FAMILY ALLOWANCES AS PARETO IMPROVEMENTS

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ABSTRACT

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Family Allowances as Pareto Improvements

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Abstract. The present paper studies the equilibria of a simple overlapping generations model of pure exchange in which money is traded. There is a continuum of agents in each generation and population growth is endogenous via voluntary decisions on children. Any monetary steady state has to satisfy the "golden rule" that the interest rate equals the growth rate. Still such a monetary steady state is pareto-inefficient. It can be shown that there exists a transfer scheme in favor of those, who raise children, which pareto-improves upon any steady state.

1. INTRODUCTION

Family allowances were known in a variety of societies. Even the ancient Romans used transfers which could be labelled "family allowances". The economic profession - since it exists - always had problems in dealing with this fact. In general these transfers were justified by distributional arguments: They help those, who raise children which every nation needs.

The literature of recent years deals with different aspects of this question. One part of the relevant papers relates family allowances to taxation and optimal taxation of the family [**Boskin and Sheshinski**, 1983]; another part relates it to optimal population [**Eckstein and Wolpin**, 1985; **Mirrlees**, 1972]; again another to labor supply [**Rosenzweig and Wolpin**, 1980]. A few authors recognize the importance of the question, when raising children has external effects [**Cigno**, 1986]. If there are no externalities, family allowances will reduce private costs of a child below social costs and the number of children would be higher than socially desirable. Only the existence of externalities would justify government intervention and transfers.

Nerlove, Razin and Sadka [1987] argue children-related transfers as responses to the inability of parents to control certain private decisions concerning the next generation. They study two special cases. The first comes from the fact that parents cannot control the actions

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of their children, once they are grown up. Parents transfer resources to their children: Directly by donations or bequest and indirectly by investments into human capital of their children. As children are differently gifted, they will obtain different increments of their income by investment in education. The optimal transfer (with the maximal rate of return on the whole transfer) will lead to high proportions of total transfers to the more gifted. As the parents will have ideas about the structure of the transfers to the next generation, e.g. an equal share for each child, the parents could only use the efficient transfer to the next generation, if they could enforce later a redistribution of income from the more gifted children with higher income to the others. As this generally will not be feasible, the size and structure of resource transfers to the next generation will not be efficient in an economic sense. The second case studied by **Nerlove, Razin and Sadka** also deals with resource transfers to the next generation. Here the externality is a result of the inability of parents to control the bequest-decision of parents-in-law of their own children.

In the common discussion children are more often than in the literature believed to generate positive and negative external effects. Such assertions occur frequently in the discussion of pay-as-you-go schemes as old age security systems. The argument is that those, who have children, subsidize the pensions of others, who did not bring up children. Consequently, some kind of externality is taken for granted.[1] This implicitly assumes that *(a)* to raise children creates a positive effect on the whole population and *(b)* there is no substitute service supplied by those, who do not raise children, which can be considered as an implicit contribution to a pension fund.

Such a common sense argument comes close to the spirit of the present paper. We will try to justify family allowances on the basis of efficiency arguments in a general equilibrium context. This, however, requires to endogenize the private decisions on the number of children.

One of the major problems, when dealing with efficiency in dynamic models with endogenous population, is a proper definition of Pareto-optimality. If population is an exogenous variable, one could simply define efficiency as the impossibility to increase the welfare of one generation without decreasing the welfare of another. But if the population dynamics is endogenously determined, one may end up comparing the welfare of people, who live, with the welfare of people, who may never live. Hence the definition of the Pareto-criterion has to be adjusted.

[1] The term "externalities" is used here in a wide sense. Although we will not allow individual utilities to depend on the entire allocation vector of the economy, we will refer to the effect of equilibrium prices on individual utilities as an "externality".

There are certainly several possible ways to solve this definitional problem: One may exclusively consider representative parents of generation t and show, how a redistribution of resources among future generations $t+i$, $i = 1, 2, \ldots$, influence the utility maximum of generation t. This is the definition used by **Nerlove, Razin and Sadka**. There is another possibility which will be employed in the present paper: We are looking at welfare situations of individuals or groups of individuals characterized by the same cost of raising children $(1-\alpha)$ across different steady states. The relation of the group size to the size of the generation is constant in a steady state, though population might grow or decline.

