SOME EVIDENCE ON HYSTERESIS IN UNEMPLOYMENT RATES*

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Abstract

We suggest an unobserved components model that can be used to evaluate unemployment data for evidence on hysteresis effects. Unemployment is decomposed into a natural rate component, assumed to be nonstationary, and a cyclical component, assumed to be stationary. Hysteresis effects are modelled by allowing lagged cyclical unemployment to affect the current natural rate. The model is estimated using quarterly Canadian, German, and U.S. unemployment series. We find large hysteresis effects in the German unemployment series and to a lesser extent in the Canadian series. Hysteresis effects in the U.S. unemployment series appear to be small although statistically significant.

1. Introduction

The behavior of unemployment rates in the OECD area over the last fifteen years has puzzled many observers. Much of the attention has focused on the fact that unemployment rates in several European countries have undergone substantial changes with no apparent tendency to revert to a stable underlying rate. An extensive body of recent work has focused on documenting and explaining the persistence of changes in unemployment.[1]

Most work assumes that changes in observed unemployment can be attributed to changes in the natural rate of unemployment and/or to changes in the cyclical rate of unemployment. Based on this framework two main classes of hypotheses have been proposed to resolve the puzzling persistence of unemployment rates. The first class assumes that the natural rate and cyclical unemployment evolve independently. Increases in the natural rate are related to structural factors including excess real wages, increased unemployment benefits, high marginal tax rates, occupational and geographic mismatch, legislative restrictions on dismissal, and minimum wage laws.[2]

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1 See the collections of papers on Europe's unemployment problem edited by Bean, Layard, and Nickell (1986), Lawrence and Schultze (1987), and Drèze and Bean (1990).

The basic assumption of the alternative explanation, the hysteresis hypotheses, is that natural rate and cyclical unemployment do not evolve independently of each other.[3] The crux of this idea is that cyclical movements in unemployment may somehow be propagated to the natural rate. That is, an increase in the rate of cyclical unemployment may lead to an increase, over time, in the level of the natural rate. Two mechanisms have been most commonly cited as giving rise to hysteresis: First, if unemployment leads to an erosion of human capital, this could lead to a reduction in the labour-force attachment of the long-term unemployed. Second, wage bargaining institutions may predominantly take into account the interests of the currently employed workers [the "insider-outsider" theory]. Blanchard and Summers (1986), Gottfries and Horn (1987) and, in a series of papers, Lindbeck and Snower [collected in Lindbeck and Snower (1988)] have provided theoretical foundations for the latter mechanism. They argue that incumbent workers develop substantial influence over wages due to the costs to the firm

2 See the references in footnote 1 and Bean and Symons (1989).

3 Virtually all commentators dismiss the possibility of an extended period of high cyclical unemployment as the cause of high unemployment over the 1980s pointing out that inflation rates were relatively stable over this period. Bernanke and Parkinson (1989), however, argue in the context of the American Depression that stable inflation is not inconsistent with extended periods of high cyclical unemployment.

of labour turnover. The insiders use this power to push wages above their reservation wages, thereby reducing total labour demand. If workers who lose their jobs lose their insider status as well, a temporary downturn leads to a smaller pool of insiders, who then raise wages as high as possible while maintaining their jobs. The natural rate of unemployment thereby rises permanently in response to a transitory shock that reduces the number of insiders.

In this paper we build on our earlier work [see Jaeger and Parkinson (1990)] and adopt an unobserved components approach to examine the empirical evidence on the relative importance of the two classes of hypotheses for explaining persistent unemployment.[4] The observed unemployment rate is decomposed into a natural rate component and a cyclical component. The decomposition used in this paper introduces hysteresis effects by allowing cyclical unemployment to have a lagged effect on the natural rate. The model is identified by assuming that capacity utilization is correlated with cyclical unemployment. Given the restricted availability of capacity utilization series, we estimate the model for three countries, namely Canada, Germany, and the United States. We find significant statistical evidence for hysteresis effects in the German unemployment series and to a lesser extent in the Canadian series. Hysteresis effects in the U.S.

4 See Harvey (1985) and Clark (1989) for other applications of unobserved components models.

unemployment series appear to be small although statistically significant. That said, in all the countries examined, natural rate shocks are themselves an important mechanism for explaining persistent movements in observed unemployment rates.

