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A THEORY OF MONEY AND BANKING

IN A GENERAL EQUILIBRIUM SYSTEM

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Research Memorandum No. 48

July 1970

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PREFACE

This is a preliminary paper on the theory of money and banking. The theorems called for are not stated with the care that is needed for fully rigorous proof. However as what is presented here represents a fundamentally new approach to the theory of money and banking, the concepts, models and examples are presented in this form now, taking the risk and responsibility that unforseen difficulties can occur.

The approach to the theory of money is via the theory of games. An n+b person game is considered where n are traders and b are bankers. There are m commodities and a substance called money in the economy. The properties of money derive from the rules of operation of the economy and the limiting behavior of the core of a coöperative game for financing cash flow.

An Appendix provides a far more formal treatment of the subject than does the main text. These who are less interested in the institutional details, the heuristics of modeling and in calculations with simple examples could proceed immediately to the Appendix for the formal statement of the basic model and the proofs of the results.

Most of the basic results are stated immediately below and are developed both in the text and Appendix.

What's about Information ??

(1) In an economy with perfect trust there is no need for money and trivially the money rate of interest is zero.

(2) In an economy in which traders do not trust each other but are willing to trust a banker (who in turn trusts the trader) a fiat money of "banker's credit" can be issued according to one of many different feasible sets of rules

(a) if the bankers (according to some appropriate criteria) are few in number they will be able to control the money supply and spend their profits on real goods and services thus creating a positive rate of interest on money.

(b) if the bankers are sufficient in number (and in the appropriate sense turn out to be very like price naming oligopolists with excess capacity) the price of paper money will be 1 - i.e. the marginal value productivity of paper money becomes zero, or the rate of interest is zero.

(c) if the rules are such that regardless of the behavior of the bankers there is not enough money this effects the intertemporal boundary conditions of the general equilibrium system and the rate of interest on paper money is positive.

These three results are obtained by the convergence of the core of an appropriate <u>no-sidepayment</u> money game which is a c-game and is in fact a market game.

(3) In an economy with perfect trust no money is needed and if a government wished to tax it would have to introduce and fix a unit of account or tax in commodities. Consider an economy without trust but with enough bankers and monetary issue that the bankers are powerless. Now we introduce a government

policy which taxes in money each period and announces that the money will be spent buying certain items at whatever their price happens to be until all the tax money has been returned to the economy. The physical resources bought by the government can then be dumped into the sea or sent to Vietnam so that they do not affect the individual utility functions of the traders with public goods supplied by the government. This in effect, enables the government to use fiat money to remove real goods and creates an interest rate for money equal to the marginal value of not having to pay taxes. The leaving out of the public goods is methodologically correct here in the sense voting and other strategic means for determining taxation and the nature of public expenditures are not being considered as part of this model.

(4) Consider an economy where there is no government taxation, but in which not only do traders not trust each other, although the traders trust the bankers, the bankers do not trust the traders. In this economy the bankers will require that the traders give security on their loans. This security will consist of real assets of long duration, preferably immovable with not too bad a physical level of depreciation. The amount of credit that can be secured against a real asset will be a "trust parameter" of the system. As trade in this model is sequential a feasible convention says that a banker will offer to extend credit as a percentage of the last price that asset traded at. In this economy even if the banking laws were such as to make it feasible for the banking system to supply enough money it would not do so unless given its "trust parameter" there were enough physical assets in the hands of the

borrowers. Thus this economy will have a positive rate of interest where the marginal productivity of money equals the marginal productivity of trust.

(5) Although inflation and deflation are hardly reflected in this static model it is conjectured that a failure by the banking system to mop up extra idle balances results in inflation. However a failure by the banking system to provide more funds when needed may not lead to deflation but to stagnation or a failure to attain Pareto optimality.

(6)A simple version of this game can be played as a parlor game; there is no uncertainty or psychological misperceptions, expectations etc. which are not modeled to the extent that a computer program version of the process could not be constructed with relative ease.

(7) There exists a simple banking system requiring fiat money and bank credit (i.e. two types of paper money) and at least 4 players (2 bankers and 2 holders of cash) that will efficiently take up almost all idle balances.

(8) In a money game the operations "minimize maximum cash needed" and then "minimize idle balances" are critical to monetary control and the selection of a unique equilibrium point.

(9) In a money game with uncertainty, for an efficient banking system a bankruptcy law is needed of prices must be able to change without bound during any period. If forward delivery contracts for <u>commodities</u> are allowed a bankruptcy law must be specified.

A THEORY OF MONEY AND BANKING IN A GENERAL EQUILIBRIUM SYSTEM

1. Introduction

The phenomena of money, banking, taxation, and public finance are complex in the extreme and deeply intertwined. In this paper an attempt is made to separate out one aspect of the general area of finance. This aspect is the role of banks and fiat money in a general equilibrium system.

The use of tax revenues for public goods are not considered. It is my belief that governmental actions cannot be viewed adequately without taking public goods into account. Decisions concerning public goods call for a non-market distribution process. Here the discussion is limited to monetary and banking modifications to a private ownership, private enterprise market system. All the conditions associated with the existence of a price system are assumed to exist 1/. In particular increasing returns, externalities and other phenomena which may destroy the existence of an efficient competitive equilibrium are ruled out.

In order to keep the model as simple as possible we consider only trade or exchange leaving out production. When the conditions for an efficient price system exist it has been shown by Rader 2/ that any model of an economy involving production and exchange can be represented by an equivalent model involving only exchange. This is done essentially by making an appropriate transformation on preference conditions which reflects the ability to produce. Thus our limitation to exchange economies alone involves no loss of generality.

2. The Basic Model

The basic model presented here is given in the form of a game that could actually be played as a cooperative game.

There are n+1 players; n players are traders and one player is a banker. All of the players have utility functions defined on the m goods available.

For the sake of simplicity in the first model, durable assets are ignored. This enables us to avoid the problem of evaluating end assets "after the game is over". We return to the important role of long term assets in Section 7. This will include a discussion of gold and other "near-money".

We consider a market that functions for T time periods. During that time trade must take place in a given order. For example it may be necessary to pay the workers and suppliers of raw materials before selling the final product to the market. The order will make somewhat of a difference to the strategic power of an individual player because it will influence his need for credit. However the general results will hold and the model is well defined for any order of trade. We modify this assumption in Section 6 and discuss it in detail.

It can be seen that the fixed time periods and order of trade assumptions will imply a fixed velocity for money. In Section 6 the velocity of money is discussed. At this point, I claim that it is not a particularly important issue in the understanding of banking, but becomes important in the understanding of bad government finance when confidence is lost in "the rules of the game".

The key assumption is that as a rule of the game <u>all</u> payments <u>must</u> be made in a substance called money, banker's credit or "banker's gold".

The banker is the only person in the society who is trusted by everyone. He is in a position to extend credit up to a point and is also in a position to accept deposits of money from the traders if the banker and a trader can work out a jointly profitable arrangement. The specifics of the rules on the banker are given below.

The requirement that all payments be made in money or banker's credit is obviously a considerable simplification of reality. People do extend credit to each other and even barter on occasions. However this rule represents an extreme in the level of distrust and anonymity. There is an old Yankee saying: "In God we trust, all others pay cash", this rule reflects that sentiment. We return to a discussion of this rule and possible modifications in Section 8.

A critical assumption in the modeling of the game concerns the rules governing the supply of money. It is assumed that at the start of the game in the initial endowment of each of the n+1 participants is an amount M_i, where $i=1,\ldots,n+1$. The initial amounts held by the traders may be regarded as the supply of fiat money in circulation. The amount held by the bank is its supply of loanable funds. In an actual parlor game of this system the bank may issue its own notes up to the amount of its loanable funds.

At the end of the game all participants (including the banker) must return to the referee the amount of money (or credit) he was given at the start. The possibility of making the game of infinite duration is discussed in Section 7. Heuristically, the fiat issue plus the loanable funds are accepted by all players as parts of the rules of the game in as much as they represent faith in the government (the referee) and the banking system.

There is a further restriction on the banker. As has already been noted the banker has a set of preferences (represented by a utility function) defined on the goods in the economy. He is not permitted to use his loanable funds to make purchases to his own consumption account. He is however, permitted by the rules to spend, at any time goods are offered for sale, an amount equal to the profits he obtains from his banking operations.

The actual game may be regarded as an n+1 person cooperative game in Characterizing Function form+) 3/ The traders all come together with the banker and cooperatively decide upon loans and deposits and the terms of loans and deposits. There is no assumption made here that the terms to players need be the same. After the agreements have been made, the strategic aspects of the game are over. All players in a coalition now participate as price takers in a general equilibrium market consisting of this coalition.

3. The General Equilibrium Market with Temporal Budget Constraints

Suppose that we were to do away with the condition that transactions needed to utilize money. Furthermore let us

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+) This form will be an adequate representation as the game is a c-game or an orthogonal coalition game 4/.

assume that everyone trusts everyone else and that we banish the banker from the economy. We are now left with a game with only one type of individual, a trader. If all are constrained to act as price takers we know that there will exist a competitive equilibrium.

Let the endowments of Trader i in the n-person economy be given by:

$A^i_{j,t}$ at the start of any period where

$i=1,\ldots, n;\quad j=1,\ldots, m\quad$ and $t=1,\ldots, T.$

The utility function for Trader i is given by:

$$\Psi_i(x^i_{1,1}, \ldots, x^i_{m,T}).$$

The competitive equilibrium prices will be a solution to:

(1) Maximize $\Psi_i\,(x^i_{1,1}, \ldots x^i_{m,T})\qquad i=1,\ldots, n$

$$\text{subject to: } \sum_{t=1}^{T}\sum_{j=1}^{m} p_{j,t}(A^i_{j,t} - x^i_{j,t}) = 0$$

$$\text{and } \sum_{i=1}^{n} (A^i_{j,t} - x^i_{j,t}) = 0 \quad \text{for } j=1,\ldots, m,\; t=1,\ldots, T.$$

Now we impose the condition that no one trusts anyone else but that by convention during the duration of the T time periods they will use some pieces of paper as symbols of trust specified by the referee. They must return the paper to the referee after trading. Furthermore the first period price of some commodity is set equal to one unit of this paper (the paper is legally acceptable for all debts public or private).

The paper is durable and lasts from period to period until it is given back to the referee at the end of the Tth time period. If the initial supplies to each of the traders were M_i then the competitive equilibrium system, instead of having one budget constraint for each trader will have T budget constraints; one for each period. The specific boundary constraints will depend upon the order of trade as well as the amount of money.

Even without writing down the constraints we may immediately note that if every trader is given "enough" money then the constrained optimization system becomes the same as the system given in (1) which has only one budget constraint at the end.

If we adhere to the convention of regarding this model as a parlor game then up to this point it will have two unsatisfactory but at least well defined features. If we give any of the players too much money they will merely keep it as idle balances until the end of the game when they are forced to give it back to the referee. This is based on the assumption that even though a trader may have surplus funds during some time period he is not permitted to lend it to another. He can do so through an intermediary, but for the moment banks do not exist hence he cannot make loans. (The possibility of inflation due to idle balances is discussed in 6.2)

If, in this model we rule out simultaneous trading (i.e. barter) then if there is no money available whatsoever then no trading can take place and the individuals merely can achieve the utility level of their initial endowments. This is obviously unrealistic. If no money or banks existed the individuals would resort to barter until they had time to organize the money and banking system.

If the referee in this parlor game wished to have it achieve n-person, full-trust, Pareto optimality, he could do so by solving the system given in (1) and determining for each player the maximum amount of money he would need (if he had to trade in money) so that only the final constraint would be effective. Thus there will be some set of numbers (M_1^*, M_2^*, ... M_n^*) for which the economy without trust but with a supply of symbols of trust gives the same outcome as the economy with trust. In this economy however most of the time most of the players will hold idle balances. The referee could avoid this by giving out and taking away fiat money all the time - but this is a poor model of process. (Except when we add consideration of taxes, this is done in 6.2.). Anyhow a bank could do it and that should be a role of banking.

In order to complete the model of the economy we add the banker. He performs the services of both lending and borrowing. If thereis no fiat money out in circulation and he has adequate loanable funds, then he has enormous power. He controls the output of the economy all the way from blocking all trade up to the values obtained from an n-person economy with trust. If the traders all have fiat money in supply greater than or equal to (M_1^*, M_2^*, ..., M_n^*) then the banker is not needed, "trust symbols" are a drug on the market and as no one needs to borrow, the banker does not need deposits. Anywhere between these two limits there will be a productive role for the banker.

We must note however that when we discuss Pareto optimality in an economy without trust and with a banker we are talking about Pareto optimality in an n+1 person economy. Furthermore it is important to remind ourselves that the banker is providing a vital productive service to the economy - that is the service of trust.

In summary, we have joined together a money and credit cooperative game with a general equilibrium system. The game is the bargaining for loan and deposit conditions and terms. Given these then there remains the solving of a set of general equilibrium systems associated with each of 2^{n+1} sets of individuals in order to determine the blocking sets for the n+1 individuals where the financial moves have been reflected in the effects on the intertemporal budget constraints on all players.+)

4. The Core of a "Money Game"

In this section we limit our discussion to the case in which there are n ($n \geq 2$) traders but only one banker. This of course will immediately introduce the aspects of monopolistic power of the banker. In Section 5, more than one banker is introduced and the power of monopoly banking atten~~t~~uates.

+) There is an additional difficulty which has not been accounted for in this analysis. That is the role of multiple equilibria. This is discussed further in Sections 5 and 6.2. and is more formally treated in the Appendix by minimizing cash needed.

First, in order to provide a little more insight into the nature of the model, before discussing the general case, we calculate an example involving two traders, two goods and a banker. Without the need to trade in money and without the banker this would give us the familiar barter model treated in the Edgeworth box diagram 5/.

