 to 2017.

2.2 Calibration

We calibrate the model at quarterly frequency. In line with the Austrian IOT, the model distinguishes 74 industries and 5 types of capital. Table 1 summarizes model parameters and calibration targets.

The steady state of the model economy is pinned down by a number of weight parameters, tax rates, and other parameters which we calibrate to directly match their counterparts in the Austrian data. The parameters and data sources are listed in table 1.

We define Keynesian households as those with equivalized disposable income below the median and gross capital income of less than 100 Euro per year. According to the EU-SILC data, this characterization applies to roughly 36% of the Austrian households. We classify the remaining 64% of the households as Ricardian. We also utilize the EU-SILC data to calibrate the differences between Keynesian and Ricardian households in hours worked, hourly wages, and government transfers.

We calibrate the eight different types of tax rates in the model such that the implied tax revenue of each tax matches the data from the National Tax List. Thus, we implicitly

[8] The IOT published by Statistics Austria are product-based, in contrast to the industry-based IOT reported by some other sources, e.g. the WIOD database. The product-based IOT provide a more suitable counterpart to the model specification. However, some other data inputs, e.g. EU-SILC data are reported at the level of industries. The differences between the classifications are within an acceptable range. For example, in case of industry-level employment, only three industries/product groups show any substantial discrepancies.

Calibration summary			
Parameter	Symbol	Value	Target/Source
Elasticities			
Intra-industry substitution	σ_I	11	10% markup
Consumption good subst.	σ_C	0.4	Molnárová and Reiter (2022)
Investment good subst.	σ_X	0.4	equals σ_C
Production factors subst.	σ_y	0.39	Molnárová and Reiter (2022)
Intermediate inputs subst. - high	σ_{MH}	0.75	Molnárová and Reiter (2022)
Intermediate inputs subst. - low	σ_M	0.05	low substitutability
Labor input types subst.	σ_l	0.5	low substitutability
Capital types subst.	σ_k	2	medium substitutability
Industry labor input subst.	σ_N	2	Molnárová and Reiter (2022)
Total labor input	η	0.5	standard
Import (Armington)	υ_A	2.4	Fenz et al. (2012), Imbs and Mejean (2015)
Export	υ_E	2.4	equals import
Taxes and transfers			
Product tax, household consumption	τ^C	0.15	tax revenues, IOT
Product tax, government consumption	τ^{CG}	0.01	tax revenues, IOT
Product tax, investment good	$\tau^{X,j}$		tax revenues, IOT
Product tax, intermediate inputs	$\tau^{M,i}$		tax revenues, IOT
Asset income, capital and profits	τ^k	0.24	tax revenues, National Tax List
Asset income, interest on private bonds	τ^B	0.47	tax revenues, National Tax List
Social insurance contribution rate	$\tau^{s,R/K}$	0.29	tax revenues, National Tax List, adjustment
Labor income, Ricardian households	$\tau_{ss}^{l,R}$	0.32	tax revenues, National Tax List, EU-SILC
Labor income, Keynesian households	$\tau_{ss}^{l,K}$	0.17	tax revenues, National Tax List, EU-SILC
Total tax revenues, percentage of value added	$TaxRev_{ss}$	48.3	total tax revenues, National Tax List
Steady state weights			
Import intensity	α_i^{it}		cost shares IOT, import shares
Production factor weights	$\mu_{\times,i}$		cost shares IOT, production factors
Intermediate inputs weights	α_{ji}		cost shares IOT, intermediate inputs
Household consumption weights	v_i		cost shares IOT, private consumption
Government consumption weights	ν_i^{CG}		cost shares IOT, government consumption
Investment good weights	$\nu_i^{X,j}$		cost shares IOT, investment by type
Export weights	ν_i^{EXPT}		cost shares IOT, exports
Disutility parameter: industry-specific labor	$\nu_i^{N,S}$		hours/wages IOT, EU-SILC
Relative productivity of Keynesian households	a_i^K		relative wages, EU-SILC
Hours per employee	HPE_i		IOT employment table
Other			
Share of Keynesian households	ω^K	0.359	EU-SILC, own definition
Discount factor	β	0.995	2% annual interest rate
Capital depreciation	δ^j		StatAT (2018)
Adjustment cost capital	κ	0.072	cond. relative volatility of investment
Productivity sensitivity to public infrastructure	γ^J	0.015	same productivity as private investment
Price stickiness	θ_i	0.75	standard
Wage stickiness	ω	0.75	relative volatility wages and consumption
Gross return on government bonds	R_{ss}^G	0.5%	low annual rate
Steady state government debt, perc. of value added	B_{ss}^G	269	declared target 60% debt-to-GDP ratio
Risk premium parameter, private bonds	γ^{NFA}	0.004	Fenz et al. (2012)
Risk premium parameter, government bonds	γ^{BG}	0	no risk premium in baseline
Steady state profit share, perc. of value added	$shProf$	4.5	IOT, StatAT (2018)
Autocorrelation, exogenous shocks	ρ^d	0.7	cumulative effect 95% after 2 years
Autocorrelation, technology shocks	ρ^A	0.95	standard
Fiscal instrument persistence	ρ^{τ_l}	0.98	slow consolidation
Fiscal instrument strength	γ^{τ_l}	0.1	slow consolidation
Number of industries	I	74	IOT classification
Number of types of capital	I^K	5	IOT classification