Section 2 discusses the concept of hysteresis in terms of an unobserved components model of the unemployment rate. Section 3 describes the estimation of the model. Empirical results are presented in section 4. Section 5 concludes.

2. The concept of hysteresis

This section develops an unobserved components (UC) model of the unemployment rate that can be used to evaluate the data for evidence of hysteresis effects. Assume the unemployment rate to be the sum of a "natural rate component", U_t^N, and a "cyclical component", U_t^C,

$$U_t = U_t^N + U_t^C. \tag{1}$$

The two unobservable components of unemployment evolve as

$$U_t^C = \phi_1 U_{t-1}^C + \phi_2 U_{t-2}^C + \epsilon_t^C \tag{2}$$

$$U_t^N = U_{t-1}^N + \epsilon_t^N + \alpha U_{t-1}^C. \tag{3}$$

The innovations ϵ_t^C and ϵ_t^N are assumed to be mutually uncorrelated shocks which are normally distributed with variances σ_C^2 and σ_N^2, respectively. Equation (2) defines the cyclical component of the unemployment rate as a stationary second-order autoregressive process. In equation (3), the natural rate component is modelled as a random walk plus a term capturing possible hysteresis effects. The cyclical unemployment rate can affect the natural rate component with a lag of one period. The coefficient α measures in percentage points by how much the natural rate increases if the economy experiences a cyclical unemployment rate of 1.0 percent for one period. The size of this coefficient is the measure of hysteresis used in this paper.

The assumption of an AR(2) process for the cyclical component of the unemployment rate in equation (2) can be relaxed in favor of more general autoregressive moving average structures. In our empirical work we typically find that AR(2) processes for the cyclical component fit the data well. Equation (3) adopts a "spillover interpretation" of hysteresis. Deviations of the actual unemployment rate from the natural rate may induce permanent shifts in the level of unemployment if the coefficient α is positive.[5] The specification also assumes that hysteresis effects are symmetric with respect to movements in cyclical unemployment. It is possible to estimate the model under the restriction that the hysteresis effect operates only with positive cyclical unemployment, i.e. $U_t^C > 0$. Such an asymmetric specification implies, however, that if α is positive the natural rate will ratchet higher and higher as the economy undergoes repeated periods of cyclical unemployment.

5 A similar specification for the natural rate component was used by Hargreaves Heap (1982) in one of the first theoretical discussions of hysteresis in labor markets. The model is also closely related to the cycle-in-trend model suggested by Harvey (1985). In Harvey's (1985) specification, however, the parameter α is a priori restricted to one.

What is the relationship of the UC approach to other work on evaluating the evidence for hysteresis effects? The commonly used definition of hysteresis suggests that unemployment rates exhibit hysteresis when actual unemployment depends on past values with coefficients summing to, or very close to, unity. More formally, hysteresis in unemployment occurs if the unemployment series has a unit root.[6] This univariate approach to defining hysteresis is, in our view, unnecessarily restrictive. As argued in the introduction, hysteresis in unemployment is essentially a phenomenon whereby changes in cyclical unemployment affect the natural rate. Obviously, natural rate shocks due to, say, changed institutional arrangements will affect the unemployment rate, but such occurrences can be examined in the context of a standard structural model without recourse to hysteresis explanations. Accordingly, if the data are to be examined for evidence of hysteresis it is necessary to utilize an approach that allows a distinction to be drawn with respect to the different sources of influences on the unemployment rate. From the perspective of the UC model, a unit root in unemployment is a necessary but not a sufficient condition for hysteresis. If unemployment data are generated by equations (1), (2), and (3), a unit root test should not reject the null if the variance of natural rate shocks is

6 See the evidence presented in Blanchard and Summers (1986) and Alogoskoufis and Manning (1988).

positive and the hysteresis coefficient α is zero. The point is that a unit root in unemployment may be induced by natural rate shocks and be entirely independent of the existence of hysteresis. By contrast, in the UC model, hysteresis in unemployment occurs if the natural rate component is also affected by movements in the cyclical component.