4.1. A Simple Market with Two Periods, Two Traders, Two Goods and a Banker

For simplicity we assume that all three players have the same utility functions given by $\psi(x_i, y_i)$. Their initial endowments are: $(A, 0, M_1)$, $(0, B, M_2)$ and $(0, 0, M_3)$. During the first period, by law the market price of the second commodity is fixed at one monetary unit.

The three players begin by deciding upon the size and terms of any loans and deposits. After this has been done a general equilibrium mechanism takes over.

The first trader (and the banker, if he has earnings from a loan) buy from the second trader to start. Then the second trader and banker buy from the first trader. After this they return M_1, M_2 and M_3 to the referee having settled up previously with the banker.

Suppose that M_1 is not sufficient for the immediate purchases of the first trader. He borrows b from the banker on terms that call for the repayment of b + r by the end of the second period.

The conditions for the general equilibrium model are now specified for all three participants if the banker does not have enough to loan he may pay the second trader an amount ϱ for the use of his deposits which he returns at the end of the last period.

<u>First trader</u>:

(1)+) maximize $\psi(x_1, y_1)$

(2) subject to $y_1 \leqq M_1 + b - r_1, \quad y_1 \geqq 0$

(3) and $p(x_2+x_3) - y_1 - r_2 = 0$

where $r_1 + r_2 = r$

<u>Second trader</u>:

(4) maximize $\psi(x_2, y_2)$

(5) subject to $px_2 = y_1 + y_3 + \varrho$ and $x_i \geqq 0$

(In this simple model the second trader will never need to borrow as he sells before he buys. He will either lend or have idle balances).

<u>Banker</u>:

(6) maximize $\psi(x_3, y_3)$

$px_3 \leqq r_1 - \varrho_1$

(7) subject to $px_3 + y_3 = r - \varrho$ where $x_3, y_3 \geqq 0$

+) Equation numbers refer to each Section.

and $\varrho = \varrho_1 + \varrho_2$ are the payments of the bank to the depositor each period.

For all of the participants:

(8) $x_1 + x_2 + x_3 = A$,

and (9) $y_1 + y_2 + y_3 = B.$

When the banker lends the amount b during the first period he may ask for the interest on the loan to be paid in two instal_ments r_1 and r_2. The payment of r_1 being immediate, amounts to an immediate discounting of the loan hence the borrower only obtains $b-r_1$. Similarly the bank could pay an amount on deposits instantly. The bank and lenders and borrowers set these terms cooperatively.

A reasonable convention to follow is that the bank may spend any earnings up to and including the current period. Thus the strategies are extremely closely related tocash flow.

We take a simple example $\psi(x,y) = \sqrt{xy}$ for all participants. The following values give the general equilibrium conditions:

(10) $x_1 = \frac{(B-r)}{2} \frac{A}{B}$ $\qquad y_1 = \frac{B-r}{2}$

(11) $x_2 = \frac{(B+\varrho)}{2} \frac{A}{B}$ $\qquad y_2 = \frac{B+\varrho}{2}$

(12) $x_3 = \frac{(r-\varrho)}{2} \frac{A}{B}$ $\qquad y_3 = \frac{r-\varrho}{2}$

$\sum x_i = A$ $\qquad \sum y_i = B$

(13) $p_1 = B/A$ and (14) $p_2 = 1$

for (15) $M_1 < B/2$ where we assume

(16) $\sum_{i=1}^{3} M_i \geq B/2.$

The condition (15) establishes that the first trader needs to borrow. If he had more money than B/2 he would not need the banker although he still might be willing to pay him.

Condition (16) states that the traders and the banker have enough money between them to cover the needs of the economy.

The general equilibrium solution to the two person trading game with the traders trusting each other is:

(17) $(\frac{1}{2}\sqrt{AB},\ \frac{1}{2}\sqrt{AB})$ $p_1 = A/B$ and $p_2 = 1.$

Without trust and with $M_1 \leq B/2$

(18) $\left(\sqrt{\frac{A(B-M_1)M_1}{B}},\ \sqrt{\frac{A(B-M_1)M_1}{B}}\right)$ which is 0 for $M_1=0$

and as in (17) for $M_1 = B/2$.

When one banker is introduced, his worth to this market is a function of M_1 and M_3. Suppose that $0 \leq M_1 \leq B/2$ and $M_1+M_3 \geq B/2$, then the contribution of the banker is given by the difference between (17) and (18).

Break M_2 into F + E where $M_1 + F = B/2$. The amount E is excess loanable funds and will never be used.

We have observed that only in equation (16) is use made of the sum M_2. This is the amount of money held by the second trader. He is given it by the referee at the start and must return it at the end. It stays as "idle balances" during the market periods or he may lend his money to the bank when he does not need it. In the example with two traders if $M_1+M_3 \gtrsim B/2$ then M_2 is never needed, otherwise it is.

Our simple example with $\psi(x,y) = \sqrt{xy}$ is picked for ease of computation with A=B=1 we obtain further simplification. In particular the equilibrium prices will always be (1,1)

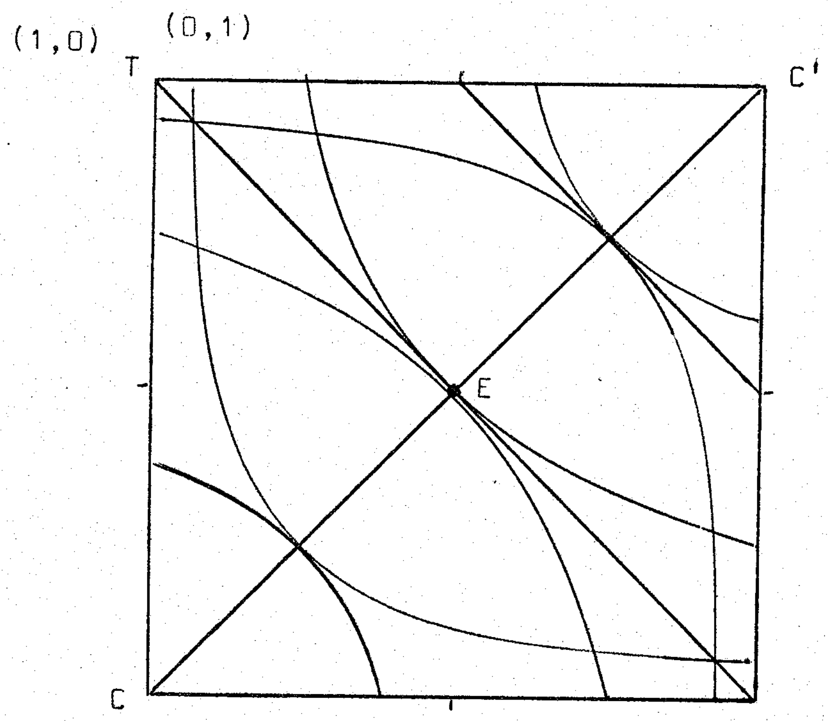

Figure 1

when resources are (1,0) and (0,1) although monetary constraints will limit the ability to buy.

As can be seen from equations (10) - (12) the optimum schedules for r_1, r_2 and ρ_1, ρ_2 are $r_1 = r_2 = \frac{r}{2}$ and $\rho_1 = \rho_2 = \rho/2$. What limits should there be on r and ρ. Only one imposed by needed logical conditions will be used. As we assume x_i, $y_i \geq 0$, $-1 \leq \rho \leq r \leq 1$.

There may be some difficulty in interpreting market behavior when $\rho_1 = -1$ and $r_1 = 1$. This is when the banker gets the use of possibly more than all the money in the economy. Both traders agree to give him 1 in exchange for nothing. He discounts their notes immediately taking $\frac{1}{2}$ each for the first period. Thus he enters the first period with a buying power of 1. He obtains the 1 the second period by having arranged a loan of 1/2 from trader 2 to trader 1.

If the above scheme is allowed then all divisions of payoffs can be achieved+). If we impose that at no time shall more than the total amount of cash and loanable funds be spent then the bank can never obtain more than that number as a payoff. For example if $M_1=M_2=0, M_3=1/2$, the bank could only obtain at most 1/2 by sequentially lending each individual 1/2 to be paid back at the end with an interest payment of 1/2 immediately.

+) In this case the banker must be in a position to create twice as much money as the economy needs without him.

In order to explore the implications of money and banking for this three person model we examine four different games. The triad (X,Y,M) stands for the initial resources of the first and second goods and then money. Each game is represented by three triads, the first gives the resources of the first trader, the next, the second trader and the third the banker.

In all of the games the price of the second good during the first period of trade is set at one unit of money. Figure 1 shows the Edgeworth box for the two traders. T is the no trade point, E the competitive equilibrium and CC' the contract curve.

<u>Game 1</u> $(1,0,0)$, $(0,1,0)$ and $(0,0,\frac{1}{2})$

In this game the banker has all of the credit. There is no money out among the traders.

<u>Game 2</u> $(1,0,\frac{1}{2})$, $(0,1,0)$ and $(0,0,\frac{1}{2})$

In this game the first trader has sufficient cash for his trades , the banker has loanable funds up to the amount $\frac{1}{2}$. As we shall see, he is not needed.

<u>Game 3</u> $(1,0,0)$, $(0,1,\frac{1}{4})$ and $(0,0,0)$

Here the second trader has some, but not enough money for the economy.

<u>Game 4</u> $(1,0,0)$, $(0,1,\frac{1}{2})$ and $(0,0,0)$

Here the second trader has enough money, the bank has nothing, but in order for any loans to be made the second trader will have to deposit money in the bank and the banker will make the loans.

We are now in a position to examine the relevant feasible sets of outcomes and the Pareto optimal surface of these games prior to constructing the characterizing functions.

First, if the traders trusted each other, did not need money and were not constrained by the rules of the game to act as mechanistic pricetakers, then the Pareto optimal surface would be a straight line AC in a two dimensional payoff space as is shown in Figures 1 and 2a. Any of the outcomes in OAC would be feasible.

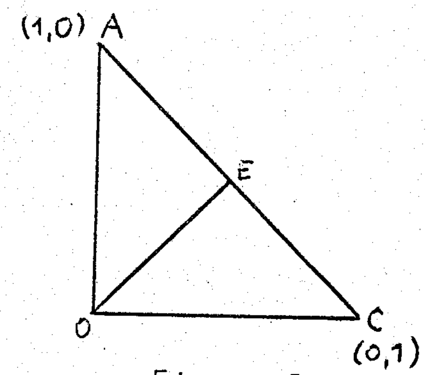

Figure 2a

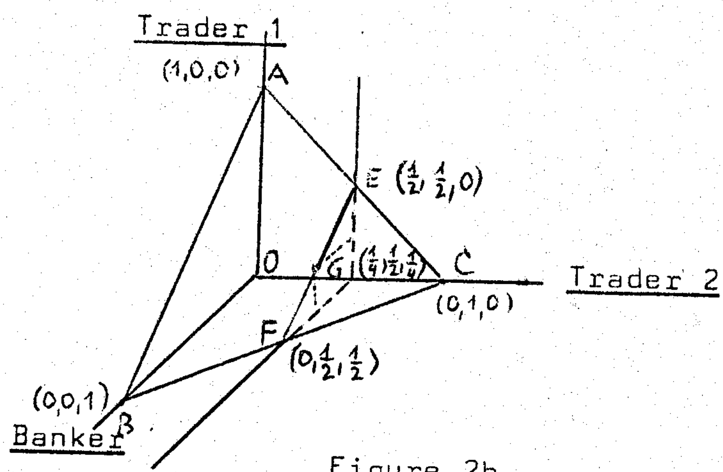

Figure 2b

If the traders do not need money, but are constrained to act as price takers then the outcome E is the only feasible outcome and hence is Pareto optimal <u>in this game</u>.

In game 1, if the traders needed money but were not constrained to using a price system (here the role of the banker would have to be regarded as that of a catalyst, or in "blessing" the transaction) then any point in the tetrahedron OABC in Figure 2b is feasible, and the Pareto optimal surface is ABC. If, however the traders and banker must first get together to decide upon loans, after which the distribution of resources is done by a price system, then (beyond no trade) the feasible set of outcomes is ABC as shown in Figure 2b. All of them are Pareto optimal.

The following notation is used for the four characterizing functions. $V(\bar{i})$ and $V(\overline{ij})$ stand for coalitions consisting of the set of player i and players i and j. In general a coalition can achieve a set of outcomes. Each member of the set is a vector containing as many dimensions as there are members of the coalition. A single outcome will be noted in a manner such as (a,b) and a set of outcomes may be noted as $\{(x,y) : 0 \leq x \leq a,\ 0 \leq y \leq b\}$. The banker is the third player.

