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INTRODUCTION

This proceedings volume contains six papers presented at the workshop on "Applied Aggregate Consumption Theory" held on May 20, 1988 at the Institute for Advanced Studies, Vienna. The topics covered several important issues in this field: econometric specification, estimation of intertemporal substitution elasticities, changing uncertainty, habit formation and the Ricardian equivalence proposition.

The seminal paper by Hall (1978) on the implications of rational expectations for the permanent income life cycle hypothesis confronted the economics profession with the hypothesis that consumption is not caused, in the sense of Granger, by any other variable once lagged consumption has been accounted for. Hall's random walk hypothesis initiated a host of empirical work. It is fair to say that the majority of studies rejected Hall's conjecture. These rather disappointing results lead Deaton (1986) to question the permanent income life cycle hypothesis as an adequate model for aggregate consumption behavior. The papers presented in this workshop can be understood as investigating several alternatives or extensions to Hall's initial model. Kugler incorporates leisure as a separate argument in a utility function subject to preference shocks; Neusser departs from certainty equivalence and investigates the role of changing uncertainty; Winder investigates intertemporal non-separable utility functions by incorporating habit formation; Thury leaves the Euler approach and tries to specify a consumption function using integral control. Finally, Flaig and Jaeger study extended versions of the life cycle - permanent income hypothesis which take into account the intertemporal implications of financing government expenditures by taxes or debt.

A prominent set of theories considers intertemporal substitution behavior as the key for understanding business cycle fluctuations and for evaluating long run effects of fiscal policy and tax reforms. In his paper on "Intertemporal Substitution in Consumption: Results from Euler-Equations under Different Stochastic Specifications" Peter Kugler presents an alternative way to estimate the elasticity of substitution. By including leisure in the utility function, one can alternatively use the static first order condition governing the substitution of consumption for leisure. The econometric specification used for estimation allows for taste shocks as well as aggregation errors and problems with seasonal adjustment procedures.

Kugler's empirical results are based on data from the USA, Great Britain, West Germany and Switzerland. He finds that the value of the elasticity of substitution depends sensitively on the type of first order condition and the estimation procedure.

Using the dynamic first order condition linking current and future consumption, the intertemporal elasticity of substitution turns out to be insignificant and the overidentifying restrictions of the model are generally rejected. Using the static first order condition linking current consumption and leisure, the estimates for the intertemporal elastictity of consumption are significantly greater than zero reaching even one in the case of the U.S. data.

Recent research on financial markets behavior has convincingly argued for the importance of information contained in higher moments of time series to explain the means of rates of return. Most work on aggregate consumption behavior builds on the premise of certainty equivalence. Klaus Neusser's paper "Consumption and Changing Income Uncertainty: A First Empirical Investigation for Austria" uses the ARCH model developed by Engle (1982) to examine the effects of changing uncertainty.

Using Austrian data, he finds that private consumption exhibits strong ARCH effects. Attempts to relate the changing conditional variance in a univariate representation of consumption to ARCH effects in income or the unemployment rate turned out to be unsuccessful. This leaves a puzzle to be clarified in future research. Neusser conjectures that the real interest rate could prove a promising candidate to identify the source of the ARCH effects in consumption.

Carlo Winder's paper "Rational Habits in the Life Cycle Consumption Function" extends the random walk model of Hall (1978) by dispensing with the assumption of separability of the life time utility function. Assuming a Gaussian process for labor income and an exponential utility function together with habit formation in consumption behavior, he shows that arbitrary ARIMA processes can be obtained by choosing appropriate patterns for habits. Interestingly, Winder's model exhibits a synthesis between forward and backward looking consumption behavior commonly treated as mutually exclusive but observationally equivalent alternatives.

The empirical analysis presented in the paper uses data for the Netherlands. Special attention is paid to structural changes in the income process and the nature of the replanning process for consumption associated with the structural changes. Winder estimates univariate representations for the income and consumption process and reports two main findings: Significant ARCH effects in the consumption process are not matched by corresponding ARCH effects in the income process (the same result as Neusser reports for Austrian data). Second, information about structural change in the income process (represented by dummies) turns out useful to identify structural change in the consumption process. Winder draws the tentative conclusion that these results could be explained by coupling the rational expectations assumption with learning behavior. Although consumers react to structural changes by adjusting consumption they need some time to learn about the nature of the structural shift (which occurs in the mean of the income process). This adjustment phase could give rise to the ARCH effects in consumption.

Gerhard Thury's paper "Dynamic Specification of Consumer Expenditure on Nondurables and Services in Austria" estimates a dynamic consumption function. His approach is based on recent advances in time series econometrics starting from a careful examination of the stochastic properties of the time series involved (order of integration, co-integration) and working from a general specification to a parsimonious and theoretically interpretable equation.

Thury finds that a simple error correction mechanism involving consumption and income only is insufficient to account for the time series properties of consumption in Austria. He introduces additionally an integral correction mechanism involving the stock of liquid assets. The preferred econmetric consumption equation passes a plethora of specification tests, encompasses a rival equation based on the first differences of the data and appears to track the data well.

The effects of government deficits on private consumption and savings have recently led to a surge of interest in the validity of the Ricardian equivalence proposition. This proposition implies that changes in public savings unrelated to changes in government expenditures do not change the national saving rate. The papers by Gebhard Flaig and Albert Jaeger deal with this question.

Gebhard Flaig's paper "Disposable Income, Government Deficit, and Private Consumption. Some Evidence for the West-German Economy", starts by observing that most of the empirical work on the Ricardian proposition does not derive the tested restrictions from an explicit theoretical framework. His paper uses the "surprise consumption function" approach based on the life cycle hypothesis with rational expectations. Additionally, he takes into account the possibility that some consumers are liquidity constrained. The central idea suggested by Flaig is to identify the innovations in labor income, government expenditures, deficits and real interest rates via a VAR model and to estimate the structural parameters linking these innovations to the change in aggregate consumption.

Flaig reports that a substantial fraction of private households is liquidity constrained thereby invalidating a crucial assumption of the Ricardian proposition. The proposition. The unconstrained households, however, react to innovations in fiscal deficits as predicted by the proposition by reducing consumption. Nevertheless, as the estimated portion of liquidity constrained households is around 70 %, the results do not support the Ricardian proposition as a plausible first order approximation.

Albert Jaeger's paper "Testing Ricardian Equivalence: Are the Data Informative" uses as a starting point the observation that a large number of empirical studies based on U.S. data finds conflicting evidence for the validity of the Ricardian proposition. He suggests a test of the proposition based on long run information. The central idea is that under Ricardian equivalence the national saving rate should be stationary in face of permanent shocks in public savings. For U.S. data, spanning 1949-84, Jaeger finds that the long run information in the data does not allow to discriminate between Ricardian and non-Ricardian behavior of consumers. This finding could possibly explain the disparate findings of different researchers. Austrian data, to the contrary, show that consumers did not react to a significant and permanent drop in the public savings rate around 1975 leading to a concurrent drop in the national saving rate.

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INTERTEMPORAL SUBSTITUTION IN CONSUMPTION: RESULTS FROM EULER-EQUATIONS UNDER DIFFERENT STOCHASTIC SPECIFICATIONS

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1. INTRODUCTION

The seminal paper of Hall (1978) introduced the so-called Euler equations approach to the empirical analysis of aggregate consumption. Instead of estimating a function relating consumption to current and expected future income and initial wealth, Hall estimated a dynamic first order condition for consumption derived from intertemporal utility maximization. The imposition of certain separability restrictions and a quadratic utility function as well as the assumption of a constant real interest rate yielded a simple AR(1) model for consumption of non-durables. This framework specializes to a random walk if the time preference rate is equal to the real interest rate. Hall's work indicated that this simple model is quite successful empirically. Under the restrictions adopted by Hall, the consumer's maximization problem is of the linear quadratic type and has, therefore, a certainty equivalence solution in the form of the permanent income consumption function. This property was exploited in subsequent empirical analyses of consumption e.g. Bilson (1980) Falvin (1981), Deaton (1986) and Campbell (1987), which in general arrived at less favourable results for the life cycle model.

The great advantage of the Euler equations approach consists of the fact that it allows the testing of the life cycle model under less restrictive assumptions straightforwardly. Thus, models with non-quadratic utility functions and uncertain real interest rates [Mankiw (1981), Hansen and Singleton (1983)], as well as non-separability over consumption and leisure [Mankiw, Rotemberg and Summers (1985)] and over time [Eichenbaum, Hansen and Singleton (1986)] were tested in the frame of the Euler equations approach. Unfortunately, these studies yielded rather disappointing results which often clearly rejected the model considered.

In this paper, we present estimation results for first order conditions for aggregate per capita consumption allowing for serially correlated error terms. Thereby, we summarize and extend the results of two recent papers concerning intertemporal substitution in consumption and labour supply (Kugler 1988 a,b) with respect to consumption. Serial correlation is mainly motivated by the probable presence of random changes in preferences: Even if we assume that individual preferences are constant over time, which is doubtful given the fact of fashions, the changing age distribution underlying the representative individual and age specific taste differences lead to time variations in preferences of the representative consumer. However, serial correlation in the error terms of first order conditions may also arise from time aggregation and other problems of measurement.

The remaining part of the paper is organized as follows: In Section 2 the equations to be estimated are briefly outlined. The empirical results obtained with quarterly data for the USA, the UK, West Germany and Switzerland are presented in Section 3. Section 4 concludes.

2. THE UNDERLYING THOERETICAL MODEL

2.1 The Theoretical Model

Our model is based on the assumption that the representative consumer maximizes expected utility of current and future consumption (C_i) and leisure (L_i) subjected to an intertemporal budget constraint. The utility function V_t is assumed to be strongly separable over time. Let $U(C_i,L_i)$ be a concave one-period utility function. The price of C_i is denoted by P_i. W_i is the wage rate at period i (the price of L_i). r_i is the nominal (one-period) interest rate, ρ is the rate of time preference and E_t is the expectation operator conditional on information available at time t. Given these definitions, we have to maximize

$$(1) \qquad V_t = E_t \sum_{i=t}^{\infty} \left(\frac{1}{1+\rho}\right)^{(i-t)} U(C_i, L_i)$$

subject to an intertemoral budget constraint.

Finding an analytical solution to this maximization problem - the consumption and the labour supply function - is an untractable task even for relatively simple specifications of $U(C_i, L_i)$. Moreover, even if an explicit solution exists as in the case of a quadratic utilitiy function, the specification of the conditional expectation of all future prices, wages and interest rates remains a gigantic task to be performed. Therefore, many recent studies on intertemporal substitution using aggregate time-series data [Hall (1985), Hansen and Singleton (1982), Mankiw (1981), Rotemberg and Summers (1985)] attempt to estimate the parameters of the utility function without specifying an explicit consumption and labour supply function.

This approach uses relationships between observed variables implied by necessary conditions for the maximization of V_t (so-called Euler equations) to estimate the parameters of the specified utility function. We consider the following two first order conditions representing the trade-offs between current and future consumption as well as those between current consumption and current leisure.

$$(2) \qquad E_t\left(\frac{\frac{1+r_t}{1+\rho}\,\frac{P_t}{P_{t+1}}\,\frac{\partial U}{\partial C_{t+1}}}{\frac{\partial U}{\partial C_t}}\right) = 1$$

$$(3)\qquad \left(\frac{\frac{\partial U}{\partial C_t}\frac{W_t}{P_t}}{\frac{\partial U}{\partial L_t}}\right) = 1$$

Equation (2) indicates that the expected utility loss of a one-unit decrease of current consumption ($\partial U/\partial C_t$) is equal to the expected utility gain of investing this nominal amount for one period and consuming the proceeds [$P_t(1+r_t)$] at the future price level (P_{t+1}). The static first order condition (3) states that the utility loss of a one-unit decrease in leisure ($\partial U/\partial L_t$) has to be compensated by the utility gain of consuming additional W_t/P_t units of consumption.

Of course, our framework allows the writing of a dynamic first order condition for leisure. As this paper deals only with consumption, this equation is not considered here.[1)]

In order to estimate the first order conditions outlined above, we have to choose a functional form of the one-period utility function
$U(C_i, L_i)$. The following additively separable specification is used

$$(4)\qquad U(C_i\ ,\ L_i) = \frac{C_i^{1-\alpha_1} - 1}{1-\alpha_1} D_{1i} + \frac{L_i^{1-\alpha_2} - 1}{1-\alpha_2} D_{2i}$$

The unobservable variables D_{1i} and D_{2i} are taste shifters which take into account that taste may change over time.
As we will see below, the specification of the utility function allows for different intertemporal elasticities of substitution in

1) For discussion and empirical analysis of this first order condition, the reader is referred to Kugler (1988a).

consumption and leisure. In addition, (4) includes a multiplicative or logarithmic utility function as a special case ($\alpha_1 = \alpha_2 = 1$).

Given this specification, we arrive at the following first order conditions

$$E_t\left[\frac{1}{1+\rho}\,\frac{D_{1t+1}}{D_{1t}}\,\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha_1}\,\frac{P_t(1+r_t)}{P_{t+1}}\right] = 1$$

$$\frac{D_{1t}}{D_{2t}}\,\frac{C_t^{-\alpha_1}}{L_t^{-\alpha_2}}\,\frac{W_t}{P_t} = 1$$

Now, the first two conditions are stated in terms of the actual values of the variables involved. $E_t(X_{t+1}) = 1$ can be expressed as $X_{t+1} = 1 + \varepsilon_{t+1}$, $E_t(\varepsilon_{t+1}) = 0$. Using this fact, we obtain

(2a) $$\frac{1}{1+\rho}\,\frac{D_{1t+1}}{D_{1t}}\,\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha_1}\,\frac{P_t(1+r_t)}{P_{t+1}} = 1 + \varepsilon_{t+1}$$

(3a) $$\frac{D_{1t}}{D_{2t}}\,\frac{C_t^{-\alpha_1}}{L_t^{-\alpha_2}}\,\frac{W_t}{P_t} = 1$$

$$E_t(\varepsilon_{t+1}) = 0$$

Equation (3a) and (4a) are simplified considerably by taking logs and using the second order Taylor approximation

$$\log(1+\varepsilon_t) = \varepsilon_t - \frac{1}{2}\varepsilon_t^2$$

Now, let us assume that in addition, ε_t is conditionally homoscedastic, i.e. $E_t(\varepsilon_t^2) = \sigma_*^2$. After some rearrangement and denoting logs by lower case letters, we arrive at the following formulation of our two first order conditions

$$(2b) \qquad \Delta c_{t+1} = k + \frac{1}{\alpha_1} [\log(1+r_t) - \Delta p_{t+1}] + v_{1t+1}$$

$$(3b) \qquad c_t = \frac{1}{\alpha_1} (w_t - p_t) + \frac{\alpha_2}{\alpha_1} l_t + \frac{d_{2t} - d_{1t}}{\alpha_1}$$

where

$$k = \frac{1}{\alpha} [\frac{\sigma_*^2}{2} - \log (1+\rho)]$$

$$v_{1t+1} = \eta_{jt+1} + \frac{1}{\alpha_1} \Delta d_{1t+1}$$

$$\eta_{t+1} = \frac{1}{\alpha_1} [\frac{1}{2}(\varepsilon_{t+1}^2 - \sigma_*^2) - \varepsilon_{t+1}]$$

$$E_t(\eta_{t+1}) = 0$$

In equation (2b), the log of the ratio of future and current consumption is related to the log of their relative price. Thus, $\frac{1}{\alpha_1}$ is the respective elasticity of intertemporal substitution in consumption. By inserting the first order Taylor approximation $\log(1+r_t) = r_t$, (2b) and (3b) are formulated as the relationship between the rate of change of consumption and the ex post real interest rate. If we assume that real interest rates and tastes are constant over time, equation (3b) specializes to the random walk model of consumption introduced by Hall (1978).

2.2 Specification of the Taste Variables and Estimation

The taste variables d_{jt} (j=1,2) are, of course, not observable. This problem could be solved by the inclusion of proxy-variables representing the changing age distribution and fashions. We proceed alternatively by adapting univariate time-series models for these preference variables. We start on the assumption that the first differences of these variables can be regarded as two (possible) contemporaneously-correlated q'th order moving average [MA(q)] processes. This assumption is motivated by the fact that most economic variables are non-stationary in levels, but stationary in first differences [integrated of order one, denoted by I(1)]. Thus, we adopt the following MA processes for d_{1t} and d_{2t}:

$$(5) \qquad \Delta d_{jt+1} = \xi_{jt+1} - \theta_{j1}\xi_{jt} - \ldots - \theta_{jq}\xi_{jt-q+1} \qquad j = 1,2$$

$$E_t(\xi_{jt+1}) = 0, \qquad E_t(\xi^2_{jt+1}) = \sigma^2_{\xi j} \qquad "$$

This specification, of course, implies that the error term of equation (3b) is in general not stationary. Thus, estimation should be based on a first differenced version of this equation

$$(3c) \qquad \Delta c_{t+1} = \frac{1}{\alpha_1}\Delta(w_{t+1}-p_{t+1}) + \frac{\alpha_2}{\alpha_1}\Delta l_{t+1} + v_{2t+1}$$

$$v_{2t+1} = \frac{\Delta d_{2t+1} - \Delta d_{1t+1}}{\alpha_1}$$

For convenience, (3c) is given for period t+1.

The error term of equation (2b) is the sum of a second order moving average process and a serially uncorrelated expectation error. The two components of these two error terms can only be contemporaneously correlated as $E_t(\eta_{t+1})$ is equal to zero. The disturbance of equation (3c) is the difference of two at most contemporaneously correlated taste shocks. According to a theorem of Granger and Morris (1976), the composite error term of (2b) is in general a q'th order moving average process,[3] i.e.

$$v_{jt+1} = (1-a_{j1}L-\ldots-a_{jq}L^q)e_{jt+1} \qquad j = 1,2$$

$$E_t(e_{jt+1}) = 0 \qquad E_j(e^2_{jt+1}) = \sigma^2_{ej} \qquad "$$

Given these assumptions, we could estimate our two equations simultaneously. However, we proceed applying single equation methods to the dynamic optimality condition (2b) as well as to the static optimality condition (3c). This approach results in two estimates of the intertemporal elasticity of substitution in consumption $\frac{1}{\alpha_1}$ which, if significantly different, may point to eventual misspecifications of the model. Equation (2b), for example, gives an estimate of $\frac{1}{\alpha_1}$ based only on the assumption of optimal trade-off between current and future consumption. Quantity constraints on the labour market do not affect the validity of this relationship. By contrast, the estimate of $\frac{1}{\alpha_1}$ obtained from equation (3c) is independent of liquidity constraints preventing optimal intertemporal substitution. In addition, expectational errors do not play any role in equation (3c), which is only based on optimal behaviour with respect to current consumption and leisure. These equations cannot be estimated using a standard generalized least squares procedure taking into account an MA(q)

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3) The sum of two independent or contemporaneously correlated $MA(q_1)$ and $MA(q_2)$ processes is MA(q), where $q = \max(q_1, q_2)$.

error term.[4] The right hand side variables and the error terms contain information of the same period and, therefore, are correlated in general. Thus, the method of instrumental variables (IV) suggests itself to estimate our equations. However, the fact that the available instrument, i.e.

$$Z_{t-i} = [\Delta c_{t-i}, \Delta l_{t-i}, \log(1+r_{t-i}) , \Delta p_{t-i} , \Delta w_{t-i}]$$

$$i > q$$

are only predetermined and not exogenous implies that the usual transformation to eliminate serial correlation - backward filtering - cannot be applied, as this transformation destroys the orthogonality property of the instruments. Therefore, we used the forward filtered estimator proposed by Hayashi and Sims (1983) using Z_{t-q-1}, Z_{t-q-2} and a constant as instruments.

Second, we may proceed on the assumption that $(d_{1t}-d_{2t})$, and therefore, the error term of the static optimality condition (3b) is, although serially correlated, stationary. This is clearly the case if d_{1t} and d_{2t} are both stationary. The theory of cointegrated processes developed by Engle and Granger (1987) and others tells us that the linear combination $(d_{1t}-d_{2t})$ may be stationary even if the individual series are non-stationary. Thus, it seems interesting to consider the case of an I(0) error term in equation (3b). If the observable variables of this equation are I(1) time-series, simple OLS estimation of (3b) is of interest. This approach was followed in an earlier paper under the restriction that $\alpha_1 = \alpha_2$ (Kugler, 1988b). In this case, (3b) reduces to a simple regression of (c_t-l_t) on (w_t-p_t). If we assume that, in addition to c_t and (w_t-p_t), l_t is an I(1) variable, we can

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4) For a detailed discussion of the estimation of these equations, the reader is referred to Kugler (1988a)

apply this approach to the three-variable regression.

In this case, the intertemporal substitution elasticity in consumption can be estimated by running the OLS cointegration regression $c_t = \hat{\beta}_0 + \hat{\beta}_1 (w_t - p_t) + \hat{\beta}_2 l_t$. Stock (1987) showed that such estimates obtained with I(1) variables and I(0) errors have unusual and interesting properties. First, $\hat{\beta}_1$ converges to $\frac{1}{\alpha_1}$ at a rate proportional to the sample size. Second, running the cointegration regression with $(w_t - p_t)$ or c_t as left-hand variable results in asymptotically equivalent parameter estimates. The cointegration estimate of the intertemporal elasticity of substitution is not only robust with respect to serially correlated taste shocks, but to all other stationary (i.e. measurement and aggregation) errors disturbing the equilibrium relationship (3b).

The non-stationarity assumption for leisure may be questioned on a priori grounds as leisure is a bounded variable. In this case, the error term of the simple regression of c_t on $(w_t - p_t)$, $\frac{\alpha_2}{\alpha_1} l_t + \frac{d_{1t} - d_{2t}}{\alpha_1}$, is an I(0) series. Therefore, c_t and $(w_t - p_t)$ are bivariately cointegrated under these circumstances. Thus, we can estimate $\frac{1}{\alpha_1}$ by simply regressing c_t on $(w_t - p_t)$.[6]

3. EMPIRICAL RESULTS

The two first-order conditions (2b) and (3c) were estimated by

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6) In addition, the estimate of $\frac{1}{\alpha}$ reported in Kugler (1988b) under the assumption $\alpha_1 = \alpha_2$ are asymptotically equivalent to this estimate of $\frac{1}{\alpha_1}$.

the Hayashi-Sims technique using quarterly time-series covering the years 1966-1985 (USA, Great Britain) and 1970-1985 (West Germany, Switzerland), respectively. Details about the data series are given in Kugler (1988 b). Briefly, C is the real private per capita (total population) expenditures on non-durables and services.[7]

Leisure L is obtained by subtracting per capita quarterly working hours from the quarterly time endowment of 1456 hours, which corresponds to a daily time budget of 16 hours. Of course, this value is arbitrary to a certain extent. However, some additional tests not reported here indicate that the estimation results are stable with respect to changes in this parameter. P is given by the consumption deflator for the respective consumption aggregate used. W is obtained by dividing labour income by aggregate working hours. The interest rate r_t is the three-month rate for treasury bills (USA, Great Britain), money market deposits (West Germany) and Euro-Swiss Franc deposits. The series are seasonally adjusted with the exceptions of interest rates.

The definition of the data series may potentially bias the estimation results. These problems are discussed in detail by Mankiw, Rotemberg and Summers (1985, 234-37). Thus, we will only briefly mention these problems and consider their relevance in the present context. First, non-durable consumption as defined statistically has durable components. Second, the use of quarterly average time-aggregate data leads to an MA(1) component in the error term, if the data are generated by a continuous process. Third, time averaging involved in seasonal adjustment introduces

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7) For West Germany and Switzerland, there is no official statistical breakdown of consumption in durables and non-durables. Therefore, privately constructed series, which are available since 1970, are used.

moving average error components. Fourth, we use gross wages, but net wages adjusted for marginal tax rates and social security contributions are appropriate from a theoretical point of view. However, in the present framework, these problems should not be of central importance as our framework allows for moving average errors in the Euler equations to be estimated. This extension of the standard intertemporal substitution model was motivated by random changes in preferences, but it also takes into account the effects of time aggregation and seasonal adjustment as well as other error sources leading to MA error components.

The estimates of the three first order conditions are presented in Table 1 and 2. Thereby, the MA order of the error term q is set to 0, 2 and 4. The maximum order of q equal to 4 is selected in order to take account of possible seasonal effects. The case q equal to 0 correcponds, of course, to the standard model with time invariant preferences. However, it can also be motivated by the assumption that the taste shifters d_{1t} and d_{2t} follow a random walk. The estimates presented were obtained by two lagged values for every instrumental variable (p=2). The use of an extended instrument set p=4 brought, however, no substantial change in the results.

Let us first turn to the results obtained for the Euler equation for consumption. In all cases considered, the standard model (q=0) is clearly rejected by the test of overidentifying restrictions. For US and British data, the MA(2) and MA(4) error term ammends this problem but the elasticities of intertemporal substitution are small and statistically insignificant. For West Germany and Switzerland, all models estimated perform very poorly, as the overidentifying restrictions are always rejected.

The estimation results presented for the static first order condition are more appealing than those for the dynamic first order Euler equations. The estimated models pass the test of

Table 1: Estimation of the Dynamic First Order Condition for Consumption with MA(q)-error term
Hayashi-Sims forward-filtered estimates (q=0)

$$\Delta c_t = k_1 + \frac{1}{\alpha_1}(r_{t-1} - \Delta p_t) + v_{1t}$$

Country		k_1	$\frac{1}{\alpha_1}$	Test of overidentifying Restrictions χ^2_9
USA	q=0	0.0025(2.37)**	0.046(0.46)	29.72***
	q=2	0.0018(2.05)**	0.051(0.40)	14.09
	q=4	0.0018(1.95)**	0.065(0.50)	18.27**
Great Britain	q=0	0.0042(3.93)***	0.157(1.45)	36.42***
	q=2	0.0045(3.97)***	0.015(0.10)	15.24*
	q=4	0.0045(3.98)***	-0.161(-1.15)	7.03
West Germany	q=0	0.0074(4.83)***	-0.235(-1.76)*	34.05***
	q=2	0.0049(4.17)***	0.0571(0.97)	26.46***
	q=4	0.0060(3.36)***	-0.00041(-0.77)	54.40***
Switzerland	q=0	0.0032(1.83)*	-0.144(-0.50)	47.07***
	q=2	0.0054(3.12)***	-0.263(-1.08)	12.73
	q=4	0.0052(2.87)***	-0.3222(-1.29)	42.35***

t-statistics are reported in parentheses

*, ** and *** indicate significances at the 10%, 5% and 1% levels, respectively

Table 2: Estimation of the Dynamic First Order Condition for Consumption with MA(q)-error term Hayashi-Sims forward-filtered estimates (q=0)

$$\Delta c_t = k_2 + \frac{1}{\alpha_2} \Delta(w_{t-1} - p_t) + \frac{\alpha_2}{\alpha_1} \Delta l_t + v_{2t}$$

Country		K_3	$\frac{1}{\alpha_1}$	$\frac{\alpha_2}{\alpha_1}$	Test of overidentifying Restrictions X^2_5
USA	q=0	$0.54.10^{-4}$(0.052)	0.567(3.76)***	-3.399(-3.14)***	7.25
	q=2	$-0.60\cdot10^{-4}$(-0.27)	0.314(1.61)	-4.446(-2.78)***	4.50
	q=4	-0.00017(-0.49)	0.383(2.39)**	-4.27(-3.01)***	4.59
Great Britain	q=0	0.0037(2.75)***	0.152(0.84)	-2.34(-1.57)	35.66**
	q=2	$-0.45\cdot10^{-4}$(-0.08)	0.382(2.15)**	-2.53(-1.51)	10.85
	q=4	$-0.43\cdot10^{-4}$(-0.72)	0.180(0.93)	-1.32(-0.69)	3.30
West Germany	q=0	0.0045(2.26)**	0.224(0.95)	-5.75(-0.32)	37.55***
	q=2	$0.97\cdot10^{-4}$(0.08)***	0.256(2.24)**	-16.93(-1.23)	6.92
	q=4	0.00061(-0.75)	0.143(1.22)	-44.8(-2.15)**	16.05*
Switzerland	q=0	0.0029(1.62)	0.182(1.99)**	-0.729(-0.36)	32.25***
	q=2	-0.00060(-0.48)	0.357(2.47)**	1.13(0.41)	2.43
	q=4	-0.00040(-0.27)	0.606(2.23)**	0.061(0.03)	11.70

t-statistics are reported in parentheses

*, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively

overidentifying restrictions. For the US data, this is even true for the model allowing no serial correlation of the error term. The estimated intertemporal substitution elasticity is statistically significantly positive and lies in the reasonable range between 0.15 and 0.6. Unfortunately, the second coefficient $\frac{\alpha_2}{\alpha_1}$, which is the ratio of the intertemporal elasticity of substitution in consumption and in leisure, is negative. However, Switzerland is the only exception to this rule. This problem does not seem to be very serious as the implied elasticity of substitution in leisure is rather small in absolute value and not significantly different from zero.

3.1 I(0) or Cointegrated Taste Variables

The results obtained by the cointegration regression estimates of the static optimality condition under the restriction $\alpha_1 = \alpha_2$ for our data set in Kugler (1988 a) are reported in Table 3.[7] Besides the estimates of the elasticity of intertemporal substitution $\frac{1}{\alpha}$, the results of three tests for cointegration are reported. Thereby, the no-cointegration or unit root hypothesis for the error of the cointegration regression is tested a) by the Durbin-Watson statistic and b) by two variants of the Dickey-Fuller t-statistic. For the UK, the USA and West Germany, there is only weak evidence for cointegration. However, it should be taken into account that the power of these tests are rather low when the residuals are stationary but strongly serially correlated. By contrast, in the Swiss case, the unit root hypothesis is clearly rejected by the Durbin-Watson statistic and the unit root t-test

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7) Results of the corresponding unit root tests for the level series and the first differences series, which are not reported here, indicate that c_t-l_t and w_t-p_t are I(1) series.

Table 3: Static First Order Condition: Results from Cointegration Regression

the restriction under $\alpha_1 = \alpha_2 = \alpha$

I: $c_t - l_t = \beta_0 + \beta_1 (w_t - p_t)$
II: $w_t - p_t = \lambda_0 + \lambda_1 (c_t - l_t)$

$\beta_1 = \frac{1}{\alpha}$, $\lambda_1 = \alpha$

		USA 1966-1985		UK 1966-1985		Switzerland 1970-1985		W. Germany 1970-1985	
		I	II	I	II	I	II	I	II
$\frac{1}{\alpha}$		0.88	1.05	0.64	0.85	0.71	0.81	0.65	0.71
R^2		0.83	0.83	0.75	0.75	0.87	0.87	0.91	0.91
Tests for Cointegration									
Durbin-Watson		0.13	0.13	0.29*	0.33**	1.10***	1.22***	0.30**	0.32**
Granger-Engle "t"-test[1)]	p=0	-1.64	-1.93	-1.44	-2.32	-5.16***	-5.24***	-1.86	-2.31
	p=4	-2.69	-2.96*	-1.60	-1.96	-1.95	-1.69	-1.16	-1.60

*, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively. Critical values are taken from Engle and Granger, Table 3.

