AN EXPERIMENT WITH TEN DUOPOLY GAMES
AND BEAT-THE-AVERAGE BEHAVIOR*)

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1. INTRODUCTION

This is a report on an experiment combined with the teaching of a seminar on competition, oligopoly and the theory of games during May and June 1970 at the Institute for Advanced Studies at Vienna. Ten teams, each with two players played for sixteen periods in the identical oligopolistic market. No player was informed of either the market structure or of any of the parameters of the market before the play.

The players were informed of their costs, production limits and inventory carrying costs. A small money prize (100 Austrian schillings) was offered to the player who managed to achieve the highest score (profits) relative to his competitor. This briefing had the effect of converting a nonconstant sum two-person game into a two-person zero-sum game where the goals of the players on a team could be described in terms of:

$$\max_{s_1} \min_{s_2} \left\{ P_1(s_1,s_2) - P_2(s_1,s_2) \right\}$$

where P_1 and P_2 are the payoff functions of each player and s_1 and s_2 stand for their strategies.

+) The authors wish to thank Prof. G. Bruckmann for his valuable comments, especially his observations on the dynamic optimal advertising policy.

2. THE GAME

2.1. THE STRUCTURE OF THE GAME

The experiment was based on a business game constructed by SHUBIK and LEVITAN [1] . A short sketch of the main features of this game is given below.

2.1.1. Scenario

The game described here is an "environment poor" game in the sense that all rules are formal and well described for the actual playing of the game. The firms and their market are completely specified by the structure of the model and the information resulting from their interplay is presented in form of balance sheets and industry statistics.

2.1.2. Number of Players

The number of players can be varied from one to twelve, in our experiment was two. There is only one product per player and one market. A team consisted of a single player.

2.1.3. Demand Conditions

The overall demand leaving out the effect of advertising and assuming that the prices charged by all firms are identical is linear. When the prices charged by the firms are not the same there is a rationing scheme which computes the contingent demand schedules. Lost sales are also recorded.

The demand equation for the product of the i^{th} firm is given by

$$F_i(p,a) = \frac{1}{n}\left[\alpha - \beta\,(p_i + \gamma\,(p_i - \bar{p}))\right] f_i(\theta, a)(1 + \eta\sqrt{\sum a_j})$$

where

$$f_i(\theta, a) = \begin{cases} \theta + (1-\theta)\dfrac{na_i}{\sum a_j} & \text{if } \sum_{j=1}^{n} a_j > 0 \\ 1 \quad \text{if } \sum_{j=1}^{n} a_j = 0. \end{cases}$$

p_i = price charged by player i,
a_i = advertising expenditure by player i,

$$\bar{p} = \sum_{i=1}^{n} p_i / n$$

α = the size of the market when all $p_i=0$,
β = price sensitivity of overall demand,
γ = inter-firm price sensitivity coefficient,
η = cooperative advertising coefficient,
θ = competitive advertising coefficient.

There are three major components of the formula:
The first deals with the effects of price and price relationship in the market; the second and third account for, respectively, the competitive and overall institutional effects of advertising on firm and industry demand. A fourth term representing trend, cycle and random elements was ommitted in the version under consideration.

2.1.4. Cost Conditions

Production is an independent variable. Production costs are linear. There is an upper bound specified on the size of production during any quarter. There are costs attached to changing the level of production. The overheads are specified as parameters of the program. Depreciation depends in a linear manner upon the amount of capacity of the firm. It was constant in this experiment because capacity was not changed. There are constant unit inventory carrying costs.

Advertising costs are merely the direct expenditures for advertising.

2.1.5. Random Variables

The introduction of random variables is optional. One may be introduced to influence overall demand and others to effect individual advertising. (Not used in this experiment.)

2.1.6. Financing

Bank loans are made automatically as the cash needs of the firm call for them. A rate of interest on loans has to be specified.

2.1.7. Ruin Conditions

The game is designed to allow the study of ruin and entry of new firms in the market. In this experiment credit lines, up to which borrowing is automatic, were set so high that no ruin was possible. Also entry of new firms was avoided.

2.1.8. Revenue Function

The following symbols are defined:

π_i = before tax net revenue for the i^{th} firm,

σ_i = actual sales for the i^{th} firm,

c = unit cost of production,

K_i = fixed costs of the i^{th} firm,

c_I = unit inventory carrying cost per period,

I_i = average inventory level of the i^{th} firm,

a_i = advertising expenditures of the i^{th} firm.

The before tax profit of the firm is:

$$\Pi_i = p_i \sigma_i - c \sigma_i - c_I I_i - a_i - K_i .$$

A tax is imposed on net profits. The tax rate is known by the players.

2.1.9. Decision Variables

There are seven possible decision variables under the control of the individual firm: price
advertising
production level
quantity offered for sale
dividends
investment
exit

Only the first three were used in the experiment described here. Every quarter the firms have an opportunity to select a new price for their product. As can be seen from the demand-equation, this version does not have any time lag effect of price. Demand is independent of previous fluctuations.

Advertising appears as an independent decision variable: Competitive and cooperative effects of advertising are controlled by different parameters. The impact of advertising may be cumulatively lagged over several periods.

The firms are required to select a production rate during each quarter. Production scheduling appears in only a rudimentary form, in the sense that there are inventory carrying costs for finished goods and there are costs associated with changing the levels of production in either direction. Half of the current production plus initial inventories are available for sale in each period.

2.1.10. Payoff and Objective Function

The players were instructed to maximize their cumulated profit differences, i.e. the largest positive difference of current net worth at the end of play. The players were also instructed not to communicate with their competitors (although the game does not provide an opportunity for cooperation even with communication).

2.1.11. Information Conditions and Briefing

The players did not know the functional forms which provide the basic structure of the game. The parameters, they did know are displayed in Exhibit 1. Briefing was done verbally in class stating only the goal of the game and some general rules of playing.

The players were informed after each period about the specific actions and the complete balance sheet of their competitor, as well as obtaining their own balance sheets and market information. At the beginning, the information for one identical sample period was given to all players. These initial values served to give them some insight in the mechansim of the game.

2.1.12. Frequency of Play

The game was played at the rate of one period a day for 16 periods. Time spent for decision could be varied by the players, as they could turn in their decisions any time before the deadline for the computer run.

2.1.13. Termination

The end of the game was decided by a random process that gave a probability of 1/2 to termination (respectively continuance) after period 15. The players were not informed about that in advance.

EXHIBIT 1

INSTITUTE FOR ADVANCED STUDIES

B U S I N E S S   G A M E

May 1970

P1

LIST OF PARAMETERS

Unit cost of production (variable Stückkosten)	150	
Unit inventory carrying cost (Lagerkosten pro Stück)	10	
Unit cost of change of production (zusätzl. Stückkosten bei Änderung d. Produktion)	20	
Administrative costs (Fixkosten, administrativ)	500,000	per period
Depreciation (Abschreibung)	250,000	per period
Income tax (Steuer)	50 %	per period
Rate of interest on loans (Zins für Darlehen)	3 %	per period

You do not know the parameters for market demand.

2.2. PARAMETERS

2.2.1. Market Parameters

α	"size of the market" was set to	1.04×10^{6}
β	price sensititity of overall market to average price	0.266×10^{4}
γ	inter-firm price sensitivity coefficient	0.3×10^{1}
η	cooperative advertising coefficient	1.212×10^{-4}
θ	competitive advertising coefficient	1.

That means that there is no competitive advertising effect and the best advertising strategy is a let-your-competitor advertise-for-you policy.+) This was done to stress the beat-the-average features of the game and to simplify the

2.2.2. Cost Parameters

c_I	Finished goods unit inventory carrying cost	10
c	average unit production cost	150
Δq	unit cost of change of production	20
ρ	discount rate	3 %
K	fixed costs or overheads (administrative only)	500,000
	depreciation	250,000
	maximum permitted production per quarter	500,000

2.2.3. Advertising Parameters

Effectiveness weights for previous and current advertising:

3 periods ago	o
2 periods ago	o
1 period ago	.5
current	.5

+) This is not quite accurate. See 3.1.1. for further discussion.

2.3. INITIAL CONDITIONS

Initial cash	$.2 \times 10^8$
initial inventory	200,000
advertising expenditures	
3 periods ago	6,000,000
2 periods ago	6,000,000
1 period ago	6,000,000
last production rate	250,000

2.4. INITIAL DECISIONS

The decisions for the sample period were made by the referee in the following way:

	Price	Advertising	Production
Player 1	240	6,000,000	250,000
Player 2	240	6,000,000	250,000

All parameters, initial conditions and initial decisions were identical for all players.

2.5. THEORETICAL SOLUTIONS

In this section the computation of the joint-maximization, non-cooperative and beat-the-average solutions for the symmetric game, as played in the experiment are given. Detailed development of the pertaining formulaeis given in [1].

These three solutions serve to provide a reasonable upper bound for cooperative behaviour (jm), a lower bound (ba) and a solution which may be regarded as a measure of "inner direction" (nc).

2.5.1. Joint Maximization

If each firm takes a highly cooperative attitude towards all the others, we may consider that each firm acts to maximize the sum of the profits of all firms. By symmetry we may assume all prices and advertising budgets equal and maximize the profit of one firm. The optimal solution gives:

Price	=	277.5
Advertising	=	2,517,768
Production	=	154,080
Net Profit	=	7,803,487

2.5.2. Non-Cooperative Equilibrium

This solution describes an individual being concerned solely with his own payoffs to the exclusion of the welfare of all others.