<u>Game 1</u> $(1,0,0),\ (0,1,0),\ (0,0,\frac{1}{2})$

$$V(\bar{i}) = \{(0)\} \quad \text{for all } i$$

$$V(\overline{12}) = V(\overline{13}) = V(\overline{23}) = \{(0,0)\}$$

$$V(\overline{123})^{+)} = \Big\{\Big(\sqrt{(\tfrac{1}{2}-r_2)(\tfrac{1}{2}-r_1)},\ \sqrt{(\tfrac{1}{2}+\rho_2)(\tfrac{1}{2}+\rho_1)},\ \sqrt{(r_2-\rho_2)(r_1-\rho_1)}\Big):$$

$$0 \leq r_i-\rho_i \leq \frac{1}{4},\quad -\frac{1}{2} \leq r_i \leq \frac{1}{2}\Big\}$$

+) We are faced with a problem in modeling which illustrates the "quasi-transferability" of money. Strictly this is a no-side payments game. However the bank can make sidepayments in the sense of the rates it charges or gives on loans and deposits. If we do not restrict these, then under certain circumstances a set of players may be in a position to "hold up" the bank for a money payment for its services in a coalition which does not involve these traders depositing any money in the bank. We could impose the condition that the banker cannot be "blackmailed" by traders for money payments, and vice-versa. If we did this then in Game 1 the characterizing function would have a different value for $V(\overline{123})$,

./p.20

Game 2: $(1,0,\frac{1}{2})$, $(0,1,0)$, $(0,0,\frac{1}{2})$

$$V(\bar{i}) = \{(0)\} \qquad \text{for } i=1,2,3$$

$$V(\overline{12}) = \{(\tfrac{1}{2}, \tfrac{1}{2})\}$$

$$V(\overline{13}) = V(\overline{23}) = \{(0,0)\}$$

$$V(\overline{123})^{+)} = \left\{(\frac{r_1}{3}, \frac{r_2}{3}, \frac{r_3}{3}) : 0 \leq r_i \leq 3 \text{ for } i=1,2,3 \text{ and } \sum_{i=1}^{3} r_i = 3\right\}$$

In Game 2 the characterizing function shows that the banker is not needed. The economic goals of the traders are not furthered by the banker's presence. This game has the unique outcome $(\frac{1}{2},\frac{1}{2},0)$ which is the point E in Figure 2b and coincides with the two trader complete trust competitive equilibrium. This is to be expected as the first trader had enough money for trade. This single point is also the core 6/. The core consists of a set of outcomes (or imputations of wealth) for which no set of individuals in that economy is effective. A set of individuals is effective with respect to an outcome or imputation if by refusing

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$$V(\overline{123}) = \left\{(\frac{1-r}{2}, \frac{1}{2}, \frac{r}{2}) : 0 \leq r \leq \frac{1}{2}\right\}$$

and the Pareto optimal set is GE as is shown in Figure 2b. The position of the second trader is improved as he is not subject to blackmail by the first.

The subsequent theory does not depend vitally on this slight difference in the modeling of the characterizing function. The less restrictive assumption is made in the exploration of the models.

+) There is now enough money for all points on ABC to be feasible.

to cooperate with the others they can obtain for themselves more than they would obtain if they accepted the outcome under consideration.

<u>Game 3</u> $(1,0,0)$, $(0,1,\frac{1}{4})$ and $(0,0,0)$

$$V(\bar{i}) = \{(0)\} \quad \text{for } i=1,2,3$$

$$V(\overline{12}) = V(\overline{13}) = V(\overline{23}) = \{(0,0)\}$$

$$V(\overline{123}) = \Big\{\Big(\sqrt{(\tfrac{3}{4}-r_2)(\tfrac{1}{4}-r_1)},\ \sqrt{(\tfrac{1}{4}+\rho_2)(\tfrac{3}{4}+\rho_1)},\ \sqrt{(r_2-\rho_2)(r_1-\rho_1)}\Big) :$$

$$0 \leq r_i - \rho_i \leq \tfrac{1}{4},\quad -\tfrac{1}{4} \leq r_i \leq \tfrac{1}{4};\ -\tfrac{1}{4} \leq \rho_i \leq \tfrac{1}{4}\Big\}$$

This is based upon the bank being unable to create money. It takes the 1/4 from the traders of the second type and pays $\rho = \rho_1 + \rho_2$ for them where $-\frac{1}{4} \leq \rho_i \leq \frac{1}{4}$; and lends them to the trader of the first type for $r = r_1 + r_2$ where $-\frac{1}{4} \leq r_i \leq \frac{1}{4}$ and $r_i - \rho_i \leq \frac{1}{4}$.

In general for the game $(1,0,0)$ $(0,1,M)$, $(0,0,0)$ where $M \leq 1/2$:

$$V(\overline{123}) = \Big\{\Big(\sqrt{(1-\{M+r_2\})(M-r_1)},\ \sqrt{(M+\rho_2)(1-\{M-\rho_1\})},\ \sqrt{(r_2-\rho_2)(r_1-\rho_1)}\Big)$$

$$r_i + \rho_i \leq M;\ r_i \geq \rho_i \quad \text{and } -M \leq r_i \leq M;\ -M \leq \rho_i \leq M\Big\}$$

The Pareto optimal surface has been shifted inwards and is no longer flat in Game 3. Its vertices are $A(\frac{1}{2},0,0)$, $B(0,\frac{1}{2},0)$ and $C(0,0,\frac{1}{4})$. The midpoint on AC is $D(\frac{\sqrt{3}}{4},\frac{\sqrt{3}}{4},0)$. These points are shown in Figure 3.

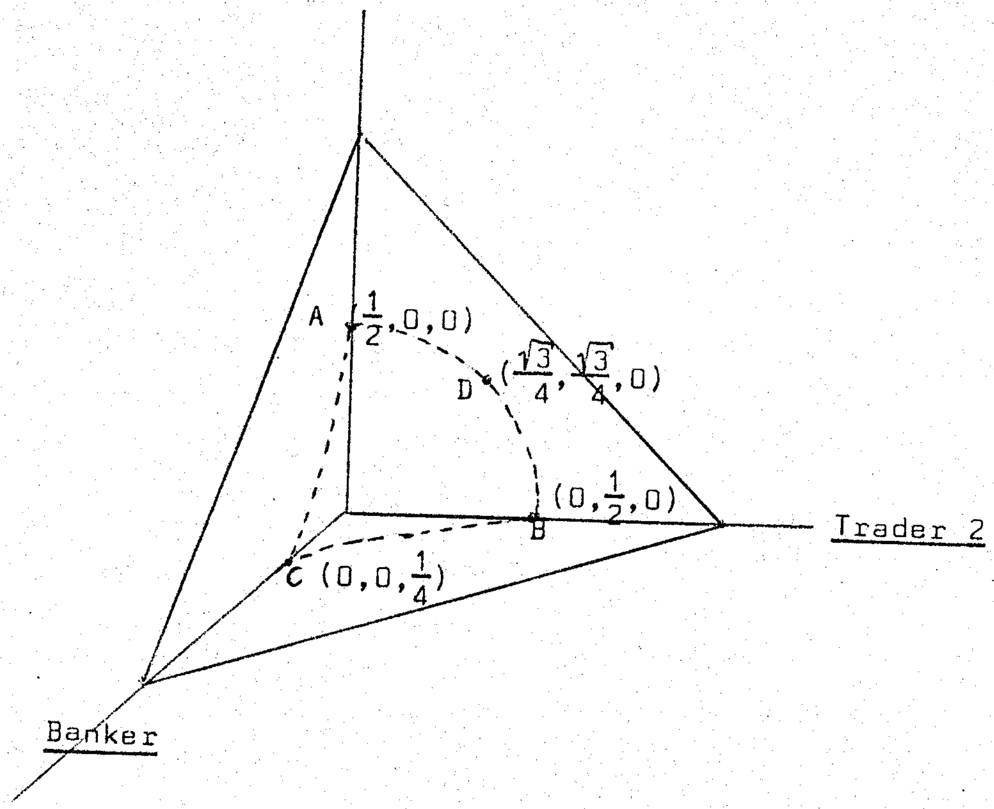

Figure 3

Game 4 $(1,0,0)$, $(0,1,\frac{1}{2})$, $(0,0,0)$

This game is strategically equivalent to Game 1 and has the same characterizing function and core. If Trader 2 were both a merchant and a banker he could finance trade. But he needs the banker and so his cash is of no help to him without banking services.

All of these games are "veto games" except Game 2. The coopera- tions of all three players is required otherwise they obtain nothing. This changes as soon as there is more than one player of the same type.

4.2. General Observations

The market with n traders and one banker always has a core. Give the banker the difference that he makes to the economy by his presence. There will be no effective set to block this imputation. The banker cannot demand more in any coalition that he is in. Any coalition without him cannot obtain more. This includes the case where he makes no difference. He obtains zero in the core.

There is an optimum amount of money needed for the economy in this model. Given that a price is fixed in terms of money during the first trading period and given that all initial endowments of fiat money and loanable funds must be returned to the referee at the end of the game and that trading takes places in fixed intervals this creates a need for a specific quantity of money. (For further discussion and an extra condition see 7.3.)

The optimum amount is calculated by solving the unconstrained competitive equilibrium without the banker with the price of a unit of the first commodity traded fixed at a unit of money. The largest amount of money ever needed to completely relax the period by period budget constraints is the largest amount that the economy will require. The banking and monetary system reqirements are that the sum of issued fiat money and loanable funds be equal to or greaterthan this amount.

If all of this amount is issued initially to the traders as fiat money, then the bank's role will be confined to being the broker, i.e. it will accept deposits and loan them out. In this system, the referee not the banks "creates money" (if discounting is controlled). Even the referee is limited if we impose the condition that the money must be given back at the end of the game and that an initial price is fixed in money terms. Suppose that the referee were to give every single trader in the economy more money than would ever be needed to relax the boundary constraints on the T period optimization problem. Most of it will merely lie around as idle balances with a price of zero. No one needs to borrow, hence the bank will pay nothing on deposits and make no profit whatsoever. The individuals might be able to inflate the price system. In 6.2. a way of removing idle balances from the economy is described.

If there is only one bank there will always be an imputation in the core that gives the bank a profit as long as there is at least one trader who does not have enough money to meet his intertemporal cash flow constraints. This is on the assumption that traders do not make loans to each other directly but are constrained to deposit and borrow via the banking system. This state of affairs changes drastically when there are two or more banks.

Suppose that there are n types of traders and a banker who has enough money M^* to supply their needs. Consider k traders of each type where $k \longrightarrow \infty$ and and where the banker now has kM^* in loanable funds.

We observe that by the formulation of the model, at the same period any good will have the same price to any customer. It also follows that the price of money will be the same to any customer of the same type during any specific time period. If it were different, a coalition between the banker and borrowers excluding the lowest price borrowers will be effective. The same is true for lenders, however with a monopolistic banker there is no effective coalition to make the borrowing rate equal the lending rate.

4.3. A Simple Example Revisited

Consider Game 1 of 4.1., the optimum amount of money needed for the game with k traders of each type and one banker is $\frac{k}{2}$. If initial conditions were $(1,0,M_1)$, $(0,1,M_2)$ and $(0,0,0)$ where $M_1 + M_2 \geq \frac{1}{2}$ the bank would act as broker.

If $M_1 \geq \frac{1}{2}$ the core is the point $(\overbrace{\frac{1}{2},\frac{1}{2},\ldots\frac{1}{2}}^{2k},0)$ and the M_2 and the excess of M_1 over $\frac{1}{2}$ lie as idle balances until given back to the referee at the end. If we wished we could consider them given to the bank as interest and charge-free loans to be returned at the end (this is not too far from the state of affairs in Swiss bank deposits on some occasions).

Given $0 \leq M_1 < \frac{1}{2}$ and k traders of each type and $\sum_{i=1}^{3} M_i \geq \frac{1}{2}$, then as $k \to \infty$ the only point in the core is:

$$\left(\sqrt{M_1(1-M_1)},\ \sqrt{M_1(1-M_1)},\ \ldots;\ \sqrt{M_1(1-M_1)},\ldots;\ k(\tfrac{1}{2}-M_1)\right)$$

The traders can enforce $\sqrt{M_1(1-M_1)}$ each by a coalition excluding the banker. Any imputation which gives a trader of either type more than this amount will be dominated via a coalition with a profile of $(k-1,k,1)$ or $(k,k-1,1)$ for a k sufficiently large. Consider $\left(\sqrt{(M_1+\frac{\varepsilon}{2})(1+\frac{\varepsilon}{2}-M_1)},\ldots k(\frac{1}{2}-M_1-\varepsilon)\right)$ as a point in the core. The coalitions with the above profiles can obtain $(\frac{1-\varrho}{2},\frac{1-\varrho}{2},\ldots,\frac{(2k-1)}{2}\varrho)$. For domination we need to select

$$\frac{1-\varrho}{2} > \sqrt{(M_1+\frac{\varepsilon}{2})(1+\frac{\varepsilon}{2}-M_1)} \text{ and:}$$

$$\frac{(2k-1)}{2}\varrho > k(\tfrac{1}{2}-M_1-\varepsilon) \quad \text{or } \varrho > (\frac{2k}{2k-1})(\tfrac{1}{2}-M_1-\varepsilon)$$

both of which can be satisfied. For $M_1 = \frac{1}{4}$ these become $\rho > 1 - \sqrt{\frac{3}{4} + \epsilon^2 + 2\epsilon}$ and $\rho > \frac{1}{4} - \epsilon$ (when k is large). This gives the banker his monopoly profit modified by the possession of cash by the traders of the first type.

A somewhat counterintuitive feature of the core emerges. The traders of each type are treated the same. They share in the good fortune that first set of traders have some cash. Symmetry applies in this direction because if the first traders have cash they can pay the second traders who then have cash to pay the first traders during the next period. Symmetry also appears in the loan negotiations. The second set of players pay half of the total interest rate of the loan to the first as they both equally need the first traders to have enough money.

If we merely switch the ownership of the initial supply of cash from traders of the first type to traders of the second type, all of the traders lose their advantage. Consider the game (1,0,0),(0,1,1/4),(0,01/4) this is almost strategically equivalent to the game (1,0,0),(0,1,0),(0,0,1/2). As the number of traders becomes large they both have the same limit core of (0,0,...,0,k/2).

Returning to Game 3 and Figure 3 the limit of the core here is (0,0,...,0,k/4). The difference between this and the previous game is manifested in the shortage of money. The banker cannot exploit the traders as effectively without ille-gally creating money.

The games selected for the calculation of examples were simple. In particular the relative prices do not change. It should be noted that in general, when the banker (or bankers) earns zero then the relative prices for actual commodities will be the same as in the barter economy. This is merely another way of saying that the economy has more than the number of trust units that it needs in the hands of the traders and hence none of the intertemporal constraints are effective.