1) t-statistic for the lagged residual of the contegration regression in a regression of its first differences on p own lagged values and the lagged level of the residual.

in a AR(1) framework. The insignificance of the augmented unit root test is of no importance here, as all the included difference terms are hardly significant at conventional levels.

The estimated intertemporal substitution elasticity lies in a rather narrow range between 0.65 and 0.85 for the UK, Switzerland and West Germany. However, the US estimate, which is 0.88 and 1.05 respectively, points to a higher degree of intertemporal substitution in the USA. In the Swiss and German cases, this elasticity estimated does not strongly depend on the direction the cointegration regression is run, whereas for the UK and the USA, larger differences are obtained. This pattern of results corresponds to the distinctly higher R^2 measure obtained for Switzerland and West Germany: Under the cointegration assumption, the small sample bias of the cointegration regression estimates is negatively related to its R^2 measure.

Now, we attempt to estimate the cointegration regression without the restriction $\alpha_1 = \alpha_2$. To this end, we first have to check whether c_t, (w_t-p_t) and l_t are I(1) series. The corresponding unit root tests are reported in Table 4. These results in general indicate that the variables of interest are I(1) series, although in some cases the "t"-statistic is rather close to the 10% critical value of the unit root test for the level series.

In the first panel of Table 5, the results obtained for the unrestricted static first order condition are given. For the UK and Switzerland, the estimate of $\bar{\alpha}_1$ is rather close to that reported in Table 3, whereas for West Germany and especially the USA, distinctly lower estimates are obtained. However, these results suffer from two problems. First, the coefficient estimate for β_2, which is the ratio of the intertemporal elasticity of substitution in consumption and in leisure is negative in all cases with the exception of Switzerland. Second, the no-cointegration hypothesis can only be rejected in the Swiss and US

Table 4: Order of Integration of log consumption, c_t, log leisure, l_t, and the log real wage, $w_t - p_t$.

Dickey-Fuller t-statistic for Unit Root Hypothesis $\Upsilon = 0$

$$\Delta X_t = \Upsilon X_{t-1} + \theta_0 + \sum_{i=1}^{p} \theta_i \Delta X_{t-i} + \varepsilon_t$$

X_t	USA 1966 - 1985		UK 1966 - 1985		Switzerland 1970 - 1985		West Germany 1970 - 1985	
	p=0	p=4	p=0	p=4	p=0	p=4	p=0	p=4
c_t	-1.11	-1.27	-0.20	-0.66	-2.34	-1.62	-1.25	-1.86
$\Delta(c_t)$	-6.55***	-4.51***	-8.83***	-3.24***	-11.66***	-2.84**	-11.61***	-2.79**
$w_t - p_t$	-1.83	-1.68	-2.02	-2.42	-2.04	-1.28	-2.53	-2.59
$\Delta(w_t - p_t)$	-6.80***	-3.33***	-8.99***	-3.71***	-15.14***	-4.31***	-8.01***	-3.67***
l_t	-0.70	-1.34	-2.48	-1.96	-0.84	-2.07	-1.45	-2.47
Δl_t	-5.16**	-3.16**	-9.19***	-3.74***	-4.91***	-2.32	-10.59***	-3.49***

*, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively.

Critical values are taken from Table 8.5.2 in Fuller (1976)

Table 5: Static First Order Condition
Results from Cointegration Regressions

$$c_t = \beta_0 + \beta_1(w_t - p_t) + \beta_2 l_t$$

	USA 1966-1985	UK 1966-1985	Switzerland 1970-1985	W. Germany 1970-1985
$\beta_1 = \frac{1}{\alpha_1}$	0.49	0.68	0.76	0.46
$\beta_2 = \frac{\alpha_2}{\alpha_1}$	-4.89	-2.54	0.37	-41.84
R^2	0.98	0.89	0.90	0.92
Durbin-Watson[1]	0.47	0.15	1.27	0.30
Granger-Engle[1] "t"-Test p=0	-3.17	-0.85	-5.66***	-1.91
p=4	-3.53*	-1.01	-1.89	-1.41

$$c_t = \beta_0 + \beta_1(w_t - p_t)$$

	USA 1966-1985	UK 1966-1985	Switzerland 1970-1985	W. Germany 1970-1985
$\beta_1 = \frac{1}{\alpha_1}$	0.81 (0.86)[4)]	0.55 (0.63)	0.79 (0.88)	0.65 (0.72)
R^2	0.86	0.76	0.90	0.91
Durbin-Watson[2)]	0.13	0.18	1.36***	0.28**
Granger-Engle[2-3)] "t" test p=0	-1.73	-1.04	-5.19***	-1.86
p=4	-2.67	-1.45	-1.86	-1.56

*, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively

1) Critical values are taken from Table 3 in Yoo/Engle (1987).

2) Critical values are taken from Table 3 in Engle/Granger (1987)

3) See footnote 2) in Table 3.

4) Estimate of $\frac{1}{\alpha_1}$ obtained by running a regression of $w_t - p_t$ on c_t.

cases. In addition, running the cointegration regression with l_t and (w_t-p_t) as left hand variables brought sometimes strikingly different parameter estimates. These unsatisfactory aspects of our results may be attributed to estimation problems. First, the favourable properties of OLS estimates of multivariate cointegration regressions hold only if no subset of the variables are cointegrated. This may be a problem with our data, as the application of the cointegration tests used also in Table 3 to all variables in pairs, yields some evidence for bivariate cointegration. Second, there are strong a priori reasons to conjecture that c_t is an I(0) variable. Under these circumstances, the unusual properties of the OLS estimate of static regression also do not apply. Although there is no particular strong evidence that l_t is a I(0) variable in Table 4, we run a bivariate cointegration regression of c_t on (w_t-p_t). As mentioned above, the estimated slope coefficient of this regression is a superconsistent estimate of $\frac{1}{\alpha_1}$ when l_t is, in contrast to c_t and (w_t-p_t), an I(0) variable. This approach results in estimates (Table 5) which are rather close to those given in Table 3 and which do not depend essentially on the direction the cointegration regression is run. However the tests for cointegration carried out do not strongly support this hypothesis. This may also be attributed to the rather low power of these tests with a highly autocorrelated I(0) residual of the cointegration regression.

4. CONCLUSION

In this paper estimates of the intertemporal elasticity of substitution in consumption and in leisure are reported for four countries (USA, UK, Switzerland and W. Germany), using quarterly time-series data. Thereby, a dynamic first order condition for consumption and a static first order condition for consumption and leisure were estimated under different assumptions with respect to the serial correlation of the error term. The residual autocorrelation is motivated by random changes in preferences of the representation consumer as well as by problems of aggregation and measurement. The results obtained for the dynamic first order condition are disappointing. Although moving average errors up to the order four are taken into account, the test for overidentifying restrictions rejects the model and the elasticity of substitution estimate is small and insignificant or wrongly signed. The estimation of the static first order condition provides us with more satisfactory results. Under the assuption of a non-stationary error term, we obtained significant estimates for the intertemporal elasticity of consumption which lie in a range between 0.15 and 0.6. Alternative estimation under the assumption of a stationary error term using cointegration regressions results in distinctly higher estimates of the intertemporal elasticity of substitution in consumption in the range between 0.5 and 1. Unfortunately, the unrestricted estimates obtained for the intertemporal elasticity of substitution in leisure in the frame of the static first order condition are often negative. Although this can be attributed to technical problems of estimation, the simple additive separable utility function may be responsible for this result. As in the latter case, our estimates are probably biased, this problem therefore deserves additional research.

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CONSUMPTION AND CHANGING INCOME UNCERTAINTY: A FIRST EMPIRICAL INVESTIGATION FOR AUSTRIA

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1. INTRODUCTION[1]

With the uprise of the theory of rational expectations, economists have started to model the uncertainty faced by economic agents in a serious way. As uncertainty is recognized explicitly in the model building stage, economic and econometric models are linked in a much closer way. In particular, the set of possible explantory variables becomes more distinct; moreover, the error term in the regression analysis receives an explicit interpretation as its properties are related to economic theory. It is then in general no longer correct to interpret the error term as capturing the remaining but neglected influences or as reflecting the approximate character of the specific model. One may therefore claim that the theory of rational expectations has provided the econometrician with a theory of the error term.

One famous example in this respect is Hall's (1978) study on consumption. There he shows that consumption ought to follow a random walk with drift and is not caused, in the sense of Granger, by any other variable included in the agent's information set once lagged consumption has been accounted for. It is fair to say that the majority of empirical studies rejected this implication of the permanent income - life cycle hypothesis with rational expectations. In particular, the study of Flavin (1981) for the US found that the reaction of consumption to income innovations is larger than implied by the corresponding change in permanent income. Flavin and most of the literature has interpreted this "excess sensitivity" of consumption as an indication for liquidity constraints (see Hayashi (1985)).

There is, however, an alternative explanation that is not based on borrowing constraints, but that abandons the assumption of certainty equivalence which has been the basis for most of the literature in this field. The impossibility to insure against individual-specific labor income risk will then induce

1 I want to thank John Hey, Albert Jäger, Robert Kunst, Roberto Mariano and Michael Wüger for their comments and suggestions. The usual disclaimer of course applies.

precautionary savings. As a result, consumption exhibits "excess sensitivity" because an income innovation will not only change permanent income but will also change the need for precautionary savings (see Barsky, Mankiw, and Zeldes (1986) and Blanchard and Mankiw (1988)).

So the purpose of the paper is to study the role of income uncertainty and to quantify its effects when the assumption of certainty equivalence is relaxed.[2] At a general theoretical level this problem has been analysed by Sandmo (1970) and Grossman, Levhari, and Mirman (1979). They show that current consumption is higher with a certain income stream than with an uncertain one if the marginal utility is convex. For empirical purposes one needs a function relating consumption to the income process. Such a function can be given in an analytic way only under assumptions about preferences and distributions. The theoretical part of the paper will explore two such specifications. Based on previous work by Hansen and Singleton (1982), and Palm and Winder (1986), it will be shown that the Euler equation of the optimization problem implies that the change or the growth rate of consumption equals a constant plus a term positively related to the conditional variance of consumption which is itself related to the conditional variance of income. Engle, Lilien, and Robbins (1987) have termed such processes as autoregressive conditional heteroskedastic in mean (ARCHM) as their conditional variance evolves according to some autoregressive scheme and, in addition, influences the mean of the underlying process.

The effects of income uncertainty are explored using Austrian quarterly time series on consumption and labor income. As the ARCH in mean model is computationnaly very costly, the paper explores in a first step the simpler autoregressive conditional heteroskedasticity (ARCH) model developped by Engle (1982). The empirical investigation will proceed in several steps. First, it will be shown that univariate models for consumption will exhibit

2 The effects of inflation and interest rate uncertainty on consumption have been investigated by Deaton (1977) and Gylfason (1982).

strong ARCH effects. The conditional variance term which appears as one determinant in the planning relation for consumption is therefore not constant. For the income process, however, no significant ARCH effects are detected. The next step then consists in an attempt to relate this changing conditional variance to a corresponding variation in the conditional variance of income and the unemployment rate.

The plan of the paper is as follows. Section 2 will present the conventional intertemporal optimization problem of the representative consumer and show how the conditional variance of consumption enters the Euler equation. The next section relates this conditional variance to the parameters of the income process which is postulated to follow an autoregressive conditional heteroskedastic model (ARCH). Section 4 then presents empirical findings for Austria. A conclusion finally ends the paper.

2. THEORETICAL FRAMEWORK

Consider a representative consumer who evaluates sequences $\{C_t\}$ of consumption services according to the following time separable intertemporal utility functional:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t) \tag{2.1}$$

where the subjective discount rate β is an element from the open interval (0,1) and where E_0 is the mathematical expectations operator conditional on information available at time 0. The period utility function U is assumed to have the usual smoothness and regularity properties. Eventually, it will be assumed that U is three times continously differentiable.

The representative consumer maximizes the utility functional (2.1) subject to the period budget constraint:

$$A_{t+1} = R_t (A_t + Y_t - C_t) \tag{2.2}$$

with A_0 given. Y_t denotes labor income of the agent as of time period t. A_t is the stock of an asset valued in units of the consumption good held at the beginning of period t, and R_t is the real gross rate of return of this asset between dates t and t+1, measured in units of time t+1 consumption good per time t consumption good.

$\{R_t\}$ and $\{Y_t\}$ are random processes. The realization R_t becomes known to the agent only at the beginning of period t+1, so that the agent knows at time t when the consumption decision for this period has be to made only the values of R dated t-1 and earlier. The process $\{Y_t\}$ is assumed to be uncontrollable and its realizations dated t and earlier are known to the agent in period t. To rule out a strategy of infinite consumption supported by infinite borrowing a suitable transversality condition is imposed.

The optimization problem yields the familiar Euler equations as first order conditions:

$$(2.3) \qquad E_t \, [\beta \, R_t \, U'(C_{t+1})/U'(C_t)] = 1 \qquad \forall \, t$$

This equation states that <u>ex-ante</u> the agent equates the marginal rate of substitution $U'(C_t)/[\beta U'(C_{t+1})]$ to the rate of transformation R_t. Thus, uncertainty affects optimal consumption only when it affects the expected marginal utility. As Grossman, Levhari, and Mirman (1979) have shown current consumption is higher with a certain income stream than with an uncertain one if and only if the marginal utility is convex (i.e. $U'''>0$). In this case, agents will postpone consumption in the face of increased uncertainty.

To make the above Euler equation econometrically tractable, one has to be explicit about the type of utility function and the distribution of random processes. Following among others Palm and Winder (1986) two sets of assumptions are considered:

<u>Assumption set 1</u>: (i) $U(C_t) = -\delta^{-1} \exp(-\delta C_t)$, $\delta>0$

(ii) $(C_{t+1}, \ln R_t)$ jointly normally distributed

<u>Assumption set 2</u>: (i) $U(C_t) = [(C_t)^{1-\delta} - 1]/(1-\delta)$, $\delta>0$

(ii) $(\ln C_{t+1}, \ln R_t)$ jointly normally distributed

The Euler equation (2.3) then becomes (In the following version a and b relate to the first and second set of assumptions):

$$(2.4a) \qquad E_t C_{t+1} = C_t + \delta^{-1} E_t \ln R_t + \tfrac{1}{2} \delta^{-1} V_t(\ln R_{t} - \delta C_{t+1}) + \delta^{-1} \ln\beta$$

$$(2.4b) \qquad E_t \ln C_{t+1} = \ln C_t + \delta^{-1} E_t \ln R_t + \tfrac{1}{2} \delta^{-1} V_t(\ln R_{t} - \delta \ln C_{t+1}) + \delta^{-1} \ln\beta$$

where V_t is the variance operator conditional on information at time t. According to the above equations the agent plans his expenditures in such a way that the level (logarithm) of consumption in period t+1 is equal to his current level (logarithm) plus adjustments positively related to the expected real gross rate of return and the perceived risk measured by the conditional variance. The conditional variance V_t is taken with respect to a linear combination of the logarithm of the real interest rate and the level (logarithm) of consumption in period t+1. It will be shown how this conditional variance is related to the conditional variance of real labor income.

Both utility functions have convex marginal utility so that an increase in uncertainty will increase savings today. The magnitude of this effect is given by δ^{-1} for these special types of utility functions. Estimating the effect of uncertainty on consumption, therefore provides another way to measure the degree of risk aversion. As the conditional variance is taken not only with respect to consumption but also with respect to the real gross interest rate, it is possible to separate the effects of income and capital uncertainty - a distinction that has been emphasized by Sandmo (1970) - by relating the first one to C_{t+1}, respectively $\ln C_{t+1}$, and the second one to $\ln R_t$. This paper concentrates on the effect of income uncertainty and takes the real interest rate as constant and equal to R.[3] The Euler equations then simplify to:

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3 In the empirical part of the paper, I will come back to the problem of capital uncertainty.

$$(2.5a) \qquad E_t\, C_{t+1} = C_t + \tfrac{1}{2}\delta\, V_t(C_{t+1}) + \delta^{-1} \ln(\beta R)$$

$$(2.5b) \qquad E_t \ln C_{t+1} = \ln C_t + \tfrac{1}{2}\delta\, V_t(\ln C_{t+1}) + \delta^{-1} \ln(\beta R)$$

Even when βR equals one, the presence of uncertainty results in a planned consumption path which is positively sloped, because agents undertake precautionary savings. This should be contrasted to the behavior under certainty equivalence (i.e. quadratic utility function) where the planned consumption path is completly flat.

The main advantage of assumption set 1 is that, without uncertainty on the gross real interest rate, it is possible to compute analytically the optimal strategy of the consumer. In particular Charpin (1987) has shown that $\{C_t\}$ follows a Gaussian process when $\{Y_t\}$ follows a Gaussian process. There is, however, the disadvantage that there is a positive probability that the optimal strategy will entail negative consumption.[4] Furthermore, seen from an empirical point of view working with growth rates rather than differences seems to be more appealing. For the second set of assumptions, however, the exact corresponding stochastic process for $\{Y_t\}$ is not known.

Seen from an econometric view point, equations (2.5a) and (2.5b) have the structure of what Engle, Lilien, and Robins (1987) have termed an ARCH in mean model. According to this model the conditional variance is not only changing over time according to some autoregressive process but is itself a determinant of the mean of the underlying consumption process. Relating $V_t C_{t+1}$, respectively $V_t \ln C_{t+1}$, to past squared error terms this model can be estimated by maximum likelihood. This technique has attracted great attention in modelling time varying risk premia in financial markets. The procedure has, however, the disadvantage that estimates are computationally hard to obtain and that they have to based on the presumption of correct specification. In this paper

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4 Charpin (1987) has shown through simulations that the probability may be actually very low.

an other, more revealing and more robust procedure is adopted. It is shown how the conditional variance is related to the income process which in this case is the only source of risk. Past squared residuals from an ARIMA model of labor income will provide valid instruments for V_t, so that it is possible to apply an instrumental variable estimator as suggested by Pagan (1986), and Pagan and Ullah (1986).

Suppose that the level (logarithm) of real labor income is an integrated stochastic process whose first difference has the following invertible moving average representation:

$$(2.6a) \qquad Y_{t+1} = \mu + Y_t + \sum_{i=0}^{\infty} \Phi_i \, v_{t+1-i}$$

$$(2.6b) \qquad \ln Y_{t+1} = \mu + \ln Y_t + \sum_{i=0}^{\infty} \Phi_i \, v_{t+1-i}$$

with $\Phi_0=1$, $\Sigma \; |\Phi_i| < \infty$. Although the level (logarithm) of income can be non-stationary some restriction on the drift term μ has to be placed to assure convergence in the definition of the expected life time wealth. The error term v_{t+1} is distributed as $N(0,h_{t+1})$ and is uncorrelated but not independent over time as the conditional variance h_{t+1} may depend on past error terms and/or some set of exogenous variables Z_t. In particular, the paper investigates the autoregressive conditional heteroskedastic (ARCH) model of order p proposed by Engle (1982):

$$(2.7) \qquad h_{t+1} = \alpha_0 + \alpha_1 \, v^2_{\;t} + \ldots + \alpha_p \, v^2_{\;t+1-p} + Z_t \, \beta$$

where the coefficients α_i, $0 \leq i \leq p$, and β have to satisfy some nonnegativity condition because the conditional variance must always remain positive.

Using the definition of expected life time wealth it is possible to derive an explicit expression of the conditional variance in terms of the parameters characterizing the income process (see Appendix):

(2.8a) $$V_t C_{t+1} = \left(\sum_{i=0}^{\infty} R^{-i} \Phi_i\right)^2 h_{t+1}$$

(2.8b) $$V_t(\ln C_{t+1}) = \{ [R(1+\mu)] / [Q_t(R-1-\mu)] \}^2 \left[\sum_{i=0}^{\infty}((1+\mu)/R)^i \Phi_i\right]^2 h_{t+1}$$

with $Q_t = (A_{t+1}/Y_t) + [R(1+\mu)/(R-1-\mu)]$. These equations relate the conditional variance of the level (logarithm) of consumption directly to the parameters characterizing the income process. A shift in those parameters, respectively changes in the conditional variance of income, will therefore affect the evolution of consumption. This is also true for a shift in the drift term μ which does not explicitly appear in equation (2.8a).[5] The presence of Q_t in equation (2.8b) is a consequence of the fact that with assumption set 2 the dichotomy between level and variance effects disappears. As wealth increases the impact of income uncertainty becomes smaller because a larger fraction of life time income holding the real interest rate constant is certain. Furthermore, changes in income translate into magnified changes in consumption, compared to the case under certainty equivalence as the need for precautionary savings changes along. This effect provides for an alternative explanation of Flavin's (1981) observed "excess sensitivity".

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5 Palm and Winder (1986) investigate explicitly shifts in μ.

3. EMPIRICAL RESULTS

The empirical investigation uses seasonally unadjusted[6] quarterly data from the Austrian national income and product account over the period 1964:1 to 1987:4. Lags are taken from previous periods. The consumption series consists of real private expenditures on nondurables and services. The income variable is real disposable labor income including transfers. Both time series are logged to take into account their steadily increasing variance which might have biased the empirical results in favor of ARCH effects.

The first step consists in the identification of a univariate time series model of consumption. Following the approach of Box and Jenkings the subsequent model has been identified:

$$(3.1) \qquad (1-L)(1-L^4)\ \ln C_t = (1+b_1 L)(1+b_s L^4)\ \epsilon_t$$

where L is the lag operator and ϵ_t a white noise error term. One might argue that the model in (3.1) is overdifferenced. But several arguments make this specification very appealing. First, the specification is rather parsimonious in that only two parameters have to be estimated. Second, the autocorrelation function of $(1-L^4)\ln C_t$ does not die out, probably as a result of a slowly changing mean when going from the seventies to the eighties. Third, the results reported in table 1 show that the residuals from (3.1) are very close to white noise. The Ljung-Box "portemanteau" statistic with 27 degrees of freedom reaches a marginal significance level of .90. Equation (3.1) will therefore serve as the basis for the further investigations.

Although the residuals seem to be uncorrelated, they may not be independent. Indeed the ARCH model suggested by Engle (1982) provides for the dependence between second moments. Engle's simplified Lagrange multiplier test provides a simple way to test

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6 The importance of correctly modelling seasonality has ben emphasized by Miron (1986).

for ARCH-effects. As can be seen from the results in table 1, the tests ARCH(1) and ARCH(4) strongly reject the hypothesis of independent errors.[7] The table also gives the estimates of the autoregressive models of order one and four for the conditional variance after one iteration of Engle's scoring algorithm. These results suggest that an ARCH model of order one is sufficient to characterize the heteroskedasticity of the residuals.8

Figure 1 shows a plot of the estimated conditional variance for the model of order one. It shows two distinct periods of increased uncertainty: 1973/74 and 1978/79, where the first one is mainly responsable for the ARCH-effects. For these periods the model (3.1) produces large residuals followed by large residuals, so that one can observe clusters of them. This is the typical feature of ARCH-models. The first period corresponds to the first oil price shock which marked a turning point for the world economy. The situation in Austria was characterized by relatively high inflation rates which did not show up in nominal interest rates. This resulted in negative real interest rates. The impact of the first oil price shock on real aggregate demand was actually very limited. Contrary to other OECD countries real (labor) income continued to grow and the unemployment rate reached an all time low of 1.5 percent. However, this is only a statement about the mean. The uncertainty associated with this rather smooth development may have increased substatially. In contrast to the period 1973/74, it is difficult to relate the years 1978/79 to any particular economic event. Austria's economic performance started to deteriorate after 1979 with a stagnation of real income, an increase in real interest rates, and a continuing rise in unemployment. The growth rate of real labor income, however,

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7 It should be emphazised that the ARCH-effect in consumption does not depend on this particular specification, in particular not on the order of differencing.

8 Note that the existence of the fourth and eighth moment of ϵ_t a condition for the some of the testing and estimation procedure requires that the coefficient of the first order ARCH model is lower than $3^{-1/2}$ and $105^{-1/4}$, respectively (Weiss (1984)). The estimated coefficient of .409 satisfies only the first but not the second requirement.

declined continously since 1973.

The theoretical framework implies that one possible source for the nonconstant conditional variance in the model for consumption expenditures is a changing uncertainty about labor income. The modelling efforts for this time series are reported in table 2. Again first and fourth differences seemed necessary to make the logarithm of real labor income stationary. This transformed variable is then best described by a forth order AR-term and a first order MA-term and a dummy for the first quarter of 1971. The residual of this model is practically white noise as indicated by the Ljung-Box statistic Q. Despite the strong ARCH-effect in the consumption equation, the conditional variance of the income process seems to be constant. The corresponding statistics ARCH(1) and ARCH(4) are far from being significant. The autoregressive coefficient for the conditional variance equation turns out to be insignificant and negative. Again, it must be emphasized that this result is not specific to the specification of the model for real labor income.

The rather disappointing role of income uncertainty in explaining the heteroskedasticity of the residuals from equation (3.1) makes it worth to test whether the change in the unemployment rate can account for the nonconstant conditional variance. This variable is especially appealing to proxy uncertainty because with a rising unemployment rate not only the probability of becoming unemployed increases but also spells of unemployment become longer. The depreciation of human capital will then transform what seemed to be a temporary income loss into a permanent one. According to the "insider-outsider" models of (see Lindbeck and Snower (1988) and the references given therein) and the models of hysteresis by Blanchard and Summers (1986), this problem becomes even more accentuated as those being unemployed are unable to bid down wages, because hiring, training and firing costs, cooperation and harassment activities, or effort response to labor turnover prevent "outsiders" from influencing the wage bargain between the firm and employed workers. This behavior will generate substantial persistence in employment and unemployment.[9] The permanent income for insiders is therefore, as a first approximation, unaffected by unemployment. The perceived uncertainty associated with this level of permanent income will, however, increase with a rise in unemployment. As business cycle fluctuations and consequently the burden of unemployment falls on a relatively small group of total employment (see Johnson and Layard (1986)), the effect of uncertainty becomes even more pronounced. As Blanchard and Mankiw (1988) argue on the basis of assumption set 1, if only α percent of consumers are subject to aggregate fluctuations, the effect on aggregate consumption is proportional to $1/\alpha$.

A further interesting aspect in assessing the role of the unemployment rate is that this variable was found by Flavin (1985) to successfully explain the excess sensitivity of consumption with respect to income. In the interpretation given before the unemployment rate is not "a proxy for the severity and prevalence of liquidity constraints" but rather a proxy for the uncertainty associated with the future income stream.

Table 3 provides estimates of equation (3.1) when lagged fourth order differences of the unemployment rate are introduced as additional regressors. From the standard deviations of the estimated coefficients as well as from the F-statistic F_U it is clear that this variable has no explanatory power in this equation. A result that is indepedent on particular specification chosen. Table 3 presents also OLS estimates of a regression of the residuals from the previous equation squared on four own lags and on lagged order differences of the unemployment rate. Although the coefficient on the first lag of $(1-L^4)U_t$ is highly significant, the negative sign stands in sharp contrast to the hypothesis mentioned above. The results of table 3 suggest that there is no direct impact of unemployment on consumption expenditures.

9 The evidence presented by Neusser (1986) for Austria that employment causes the wage - and not the other way round -, can be interpreted in this framework. The history of employment provides information on future demand conditions which induces currently employed workers to set their wage demands so as to maximize the expected rent from their insider status.

One may think that official unemployment rates are a rather poor proxy for uncertainty. For this reason I investigated whether survey data on consumer climate can be used as explanation.[10] Two series are particularily relevant in this respect: "present security of the own job" (OWN) and "overall unemployment next year" (GEN). An increase in those indicators reflects an improved assessment. Regressing the residuals from equation (3.1) squared on own lags and on these indicators gives the results reported in the table 4. Two versions have been estimated depending on whether the current value is included. They show that neither indicator is able to explain the heteroskedasticity in the residuals.[11]

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10 The data have been kindly provided by Michael Wüger.

11 These variables are not significant in alternative consumption functions either (see Breuss and Wüger (1986)).

Table 1: Univariate model of consumer expenditures on nondurables and services[a]

estimation period: 1964:1 - 1987:4

$$(1-L)(1-L^4)\ \ln C_t = \underset{(.079)}{(1-.675L)}\underset{(.092)}{(1-.505L^4)}\ \epsilon_t$$

$R^2 = .442$ SE = .0167 Q(27) = 17.99 AIC = -8.163 BIC = -8.109

ARCH(1) = 9.46*** ARCH(4) = 11.82**

ARCH-model of order one:

$$h_t = \underset{(.000076)}{.000151} + \underset{(.113)}{.409}\ \hat{\epsilon}^2_{t-1}$$

ARCH-model of order four:

$$h_t = \underset{(.000100)}{.000148} + [\underset{(.138)}{.419L} - \underset{(.144)}{.011L^2} - \underset{(.170)}{.141L^3} + \underset{(.151)}{.160L^4}]\ \hat{\epsilon}^2_t$$

a The model for h_{t+1} is estimated by computing one iteration of Engle's (1982) scoring algorithm.

Estimated standard errors are given in parenthesis.
*,**, and *** indicate that the corresponding test statistic is significant at the 10, 5, 1 percent level, respectively.