Price	=	225
Advertising	=	486,372
Production	=	222,630
Net Profit	=	7,173,855

2.5.3. Beat-the-Average

A beat-the-average solution is an equilibrium solution of the transformed game with payoffs.

$$P_i = \Pi_i - \frac{1}{n-1} \sum_{j \neq i} \Pi_j$$

In the case of a duopoly this amounts to

$$P_i = \Pi_i - \Pi_j$$

Price	=	204
Advertising	=	0
Production	=	248,000
Net Profit	=	5,700,999

2.5.4. The Efficient Point or Competitive Equilibrium

If the firms were run by a central agency using marginal cost pricing, then:

Price	=	155
Advertising	=	0 ?
Production	=	308,160
Net Profit	=	-750,000

There are several conceptual problems with this solution. If advertising influences overall demand, as it does here, should the central agency advertise for the public benefit? We will assume zero advertising, but note that this is merely avoiding a deep conceptual difficulty, and is not a solution to it.

The net profit when p = 155 is negative. In a steady state the administrative costs and overheads must be paid. In the long run they too could be regarded as variable costs. For a steady state "breakeven point" price must be set at approximately p = 157,34.

2.5.5. The Starting Point

From the above calculations we see that the starting point for a player was such that his price was above the noncooperative equilibrium, but below the joint maximum. His advertising was far above the joint maximum. His production and inventory were too large for even the "beat-the-average" steady state. These are illustrated in Figure 1.

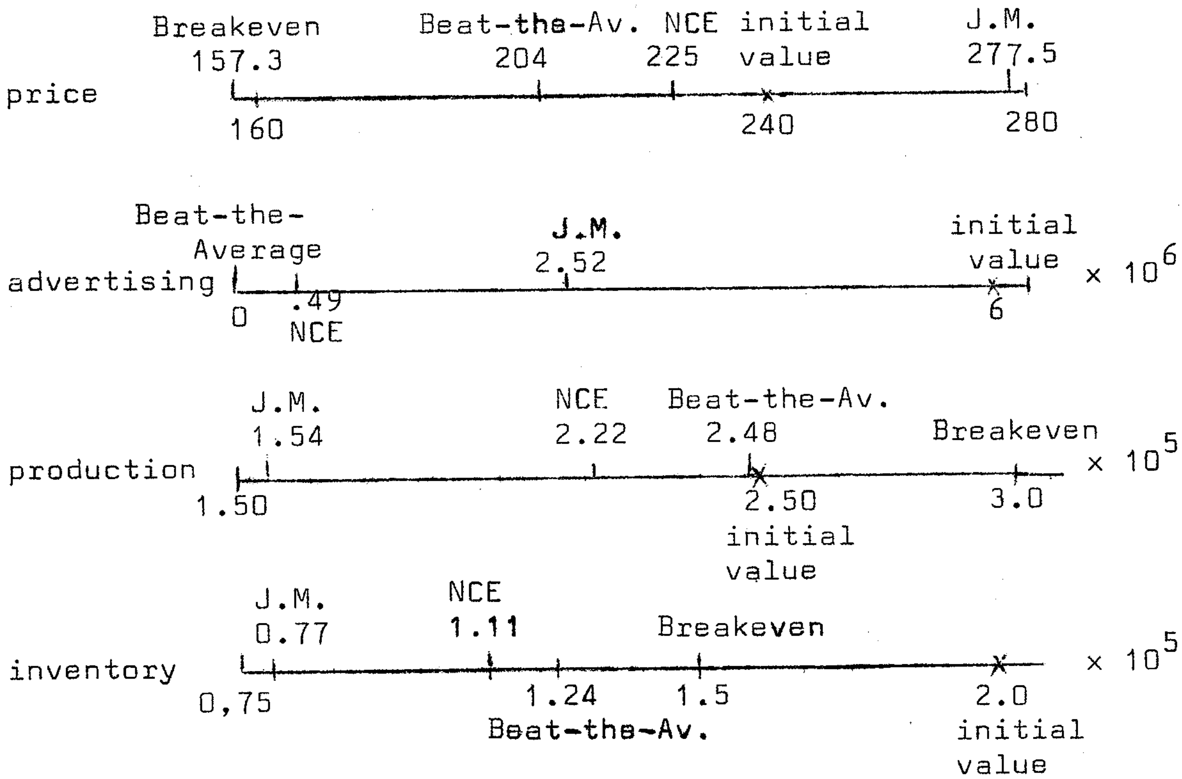

Breakeven
157.3
Beat-the-Av.
204
NCE
225
initial
value
J.M.
277.5
price
160
240
280
Beat-the-
Average
J.M.
2.52
initial
value
advertising
0
.49
NCE
6
× 10^6
J.M.
1.54
NCE
2.22
Beat-the-Av.
2.48
Breakeven
production
1.50
2.50
initial
value
3.0
× 10^5
J.M.
0.77
NCE
1.11
Breakeven
inventory
0,75
1.24
1.5
Beat-the-Av.
2.0
initial
value
× 10^5

Figure 1

3. DESCRIPTION OF EXPERIMENT

3.1. OVERALL PERFORMANCE

This section is devoted to the description of the players' behaviour in total. This is done by observing how aggregate variables changed from period to period and whether they had a tendency towards the theoretical solutions. The distribution of the variables at different times is examined in order to see if there is a reduction in variance towards the end. Then from the aggregate data something like an idealized player is constructed, the behaviour of which is described.

3.1.1. Aggregation

The graphs show for several variables

(a) the average value for that variable per period (continuous line) plus a 1σ band width (dotted lines). Also the theoretical solutions are plotted in,

(b) a histogram showing the distribution of that variable for the last 8 periods,

(c) a histogram showing the distribution of that variable for the last 4 periods.

Market Variables

The price of the sample-period was much too high compared to the theoretical solution for beat-the-average behaviour. On an average scale the players realized that and quickly came down to a suboptimal price; after that they settled in a somewhat cyclical way around the theoretical price. This is shown in Figure 2.

From the distribution it can be seen that the variance decreased towards the end. The mode still represents a suboptimal price indicating that most players did not fully recover from the strong fall in price at the beginning. But the long upper tails show that digression from the optimum to the right more frequently occurred than to the left.

Corresponding cycles in the opposite direction of aggregate sales can be observed. See Figure 3.

The optimal steady state solution of advertising expenditures was soon guessed by the players. In the last quarter of the play advertising virtually ceased.+)

+) However G. Bruckmann observed if one's opponent has stopped advertising and is charging a higher price than yourself, it pays to advertise a small amount to take advantage of this situation. An example shows this:

$$\Pi_1 - \Pi_2 = (1+\eta)\sqrt{(a_1+a_2)}\left[S_1(p_1-150) - S_2(p_2-150)\right] - a_1+a_2,$$

where S_i = the sales of the i^{th} firm.

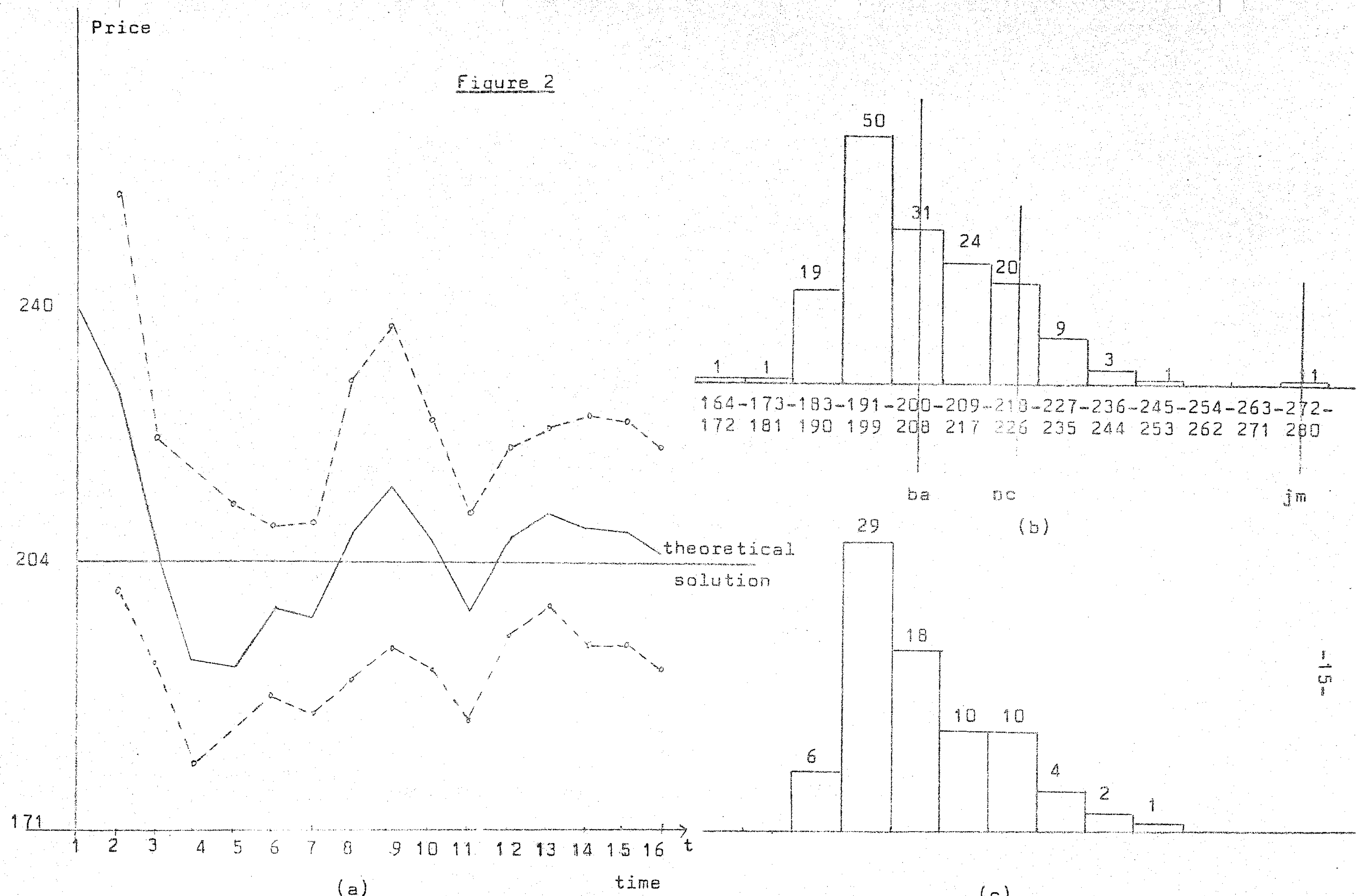
For a_1 to be optimal, given a_2 we have:

$$\frac{\partial(\Pi_1-\Pi_2)}{\partial a_1} = (1+\eta)\,\frac{S_1(p_1-150)-S_2(p_2-150)}{2\sqrt{a_1+a_2}} - 1 = 0 \quad \text{or}$$

$$a_1 = \left\{\frac{(1+\eta)}{2}\left[S_1(p_1-150)-S_2(p_2-150)\right]\right\}^2 - a_2$$

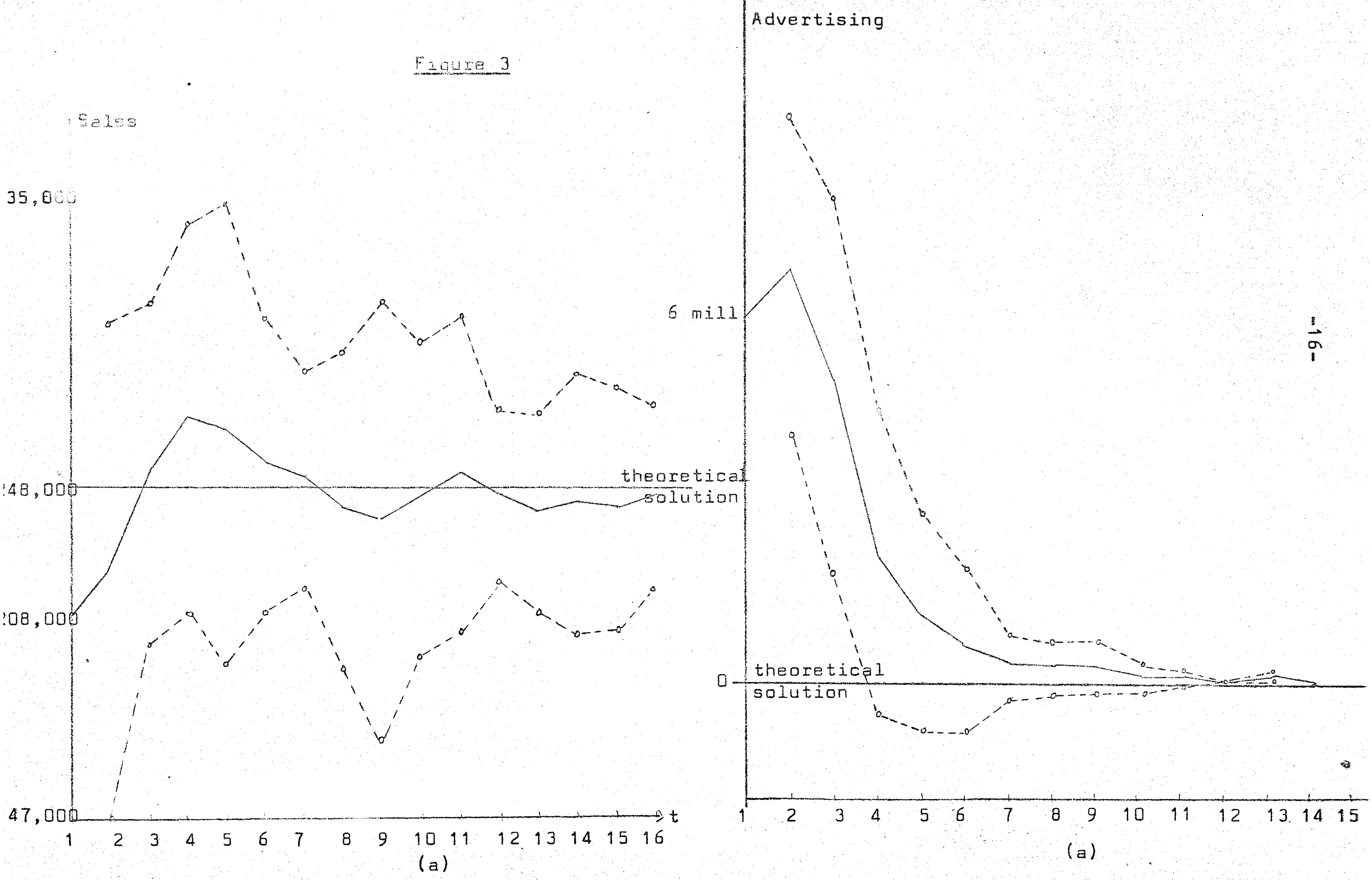
This was the reason given for his team maintaining a small amount of advertising when he expected to charge a lower price than his competitor.

Figure 2


Price
240
204
171
theoretical solution
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 t
time
1 1 19 50 31 24 20 9 3 1 1
164-172 173-181 183-190 191-199 200-208 209-217 218-226 227-235 236-244 245-253 254-262 263-271 272-280
ba
nc
jm
6 29 18 10 10 4 2 1

(a) (b) (c)

Figure 3


Sales
35,000
248,000
208,000
47,000
theoretical solution
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
t
(a)
Advertising
6 mill
0
theoretical solution
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
(a)

Internal Variables of the Firm

In a steady-state the theoretical solution requires production to be equal to sales and inventory to half of production. From the diagrams it can be seen that aggregate production fluctuated moderately around the theoretical value, inventory on the other hand was considerably higher (see Figure 4).

The main feature is the rise in production at the beginning. The hope of the players that by rising production and lowering price they were on the right way was not fulfilled. The decrease in price was too small. The players found themselves stuck with an inventory even higher than sales and had to try to reduce their inventory-holdings by curbing production and heading for higher sales. (In another related experiment inventory levels were found to be far higher than expected [2]).

Coordination

To reflect the degree of coordination between the market decisions and the internal decisions of the firm three further variables were constructed. These are shown in Figure 5.