When, owing to oligopoly power, or to an actual shortage of money (failure of the referee to name a big enough number), then bankers make a positive profit (using the limit of the core as our solution criterion) their purchasing power then enters the economy and the number of players with effective demand increases. Obviously when we think of the number of bankers needed relative to the size of the economy, the influence that they will have in general on changing the relative price structure will be slight in most instances.

5. Two or More Banks in General Equilibrium

5.1. A Preliminary Comment

It is my belief that there are two conflicting poor styles in economic theory. The first style has its proponents call a few casual conjectures, a theory and a casually stated assertion, a theorem. The proponents of the other style believe in a procedure which is also not necessarily desirable. Conditions must be stated so clearly and theorems proved so carefully from the very start that statements involving insight and intuition are utterly inhibited if one tries to obey all of the methodological rules of the fraternity.

In this discussion of the role of money and banking in a general equilibrium system I am acutely aware that eventually levels of rigor which I have not met, need to be met. My stress is on the formulation of the appropriate models in the belief that the needs for complete rigor can be met later without substantially changing the argument in this paper. In order to stress this point, in the main text nothing is labeled as a theorem. In the Appendix the more formal model and several theorems are given. Even there I label them as "C-Theorems" where the "C" indicates that I recognize that even these may need a somewhat tighter formal demonstration than is provided.

5.2. The Core of a General Equilibrium System with Banking

A "Money Game" with n traders and b bankers as described in Sections 2 and 3 always has a core. This follows immediately from the observation that the game may be regarded as a market game 7/ 8/ with n+b traders trading in T commodities where the commodities are money with different dates. This is shown formally in the Appendix.

5.3. The effect of Two or More Banks

The effect of oligopolistic competition among the banks is essentially equivalent to oligopolistic firms with constant average costs (the cost of "producing" the line of loanable funds given to them by the referee) and a capacity limit (the size of their loanable funds).

Two banks and "many" banks are all that are needed to demonstrate the most relevant properties.

Let the supply of money issued at the start to the n traders be: $M_1, M_2, M_3, \ldots, M_n$ and the supply of loanable funds granted to the banks be B_1 and B_2.

Excluding the bankers from the economy we can calculate the largest amount of money ever needed by each trader so that the n traders could obtain an unconstrained Pareto optimum. We call these amounts M_i^*, they will of course depend upon the order of trade and the initial distribution of fiat money - but given these fixed - the calculation can be made. The power of the bankers will depend heavily upon the size of M_i^*, the sizes of M_1, M_2, M_3,... and B_1 and B_2 and their distribution. There are several different cases and we list them below:

<u>Case 1</u> $M_i \geq M_i^*$. Here the power of the banks is zero. They are not needed. There is enough fiat money in the economy for the traders to achieve optimality among themselves without banking services.

If we wished to introduce other frictions such as danger of theft, account keeping, transaction simplification and so forth then the banks might still mop up the idle balances that are around. But the price paid for deposits would be slightly negative and equal in value to the other banking services being performed.

<u>Case 2</u> There will be a number $M^* \leq \sum_{i=1}^{n} M_i^*$ that is the largest amount of money needed in the economy if traders with short term surpluses can lend traders with short term deficits via the banks.

Suppose that $\sum_{i=1}^{n} M_i \geqq M^*$ then the banks are needed only to act as the agents through which loans are made. However it can be seen that the rate on deposits will always equal the rate on loans hence the banks make no profit. Suppose that this were false. Then there might exist two different rates, but then it would always be possible for the bank with the lower "spread" to form a coalition to dominate any imputation suggested by a coalition with the other banker by taking some of the latter's customers and splitting the difference in the spread between himself and them. As long as the spread is greater than zero, by undercutting the spread of the other even by a slight amount the bank should be able to entice away all of the other's customers. The banks are selling a homogeneous commodity each with enough capacity to saturate the market.

The core of this game gives the banks an imputation of 0 each. Although the deposit and loan rates are the same and the banks make no profit, this does not mean that the rates are also zero. If one private individual has been able to corner all the fiat money supply then he can certainly demand from the banks an extremely large rate on his deposits, but the banks will pass on this rate to the customers.

<u>Case 3</u> $B_1 \geq M^*$ and $B_2 \geq M^*$: This case is not unlike Case 2 inasmuch as the banks will make a zero profit in the core. There is a zero rate on loans and deposits.

<u>Case 4</u> $B_1 + B_2 \geq M^*$ but $B_1 < M^*$ and $B_2 < M^*$. There are several cases depending upon the supply of cash in the hands of the traders. We will deal only with the simplest. Suppose that the traders had no cash whatsoever. Any vital coalition must have a banker, however although two bankers are desirable as $B_i < M^*$, because $B_1+B_2 \geq M^*$ they have surplus funds between them. The coalitions containing one banker and all traders will be effective in cutting down the lengths of the core. It will shrink with the size of the B_2, until when the $B_i=M$ the bankers (not unlike Bertrand duopolists) make

no profit in the core. As long as the $B_i < M$ then there exist points in the core at which the bankers can make a profit by paying less on deposits than they charge for loans. Although there is more money available than the economy needs, they control it all together and as a pair they can jointly exploit their joint monopoly power measured by the gap $M^* - \max[B_1, B_2]$.

<u>Case 5</u> $B_1+B_2 < M^*$ but $B_1+B_2+\sum_{i=1}^{n} M_i \geq M^*$ and $\sum_{i=1}^{n} M_i < M^*$.

This splits into three cases:
(i) $B_j+\sum_{i=1}^{n} M_i \geq M^*$ for j=1 or 2

(ii) $B_j+\sum_{i=1}^{n} M_i \geq M^*$ but $B_k+\sum_{i=1}^{n} M_i \geq M^*$

and (iii) $B_j+\sum_{i=1}^{n} M_i < M^*$.

In the first case it can be seen immediately that the banks cannot earn anything in the core. A coalition of one bank and all traders blocks this. In the second case the big bank can make a profit in the core, and in the third case both can make a profit.

<u>Case 6</u> $B_1+B_2+\sum_{i=1}^{n} M_i < M^*$: In this instance there is a money shortage in the economy. Extra money has a positive marginal value productivity. There exist points in the core at which the rate on deposits does not equal the rate on loans. Furthermore at the worst point in the core (from the view point of the bankers) the rate on loans is not lower than the marginal value productivity of money.

We now state generally and sketch the proof of a basic theorem. Let there be a market lasting for T time periods. Let there be n+1 types of players. The first n types are traders of different types and the last type consists of bankers. Let there be m commodities in the economy. Each individual of type i has a utility function defined over the amounts of each commodity j consumed in time t, i.e. a function of the form: $\psi_i(x^i_{1,1}, x^i_{2,1}, \ldots x^i_{m,1}, \ldots x^i_{m,T})$. We assume convexity and twice differentiability of these functions. At the start of trade each individual is given his schedule of "Robinson Crusoe" conditions, i.e. those amounts of the various goods that he will have at the start of any period without trade. Trade is sequential, i.e. only one type of trader is permitted to sell his goods at any time (transsshipments are also ruled out - in the sense that a trader of a specific type cannot buy from another from the same type and sell those goods to a different customer in the same period). All transactions have to be carried out with goods going in one direction and cash going in the other. After trade the goods are consumed and do not appear as transformed durables during the next period. Except at least one commodity can be shipped forward for later consumption.

There is an m+1st commodity called money, cash or banker's credit. This commodity is durable and is handed out to the traders and bankers at the start of the game. It can be transferred between the players, indeed it is their only legal tender. After the game is over each player must return to the referee the amount that was originally issued to him.

The banker is a privileged player only he can lend money and accept deposits. All k(n+1) players come together at the start to play in a cooperative nosidepayment game. This game consists of deciding upon loans, deposits and schedules of payments among the players. After these have been decided all of the players (bankers included) enter into trading markets as passive price takers. The traders can buy or sell according to their initial physical endowments and loan and repayment schedules, whereas the bankers are permitted to spend the agreed upon profits from their loan and deposit activities, however, no sooner than they are obtained.

The price of the first commodity to be traded is fixed during the first period at one unit of money per unit.

Given that there is sufficient money in the economy (where "sufficient" is defined below) the core of the game with k(n+1) players converges to a subset of the competitive equilibrium points of the game with only the kn traders as players and no monetary restrictions on trading. i.e. in the limit the bankers in the game with k(n+1) players obtain nothing.

Suppose that the trading market unconstrained by monetary conditions has s competitive equilibrium points. Suppose that an order of trade has been established, then there will be a minimum cash requirement associated with each of these equilibrium points. i.e. an amount such that the unconstrained Pareto optimum could be obtained if this money were available to the kn traders. Call these amounts of money M_1^*, $M_2^*, \dots M_s^*$, where $M_1^* \leq M_2^* \leq M_3^*, \dots \leq M_s^*$, i.e. M_1^* is associated with the equilibrium point which requires the least amount of money to finance and M_s^* is the largest amount of money ever needed. We say that an economy has sufficient money if it has at least M_1^* .

Comment: If the monetary requirements of the different equilibria are strictly different, then by selecting the amount of money be less than M_2^* it is conjectured that the core will consist of only one equilibrium point. That equilibrium point which is characterized by the property that it requires the minimum amount of "trust".

Suppose that the amount of money available is more than or equal to M_s^*. Consider any imputation which gives a banker a finite profit. The coalition of all traders and all but one banker will either have enough or too much money in which case the bankers' profits must immediately be zero in the core. If this coalition does not have enough then as its size increases it can come arbitrarily close to the imputation that can be achieved with the extra banker. Thus there will alsways exist some k such that the coalitions of all players leaving out one banker with a finite profit dominates any imputation in which he obtains the profit.

Once it has been shown that the bankers make no profit it remains to establish that not only are the deposits and loan rates the same but that they are zero. This can be done by considering the domination possibilities of the different sets of size $k(n+1)-1$ where we exclude a single player of any type if he is being paid a positive rate on his monetary transactions.

<u>Comment</u>: If we restricted the amount of money to be less than M_s^* we will not obtain the above results. Given that the game is a market game both parts of this observation immediately follow from Scarf's theorem 9/. If the amount of money available in the economy is less than M_1^* the core will converge to a set of competitive equilibria in a $k(n+1)$ person economy where the bankers will each obtain as their payoffs an amount equal

to the marginal value productivity of their initial holdings of loanable funds.

Game theoretic considerations show immediately than when there is more than enough money in the economy, then depending upon the amount and distribution as numbers grow, at speeds varying from immediately to quite slowly the bankers will obtain nothing. The marginal worth of money is zero and the rate of interest on loans and deposits is zero. When there is insufficient money there will be at least one imputation in the core which gives the bankers some spending power. As soon as this happens the appropriate market game model is one with at least n+1 commodities and k(n+1) players, and as $k \to \infty$ then the core of this game converges to its competitive equilibrium points as has been shown by the theorem of Scarf. Thus a price for money emerges which is not quite the price dictated by the marginal value productivity in the kn person game but by the marginal value productivity in the k(n+1) person game.

As can be seen by the example in 4.2., the payments among the different groups do not quite conform to our ideas concerning interest rates on loans and deposits. In particular in the example below the banker is able to levy a "tax" on players of the second type for what appears to be no service whatsoever. This is not quite accurate. If the banker fails to lend to the traders of type 1, then both types of traders suffer, hence in a cooperative game both should be willing to bear part of the cost of the loan.

5.4. A Simple Example Revisited

Consider Game 3 where the initial distribution of resources was (1,0,0), (0,1,0) and (0,0,1/4). Let there be k players of each type. The core of this game is $(\frac{1}{3},\frac{1}{3},\ldots;\frac{1}{3},\frac{1}{3},\ldots;\frac{1}{12},\frac{1}{12},\ldots)$,

The core is part of the line

$$\left\{\left(\sqrt{(\tfrac{3}{4}-\tfrac{\rho}{2})(\tfrac{1}{4}-\tfrac{\rho}{2})},\ \ldots;\ \ldots;\ \tfrac{\rho}{2},\tfrac{\rho}{2},\ldots\right):\ 0 \leq \rho \leq 1/4\right\}.$$

The coalition $V(\overline{112})$ can obtain $\{(\frac{1}{2}-\eta,\frac{1}{2}-\eta,\frac{\eta}{2},\frac{\eta}{2}):\ 0 \leq \eta \leq 1/2\}$ which dominates imputations with $\rho \gtrless 1/6$.

The equivalent model is given by:

$$\psi_1(x,y,m) = \sqrt{x\min[y,m]}$$

$$\psi_2(x,y,m) = \sqrt{\min[x,m]\,y}$$

and

$$\psi_3(x,y,m) = \sqrt{\min[x,m]\min[y,m]}$$

The price of money is $p_3 = 1$ both traders pay 1/6 for a loan of $(\frac{1}{4}-\frac{1}{12}) = 1/6$.

5.5. Money and the Rate of Interest in an Economy with Banking

We are now in a position to summarize our observations from the investigation of the "money game".

(1) The rate of interest will be zero if there is always "enough" money and there is no oligopolistic power held by the banks. The outcome will be Pareto optimal and a competitive equilibrium point in the n trader game.

(2) Even if there is enough money, but the banks have oligopoly power, then the outcome will be Pareto optimal and a competitive equilibrium point in an n+b person market. The marginal value productivity of money if appropriately distributed would be zero, but the banks by the use of oligopoly power can create a shortage and hence create a positive shadow price for more money capacity in the economy.

(3) If there is not enough money then the banks can always obtain at price at least equal to the marginal value productivity of money.

(4) "Enough money" is enough to satisfy the demands during the T time periods without hitting an intertemporal budget constraint. There will be some minimal amount which will just do this, if correctly distributed.

(5) In this formulation the quantity of money is constant and velocity fluctuates in the sense that almost always there are large sums lying about as idle balances. Although these appear to be counterintuitive results, I believe them to be correct. A flexible reserve banking system is needed to get rid of idle balances (as is argued in 6.2.) and velocity is more or less an irrelevant question (as is argued in 6.1. and in 6.2.)