The theoretical framework of our arguments is adapted from **Samuelson's** [1958] seminal paper. Assume a life-cycle model with the usual assumptions: Utility functions with two arguments, consumption in the first and second period of life; consumption in the second period can be provided by savings and/or benefits from children. The number of children is endogenous. (A paper examining the properties of this type of model is [**Felderer**, 1990].) One assumption that may be new in the mainstream of overlapping-generations models is that a generation is assumed to consist of a continuum of individuals, who distinguish themselves only by differences in the cost of raising children. The model does not formulate explicitely a representative firm nor production, but considers only two markets: A good-market and a capital (or: money) market. Thus the rate of interest is the only endogenous price. Interest is paid, in this framework, on holdings of unbacked government-issued currency - a treatment that is now standard in the literature [see: **Sargent**, 1987, chp.7; **Wallace**, 1980]. Under these assumptions it can be shown that the growth rate of the population will in a steady state always be equal to the interest rate. But, contrary to Samuelson's finding that the golden rule path is pareto-optimal, with endogenous population such a steady state is *not* efficient. In particular it can be shown that transfers from those, who decide not to have children, to those, who raise children, are pareto-improving.

The intuition which is underlying the result on pareto improvements by family allowances can be characterized as follows: Consider two representative families, one, who prefers to secure their old age by saving, another by raising children, who will support their parents, when they are old. The children as adults will increase the rate of return on savings which will compensate the childless family for any transfers made. Thus all are better off with family allowances.

The following Section 2 contains the formal analysis and Section 3 summarizes and draws some conclusions.

2. The Model

Consider an overlapping generations model, where each generation t, $t = 1, 2, \dots,$ (actively) lives for two periods, t and $t+1$. In fact the model can also be interpreted as one, where each generation t lives for three periods ($t-1$, t, and $t+1$), but where an individual of generation t is entirely passive during its childhood in period $t-1$ (it is only fed and raised by its parents of generation $t-1$, but does not actively decide on anything relevant).

An individual of generation t is identified with a point (α, t), where α is in the interval between zero and one for each generation t: $(\alpha, t) \in [0, 1] \times \{1, 2, \dots\}$, such that there is a continuum of individuals in each generation. Individuals (α, t) have preferences, given by a utility function $u^{(\alpha, t)}(c_t^1, c_{t+1}^2)$, defined on consumption c_t^1 in period t, when they are "young", and consumption c_{t+1}^2 in period $t+1$, when they are "old". An individual (α, t) receives in period t an endowment $w_t > 0$, pays in period t a subsidy $b_t > 0$ to its parents, receives in period t a transfer (or pays a tax) $\varepsilon T_t(\alpha)$, $\varepsilon > 0$, and can in period t decide on its savings, s_t^α, and on the number of children, e_t^α, it wishes to raise.[2] Raising children has different costs for different individuals (α, t) and (α', t), $\alpha \neq \alpha'$, which is represented by a cost function $C^\alpha(\,.\,)$. To keep the analysis simple, the costs of raising children will be treated as deductions from endowments during youth.

There is only one non-durable consumption good ("chocolate") in this economy, of which individuals receive a positive endowment in their youth and none, when they are old. Thus the endowment structure is such that an old individual apparently has nothing to offer in return for its consumption in the second period of its life. To avoid an autarky equilibrium, here the social contrivance of "chocolate wrapper" (also called: money) is invoked: Savings are purchases of unbacked government-issued currency without any intrinsic value, called "money", which will serve as a store of value and is traded against consumption goods by the aged [**Samuelson**, 1958; **Cass and Yaari**, 1966; **Sargent**, 1987, chp.7; **Wallace**, 1980]. In this sense the equilibria to be studied in the sequel are monetary equilibria. This is quite a simplistic theory of money, but it is one which originally fueled a widespread interest in overlapping generations models [a good reference may be: **Balasko and Shell**, 1980, 1981a and 1981b]. Since this is not a paper on the theory of money, but rather on old-age security systems, the above convention on a monetary

[2] Since the "name" of an individual, (α, t), consists of two parts, its "first name" α and its "generation name" t, we should actually write $s_t^{(\alpha, t)}$ and $e_t^{(\alpha, t)}$. To keep the notation compact, we supress the "generation name", when no confusion can arise.

economy seems quite appropriate. Thus for all what follows we will treat the economy *with* the social contrivance of money. (We could also assume that all individuals of each generation t have access to a storage technology which yields a return of R_{t+1} per unit of input next period.)