$$EX_t = (\text{Inventory}_{t-1} + 1/2\ \text{Production}_t) - \text{Sales}_t$$

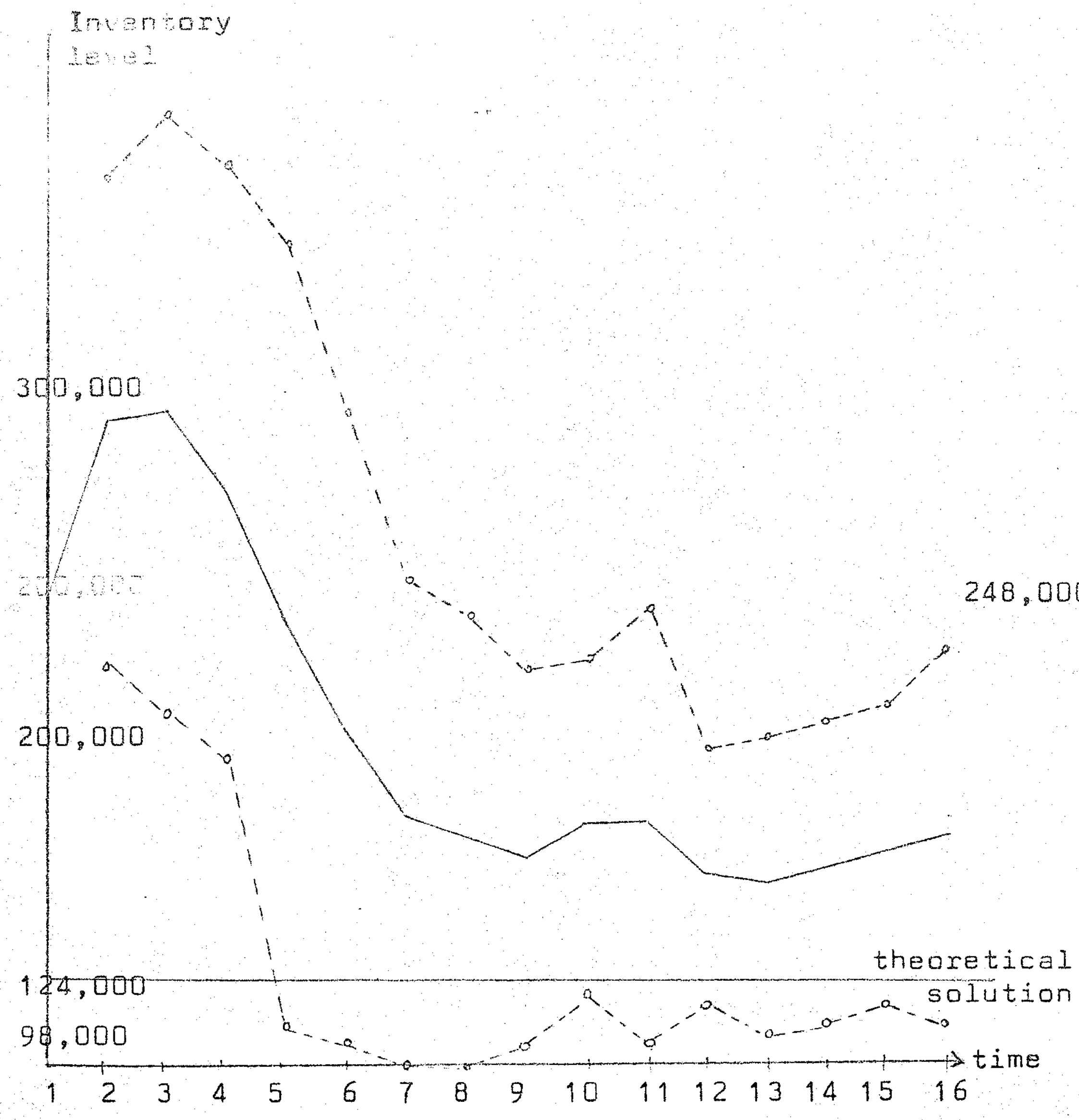
This is the excess available stock in each period. In a steady state the optimal solution is, of course, zero.

LOST SALES This is the deficiency of stock in each period. In a steady state the optimal solution is again zero.

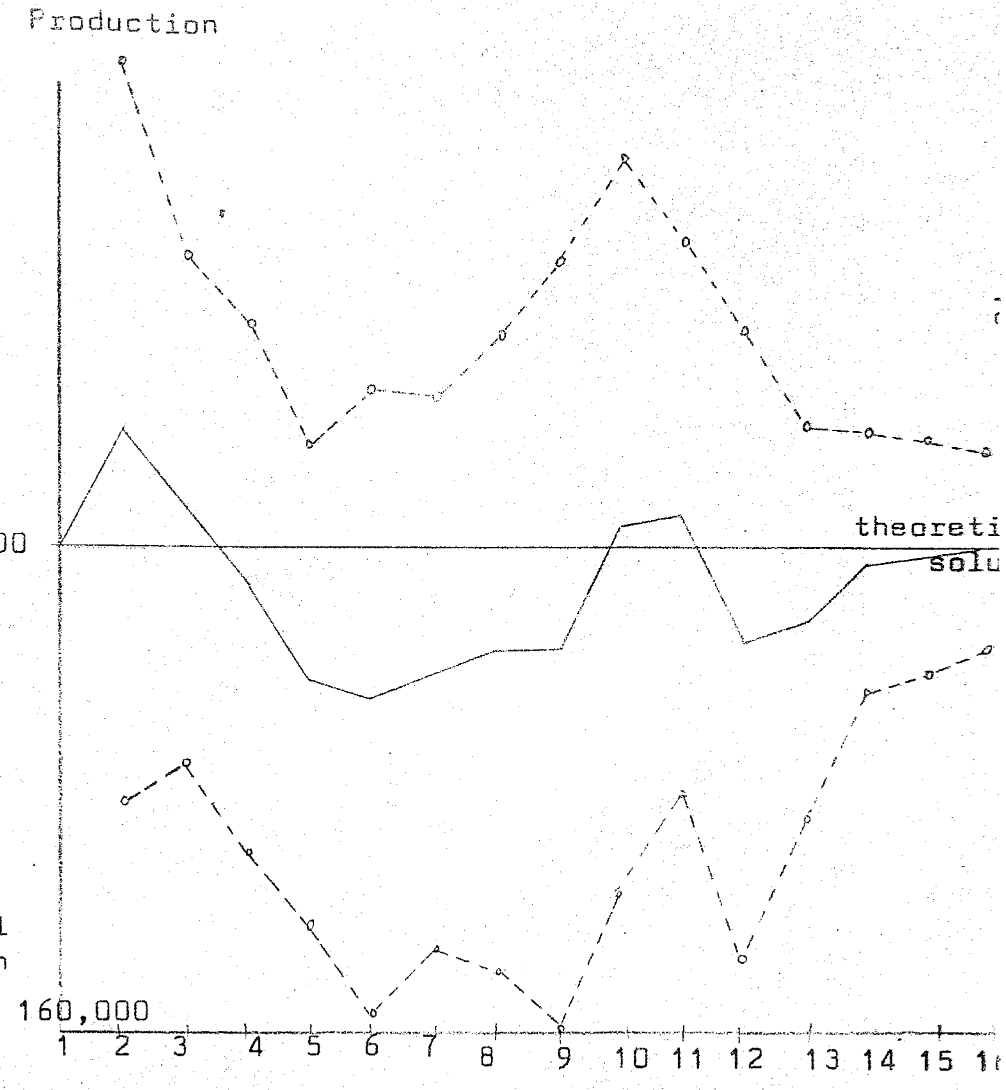
These two variables show the deviation of inventory-holdings and production scheduling from optimal in either direction. To have a handy combined measure a third variable is introduced.

$$IP_t = \begin{cases} EX_t & \\ \text{LOST SALES}_t & \text{if } EX_t = 0 \end{cases}$$

Figure 4

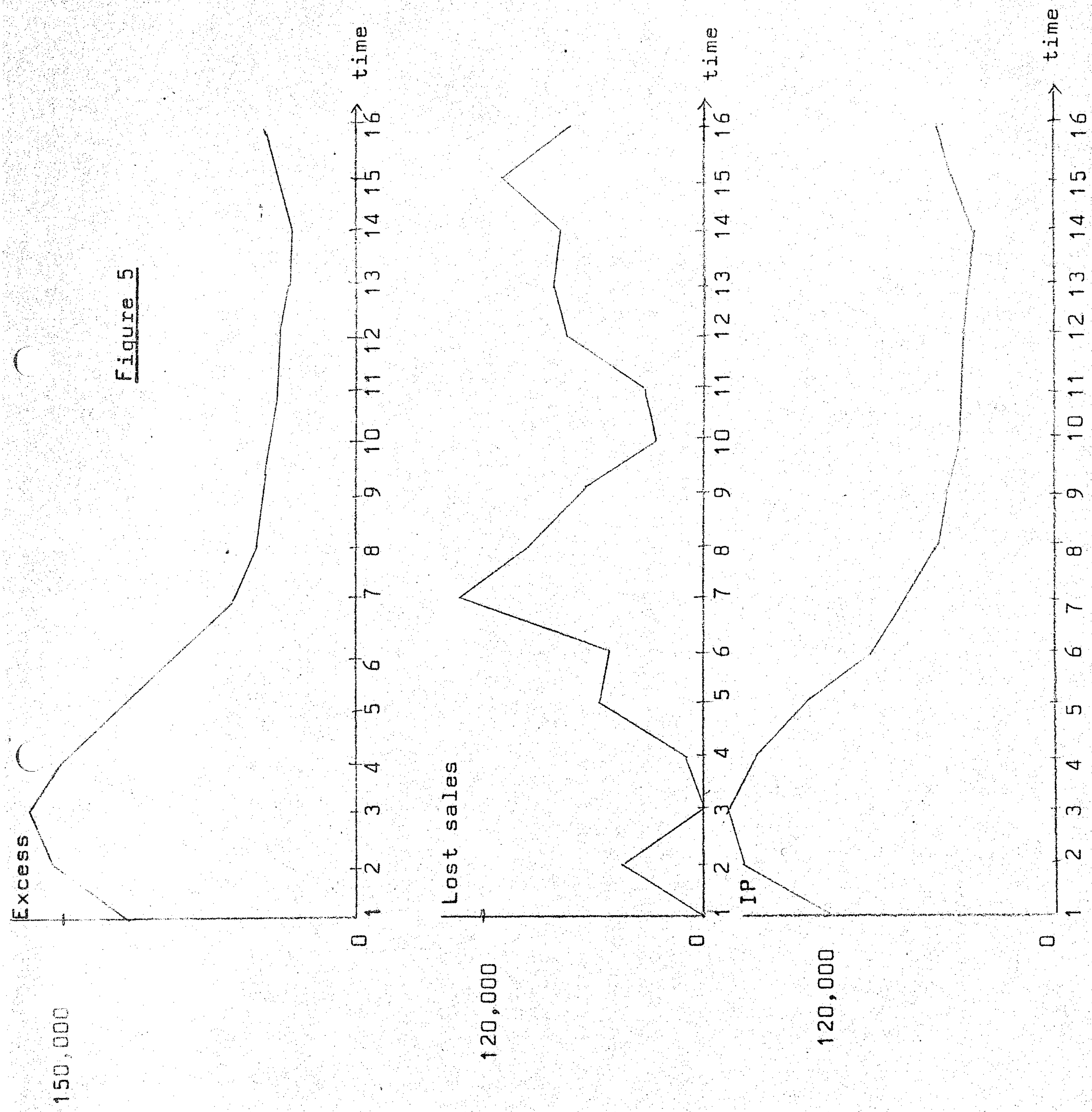

Inventory level
300,000
200,000
124,000
98,000
theoretical solution
time
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

(a)


Production
248,000
160,000
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

(a)

Figure 5



Excess
150,000
0
time
Lost sales
120,000
0
time
IP
120,000
0
time
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

From the diagrams it can be seen that the magnitude of excess stock is by far greater than of LOST SALES. This can be explained by the fact that incurring LOST SALES affects profit (let alone profit difference) much more seriously than carrying excess stock. In the second half of the play a convenient balance between risks and costs seems to be reached.