(6) If the economy were growing constantly during the length of the market periods, in this formulation this would imply a constant growth in the need for money. If the referee issued enough to cover the needs for the whole time at the start, then almost all of the money would lie around in idle balances until the last period when it would all be needed to satisfy the last intertemporal budget constraint. Reserve banking prevents this.

(7) There appears to be no economic reason for there to be a shortage of money in a society without taxes and which trusts the government. Thus the issue of fiat and the availability of loanable funds should always be up to the point of zero marginal value productivity of money or a zero shadow price for monetary capacity.

(8) Even with a monetary shortage we do not obtain quite the same thing as an interest rate for loans and an interest rate for deposits. The rules of the game described influence the strategy spaces in such a way that a trader may have to pay a banker even if he is neither a lender of borrow. The core reflects a reward of the overall marginal value productivity of the bankers' role. It reflects the paradoxical but valid point that both the merchant and the customer want the customer to have enough money to pay his bills otherwise they both starve. One way in which these strange payment patterns could be regarded is that all of the traders are willing to pay a "seigniorage" charge for the creation of money, or more precisely in this case, for the services of the banks in facilitating trade.

One might construct a model with the added rule (as was suggested in the footnote to Game 1) that a trader who neither deposits or borrows from the bank cannot make a payment to the banker and that a trader who is only a depositor should not make payments to the banker.

(9) The device of introducing a finite end to the game by an end of the game taxation or redemption of currency enables this analysis to go through. It is conjectured that it will hold for the appropriate infinite version.

6. The Order of Trade, Taxation and the Velocity of Circulation

6.1. Habit, Order of Trade and Velocity of Circulation

Our analysis in the previous sections has depended critically upon having the individuals trade in order and trade for money with the banker being in the "position of trust". In one sense the theory is extremely institutional, yet in another sense it is completely general. The fixed order of trade was only a device to "hook money into the system". Once we have created the formal mechanism for doing so, then as regards a specific economy undoubtedly order of trade, custom, bill paying habits, degree of mechanization of the financial system, existence of credit cards, stamps, tokens etc. all do make a difference to the financial system of that country. These differences weaken to the control of the banking system and offer alternative ways for institutionalizing or partially institutionalizing "trust".

Formally we may take the model used in the previous sections and enlarge it by introducing random moves into the order of trading. From the view point of actual computation the model becomes far more cumbersome, however from the view point of existence, nothing basic has been changed. The strategies of the individuals in the financial game now become somewhat more complex because the financing must take care of contingencies. Bad winter weather, or getting up late may change the order of payments.

In the game we outlined previously the referee issued cash to the traders and a line of credit to the bankers. We can just as easily construct a somewhat more complicated game where the total amount of money during the process will be variable.

This is done by the simple expedient of setting up reserve levels for the banks. All loans must be paid by the last period of the game and all chips or gold or federal reserve funds must be returned to the referee, however during the actual game, a bank may make loans up to a point that is a function of its deposits. In a cooperative game this increases the strategic importance of the depositors and saves the referee a fair amount of calculation in the sense that under the other scheme if he issued too little money the economy is stuck without it. Here however there is a magnifying effect that enlarges the supply temporarily when needed.

It is easy to observe that the very nature of the "money game" offers the imaginative, vast opportunities to create new financial instruments. A new form of financial note or a new banking law will amount _in abstracto_ to a modification of the strategy space in the financial game.

In Figure 4 the trading patterns for a game with two traders and a banker are shown. This is the equivalent of the bilateral monopoly barter model, but with money added. Suppose that they randomize to see who sells to whom first. This is indicated by the move ascribed to N or "Nature". On each branch of the tree belonging to a player there are two numbers in a circle. These contain goods and cash flow information.

The first number gives the delivery date for the goods (1 stands for this period, k for the k th period from now counting this as the first). The second number gives the payment date. Thus the pair 1/2 means that the goods are to be received now but paid for next period. All goods must be delivered and payments settled before the end, hence in this tree no number in the upper branches can be larger than 2 and no number in the lower branches is larger than 1. We could also introduce a random

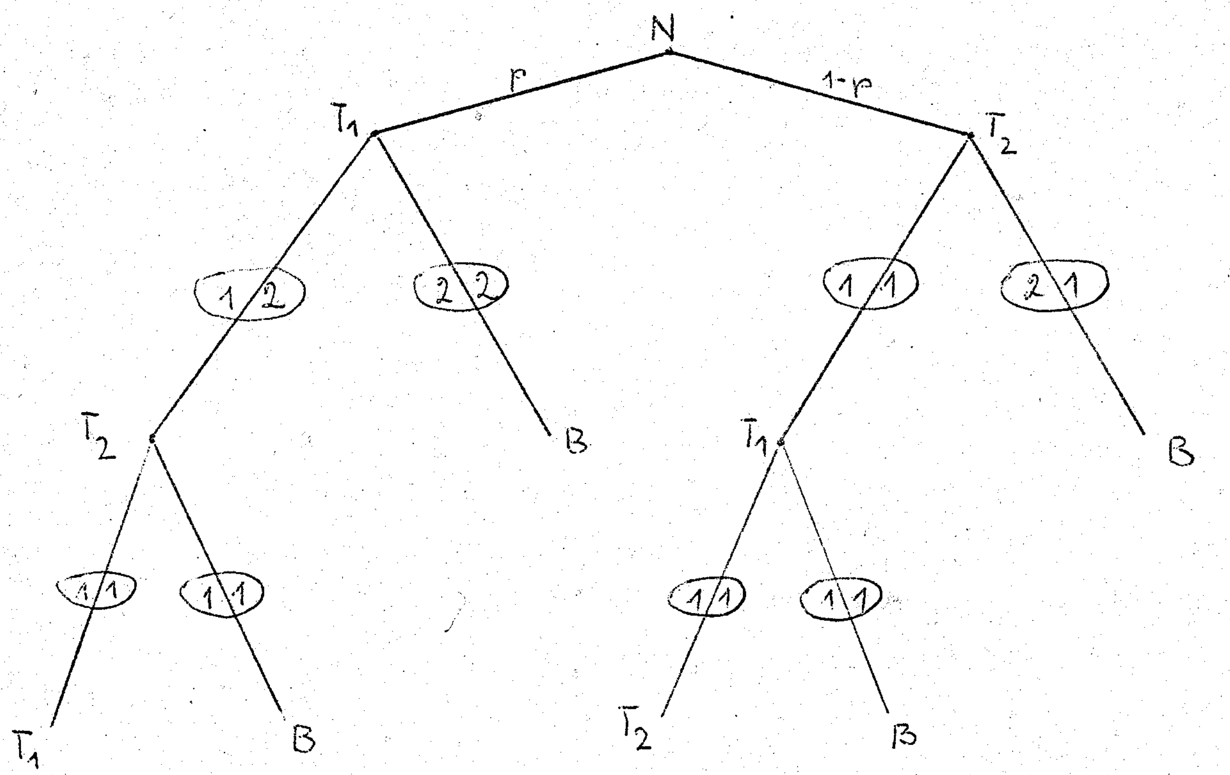

Figure 4

variable to select these numbers. The meaning of this would be to give the probability that an individual might obtain credit from the firm he is dealing with and to give the length of time for which it is granted. Similarly the delivery dates give the firm added flexibility. On some occasions the firm may be paid before delivery.

There are 32 different two trader games possible, leaving out the distribution of money and loanable funds among the traders and the banker. If we were to include these the number becomes much larger, however fortunately, as is shown in 6.5. the number that are fundamentally strategically different is far fewer.

Another institutional feature which fits into this scheme would be the attaching of terms to the granting of credit by a firm to a customer. Discounts on payments before 30 days and better terms for larger or more trusted customers can also easily be modeled in the same formal set-up.

Here we must stress the difference between the general demonstration of existence (even if it is a constructive demonstration) and the empirical problems of measurement and control. Features such as the degree of trust that a merchant has for his customers are not completely but are considerably exogenous to economic phenomena, unless one adopts an attitude that all human behavior can be explained by economic considerations. The theory suggested here offers an expla nation of the money and banking system given institutional and cultural features such as level of trust, habits and laws.From an economic point of view complete trust might be optimal and would do away with the need to construct much of this apparatus, however as we observe, complete trust is not the custom.

What role does the velocity of money play in this scheme? The answer appears to be a very minor role. It depends upon the details of the rules of the game and one's definition of money. If we count fiat money plus bank loans plus the bank's spendable funds (i.e. its retained profits) then in the model given here the velocity of money is constant at 1 per period, however the amount of money fluctuates from period to period. If we count fiat money plus the loanable funds of the banks then the amount of money is constant but the velocity changes as most of the time there will be idle balances around. Specifically in a game of duration of T time periods all of the money in the constant supply version need not be active for more than two periods. They are the period when the intertemporal constraints call for the largest amount and the end of the game when this amount has to be given back to the referee.

In this model it is important to note the strategic distinction between time and current account deposits. We actually have no current account deposits, those are regarded as cash. Any deposit to the bank constitutes an agreed upon loan to the bank and the terms must be specified in the strategies. In this sense a current account deposit could be regarded as a loan to the bank that is instantly repaid to the lender, hence is counted in fiat money. In general however a deposit is not available for one of more periods.

If loanable funds are related to the amount of time deposits in the bank then when more money is needed cash must flow into the bank in the form of time deposits and will hence be "sterilized" from being part of the money supply, then loans can be increased, hence if we count loans plus fiat money in circulation there will be a variable money supply and a constant velocity of 1 per period.

Other ways in which velocity may be influenced include "monetary-barter-deals" which have been ruled out in the game described in the earlier sections but can be modeled. An example of such a transaction would be where a 500,000 building is exchanged for a $ 500,000 piece of land with both parties reporting the monetary aspect of the exchange but neither using money nor credit. This calls for trust between the dealers. Other influences which will be discussed further in Section 7 include "monetizing" private notes on indebtedness when A pays B with a note on C.

Minor temporary influences on velocity include the possibility of the English gentleman paying his tailor on time, changing the number of "settlement days" or the contango; or the prompt settlement of doctor, dentistand lawyer's bills and the reduction of the float. One may also change the methods of payments of employees from monthly to weekly or vice-versa.

These all involve delicate adjustments to cash flow. From the point of view of becoming a millionaire they are undoubtly extremely important. For example the length of time between the issue of trading stamps and their redemption, or the issue of traveller's checks and their redemption has been critical to the financial success of these schemes. However interesting as they are, they are merely a few of the host of institutional differences that influence the functioning of a monetary system, but do not make a fundamental overall difference.

Paying bills faster or slower may be a nuisance. There are definite transaction costs accociated with bookkeeping, bank hours, the speed with which you can run to the store etc. The increase of velocity involved in hyperinflation is another matter and involves basic confidence in the society. This is discussed in 6.4. Sticking to the "money and banking game" it is my belief that the velocity of money question is a basically irrelevant and uninteresting question which has managed to hamper the analy sis of the monetary system for many years.

6.2. Dynamics Inflation and a Flexible Reserve System

In this paper, in one sense, no attempt has been made to model a "real economy". In a real economy the price level is fixed by some mysterious process of tâtonnement and if there is too little cash around it is meant to adjust by cutting prices if there is too much cash we should be in a position to describe the mechanics of the upward spiral.

It is my opinion that although the problem of the detailed dynamics of price adjustment has many fascinating facets, both as a mathematical exercise and as a study in institutions, it is not necessarily the key to the understanding of monetary phenomena. Specifically in the models investigated here, the details of the dynamics of adjustment are explicitly excluded from the model.

I consider that I am trying to construct a game which can be played and must specify all of the rules. In order to do so the traders need to know what prices are in the market and what prices are expected to be. I assume that the referee can calculate the equilibrium points in this trading game; he selects the one with a minimum cash need and announces the current and future market prices. So far there is no uncertainty in the game I have described. Distrust and uncertainty are <u>not</u> the same thing. There may be a religious fetish that calls for payment in money which has nothing to do with uncertainty. Uncertainty is dealt with later. However here for the game I am describing we may assume that the referee announces prices having calculated the general equilibrium.

A key problem in central banking is the control of the price level in the economy inasmuch as it is influenced by monetary phenomena. Thus a control view of the economy may not call for a detailed appreciation of the finer points of the tâtonnement <u>provided</u> that the banking system moves <u>faster than the traders</u>.

I believe that the dynamics of price movements are highly dependent upon institutional details which rarely if ever, figure in general equilibrium models. I also believe that if a banking system moves fast enough, for many important questions concerning the control and functioning of a monetary system we do not need to know very much more than the banks can move faster than the traders. Thus within the game I do not assert that traders might not try to influence prices if, for example, large sums of idle balances were lying around, I merely claim that if this were the case the banking system could soak up the balances before they influenced the equilibrium prices.

An Efficient Reserve Ratio Banking System

The minimum number of people needed in an economy in order to be able to design an efficient banking system that can shrink the supply of idle balances in the hands of the public is four. Furthermore the system will require two forms of paper money which we can call "fiat money" and "bank credit".

The design of the system can be described heuristically as follows: We must put two pairs of Bertrand duopolists face to face in such a manner that they cancel out each others' monopoly power but provide a banking system that enables the amount of fiat money plus bank credit in the hands of the public to follow the varying intertemporal requirements for funds in such a manner that the amounts of idle balances can be reduced to an arbitrarily small amount.