CONDITIONAL VARIANCE

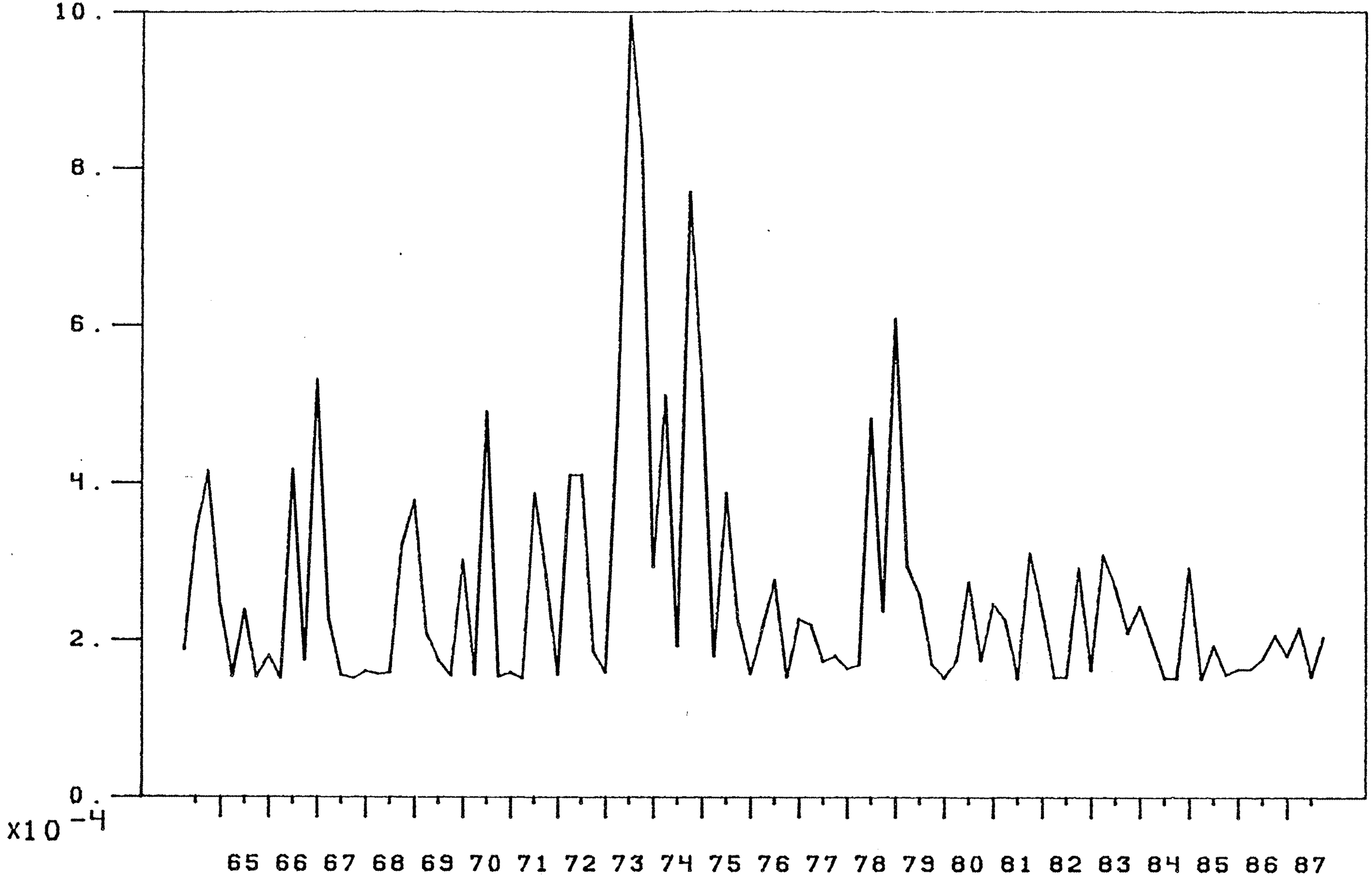

FIGURE 1

Table 2: Univariate model for labor income (Y)[a]

estimation period: 1964:1 - 1987:4

$$\underset{(.099)}{(1-.389L^4)}\,(1-L)(1-L^4)\ln Y_t \;+\; \underset{(.013)}{.046}\,D71{:}1 \;=\; \underset{(.097)}{(1-.355L)}\,\epsilon_t$$

$R^2 = .281$ SE = .0145   Q(27) = 11.98   AIC = -8.431   BIC = -8.351

ARCH(1) = .22   ARCH(4) = 1.38

ARCH-model of order one:

$$h_t = \underset{(.000054)}{.000212} - \underset{(.165)}{.022}\,\hat{\epsilon}^2_{t-1}$$

a The model for h_{t+1} is estimated by computing one iteration of Engle's (1982) scoring algorithm.

Estimated standard errors are given in parenthesis.
*,**, and *** indicate that the corresponding test statistic is significant at the 10, 5, 1 percent level, respectively.

Table 3: The role of unemployment (U) in the equation (3.1)[a]

Estimation period: 1964:1 - 1987:4

$$(1-L)(1-L^4)\ \ln C_t = \underset{(.085)}{(1-.679L)}\underset{(.100)}{(1-.501L^4)}\ \epsilon_t$$

$$-\ [\underset{(.0052)}{.0053} - \underset{(.0087)}{.0100L} + \underset{(.0082)}{.0001L^2} + \underset{(.0047)}{.0046L^3}]\ (1-L^4)U_{t-1}$$

$R^2 = .462$ SE = .0168 $Q(27) = 20.09$ AIC = -8.117 BIC = -7.957

ARCH(1) = 8.04*** ARCH(4) = 11.47** $F_U(4,90) = .79$

ARCH-model of order 4 and change in unemployment rate:

$$\hat{\epsilon}^2_t = \underset{(.000006)}{.00014} + [\underset{(.110)}{.239L} + \underset{(.112)}{.141L^2} + \underset{(.111)}{.037L^3} + \underset{(.107)}{.088L^4}]\ \hat{\epsilon}^2_t$$

$$-\ [\underset{(.00012)}{.00029} - \underset{(.00016)}{.00016L} - \underset{(.00016)}{.00015L^2} + \underset{(.00013)}{.00013L^3}]\ (1-L^4)U_{t-1}$$

$F_U(4,90) = 1.80$

a Estimated standard errors are given in parenthesis.
*,**, and *** indicate that the corresponding test statistic is significant at the 10, 5, 1 percent level, respectively.

Table 4: Consumer climate data

OWN: assessment of present own job security
GEN: assessment of overall unemployment next year

Estimation period: 1975:1 - 1987:3

$$\hat{\epsilon}^2_t = \underset{(.000830)}{-.00077} + [\underset{(.135)}{.091L} + \underset{(.116)}{.239L^2} + \underset{(.109)}{.074L^3} - \underset{(.107)}{.274L^4}]\, \hat{\epsilon}^2_t$$

$$+ [\underset{(.000008)}{.000016} - \underset{(.000010)}{.000018L} + \underset{(.000010)}{.000011L^2} - \underset{(.000010)}{.000018L^3} + \underset{(.000007)}{.000015L^4}]\, OWN_t$$

$$F_{OWN}(5,41) = 1.58$$

$$\hat{\epsilon}^2_t = \underset{(.000830)}{.00001} + [\underset{(.135)}{.045L} + \underset{(.116)}{.196L^2} + \underset{(.109)}{.099L^3} - \underset{(.107)}{.235L^4}]\, \hat{\epsilon}^2_t$$

$$- [\underset{(.000007)}{.000003L} - \underset{(.000010)}{.000010L^2} + \underset{(.000010)}{.000017L^3} + \underset{(.000007)}{.000011L^4}]\, OWN_t$$

$$F_{OWN}(4,42) = .87$$

Estimation period: 1978:1 - 1987:3

$$\hat{\epsilon}^2_t = \underset{(.00019)}{.00015} + [\underset{(.175)}{.066L} + \underset{(.176)}{.439L^2} - \underset{(.179)}{.064L^3} - \underset{(.186)}{.291L^4}]\, \hat{\epsilon}^2_t$$

$$+ [\underset{(.000008)}{.000000} - \underset{(.000010)}{.000002L} + \underset{(.000010)}{.000005L^2} - \underset{(.000010)}{.000005L^3} + \underset{(.000007)}{.000003L^4}]\, GEN_t$$

$$F_{OWN}(5,29) = .30$$

$$\hat{\epsilon}^2_t = \underset{(.00019)}{.00014} + [\underset{(.172)}{.063L} + \underset{(.173)}{.436L^2} - \underset{(.176)}{.065L^3} - \underset{(.176)}{.302L^4}]\, \hat{\epsilon}^2_t$$

$$- [\underset{(.000004)}{.000003L} - \underset{(.000005)}{.000005L^2} + \underset{(.000005)}{.000005L^3} - \underset{(.000003)}{.000003L^4}]\, GEN_t$$

$$F_{OWN}(4,30) = .37$$

4. CONCLUSIONS

Using Austrian data, the paper has demonstrated that uncertainty, measured by the conditional variance of consumption, is not constant over time and should therefore be incoporated in testing Hall's hypothesis. Particularily, Flavin's excess sensitivity test may suffer from this misspecification. It was then tried to relate this observed uncertainty to a changing variance in the income process and/or to the unemployment rate which was thought to be a proxy for the risk associated with a given income stream. This attempts turned out to be unsuccessful. The reason for this negative result is that the periods of increased uncertainty are concentrated in the years 1973/1975 and to a lesser extent in the years 1979/1980 and 1983/84 where neither the unemployment rate nor real labor income showed in Austria much of a reaction to the supply shocks that were hiting the world economy. Relating the heteroskedasticity of the consumption process to consumer climate data such as own or overall job security turned out to be also unsuccessful.

This result is, however, not the end of the story, because the underlying intertemporal model also suggests an other source of uncertainty which is associated with the real interest rate. The effects of inflation volatility emphasized by Deaton (1977) could be regarded as a hint in this direction. Additionally, the real interest rate did undergo major shifts during the time periods associated with increased uncertainty so that this seems to be a promissing path for future investigations.

APPENDIX

The expected life time wealth W_{t+1} is defined by:

$$\text{(I)} \quad \sum_{i=0}^{\infty} R^{-i} E_{t+1} C_{t+1+i} = A_t + \sum_{i=0}^{\infty} R^{-i} E_{t+1} Y_{t+1+i} = W_{t+1}$$

<u>The first set of assumptions</u> gives the general Euler equation:[12]

$$\text{(II)} \quad E_{t+1}C_{t+1+i} = C_{t+1} + \tfrac{1}{2}\delta\, V_{t+1}(C_{t+1+i}) + i\delta^{-1}\ln(\beta R), \quad i\geq 1$$

Substituting this equation into the expected life time wealth W_{t+1} gives:

$$\text{(III)} \quad W_{t+1} = [R/(R-1)]\, C_{t+1} + [R/(R-1)^2]\, \delta^{-1}\ln(\beta R)$$
$$+ \tfrac{1}{2}\delta \sum_{i=1}^{\infty} R^{-i}\, V_{t+1}(C_{t+1+i})$$

By taking the conditional variance of the above equation with respect to the information set available at time t one gets:

$$\text{(IV)} \quad V_t(C_{t+1}) = [(R-1)/R]^2\, V_t(W_{t+1})$$
$$= [(R-1)/R]^2\, V_t(A_{t+1} + \sum_{i=0}^{\infty} R^{-i} E_{t+1} Y_{t+1+i})$$

As A_{t+1} and $E_t\, Y_{t+i+1}$, $i\geq 0$, are known konstants, their variance conditional on information available at time t is zero. Therefore

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12 For the first set of assumptions I follow the derivation of Palm and Winder (1986),

$$V_t(C_{t+1}) = [(R-1)/R]^2 V_t[\sum_{i=0}^{\infty} R^{-i}(E_{t+1}Y_{t+1+i} - E_tY_{t+1+i})]$$

$$\text{(V)} \qquad = [(R-1)/R]^2\ V_t(\sum_{i=0}^{\infty} R^{-i}\ (\Phi_0+\ldots+\Phi_i)\ v_{t+1}\ )$$

$$= [\sum_{i=0}^{\infty} R^{-i}\ \Phi_i\ ]^2\ h_{t+1}$$

which is equation (2.8a) in the text.

Using the <u>second set of assumptions</u> the general Euler equation becomes:

$$\text{(VI)} \qquad E_{t+1}\ \ln C_{t+1+i} = \ln C_{t+1} \quad + \quad \tfrac{1}{2}\delta\ V_{t+1}(\ln C_{t+1+i})$$

$$+ \quad i\delta^{-1}\ \ln(\beta R) \qquad\qquad i\geq 1$$

Inserting this equation into the expected life time budget (I) yields:[13]

$$\text{(VII)} \qquad W_{t+1} = \sum_{i=0}^{\infty} R^{-i}\ E_{t+1}\ C_{t+1+i}$$

$$= \sum_{i=0}^{\infty} R^{-i}\exp[E_{t+1}\ln C_{t+1+i} + \tfrac{1}{2}\ V_{t+1}(\ln C_{t+1+i})]$$

$$= C_{t+1}\ \{\sum_{i=0}^{\infty} R^{-i}\ \exp[\ \tfrac{1}{2}(\delta+1)\ V_{t+1}(\ln C_{t+1+i}) + i\ \delta^{-1}\ \ln(\beta R)\ \}$$

Taking logarithms and then the conditional variance yields:

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13 In contrast to the first set of assumptions the following manipulations are still possible when the interest rate is stochastic.

$$\text{(VIII)} \qquad V_t(\ln C_{t+1}) = V_t\{ \ln[A_{t+1} + \sum_{i=0}^{\infty} R^{-i} E_{t+1} Y_{t+1+i}]\}$$

$$= V_t\{\ln[(A_{t+1}/Y_t) + \sum_{i=0}^{\infty} R^{-i} E_{t+1}(Y_{t+1+i}/Y_t)]\}$$

Using the law of motion for the logarithm of income (equation (2.6b)), the approximation suggested by Campbell and Deaton (1987) is:

$$\text{(IX)} \qquad Y_{t+1+i}/Y_t \approx (1+\mu)^{1+i} [1 + \sum_{k=1}^{1+i}(\Delta\ln Y_{t+k} - \mu)]$$

Inserting this approximation into equation (VIII) then yields:

$$\text{(X)} \qquad V_t(\ln C_{t+1}) = V_t\{\ln[(A_{t+1}/Y_t) + \sum_{i=0}^{\infty} R^{-i}(1+\mu)^{1+i} E_{t+1}(1 + \sum_{k=1}^{1+i}(\Delta\ln Y_{t+k} - \mu))]\}$$

$$= V_t\{\ln[(A_{t+1}/Y_t) + [R(1+\mu)/(R-1-\mu)] + [R(1+\mu)/(R-1-\mu)] \sum_{i=0}^{\infty}[(1+\mu)/R]^{i}(E_{t+1}\Delta\ln Y_{t+1+i}-\mu)]\}$$

This expression can then be approximated by

$$\text{(XI)} \qquad V_t(\ln C_{t+1}) = [R(1+\mu)Q_t^{-1}/(R-1-\mu)]^2 \; V_t\{\sum_{i=0}^{\infty}[(1+\mu)/R]^{i} (E_{t+1}\Delta\ln Y_{t+1+i} - \mu) \}$$

where Q_t is defined as $(A_{t+1}/Y_t) + [R(1+\mu)/(R-1-\mu)]$. Since the conditional variance of the sum over i of $[(1+\mu)/R]^i\ [E_t \Delta lnY_{t+1+i} - \mu]$ is zero it can inserted into the above equation which then becomes:

$$\text{(X)} \qquad V_t(lnC_{t+1}) = [R(1+\mu)Q_t^{-1}/(R-1-\mu)]^2\ [\sum_{i=0}^{\infty}[(1+\mu)/R]^i \Phi_i]^2 h_{t+1}$$

which is equation (2.8b) in the text.

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RATIONAL HABITS IN THE LIFE CYCLE CONSUMPTION FUNCTION

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ABSTRACT

The basic implication of the life cycle consumption hypothesis is the separation of the income and consumption profiles. In this paper we extend the results of Hall (1978). It is shown that for the exponential utility function an arbitrary autoregressive integrated moving average (ARIMA) process of consumption can be obtained by choosing an appropriate pattern of rational habits. As special cases we discuss how a model in the four period difference operator and how a specification with seasonal dummy variables can be obtained by imposing a specific structure on the preferences. In the empirical part we illustrate how the perturbations resulting from structural changes in the income process can be incorporated in the stochastic process for seasonally unadjusted data on consumption for the Netherlands.

Keywords: Life cycle consumption hypothesis, rational habits, ARIMA processes, seasonality, Lucas critique.

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Section 1 Introduction.

Most modern theories of the consumption function are formulated to reconcile the low marginal propensity to consume with the relative stability of the average propensity to consume observed over longer periods, a phenomenon that can not be explained by the Keynesian consumption function. Loosely speaking, in recent approaches constraint variables are introduced, which mitigate the impact of current real disposable income. Important examples are Modigliani and Brumberg's (1955) life cycle consumption hypothesis, which stresses the role of wealth and Brown (1952) who finds a significant impact of previous consumption, which may reflect the influence of habits. Among the many articles that deal with extensions and refinements of the life cycle theory, an important contribution is due to Hall (1978). He formulates the life cycle hypothesis as an intertemporal decision problem with time additive utility function and shows that the first order conditions for an intertemporal optimum imply a first order autoregressive process for the marginal utility of consumption. Obviously, in this context an overwhelming impact on current consumption is reserved for previous consumption. Recently, Muellbauer (1986) has put forward a synthesis between Modigliani and Brumberg's life cycle hypothesis and Brown's model of habit formation. He investigates two kinds of habit formation, myopic and rational, and gives empirical evidence for the U.S. which is in favour of myopic habit formation. He also gives an impressive historical overview of the literature and reviews empirical evidence in the cross-section context. For a lucid exposition I refer to that paper.

In this paper we investigate the life cycle model under rational habit formation. The principal implication of the life cycle model is the separation of the consumption and income profiles. Consequently, the dynamics of consumption are basically determined by the structure of the preferences. This observation reveals that Hall's (1978) finding is an immediate implication of the postulated separability of the life time utility function. When we impose a different structure on the preferences a different stochastic process for consumption will arise. We will show that for the exponential utility function an arbitrary autoregressive integrated moving average (ARIMA) process for consumption can be obtained by choosing an appropriate pattern of rational habits. The model provides us with a theoretical framework for interpreting a broad category of stochastic processes for consumption. The major advantage of interpreting ARIMA processes within the context of intertemporal decision-making is that it enables one to investigate the effects of policy interventions in the rigorous way indicated by Lucas (1976). The results of this paper illustrate how simple ARIMA schemes for consumption may be used not only for forecasting purposes, but also for policy analysis. An illustrative example is for instance when one wants to predict the effects on consumption of a change in the tax rate on income. Given the low cost of specifying and estimating ARIMA processes the results of this paper may be of practical importance.

We investigate the life cycle consumption model extended for the presence of habits, in which the planning horizon is assumed to be infinite. We adopt the infinite horizon formulation because it is more convenient. Gale (1967) gives a more positive argument for the choice of the infinite horizon formulation. He argues that the choice of an infinite plan will affect very crucially what one does the very near future and describes the situation figuratively as follows

> "One is guiding a ship on a long journey by keeping it lined up with a point on the horizon even though one knows that long before that point is reached the weather will change (but in an unpredictable way) and it will be necessary to pick up a new course with a new reference point, again on the horizon rather than just a short distance ahead" (p.2).

Throughout the paper we make the assumption of rational expectations, that is, we assume that the subjective distribution of the income process used in the utility maximization problem coincides with the actual distribution of income. The interest rate is assumed to be constant. When a specific functional form of the utility function is required, we use the exponential utility function.

The paper is organized as follows. In section 2 we analyze the model. The framework is similar to that of Palm and Winder (1987a). The main difference is that we extend the preference structure for the presence of habits. In line with the authors who investigate the Permanent Income Hypothesis (see e.g. Flavin (1981) and Campbell (1987)), we assume that the consumer uses <u>only</u> information on expected future labor income. This assumption facilitates the analysis considerably and differs from the one made by Hall, that consumers take into account the complete distribution of labor income. It will be shown that by choosing the pattern of habits an arbitrary ARIMA process for consumption can be obtained. In an example we discuss a special form of rational habits which yields a model in the four period difference operator Δ_4. In many studies the use of this filter has proved satisfactory in removing seasonal fluctuations. Recently, Miron (1986) has suggested that improper handling of seasonality might be the explanation for the frequent rejections of the life cycle model. He treats the seasonality as an unobserved component for which a model is postulated, whereas we model the seasonality as a special form of rational habits. We will also indicate how a model with seasonal dummy variables may be interpreted as resulting from seasonal shocks to the preferences.

In section 3 we examine quarterly seasonally unadjusted data on consumption for the Netherlands to see whether the data suggest the presence of habits. The chosen model is very similar to the one of Davidson and Hendry (1981), which they present as the analogue of Hall's model. Under the assumption that income is exogenous, the stochastic process of consumption is simply a transformation, accomplished by the intertemporal utility maximization problem, of the stochastic properties of income. This observation shows that the theoretical model generates a number of restrictions between the processes for consumption and income. In line with Lucas (1976) we pay attention to the implications of structural changes in the process of the income variable for the model for consumption. The empirical evidence suggests the presence of rational habits, but also shows that the implications of the model are not fully in agreement with the information in the data. It is argued that a possible remedy to bring the theoretical model into agreement with the empirical evidence may be the relaxation of the rational expectations assumption.

Finally, section 4 concludes the study.

Section 2 Theory.

In this section we discuss the implications of the life cycle consumption model when the preference structure exhibits rational habits. In the first instance we assume that the utility function depends on a finite number of past realizations of consumption. At time t the representative consumer is assumed to maximize his life time utility subject to the life time budget constraint

$$\text{MAX} \sum_{i=0}^{\infty} \beta^{i} U(\Phi(L)c_{t+i}) \tag{2.1}$$

$$\text{S.T.} \quad \sum_{i=0}^{\infty} (1+r)^{-i} c_{t+i} = (1+r)a_{t-1} + \sum_{i=0}^{\infty} (1+r)^{-i} E(y_{t+i}|I_t) \quad ,$$

with $U'>0$ and $U''<0$, where U' and U'' are the first and second derivatives of U. Real consumption and real labor income are denoted by c_{t+i} an y_{t+i} respectively, a_{t+i} is real financial wealth, β is the time preference parameter $(0<\beta<1)$, r is the real rate of interest, which is assumed to be constant $(0<r<1)$ and $\Phi(L)$ is a polynomial of order p in the lag operator L

$$\Phi(L) = 1 - \varphi_1 L - \ldots - \varphi_p L^p$$

with factorization

$$\Phi(L) = (1-\pi_1 L)(1-\pi_2 L)\ldots(1-\pi_p L). \tag{2.2}$$

The subsequent analysis will show that we have to impose the condition that the roots of $\Phi(L)=0$ must lie on or outside the unit circle, that is $|\pi_i|\leq 1$, $i=1,\ldots,p$. E denotes the expectations operator, and I_t is the set of information available at time t. The only source of uncertainty concerns future labor income and it is assumed that the consumer knows the value of y_t when taking a decision about c_t. Hence, $E(y_t|I_t)=y_t$.

Model (2.1) shows that the current decision c_t is affected by past choices of consumption. Winder (1987) investigates the problem (2.1) with finite time horizon and Palm and Winder (1987a) examine the model with finite time horizon without habits, that is $\Phi(L)=1$. Muellbauer (1986) discusses the stochastic version of model (2.1) in which expected utility is maximized for the case that $\Phi(L)$ is of order 1. In this paper we assume that the consumer uses only information on the first (conditional) moments of the income process.

A few comments are in order. Firstly, the life time budget constraint in (2.1) results from successive substitution of the period by period budget constraints

$$a_{t+i} = (1+r)a_{t+i-1} + E(y_{t+i}|I_t) - c_{t+i}, \quad i=0,1,\ldots$$

and the boundary condition

$$\lim_{i\to\infty} (1+r)^{-i}a_{t+i} = 0,$$

which is the transversality condition (see d'Autume and Michel (1987)).

Secondly, the life time budget constraint as formulated in (2.1) is meaningless, unless the infinite sums converge. This leads to the requirement that c_{t+i} and $E(y_{t+i}|I_t)$ are of exponential order less than $(1+r)$. A sequence z_{t+i} will be termed of exponential order less than $(1+r)$, when there exist i_0 and $K>0$ such that for every $i>i_0$

$$|z_{t+i}| < Kx^i \text{ for some } x \in [1,1+r). \tag{2.3}$$

It is also required that the life cycle utility determined by (2.1) does not diverge to infinite. When U is bounded from above, the convergence of the target function is guaranteed.

Thirdly, although the discussion of model (2.1) is presented within the context of rational habit formation, the preference structure determined in (2.1) may be used to model consumption of durable goods. When we lump all goods together and assume an average life time of N periods, a depreciation rate $\delta=N^{-1}$ and that the stock of durable goods yields a consumption service flow which is proportional to its magnitude, it follows that

$$s_{t+i} = \theta K_{t+i} = \theta N^{-1} \sum_{j=0}^{N-1} (N-j)L^j \; c_{t+i} \; ,$$

where s_{t+i} and K_{t+i} denote the service flow and the stock of durable goods in period t+i respectively, and θ is the proportionality factor. Given an intertemporally additive life time utility function with arguments s_{t+i}, we conclude that (after normalization) a special case of model (2.1) arises. Notice that other schemes of depreciation may be considered and that the extension to

$$s_{t+i} = \theta(L)K_{t+i}$$

is straightforward, as long as the lag polynomial $\Phi(L)$ in (2.1) satisfies the regularity conditions that the roots of $\Phi(L)=0$ lie on or outside the unit circle.

The first order conditions of (2.1) consist of a system of difference equations

$$\frac{\partial}{\partial c_{t+l}} \left\{ \sum_{i=l}^{l+p} \beta^i U(\Phi(L)c_{t+i}) \right\} = \frac{1}{1+r} \frac{\partial}{\partial c_{t+l-1}} \left\{ \sum_{i=l-1}^{l+p-1} \beta^i U(\Phi(L)c_{t+i}) \right\} \quad l=1,2,\ldots \tag{2.4}$$

Determining the solution of (2.1) corresponds to solving the $(p+1)^{th}$ order difference equation (2.4) subject to the (p+1) initial conditions $c_{t-1}, c_{t-2}, \ldots, c_{t-p}$ and

$$\sum_{i=0}^{\infty} (1+r)^{-i} c_{t+i} = (1+r)a_{t-1} + \sum_{i=0}^{\infty} (1+r)^{-i} E(y_{t+i} | I_t). \tag{2.5}$$

Substituting

$$\frac{\partial}{\partial c_{t+l}} \left\{ \sum_{i=l}^{l+p} \beta^i U(\Phi(L)c_{t+i}) \right\} = \sum_{i=l}^{l+p} \beta^i U'(\Phi(L)c_{t+i}) \frac{\partial(\Phi(L)c_{t+i})}{\partial c_{t+l}}$$

and

$$\frac{\partial}{\partial c_{t+l-1}} \left\{ \sum_{i=l-1}^{l+p-1} \beta^i U(\Phi(L)c_{t+i}) \right\} = \sum_{i=l}^{l+p} \beta^{i-1} U'(\Phi(L)c_{t+i-1}) \frac{\partial(\Phi(L)c_{t+i})}{\partial c_{t+l}}$$

into (2.4) leads to

$$\sum_{i=1}^{l+p} \beta^{i}[U'(\Phi(L)c_{t+i}) - \frac{1}{\beta(1+r)} U'(\Phi(L)c_{t+i-1})] \frac{\partial(\Phi(L)c_{t+i})}{\partial c_{t+l}} = 0 \quad ,l=1,2,\ldots . \tag{2.6}$$

A sufficient condition for (2.6) to hold true is

$$U'(\Phi(L)c_{t+i}) = \beta^{-1}(1+r)^{-1}U'(\Phi(L)c_{t+i-1}) \quad ,i=1,2,\ldots . \tag{2.7}$$

To arrive at an operational model, it is necessary to choose a specific functional form for the utility function U. In this paper we will investigate the exponential utility function

$$U(c) = -\gamma^{-1}\exp(-\gamma c),\ \gamma>0. \tag{2.8}$$

Obviously, the utility function (2.8) is bounded from above. For the exponential utility function the optimal consumption path corresponds to the solution of the <u>linear</u> difference equation of order (p+1)

$$\Phi(L)(1-L)c_{t+i} = \gamma^{-1}\ln[\beta(1+r)] \quad ,i=1,2,\ldots \tag{2.9}$$

with initial conditions $c_{t-1}, c_{t-2}, \ldots, c_{t-p}$ and the life time budget constraint (2.5).

A convenient alternative procedure to obtain a closed-form solution of the utility maximization problem (2.1) is imposing the restriction $\beta(1+r)=1$. In Winder (1988, ch.3) it is shown that for the life cycle model (2.1) without habits, $\beta(1+r)=1$ is a sufficient condition to obtain Friedman's (1957) Permanent Income Hypothesis, studied by for instance Flavin (1981) and (with some refinements) Campbell (1987). Using expression (2.7), it follows that for any utility function U satisfying $U'>0$ and $U''<0$, the first order conditions consist of a system of linear <u>homogeneous</u> difference equations of order (p+1). As the resulting difference equation arises as a special case of (2.9), we conclude that the assumption $\beta(1+r)=1$ is more restrictive than the choice of the exponential utility function. It should be obvious that the subsequent analysis remains valid for an arbitrary utility function U satisfying $U'>0$ and $U''<0$ under the alternative restriction

$\beta(1+r)=1$.

The literature on linear difference equations (see e.g. Sargent (1979)) provides us immediately the form of the solution of (2.9). When we assume in the first instance that the roots of $\Phi(L)=0$ are distinct and do not lie on the unit circle, (2.9) yields as a solution

$$c_{t+i} = k_0 i + \sum_{j=1}^{p} k_j \pi_j^i + k_{p+1} 1^i \tag{2.10}$$

with k_j ,$j=1,2,\ldots,p+1$, determined by the (p+1) initial conditions, and π_i , $i=1,\ldots p$, given by (2.2). Imagine the situation in which one of the roots, say $1/\pi_1$, lies inside the unit circle. For $|\pi_1|\geq 1+r$ we have a solution of the difference equation which is in contradiction with the requirement that c_{t+i} is of exponential order less than (1+r). As we do not want to exclude any value of $r\in(0,1)$, we require that the ultimate result has to hold for every $r\in(0,1)$. We conclude that all the roots must lie on or outside the unit circle. In general, when the first n roots are equal and do not lie on the unit circle, the difference equation yields the solution

$$c_{t+i} = k_0 i + \sum_{j=1}^{n} k_j i^{j-1} \pi_1^i + \sum_{j=n+1}^{p} k_j \pi_j^i + k_{p+1} 1^i .$$

This shows that the case of multiple roots does not lead to incompatibility with requirement (2.3), as long as the roots lie outside the unit circle. Obviously, multiple roots equal to 1 do not lead to difficulties. In conclusion, to avoid a contradiction between condition (2.3) for any value of the real interest rate between 0 and 1, and the solution of the difference equation, it is necessary to impose the restrictions

$$|\pi_i|\leq 1 \quad , i=1,2,\ldots,p. \tag{2.11}$$

We proceed by examining (2.9). It is convenient to define auxiliary variables c^*_{t+i} as

$$c^*_{t+i} = \Phi(L)c_{t+i} \quad ,i=0,1,\ldots .$$

Solving the difference equation (2.9) subject to the (p+1) initial conditions is equivalent to solving the linear first order difference equation

$$c^*_{t+i} = c^*_{t+i-1} + \gamma^{-1}\ln[\beta(1+r)], \quad i=1,2,\ldots, \tag{2.12}$$

with one boundary condition, namely the life time budget constraint expressed in terms of c^*_{t+i}. Using $a_{t-1}=(1+r)a_{t-2}+y_{t-1}-c_{t-1}$, it can be easily shown that

$$\sum_{i=0}^{\infty}(1+r)^{-i}c_{t-1+i} = (1+r)a_{t-2} + \sum_{i=0}^{\infty}(1+r)^{-i}E(y_{t-1+i}|I_t)$$

is equivalent to (2.5). By repeated argument we find for the transformed life time budget constraint

$$\sum_{i=0}^{\infty}(1+r)^{-i}c^*_{t+i} = (1+r)\Phi(L)a_{t-1} + \sum_{i=0}^{\infty}(1+r)^{-i}E(\Phi(L)y_{t+i}|I_t)\ . \tag{2.13}$$

It can be easily checked that (2.13) is only equivalent to (2.5) in case of an infinite planning horizon. Winder (1987) discusses an alternative procedure which enables us to tackle the model with finite time horizon. Expression (2.12) can be rewritten as

$$c^*_{t+i} = c^*_t + i\gamma^{-1}\ln[\beta(1+r)], \quad i=1,2,\ldots\ , \tag{2.14}$$

and substitution of (2.14) into the life time budget constraint (2.13) yields

$$c^*_t\,\frac{1+r}{r} + \gamma^{-1}\ln[\beta(1+r)]\,\frac{1+r}{r^2} = (1+r)\Phi(L)a_{t-1} + \sum_{i=0}^{\infty}(1+r)^{-i}E(\Phi(L)y_{t+i}|I_t) \tag{2.15}$$

Substituting $c^*_t=\Phi(L)c_t$, formula (2.15) expresses the decision c_t as a function of income, future income expectations, wealth and past consumption. In line with Brown (1952) the latter may be interpreted as the influence of habits.