3. Estimation

The UC model of section 2 is, in general, not identified, i.e. the second moments of the observed data do not uniquely determine the parameters of the model. To illustrate the identification issue, assume for simplicity that $U_t^C = \epsilon_t^C$ instead of the more general specification given by equation (2). Then

$$\Delta U_t = \epsilon_t^N + \epsilon_t^C - (1-\alpha)\epsilon_{t-1}^C, \quad (4)$$

where ΔU_t denotes the first difference of the unemployment rate. From the data, we can determine the variance and the first autocovariance of ΔU_t. But this second-moment information is insufficient to determine uniquely the three model parameter α, σ_C^2 and σ_N^2. Thus, to identify the model, information from an additional variable that is related either to the cyclical or the natural rate component of the model is needed. We assume that capacity utilization (CU_t) is a stationary cyclical indicator correlated with the cyclical component of unemployment and the relationship between the two variables is specified as

$$CU_t = \beta_0 U_t^C + \beta_1 U_{t-1}^C + v_t. \tag{5}$$

The shock v_t denotes a normally distributed error term with variance σ_v^2 which is uncorrelated with the other two shocks in the UC model. Using again the example $U_t^C = \epsilon_t^C$, it is easily checked that the second moments of capacity utilization and the first difference of unemployment provide sufficient information to determine the parameters of the extended model. Simply counting whether the number of second moments is sufficient to determine the unknown parameters provides only a necessary condition for identifiability. Nowak (1989) contains a general formal discussion of identifiability of parameters in multivariate unobserved components models. His theorem 1(b) can be used to verify that the UC specified in this paper is indeed identified.[7]

To estimate the model, we first write it in state space form. The transition equations of the state space model are given by

7 The important assumption needed for assuring global identifiability is that the cyclical unemployment component is ARMA(p,q) with $p > (q+1)$.

$$\begin{bmatrix} U_t^N \\ U_t^C \\ U_{t-1}^C \end{bmatrix} = \begin{bmatrix} 1 & \alpha & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} U_{t-1}^N \\ U_{t-1}^C \\ U_{t-2}^C \end{bmatrix} + \begin{bmatrix} \epsilon_t^N \\ \epsilon_t^C \\ 0 \end{bmatrix} \qquad (6)$$

The measurement equations of the model are

$$\begin{bmatrix} U_t \\ CU_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & \beta_0 & \beta_1 \end{bmatrix} \begin{bmatrix} U_t^N \\ U_t^C \\ U_{t-1}^C \end{bmatrix} + \begin{bmatrix} 0 \\ v_t \end{bmatrix} \qquad (7)$$

Finally, the variance-covariance matrix (Ω) of the three shocks of the model is given by

$$\Omega = \begin{bmatrix} \sigma_N^2 & 0 & 0 \\ 0 & \sigma_C^2 & 0 \\ 0 & 0 & \sigma_v^2 \end{bmatrix} \qquad (8)$$

All shocks are assumed to be uncorrelated over time and normally distributed.

The parameters of the model are estimated by maximum likelihood using the prediction error decomposition approach [see Harvey (1989)]. The one-step prediction errors (w_t) for unemployment and capacity utilization are calculated via the state space form using the Kalman filter. Given the prediction errors and their variance-covariance matrix (F_t) at time t, we form the log-likelihood

$$\log L = -\frac{T}{2}\log(2\pi) - \frac{T}{2}\log(|F_t|) - \frac{1}{2}\sum_{t=1}^{T} w_t' F_t^{-1} w_t, \qquad (9)$$

where $|F_t|$ denotes the determinant of F_t. The log-likelihood is maximized by the method of scoring with modified step size suggested in Berndt et. al. (1974).[8]

4. Empirical results

The empirical analysis uses seasonally adjusted quarterly data for Canada, Germany, and the United States. The time range is 1960(1) to 1990(4) for Germany and the United States and 1961(1) to 1990(4) for Canada. This rather restricted sample of countries was dictated by the availability of a proper capacity utilization series.[9] All data series are

8 The program used for estimation is written in GAUSS 1.49 and available from the authors on request.

9 For example, there is only a series giving the "percentage of firms working at full capacity" available for the United Kingdom. For some countries, available capacity utilization series start only in the early 1970s.

taken from the OECD Main Economic Indicator Database. Because the German data were seasonally unadjusted, we have adjusted the series using the additive version of Census X-11. The series for the three countries are plotted in Figures 1a, 2a, and 3a. Note that the plots for the capacity utilization series are based on mean adjusted series.