Suppose that there are two traders who could, by trade with trust, obtain a joint gain of 1 +). They are constrained to trade with money because of the rules of the game. Give each of them an amount $\mathcal{E}$ in fiat money for its price in the first period. Suppose that in order to be able to achieve the full utility gain the traders will at some point need an amount M^* of fiat money or credit, where the amount is considerably larger than the amount they have. We now introduce two bankers and set the reserve ratio for each of the bankers at $M^*/\mathcal{E}$. In other words if a banker can obtain $\mathcal{E}$ in deposits of fiat money he can extend his bank credit up to M^*. The characteristic function of this four person game is as follows:

$$V(\bar{1}) = V(\bar{2}) = V(\bar{3}) = V(\bar{4}) = 0$$

$$V(\overline{12}) = f(\mathcal{E})<1, \; V(\overline{13}) = V(\overline{23}) = V(\overline{14}) = V(\overline{24}) = V(\overline{34}) = 0$$

$$V(\overline{123}) = V(\overline{124}) = 1; \quad V(\overline{134}) = V(\overline{234}) = 0$$

$$V(\overline{1234}) = 1$$

where 1 and 2 are traders and 3 and 4 are bankers. The core of this game is $(1-x,x,0,0)$ and the maximum amount of cash ever in the hands of the public is $2\mathcal{E}$. The bankers obtain nothing in the core because they have so much excess capacity that they bid each other down to their marginal costs of zero and the traders obtain nothing for their deposits

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+) This example is presented in characteristic instead of characterizing function form because it is quicker and easier to do so. It is easy to see that it holds in the more general form.

because they too have excess capacity in the amount of paper money of type 1 needed to create a sufficiency of paper money of type 2.

This system has the property that "money" is created and destroyed virtually costlessly as it is needed. In reality there will be many more traders than bankers. This will help the functioning of the system as we can spread the 2€ of fiat money among many.

In a well-run monetary system for a modern economy we should expect that the ratio between fiat money and bank credit is small, i.e. there should be far more bank credit than fiat money. Detailed frictional considerations may call for us to hold a certain amount of cash, however as they are removed we might expect to approach arbitrarily close to a"cashless society".

There are many institutional variations that can be used in the design of a reserve system, such as allowing the deposits from another bank to count in the reserves of a bank. The principle however remains the same and we do not discuss the institutional details which may make one form slightly more desirable over another as we go from country to country.

The ability to pull in idle balances costlessly and to "destroy them" helps to enable a controlling body to nail down a price system. In reality this may be rather hard to do and calls for a better system of forecasting and more sensitive system than exists.

6.3. The Rules of the Game, Taxes and the Rate of Interest

In Section 5 it was argued that in an economy with a non-oligopolistic banking system and an adequate money supply the rate of interest would always be zero and the earnings of the banking system would be zero (excluding bookkeeping and other minor services rendered). Further it was argued in 6.1. and 6.2. that in a formal finite game, either one could fix a large enough amount of money in advance for all future needs so that velocity changes and the money supply is fixed, or one could keep the velocity fixed and have the money supply vary. Beyond that it was argued that other variations in the velocity of money come about from changing the rules of the game. In particular phenomena such as hyperinflations depend upon considerably different rules than the ones described so far.

Suppose that the referee or the government does not like to see idle balances lying around and that it does not have a flexible booking system. It could put in a tax and subsidy program to mop them all up. A simple example is illustrated in Figure 5. Here we have the two traders of the previous examples trading for three periods. The first is given 2 units of money and the second none. There is no banker needed. The first must return the 2 units to the referee after the end of the third period. The first always buys first from the second. Their endowments of goods are (2,0),(0,2); (4,0),(0,4) and (1,0),(0,1).

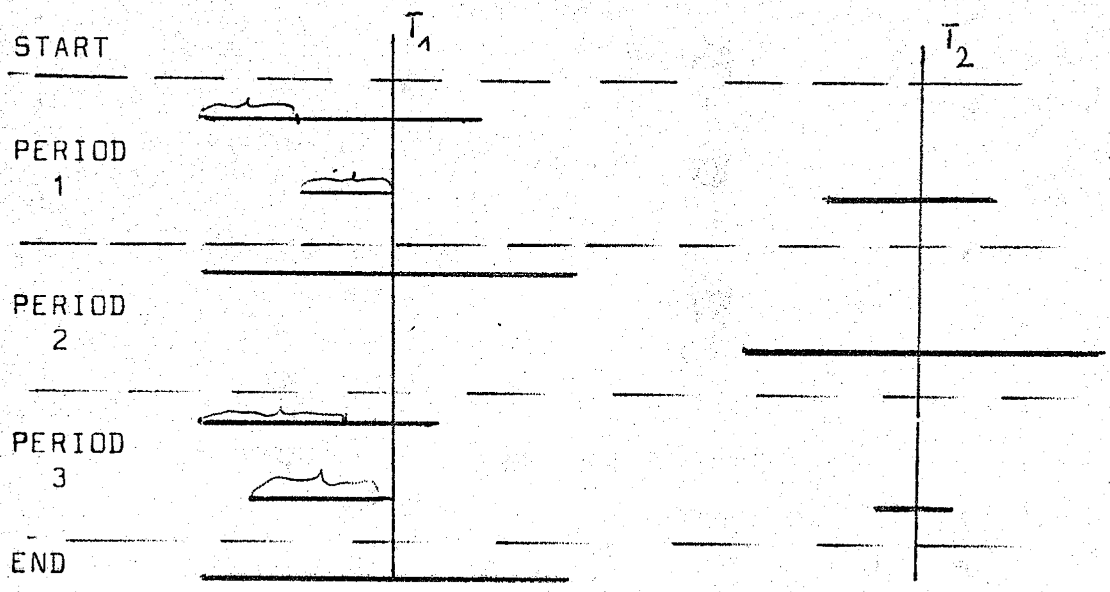

Figure 5

The left side of each line gives the supply of money available to an individual at any time and the right side indicates the demand of the individual for money at that time. The difference in length between the lines on each side indicates an excess of supply or of demand. In this example the part of the lines for the first trader with brackets on them indicate idle funds. In this economy the largest need for money occurs during the second period. The government could have taxed the first trader one unit in Period 1, given it back at the start of Period 2 and taxed him 1 and 1/2 units at the start of Period 3 crediting them against its final tax collection of 2 after Period 3.

This tax policy serves no particular purpose except it provides an anti-inflationary "neatness" in the sense that there are no idle funds left about. The government is not using the money to appropriate resources for its own purposes. In this paper I do not propose to enlarge upon the role of goverment finance, but its taxation plays a critical role in determining the marginal value of money hence we need to consider taxation. Taxes, unlike those in the example given above are usually spent by the government for the purchase of real resources. These resources may be used for the construction of public goods which in turn enter into the preferences of the individuals being taxed.

As the problems of public finance of public goods are being avoided we may modify the money game model as follows. The government or referee is a force outside of the game, not a player, i.e. it is part of the rules. In particular the rules inform all players of their tax liabilities (care must be taken here in making sure that the demands are feasible). Furthermore they inform all players of the amounts of money that the government intends to spend on various commodities at specific times. We may imagine that the commodities are being used to fight a war or are dumped into the ocean so that they do not enter into the utility functions of the players. An economy with n traders and a banker was treated as an n+1 person game. Here an economy with n traders, the government and taxation, in this formulation, is still an n-person game. We see immediately from the analogy of this model with the model with a banker that if the government can use its money to purchase goods this is similar to the money shortage case or the monopolistic bank. The traders are no longer able to achieve the old barter game Pareto Optimal surface. The mere fact that real resources have been withdrawn from this economy by monetary methods has created a positive marginal value for money. Furthermore this will depend upon the manner in which the taxes were spent as each spending pattern represents a different set of changes in the period by period boundary conditions on the n-person general equilibrium model. Relative prices will also depend on the pattern of government spending.

The rate of interest we may determine is not the price of borrowing a unit of money during one period and returning it the next. It is the shadow price of the addition of a unit of monetary capacity to the whole economy over the T time periods, i.e. it is the marginal value of an extra unit of money that does not have to be paid back. In other words the marginal value of reducing an individual's taxes by a unit.

The effect of the government on this finite market is manifested in the change on the intertemporal boundary conditions in the sense that at each period the players are required to pay up a certain number of chips which then leave them with tighter budget constraints. However over the length of the game the government or referee returns all of these chips via the market machanism. Hence the supply of money at the very end equals the supply of money at the start, so all players can meet the condition of returning initial stocks to the referee.

Interim financing in this market is a matter of making sure that there is an adequate money supply so that the traders can at least achieve the sub-Pareto optimal surface that would be available to them if, after having given the government its due (i.e. modified their budget constraints and the demand conditions) they maximized ignoring the money payment constraints on intertemporal trade (using as numeraire the first good traded with a price of one unit of money, thereby keeping the system pegged to money).

All of our observations have been based upon games with finite duration. It is my belief that the models are reasonable approximations of an economy. Most loans are for finite duration and settlement and tax days do roll around for most of us. Nevertheless although I conjecture that my results will hold substantially the same for the infinite case, I suspect that new phenomena will appear. In an economy with taxation and tax lawyers the possibilities of the paradoxes of the infinite are undoubtedly exploited to the hilt. Fortunes are saved in taxes by indefinitely delaying payments. For example stock swaps are often used to put off paying capital gains taxes. The problems of games with infinite length are discussed further in Section 7.

6.4. Taxation, Inflation and Hyperinflation

Taxation and Inflation may be looked upon as twin brothers. In the first the philosophy can be characterized as "tax first spend later". In the second it is spend first and tax by doing so. The first can be more selective in its targets than the second.

A simple example is given to show their relationship. Consider the two traders in our previous examples: they are trading for three time periods. At each time period their initial physical resources are (2,0) and (0,2). Trader 1 buys first. Each starts with a unit of money which must be returned to the referee after the third period. They have more than enough cash for their trade hence do not need a banker. Each has a utility function of the form $\sqrt{xy}$ in each period and they are additive.

We now consider three different sets of government actions as given. (1) no taxes, (2) tax nothing in the first period then 3/4 on each in the next two periods spending the money equally on both products each time immediately. (3) do nothing the first period; print and spend 15 units during the second period. Spending 3 on Good 2 and 12 on Good 1; print and spend 240 units the next period. 48 on Good 1 and 192 on Good 2. Under the third policy the new money is dumped into the economy as legal tender and does not have to be returned by the players to the referee after the game is over. They can throw it away as it is worthless (except as a work of institutional art, but we will forget this). Table 1 contrasts the three equilibrium states. The first column indicates a trader's utility level obtained each period, the second notes the amount of money he has after taxes (this assumes taxes are paid at the start of each period) and the third column shows equlibrium prices.

Policy 1			Policy 2			Policy 3			
Trader's utility	After tax money	Price level	Trader's utility	After tax money	Price level	Trader's utility	After tax money	Price level Prod.1	Price level Prod.2
1	1	1	1	1	1	1	1	1	1
1	1	1	1/4	1/4	1	1/4	1	16	4
1	1	1	1/4	1/4	1	1/4	16	256	64

Table 1

It can be seen that Policy 2 and Policy 3 yield the government precisely the same physical goods. In the first case the price level stays constant and in the second case the price level is going up exponentially as 4^{t-1} as the traders hasten to get rid of the torrent of progressively more and more worthless currency under which they are being buried. The general equlibrium model, because it is sequential is still nevertheless well-defined, the velocity of money is constant but the supply is exploding.

It is my contention that although Policy 3 shows an enormous inflation it does not quite catch the flavor of a hyperinflation in the sense that everyone was assumed to be obeying all the rules. A monetary system is manmade and is part of the legal framework of a country. Furthermore many aspects which are minor in normal times, but which are important in times of crisis or breakdown, are determined by custom and habit. At some level of inflation the rules are abandoned. Quasi-barter and then full barter become more common, people run to buy supplies and try to avoid paying debts.

We note that we could set up a game in which one individual is not required to return some of his chips. This would inflate the price system and on the margin give him the rate of interest.

6.5. A Simple Example Revisited

Returning to Figure 4, suppose that the initial endowments of the two traders were (1,0,1/2) and (0,1,0) we can examine how the granting of credit by firms weakens the power of the banking system. Although there are 32 cases many are distinguished by delivery dates on goods. This can make a difference to utilities gained, but does not influence the payment conditions. There are 8 different credit conditions prevailing of which the banker is needed in 4. Each time it is the second trader who will need credit. Thus in their joint strategy prior to trading contingent loan conditions need to be made.

7. ASSETS AND NEAR MONEY

7.1. Assets as Security against Loans

The previous models have all been based upon the implicit assumption that not only do all traders trust the banker but not each other, but that the banker trusts all players. In our model of the money game all loans made by the bankers are unsecured. In most economies loans are secured by physical assets or by financial assets which in turn are claims on physical assets guaranteed by the law. Furthermore all borrowers are not equal in the eyes of the bank. Lending rates are cheaper for "prime names" than they are for unknown individuals. A fine reputation is, on occasion, bankable.

The terms of borrowing from a banker are a function of trust in the honesty and ability of the customer and these in turn depend on the knowledge that the banker has of the individual. In general, the less the trust, the more the security a banker will require be put up for a loan.

Virtually every item in existence may have some use as security for a loan. However some are obviously better than others. It is harder to flee the district carrying your house than it is carrying your jewels. Ripe Camembert cheese is a thing of beauty but in general is not too good as security against a long term loan. Items which require elaborate maintenance and have a high physical depreciation are also not good items for security (it is probably easier to pawn one's teenage daughter than one's grandmother).

The financial infrastructure of most economies recognizes the need to provide methods to utilize even the assets of the anonymous poor for security. Thus some countries have state pawnbrokers. The more fungible an item is, the wider will its market be. Thus, as my uncle once pointed out to me, a silver cigarette case without one's initials engraved on it has a higher pawnable value than one that is engraved.

Immovables of a reasonably high value historically, relative to other assets with a long durability make good security items. Item such as jewels and art treasures are also good items although their protection may be costly. Stock certificates and other ownership papers, and bonds also are good as security against a loan.