To investigate the dynamics in consumption, it is convenient to relate c_t to c_{t+1}. Carrying out the same operations as before for the model solved for period t+1 leads to

$$c^{*}_{t+1}\,\frac{1+r}{r} + \gamma^{-1}\ln[\beta(1+r)]\,\frac{1+r}{r^2} = (1+r)\Phi(L)a_t \quad + \sum_{i=0}^{\infty}(1+r)^{-i}E(\Phi(L)y_{t+1+i}\,|\,I_{t+1}) \tag{2.16}$$

Dividing (2.16) by 1+r, substituting $a_t=(1+r)a_{t-1}+y_t-c_t$ and subtracting (2.15) yields

$$c^{*}_{t+1}-c^{*}_{t}=\gamma^{-1}\ln[\beta(1+r)]+\frac{r}{1+r}\sum_{i=0}^{\infty}(1+r)^{-i}[E(\Phi(L)y_{t+1+i}\,|\,I_{t+1})-E(\Phi(L)y_{t+i+1}\,|\,I_t)]\ .$$

When we substitute $(1-L)c^{*}_{t+1}=\Phi(L)\Delta c_{t+1}$ and define the consumption innovation ε_{t+1} as $\varepsilon_{t+1}=c_{t+1}-E(c_{t+1}\,|\,I_t)$, we have

$$\Phi(L)\Delta c_{t+1} = \gamma^{-1}\ln[\beta(1+r)] + \varepsilon_{t+1} \tag{2.17}$$

with

$$\varepsilon_{t+1} = \frac{r}{1+r}\sum_{i=0}^{\infty}(1+r)^{-i}[E(\Phi(L)y_{t+i+1}\,|\,I_{t+1})-E(\Phi(L)y_{t+i+1}\,|\,I_t)]\ . \tag{2.18}$$

Consumption follows an autoregressive integrated (ARI) stochastic process. As unit roots are permitted the order of integration may be larger than one for an appropriate choice of the lag polynomial $\Phi(L)$ (obviously, integration of order zero is excluded). Notice also that a model in the s-period difference operator Δ_s may be obtained by the choice of $\Phi(L)=1+L+...+L^{s-1}$.

Unanticipated changes in the process of the exogenous variable y_t have definite effects on the model for consumption. In line with Lucas (1976) the implications can be traced by using the closed form solutions (2.15) and (2.17). The empirical analysis of the next section will illustrate how structural changes can be handled.

As an illustration we give two examples. The first one corresponds to the model discussed by Muellbauer (1986), where it is assumed that the current consumption decision is only influenced by previous consumption. In the second one we use a lag polynomial with unit roots. As it generates a model in the annual difference of consumption, it illustrates the rich possibilities of the chosen polynomial for modeling seasonally unadjusted consumption series.

Example 1 $\Phi(L)=1-aL$.

Substitution of $\Phi(L)=1-aL$ into (2.17) and (2.18) leads after some rearranging to

$$\varepsilon_{t+1} = (1-\frac{1}{1+r})(1-\frac{a}{1+r}) \sum_{i=0}^{\infty}(1+r)^{-i}[E(y_{t+i+1}|I_{t+1})-E(y_{t+i+1}|I_t)]$$

and

$$(1-aL)\Delta c_{t+1} = \gamma^{-1}\ln[\beta(1+r)] + \varepsilon_{t+1} \quad .$$

Example 2 $\Phi(L)=1+L+L^2+L^3$.

Noting that $(1+L+L^2+L^3)(1-L)=1-L^4$, we have for the consumption process

$$\Delta_4 c_{t+1} = \gamma^{-1}\ln[\beta(1+r)] + \varepsilon_{t+1} \ ,$$

where ε_{t+1} is equal to

$$\varepsilon_{t+1} = (1 - \frac{1}{(1+r)^4}) \sum_{i=0}^{\infty}(1+r)^{-i}[E(y_{t+i+1}|I_{t+1})-E(y_{t+i+1}|I_t)] \qquad (2.19)$$

as can be verified by substitution of $\Phi(L)=1+L+L^2+L^3$ into (2.18).

Box and Jenkins (1970) advocate the use of the Δ_4-filter to achieve stationarity of quarterly seasonally unadjusted series. In many studies it has proven to be an effective way to eliminate the seasonal fluctuations. The consumption function of Davidson, Hendry, Srba and Yeo (1978) is an illustrative example. Example 2 illustrates how a model in annual differences may be obtained by an appropriate choice of the lag polynomial $\Phi(L)$. Hendry and Von Ungern Sternberg (1981) however, present estimation results which show that the use of the Δ_4-operator may not be sufficient. They find significant coefficients for the seasonal dummy variables. Hansen and Singleton (1983) discuss the possiblity of a preference structure which is liable to shocks. A deterministic seasonal pattern may be incorporated in the maximization problem (2.1) and be interpreted as "seasonal shocks to the preferences" or, equivalently, as taste shifters. More particularly, when we assume that the consumer solves every time period t the optimization problem

$$\text{Max} \quad \sum_{i=0}^{\infty} \beta^{i} U(\Phi(L)c_{t+i} - s_{t+i})$$

$$\text{S.T.} \quad \sum_{i=0}^{\infty} (1+r)^{-i} c_{t+i} = (1+r)a_{t-1} + \sum_{i=0}^{\infty} (1+r)^{-i} E(y_{t+i}|I_t)$$

it can be shown that with the utility function (2.8) the following consumption model results

$$\Phi(L)\Delta c_{t+1} = \gamma^{-1}\ln[\beta(1+r)] + s_{t+1} - s_t + \varepsilon_{t+1} \tag{2.20}$$

with ε_{t+1} given by (2.18). When $(1+L+L^2+L^3)s_{t+i}=s^0$, that is the sum over four subsequent quarters is equal to s^0, the consumption model (2.20) displays a deterministic seasonal pattern. Notice that in the model without habits, that is $\Phi(L)=1$, and $s^0>0$, the interpretation of the seasonal component as "subsistence" or "necessary" consumption is straightforward. Instead of deriving satisfaction from total consumption, the consumer is assumed to attach utility to consumption <u>in excess</u> of the necessary seasonal component of total consumption. Expression (2.20) shows that in that case we have a seasonal random walk with drift. This model has been found to fit many economic series remarkably well, see e.g. Pierce (1978). For the lag polynomial used in example 2, the result corresponding to (2.20) is

$$\Delta_4 c_{t+1} = \gamma^{-1}\ln[\beta(1+r)] + s_{t+1} - s_t + \varepsilon_{t+1}$$

with ε_{t+1} given by (2.19). This illustrates how a model in annual differences with a deterministic seasonal pattern may be obtained.

In the models discussed above the consumer is assumed to use a finite memory with respect to past realizations of consumption. Pollak (1970) mentions the possibility of a preference structure that depends on an infinite number of past realizations of consumption and the utility function in (2.1) may be generalized by replacing the utility function argument $\Phi(L)c_{t+i}$ by $\Phi(L)\Theta(L)^{-1}c_{t+i}$, where $\Theta(L)$ is a finite order lag polynomial. It seems reasonable to impose the additional restriction that the roots of $\Theta(L)=0$ lie outside the unit circle. By this restriction we are assured that the consumer attaches declining weights to the very past of consumption. When we carry out the same operations as before, the first order conditions yield

$$\Theta(L)^{-1}\Phi(L)(1-L)c_{t+i} = \gamma^{-1}\ln[\beta(1+r)] \ . \tag{2.21}$$

The only difference between (2.21) and its "finite memory" counterpart (2.9) is that we have $\Theta(L)^{-1}\Phi(L)$ instead of $\Phi(L)$. Solving the model for period t and period t+1 leads to the ultimate result

$$\Phi(L)(1-L)c_{t+1} = \Theta(L)\gamma^{-1}\ln[\beta(1+r)] + \Theta(L)\varepsilon_{t+1} \tag{2.22}$$

where ε_{t+1} is given by (2.18). Hence, consumption will follow an arbitrary invertible ARIMA process.

The model with infinite memory may be used to describe consumption behaviour with respect to durable goods with an infinite life time. When we assume that the stock of durable goods K_{t+i} evolves according to

$$K_{t+i} = (1-\delta)K_{t+i-1} + c_{t+i} \quad ,$$

we have for the service flows s_{t+i}

$$s_{t+i} = \theta K_{t+i} = (1-(1-\delta)L)^{-1}\theta c_{t+i} \ .$$

Hence, s_{t+i} depends on the acquisition of durable goods infinitely far into the past. Given an intertemporally additive utility function with arguments s_{t+i}, the resulting model is a special case of (2.22).

In line with the analysis of Hall (1978) the life cycle model has been extensively tested by examining the predictive power of information of the past. The discussion of the model in this section illustrates that checking the significance of past realizations of consumption is not so much a test of the life cycle model as a test of the specific form of rational habit formation. Many authors (see e.g. King (1983)) have stressed that an empirical analysis of the life cycle theory tests the joint hypothesis of the life cycle model and the chosen functional form of the utility function. Obviously, the results of this section suggest that ignoring habits may explain the frequent rejection of the life cycle hypothesis. We may ask the question whether one <u>can</u> infer from an empirical analysis that the life cycle model is inappropriate. King (1983) raises this issue and notices the problem of determining the distinctive characteristics of the life cycle model and the absence of a coherent alternative model with which it can be compared. In the next section we will test the life cycle model by examining the implied relationships between the consumption and income processes and we will argue that the life cycle hypothesis studied in this section is not in full agreement with the information in the data for the Netherlands.

Section 3 Empirical results.

This section asks whether data series on consumption for the Netherlands suggest the presence of habits. Palm and Winder (1987a) investigate the life cycle model without habit formation and test the model using seasonally adjusted quarterly data on total and on nondurable consumption per capita. The empirical evidence is consistent with the hypothesis of absence of habits.

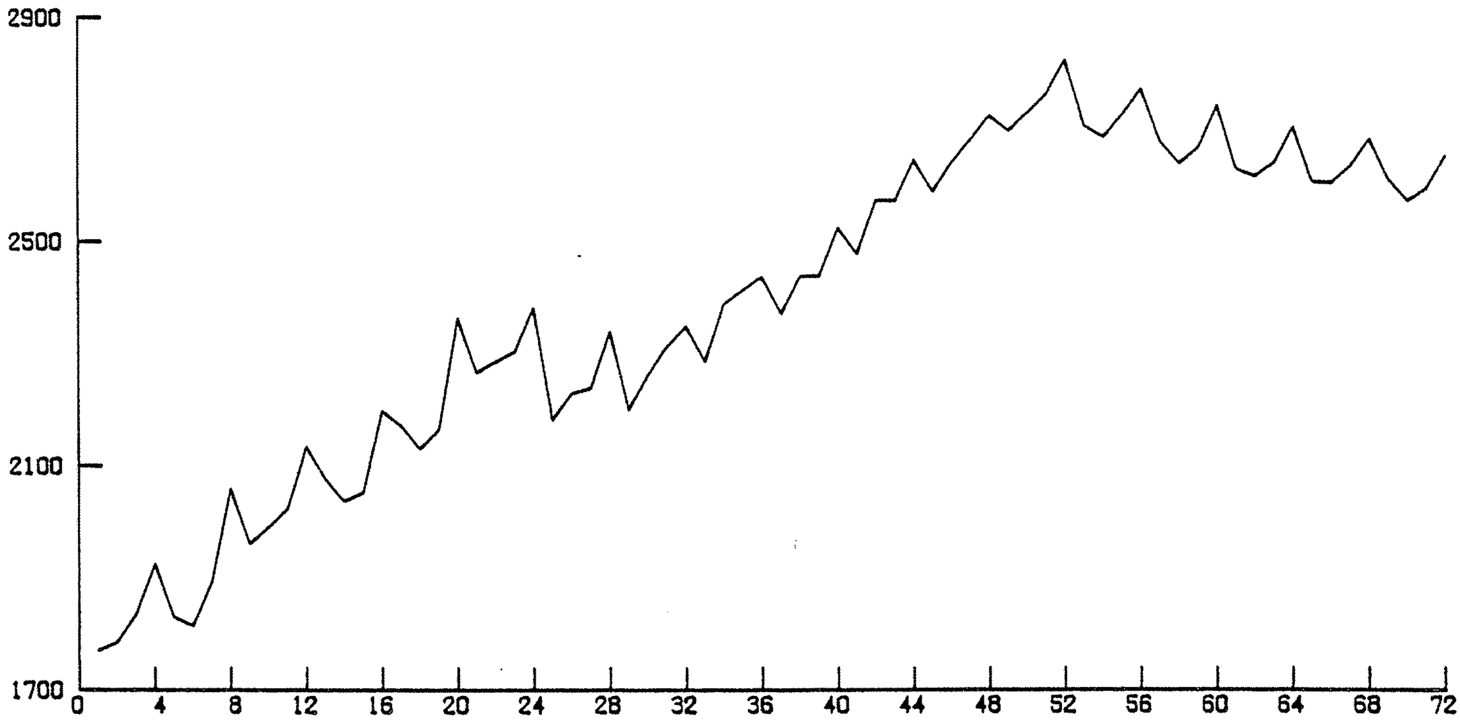
Palm and Winder (1987b) study the stochastic life cycle model without habits and examine the model using seasonally unadjusted quarterly data on nondurable consumption per capita. The seasonality is modeled jointly with the dynamics implied by the life cycle model. They assume that observed consumption may be decomposed in a life cycle component and a seasonal component for which they postulate that the sum of four subsequent quarters is white noise. In their ultimate model, $\Delta_4 c_t$ follows a restricted MA-process of order 3. In the empirical analysis the model passes the tests and provides a satisfactory description of the serial correlation properties of the data. In the light of the analysis of section 2, it should be clear that the possibility of alternative stochastic processes that are data coherent and theory consistent can not be excluded. Therefore we have chosen for a new examination of that series.

As the stochastic behaviour of consumption is implied by both the intertemporal maximization problem and the stochastic process of income, it is natural to start the analysis by examining the income process. Unfortunately, we do not have quarterly seasonally unadjusted data on labor and transfer income at our disposal. Therefore, we decided to use the same income series as Palm and Winder (1987a). Although we would have preferred to use a seasonally unadjusted series, the use of an adjusted series is not prohibitive since the rational consumer is capable to anticipate on and incorporate in his consumption decision the seasonal fluctuations of income (see also Miron (1986)). A short decription of the data is given in Appendix A. The data series used are given in figures 1 and 2.

Inspection of figure 1 clearly shows that the change in income is not stationary. In Palm and Winder (1987a) an extensive analysis of the series is carried out. For a detailed discussion we refer to that paper. Here we confine ourselves to reproducing the Maximum Likelihood (ML) estimates of

Fig.1 Real labor and transfer income per capita in the Netherlands, 1968(1)-1984(4).

Fig.2 Real nondurable consumption per capita in the Netherlands, 1967(1)-1984(4).



the income process:

$$\Delta y_t = \underset{(7.81)}{40.46}d_{1t} + \underset{(8.56)}{25.19}d_{2t} - \underset{(3.81)}{13.01}d_{3t} + \nu_t - \underset{(3.72)}{.428}\nu_{t-1} \tag{3.1}$$
$$\sigma^2_\nu = 809.6$$

where d_{1t}=1 for 1968(2)-1970(4)

d_{2t}=1 for 1971(1)-1978(4)

d_{3t}=1 for 1979(1)-1984(4)

and t-values are reported between parentheses. In Table 1 the values of a number of test statistics are given.

Table 1 Test statistics for model (3.1).

p	BP	LB
4	1.03	1.22
8	2.98	3.40
12	5.38	6.37
16	5.66	6.75
$\eta(1)$	.15	
$\eta(4)$	3.22	
S_1	.26	
S_2	.07	

Inspection of the residuals does not reveal any significant correlation. We find three outliers for 1974(2), 1978(4) and 1982(1). The Box-Pierce (BP) and the Ljung-Box (LB) test statistic based on s residual autocorrelations has been computed for s=4,8,12 and 16. A Lagrange Multiplier (LM) test has been carried out for the null hypothesis that ν_t in (3.1) has a constant variance against the alternative hypothesis that the disturbance has an autoregressive conditional heteroscedastic (ARCH) (see Engle (1982) and Weiss (1984)) structure of order 1 and 4 respectively. The values are reported in table 1 as $\eta(1)$ and $\eta(4)$. Finally, the normality has been checked using the test statistics put forward by Lomnicki (1961). When we define

$$m_j = T^{-1}\sum_{t=1}^{T}\nu_t^j,\ j=2,3,4 \text{ and } G_1=m_3m_2^{-3/2}\ ,\ G_2=m_4m_2^{-2}-3,$$

then if ν_t is Gaussian and stationary, for large T, both G_1 and G_2 are normally distributed with zero means and variances that depend on the autocorrelations of ν_t. The values of $S_1 = G_1/\sqrt{\text{var}G_1}$ and $S_2 = G_2/\sqrt{\text{var}G_2}$, based on the first 36 autocorrelations are given in Table 1. All test statistics yield insignificant values and we conclude that specification (3.1) with the normality assumption of ν_t provides a fairly good description of the income process.

Inspection of figure 2 immediately reveals that the consumption process is not stationary. In the first stage we investigate the correlation structure of the annual difference of consumption over the subperiods 1967(1)-1979(4) and 1980(1)-1984(4). The autocorrelation function (ACF) suggests that an AR(1)-process for $\Delta_4 c_t$ might be compatible with the information in the data. Therefore, we chose for the lag polynomial $\Phi(L)$ in (2.1)

$$\Phi(L) = (1 - aL)(1 + L + L^2 + L^3) \ . \tag{3.2}$$

Substituting (3.2) into (2.17) and (2.18) yields after some rearrangements

$$(1 - aL)\Delta_4 c_t = \gamma^{-1}\ln[\beta(1+r)] + \varepsilon_t \tag{3.3}$$

with

$$\varepsilon_t = (1-\frac{a}{1+r})(1 - \frac{1}{(1+r)^4}) \sum_{i=0}^{\infty}(1+r)^{-i}[E(y_{t+i}|I_t)-E(y_{t+i}|I_{t-1})] \ . \tag{3.4}$$

When we assume that the change in income is generated by a stationary and invertible ARMA process it is straightforward to show that the consumption innovation is a linear transformation of the income innovation. The proportionality factor depends not only on the parameters of the income process but also on those that reflect the impact of rational habits. Notice that the model does not imply that consumption is smoother than income. For the life cycle model without habit formation, it can be easily shown that the variance of the consumption innovation may be larger as well as smaller than that of the income innovation (see also Deaton (1985) and Campbell and Deaton (1987)). This property remains valid in the more comprehensive framework investigated here.

The drift parameter of the consumption process (3.3) depends only on parameters that characterize consumer behaviour and the change of the slope of the consumption line from positive to negative is not in accordance with the theoretical model. It can only be interpreted within this framework by the assumption of a structural change of the parameters. In line with Palm and Winder (1987a) we assume that the preference parameter β has changed as a result of the increased uncertainty about the future. The consequences of a decrease of β to $\beta*$ can be traced by using the closed form solutions (2.15) and (2.16). They are a persistent downward adjustment of the drift parameter in (3.3) after an increase of the drift parameter in the current period of size $\gamma^{-1}r^{-1}\ln[\beta\beta*^{-1}]$.

Under the assumption that the changes in the drift parameter of the income process were not anticipated, the model for consumption (3.3) needs revision. Let us assume that the constant term δ moves to $\delta*$. Using expressions (2.15) and (2.16) it can be shown that it will give rise to a step change in the consumption model (3.3) equal to $(\delta*-\delta)(1+r-a)(1-(1+r)^{-4})(1+r)r^{-2}$. Therefore, both in 1971(1) and 1979(1) we should expect a negative adjustment in the drift parameter of consumption. A similar mechanism was found in Palm and Winder (1987a). For the model without habits, the perturbation takes the form of an innovational outlier. Notice that as the underlying time series in in that case a random walk the innovational outlier is equivalent to a level change (see e.g. Tsay (1988) and Box and Tiao (1965)). The disturbance in the model with habit formation investigated here implies on the other hand a gradual response before the permanent change is reached. Obviously, this mechanism reflects the role of habits. Surprisingly, the consequences for the stochastic process of consumption are in both cases the same: introduction of one dummy variable obviates the problem.

The following estimation equation is in accordance with the theoretical model and the empirical findings for the income process

$$\Delta_4 c_t = \underset{\substack{(7.32)\\ [6.99]}}{.651}\Delta_4 c_{t-1} + \underset{\substack{(2.92)\\ [2.57]}}{25.85}d_{1t} - \underset{\substack{(1.68)\\ [2.84]}}{14.50}d_{2t} + \underset{\substack{(.72)\\ [4.17]}}{27.52}d_{3t} + \underset{\substack{(.83)\\ [4.94]}}{31.42}d_{4t} + \underset{\substack{(.56)\\ [3.33]}}{21.12}d_{5t}$$

$$\sigma^2_\varepsilon = 1413.6 \qquad (3.5)$$

where $d_{1t}=1$ for 1967(2)-1979(4)

$d_{2t}=1$ for 1980(1)-1984(4)

$d_{3t}=1$ for 1971(1)

d_{4t}=1 for 1979(1)

d_{5t}=1 for 1979(4).

The dummy variables d_{3t} and d_{4t} are included as a result of the structural changes in the income process, whereas d_{2t} and d_{5t} emerge because of the change in the time preference parameter which is timed at the turning point of the consumption series. The values between parentheses are the t-values calculated in the conventional way and those reported between square brackets correspond to the t-ratios calculated as in White (1980) (see also Domowitz and White (1982) and Bierens (1984)). The latter are robust with respect to any form of heteroscedasticity.

The residuals of the model have been analyzed. They do not exhibit significant serial correlation. The ACF takes only significant values for r_{16} and r_{17} and the BP and LB test statistics yield values that are insignificant at commonly used significance levels. The observations for 1972(4) and 1973(1) exceed twice the standard error of the residuals in magnitude. Before we saw that normality and homoscedasticity of the process for Δy_t are not rejected. The theoretical implications are that $\Delta_4 c_t$ follows a normally distributed homoscedastic process. Inspection of the values reported in Table 2 shows that the empirical findings are not in agreement with the theory. In particular, the significant values of the LM test statistic for the hypothesis of homoscedasticity against the alternative hypothesis of an ARCH structure is in contradiction with the empirical results for the income process. In Palm and Winder (1987a) an economic argument is given for the plausiblity of the appearance of ARCH processes. They argue that when we are prepared to relax the assumption of fully rational expectations, we may find consumption innovations that can be modeled as an ARCH process although the income innovations are homoscedastic. We have seen that a structural change in the income series leads to the inclusion of a dummy variable in the consumption model. When the consumer incorrectly assesses a shift in the income process, he will become aware of this after a while, and adjust his consumption level accordingly. This will lead to an outlier which can be modeled by a dummy variable. A nice feature of ARCH processes is that they can handle outliers arising in clusters. The significant values of the LM test statistics may be interpreted as a confirmation of temporarily incorrect assessment of the expected value of future income by the consumer.

However, when we stick to the assumption of rational expectations there exists a one to one correspondence between the stochastic properties of the two series. In that case we have to judge the significant values of the LM test for consumption as being in contradiction with the properties of the process for income.

As the regressors include a lagged dependent variable, the presence of heteroscedasticity of an ARCH type implies that ordinary least squares (OLS) will no longer give correct standard errors (see e.g. Weiss (1984)). The consistency of the OLS estimates is however not affected. The reported t-values in (3.5) illustrate that ignoration of the heteroscedasticity may lead to incorrect inference. As the presence of ARCH structures jeopardizes the validity of the BP and LB test statistics, we have also carried out a test for serial correlation in the residuals put forward by Bierens (1984). When the data generating process is strictly stationary, this test is consistent with respect to any deviation from the null hypothesis. The values of the simplified form of the test statistics for the null hypothesis that the errors are martingale differences against the alternative hypothesis that the null is false, are reported in Table 2 as $\tau(L_n, \varepsilon)$, where L_n and ε are chosen in line with Bierens' simulation results. For details we refer to Bierens (1984, sections 7 and 8). Obviously, the results indicate no deviation from the hypothesis of zero residual serial correlation.

Table 2 Test statistics of model (3.5)

p	BP	LB
4	4.03	4.22
8	10.10	10.56
12	11.57	12.10
16	18.71	19.56
$\eta(1)$	8.97	
$\eta(4)$	9.72	
S_1	-.26	
S_2	.27	
$\tau(20,.5)$	100.83	
$\tau(20,1.5)$	3.54	

We proceed by examining the point estimates. Substitution of $y_t - E(y_t|I_{t-1})=\nu_t$ and $E(y_{t+i}|I_t)-E(y_{t+i}|I_{t-1})=(1-\theta)\nu_t$, $i\geq 2$ into (3.4) leads to

$$\varepsilon_t = (1+r-a)(1 - \frac{1}{(1+r)^4})(1 - \frac{\theta}{1+r}) \frac{1}{r} \nu_t$$

and hence

$$\sigma^2(\varepsilon_t) = (1+r-a)^2(1 - \frac{1}{(1+r)^4})^2(1 - \frac{\theta}{1+r})^2 \frac{1}{r^2}\sigma^2(\nu_t) \; . \qquad (3.6)$$

With the point estimates $\hat{\theta}=.428$ and $\hat{a}=.650$, expression (3.6) shows that the variance of the consumption innovation may be smaller as well as larger than the variance of the income innovation. For small values of r the proportionality factor is smaller than 1. With r=.05 we find for instance .71. Hence, it seems reasonable to expect the variance of the consumption innovation to be smaller than that of the income innovation. Comparison of the reported values in (3.1) and (3.5) shows that the estimates contradict this implication. When the appropriate income series is the seasonally unadjusted one, a plausible explanation might be that the method of seasonal adjustment has led to a smoothed series. Consequently, the relevant residual variance would be larger than the value given in (3.1).

Notice that the appearance of an ARCH process does not obviate the contradiction. Taking into account the effects of an ARCH structure will increase the variance of the consumption innovation (see Engle (1982), theorem 2). For the appraisal of the step changes, we have to keep in mind that the dummy variables absorb the joint effect of both the adjustment of the consumption level and the transformed income innovation. From (3.1) we have an estimate of the income innovation and the MA-parameter θ from which we can infer a negative sign of the coefficient of d_{3t} and a positive one of d_{5t}. This implication is confirmed for d_{5t}, but violated for d_{3t}. Because of the opposite sign of the adjustment of the constant term and the estimate of the transformed income innovation, we can not determine a priori the sign of the coefficient of d_{4t}. With respect to the evaluation of the size and sign of the parameter estimates it should be remarked that the analysis is highly tentative. Apart from the fact that we use the point estimates of a, θ, $\sigma^2(\nu)$ and the income innovation, a reinterpretation of the formulae is required as we estimate the model from aggregate per capita data.

Finally, notice that Davidson and Hendry (1981) investigate the log-linear version of model (3.5), which they present as the analogue of Hall's (1978) consumption function. Their analysis tempted Hall (1981) to comment

> "I found their tests unconvincing because of their treatment of seasonality".

The foregoing analysis suggests that their model is not the analogue of Hall's model, but also that their specification is not necessarily incompatible with the life cycle theory extended for the presence of habits.

Section 4 Summary and concluding remarks.

In this paper we considered the life cycle model under rational habit formation. It was shown that for the exponential utility function an arbitrary ARIMA process for consumption is obtained by choosing an appropriate pattern of rational habits. We made the assumption that the planning time span is infinite. Winder (1987) investigates the life cycle model under rational habit formation in which the consumer uses a finite planning horizon and shows that the only difference with the result obtained in section 2 is that the drift parameter and the variance of the ARIMA process of consumption become age/time dependent.

The empirical analysis carried out in section 3 illustrated how the life cycle model extended for the presence of habits may be used to describe data on quarterly seasonally unadjusted consumption. Special attention was paid to the implications of structural changes in the income process, which because of replanning, will have an impact on the process of consumption. For the model with infinite planning horizon investigated in section 3, it was shown that the only adjustment consisted of the inclusion of one dummy variable in the consumption model. This illustrates the gradual adaptation of the consumption level to the new perspectives, which is of course a result of the presence of habits. We conclude that the framework of intertemporal maximization is an appropriate one for interpreting outliers in the consumption process. Moreover, the relationship with the appearance of structural changes in the income process may be of some use in detecting the time point of the occurrence of a structural break. A preliminary analysis along the lines of Tsay (1988) is expected to yield useful information in this respect. As we used the income process specified in Palm and Winder (1987a), we refrained from this possibility.

The observation that the stochastic process of consumption is a transformation of that of income, led to the examination of a number of implications of the theoretical model. As the empirically observed heteroscedasticity of the ARCH type in the consumption process is in contradiction with the homoscedasticity of the income process, we concluded that the model was not in full agreement with the information in the data. The analysis illustrated the importance of an examination of the stochastic properties of consumption in relation with those of income. When we would have confined ourselves to the examination of the Euler equations, the incompatibility would not have come to light. As a possible explanation of the observed inconsistency we suggested a relaxation of the rational expectations assumption. An alternative explanation might be the inappropriateness of the income series. We used a seasonally adjusted series, and it seems not imaginary that the used adjustment method has eliminated the heteroscedasticity. When the stochastic process of the seasonally unadjusted income series exhibits heteroscedasticity of the ARCH-type, the empirical results of section 3 are not necessarily at variance with the theoretical model. Unfortunately, the lack of seasonally unadjusted income data hampers a further analysis. Obviously, the interpretation of the inconsistency as an indication of some kind of misspecification can not be ignored, too.