When old in period $t+1$ an individual (α, t) receives the return on its savings, denoted $R_{t+1}s_t^\alpha$, and the benefits from its children, $b_{t+1}e_t^\alpha$. Thus the core decision variables of an individual (α, t) are the decisions on savings, s_t^α, and the number of children, e_t^α, (treated as a continuous variable here) in period t. Both these decisions yield certain returns to individuals, when they are old, and can, therefore, be viewed as two alternative investment opportunities. The return to the investments is determined by the equilibrium. The latter feedback from the equilibrium conditions provides the driving force to the results to be presented below.

For the sake of simplicity the following assumptions are adopted (partial derivatives of functions will be denoted by subscripts):

(A.1) *(i)* (Identical preferences)

$$u^{(\alpha, t)}(c_t^1, c_{t+1}^2) = u(c_t^1, c_{t+1}^2).$$

(ii) (Constant marginal costs)

$$C^\alpha(e) = (1-\alpha)e, \quad \alpha \in [0, 1].$$

(iii) (Properties of the utility function) The utility function u is strictly quasi-concave and satisfies

$$u_1 > 0, u_2 > 0, \ u_{11} < 0, \ u_{21} \geq 0,$$
$$u_2(\,.\,, x) \longrightarrow_{x \to 0} +\infty.$$

The assumption $u_{11} < 0$ is simply decreasing marginal utility of consumption during youth. Assuming $u_{21} \geq 0$ means to assume persistence in consumption habits in the sense that a higher consumption during youth raises (weakly) the marginal utility of consumption, when the consumer is old. This excludes the case that someone, by consuming more during his youth, reduces his urge to consume more, when old, which seems like an empirically implausible case. However, the sign restrictions on u_{11} and u_{21} are not strictly necessary. They could easily be replaced by the assumption that consumption during old age is a normal good, i.e. $dc_{t+1}^2/dw_t > 0$ at the optimum. Such a normality assumtion would imply that $(1-\alpha)u_{11} - b_{t+1}u_{21} < 0$, which is all we will need for our results.

The composition of a generation t of individuals with respect to the parameter α is given by a function $\mu_t\colon [0, 1] \to \Re_+$ which is assumed to satisfy all the properties of a distribution function except for size:

(A.2) The function $\mu_t\colon [0, 1] \to \Re_+$ is assumed non-decreasing and satisfies $\mu_t(\alpha) > 0$, $\forall 0 < \alpha \leq 1$, $\mu_t(1) = N_t > 0$, $\forall t = 1, 2, \ldots$, where N_t measures the size of the generation, and μ_t is assumed differentiable, such that a density function exists.[3]

If $e_{t-1}(\alpha, .)$ denotes the function assigning to each individual $(\alpha, t-1)$ its (optimal) decision on the number of children, then the population dynamics is given by

$$N_t = \int_0^1 e_{t-1}(\alpha, .)\, d\mu_{t-1}(\alpha).$$

The next assumption rules out that the distribution of the cost parameter α varies with the decisions on the individual number of children as the population grows:

(A.3) Assume that the composition (with respect to the parameter α) of each generation t of individuals, given by μ_t, is independent of the composition of any generation τ, for all $\tau \neq t$, $\tau = 1, 2, \ldots, t = 1, 2, \ldots$, i.e. the sequence $\{\mu_t/N_t\}_{t=1}^{\infty}$ is exogenously given.

This assumption means that the characteristics-parameter (one minus the per unit cost of raising children) $\alpha \in [0, 1]$ is *not* inherited. Another assumption which is not explicitely numbered, but always adopted, is, of course, rational expectations which here takes the form of perfect foresight.