The capacity utilization series refer to the manufacturing sector, and the plots suggest a close inverse relationship between these series and the unemployment rates for all three countries. Capacity utilization is not the only variable which could be used for identifying the UC model. Any variable which is related either to the cyclical or natural component should in principle suffice. As a matter of implementation, however, we have found it simpler to use a stationary variable, such as capacity utilization, rather than a nonstationary variable like real output. Another variable that is conceivably correlated with cyclical unemployment is unanticipated inflation. That is, both the Fischer-Taylor contracting model and the Lucas aggregate supply curve model suggest that unanticipated inflation can directly influence the level of employment and hence, except in cases with perfectly elastic labour supply, unanticipated inflation would be correlated with the level of cyclical unemployment. Similarly, if technological change has only permanent effects then the natural rate and the Solow residual could conceivably be correlated. However, while it is conceivable that real output, unanticipated inflation and the Solow residual might all be alternatives to capacity utilization as an information variable, each of these alternatives carries with it certain difficulties: GNP needs first to be decomposed into permanent and transitory components while both unanticipated inflation and the Solow residual are subject to severe measurement problems.

This is not to suggest that the utilization measure is without drawbacks. One criticism that might be made is that to the extent that statisticians force the utilization measure to be stationary, and construct the measure using actual unemployment rates as part of the information set, our procedure of identifying the model may be illusory. We believe this criticism is not damaging to our results. First, for Germany the utilization measure is drawn directly from a survey of firms. For Canada and the U.S. the capacity utilization measures is indeed constructed, but a closer look at the method of construction does not suggest that the criticism has much validity. For example, the U.S. capacity utilization series is constructed by dividing quarterly production indices by estimated capacity, where estimated capacity is constructed using, inter alia, disaggregated data on firm operating rates and yearly changes in capacity from the annual McGraw Hill survey and the Census Bureau's survey of fourth quarter utilization rates by establishment. Shapiro (1989) has argued that most of the within-year variation in measured U.S. capacity utilization is due to the within-year change in production, so that period-by-period changes in utilization contain essentially no information beyond that contained in changes in the disaggregated production series.

The series for unemployment and capacity utilization used below are expressed as percentage rates and are therefore bounded economic variables. The UC model specification and the error distribution assumptions do not restrict the variables between their natural bounds. A logistic transformation of the data was considered to account for this problem [see Wallis (1987)]. The results in terms of statistical significance of parameters are unaffected by the transformation. Transforming the series has, however, the distinct disadvantage that the interpretation of the hysteresis parameter as a straightforward measure of hysteresis is lost.

Table 1 presents statistical evidence on the occurrence of unit roots in the unemployment and capacity utilization series based on augmented Dickey-Fuller tests. The lag length for the Dickey-Fuller regressions was determined by maximizing Akaike's information criterion. The null hypothesis of a unit root in the unemployment series is not rejected for any of the three countries even at the 10 percent significance level. For capacity utilization rates, the null of a unit root is rejected at the 10 percent level for all countries. We interpret the unit root test statistics as evidence indicating that our assumptions with respect to the low frequency properties of unemployment rates and capacity utilization series are not strongly at variance with the data.[10]

Table 2 reports the estimation results for the unobserved components model. The estimates for the model parameters and the value of the likelihood are given in three columns headed by a country name. Additionally, the table reports the likelihood ratio test statistic for the null hypothesis of "no hysteresis" and a diagnostic test for general serial autocorrelation in the prediction errors. The hysteresis test statistic is distributed chi-square with 1 degree of freedom under the null. The serial autocorrelation test statistic is the conventional Ljung-Box portmanteau statistic for general serial autocorrelation.

The estimates of the hysteresis parameter indicate substantial hysteresis for Germany and to a lesser extent for Canada. For the United States, the estimated hysteresis parameter is quite small but significant at the 10 percent level. For Germany, a cyclical unemployment rate of 1 percent

10 Unit root test results for the U.S. unemployment rate are sensitive to the inclusion of a deterministic time trend in the Dickey-Fuller regression [see e.g. the results in Clark (1989)]. We find the assumption of a deterministic upward or downward trend in unemployment rates unattractive on an a priori basis.

maintained for one quarter leads to a permanent increase of the actual unemployment rate of about 0.20 percent. Thus, extended episodes of cyclical unemployment will lead to substantial changes in the natural rate. Figures 1b, 2b, and 3b plot the smoothed estimates of the natural and cyclical rate components derived from the results in table 2.