The principal implication of the life cycle model is the complete separation of the life time consumption and income profiles. Consequently, the dynamics of consumption is basically determined by the preference structure. This observation reveals that structural changes in the ARMA parameters of the consumption process can only be interpreted within this framework by the assumption of a change in the preference structure. Using the property that the consumption innovation is a transformation of the income innovation, it becomes clear that structural changes in the income process will affect <u>persistently</u> only the properties of the consumption innovation. It may be desirable however to establish a more direct link between the consumption and income processes. For the series investigated in section 3 for instance, one may want to relate the change in the drift of the consumption process to a change in the drift of the income process. The model with moving planning horizon put forward by Palm and Winder (1987a) furnishes such a link. In the model with moving planning horizon the consumer is assumed to solve an intertemporal decision problem in which a planning time span is used that does not coincide with the expected life time. When the time horizon deviates from the life time, a mechanism that describes the adjustment of the planning horizon as time goes on has to be introduced in the model. Palm and Winder (1987a) assume that the consumer uses a planning time span of constant length. Surprisingly, as a result of adjusting the planning horizon an error correction term has to be include in the consumption function. Winder (1987) investigates the model with moving planning horizon for the special form of rational habit formation that yields a model in the four period difference operator and shows that when the annual change in income is generated by an autoregressive process of order 1, the model leads to a relationship between income and consumption that is identical to the mechanism underlying the consumption function of Davidson, Hendry, Srba and Yeo (1978). More specifically, in each quarter of a year the consumer spends the same as he spent the corresponding quarter of the previous year, modified by a proportion of the annual change in income and of the change of the annual change in income and by the error correction term.

Davidson and Hendry (1981) among others have stressed the (almost) observational equivalence of models based on forward looking behavior and those based on feedback control rules. The models studied in this paper and Winder (1987) provide a new illustration of this observation. The only possibility to discriminate between these two interpretations seems to occur when structural breaks appear in the forcing variables. When the agents display full capacity of anticipatory behavior, the model for consumption differs from that of an agent who bases his decision on a feedback rule. The empirical analysis carried out in section 3 shows that the dummy variables are included to capture the effects of perturbations of future perspectives. In applied work it will usually be difficult to determine the moment and nature of the structural changes. Thus, the lack of experimental data may hinder a thorough examination. Notice that the life cycle model extended for the presence of habits effectively establishes a synthesis between forward looking and backward looking behaviour. In making his decision, the consumer is assumed to incorporate information on expected <u>future</u> labor income and information consisting of <u>past</u> realizations of consumption.

Finally, it should be remarked that the question what habits and seasonality are remains unanswered. Muellbauer (1986) raises this issue and concludes that the use of aggregate data is unlikely to help very much in distinguishing exactly what habits represent. However, some kind of behavioral persistence seems not unreasonably and in this paper we have shown how this may be incorporated in the preference structure of an economic agent.

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Appendix A Sources of the data.

The quarterly series on nondurable consumption per capita in prices of 1980 for the period 1967(1)-1984(4) has been computed as the sum of consumption expenditures per capita on food, beverages, services and other nondurables. Monthly indices on these series are published in Centraal Bureau voor de Statistiek, Maandstatistiek Binnenlandse Handel en Dienstverlening, Staatuitgeverij, 's Gravenhage. Annual figures on expenditures which are published in Centraal Bureau voor de Statistiek, Nationale Rekeningen, Staatsuitgeverij, 's Gravenhage, have been used to transform the indices into monthly expenditures per capita expressed in prices of 1980. The monthly figures have been aggregated into quarterly data. The observations on consumption in the first and fourth quarter of 1975 are replaced by an average of the corresponding quarters in 1974 and 1976. In Centraal Plan Bureau, Centraal Economisch Plan 1976, Staatsuitgeverij, 's Gravenhage, the irregular behavior in 1975 is explained as an advance of sales in the first quarter from the second and third quarters. The high level of consumption in the fourth quarter is due to an increase of sales as a result of a change in the excise tax at the beginning of 1976.

Quarterly data on labor and transfer income for 1968(1)-1984(4) have been kindly provided by the Centraal Plan Bureau. To obtain per capita figures in 1980 prices, the nominal series has been deflated by the price index of total consumption and has been divided by the size of the population.

DYNAMIC SPECIFICATION OF CONSUMER EXPENDITURE ON NONDURABLES AND SERVICES IN AUSTRIA

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Recent developments in econometrics (concept of co-integration, dynamic specification) paired with an increasing supply of the corresponding computer software have led to a renewed interest in one of the most thoroughly researched topics in quantitative economics, namely the relationship between consumer expenditure and disposable income. Jäger and Neusser (1987a,b) were the first to apply these new concepts to Austrian consumption data.

1. THE DATA

It should be mentioned right at the beginning that, throughout the paper, we are only concerned with consumer expenditure on nondurables and services. Purchases of durables and cars have more investment character and should be modelled therefore accordingly. Figure 1 shows the graphs of real consumer expenditure and real disposable income in billions of AS from 1957:1 to 1986:4. The salient features of the data are the strong trends in both consumer expenditure and disposable income, and the magnitude and stability (although the seasonal shape has tended to become somewhat "elongated" over time) of the seasonal pattern in consumer expenditure. As shown in figure 2, the average propensity to consume has fallen steadily over time from around 0.9 to 0.8.

Figure 1

Real Consumer Expenditure and Real Disposable Income

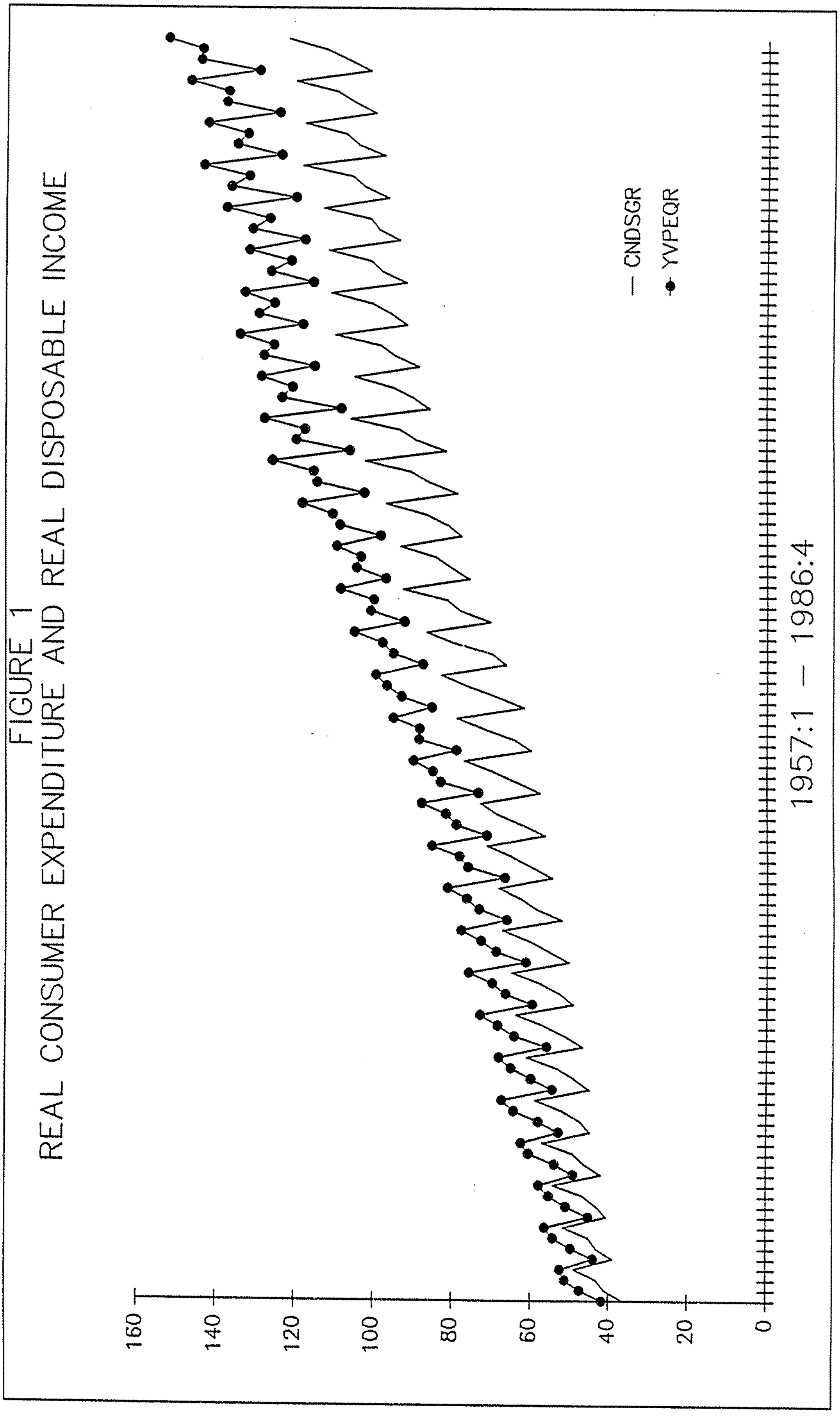
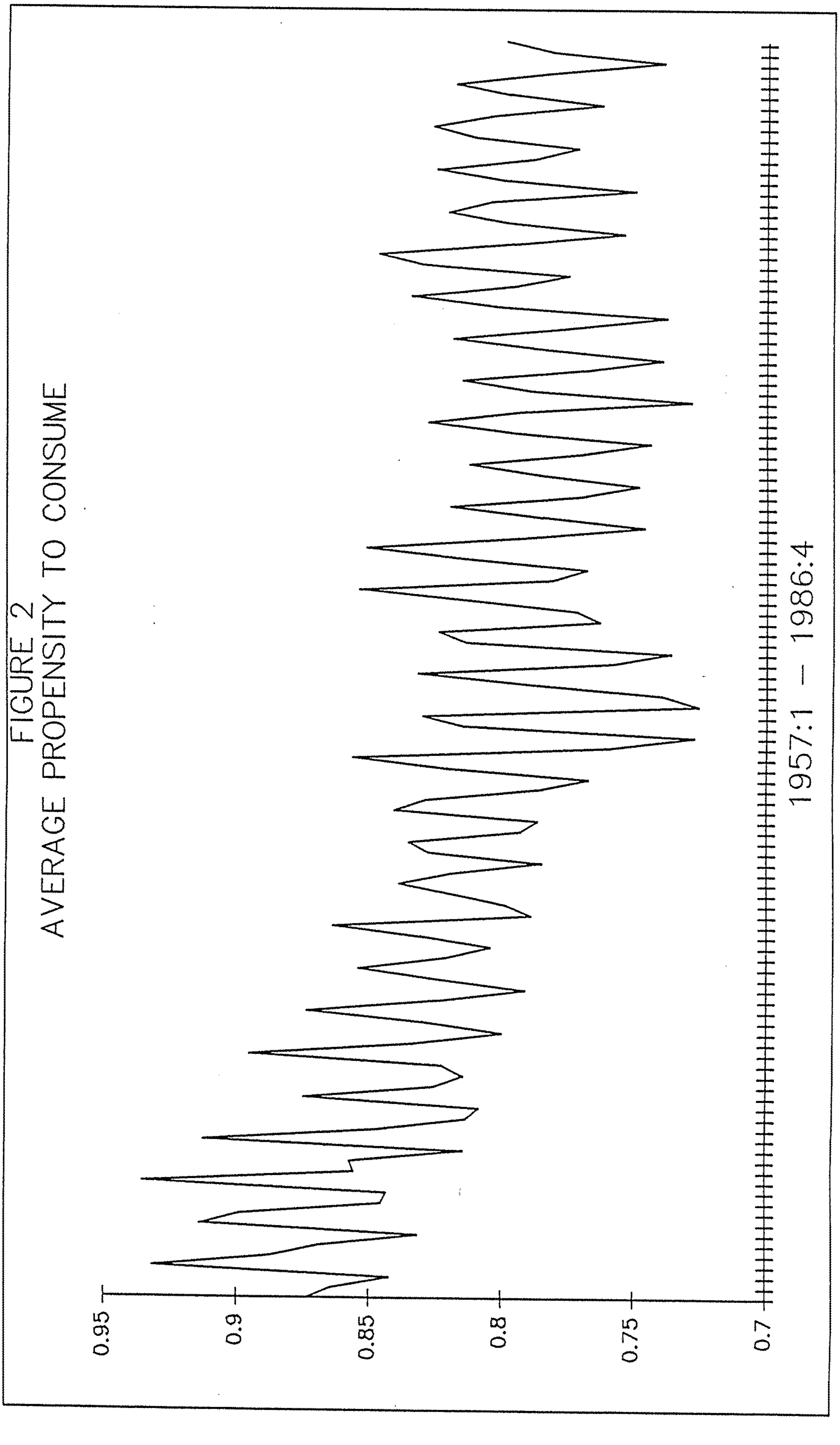


Figure 2

Average Propensity to Consume



2. TESTING FOR STATIONARITY AND CO-INTEGRATION

Visual inspection of the data may be quite helpful at the outset of an analysis, but it should be replaced by formal testing in due course. Near-stationarity of the time series under analysis is an indispensable prerequisite of any successful research. In order to test for stationarity, the socalled Dickey-Fuller tests are generally applied in the literature. For a discussion of these tests see Fuller (1976). It must be mentioned, however, that these tests are not unquestioned in the profession (see, for example, Dolado and Jenkinson (1987)). We believe that they can at least provide some evidence whether a time series is stationary or not.

Given all the problems connected with these tests, we decided to compute them for two types of series: seasonally unadjusted and seasonally adjusted (1). It is namely far from clear how one should proceed in the case of seasonal data. For stationarity and co-integration tests, the use of seasonally adjusted data seems to be preferable. The test procedures were developped originally for this data type and, above all, seem to be extremely sensitive to the addition of further variables to the underlying regression equations. And one must include seasonal dummies when one is working with seasonally unadjusted data. On the other hand, the use of seasonally unadjusted data is often preferable for modelling economic time series. It is well known in the literature that seasonal adjustment might distort the underlying economic relationships (Wallis (1974)).

Table 1

Stationarity Tests for Consumer Expenditure and Disposable Income

Variable	Dickey-Fuller	Augmented Dickey-Fuller
Consumer Expenditure		
(i) seasonally unadjusted		
levels	-.1914	.2871
first differences	-17.9203	-5.5128
(ii) seasonally adjusted		
levels	-.1872	.0625
first differences	-17.5668	-6.9187
Disposable Income		
(i) seasonally unadjusted		
levels	-.3089	.0007
first differences	-38.1524	-5.1397
(ii) seasonally adjusted		
levels	-.1273	.1669
first differences	-17.9190	-5.4806
5 percent critical values (2):	-3.37	-3.17

The outcome of our Dickey-Fuller tests is presented as table 1. Fortunately, the message of this table seems to be uncontroversial, irrespective of the data type. Consumer expenditure and disposable income are definitely non-stationary in levels, but stationary in first differences. Or, more formally, both time series are integrated of order 1, generally denoted by I(1) in the relevant literature. Thus, differences (perhaps of the logarithms in order to, additionally, stabilize their variance) of the raw data might constitute a promising starting point for the equation identification stage.

The next question is now whether there exists a linear combination of the above time series, which is integrated of order 0. If this is the case, consumer expenditure and disposable income are said to be co-integrated. Or, more formally, if two variables x_t and y_t are both I(d), then it is generally true that the linear combination $u_t = x_t - \delta y_t$ will be also I(d). However, it is possible that $u_t \sim I(d-b)$, $b>0$. The concept of co-integration was introduced by Granger (1981). For a more recent discussion and applications to economic time series see Engle and Granger (1987). It is shown there that the concepts of co-integration and error correction are closely related. If two series are co-integrated, a very special constraint operates on the long run components of these series. In the d=b=1 case, x_t, y_t are both I(1) with dominant long run components, but u_t is I(0) without an especially strong low frequency component. Consequently, the co-integrating constant δ must be such that the bulk of long run components of x_t and y_t cancel out. The economic interpretation is that there exists some form of equilibrium relationship between x_t and y_t. The existence of such an equilibrium relationship is now one of the basic assumptions of an error correction model (see, for example, Davidson, Hendry, Srba and Yeo (1978)).

The outcome of our tests for co-integration between consumer expenditure and disposable income is reported as table 2. Once again, we perform these tests for seasonally adjusted and seasonally unadjusted data. The 5 percent critical values for the Durbin-Watson (DW), Dickey-Fuller (DF) and Augmented Dickey-Fuller

Table 2

Co-Integrating Regressions for Consumer Expenditure, Disposable Income and Liquid Assets

Seasonally adjusted data

constant	6.301 (.378)	constant	7.448 (.812)
real income	.738 (.004)	real income	.718 (.013)
		real liquid assets	.008 (.005)
R^2 =	.997	R^2 =	.997
DW =	1.080	DW =	1.080
DF =	-6.581	DF =	-6.607
ADF =	-3.702	ADF =	-3.604

Seasonally unadjusted data

constant	4.768 (.603)	constant	10.536 (1.030)
real income	.739 (.006)	real income	.620 (.019)
		real liquid assets	.046 (.007)
R^2 =	.994	R^2 =	.995
DW =	1.270	DW =	1.350
DF =	-7.429	DF =	-7.732
ADF =	-2.475	ADF =	-6.838

5 percent critical values: .386 (DW) -3.37 (DF) -3.17 (ADF)
Numbers in parentheses are standard errors.

(ADF) statistics given in Engle and Granger (1987), who recalculated the tables of Fuller (1976), are 0.386, -3.37, and -3.17, respectively. For seasonally adjusted data, we can reject the null hypothesis of no co-integration, irrespective of the fact whether we include real liquid assets (defined as cumulated savings minus purchases of consumer durables and housing expenditure and deflated by the consumer price index) as additional regressor or not. For the seasonally unadjusted data, the empirical evidence is less clear. In the co-integrating regression of consumption on income, the ADF statistic does not allow a rejection of the null hypothesis of no co-integration. Jäger and Neusser (1978a) encounter a similar problem, although of a less severe nature (their ADF statistic is only slightly below the critical value). They reject the null by argueing that the DW and DF statistics clearly reject it, and that the ADF statistic only fails marginally. In our opinion, this line of reasoning is not fully convincing. The DF and the ADF statistics are not really independent. Either the one or the other is appropriate depending on the fact, whether the additional lagged terms in the ADF regression are significant or not. We rather believe that the whole problem is caused by the ommission of real liquid assets from the co-integrating regression. Inclusion of this variable leads immediately to a rejection of the null hypothesis of no co-integration.

3. ESTIMATION OF A CONSUMPTION EQUATION

The results of our above stationarity and co-integration tests tell us that a combination of differences and levels of the relevant basic variables should produce a satisfactory equation for expenditure on nondurables and services in Austria. Thus, an error correction model of the form proposed by Davidson et al. (1978) might be a good starting point for our further analysis.

Before we can actually start, however, several technical problems have to be solved. First of all, we must decide whether we intend to work with seasonally adjusted or seasonally unadjusted data.

The results of first tests with seasonally adjusted data were in no respect promising. They fully corroborate the often heard opinion that seasonal adjustment distorts the underlying relations between economic variables. Therefore, we decided to work with seasonally unadjusted data. The next question was then whether to take first or fourth (i.e. annual) differences. The graph of the first differences of consumer expenditure looks frightening. It is no surprise, therefore, that Jäger and Neusser (1987b) include 4 lags of the dependent variable as additional regressors into their estimated error correction model for the first differences of consumer expenditure. We must confess, however, that we consider this solution more as a treatment of symptoms than as a cure of the underlying problems. In the light of this evidence, we decided finally to work with annual differences. Additionally, we transformed the raw data of our basic variables into natural logarithms. We know from figure 2 above that average propensity to consume is far from constant. Consequently, a linear relation between the original series seems to be inadequate. A log-linear relation, however, might be consonant with the observed varying propensity to consume.

Our first attempts to estimate an error correction model for the annual differences of the logarithm of real expenditure on nondurables and services were far from being successful. Seen in the light of recent theoretical advances, this fact did not discourage us. It is shown in Hendry and von Ungern-Sternberg (1980) that the conventional ECM is incomplete. It incorporates, in the terminology of Phillips (1954 and 1957), derivative (in a consumption function usually the change in income) and proportional (the consumption/income ratio) control mechanisms, but no integral ($\Sigma_{j<t}C_j/Y_j$) control. In a consumption function, we would consequently observe cumulative underadjustment if income is steadily increasing or decreasing. If C_t is an expenditure and Y_t an accrual then some stock of assets, A_t, is implicitly altering and, for decreases in Y_t, is essential for financing the 'overspending'.

3.1 Integral Correction Mechanisms

Previous researchers have already included integral variables in consumption functions but, usually, in a rather ad-hoc fashion. Hendry and von Ungern-Sternberg (1980) offer a formal extension of the traditional error correction model to allow for integral correction mechanisms. Their starting point is the following one-period loss function, where c_t denotes expenditure, y_t income and a_t is the integral of past discrepancies between y and c (here and in the following, lower case letters denote $\log_e$ of corresponding capital letters):

$$lf_t = \theta_1(a_t^p - a_t^e)^2 + \theta_2(c_t^p - c_t^e)^2 + \theta_3(c_t^p - c_{t-1})^2 - 2\,\theta_4(c_t^p - c_{t-1})(y_t - y_{t-1}) \qquad (1)$$

The first two terms are the relative costs attached to discrepancies between planned values (c_t^p and a_t^p) and their respective steady-state outcomes (c_t^e and a_t^e). Further, to avoid 'bang-bang' control in response to random fluctuations, econmic agents attach costs to changing c_t^p from c_{t-1}. But, on the other hand, it is not reasonable to quadratically penalize changes in c_t^p when it is known that c_t^e has changed. Thus, the third term in (1) is an offset term which allows more adjustment at a given cost when c_t^e has changed. Optimization of such a loss function for expenditure on nondurables and services results in the following estimation equation:

$$\Delta_4 c_t = \beta_0 + \beta_1\,\Delta_4 y_t + \beta_2(c-y)_{t-1} + \beta_3\left(\sum_0^3 l_{t-j-1} - \sum_0^3 y_{t-j-1}\right) + u_t \qquad (2)$$

Here, c_t denotes the logarithm of consumer expenditure on nondurables and services in constant prices, y_t the logarithm of disposable income in constant prices and l_t the logarithm of liquid assets as defined above.

3.2 An Unrestricted AR-DL Formulation

Before we try to estimate (2) with Austrian data, it seems worthwhile to estimate an unrestricted autoregressive-distributed lag equation relating the basic variables of (2):

$$c_t = \sum_{i=0}^{n} (\alpha_i c_{t-i-1} + \beta_i y_{t-i} + \Gamma_i l_{t-i-1} + \delta_i Q_{it}) + \mu E_t + e_t \qquad (3)$$

By E_t, we denote here an Easter dummy, which indicates whether Easter falls into the first or second quarter of a year. For details see Thury (1987). Since we are working with quarterly data, we set n=3 for c, l and Q and n=4 for y. Such an unrestricted equation provides helpful information in several respects. It informs us about the significance of the different explanatory variables. It indicates the existence of common factors in the different lag polynomials. It tells us whether long run restrictions implied by economic theory are consistent with the data. And, finally, it constitutes a useful baseline against which to compare restricted equations. If the latter fit significantly worse, they cannot be considered as acceptable simplifications of the former.

The estimates obtained from (3) are reported in table 3. Clearly, some of the individual coefficients are badly determined, but itcan be seen that each basic variable is highly significant. The null hypothesis, that the static long run restrictions implied by economic theory are invalid, is rejected by the Wald test. All long run coefficients are significant. The COMFAC test, finally, indicates that there exist common factors in the polynomials of the different basic variables.

Table 3

Unrestricted Estimates of (3)

j	0	1	2	3	4	Σ
c_{t-j}	-1.000	.200	-.257	.033	.612	-.412
		(.112)	(.106)	(.107)	(.078)	(.104)
y_{t-j}	.297	.062	.022	-.056	.003	.328
	(.071)	(.082)	(.080)	(.082)	(.076)	(.099)
l_{t-j-1}	.	.030	-.108	.038	.054	.014
		(.0379	(.047)	(.047)	(.035)	(.006)
E_t	.307	.	.	.	.	.307
	(.063)					(.063)
Q_{jt}	.217	.272	.236	.265	.	.
	(.064)	(.062)	(.064)	(.062)		

R^2 = .998 σ% = 1.26 DW = 2.08

AR F(6,92) = 2.64 (2.20) ARCH F(6,86) = 1.19 (2.21)

Test on the Significance of each Basic Variable

c	F(4,98) = 39.373 (2.465)
y	F(5,98) = 6.796 (2.307)
l	F(4,98) = 4.534 (2.465)
E	F(1,98) = 24.081 (3.938)
Q_j	F(4,98) = 5.561 (2.465)

Static Long Run Restrictions

$$c = \underset{(.055)}{.797}\, y + \underset{(.017)}{.033}\, l + \underset{(.240)}{.745}\, E + \underset{(.241)}{.643}$$

Wald Test $Chi^2(4)$ = 405927.9

COMFAC Wald Tests
Summary Table

COMFAC Order	Chi^2 df	Value	Incremental Chi^2 df	Value
1	2	.063	2	.063
2	4	2.005	2	1.942
3	6	6.606	2	4.602

COMFAC F(6,98) = 1.11 (2.19)

3.3 An Error Correction Model with Integral Control

We turn now to the estimation of a parsimonious reparameterization of our above unrestricted equation. After lengthy and tediuos experimentation, we found that only an equation, which incorporates derivative ($\Delta_4 y_t$), proportional ($c_{t-4}-y_{t-4}$) and integral ($\Sigma_0 l_{t-i}-\Sigma_0 y_{t-i}$) control mechanisms, can be considered as adequate simplification of the general autoregressive-distributed lag model. Thus, we estimated the following equation

$$\begin{aligned}
\Delta_4 c_t = \underset{(.0519)}{.3170}\ \Delta_4 y_t - \underset{(.0133)}{.0564}\ y_{t-4} - \underset{(.0522)}{.2114}\ (c-y)_{t-4} \\
+ \underset{(.0050)}{.0116}\ (l-y)_{t-1} - \underset{(.0999)}{.2126}\ \Delta_1\Delta_4 p_t + \underset{(.0307)}{.1999}\ \Delta_4 E_t \\
+ \underset{(.0120)}{.0428}\ \Delta_4 D73{:}2_t + \underset{(.0120)}{.0388}\ \Delta_4 D73{:}4_t + \underset{(.0082)}{.0204}\ \Delta_4 D78{:}4_t \\
- \underset{(.0060)}{.0187}\ Q_{1t} - \underset{(.0063)}{.0217}\ Q_{2t} - \underset{(.0043)}{.0103}\ Q_{3t} + \underset{(.0574)}{.2456}
\end{aligned} \tag{4}$$

$T = 108 \quad R^2 = .7138 \quad \sigma\% = 1.1338 \quad DW = 1.76 \quad RHO(4) = \underset{(.1172)}{-.1852}$

$\text{CHOW } F(8,94) = \underset{(2.04)}{.30} \quad \text{Normality } Chi^2(2) = \underset{(5.99)}{.04} \quad \text{LM } F(6,88) = \underset{(2.20)}{.91}$

$\text{ARCH } F(6,82) = \underset{(2.21)}{.36} \quad X_i{*}X_j\ F(22,72) = \underset{(1.68)}{1.52} \quad X_i^2 F(21,72) = \underset{(1.70)}{.72}$

In (4), l and y denote $\sum_0^3 l_{t-i}$ and $\sum_0^3 y_{t-i}$, respectively, and p_t stands for the logarithm of the implicit consumption deflator. D73:2, D73:4 and D78:4 are meant to capture distortions which were caused by the introduction of VAT in 1973:1 and the change in VAT rates in 1978:1.

Comparing the equation standard errors, σ%, we see that (4) is indeed a fully acceptable simplification of the general autoregressive-distributed lag equation (3). The various diagnostic checks, given below (4) with their corresponding 5 percent critical values in parentheses, reveal no model distribution. The residuals are also free of autoregressive conditional heteroscedasticity as Engle's ARCH test demonstrates. The presence of heteroscedasticity due to missing squares of the regressors can also be rejected. Functional form mis-specification due to missing crossproducts of the regressors does not occur.

Further insight is provided by the following graphs. Figure 3 shows a time series graph of the scaled residuals from (4). Scaling is achieved by dividing the residual series by the equation standard error σ. There is no evidence of a recognizable cyclical pattern in the residual series, and only few values are in excess of two times the equation standard error. Figure 4, finally, presents a time series graph of the actual and fitted values obtained from (4). We see immediately that even the annual differences of quarterly consumer expenditure still show a considerable amount of erratic variation. For such a series, the tracking performance of (4) seems to be quite satisfactory.