Since for $\varepsilon > 0$ any α-type individual may receive transfers or may have to pay taxes, the transfers have to be financed from taxation. Thus as to the transfer scheme $T_t(\alpha)$, it has to be assumed that it breaks even for all $\varepsilon > 0$, although ε is allowed to become arbitrary small.

(A.4) $\int_0^1 T_t(\alpha)\, d\mu_t(\alpha) = 0$ and the function T_t is continuously differentiable, $\forall t = 1, 2, \ldots$.

[3]This assumption is in fact stronger than actually needed. As the interested reader can easily check, it is sufficient for our results that μ_t is non-decreasing and right-continuous instead of differentiable. Then the Riemann-integral may have to be replaced by a Lebesgue-Stieltjes integral with respect to a unique measure on (the Borel-sets of) the unit interval, but none of our results will be changed by this.

The individual's problem is to maximize $u(c_t^1,\, c_{t+1}^2)$ subject to the two budget constraints for the two periods of life. From assumption (A.1), (ii) and (iii), it follows that for all individuals of type α and generation t at the utility maximizing choices

$$\begin{aligned} c_t^1 &= w_t - b_t - s_t^\alpha - (1-\alpha)e_t^\alpha + \varepsilon T_t(\alpha)\,, \\ c_{t+1}^2 &= R_{t+1}\, s_t^\alpha + b_{t+1}\, e_t^\alpha\,. \end{aligned}$$

Consider now the problem of an individual α which is young in period t, for any $t = 1,\, 2,\, \ldots$:

$$\begin{aligned} &\max_{(s_t^\alpha,\, e_t^\alpha)} u(w_t - b_t - s_t^\alpha - (1-\alpha)e_t^\alpha + \varepsilon T_t(\alpha),\, R_{t+1}s_t^\alpha + b_{t+1}e_t^\alpha), \\ &\qquad\text{s.t.:}\quad s_t^\alpha \geq 0, \quad e_t^\alpha \geq 0. \end{aligned}$$

Note that $s_t^\alpha \geq 0$ implies that there are no credit facilities, but only savings (storage). The above problem yields the first order conditions

$$\begin{aligned} -u_1 + R_{t+1}u_2 \leq 0, &\quad \text{compl.} \quad s_t^\alpha \geq 0, \\ -(1-\alpha)u_1 + b_{t+1}u_2 \leq 0, &\quad \text{compl.} \quad e_t^\alpha \geq 0. \end{aligned}$$

The current assumptions yield a particularily transparent behavorial rule: Each generation will consist of two types of subpopulations, one relying exclusively on savings to finance their consumption, when they are old, and the other one relying exclusively on children to support them, when they are old. The magnitude of the parameter α determines this separation, such that individuals with a low value of α will tend to turn to the capital market, while individuals with a high value of α will tend to have children. From the complementary slackness conditions one obtains

$$\begin{aligned} s_t^\alpha > 0 &\implies R_{t+1}u_2 = u_1 \implies b_{t+1}u_2 \leq (1-\alpha)R_{t+1}u_2 \implies \\ &\implies b_{t+1} \leq (1-\alpha)R_{t+1}, \quad \text{and} \\ e_t^\alpha > 0 &\implies b_{t+1}u_2 = (1-\alpha)u_1 \implies R_{t+1}\frac{1-\alpha}{b_{t+1}}u_1 \leq u_1 \implies \\ &\implies (1-\alpha)R_{t+1} \leq b_{t+1}\,. \end{aligned}$$

These implications can also be written as

$$\begin{aligned} b_{t+1} > (1-\alpha)R_{t+1} &\implies s_t^\alpha = 0, \quad \text{and} \\ b_{t+1} < (1-\alpha)R_{t+1} &\implies e_t^\alpha = 0. \end{aligned}$$

But from the boundary behavior assumption in (A.1), (iii), it follows that either $e_t^\alpha > 0$ or $s_t^\alpha > 0$ (or both), such that

$$s_t^\alpha > 0 \text{ and } e_t^\alpha = 0, \quad \forall \alpha < 1 - b_{t+1}/R_{t+1}, \text{ and}$$
$$s_t^\alpha = 0 \text{ and } e_t^\alpha > 0, \quad \forall \alpha > 1 - b_{t+1}/R_{t+1}.$$