Profit Variables

Two markedly opposite trends in level of profits and profit differences can be observed. After some periods of experimentation profit level shows an upward trend. Players learnt how to make higher profit, but also did their competitors as can be seen from the decreasing variance. Great differences of profits were achieved at the beginning, due to the ignorance of the game and of the behaviour of the competitor. After discovering some of the mechansim of the game the players were able to employ better, though not optimal, strategies.

To neutralize the effect of advertising on profits adjusted profit variables were constructed in the following way: advertising expenditures per period of each player were added to the net profit in that period, thereby approximating the profit he would have obtained, if he had outguessed the optimal advertising expenditure. Adjusted profits and profit differences are plotted in broken lines.

It can be seen that the influence of the ignorance about advertising on profit differences is negligible because the negative effect is canceled. Profit levels on the other hand are strongly influenced. The effects are shown in Figures 6 and 7.

Figure 6

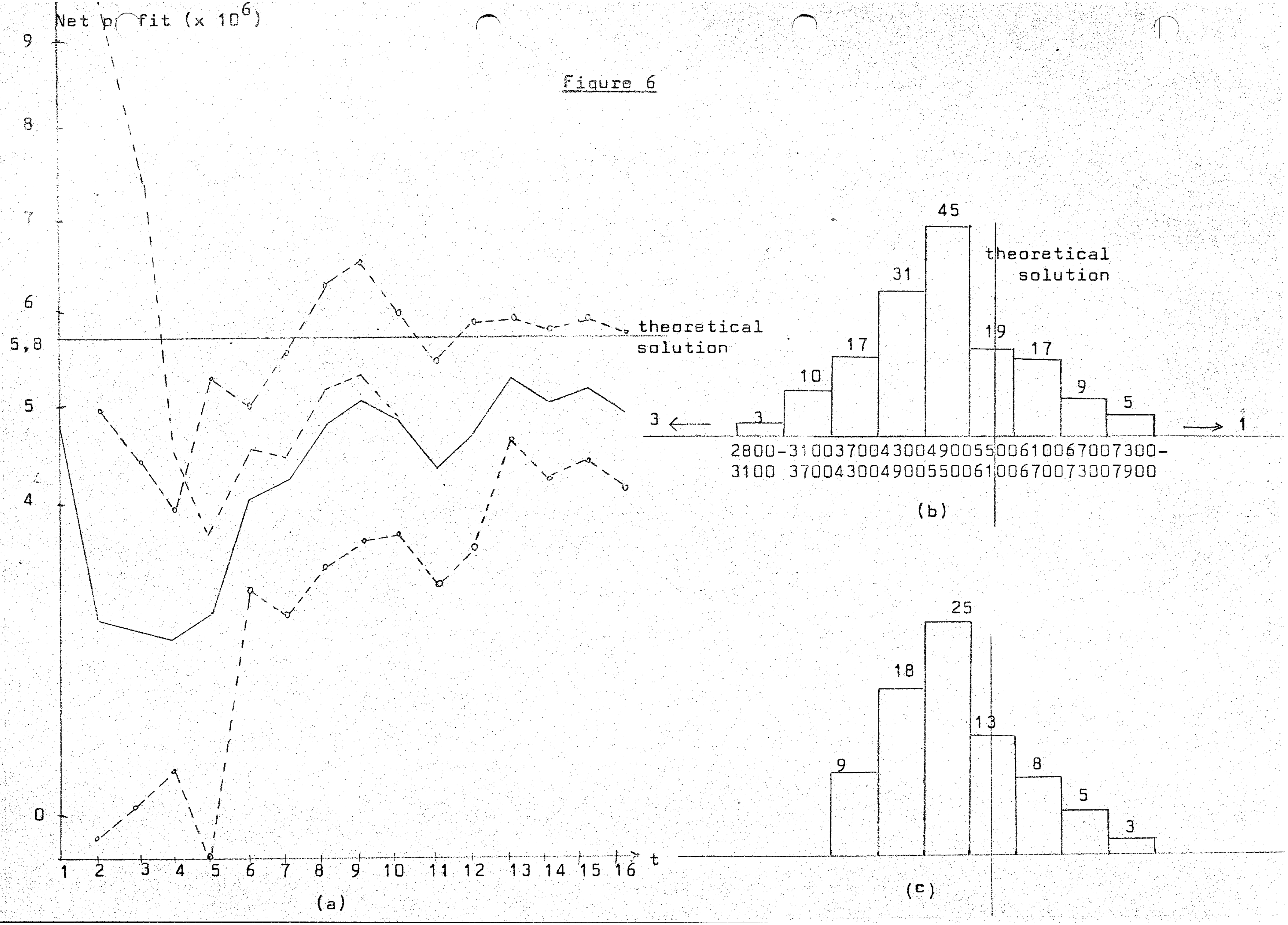
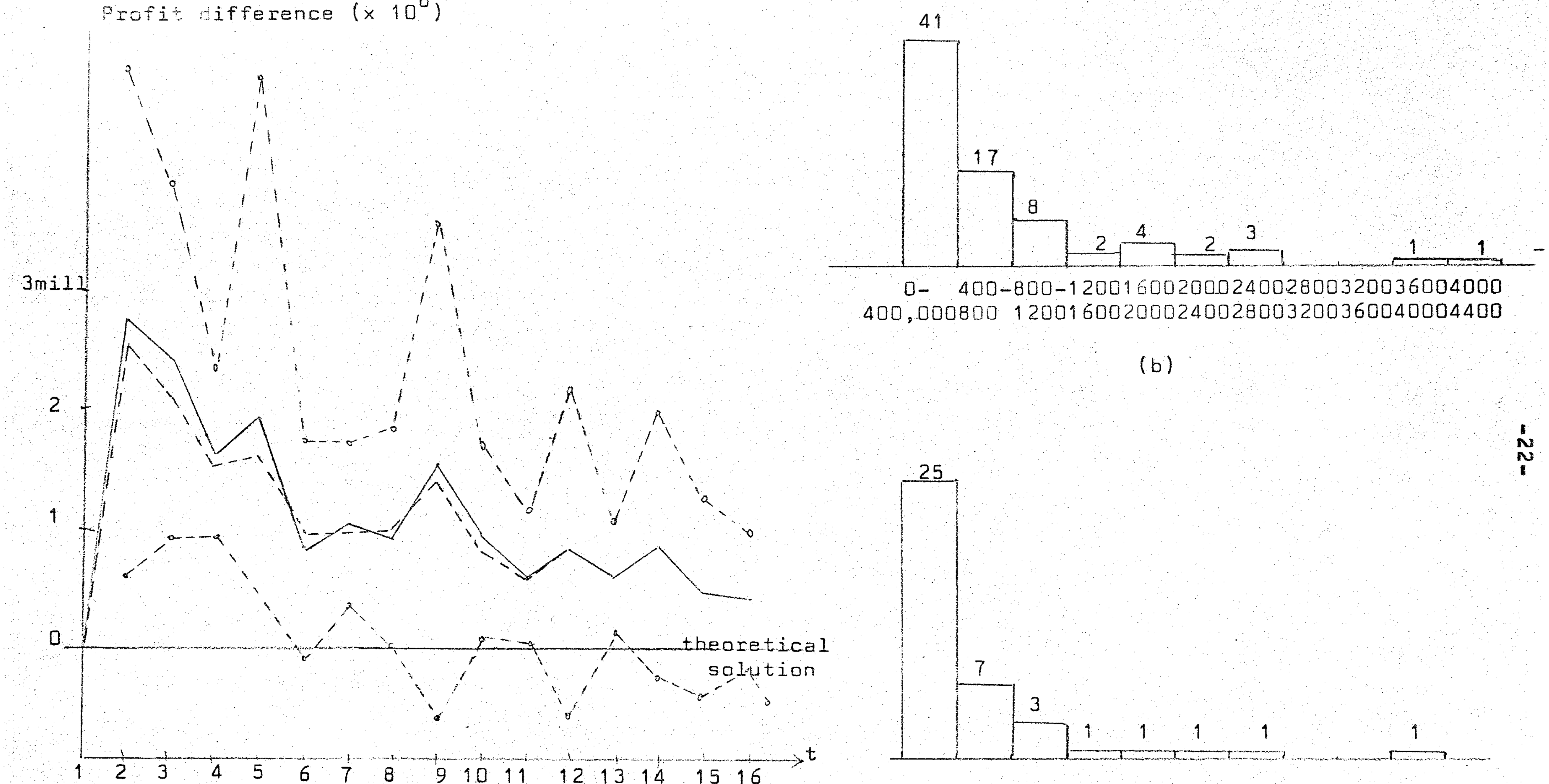