Figure 3

Scaled Residuals

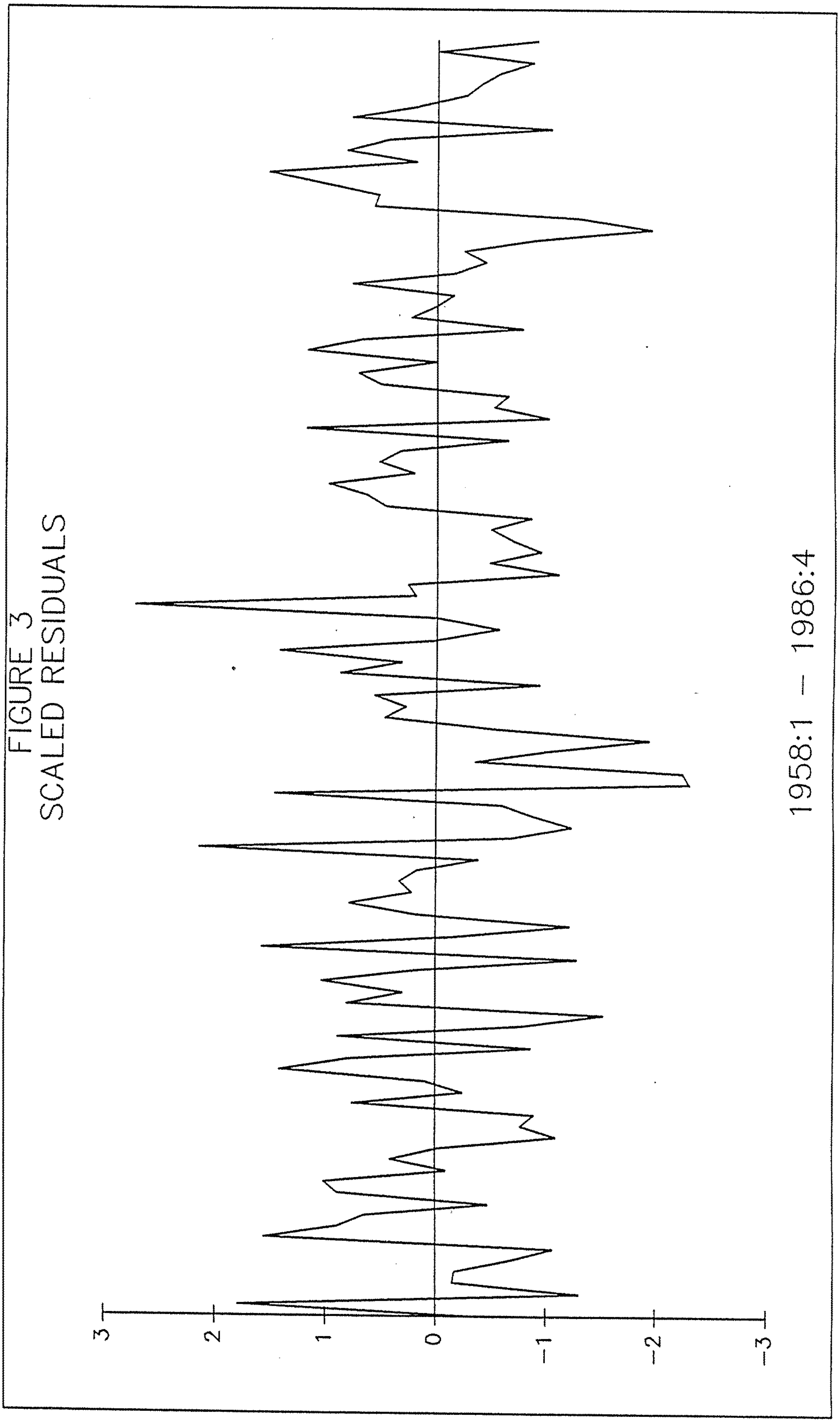
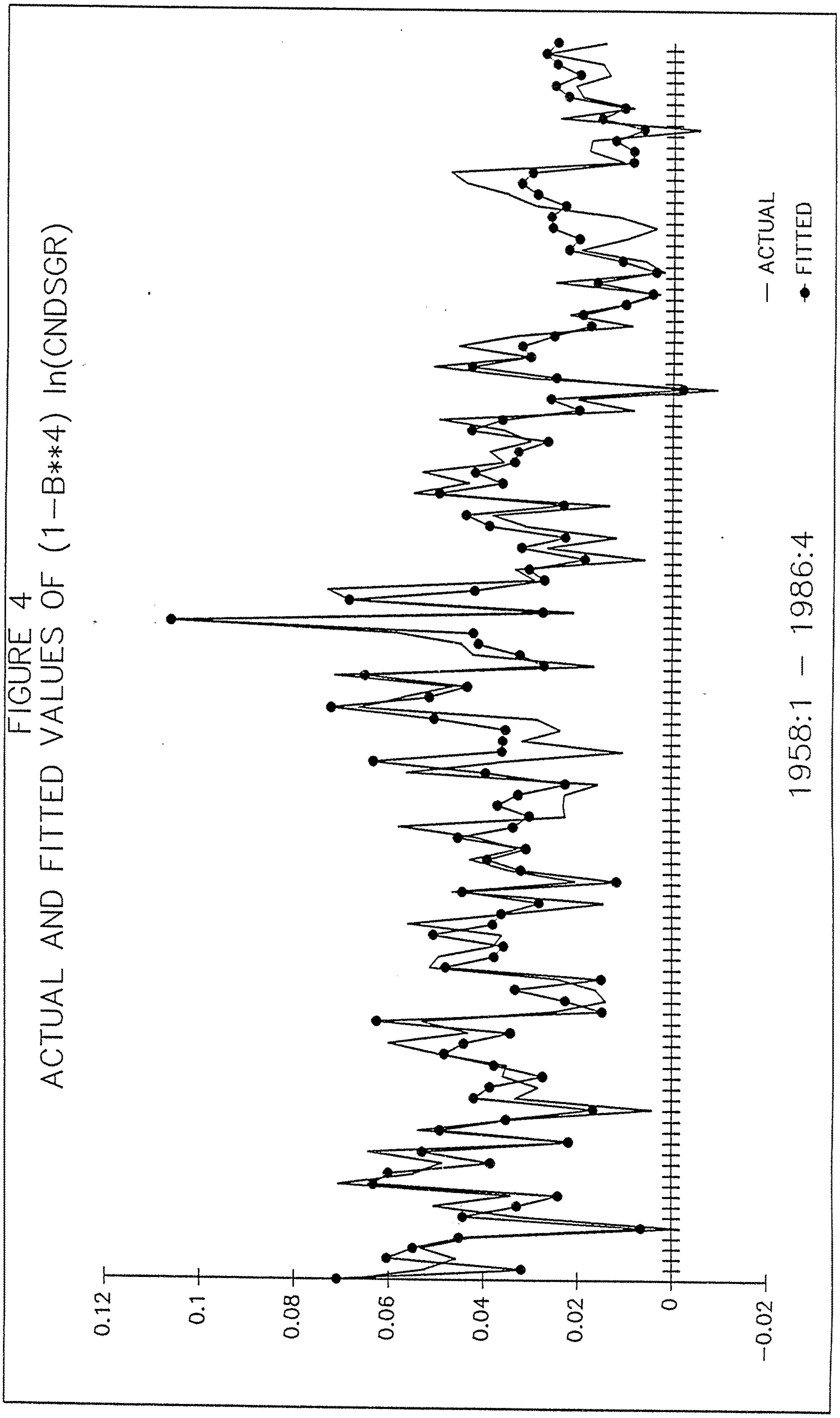


Figure 4

Actual and Fitted Values from (4)



So far, we have only discussed the statistical properties of (4). This equation has also interesting economic implications. The usual steady-state assumption that C = KY seems questionable in the light of the observed seasonal behaviour of C/Y (see fig. 2). A steady-state assumption of the form $C = K_1Y$, where K_1 is allowed to vary seasonally, seems to be more consistent with this evidence. And indeed, seasonal dummies were significant when included into the equation. A similar result was obtained by Hendry and von Ungern-Sternberg (1980) in a study of UK consumer expenditure. Next, the significant coefficient of lagged disposable income indicates that the long run income elasticity of consumer expenditure deviates definitely from unity. The static long run solution (the growth rates of all variables being equal to zero) is given by

$$C = K_1 Y^{.6785} L^{.0608}, \qquad (5)$$

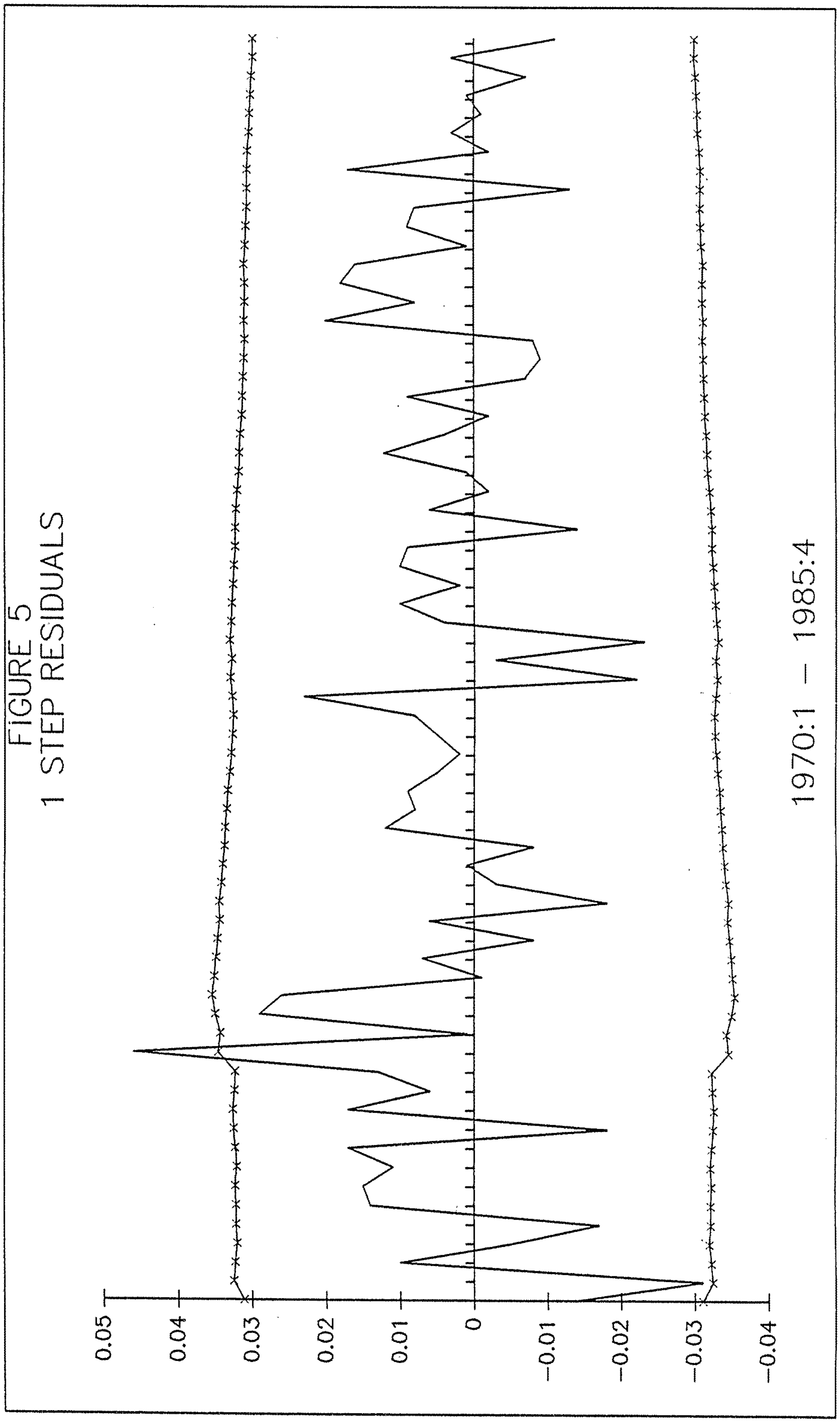
where $K_1 = \exp[.5267\ Q_1 + .6602\ Q_2 + .5728\ Q_3 + .6432\ Q_4]$ and, therefore, varies seasonally. Finally, (3) can be solved for the weights θ_i of the loss function above. Normalizing $\theta_1 = 0.1$, we obtain $\theta_2 = 1.274$, $\theta_3 = 4.951$ and $\theta_4 = 0.736$. We note that Austrian economic agents give a large weight to the avoidance of 'bang-bang' control.

3.4 Recursive Estimation and Encompassing Tests

Recursive estimation is a convenient method to obtain further insight into the development of our final consumption equation over time. It highlights deteriorations or improvements in the individual parameter estimates or general goodness of fit during the sample period. The most interesting output, which can be obtained from recursive estimation, is a graph of 1-step residuals. We use the 52 observations from 1957:1 to 1969:4 for the initialization of the recursive estimation process for (4).

Figure 5

1-Step Residuals



No significant deterioration in the goodness of fit over the sample period is noticeable.

Figures 6 to 9 provide information about the behaviour of the estimated coefficients of (4) over time. We notice that, during the early seventies, the estimated coefficients (and here especially that of our integral control variable $(l-y)_{t-1}$) tend to fluctuate moderately when new observations are added to the sample. From the mid-seventies onwards, however, the individual parameter estimates hardly fluctuate anymore and also exhibit no visible tendency to drift.

Figure 6

Coefficient of $\Delta_4 y_t$

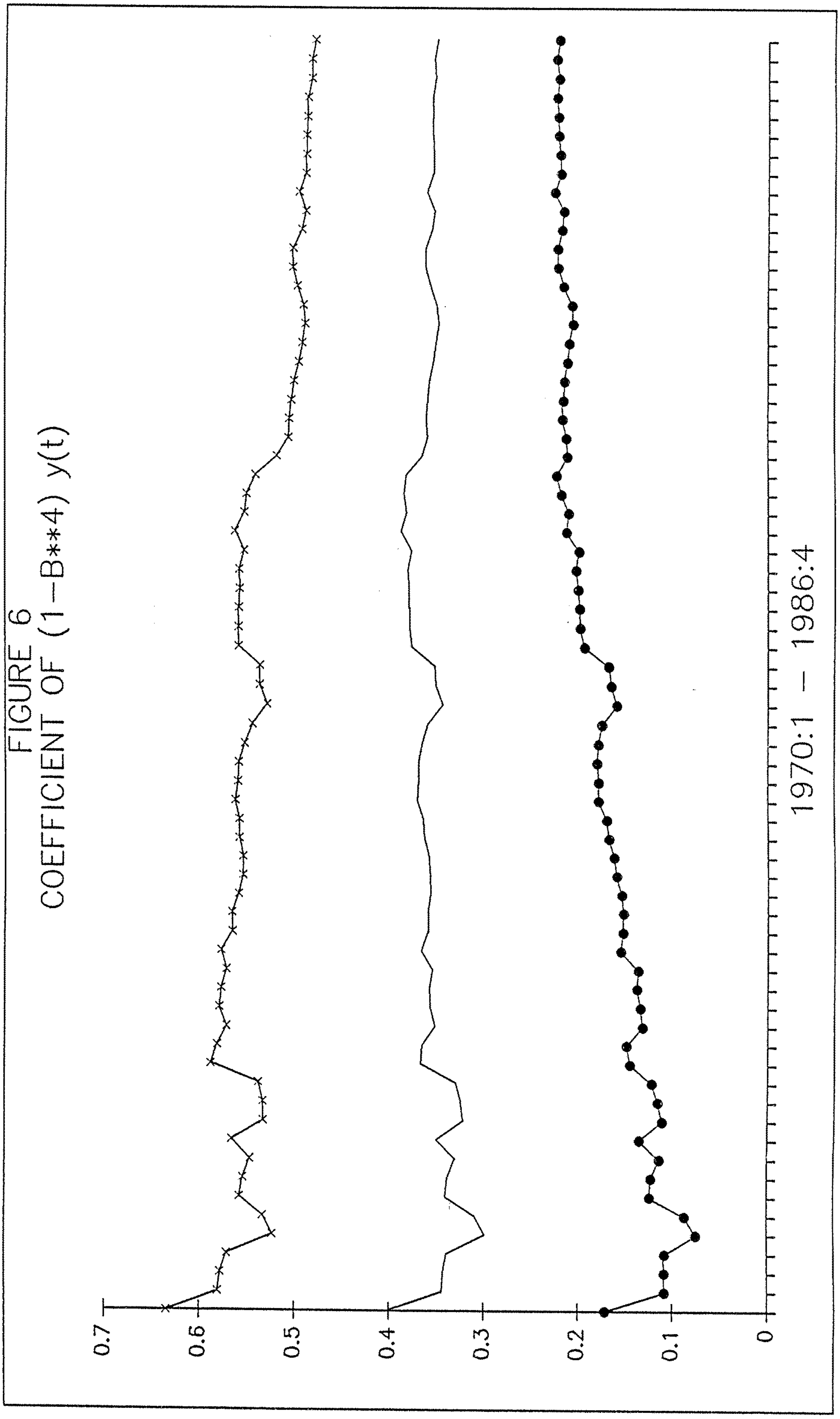


Figure 7

Coefficient of y_{t-4}

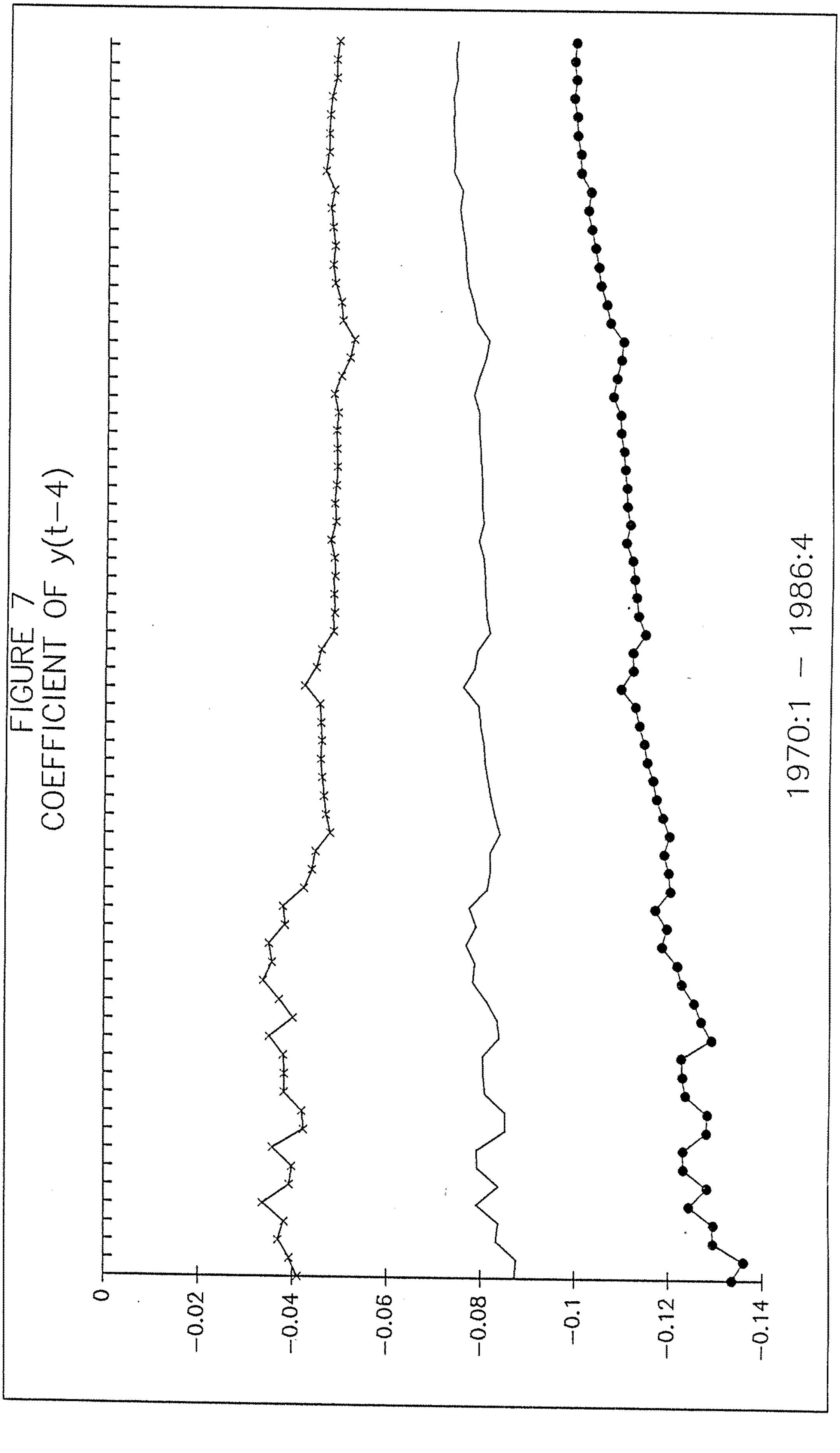


Figure 8

Coefficient of $(c-y)_{t-4}$

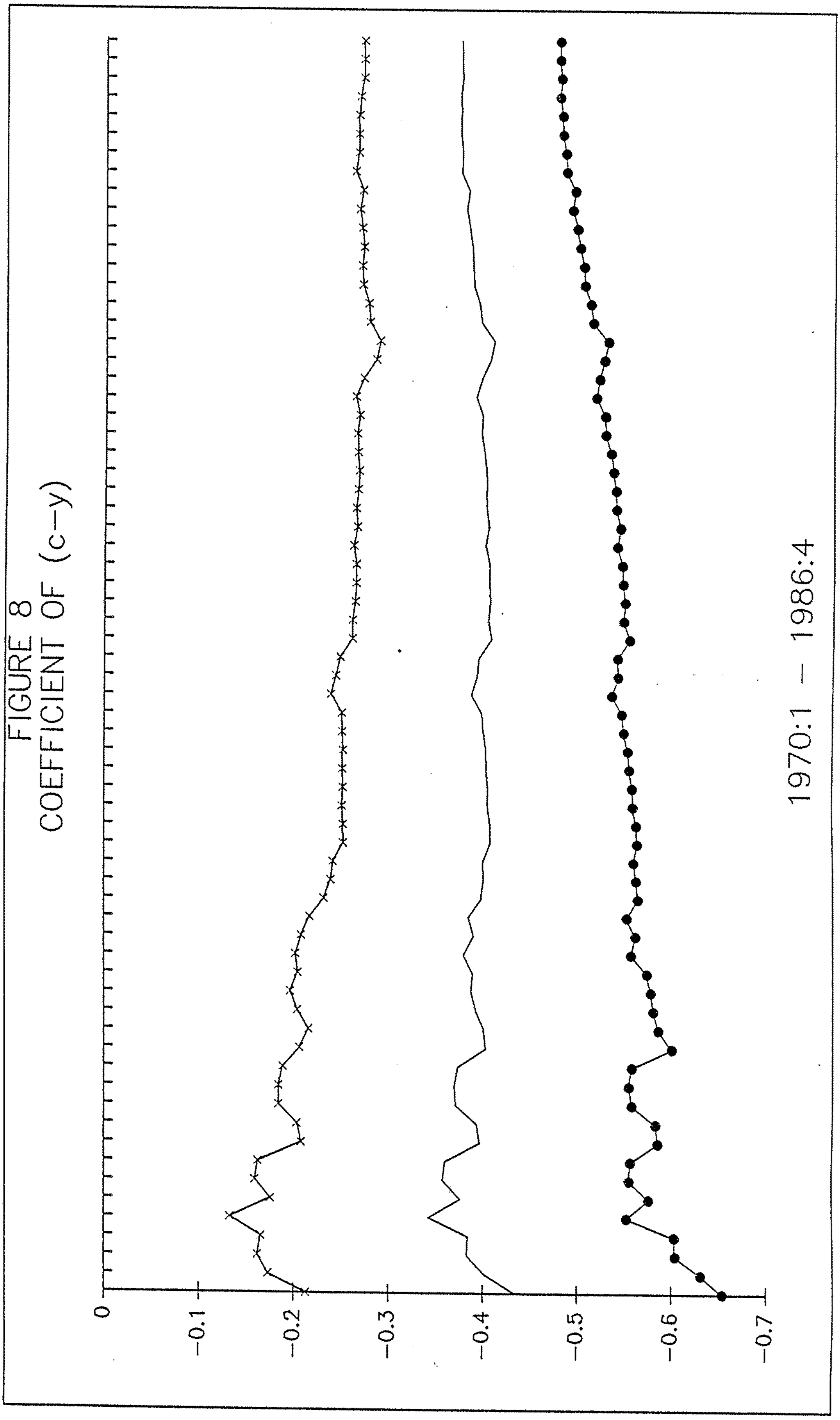
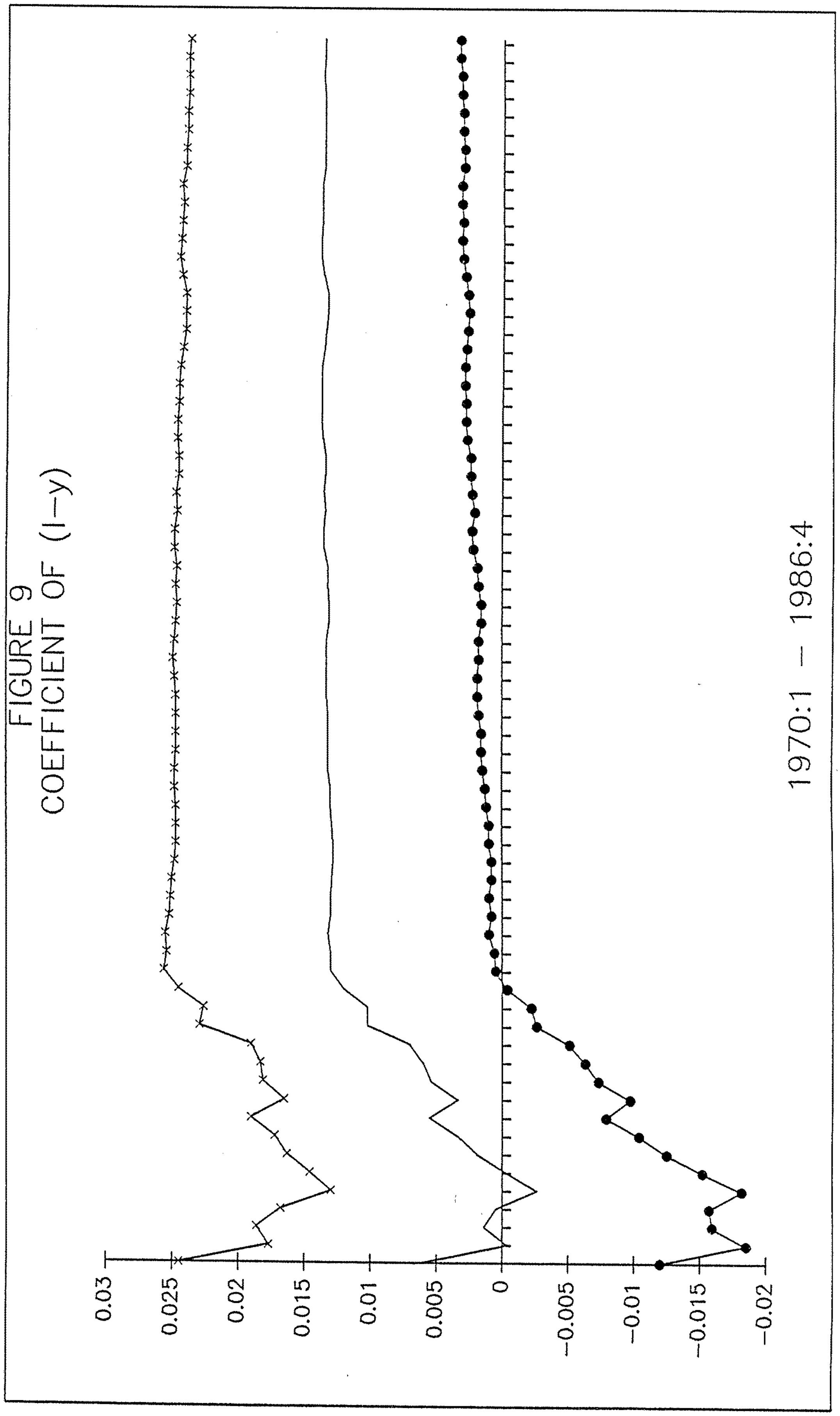


Figure 9

Coefficient of $(l-y)_{t-1}$



The computation of different Chow test statistics offers a formal testing procedure for parameter constancy. The outcomes of these alternative Chow tests are shown in figure 10 to 12. The values in these graphs were scaled by their respective 5 percent critical values, so that values in excess of 1 are significant. We see that the outcome of the different Chow tests does not point to any problems with structural change. The significant values in 1973 are obviously caused by measurement errors after the introduction of VAT in 1973:1. We, therefore, added several dummy variables to our consumption equation (D73:2, D73:4) in order to remove these distortions.