Table 1: Baseline calibration summary

assume that the average tax rates equal the marginal tax rates which influence the economic decisions. Such assumption is not innocuous, but relatively common in the DSGE literature, see Coenen et al. (2008), Gadatsch et al. (2016), Adolfson et al. (2013).[9]

Other parameters The majority of the remaining parameters are calibrated to values used in the existing macroeconomic literature. The parameters and economic quantities entering the stationary model specification must be adjusted for inflation and economic growth. We follow the long term projection of the OECD (2018) and assume that trend inflation is 2% annually and population growth is 0.5% annually. We choose a comparatively more conservative labor productivity trend growth of 1% annually. As a result, total effective labor force grows approximately by 1.5% annually.

The household discount rate β is set such that it implies the steady state real net interest rate of 2%. We set the risk premium parameter on private bonds, γ^{NFA} to 0.004 in line with Fenz et al. (2012). We conservatively assume that the effective real interest rate on government bonds is moderately above the OECD long-term projection (OECD 2018), namely 0.5% annually above the trend growth and inflation. In the baseline calibration, we assume no risk premium connected to the government bonds, $\gamma^{BG} = 0$. We calibrate the steady state debt-to-GDP ratio B^G to 60% in the baseline calibration, which is a declared long run target of the Austrian government. We discuss the results for alternative values of debt-to-GDP ratio.

The elasticity of substitution between firm goods σ_I is set such that the steady state markups equal 10%, which is a standard value used in the New Keynesian DSGE models. We choose the steady state firm profits such that the implied capital depreciation rates approximately match the rates reported by the Statistics Austria adjusted for growth. The sensitivity of productivity to public infrastructure γ^J is chosen such that the effectivity of public investment is the same as for private investment. The value is within the range considered in the international literature, e.g. Stähler and Thomas (2012). The calibration implies government investment multipliers (short and long run) in line with the existing DSGE models.[10] The capital adjustment costs parameter κ is calibrated such that the response of investment to an aggregate technology and demand shocks is within the range usually considered in the empirical literature. In line with the literature standard we set the Calvo parameter for price rigidity θ_i to 0.75 for all industries. We calibrate the wage rigidity parameter ω such that it delivers a reasonable compromise between the volatility of average wages relative to output and fluctuations of consumer price inflation.

We calibrate the import elasticity parameter υ_A to 2.4, in line with the estimated DSGE model of the Austrian economy in Fenz et al. (2012) and within the range typically estimated in the international literature, see for example Imbs and Mejean (2015). Although export elasticities of a small open economy are likely to be higher than import elasticities, we use the same value for υ_E. Values of the Frisch elasticity of labor supply η have been the subject of extensive discussion in both academic and applied macroeconomic literature. The New Keynesian literature in the recent years mostly focused on values close to 0.5, see e.g. de Walque et al. (2015).

[9] Some recent studies assume marginal tax rates to be equal to the average tax rates with the exception of labor income tax, see Brinca et al. (2016), Stähler and Thomas (2012). In our model, the labor income tax rates differ between Keynesian and Ricardian households, capturing a part of the heterogeneity in the effective tax rates.

[10] See de Walque et al. 2015, Stähler and Thomas 2012, Roeger and in 't Veld 2009

Our model also features several elasticities which pin down the substitution between production factors and outputs across industries, which are less standard in the macroeconomic literature. Molnárová and Reiter (2022) identify the values of the substitution elasticities using a New Keynesian model of the U.S. economy. As the necessary industry-specific longitudinal data are not available for Austria, we calibrate the substitution elasticities based on these previous results.