Net p fit (x 10^6)
9
8
7
6
5,8
5
4
0
theoretical solution
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 t
(a)
45
31
17
10
3
19
17
9
5
theoretical solution
3
1
2800-3100 3100-3700 3700-4300 4300-4900 4900-5500 5500-6100 6100-6700 6700-7300 7300-7900
(b)
25
18
9
13
8
5
3
(c)

Figure 7


Profit difference (x 10^6)
3mill
2
1
0
theoretical
solution
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
t
41
17
8
2
4
2
3
1
1
0- 400-800-1200 1600 2000 2400 2800 3200 3600 4000
400,000 800 1200 1600 2000 2400 2800 3200 3600 4000 4400
25
7
3
1
1
1
1
1

(a) (b) (c)

3.1.2 A Macro-Model

On the aggregate data as described in 3.1.1 the standard linear regression model was applied. This can serve as an explanation of the behaviour of the typical player in this experiment. The resulting equations can easily be interpreted in a causal way, as if the mass itself or its representative acted according to a specific behavioral pattern.

The equations were estimated in relative differences of the first order, thereby stressing the dynamic features of the game.

The reaction equations for the two main decision variables are given below. The standard deviation as a percentage is given below the coefficients. The multiple correlation coefficient is also stated.

$$PRI = \underset{(407)}{.126} - \underset{(26)}{.435}\ PRO - \underset{(21)}{.354}\ INV_{-1} + \underset{(18)}{.923}\ PRI_{-1} - \underset{(22)}{.158}\ PRF_{-1}$$

$$R^2 = .878$$

$$PRO = \underset{(109)}{.440} - \underset{(21)}{.752}\ PRI + \underset{(8)}{.913}\ INV - \underset{(10)}{.768}\ INV_{-1} \qquad R^2 = .935$$

The price decision is governed by the following factors. If production is changed this is accompanied by a countervailing change in price, otherwise too large inventories may be expected. If inventories had risen in the last period, this is a signal to the player to lower his price. There is considerable momentum to price changes. Price drifts of the last period are carried forward to the next one. The negative coefficient of PRF_{-1} is a mark of cautious play. Hhigh profits in the past apparently motivated the player to protect himself andcome down with his price.

The production decision depends on the simultaneously made price decision and changes in the opposite direction. The wish to build up or reduce inventories is the second component in the production-decision. The change of inventories in the previous period enters negatively. If the player accumulated inventory in the previous period he will decrease production in the current period as expected.

Heuristics and Game Theory

This particular experiment was so designed that from the game theoretic point of view it can be regarded as a set of two-person zero-sum games, each with a pure strategy solution, given the goal set for the players. The (static) game theory solutions serve to provide bench marks against which we can compare actual behavior. The two "macro" or behavioral equations can be regarded as heuristics or rules of thumb which could be used to construct an artificial player for this game. Using the two equations above combined with the structural equations for sales and profits, it is possible to calculate the steady state predicted by this type of behavior.

The game theoretic approach, by its assumption of full knowledge, ability to compute and no learning or change in values and perception during the course of the game both provides a contrast to a behavioral approach and provides several clues to measurement. For example, we would expect that inventories would be higher than the game theoretical solution and that lost sales would appear because they are called for by virtually any dynamic process of adjustment where the players have to experiment to learn more about both the market structure and each others' behavior.

3.2. INDIVIDUAL PERFORMANCE

3.2.1. Decision Variables

The distribution of average price per player, average production per player and average advertising per player are shown in Figure 8, 9, and 10. Also the theoretical solutions are plotted in. The players can be identified by the numbers in the histogram. Players 1/11, 2/12 etc. form a team. The appearance of numbers within the same class indicates the ranking (increasing order).