Thus a low value of α (high marginal costs of raising children) characterizes those individuals, who prefer to save rather than have children to support them, when they are old, while a high value of α (low marginal costs) characterizes individuals, who rely on children and do not save. This even carries a little further: As a next step we verify that indeed those with a higher α (lower marginal costs) will have more children at the optimum:

$$\begin{aligned}
&\alpha > 1 - b_{t+1}/R_{t+1} \implies e_t^\alpha > 0 \implies -(1-\alpha)u_1 + b_{t+1}u_2 = 0 \implies \\
&\implies [(1-\alpha)^2 u_{11} - (1-\alpha)b_{t+1}(u_{12}+u_{21}) + b_{t+1}^2 u_{22}]\, de_t^\alpha + \\
&\quad +[u_1 - (1-\alpha)(e_t^\alpha + \varepsilon T_t')u_{11} + b_{t+1}(e_t^\alpha + \varepsilon T_t')u_{21}]\, d\alpha = 0, \\
&\implies \left.\frac{de_t^\alpha}{d\alpha}\right|_{e_t^\alpha > 0} = \frac{(e_t^\alpha + \varepsilon T_t')[(1-\alpha)u_{11} - b_{t+1}u_{21}] - u_1}{(1-\alpha)^2 u_{11} - (1-\alpha)b_{t+1}(u_{12}+u_{21}) + b_{t+1}^2 u_{22}}.
\end{aligned}$$

Moreover, if $e_t^\alpha > 0$, then at the optimal choice the second order condition

$$(1-\alpha)^2 u_{11} - (1-\alpha)b_{t+1}(u_{12}+u_{21}) + b_{t+1}^2 u_{22} < 0$$

has to hold, such that under (A.1), (iii), $e_t^\alpha + \varepsilon T_t' \geq 0$ implies $de_t^\alpha/d\alpha > 0$, as required. Clearly, for $\alpha < 1 - b_{t+1}/R_{t+1}$ one has $e_t^\alpha = 0$ and, therefore, $de_t^\alpha/d\alpha = 0$.

It is thus possible to define

$$e_t(\alpha, R_{t+1}, b_{t+1}) = \begin{cases} 0, & \text{if } \alpha \leq 1 - b_{t+1}/R_{t+1}, \\ \text{solves } b_{t+1}u_2 = (1-\alpha)u_1, & \text{otherwise,} \end{cases}$$

and analogously

$$s_t(\alpha, R_{t+1}, b_{t+1}) = \begin{cases} 0, & \text{if } \alpha > 1 - b_{t+1}/R_{t+1}, \\ \text{solves } R_{t+1}u_2 = u_1, & \text{otherwise.} \end{cases}$$

Equilibrium requires that the market for consumption goods clears in every period, given the optimal choices by all active individuals, i.e. total consumption by the young *and* old generation has to equal total endowments of the young generation minus what the young spend on children:

$$N_t w_t - N_t b_t - \int_0^1 s_t(\alpha,\, R_{t+1},\, b_{t+1})\, d\mu_t(\alpha) +$$
$$+ R_t \int_0^1 s_{t-1}(\alpha,\, R_t,\, b_t)\, d\mu_{t-1}(\alpha) + b_t \int_0^1 e_{t-1}(\alpha,\, R_t,\, b_t)\, d\mu_{t-1}(\alpha) =$$
$$= N_t w_t\,.$$

Using $N_t = \int e_{t-1}\, d\mu_{t-1}$, this reduces to the *equilibrium condition*

$$R_t \int_0^1 s_{t-1}(\alpha,\, R_t,\, b_t)\, d\mu_{t-1}(\alpha) = \int_0^1 s_t(\alpha,\, R_{t+1},\, b_{t+1})\, d\mu_t(\alpha)\,.$$