An interesting and moderately important distinction must be made between the instances in which an individual retains the use of the item he is pledging as a security and when he surrenders the item to the lender who in general puts it into safekeeping (or in some cases may use it himself). Houses are usually mortgaged but are used by their owners. Jewelry that is pledged is usually held in safekeeping. Stock certificates are admirable for security as even though they are surrended thereby giving the lender the security he requires, being paper the individual derives no direct utility from holding the paper <u>per se.</u>

Sometimes a loan is secured by the signature of a third party, This is another example of the value of a good name. "Unto him who hath, it shall be given" makes good banking sense.

How is the size of a loan to be related to the security that is pledged? The common practise provides an indication and a method that can be modeled mathematically. As we consider that trade takes place sequentially and in the economy the price of a house has usually been established before a loan on the house is made, a way to fix upon the maximum size of a loan is that it will be some fraction α (where $0 \leq \alpha < \infty$) of the last established price of the item pledged (or on an "independent evaluation"). The α is usually less than one and will vary for different securities. From the viewpoint of a mathematical model of the lending process almost any convention will provide a consistent model. There is a large institutional element which enters into picking the rule and in one sense the α can be regarded as a complex parameter reflecting the experience, customs and degree of trust of the banking system.

7.2. The Rate of Interest in an Economy with Assets and Distrustful Bankers

The model for this case is merely sketched here but is given in the Appendix. Suppose that there are n individuals and b bankers. Furthermore let us assume that there are in the economy which lasts T time periods at most mK(T+1) goods. Associated with each of the new goods is a related good, 1,2,3.... periods older, including some which may attain an age of beyond T and may be left over at the end of the market. An individual is supplied exogenously at the start of each time period with an additional endowment of new goods. All goods in his possession (even if used for security) are utilized by an individual. In the act of utilizing a good the individual converts it into a related good which is one period older. Thus an individual can buy a 1960 automobile in 1970 use it for a year and be in possession of a new good - an eleven year old automobile as contrasted with the ten year old automobile he previously owned.

Let $\hat{y}^i_{j,k,t}$ be the amount of good j of age k in possession of the trader i at the start of time t

$$(1) \qquad \hat{y}^i_{j,k,t} = x^i_{j,k,t} + y^i_{j,k-1,t-1} + d^i_{j,k-1,t-1} - s^i_{j,k-1,t-1}$$

Equation (1) states that this amount equals the amount of additional endowment of this good plus the previous supply of the good dated one time period earlier which by its use has now been converted into this commodity. The previous supply is the amount at the start of the previous period plus purchases minus sales (these are assumed to take place at the start of the period).

Suppose that each individual has a utility function of the following form:

$$(2) \qquad U_i = \Psi_i(y^i_{j,k,t};\ \hat{y}^i_{j,k,T+1}) \qquad \text{where } j=1,\dots,m;\ k=1,\dots,z \text{ and } t=1,\dots,T.$$

The term $\hat{y}^i_{j,k,T+1}$ is the asset evaluation attached to an individual's utility at the end of the game. It can be looked at as the referee's liquidation evaluation of the individual's final possessions.

It would be desirable to derive the shape of U_i in a non-arbitrary manner from an infinite process. It could be then regarded as the value attached to having this initial distribution of assets in the game that is about to start at time T+1. It is the value of an inheritance by an individual who has the same utility function as the previous player and is going to play the next round of the game.

Given the function in (2) however, without solving an infinite process we are now in a position to set up a finite money game of the variety established before. Suppose that the banking system has been arranged so that if the bankers trusted the traders and required no security on their loans, then there would be enough credit in the economy to achieve Pareto optimality. Now however, the rules of the game, limit the strategies of the players when coalitions of bankers and traders form inasmuch as loans must now be "secured". If there are "enough" assets then the prices that satisfy the unconstrained general equilibrium will also satisfy the new system which carries an addition set of inequalities representing the need of the traders to put up at least enough security to cover their loans.

If there are not enough assets in the hands of the traders for securing loans then the additional inequalities will become effective on the intertemporal constraints and the system will be unable to achieve the unconstrained Pareto optimality. This represents a real misallocation of resources and creates a positive rate of interest for "trust" or a positive return for the "moneyness property" of an asset.

Trust has been modeled as part of the rules of this game. Why should the bankers not lend more. An answer to this question requires the consideration of uncertainty and the role of bankers as evaluators. We return to this in Section 8.

When there are enough assets then the marginal value productivity of moneyness of an asset is zero as is the marginal value productivity of more trust. We may observe immediately that a highly industrialized society with plenty of large immovable assets should have a more flexible banking system than one which does not have the assets and has the same degree of trust.

Another implication of this system concerns the role of common stock or ownership certificates for jointly held large assets. The individual will obtain his derived utility from the ownership of the common stock from the effect that the production process has on the economy (assuming constant or decreasing returns). He does not obtain his utility from the stock certificate, hence when he gives his banker the stock certificates he is not losing the use of the assets - this contrasts with giving the pawnbrokers the family heirlooms.

7.3. Gold, Loanable Funds, Reserves and Fiat Money: "The Gold Game"

Consider the game described here played with a specific monetary commodity such as gold. From the viewpoint of the model there are three features which must be taken into account. They are: (1) the money use of gold, (2) the conditions on the supply of gold and (3) the utility or value of gold as a nonmonetary commodity. The last two properties of gold merely add computational complexity to using gold as a monetary medium, but do not change its basic characeristics. Suppose we assume that gold is in fixed finite supply and that it has no utilitarian value beyond being a "symbol of trust". Furthermore let us assume that we have fixed one price in terms of gold; say for example the price of bread during the first period of trade. The gold standard economy now becomes the equivalent of a fiat money economy where the central bank has tied its own hands and has no control over trade and prices. If there is a great abundance of gold (supplied, say by some handy Incas) then prices will be bid up and a considerable inflation will result.

If there is a shortage of gold, in this game economic stagnation will result. After a bout of stagnation in a real economy (as opposed to the game described here) we would expect that the traders would abandon the strict rules and ways of avoiding the economic suicide applied by rigid adherence to the gold standard would be adopted.

The "Gold Game" has a fixed monetary supply, thus velocity varies. If idle balances lie around too long we suspect that they will be used to bid up prices. The bankers do not have the opportunity to extend credit beyond the amount of gold they have on hand. The only way that they can go beyond the rather stringent limits is by "cheating" or violating the rules of the simple game. This can be done if the original bankers happen to be goldsmiths in whose trust other private citizens have deposited sums of gold for safekeeping (for no other reason than goldsmiths are usually better able to protect sums of gold against robbers than are other private citizens engaged in other occupations). An enterprizing goldsmith might be willing to take advantage of the law of large numbers and lend gold that is not his on the basis that it is a fungible commodity and that within a reasonable expectation he can return to those owners calling for their gold the amounts required.

If the goldsmith fails to do so he might get a hand chopped off or go to debtors prison. The penalties for being caught short appear to have been worth risking for many hundreds of years. They were best summarized by Daniel Drew in the last century. He observed: "He who sells what isn't hisn, must pay up or go to prison."

In summary, gold appears to be a universally acceptable, almost rigid supply near-fiat money whose monetary properties are further obscured by its value in jewelry, to dentists and in other industrial processes. Without "cheating" the gold game calls for 100 % covered reserves as a backing for loanable funds.

7.4. Financial Gimmicks and the Money Supply

Credit cards, trading stamps, food coupons, special export currencies, stamp collections and art are but a partial list of the highly transportable financial instruments available to a sophisticated economy. Each one has its own roles concerning its strategic possibilities in a "money game". For example it is often a crime to use someone else's credit card even with his permission. This being the case one cannot really define the supply of money in a simple way without reflecting the strategic possibilities involved in a modern economy.

It is of interest to observe that in the United States the trading rules concerning trading stamps given away with purchases in many retail establishments, pose not merely academic questions, but have been the subject of important litigation. Can an independent broker set himself up in business to establish a market among the different types of stamps? The companies issuing the stamps wanted to prevent this (thereby extending the "length of the float") but it appears to be legal.

Given the special role of gold in international economics, should we count the amount of gold in existence in an economy as part of the money supply? Once more we must look at the rules of the game in detail. In the United States it is <u>not legal</u> for a private citizen to hold gold in ingot form (although he could get a friendly Swiss coin press to transmute it into Tzarist gold pieces which he could hold legally)! In Tangiers, Macao and Hong Kong, gold bullion is regarded as a very acceptable form of money.

A reasonable rule of thumb appears to be: count as part of the money supply all gold that is available to the economy minus the costs of transforming it into bullion form. Thus in general the amount of gold in the citizens' teeth is not part of the monetary supply unless one intends to operate Bergen-Belsen or Treblinka.

Occasionally enthusiatic representatives of special interests have suggested bimetalism as a way of increasing the money supply. Thus instead of using gold alone we might wish to use both gold and silver where the two would be linked by a fixed ratio. If either or both items had no other use than as a monetary medium there would be nothing particularly wrong in doing so. However if both have a utilitarian value it is fairly obvious that any attempt to do so would fix one price too many in the economy and an inefficient allocation would result.

7.5. Stocks, Bonds and Government Debt

In the formal model presented in the Appendix we have considered only a trading game, having made use of the Rader transformation. In order to talk meaningfully about stocks and bonds it is desirable to consider production explicitly. If there is no uncertainty in our model then there will be no difference between bonds and stocks. In Section 8 we introduce uncertainty and hence make the distinction there. However here we lump them together. If all production process were strictly homogenous of order one and were run by Arrow-Debreu automata (called "profit-maximizing managers") then their profits would be zero and the dividends paid to a stock holder of the joint enterprise would also be zero.

Suppose that we adopt the attitude that at least a few of the firms in the society display increasing costs, thereby implying that they have some nonmarketable commodity called "management" or "organization" which is a peculiar property of a particular institution. Such an institution will make a positive profit in a general equilibrium system and hence will be able to pay a dividend. This establishes a value for shares in the ownership of that organization (and without uncertainty for bonds as well, as they are equivalent).

Stocks and bonds are pieces of paper which are symbols of ownership. It does not matter where the pieces of paper happen to be locked up as far as the process of paying dividends is concerned. Thus, as has been noted previously, to all intents and purposes stocks and bonds offer an individual in a modern economy excellent instruments to post as security against a loan. They are so excellent that in some countries they are called "securities". While they are posted as security the individual still obtains their productive use.

If for some political reason a government wishes to enlarge its battery of financial weapons it may go beyond taxing and printing and decide to issue debt as well. In particular it could even issue bonds of any redemption date including an infinite redemption date. If it insists upon taxing its citizens,then a perpetuity can bear an interest rate equal to the marginal value utility of not having to pay taxes up to the tax value of the paper. In some countries it is permitted to pay taxes by surrendering goverment bonds.

8. UNCERTAINTY, NUMBERS, ANONYMITY AND TRUST

8.1. Numbers, Social Mores, Trust, Information and Limit Theorems

The functioning of a modern market economy calls for decentralization and anonymity combined with cooperation. The "money game" is a cooperative game whereas the market is basically an anonymous mechanism. A modern society tries as much as it can to make financial as well as market moves anonymous acts. Problems involving evaluation and trust make it difficult to do so. In different societies various forms of insurance have been devised to ameliorate the difficulties caused by uncertainty, poor evaluation, lack of trust and inability to calculate.

The methods needed to handle uncertainty and impersonal dealings are highly dependent upon the numbers of individuals in the economy. In general, the more people there are, *ceteris paribus*, the more unlikely will there be personal dealings between two individuals selected at random.

The importance of personal knowledge, trust and evaluation of individuals' wisdom, honesty and sense of responsibility cannot be overemphasised in financial dealings. The history of finance provides myriads of examples of different instruments handtailored to take into account varying degrees of knowledge and trust among individuals. Even the issue of banknotes reflects this. In the 19th century banks used to issue their own banknotes in the United States. Various banknotes sold at a discount related to their distance from the bank and to the reputation of the bank. Today in the normal course of business virtually every degree of anonymity and personal contact is present in the transactions. When an individual buys a pack of cigarettes at a strange store for cash there is anonymity on both sides and trust is achieved by the trust-unit function of coin or fiat bills. In some parts of the world there is still a predeliction for silver to cover such transactions.

When an individual buys gasoline he may pay by cash or use a credit card which has his name on it, or if he is a "local" and knows the man who runs the station he may pay by personal check on a local bank. In buying jewellery at a boutique in a tourist hotel in Brazil a United States citizen may find that with a little identification, the store will accept a personal check.

An important business deal may be sealed by a handshake by brokers or traders in the same trade. Loans may be based on "your good name", on assets put up by the borrower or on assets or a guarantee by a third party.

In general all of these arrangements depend upon the information state. All other things being equal (1) the more people there are in the society the more anonymous dealings tend to become. (2) the more change there is in price levels and other states of society the more information is required to describe or interpret the environment.

General equilibrium theory, as it is usually expounded, ignores the effect of numbers. An existence proof for the price system is presented which holds equally true for 2 or 2,000,000, The fundamentally different feature of the game theory approach to the study of markets is that it is done by means of limit theorems in which numbers play a critical role. If fundamentally different behavior is expected from 2 individuals than when there are 2,000,000 present this must be deduced from the process or explicit assumptions must be made about what is assumed to change as numbers increase.

In the earlier sections the core of a money game was used to study the properties of money in a society without uncertainty. In Section 8.3. when uncertainty is introduced explicitly into a money game we must take care to be explicit about the information conditions and the insurance conditions that we deem to be present.

Radner has recently 10/ considered competitive equilibrium conditions where individuals are assumed to have limited computational abilities. He however does not relate the lack of calculation power to the number of individuals in the society. It is my belief that this link is extremely important.