AVERAGE PRICE PER PLAYER

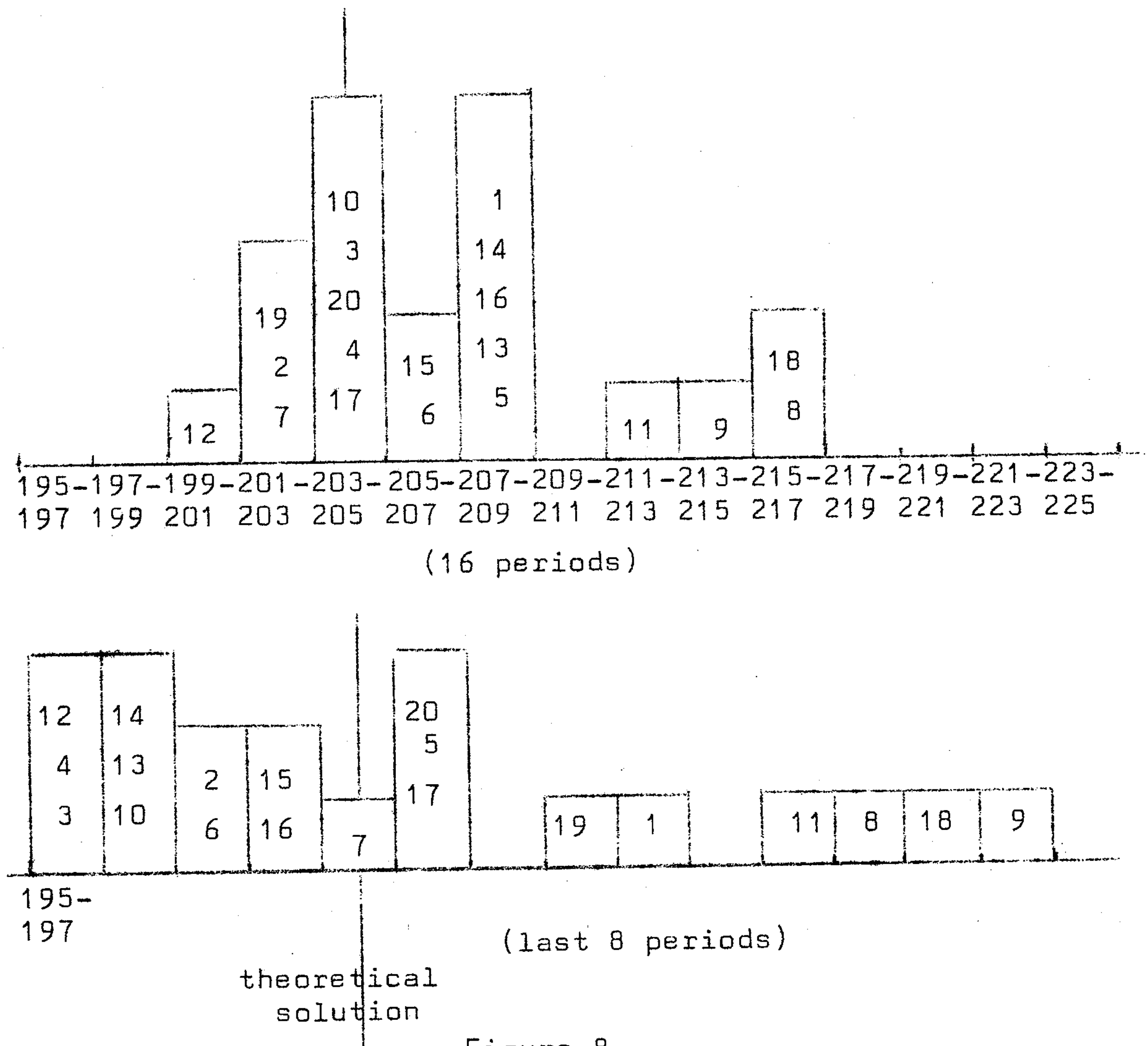


Figure 8

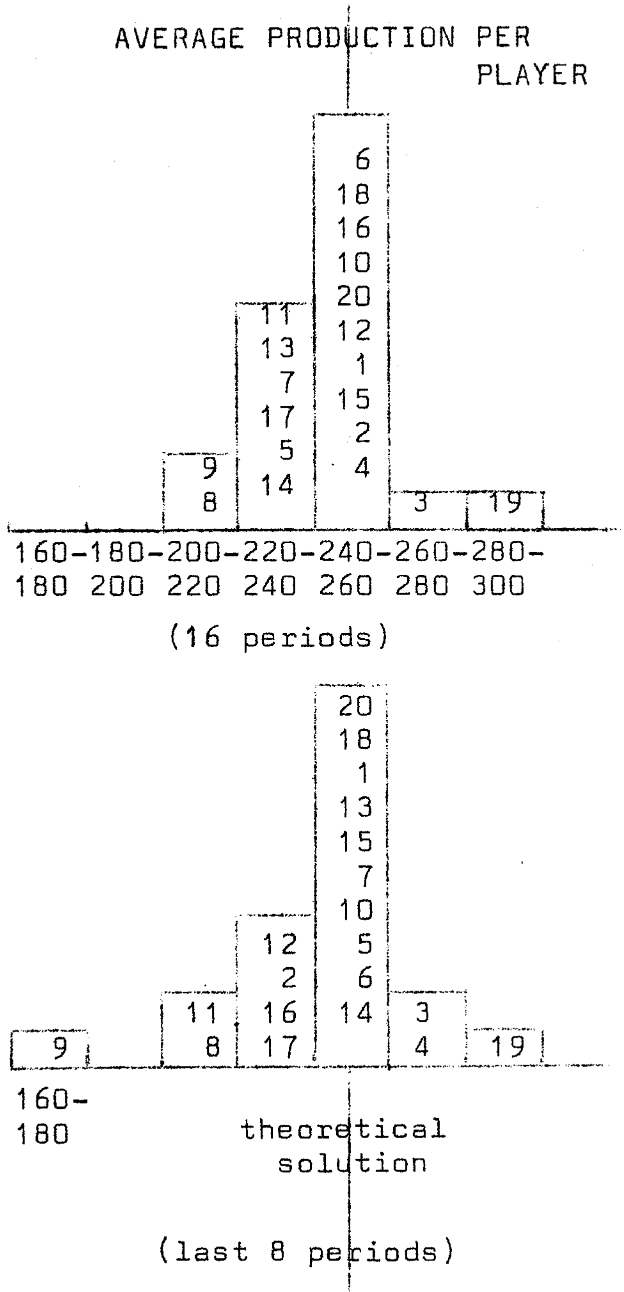


Figure 9

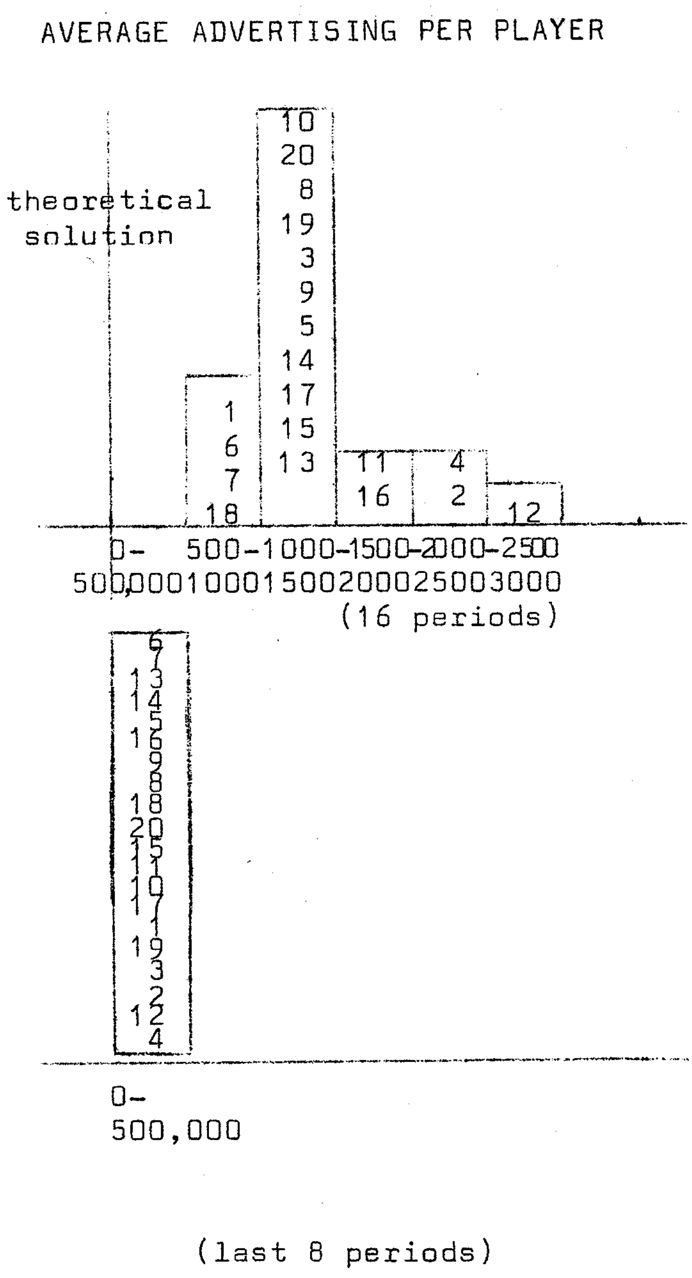


Figure 10

Two types of players can be distinguished:

high production - low price
low production - high price

(KENDALL-rank correlation coefficient -.530 between average price per player and average production per player for 16 periods.
-.379 for 8 periods.

Significance level: .317 (95 %)).

The average amount of advertising is correlated significantly to neither of these types.

In general, the second type (low production - high price) did better with regard to profit level, as can be seen from the correlation coefficients in Figure 11.

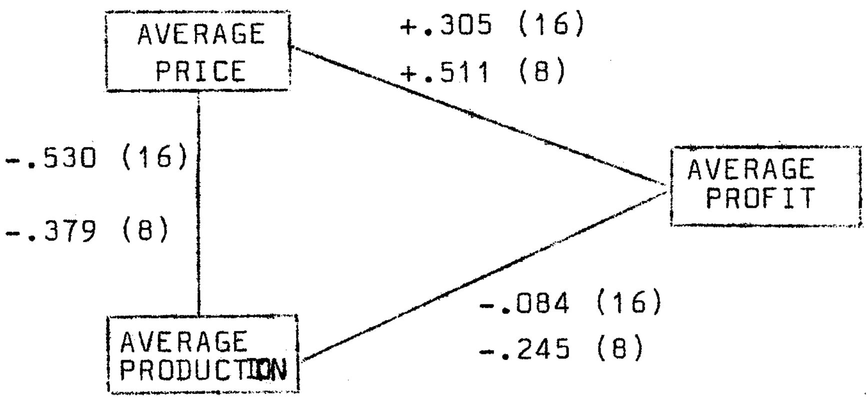


Figure 11

3.2.2. Profit Making

Average profit per period (or equivalently current net worth at the end of the play) and average difference between profit of the player and profit of his competitor (or equivalently current net worth difference at end of the play) are significantly positively correlated (KRCC .389 for 16 periods;
.305 for 8 periods;
significance level .317 (95 %).

Neutralizing the effect of advertising similar relationships hold between the adjusted profit variables: (KRCC .252 for 16 periods, .305 for 8 periods).

Advertising did not distort the result seriously:
KRCC .447 between unadjusted and adjusted profit level
.421 between unadjusted and adjusted profit difference.

To track the influence of stability of each player on profits the coefficient of variation was calculated for some variables. In Figure 12 one can see the following ralationships:

VP	coefficient of variation in price
VPR	coefficient of variation in profit level
VINV	coefficient of variation in inventory
AVPR	average profit per player
AVPRD	average profit difference per player

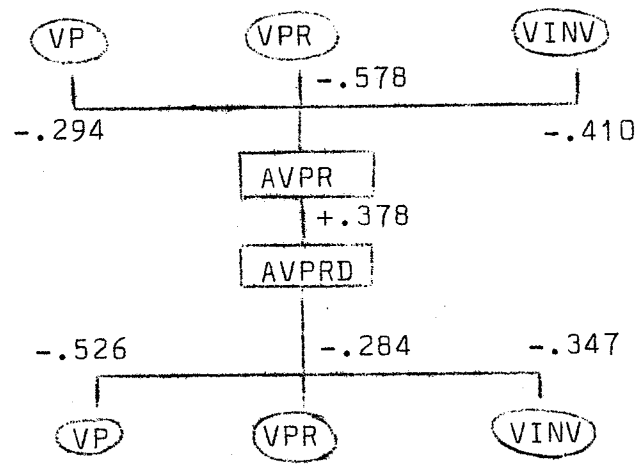


Figure 12

The accumulation of high profits was associated with stable accumulation. Great variations of profits per period were a mark of unsuccessfull play. The correlation between fluctuation of inventory holdings and profits indicates that deviation from a steady state, not only from the optimal one, did not pay. Wild fluctuations in price-level affected profits negatively.

Because of the positive correlation between profit and profit difference similar relationships hold for the accumulation of profit differences.

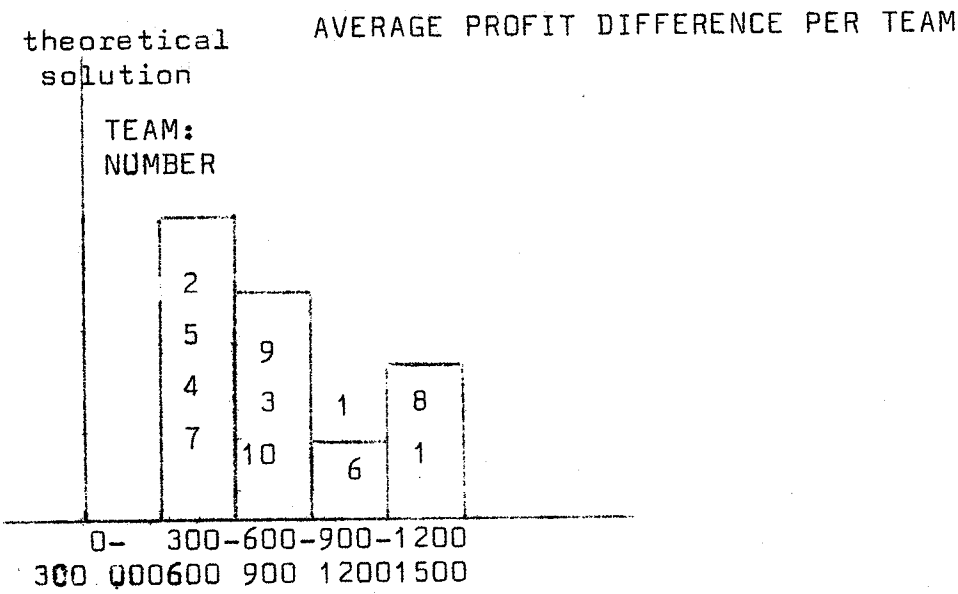
3.2.3. Individual Differences

No test for individual differences in the players prior to playing was made. Were these to be performed, a fruitful inter-link between the economic (and game-theoretic) experimentation and socio-psychological experimentation might be forged by an investigation of the distribution of individual behavior in relation to the tests.