The diagnostic test statistics reported in table 2 indicate serial autocorrelation in the prediction errors for Canada and the United States. This finding suggests that the estimation results should be interpreted with some caution. To check for subsample stability we re-estimated the models for the two subperiods 1960(1) to 1973(4) [1961(1) to 1973(4)) in the case of Canada] and 1974(1) to 1990(4). The estimation results for the hysteresis parameter and the diagnostic test statistics are presented in table 3. In general, hysteresis effects appear to be weaker in the pre-1973(4) subsample. For Canada, the effect disappears completely. The serial autocorrelation test statistics for the subsample estimates are all insignificant with one exception.

5. Conclusions

In this paper, we use an unobserved components model of the unemployment rate to provide an operational definition of hysteresis. Unemployment is decomposed into a natural rate component, assumed to be nonstationary, and a cyclical component, assumed to be stationary. We interpret the central feature of hysteresis to be that these two components do not evolve independently. In particular, hysteresis effects are modelled by allowing the lagged cyclical unemployment rate to affect the current natural rate. To identify the decomposition without imposing orthogonality on the natural and the cyclical component of unemployment, we assume that cyclical unemployment is correlated with the capacity utilization rate.

We report estimates of the model for Canada, Germany, and the United States. For Germany, and to a lesser extent Canada, we find that hysteresis effects are important in accounting for persistent movements in unemployment rates. Hysteresis effects appear to be relatively unimportant in explaining movements in the U.S. unemployment rate.

Table 1

Tests for unit roots.[a]

$$\Delta X_t = \text{Constant} + aX_{t-1} + \sum_{i=1}^{m} b_i \Delta X_{t-i} + \epsilon_t .$$

A. Unemployment rates

	Constant	a	Lag length	ADF-test
Canada 1961(1)-90(4)	0.149 (0.092)	-0.020 (0.013)	1	-1.53
Germany 1960(1)-90(4)	0.051 (0.034)	-0.050 (0.006)	3	-1.46
United States 1960(1)-90(4)	0.188 (0.106)	-0.032 (0.017)	4	-1.86

B. Capacity utilization rates

Country	Constant	a	Lag length	ADF-test
Canada 1961(1)-90(4)	5.186 (2.032)	-0.064 (0.023)	1	-2.83
Germany 1960(1)-90(4)	7.673 (2.491)	-0.009 (0.034)	2	-3.09
United States 1960(1)-90(4)	6.973 (1.991)	-0.085 (0.024)	1	-3.51

[a] Lag length is determined by maximum A.I.C.. Critical values for Augmented Dickey-Fuller (ADF) test statistic from Fuller (1976, p. 373) are -3.51 (1%), -2.89 (5%), and -2.58 (10%). Numbers in parentheses are standard errors.

Table 2

Estimates for unobserved components models.

Parameters	Countries		
	Canada	Germany	United States
ϕ_1	1.696	1.632	1.657
ϕ_2	-0.752	-0.691	-0.728
α	0.085	0.202	0.032
β_0	-0.746	-0.431	-0.620
β_1	0.156	0.210	0.197
σ_N	0.2774	0.1966	0.1781
σ_C	0.1682	0.1241	0.2099
σ_v	0.0306	0.0672	0.0011
Likelihood	48.65	94.21	111.16
Hysteresis[a] test(α=0)	6.59**	26.14***	3.83*
Diagnostic[b] tests			
Q_U	22.95**	11.71	21.44**
Q_{CU}	18.07*	7.01	18.29*

[a] The likelihood ratio test statistic for the null hypothesis "no hysteresis (α=0) is distributed chi-square with 1 degree of freedom under the null.

[b] The Ljung-Box statistics Q_U and Q_{CU} for general serial autocorrelation in the prediction errors are chi-square distributed with 11 degrees of freedom under the null.

***Significant at 1%; **significant at 5%; *significant at 10%.

Table 3

Estimates for subsamples.