Figure 10

1 Step Chow Tests

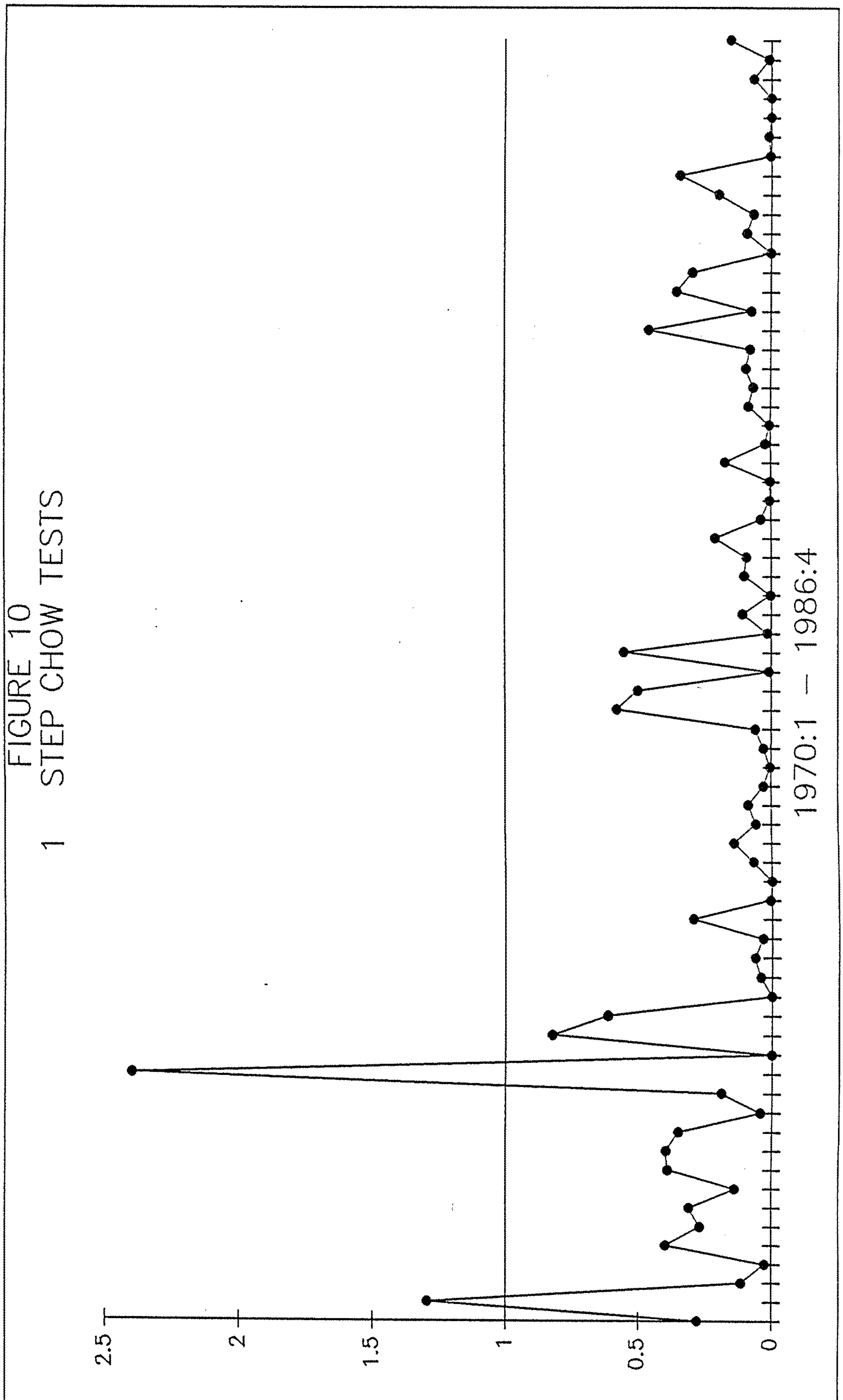


Figure 11

N Step Chow Tests

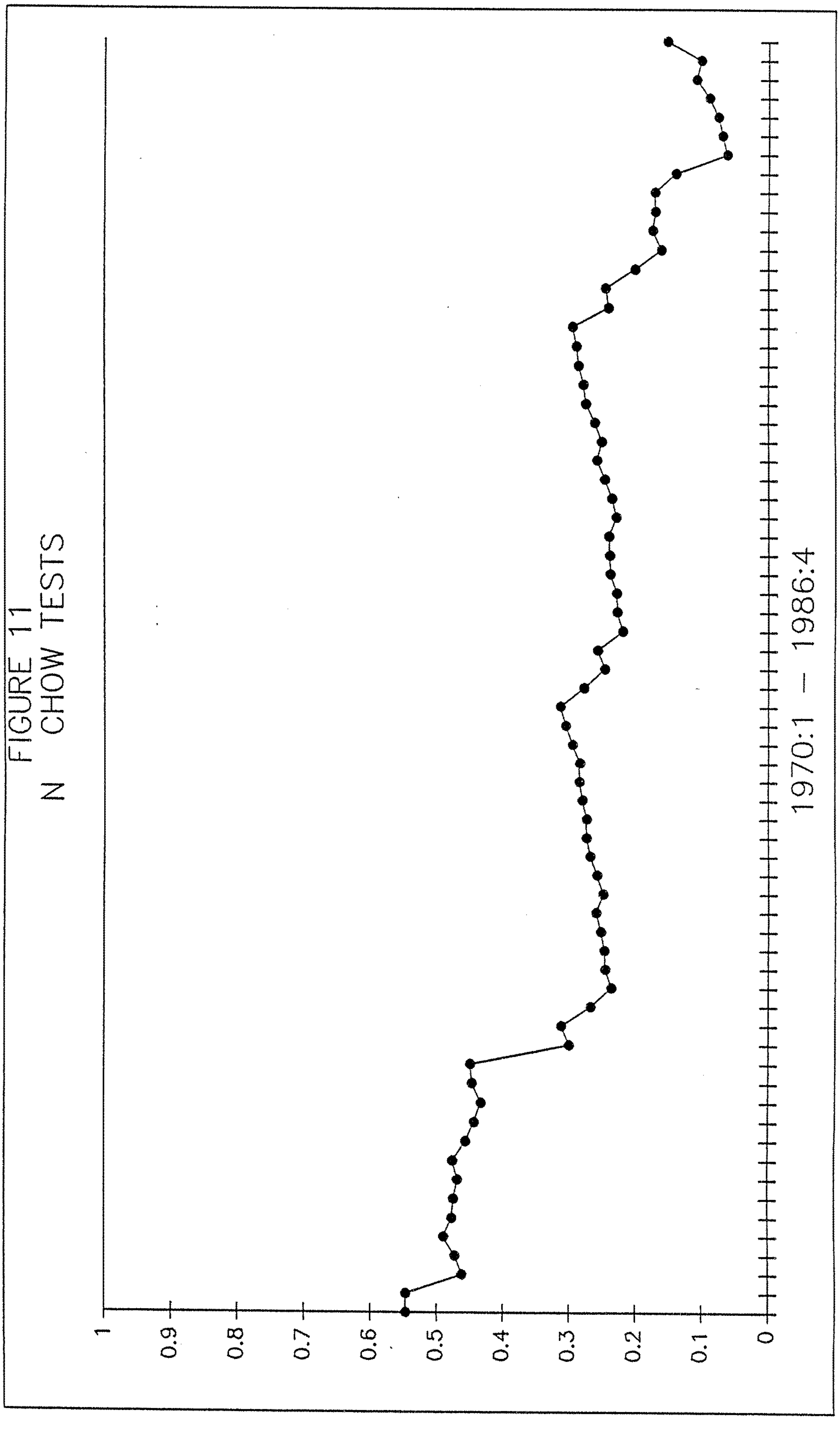
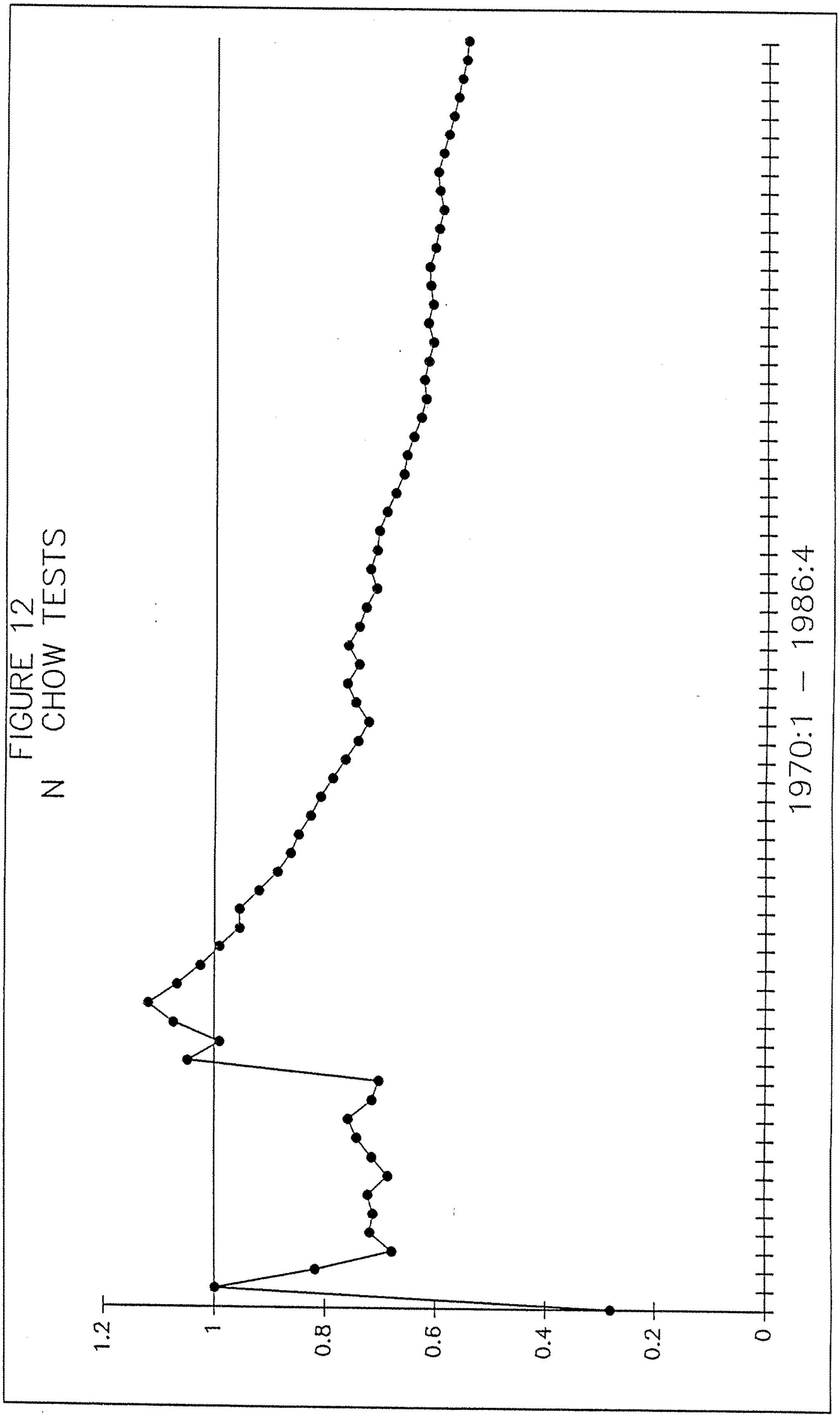


Figure 12

N Step Chow Tests



OLS estimation requires that the current explanatory variables are weakly exogenous. It was shown in Hendry (1985) that tests for parameter constancy test indirectly also for weak exogeneity. Since we detected no problems with the constancy of our estimated parameters, we can legitimately assume that our explanatory variables (and here, above all, disposable income) are indeed weakly exogenous with respect to consumer expenditure.

Encompassing testing, finally, is meant to investigate whether a particular model is able to explain the characteristics of sensible rival models. An introduction into the technical details of the encompassing approach can be found in Mizon (1984). One of the striking features of our consumption equation is the combined use of differences and levels of the explanatory variables. It seems therefore only natural to test it against a simple specification in differences of the variables alone. Thus, model 2 is given by

$$\Delta_4 c_t = a_0 + a_1 \Delta_4 c_{t-4} + a_2 \Delta_4 y_t + a_3 \Delta_4 y_{t-4} + a_4 \Delta_4 E_t + v_t \quad (6)$$

The outcome of encompassing testing is given in table 4. All four test statistics provide significant evidence that model 1 encompasses model 2, but not vice versa. Consequently, a specification in differences alone will have serious deficiencies.

Table 4

Encompassing Tests

Model 1 versus Model 2			Test	Model 2 versus Model 1		
Test statistics	Distribution of test statistics	5 percent critical values		Test statistics	Distribution of test statistics	5 precent critical values
-1.22	N(0,1)	1.96	Cox	-15.09	N(0,1)	1.96
1.13	N(0,1)	1.96	Ericsson IV	10.42	N(0,1)	1.96
3.18	$Chi^2(3)$	7.81	Sargan	51.55	$Chi^2(8)$	15.50
1.06	F(3,104)	2.69	Joint Model	11.08	F(8,104)	2.03

It should be clearly understood, however, that this does not mean that our model is absolutely correct. Theoretically, there may exist other, still more sophisticated specifications for a consumption equation.

4. CONCLUDING REMARKS

We established by co-integration tests that consumer expenditure on nondurables and services and disposable income are co-integrated. Consequently, an error correction model should be an adequate specification for consumer expenditure. Our first attempts to estimate such a model for the Austrian data were however rather disappointing. The explanatory power and the fit of this model were very poor. Several modifications of the original model were necessary in order to obtain a more data consistent specification. We added a liquidity variable as integral control mechanism to the model. We allowed the long run income elasticity of consumer expenditure to deviate from unity. And, finally, we switched from a model with an in the long run constant average propensity to consume to a model, which allows for seasonal variations in this magnitude. These modifications led to a consumption equation with satisfactory statistical properties, a plausible economic interpretation and interesting data coherent dynamic features.

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6. NOTES

(1) All computations were done by Version 5.01 of PC-GIVE. Only the seasonal adjustment was done by RATS using the approach of frequency domain deseasonalization (see Sims (1974)).

(2) We use here critical values which were recalculated by Engle and Granger (1987).

DISPOSABLE INCOME, GOVERNMENT DEFICIT, AND PRIVATE CONSUMPTION. SOME EVIDENCE FOR THE WEST-GERMAN ECONOMY

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1. Introduction *)

One of the most debated issues in modern macroeconomics concerns the impact of government deficits and public debt on real economic variables. Two competing theories can be identified. The traditional (Keynesian) theory states that private households are relatively short–sighted and consumption is determined for the most part by current disposable income. When government expenditure are financed by selling bonds rather than by collecting taxes, the higher disposable income of private consumers will stimulate private consumption, raise real interest rates and lower long–run capital formation. To a lesser extent this reasoning is valid also in a conventional life–cycle model with a finite planning horizon. A second theory, often called the Ricardian equivalence hypothesis, states that consumers are far–sighted and know that deficits today imply higher taxes tomorrow, given the time path of government expenditure. According to this theory, private individuals have a clear idea what the optimal intertemporal allocation of resources is, and any attempt by the government to modify the national saving rate will be frustated by offsetting private behaviour.

Recent theoretical work convincingly shows that the Ricardian view only holds under special restrictive assumptions (for a survey see Bernheim 1987; Leiderman, Blejer 1988). To mention the most important:

a) Agents must have an infinite planning horizon. This can be justified by the assumption that subsequent generations are operatively linked by altrustically motivated transfers (Barro 1974). This rules out that bequests are made for the "strategic" purpose of influencing the behaviour of children (see Bernheim, Shleifer, Summers 1985), or that the level of bequests and not the utility of decendents is an argument in the utility function. Whereas in theoretical discussions the role of intergenerational links is crucial, from an empirical standpoint it is probably not as important, if a substantial part of the deficit is repaid during the lifetime of the existing generation (Poterba, Summers 1986).

b) Private households are not liquidity constrained. If some households are unable to borrow enough money to finance their optimal consumption, a tax cut would stimulate their consumption even if households know that they have to pay higher taxes in the future.

c) Only lump–sum taxes are levied. If a deficit is caused by lower taxes and is repaid by collecting distortionary taxes on property and labour income, intertemporal substitution effects will change consumption even if households have an infinite planning horizon.

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d) The effect of uncertainty is ignored. If income and the timing and type of taxes are uncertain, deficit financing may have an expansionary effect on consumption (Barsky, Mankiw, Zeldes 1986; Feldstein 1986).

Considering all these objections, it seems unlikely that the Ricardian equivalence hypothesis is a fruitful approach to study the impact of fiscal policy. But proponents of Ricardian equivalence claim that this proposition is nevertheless a valid approximation to private sector consumption behaviour.

Many empirical studies have been done to test the Ricardian equivalence hypothesis. But, as will be argued in section 2, the theoretical foundations of much of this work seem to be weak. Therefore a theoretical model is formulated which is consistent with the basic assumption of rational consumers who optimize an intertemporal utility function. In order to allow for imperfect capital markets, a simple version of liquidity constrained behaviour is combined with the basic model. In section 3 the estimation method and the empirical results are presented and section 4 concludes with some final comments.

2. Theoretical considerations

a) Some conventional test procedures

A standard approach in testing the Ricardian equivalence hypothesis is given by the following consumption function:

$$(1) \qquad C_t = \alpha_1 YD_t + \alpha_2 F_t + \alpha_3 W_t + \alpha_4 D_t$$

where C_t denotes private consumption, YD_t disposable income, F_t government deficit, W_t private non–human wealth including public debt and D_t public debt. Modigliani/Jappelli/Pagano (1985) identify the conventional (myopic or Keynesian) model with the restrictions $\alpha_2=\alpha_4=0$ and the strong Ricardian equivalence model with $\alpha_2=-\alpha_1$ and $\alpha_4=-\alpha_3$. Analogous restrictions are tested by Tanner (1979), Kochin (1974), Heri (1987), Koskela/Viren (1983), among others.

Using the government budget constraint $F_t = G_t + Z_t - T_t$ and the definition of private disposable income $YD_t = Y_t - T_t + Z_t$, where G_t denotes government expenditure, Z_t interest payments on public debt, T_t taxes minus transfers and Y_t net national product, equation (1) can be reformulated as:

$$(2) \qquad C_t = \alpha_1 Y_t + \alpha_2 G_t + (\alpha_1 + \alpha_2) Z_t - (\alpha_1 + \alpha_2) T_t + \alpha_3 W_t + \alpha_4 D_t$$

Given the restrictions as formulated by Modigliani/Jappelli/Pagano, the Ricardian equivalence hypothesis implies that interest payments on public debt and taxes have no effect on consumption when net national product Y and government expenditure G are used as explaining variables and that the parameter on G should have the negative value of the parameter on Y. Equation (2) was the basic approach for the models estimated by Feldstein (1982), Kormendi (1983), Aschauer (1985) and Sarantis (1985).

Despite the popularity of this approach it should be noted that the basic specification of equation (1) and (2) is fundamentally inconsistent with a theoretical framework in which Ricardian equivalence can hold and therefore no valid conclusion can be drawn from the estimated coefficients. The very least condition for Ricardian equivalence to hold is that private households maximize an intertemporal utility function and form rational expectations. Ignoring uncertainty and assuming a standard utility function, e.g. a quadratic or one with constant absolute or relative risk aversion, the consumption function for a rationally acting household can be written as:

$$(3) \qquad C_t = \Gamma_{0,t} + \Gamma_{1,t} \cdot (W_t + \sum_{i=0}^{\infty} E_t R_{t+i} (YL^g_{t+i} - T_{t+i}))$$

where E_t is the conditional expectation operator, YL^g is real gross labour income and T denotes taxes net of transfers. R_{t+i} is the discount factor defined as:

$$(4) \qquad R_{t+i} = (1+r_t) \prod_{j=0}^{i} (1+r_{t+j})^{-1}$$

where r_{t+j} is the real interest rate in period t+j. The time varying parameters $\Gamma_{0,t}$ and $\Gamma_{1,t}$ depend in a complicated way on expected real interest rates, on the rate of time preference and the parameters of the utility function.

If the Ricardian equivalence hypothesis holds, an alternative but equivalent decision rule can be derived by substituting the intertemporal government budget constraint

$$(5) \qquad D_t + \sum_{i=o}^{\infty} R_{t+i} G_{t+i} = \sum_{i=o}^{\infty} R_{t+i} T_{t+i}$$

into equation (3). We get (see Aschauer 1985 or Jäger 1987):

$$(6) \qquad C_t = \Gamma_{0,t} + \Gamma_{1,t}((W_t - D_t) + \sum_{i=o}^{\infty} E_t R_{t+i}(YL^g_{t+i} - G_{t+i}))$$

Under Ricardian equivalence both equation (3) and (6) are valid descriptions of private consumption behaviour. The equations show that for a rational household the expected values of future net labour income (YL^g–T) depend on the stock of public debt D and the expected time path of real government expenditure G.

In the light of consumption functions which are consistent with intertemporal optimization, the traditional approach represented by equation (1) or (2) is seriously deficient: 1) The real interest rate is ignored. 2) The chosen income measure "disposable income" seems to be inappropriate since in a life–cycle model the correct measure is labour income. 3) Only current values of labour income and government expenditure are included as independent variables. The approximate validity of this specification depends on the stochastic nature of the variables and their dynamic interrelations. Suppose a reduction of taxes and the accompanying deficit signals that the government is planning to cut expenditure in the future. Even if Ricardian equivalence is valid, private households will probably increase their consumption. Since in this case the estimated value of α_2 in equation 1 is positive, the Ricardian equivalence hypothesis would be erroneously rejected. Likewise in equation (2) the coefficient on taxes would be negative. Feldstein and other authors would erroneously conclude that the equivalence hypothesis is violated. These observations reflect the fundamental insight of life–cycle models that consumption depends on permanent income, but not on current values of income, deficits etc.

In the next section we formulate a model which takes into account the dynamic properties of labour income, deficit and government expenditure and which is consistent with rational, intertemporally maximizing behaviour.

b) The "surprise consumption function" – approach for testing Ricardian equivalence

A natural method to estimate and test models of consumption behaviour based on intertemporal optimization and rational expectations is to formulate a surprise consumption function. In this approach the change in consumption between two succeeding time periods is decomposed in a forecastable part given by the first order condition for intertemporal optimization (the "Euler–equation") and a part which is unforecastable and depends on new information about future net labour income and real interest rates. This approach was developed by Hall (1978), Bilson (1980), Flavin (1981), Muellbauer (1982), Wickens, Molana (1985) and others and applied to test the stochastic version of the life cycle theory. But surprisingly few studies of the effects of deficits have used this approach. The essence of the model is to test whether households revise their estimate of permanent income if government deficit has changed in a way unexpected in the preceding period (see for example Aschauer 1985; Flaig 1987a or Poterba, Summers 1987).

In order to get an estimable formulation of this idea a representative household is assumed to maximize an additive time separable utility function of the following form:

$$(7) \qquad L = E_t \sum_{i=o}^{\infty} (1+\delta)^{-i}\, U(C_{t+i})$$

E_t is the expectation operator, δ is the rate of time preference and $U(\cdot)$ is the instantaneous utility function. Utility maximization is constrained by the period to period budget equation

$$(8) \qquad W_{t+i} = (1+r_{t+i})W_{t+i-1} + YL^n_{t+i} - C_{t+i} \qquad\qquad i = 0,1,...$$

W_{t+i} is real non–human wealth at the end of period t+i, YL^n_{t+i} is real net labour income, C_{t+i} is consumption and r_{t+i} is the real interest rate.

The first order condition is given by

$$(9) \qquad E_t \left\{ \frac{1+r_{t+1}}{1+\delta} U'(C_{t+1}) \right\} = U'(C_t)$$

If we approximate the instantaneous utility function by

$$(10)\quad U(C_{t+i}) = -\gamma\exp(-C_{t+i}/\gamma)\,,$$

assume certainty equivalence and replace the expected value of C_{t+1} by its realization plus an innovation ϵ_{t+1} with $E_t\epsilon_{t+1}=0$, we get:

$$(11)\quad C_{t+1} - C_t = \beta_o + \beta_1 E_t r_{t+1} + \epsilon_{t+1}$$

with $\beta_o = \gamma\ln(1+\delta)^{-1}$, $\beta_1 = \gamma$ and $r_{t+1} \approx \ln(1+r_{t+1})$.

An almost identical version of this equation can be derived without the questionable assumption of certainty equivalence if $(C_{t+1}, \ln(1+r_{t+1}))$ is normally distributed (see Palm/Winder 1986). In this case β_o depends also on the variances of C and r.

ϵ_{t+1} is the unexpected change in consumption, $C_{t+1}-E_tC_{t+1}$, and reflects unexpected changes in the level or in the optimal intertemporal allocation of permanent income. In order to get a tractable model, households are assumed to use current and past information on real net labour income YL^n, government expenditure G, government deficit F and the real interest rate r in their prediction of future values of net labour income and the real interest rate and hence in their calculation of permanent income. We define a vector $X_t = (\Delta YL^n_t, \Delta G_t, \Delta F_t, r_t)'$ with Δ the difference operator and assume that a multivariate moving average representation exists of the form

$$(12)\quad X_t = a_o + A(L)u_t$$

where $u_t \equiv (u^Y_t, u^G_t, u^F_t, u^r_t)'$ is a vector of white noise innovations with $Eu_t = 0$ and $E\,u_tu_t' = \Sigma$. Σ is a symmetric, positive–definite variance–covariance matrix. A(L) is a matrix polynominal in the lag–operator L:

$$(13)\quad A(L) = A_o + A_1L + A_2L^2 + A_3L^3 + \ldots$$

A_i is a (4x4)–matrix and we use the identifying restriction A_0=I. Since the optimal variance minimizing revision of the expectations of all future values of labour income and interest rates is a linear function of the innovations u_t, if follows that ϵ_t also is approximately a function of u_t (see Wickens, Molana 1985 or Flaig 1987b):

$$(14) \quad \epsilon_t = b_1 u_t^Y + b_2 u_t^G + b_3 u_t^F + b_4 u_t^r$$

The parameter b_i, i = 1,2,3,4, measures the reaction of consumption to unexpected movements of the variables Y, G, F and r, respectively. The change in consumption is due to the revision of permanent income and of the propensity of current consumption with respect to permanent income.

In virtually all studies on Ricardian equivalence it is assumed that a government deficit is at least partially financed by future higher taxes and not totally by lower government expenditure. Our empirical estimates (see section 3) indicate that an unexpected deficit typically had led to a reduction of net labour income in subsequent years. Therefore the coefficient of u^F should be negative to be consistent with Ricardian equivalence. Since the null hypothesis is b_3=0, this is not a strong test of Ricardian equivalence, but rather one of the hypothesis that households don't care about government deficit when deciding about their consumption. $b_3 < 0$ is a necessary, but not a sufficient condition for the validity of Ricardian equivalence.

c) Liquidity constraints and the final form of the consumption function

One popular argument against Ricardian equivalence is that many households are liquidity constrained. In this case, a tax cut would rise current disposable income and stimulate private consumption even if the households know that they will have to pay higher taxes in future years and their permanent income remains unchanged. Ideally one should explicitly model capital market imperfections and explain liquidity constraints as endogeneous variables depending on households' income and non–human wealth. Especially with aggregated data this seems to be a very difficult, if not impossible task. Therefore we follow Hall, Mishkin (1982) and Hayashi (1985) in characterizing liquidity constrained consumption behaviour by a marginal propensity of consumption out of labour income of one, i.e. $C_t - C_{t-1} = YL_t^n - YL_{t-1}^n$.

We model the aggregate change in consumption as a weighted average of unconstrained and constrained behaviour. Let λ_t be the weight given to consumption which is governed by the rules of intertemporal optimization and $(1-\lambda_t)$ be the weight given to liquidity constrained consumption. The change of aggregate consumption can then be written as:

$$(15)\quad C_t - C_{t-1} = \lambda_t(\beta_o + \beta_1 E_{t-1} r_t + \beta_2 u_t^Y + \beta_3 u_t^G + \beta_4 u_t^F + \beta_5 u_t^r)$$
$$+ (1-\lambda_t)(YL_t^n - YL_{t-1}^n) + \eta_t$$

where η_t denotes transitory consumption changes.

In the empirical part we model λ_t alternatively as a constant λ or as given by the logistic function

$$(16)\quad \lambda_t = (1+\exp(-\lambda_o - \lambda_1 t))^{-1}.$$

Equation (16) reflects the idea that the importance of liquidity constraints is reduced by the secular growth of real income which is approximated by a simple time trend.

The estimated consumption function can then be formulated as:

$$(17a)\ C_t - C_{t-1} = \lambda(\beta_o + \beta_1 E_{t-1} r_t + \beta_2 u_t^Y + \beta_3 u_t^G + \beta_4 u_t^F + \beta_5 u_t^r - (YL_t^n - YL_{t-1}^n)) +$$
$$(YL_t^n - YL_{t-1}^n) + \eta_t$$

or alternatively as

$$(17b)\ C_t - C_{t-1} = (1+\exp(-\lambda_o - \lambda_1 t))^{-1}(\beta_o + \beta_1 E_{t-1} r_t + \beta_2 u_t^Y$$
$$+ \beta_3 u_t^G + \beta_4 u_t^F + \beta_5 u_t^r - (YL_t^n - YL_{t-1}^n)) + (YL_t^n - YL_{t-1}^n) + \eta_t$$

As should be clear from the previous discussion, a necessary condition for Ricardian equivalence to hold is $\lambda_t=1$ (for all t) and $\beta_4<0$. But – as already noted – this is not a test of the one–to–one equivalence of tax – and debt – financing of government expenditure. Thus even if $\lambda_t=1$ and $\beta_4<0$, the decision how to finance government expenditure may have real effects. But if $\lambda_t<1$ and/or β_4 is not significantly different from zero, Ricardian equivalence is clearly rejected.

3. Estimation procedures and empirical results

a) Construction of the surprise variables and of the expected real interest rate

As can be seen from equation (17), we need some measure for the unexpected movements of YL^n, G, F and r and for the expected real interest rate. After examining the autocorrelation and the partial autocorrelation function and the results of Stock–Watson and Dickey–Fuller tests it seems legitimate to treat the elements of the vector $X_t=(\Delta YL^n_t, \Delta G_t, \Delta F_t, r_t)'$ as stationary variables. For a description of the variables see the data appendix. ΔYL^n, ΔF and the change of consumption are shown also in figure 1.

Some experimentation showed that X can be represented by the following vector–autoregressive model with a lag length of two years:

$$(18)\quad X_t = \mu + B_1X_{t-1} + B_2X_{t-2} + u_t$$

μ is a (4x1)–vector and B_i is a (4x4)–matrix. A system with a lag length of one year exhibits significant autocorrelation of the residuals. In a system with a lag length of three years parameters on lags of three years are all insignificant. Table 1 presents the results of a VAR–estimation of this system using yearly data for the period 1955–1986. We will not comment on all parameters, but concentrate on the effects of government deficit. Contemporaneously the innovation in deficit is positively correlated with the innovations in net labour income and the real interest rate and negatively with the innovation in government expenditure. The positive correlation between u^Y and u^F can plausibly be explained by changes of tax laws which simultaneously rise net labour income and deficit but not by assuming a causal dependence of deficit on income since the latter should imply a negative correlation. The negative correlation between u^G and u^F indicates an active role of deficits in the governmental decision process. If deficits would only passively respond to government expenditure, the correlation should be positive. An innovation in the deficit leads in

Fig. 1: Changes in real consumption C (———), real labour income YL^n (---) and real government deficit G(— —) (in thousands of DM; per capita values in prices of 1980)

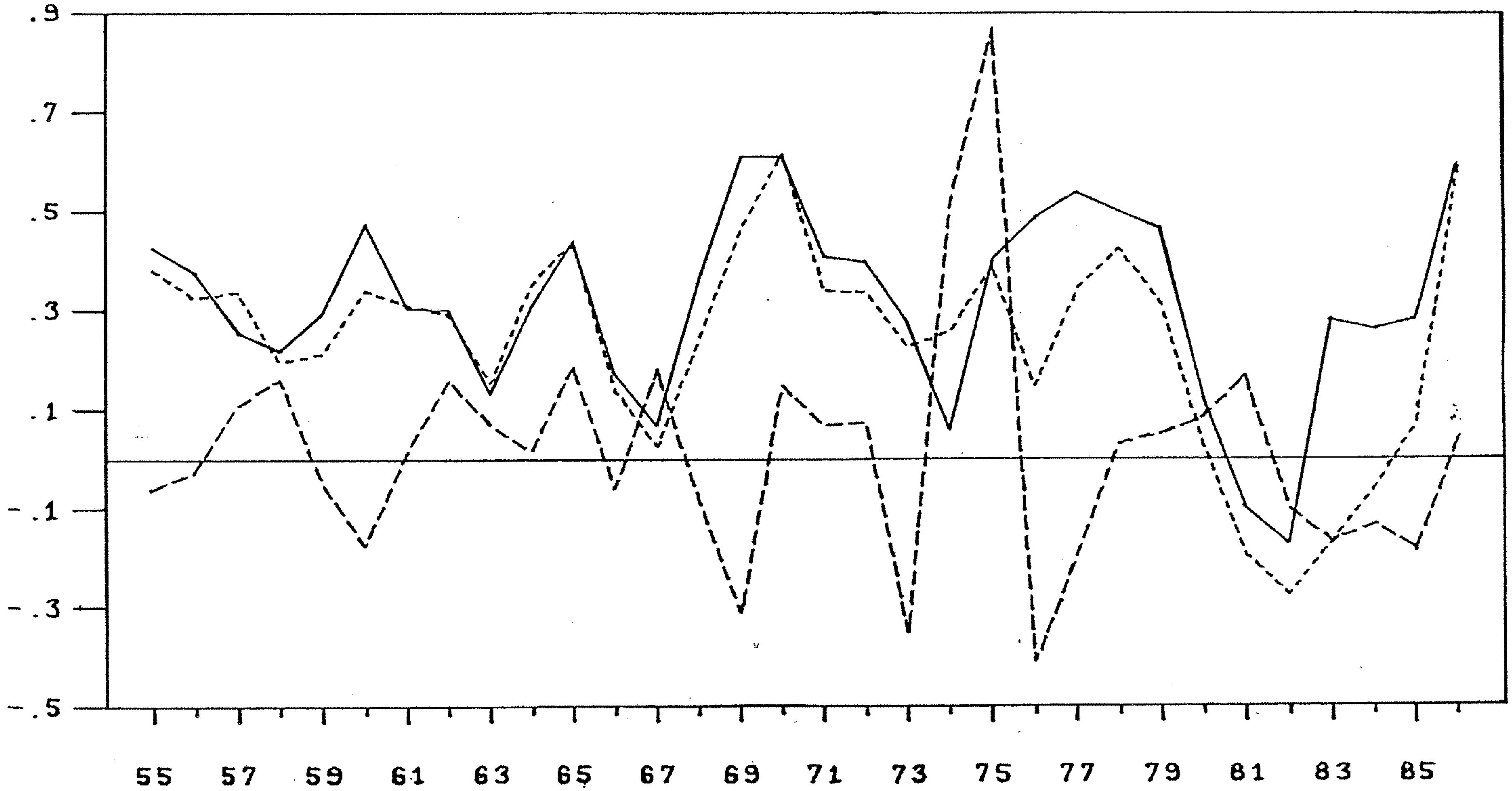

Table 1: VAR–system for ΔYL^n, ΔG, ΔF and r

	Dependent variable			
	ΔYL^n	ΔG	ΔF	r
Constant	–0,004 (01,)	–0,056 (1,9)	0,003 (0,1)	0,900 (2,2)
ΔYL^n_{-1}	0,570 (2,5)	0,407 (3,2)	–0,257 (0,9)	2,989 (1,7)
ΔYL^n_{-2}	–0,334 (1,5)	–0,251 (2,0)	0,427 (1,5)	0,524 (0,3)
ΔG_{-1}	0,526 (1,6)	0,458 (2,4)	1,226 (2,8)	0,229 (0,3)
ΔG_{-2}	0,440 (1,2)	–0,057 (0,3)	–0,832 (1,7)	–1,053 (0,3)
ΔF_{-1}	–0,272 (2,2)	–0,201 (2,7)	–0,051 (0,3)	1,016 (1,0)
ΔF_{-2}	–0,040 (0,3)	–0,044 (0,6)	–0,301 (2,0)	0,348 (0,4)
r_{-1}	0,089 (2,8)	0,010 (0,6)	–0,028 (0,7)	0,696 (2,8)
r_{-2}	–0,022 (0,7)	0,002 (0,1)	–0,046 (1,1)	0,037 (0,2)
R^2	0,587	0,659	0,445	0,467
DW	1,51	1,69	1,82	1,74

Correlation matrix of contemporaneous residuals:

	u^Y	u^G	u^F	u^r
u^Y	1.00	0.36	0.34	0.34
u^G		1.00	–0.19	0.09
u^F			1.00	0.29
u^r				1.00

Note: t-values are shown in parentheses
Estimation period: 1955–1986 (Yearly data)

subsequent years to a reduction of labour income (mainly because of reductions of transfers) and a reduction of government expenditure.

b) Estimation of the consumption function

In the subsequent empirical part we use two alternative methods to estimate the parameters of the surprise consumption function (17). Some strong assumptions with regard to the stochastic nature of u and η have to be made to ensure the identification of the structural coefficients and their consistent estimation (Abel, Mishkin 1983). A necessary assumption is that transitory consumption η_t must be uncorrelated with the innovations u_t. This assumption rules out contemporaneous feedback from consumption to income, deficit etc. and also the dependence of u and η on a third unobserved factor.