The focus of the analysis will be on long-term positions of the system which are repetitive in the sense that exogenous parameters do not change anymore. Define a *steady state* of the economy as a sequence $\{w_t,\, b_t,\, R_t,\, T_t,\, \mu_t\}_{t=1}^{\infty}$ such that

$$w_t = w > 0,\ b_t = b > 0,\ R_t = R > 0,\ T_t = T,$$
$$\mu_t / N_t = \mu, \quad \mu\colon [0,\, 1] \to [0,\, 1],$$

where μ satisfies (A.2), for all $t = 1,\, 2,\, \dots$. Assume for the moment that a steady state exists. Then in such a steady state one has

$$\forall \alpha \leq 1 - b/R\colon\ e_t(\alpha,\, R,\, b) =: e(\alpha,\, R) = 0, \text{ and}$$
$$s_t(\alpha,\, R,\, b) =: s(\alpha,\, R) \quad \text{solves}$$
$$-u_1(w - b - s(\alpha,\, R) + \varepsilon T(\alpha),\ R\, s(\alpha,\, R)) +$$
$$+ R\, u_2(w - b - s(\alpha,\, R) + \varepsilon T(\alpha),\ R\, s(\alpha,\, R)) = 0,$$

and

$$\forall \alpha > 1 - b/R\colon\ s_t(\alpha,\, R,\, b) =: s(\alpha,\, R) = 0, \text{ and}$$
$$e_t(\alpha,\, R,\, b) =: e(\alpha,\, R) \quad \text{solves}$$
$$-(1 - \alpha) u_1(w - b - (1 - \alpha) e(\alpha,\, R) + \varepsilon T(\alpha),\ b\, e(\alpha,\, R)) +$$
$$+ b\, u_2(w - b - (1 - \alpha) e(\alpha,\, R) + \varepsilon T(\alpha),\ b\, e(\alpha,\, R)) = 0\,.$$

The parameter b is supressed in the above representation, because it is exogenously given and does not have to adjust to clear some market. In fact the only (relative) price which is endogenously determined by the equilibrium condition in each period is the interest factor R, because the price of consumption goods serves as the numeraire.

Next, for a given steady state define *indirect utility functions*

$$V(\alpha,\, R) := u(w - b - s(\alpha,\, R) - (1-\alpha)e(\alpha,\, R) + \varepsilon T(\alpha),\\ Rs(\alpha,\, R) + be(\alpha,\, R)),$$

such that by the envelope theorem

$$dV = s(\alpha,\, R)u_2\, dR + T(\alpha)u_1\, d\varepsilon, \quad \forall\, \alpha \in [0,\, 1].$$

The latter will allow us to vary $\varepsilon > 0$ in order to identify whether a pareto-improvement is possible for some given transfer scheme $T(\alpha)$ by magnifying the transfers/taxes.

But first existence and uniqueness of a steady state have to be established:

THEOREM. *There exists a unique steady state interest factor $R^* > 0$. Moreover, this unique steady state satisfies the "golden rule"*

$$R^* = \int_0^1 e(\alpha,\, R^*)\, d\mu(\alpha),$$

that the interest rate equals the growth rate of the economy.

PROOF: In a steady state the equilibrium condition can be rewritten as

$$R\, N_{t-1} \int_0^1 s(\alpha,\, R)\, d\mu(\alpha) = N_t \int_0^1 s(\alpha,\, R)\, d\mu(\alpha),$$

or, by using $N_t = \int e_{t-1} d\mu_{t-1} = N_{t-1} \int e\, d\mu$, as

$$R = \int_0^1 e(\alpha,\, R)\, d\mu(\alpha) \iff R = \int_{\max(0,\, 1-b/R)}^1 e(\alpha,\, R)\, d\mu(\alpha)\,,$$

which verifies the second claim of the Theorem for all steady states. As the next step, it is demonstrated that indeed a unique steady state return on savings, R^*, exists. Let $LHS(R) = R$ and

$$RHS(R) = \int_{\max(0,\, 1-b/R)}^1 e(\alpha,\, R)\, d\mu(\alpha)\,.$$

Observing that for all $\alpha > 1 - b/R$ the function $e(\alpha, R)$ is constant in R, because these α-types do not save, it is easily seen that $RHS(R)$ is monotone decreasing and continuous in R. Now

$$LHS(0) = 0 < RHS(0) = \int_0^1 e(\alpha, R)\, d\mu(\alpha), \quad \text{and}$$
$$\lim_{R\to\infty} LHS(R) = +\infty > \lim_{R\to\infty} RHS(R) = 0,$$

such that a unique point of intersection between the graph of LHS and the graph of RHS exists by continuity. ▮

The main objective of what follows is to compare steady states with different transfer schemes εT in order to identify any potential pareto-improvements across steady states. The question addressed can be stated as follows:

Is it possible to find a function T: $[0, 1] \to \Re$ which satifies (A.4), such that, for some small $\varepsilon > 0$, a steady state with a slightly higher ε makes all individuals of type $\alpha \in [0, 1]$ better off, for all $\alpha \in [0, 1]$?