The elasticity parameter σ_N determines the reallocation of labor across industries. In Molnárová and Reiter (2022), the value of σ_N is significantly above one, identified based on the relative unconditional volatility of industry hours. For Austria, we calibrate the parameter $\sigma_N = 2$, implying somewhat less flexible adjustment of the labor supply compared to the U.S. They identify the elasticity of substitution between production factors σ_y based on the relative unconditional volatility of factor shares in the U.S. economy. The results of the model are very robust with respect to elasticities of substitution between industry goods σ_M, σ_X and σ_C. The values are comparable with other industry-level models, see e.g. Atalay (2017), Huo et al. (2019). The model results are very robust with respect to the value of the elasticity of substitution between labor and capital types σ_l and σ_k. We set the elasticity of substitution between labor types σ_l to a high value of 0.5, which implies relatively stable labor income shares of Ricardian and Keynesian households. We calibrate the parameter σ_k somewhat arbitrarily to a medium value of 2.

Stability of forward-looking DSGE models requires a fiscal instrument that adjusts endogenously in response to government spending. In the baseline calibration, we set the response of the fiscal instrument (labor income tax rate or lump sum tax) to be extremely slow, effectively leading to the same outcomes as budget-financing rule. Parameters ρ^{τ_l} and γ^{τ_l} are set to be as close to one (resp. zero) as possible in order to avoid problems with the model stability.

3 Specific remarks on the estimation of the effects of the EU oil embargo on the Austrian economy

This section summarizes the information on the use of ATMOD model for estimating the effects of the EU embargo on Russian oil imports in May 2022.

3.1 Assumptions

3.1.1 Specification of the oil embargo shock

The oil imports in the model belong to the IOT industry *Coal and lignite; crude petroleum and natural gas; metal ores* (we further refer to this industry as KEEE, abbreviating the German expression Kohle; Erdöl und Erdgas; Erze). We model the impact of the oil embargo as temporary exogenous increase in the price of imports of the KEEE industry, shock $d^{pF}_{i,t}$. Moreover, in our baseline exercise, we assume that the rest of the world is affected symmetrically to Austria:

- Prices of imports from the rest of the world in each industry increase proportionally to the domestic prices. With the exception of the KEEE industry imports, it holds that

$$p^F_{i,t} = p^H_{i,t}. \tag{180}$$

- The RoW demand for Austrian exports falls proportionally to Austrian GDP. Simultaneously to the oil import prices we shock aggregate exports d_t^{EXP}.

3.1.2 Calibration alterations

The recent international literature has highlighted the importance of some parameter values, especially elasticities of substitution between goods, for the estimated impact of shortages of crude materials and fuels, see for example Bachmann et al. (2022). Thus, we alter the baseline values of several key model parameters in order to ensure that the estimates are conservative and reflect the low substitutability of oil in the short run. Compared to the standard model calibration summarized in Table 1, we make the following adjustments:

- For consumers, the elasticity of substitution between consumption goods σ_C is set to a low value of 0.05.

- For firms, we assume that it is difficult to substitute away from the intermediate inputs from KEEE industry and from a group of KEEE-related downstream industries[11]. These industries belong to the low-elasticity set I^L with the elasticity of substitution σ_M being 0.05. Moreover, the elasticity of substitution between intermediate inputs and capital-labor composite σ_y is set to a low value of 0.2 and 0.05 for I^H and I^L industries, respectively.

- The Armington elasticity of substitution between domestic and foreign goods $\upsilon_A = \upsilon_E$ is set to a relatively low value of 0.9.

- We assume that prices are flexible in the KEEE-related industries. In all other industries, price rigidity is temporarily weaker following the shock, with $\theta_i = 0.5$.

- The external central bank reacts to the international price inflation, replacing 66 with the standard Taylor rule following

$$R_t = R^* + \rho_\pi(R_{t-1} - R^*) + (1 - \rho_\pi)\gamma_\pi(\pi_t^F - \pi^*) + \log(d_t^M), \tag{181}$$

  with $\rho_\pi = 0.6$ and $\gamma_\pi = 1.1$.

3.1.3 Industries and capital types

The model distinguishes 74 industries and 1 type of capital.

3.2 Outputs

We report the deviations of value added at constant prices, consumer price inflation, nominal wages, total labor input, and the components of value added from the scenario with no embargo. The results are computed for the first year following the embargo. The changes are reported in percent deviations from the no-embargo scenario, with the exception of inflation, which is reported in percent point changes.

[11] We define the group of KEEE-related industries as those industries, that directly use KEEE products as intermediate inputs. The group includes Coke and refined petroleum products; Chemicals and chemical products; Basic metals; Electricity, gas, steam and air conditioning; and the KEEE industry itself.

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