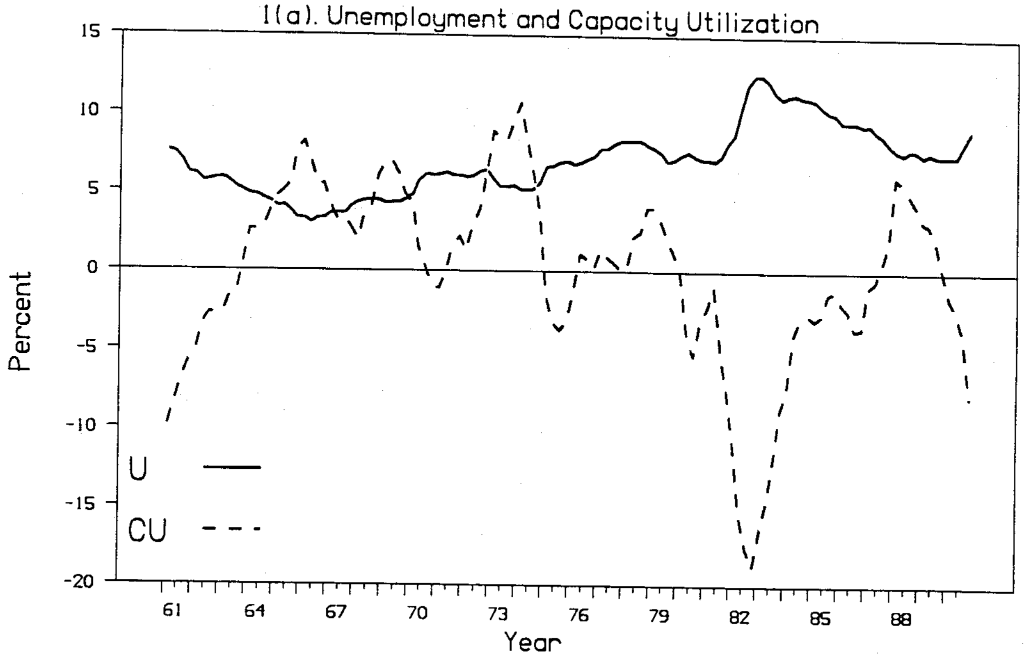
	Canada	Germany	United States
A. Pre-1973(4)			
Hysteresis parameter (α)	0.017	0.138	0.067
Hysteresis[a] test (α=0)	0.14	5.50**	4.48**
Diagnostic[b] tests			
Q_U	1.58	5.09	8.58
Q_{CU}	2.05	2.03	4.35
B. Post-1973(4)			
Hysteresis parameter (α)	0.196	0.182	0.043
Hysteresis[a] test (α=0)	12.36***	16.74***	3.60*
Diagnostic[b] tests			
Q_U	10.84	13.91*	9.13
Q_{CU}	4.37	7.11	6.68

[a] Distributed chi-square with 1 degree of freedom under the null.

[b] The Ljung-Box statistics Q_U and Q_{CU} for general serial autocorrelation in the prediction errors for the subsamples are chi-square distributed with 7 degrees of freedom under the null.