Estimation method I is a two-step procedure. In the first step the VAR-system (18) is estimated by OLS and the vector of empirical residuals $\hat{u}_t$ is calculated. From the last equation of (18) we construct a measure of the expected real interest rate:

$$(19) \quad \hat{r}_t = \hat{\mu}(4) + \hat{A}_1(4,.)X_{t-1} + \hat{A}_2(4,.)X_{t-2}$$

where a "^" indicates an estimated value; $\hat{\mu}(4)$ is the fourth element of the vector $\hat{\mu}$ and $\hat{A}_i(4,.)$ is the fourth row vector of $\hat{A}_i$. In the second step the consumption function is estimated by nonlinear least squares (NLS), where $\hat{u}^Y$, $\hat{u}^G$, $\hat{u}^F$, $\hat{u}^r$ and $\hat{r}$ are substituted for u^Y, u^G, u^F, u^r and $E_{t-1}r_t$. This procedure is easy to implement and yields consistent parameter estimates. But it is not fully efficient and may produce inconsistent estimates of the variance-covariance matrix of the regression parameters (see Pagan 1984 for a discussion of linear models).

Estimation method II involves joint nonlinear estimation of a system consisting of equation (17) and (18), imposing the constraint that the same parameter are present in both equations. This procedure is essentially a minimum distance estimator (MDE) which was proposed by Mishkin (1982) to test models with rational expectations and/or market efficiency. This estimator minimizes the criterion

$$(20) \quad S = \sum_{t=1}^{T} \nu_t' \Omega^{-1} \nu_t$$

where T is the number of observations and ν_t is the stacked vector of residuals defined as $\nu_t' = (u_t', \eta_t)$. Ω^{-1} is a weighting matrix with the following structure:

$$(21) \quad \Omega = \begin{pmatrix} \Sigma_{uu} & 0 \\ 0' & \sigma_\eta \end{pmatrix}$$

Σ_{uu} is the variance–covariance matrix of the residuals u and σ_η is the variance of η. 0 is a column vector of zeros with the same number of rows as Σ_{uu}. This structure reflects the zero correlation between u und η and ensures the identification of the structural parameters.

The estimation proceeds as follows: We get an estimate $\hat{\Omega}$ for Ω using the residuals from unrestricted OLS–estimations. Then equations (17) and (18) are estimated as a system by minimizing the criterion function (20). After convergence a new matrix $\hat{\Omega}$ is calculated and the system is reestimated . The iterative procedure is continued until there is little change in $\hat{\Omega}$ (less than 1 % in each component). This method is consistent and asymptotically efficient. All estimation is done with program GAUSS using the Berndt/Hall/Hall/Hausman–procedure to approximate the variance–covariance matrix of the estimated parameters.

c) Estimation results

Table 2 presents the results for the consumption function (17a). The dependent variable is the change in real consumption expenditure. In column 1 and 2 the restricted version with $\lambda=1$ (this implies that there are no liquidity constraints) is displayed. As in all other variants of the consumption function, the standard error of regression is lower with the MDE–estimator than with the NLS–estimator. All parameters show the expected sign: The expected real interest rate has a positive impact on consumption change (this is the intertemporal substitution effect), unexpected changes in real labour income have a positive effect and unexpected changes in government expenditure, in deficit and in the real interest rate have negative effects. But the significance of the parameters is in some cases rather low. The version of consumption function (17a) with an estimated λ–parameter shows a similar picture (columns 3 and 4). The estimated value of λ indicates that 70 % of total consumption is due to consumers who are liquidity constrained. This result is not compatible with the Ricardian equivalence hypothesis.

Table 2: Results for consumption function (17a) 1955:1986 (Yearly data)

	Estimation method			
	NLS	MDE	NLS	MDE
λ	1,000*	1,000*	0,301 (2,9)	0,291 (3,0)
β_0	0,269 (6,9)	0,202 (6,9)	0,365 (4,0)	0,310 (5,1)
β_1	0,050 (1,9)	0,120 (4,8)	0,146 (2,0)	0,213 (2,3)
β_2	0,976 (3,1)	1,095 (11,4)	0,919 (1,4)	1,033 (3,0)
β_3	–0,222 (1,3)	–0,245 (1,9)	–0,737 (0,7)	–0,861 (1,6)
β_4	–0,302 (1,3)	–0,339 (3,8)	–1,003 (1,8)	–1,169 (2,7)
β_5	–0,009 (0,2)	–0,024 (1,9)	–0,029 (0,4)	–0,061 (1,1)
SE	0,165	0,070	0,100	0,070
ρ_1	0,35	–0,11	0,10	–0,09
Q(4)	6,84	1,29	0,90	0,48

Notes:

* a–priori values

t–values are shown in parentheses

Estimation method NLS: The empirical residuals from the VAR–system are used as regressors.

Estimation method MDE: The VAR–system and the consumption function are simultaneously estimated by a minimum distance estimator.

SE = Standard error of estimation; ρ_1 = First order autocorrelation coefficient of residuals; Q(4) is the Box–Pierce statistic

Table 3: Results for consumption function (17b) 1955–1986 (Yearly data)

	Estimation method	
	NLS	MDE
λ_o	–2,647 (2,8)	–3,808 (6,3)
λ_1	0,549 (2,3)	1,044 (4,7)
λ_o	0,407 (2,7)	0,310 (3,0)
λ_1	0,231 (1,8)	0,380 (2,2)
λ_2	1,287 (1,3)	1,996 (2,8)
λ_3	–0,333 (0,2)	–0,441 (0,5)
λ_4	–1,116 (1,5)	–0,771 (1,6)
λ_5	–0,064 (0,6)	–0,111 (1,4)
SE	0,089	0,056
ρ_1	0,04	–0,18
Q(4)	2,35	2,46

Notes: See table 2.

Table 3 reports the results of our preferred consumption function. Both NLS– and MDE–results have the lowest standard error of regression among all models and the residuals don't show significant autocorrelation.

Using the estimates for λ_0 and λ_1 we can compute the weights given to liquidity constrained behaviour. The weight varies from 91,5 % in 1955 to 67,4 % in 1985 for the NLS–estimates and from 96,4 % in 1955 to 32,4 % in 1985 for the MDE–estimates. This estimate indicates a substantial degree of liquidity constraints.

The behaviour of those consumers who are not liquidity constrained can be described as follows. The expected real interest rate has a positive impact on consumption changes. Intertemporal substitution plays an important part in explaining consumption behaviour. If future taxes affect expected real interest rates this can be a channel through which deficits may produce real effects. The parameter on income surprises is positive and rises dramatically (in the MDE estimation from 1.03 to 1.99). At first sight, this seems to be a very high value since it implies a higher variance of changes in consumption than of innovations in real labour income. But this may be not an a–priori unreasonable value if first differences of income follow a stationary process (see the discussion of "excess sensitivity" versus "excess smoothness" of consumption in Campbell/Deaton 1987). Surprises of government expenditure have a negative but insignificant coefficient. The coefficient on deficit surprises is negative with a significance level of about 5 %. This implies that households are revising their estimate of permanent income downwards when they observe an unexpected deficit because they fear higher taxes in the future.

The effect of an unexpected change in net labour income on the change in aggregate consumption, when the innovations in all other variables are zero, is given by $\lambda_t + (1-\lambda_t)u^Y$. Using the parameters of the MDE–estimation in table 3 the numerical value of this expression is 1,67 for the year 1985. If the unexpected income change is due to a tax cut financed by a deficit, the change in consumption is given by $\lambda_t + (1-\lambda_t)(u^Y + u^F)$. The numerical value of 1,15 is about one third lower than in the first case but still of a considerable magnitude.

At this point, some comments on the presented results and interpretations are in order:
1) The data are highly time aggregated. Since the theory of intertemporal maximization

refers to consumption changes between two points of time, using yearly averages may pose some problems. 2) Due to data limitations, the dependent variable is consumption expenditure. This implies a unsatisfactory treatment of durables. 3) The estimated model do not allow for substitutability between public and private consumption and ignores the effects of labour supply on consumption. 4) Unexpected changes of government deficit may not measure the deliberate decision how to finance government expenditure but simply reflect bad economic conditions (e.g adverse supply shocks). But as argued in section 3a, the contemporaneous correlations between the innovations in YL^n, G and F indicate that the deficit is at least partially under control of the government. The negative coefficient on deficit surprises can therefore not explained by "spurious correlation".

The last comment concerns the formulation of the model which generates the vector of surprises and the expected real interest rate (equation 18). We have used a VAR–system. Some might argue that there exist long–run "equilibrium conditions" between the variables YL^n, G, F and r. In this case better estimates of the innovations could probably be achieved if one would apply the recently developed cointegration techniques (see Engle, Granger 1987). Some attempts to use this approach were not very successful. First, it is not easy to determine the correct degree of integration (see Evans, Savin 1984 and Schwert 1987 for a discussion of the low power of some proposed tests, e.g. Dickey–Fuller–test). For instance, a Dickey–Fuller test does not reject the null hypothesis of a random walk for the real interest rate r; but the alternative that r is stationary could also be maintained. Second, using a cointegration model für YL^n, G, F and r results in worsening of the estimation results for the consumption function: The standard error of regression is now higher, the t–values are lower, and in the case of a logistic function for λ the estimates for λ_0 and λ_1 imply a weight for liquidity constraint consumption of 99,1 % in 1955 and 0,3 % in 1985. Such values are highly implausible. Given these results, the chosen procedure seems to be a reasonable strategy to model the income process.

4. Conclusions

In this paper an empirical test of the impacts of government deficit on the change in private consumption is presented. This test uses a surprise consumption function approach and is consistent with intertemporal optimizing behaviour but allows that some households are liquidity constrained. The most important results can be summarized as follows: 1) A substantial fraction of private households is liquidity constrained. Their consumption tracks current labour income. 2) For the other households the empirical results are consistent with life cycle optimization: Expected real interest rates increase planned changes in consumption and unexpected movements of income, government expenditure, deficits and real interest rates induce unexpected consumption changes. The estimated regression coefficients are not statistically significant in all cases, but have always the expected sign. Most important with respect to the subject of this paper rational households reduce their consumption when they observe an unexpected government deficit.

When an economy suffers Keynesian unemployment, running a deficit may be a welfare–enhancing policy in the short–run. Since the long–run consequences of deficit–financing may be a reduction of the capital stock and hence real income (see for instance the simulation results in Auerbach, Kotlikoff 1987) such a policy can have high social costs. Clearly, the model considered in this paper can give answers only to relatively short–run problems. This is a limitation of the present study. But it is extremely difficult to measure the long–run consequences of deficits given the level of noise in macroeconomic data.

Data appendix:

YL^n: Real net labour income
This variable is constructed as the sum of nominal net labour income plus transfers from the government to the household sector, divided by the price index of consumption goods.

G: Real government expenditure for goods and services (including investment goods).

F: Real government deficit
This variable is defined as nominal government deficit (including social security), divided by the price index of consumption goods.

C: Real private consumption expenditure

r: Real interest rate (in percent)
$r_t = RS_{t-1} - WP_t$, where RS is the yearly average of the interest rate on saving deposits (mean rate of 3 and 12 month notice) and WP is the inflation rate of consumption goods.

The variables YL^n, G, F and C are all per capita values (in thousands of DM). Source for YL^n, G, F, C and the price index: Statistisches Bundesamt, Fachserie 18: Volkswirtschaftliche Gesamtrechnungen. Source for r: Deutsche Bundesbank, Monatsberichte.

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LONG RUN PRIVATE CONSUMPTION BEHAVIOR AND RICARDIAN EQUVIALENCE

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ABSTRACT

Recent empirical work on the Ricardian equivalence proposition has produced sharply conflicting evidence. In this paper, I use a standard version of the permanent income hypothesis to derive a test for Ricardian equivalence that exploits long-run information in time series data. The test is based on the idea that "Ricardian consumers" will base their consumption decisions on an income concept which takes into account the intertemporal government budget constraint. Under Ricardian equivalence, consumption should be cointegrated with the modified income concept. It is possible, however, that time series data do not contain sufficient information for discriminating between Ricardian and non-Ricardian restrictions on long-run consumption behavior. The suggested test explicitly allows for this possibility.

Evidence from cointegration tests supports the conclusion that U.S. data spanning 1949-84 do not contain sufficient information to accept or reject the derived long-run implication of Ricardian equivalence. Austrian data, on the contrary, are shown to contradict Ricardian equivalence. The discriminatory power of the data in the Austrian case can be traced to a permanent shift in the mix between tax and bond financing of public expenditures around 1975.

I. INTRODUCTION

How changes in public deficits and debt affect private sector behavior is a perennial question in macroeconomics. The Ricardian equivalence proposition (REP), forcefully restated by Robert Barro (1974), holds that a change in the mix between tax and bond financing of a given public expenditure stream induces an offsetting change in private savings, leaving the economy's total savings unaffected.

Recent empirical tests of the REP using U.S. data rely on estimating consumption functions which include variables measuring public sector activities (taxes, transfers, expenditures, etc.) among the regressors. Restrictions on the parameters of these variables are employed to test whether consumers behave in a Ricardian or non-Ricardian manner. The empirical work following this approach has produced sharply conflicting evidence. Roger Kormendi (1983, p. 1007) reports "decisive rejection" of a specification assuming non-Ricardian behavior. Franco Modigliani and Arlie Sterling (1986, p. 1179), on the contrary, conclude that the data are "strikingly and unmistakenly consistent" with the assumption of non-Ricardian behavior. Other authors have adopted an agnostic attitude. James Poterba and Lawrence Summers (1987, p. 378), for example, point out that until recently U.S. economic history "has provided relatively few powerful tests" of the REP. A comprehensive survey of this research is contained in Douglas Bernheim (1987).

In this paper, I derive a novel test for Ricardian equivalence that exploits the long-run information in time series for private consumption, income, and budget deficits. I assume that a standard version of the permanent income hypothesis (PIH) describes consumption behavior. Under the PIH, consumption and a modified measure of disposable income which takes into account the government budget constraint should be cointegrated. Recent tests of the PIH have generally ignored this implication of infinitely forward looking consumption behavior. John Campbell's (1987) "rainy days" test of the PIH, for example, is based on the assumption that consumption and conventially defined disposable income are cointegrated, thereby effectively ignoring the government budget constraint. It can be shown, however, that these two implications will be observationally equivalent if no permanent shift in the mix between tax and bond financing took place in the time period used for testing. In this case, both measures of income will be cointegrated with consumption and the long-run information in the data will not be informative for testing the REP.

The finding that the data do not contain long-run information for testing Ricardian equivalence should go a long way towards explaining the disparate conclusions reached by different authors. I report results of cointegration tests for U.S. data covering the period 1949-84, supporting the thesis that the data are not informative. For illustration, I also report results for Austrian data that clearly reject the hypothesis that consumers take into account the government budget constraint. The discriminatory power of the data in the Austrian case can be traced to a permanent shift in the mix between tax and bond financing of public expenditures around 1975.

The paper is organized as follows: Section II discusses the theoretical framework. Section III reports the empirical findings for the U.S.A. and Austria. Finally, section IV contains a brief discussion of the results.

II. THEORETICAL CONSIDERATIONS

I assume that the standard life cycle-permanent income hypothesis with infinite horizon and rational expectations describes private consumption behavior

$$(1) \quad C_t = \Gamma(r/(1+r)) \, [A_t + \sum_{i=0}^{\infty}(1/(1+r))^i \, E_t(Y_{t+i} - T_{t+i})].$$

In (1), C_t is real private consumption, A_t real non-human assets including real public debt, Y_t real gross labor income, T_t taxes net of transfers, r is the after-tax real interest rate taken to be constant and Γ is a proportionality factor. E_t denotes the mathematical expectations operator conditional on full public information at time t. The expression in brackets times r/(1+r) denotes permanent income defined as the annuity value of the sum of human and non-human wealth.

Decision rule (1) for private consumption is based on a set of rather restrictive assumptions. First, real interest rates are assumed constant and consumers can freely borrow and lend at this rate in a fictitious capital market subject only to their life time budget constraint. Second, the utility function is time-separable in private consumption and excludes public consumption as an argument. Third, the optimization problem of the representative consumer exhibits certainty equivalence, thereby excluding precautionary motives. And fourth, the planning horizon stretches into infinity and decisions are based on all relevant information about the future course of income and taxes.

The last assumption implies that private agents take into account the government budget constraint when deciding on consumption and saving

$$(2) \qquad B_t = \sum_{i=0}^{\infty}(1/(1+r))^i E_t T_{t+i} - \sum_{i=0}^{\infty}(1/(1+r))^i E_t G_{t+i}.$$

Here, B_t denotes real public debt and G_t real government consumption. This equation can be derived by iterating the government budget identity, $B_{t+1} := (1+r)[B_t+G_t-T_t]$, forward in time and imposing a solvency constraint.[1]

Inserting (2) in (1) results in

$$(3) \quad C_t = \Gamma(r/(1+r)) \, [(A_t-B_t) + \sum_{i=0}^{\infty}(1/(1+r))^i \, E_t(Y_{t+i}-G_{t+i})].$$

Equation (3) is the decision rule for consumption of private households under the standard version of the PIH used in this paper. Consumers will not consider the public debt as net private wealth and they deduct consumption of real resources by the public sector from gross income instead of taxes. In other words, Ricardian equivalence is assumed to hold.

Recent literature on testing the PIH, e.g. John Campbell (1987), has generally ignored the implication that consumers with infinite horizon and rational expectations will take into account the government budget constraint for calculating permanent income. They have relied on equation (1) or some "Euler equation" analogue of it to derive testable implications. In the next step, I show that equations (1) and (3) have rather different implications.

For the further derivations, I identify disposable income of private households (YD) as conventionally measured with the following expression

$$(4) \quad YD_t = (r/(1+r))A_t + Y_t - T_t.$$

Similarly, a measure of "Ricardian disposable income" (YD^*) can be defined as

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1 The government budget identity excludes financing of the budget deficit by printing money. This assumption is probably not too restrictive for countries with low inflation rates.

$$(4)' \quad YD^*_t = (r/(1+r))(A_t - B_t) + Y_t - G_t.$$

Multiplying (4) and (4)' by the proportionality constant Γ and subtracting these expressions from (1) and (3) respectively, results after some algebraic manipulations in the following equations

$$(5) \quad C_t - \Gamma YD_t = \Gamma \sum_{i=1}^{\infty} (1/(1+r))^i \, E_t[\Delta Y_{t+i} - \Delta T_{t+i}]$$

and

$$(5)' \quad C_t - \Gamma YD^*_t = \Gamma \sum_{i=1}^{\infty} (1/(1+r))^i \, E_t[\Delta Y_{t+i} - \Delta G_{t+i}].$$

An equation similar to (5) was first studied by Campbell (1987). To repeat, (5) is based on an unexplained failure of consumers to take into account the government budget constraint whereas (5)'is based on the full set of restrictions. As pointed out by Campbell, equations of this type possess a nice economic interpretation: The linear combination on the left hand side corresponds to a measure of savings (exactly so if Γ is 1) and is an optimal predictor of expected future changes in the income measures on the right hand side.

Using the concept of cointegration recently developed by Robert Engle and Clive Granger (1987), testable long-run implications can be derived from (5) and (5)'. Under the maintained hypothesis that gross labor income, net taxes, and government expenditures are stationary after first differencing, the sums on the right hand sides of (5) and (5)' will also be stationary. Given that consumption and the disposable income variables are non-stationary variables[2] in levels, (5) implies that C_t is cointegrated with YD_t

2 If Γ is smaller than 1, consumption as well as disposable income will not be stationary after first differencing (see Campbell (1987, p. 1254). But the linear combination between the two variables will still be stationary.

and (5)' implies that C_t is cointegrated with YD^*_t.[3]

These results suggest the following test: Given the standard permanent income hypothesis is valid, C_t and YD^*_t should be cointegrated according to (5)'. But this result does not imply that C_t and YD_t can not be cointegrated. The Ricardian measure of disposable income, YD^*_t, is simply the sum of conventially measured disposable income, YD_t, and the government budget deficit. C_t and YD_t could as well be cointegrated if the deficit was stationary in the period under observation. In this case, the long-run information in the data will not be sufficiently powerful to accept or reject Ricardian equivalence. In summary, the cointegration tests could give rise to four possibilities:

(i) C_t is cointegrated with YD_t as well as YD^*_t. The data generated by this economy should be judged uninformative with respect to the validity of the REP.

(ii) C_t cointegrates with YD_t but not with YD^*_t. The standard version of the PIH which implies Ricardian equivalence has to be rejected.

(iii) C_t cointegrates with YD^*_t but not with YD_t. PIH as well as REP can not be rejected.

(iv) C_t neither cointegrates with YD_t nor with YD^*_t. The assumed behavioral hypothesis for long run consumption behavior is rejected and we can infer nothing concerning the validity of the REP within the testing framework of this paper.

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3 Engle and Granger (1987) call a variable X_t integrated of order one if it is stationary after first differencing. Two variables X_t and Y_t are called cointegrated if they are (i) individually integrated of order one but (ii) a linear combination between the two variables is integrated of order zero. For a more exact definition see Engle and Granger (1987, pp. 252-53).

III. EMPIRICAL RESULTS

The empirical analysis is based on Austrian and U.S. data. The data for the USA are taken from Modigliani and Sterling (1986). These authors perform several adjustments of the official data as described in the data appendix of their paper. The Ricardian measure of income employed in this paper was constructed by simply deducting the public deficit variable as listed by Modigliani and Sterling from the conventionally defined income variable. The Austrian data were constructed as follows: The income variable corresponds to the disposable income of private households. The Ricardo definition of income corresponds to the sum of convential disposable income and public savings as reported in the national income accounts.[4] This definition effectively pools private and public savings behavior. The consumption concept is total private consumption. The deflator of total private consumption was used to convert nominal into real magnitudes.

As a first step, in table 1 I report results of Dickey-Fuller tests for unit roots in the time series used for testing cointegration. The null-hypothesis of non-stationarity in levels of the three variables in lines (1) to (3) for both countries can not be rejected (results are not reported). Lines (1) to (3) contain test statistics for unit root tests after first differencing of the variables. A comparison of the statistics with the critical values listed at the bottom of table 1 reveals that all variables with the possible exception of the U.S. consumption series can be judged stationary after first differencing with high confidence.

In lines (5) and (6), Dickey-Fuller tests for the level as well as the first difference of the public deficit variable (D_t) are reported. If permanent shocks in this variable occured during the observation period, D_t should be non-stationary in levels. This is the case for the Austrian time series. The U.S. fiscal deficit

4 Note that public savings as calculated in the national income accounts is the appropriate "public deficit" variable because it does not depend on public investment expenditures.

appears to be stationary in levels according to the test statistics in line 5.

The results of the tests for cointegration in table 2 can be summarized as follows: For the Austrian data, consumption is cointegrated with conventially defined disposable income (equation 1) but not with the Ricardian definition of income (equation 2). The Dickey-Fuller tests reject the null hypothesis of no cointegration for convential disposable income at very low significance levels. In the cointegrating regression using the Ricardian income concept, the null hypothesis can not be rejected and the DW test statistic drops markedly. From the regressions for the U.S. data (equations 3 and 4) the conclusion is that cointegration seems to hold for both income concepts.[5] Here the test statistics for cointegration as well as the coefficient estimates are very similar across the equations demonstrating that the data do not contain enough information to discriminate between Ricardian and non-Ricardian behavior.

The results for the United States reflect the fact that the time series for the U.S. fiscal deficit was stationary over the period of observation. As more data on the U.S. experience with historically high peace-time deficits in the 80s become available, the data might well reject the REP as asserted by Poterba and Summers (1987), who evaluate data up to 1986 on an informal basis. The information content in the Austrian case can be traced to a change in the mix between bond and tax financing around 1975. Although, the boosting of the public deficit was originally planned as a transitory measure to stabilize aggregate demand in the face of the first oil shock, it became permanent in the aftermath. As a result, public savings in percentages of private disposable income declined from 10.6 % (mean 1960-75) to 4.2 % (mean 1975-86). The permanent shock in the Ricardian measure of disposable income was obviously ignored by Austrian consumers who

5 As pointed out by Anindya Banerjee et.al. (1986), OLS-estimates of cointegration vectors can exhibit considerable small-sample bias despite the "superconsistency"-result derived for this estimator by James Stock (1987). They suggest R^2 as an index for judging whether the bias is small. Informally, an estimate higher than .95 should be assurance that the bias is small. The reported R^2-statistics in table 2 are always higher than .95.

based their consumption plans on disposable income as conventially measured instead.

Table 1: Dickey-Fuller tests for unit roots*

		Country/Test Statistic			
		Austria 1960:86		USA 1949:84	
	Variable	DF	ADF	DF	ADF
(1)	ΔC_t	-6.00	-3.62	-2.84	-3.19
(2)	ΔYD_t	-4.51	-4.38	-5.47	-3.76
(3)	ΔYD^*_t	-3.73	-3.05	-5.31	-4.71
(4)	D_t	-1.73	-2.35	-3.16	-4.99
(5)	ΔD_t	-3.72	-4.53	-	-

* These tests are based on the regression $\Delta X_t = \mu + \alpha X_{t-1} + \sum_{i=1}^{n} \beta_i \Delta X_{t-i}$. The Dickey-Fuller test (DF) assumes n=0 and uses the t-statistic on α. The augmented Dickey-Fuller tests (ADF) were performed setting $n = 1$ and again use the t-statistic on α. Critical values from Fuller (1976) are: 1% -3.75; 5% -3.0; 10% -2.63; (sample size = 25).

Table 2: Cointegration tests*

Austria 1960:86

(1) $C_t = 5.746 + .892\ YD_t$ $\quad R^2 = .997$
$\qquad (4.219) \quad (.010)$

DW = 1.59 $\quad$ DF = -4.02 $\quad$ ADF = -3.37

(2) $C_t = -27.541 + .910\ YD^*_t$ $\quad R^2 = .984$
$\qquad (10.252) \quad (.023)$

DW = .45 $\quad$ DF = -2.02 $\quad$ ADF = -2.19

USA 1949:84

(3) $C_t = -.201 + .976\ YD_t$ $\quad R^2 = .990$
$\qquad (.058) \quad (.017)$

DW = .911 $\quad$ DF = -3.36 $\quad$ ADF = -3.14

(4) $C_t = -.319 + .997\ YD^*_t$ $\quad R^2 = .969$
$\qquad (.108) \quad (.031)$

DW = .818 $\quad$ DF = -2.86 $\quad$ ADF = -3.33

* The Engle and Granger (1987, p.269) critical values for the null hypothesis of no cointegration are Durbin-Watson (DW) 1 % .511, 5 % .386, 10 % .322; Dickey-Fuller (DF) 1 % -4.07, 5 % -3.37, 10 % -3.03; Augmented Dickey-Fuller (ADF) 1 % -3.77, 5 % -3.17, 10 % -2.84. These critical values are based on sample sizes of 100. Engle and Yoo (1987) present critical values for cointegration tests for sample sizes of 50. Their values are only slightly below those listed above.

IV. DISCUSSION

The empirical results in the preceding section suggest that the typical U.S. data set does not contain sufficient long-run information to accept or reject the REP whereas Austrian data clearly reject the proposition. Most of the research on this issue, summarized conveniently by Bernheim (1987, p. 278), has typically relied on regressing consumption on income, wealth, and variables like taxes, government expenditures, transfers, and public debt. If these regressions are performed in levels, the non-stationarity of the listed variables could make inference from the regression results hazardous. The usual asymptotic distribution theory can not be invoked and if consumption is cointegrated with a subset of the right-hand variables, the coefficients on the other non-stationary variables should be zero by construction. First differencing, as recommended by Kormendi (1983), would not be an appropriate remedy, however, as this operation destroys the long-run information in data. And this type of information is likely to be most helpful to distinguish between Ricardian and non-Ricardian behavior in view of the notorious difficulties to discriminate between macroeconomic hypotheses on the basis of noisy short-run fluctuations in time series. Given these considerations, the mixed evidence reached by different authors working with U.S. data should not come as a surprise.

Empirical evaluation of the REP faces some awkward questions as far as the quality and economic meaningfulness of the measures of public sector activities is concerned. Well known non-sensical accounting conventions are discussed extensively by Robert Eisner (1986). Further research should therefore investigate whether the empirical results of the cointegration tests suggested in this paper are robust with respect to different measures of public sector activities.

A rather different issue is the question what can be learnt from rejections of the REP on the basis of cointegration tests. I have argued in this paper that Ricardian equivalence is a direct implication of the standard PIH. Therefore, rejections of the REP should lead to a reconsideration of the assumptions the PIH is derived from. Now, the standard version of the PIH as interpreted in literature is generally considered as being inconsistent with at least some features of the data (see e.g. Angus Deaton (1986)). Recent research concentrates on the separability assumption (e.g. John Muellbauer (1986)) or the assumptions of perfect capital markets (e.g. John Campbell and Gregory Mankiw (1987)) as the critical assumptions to be reconsidered. Rejection of the REP, however, points to the assumptions of an infinite planning horizon and rational expectations as the most likely culprits for the failure of the PIH. Finite horizons have always been considered as one of the possible stumbling blocks for Ricardian equivalence. Reconsideration of rational expectations as a working hypothesis is plausible because of the information assumptions necessary to derive the REP.

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