Of course this notion of welfare comparisons completely neglects the adjustment process from a given steady state to a new one. In fact it even completely neglects the conceptional difficulty of defining pareto-optimality for an overlapping generations model with endogenous fertility: Since with endogenous fertility welfare comparisons may have to refer to individuals, who will never be borne at the equilibrium, pareto-improvements for this class of models cannot be defined in the usual way. Here we propose to circumvent this difficulty by concentrating exclusively on steady states and comparing the welfare of certain groups of individuals (with the same α) across steady states. This is feasible, because steady states are unique.

In our view this is a proper extension of the notion of pareto-efficiency to models with endogenous fertility: By making the focus of the analysis a representative individual of *type* α rather than their number, $\mu_t(\alpha)$, the problem of comparing utilities of born with the ones of unborn disappears, because that an extra individual of type α is born (or not born) only changes $\mu_t(\alpha)$, but not the utility of a representative individual of type α (ceteris paribus). Thus, if the above question can be answerded in the affirmative, then to each steady state equilibrium allocation across all generations there exists an alternative allocation (the steady state allocation corresponding to a slightly higher ε) which is feasible and in which all *types* are better off across all generations. Except for the substitution of "types" for "individuals" this is literally the definition of pareto-inferiority. Of course, this ignores any dynamic adjustment process from some given equilibrium to an alternative one. But ignoring such adjustments is the only correct way to define the pareto-criterion: In any model of an economy the pareto-criterion is defined with respect to the allocation over the *whole* economy. In an OLG model the allocation consists of a specification of consumption bundles for *all* generations, and thus adjustment processes must be ignored.

To be able to compare steady states with different ε's it is necessary to identify the reaction of the equilibrium return on savings R^* to a small variation of ε. This is accomplished by identifying the change of e_t^α in reaction to a change in ε. For all $\alpha > 1 - b_{t+1}/R_{t+1}$ one has

$$\begin{aligned}
&[(1-\alpha)^2 u_{11} - (1-\alpha)b_{t+1}(u_{12}+u_{21}) + b_{t+1}^2 u_{22}]\, de_t^\alpha + \\
&+[-(1-\alpha)T_t(\alpha)u_{11} + b_{t+1}T_t(\alpha)u_{21}]\, d\varepsilon = 0, \\
&\Longrightarrow \frac{de_t^\alpha}{d\varepsilon}\bigg|_{e_t^\alpha>0} = \frac{T_t(\alpha)[(1-\alpha)u_{11} - b_{t+1}u_{21}]}{(1-\alpha)^2 u_{11} - (1-\alpha)b_{t+1}(u_{12}+u_{21}) + b_{t+1}^2 u_{22}}, \\
&\Longrightarrow \operatorname{sign}\Big(\frac{de_t^\alpha}{d\varepsilon}\bigg|_{e_t^\alpha>0}\Big) = \operatorname{sign}(T_t(\alpha)),
\end{aligned}$$

again using the second order condition for a maximum, when $e_t^\alpha > 0$, and (A.1), (iii).