***Significant at 1%; **significant at 5%; *significant at 10%.

FIGURE 1. TIME SERIES: CANADA



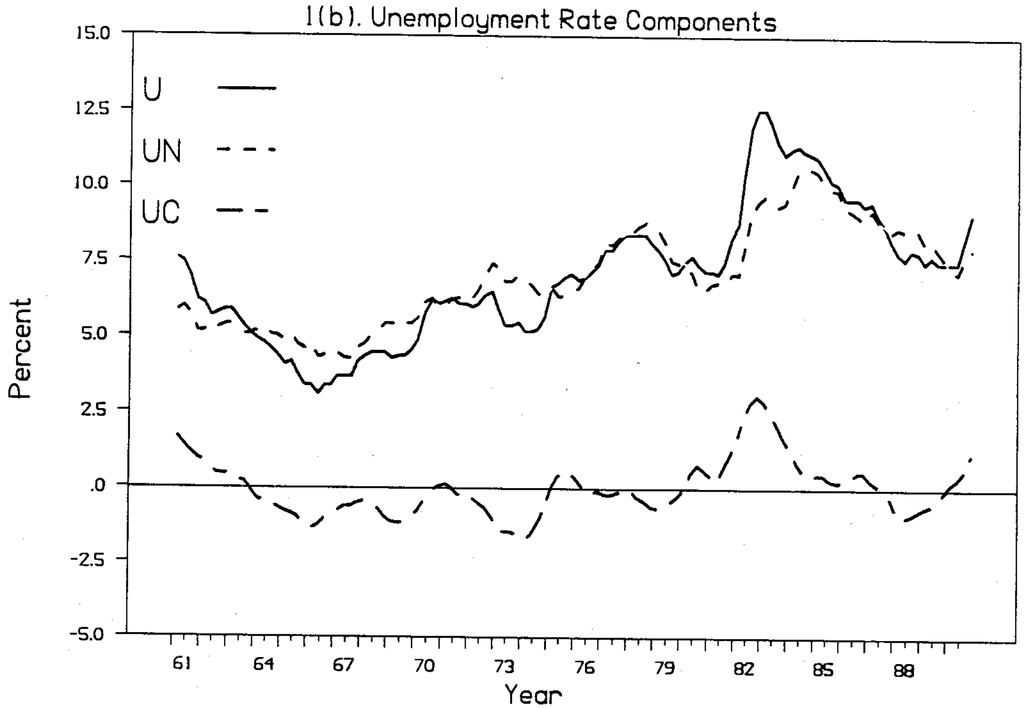


FIGURE 2. TIME SERIES: GERMANY

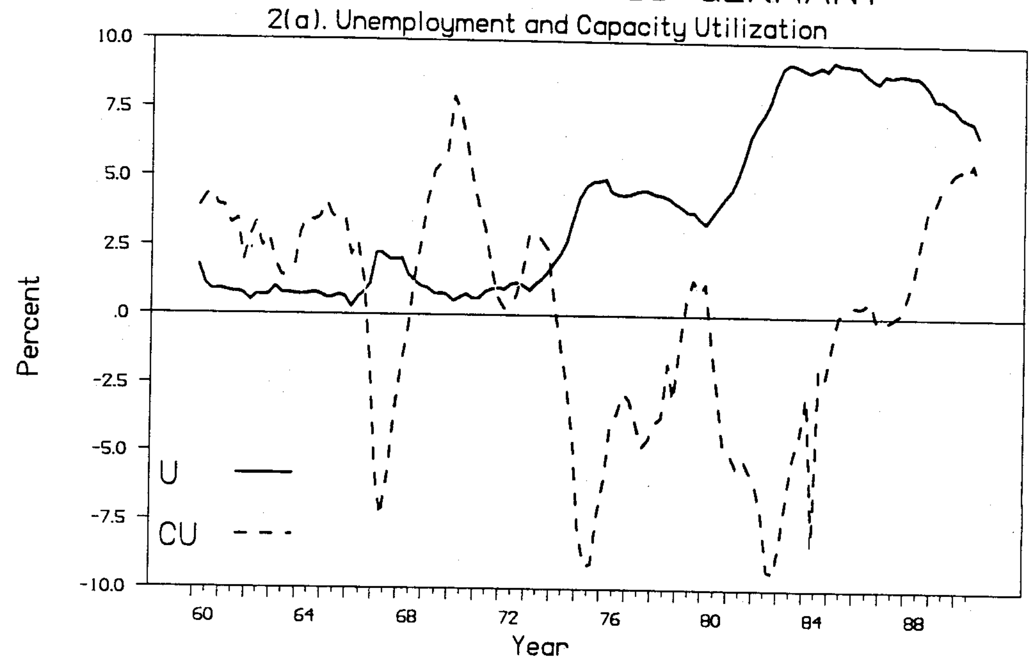