3.3. TEAM PERFORMANCE

3.3.1. Profit Variables

Diagrams of the distribution of average profit difference per team and average joint profits are given in Figure 13.



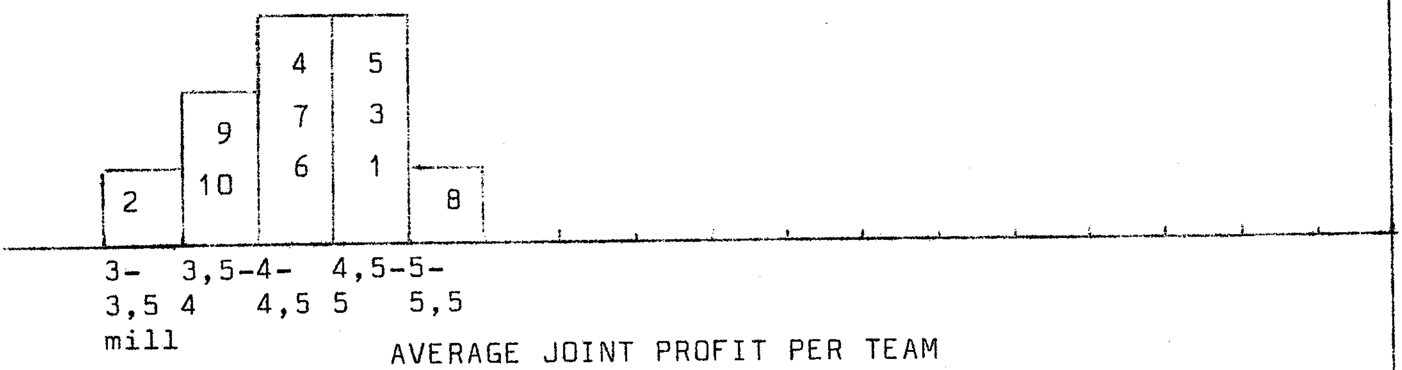


Figure 13

Difference of net profits and joint profits are correlated positively (KENDALL rank correlation coefficient .466, significance level .486 (95 %). That means that small differences of profits did not occur because both duopolists played brilliantly but could not outdo each other, but indicate also poor performance of both competitors.

The distribution of average profit per player is given in Figure 14.

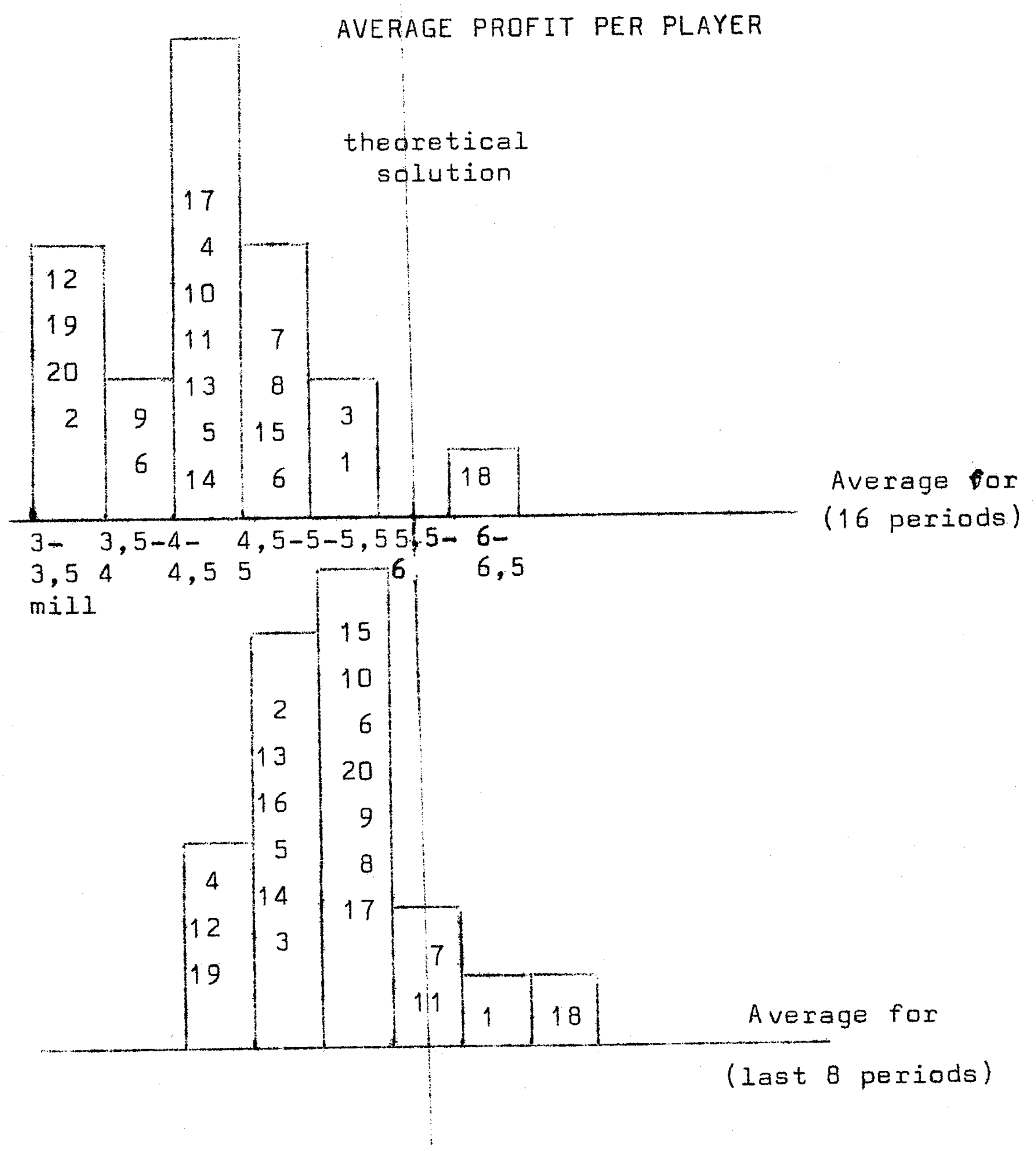


Figure 14

3.3.2. A Sign Test

The sign of the difference of net profits per period was computed for all teams. Six teams show a random distribution. The remaining four are those that came first, second, third, and fifth with respect to the maximization of difference of cumulated net profits.

Adjusting for advertising only two of these teams remain, in the other two dominance was due to the wrong advertising policy of the competitor.

This shows that in the winning teams one player was systematically outdoing his competitor. A similar result is this: Considering only the second half of the play, and thereby eliminating the effects of ignorance at the beginning, in all teams but one the same player managed to accumulate a positive difference of profit as in the complete game.

4. CONCLUDING REMARKS

Compared with a matrix game, this business game is quite rich, complicated and has a complex environment. By the device of briefing the players to try to maximize the difference in teir scores, the game is turned into a two-person zero-sum game with a saddle point. The strategy space is multidimensional. An extremely strong Prisoner's Dilemma game is built in on advertising and the evidence obtained was that this was reflected in the actual play. In pravious experiments 3/ where the players were trying to maximize individual payoffs and the game was truly non-constant sum this effect was also observed.

The economic solution predictions were quite good in predicting the range to which price and advertising proceeded. Profits were lower than the steady state prediction. This could be explained by the dynamic aspects of the game which were manifested in the learning and control problems of the players. Inventories were in general too high. It took time to work out the implications of advertising. The value of production scheduling and its relationship to management of prices was probably not appreciated by many of the players.

REFERENCES

[1] LEVITAN, R. and M. SHUBIK, "A Business Game for Teaching and Research Purposes," Part 1, CFDP No.115, Cowles Foundation of Research in Economics at Yale University May 16, 1961.

__________, Part 2a, CFDP No. 115, June 2, 1961.

__________, Part 3, CFDP No. 115, September 6, 1961.

__________, Part 4, "Mathematical Structure and Analysis of the Nonsymmetric Game," CFDP No. 219, March 2, 1967.

[2] FEDER, P. and WOLF, G., Data analysis in a business game. Report 40. Department of Administrative Sciences. Yale University. 1971.

[3] SHUBIK, M., WOLF, G., and EISENBERG, H., An experimental oligopoly business game: part I, some experiments. Technical Report 33, Department of Administrative Sciences. Yale University, 1970.