From the equilibrium condition $R - \int_{\max(0,\,1-b/R)}^{1} e(\alpha,\, R)\, d\mu(\alpha) = 0$ one obtains

$$\frac{dR^*}{d\varepsilon} = \int_{\max(0,\,1-b/R)}^{1} \frac{\partial e(\alpha,\, R)}{\partial \varepsilon}\, d\mu(\alpha),$$

using $e(1-b/R,\, R) = 0$ and $\partial e/\partial R = 0,\ \forall \alpha > 1 - b/R$. In a steady state equilibrium the change of indirect utilities can now be written as

$$\begin{aligned}
\frac{dV}{d\varepsilon} &= s(\alpha,\, R^*)u_2 \frac{dR^*}{d\varepsilon} + T(\alpha)u_1, \quad \forall \alpha \le 1 - b/R^*, \\
\frac{dV}{d\varepsilon} &= T(\alpha)u_1, \quad \forall \alpha > 1 - b/R^*.
\end{aligned}$$

Choose now the transfer scheme $T\colon [0,\, 1] \to \Re$ such that

$$\begin{aligned}
&\text{(i)} \quad \int_0^1 T(\alpha)\, d\mu(\alpha) = 0, \\
&\text{(ii)} \quad T(\alpha) > 0, \quad \forall \alpha > 1 - b/R^*, \\
&\text{(iii)} \quad 0 \ge T(\alpha) > -s(\alpha,\, R^*)\frac{u_2}{u_1}\frac{dR^*}{d\varepsilon},\ \forall \alpha \le 1 - b/R^*.
\end{aligned}$$

Such a choice is possible, if and only if $dR^*/d\varepsilon > 0$. But (ii) implies that $\partial e(\alpha, R^*)/\partial\varepsilon > 0, \forall\alpha > 1 - b/R^*$, such that $dR^*/d\varepsilon > 0$. Thus a function T satisfying the above requirements (i) - (iii) exists.

Given such a function T it is straightforward to show that slightly increasing ε makes all $\alpha \in [0, 1]$ strictly better off:

$$\frac{dV}{d\varepsilon} = T(\alpha)u_1 > 0, \quad \forall\alpha > 1 - b/R^*,$$
$$\frac{dV}{d\varepsilon} = s(\alpha, R^*)u_2 \frac{dR^*}{d\varepsilon} + T(\alpha)u_1 > s(\alpha, R^*)u_2 \frac{dR^*}{d\varepsilon} - $$
$$- s(\alpha, R^*)u_2 \frac{dR^*}{d\varepsilon} = 0, \quad \forall\alpha \leq 1 - b/R^*.$$

This is the desired result which can be summarized as follows:

PROPOSITION. *For any transfer scheme $T(\alpha)$ which redistributes from types with a low value of α to types with a high value of α and breaks even (satisfies (i) - (iii) above), a steady state, where this transfer scheme is slightly magnified, is strictly better for all $\alpha \in [0, 1]$.*

Hence a redistribution from those, who save, but do not have children, to those with a high propensity to raise children raises the welfare of all in the sense that the corresponding steady state with the magnified transfer scheme is better for all. The intuition behind this result is that "investment" into children is *productive* in the sense of raising the return on savings. And the increase of the return on savings (more than) compensates the childless, who subsidize via the transfer scheme those, who raise children, but do not save.

The latter intuition already suggests, why the example with pure exchange is particularily simple. Since the driving force behind the result for the pure exchange economy is the real productivity of investments into children, the picture may change, if savings are used to run productive firms. In the latter case the possible multiplicity of equilibria may require extra conditions in order to generate an analogous result to the one presented here.[4] The present paper only offers a first step in the analysis and we feel that the case with production remains an important field for future research. Such considerations are, however, beyond the scope of the present paper.

[4]First attempts to cope with the case with production show that there are three aspects which complicate the analysis: (1) The feedback from the equilibrium interest rate to wages which substitute for endowments in such a model; (2) the scheme according to which the firms are financed (note that savings can at best be turned into one-period loans); (3) multiplicity of equilibria such that some equilibria will display a "perverse" slope of the interest rate with respect to parameters.

3. Conclusions

The present paper argues the desirability of family allowances as means of pareto-improvements in an overlapping-generations economy of pure exchange with endogenous population growth. It is shown that there exists a unique steady state of such an economy and this steady state satisfies the "golden rule" that the growth rate equals the interest rate. Despite this fact the unique steady state is inefficient. This is demonstrated by showing that there exists a transfer scheme in favor of those, who raise children, and financed by those, who do not, which breaks even and is socially desirable. This holds, because an increase of population growth will increase the steady state interest rate. The latter effect compensates the donors via increased returns on their savings.

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