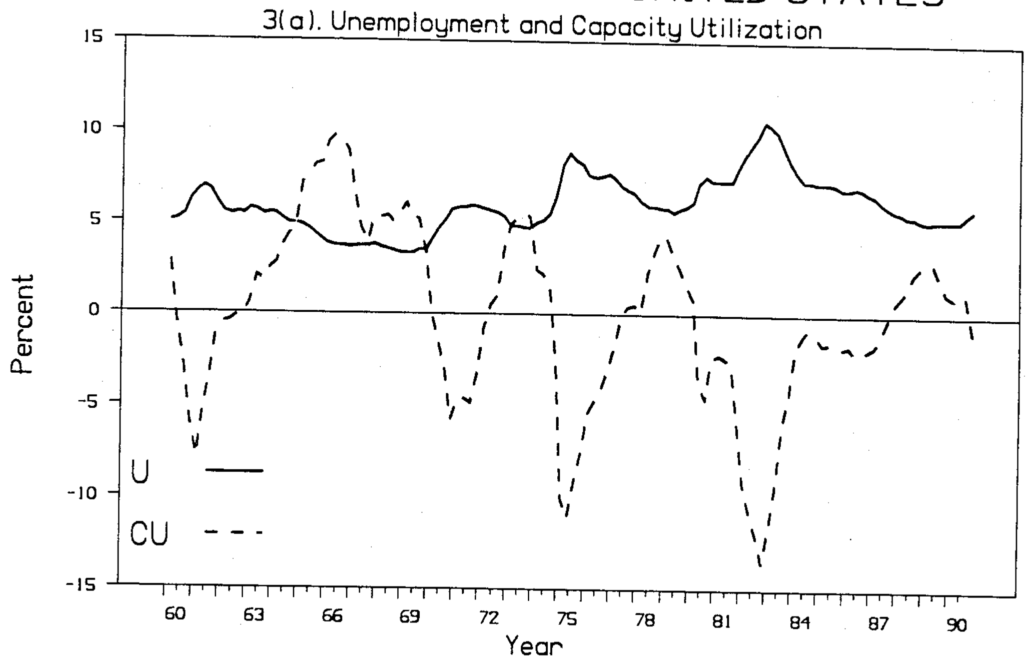
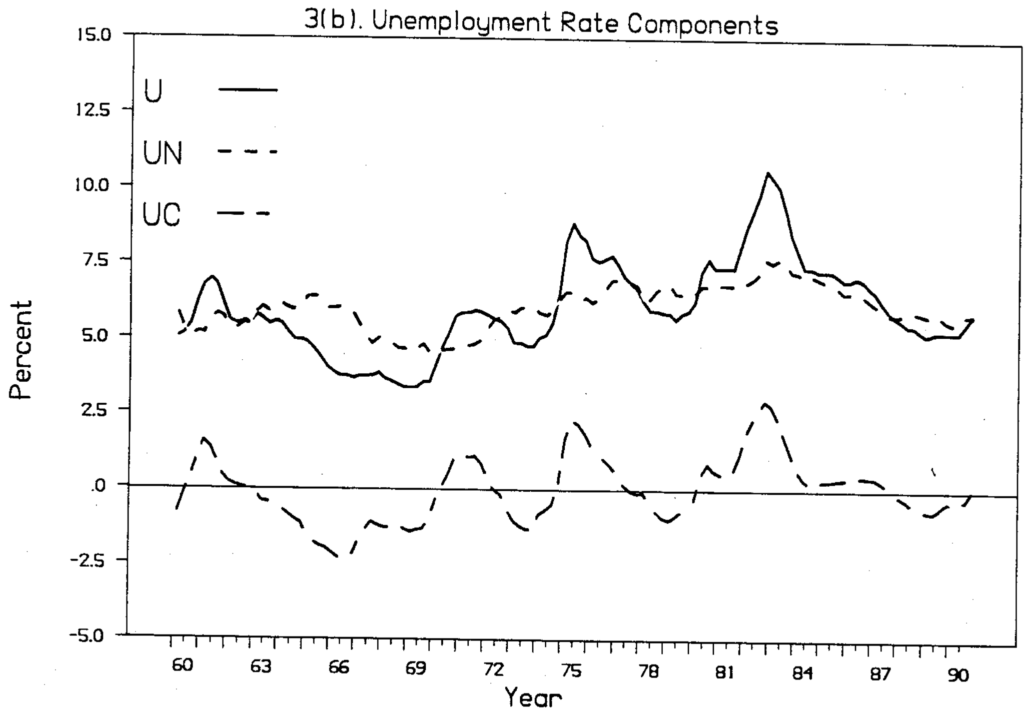


FIGURE 3. TIME SERIES: UNITED STATES





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