8.2. Types of Uncertainty

In the world of finance, I suggest that there are six different types of uncertainty which must be accounted for. They are;

(1) Statistical uncertainty with uncorrelated events,
(2) Statistical uncertainty with correlated events,
(3) Rare events,
(4) Statistical uncertainty concerning personal traits,

(5) Uncertainty concerning strategic behavior,

and (6) Uncertainty concerning conformity to the rules of the game.

The first is the wellknown type of uncertainty for which in general, it is easy to buy insurance. Fire insurance serves as a reasonably good example.

The second involves "acts of God" plagues and a host of events for which the weasel-wording and the fine print of most insurance contracts have an out.

A Bayesian statistician might argue that there is fundamentally no difference between an event such as the failure of the tomato crop next year due to bad weather and the failure of the Edsel automobile or some other new product; however although it is relatively easy to obtain insurance for the first, it is virtually impossible to obtain it for the second.

In the granting of credit the evaluation of things and possible events is only part of the process. The evaluation of character, trust-worthiness etc plays an extremely important role. From a purely formal point of view we could merely say that there is no difference in the two. It is my belief that the actual methods of evaluation are condiderably different and that the difference should be noted.

Game theoretic uncertainty is yet another type of uncertainty and arises from the strategic behavior of the players in conditions in which less than perfect information 11/ prevails in a game of strategy.

The last, but extremely important form of uncertainty concerns the adherence to the rules of the game. All game theoretic models assume that the rules are obeyed. A financial and monetary system is a tenuous and sophisticated construct of a society and depends to a great degree upon the explicit and implicit adherence to the laws, practises and customs. On occasions, people cheat somewhat against the rules. If enough cheat for a long enough time, the system breaks down and a different game is played. This is especially important to keep in mind when we try to model the rules concerning credit.

8.3. Uncertainty, Bankruptcy and the Rules of the Game

In this section we commence with the examination of statistical uncertainty in a general equlibrium framework. We pursue our investigation by means of examining a series of simple examples. A general treatment is deferred to a later date.

8.3.1. The Credit and Insurance Game with Two Traders

Consider a market that lasts for two periods. There are two traders T_1 and T_2. Trader 1 buys first from Trader 2 during each period. There is an efficient banking system available to supply the monetary needs of the traders at a zero monetary rate of interest, excluding risk.

There is risk present in the economy. The traders are informed about the risk which is manifested in the variability of the potential supplies during the second period.

In particular let the utility functions of the traders be given by:

(1) $$U_1 = 10 \min \left[x_1^1, y_1^1\right] + \min \left[x_2^1, y_2^1\right]$$ and

(2) $$U_2 = \min \left[x_1^2, y_1^2\right] + 10 \min \left[x_2^2, y_2^2\right]$$

where x_t^i and y_t^i are the amounts of commodities X and Y consumed by Trader i during period t. Here we observe that Trader 1 has a considerable time preference for the first period, while Trader 2 has a marked time preference for the second period . We assume that Y cannot be inventoried, but must be consumed immediately.

Figure 6 illustrates the supply conditions in the market. In the first period Trader 1 has nothing and Trader 2 has (10,10). Whereas in the second period Trader 1 has nothing with a probability of (1-p) and (10,10) with a probability of p, and Trader 2 has nothing.

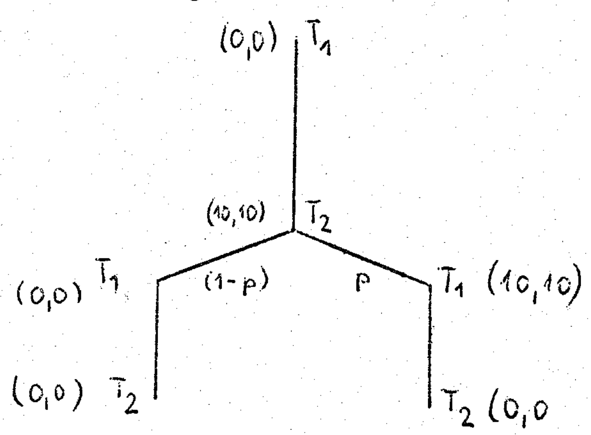

Figure 6

It is easy to observe that if the economy were favorable the traders could take advantage of their difference in time preferences and each achieve (in his own utility scale) a score of 100. This is shown as the point E in Figure 7a. However there is risk present and we must describe how the traders behave in the presence of risk and what we mean by Pareto optimality in the presence of risk.

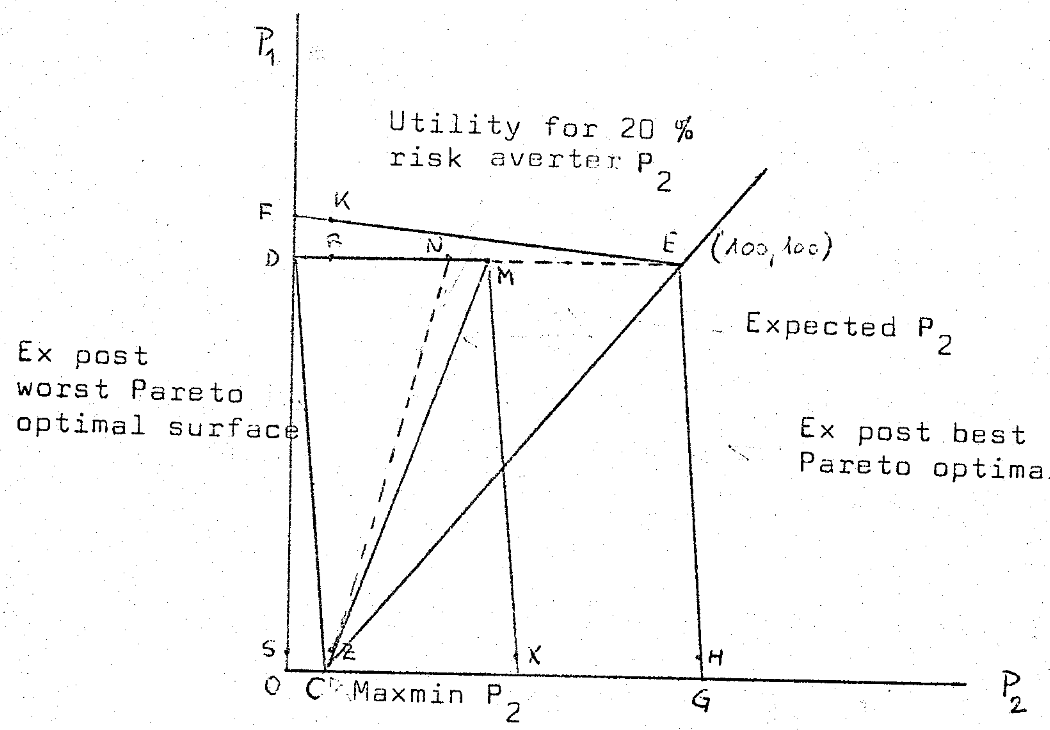

Figure 7a

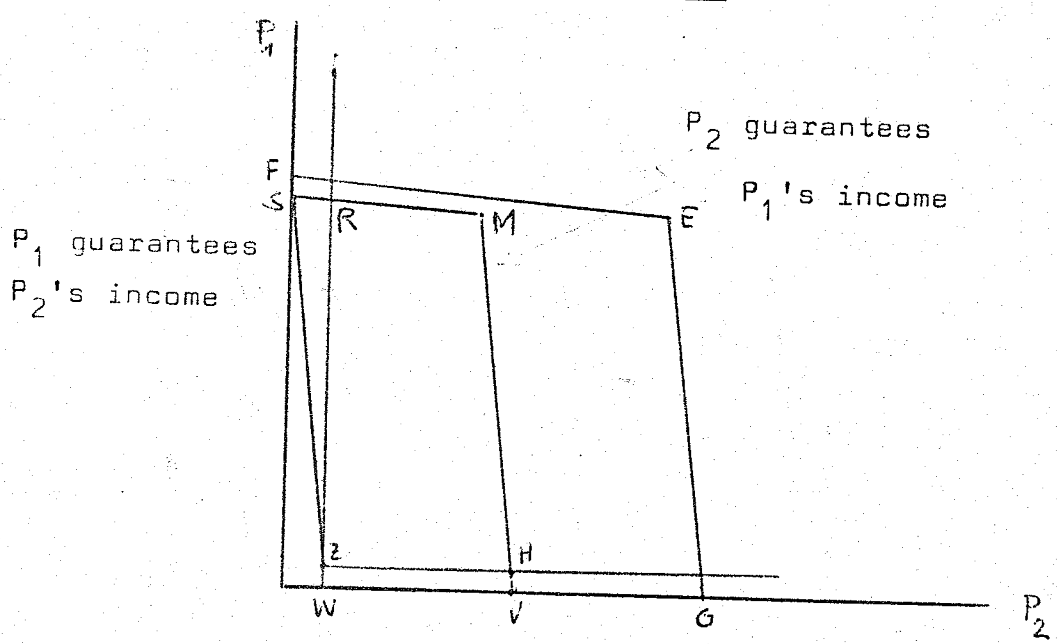

Figure 7b

The line CD in Figure 7a is the ex post worst Pareto optimal surface that could be achieved by the economy if there were a "crop failure" or,in less picturesque words, if there were no resources during the second period. The whole of the line CD could be achieved if there were no restrictions on barter or on gifts, however only the point C individually rational for Trader 1 as he can obtain it without any form of cooperation.

The boundary given by FEG is the ex post best Pareto optimal surface. If no trading restrictions were imposed on the traders it could be achieved. The Pareto optimal surface and the contract curve are not quite the same here as Trader 2 can obtain 10 for himself; furthermore Trader 1 has an expected utility gain of 5 if he is risk linear (in this example for most purposes of illustration we assume risk-linearity and also set $p = 1/2$). Hence the contract curve is given by the segment KEH.

If we impose specific conditions on the type of trading then all points are not necessarily feasible. In particular if we wish to use a monetary mechanism how is trade to take place in this two person economy?

Suppose that a bank had lent Trader 1 20 units of money and that the price of the commodity Y had been fixed at one unit of money per unit for the first period. Trader 1 could buy all of the supplies of Trader 2 during the first period. If the economy were prosperous (probability p) Trader 2 would buy all of the supplies of Trader 1 and they would achieve the distribution indicated by the point E. If the economy were not properous Trader 1 would achieve a very favorable outcome indicated by D and go bankrupt to the bank for 20 while Trader 2 would left with 20 and nothing to by. Suppose that Trader 2 had preferences that were risk linear. His expected utility gain from the trade is given by M. If he has a 20 % bias towards risk aversion then his expected utility gain is denoted by the abcissa of N.

If money is used in this market, then there is a constant possibility of (1-p) that the first trader will go bankrupt. The lines CM and CN show the expected utility gain for the traders with risk linearity (CM) and with a relatively high risk aversion (CN). In either case we observe that the second trader is well served to select the means of trading that permits the first trader to go bankrupt for even the largest amount. This will yield the first trader a utility of 100 and the second will obtain some point of the line RM depending upon his risk aversion (someone who welcomed risk would obtain a payoff on ME). The full expected Pareto optimal surface includes MV.

Figure 7b shows the expected Pareto optimal surface if both traders are risk linear. The expected Pareto optimal surface is SMV. SZ shows the expected income to Trader 1 on the assumption that he guarantees the income level of Trader 2. MV shows the expected income of Trader 2 on the assumption that he guarantees the level of Trader 1's income.

SZ is almost irrelevant in the sense that Trader 2 can guarantee himself the maximum on that curve without cooperating with Trader 1. However if we wish to explore all of the relevant strategic possibilities we need to consider the cooperative coalition with one of the traders maximizing his expectations subject to a guaranteed payoff for the other.

The part of the expected Pareto optimal surface that is jointly rational *ex ante* is given by RMH. If we are willing to consider a cardinal utility scale for each trader in his evaluation of risky prospects, then any trading method which reaches SMV is efficient. Any attitude towards risk can be reflected in the preferences and will change the location of SMV. If we do not wish to consider cardinal scales then we must be prepared to accept virtually any outcome to the right of the line RW.

Any method of trading which yields a result along the curve SMV is Pareto optimal! For the afficionados of the price system only, we can achieve a solution arbitrarily close to M by the use of the price system. Suppose that instead of giving Trader 1 (0,0) during the second period with probability (1-p), we give him $(\varepsilon,\varepsilon)$ where is arbitrarily close to zero. Then a price system of $(1,1,\frac{10}{\varepsilon},\frac{10}{\varepsilon})$ will clear the market under all circumstances. As $\varepsilon \rightarrow 0$ the prices in the second period increases without bound. For $\varepsilon=0$ if we wish to preserve our market without bankruptcy then we could introduce the convention that

$$\lim_{\varepsilon\to 0} \frac{10}{\varepsilon} \left\{ \varepsilon + \varepsilon \right\} = 20$$

so that Trader 2 happily spends his 20 units of money on nothing.

Is the price system under risk "institution free"? Certainly not, it is merely one of many alternative cooperative economic conventions for a society to assign responsibility for a failure of process. In this case paradoxically Trader 2 absorbs all of the risk!

Failure of process could be caused by an exogenous random factor such as weather causing a crop failure, or by inefficiency, ineptitude or fraud. We only consider the first at this point.

Let us consider several different conventions for handling trade through time with risk.

(1) No Credit: This yields a result which is not Pareto optimal. In this case with risk-linear utility scales and $p=\frac{1}{2}$ the expected payoffs are (5,10) as is shown at the point Z.

(2) Private Credit: Suppose that the price of Y in the first period is fixed at $p_{2,1}=1$. If Trader 2 extends one unit of credit, Trader 1 can achieve a utility level of 5 and together their expected payoff is (5,59.5) as is shown in Figure 7b at H. The price system is (1,1,1/10,1/10) if the economy prospers, otherwise it is $(1,1,\infty,\infty)$. Under this arrangement virtually all of the gain goes to the second trader.

(3) Bank Credit: If there were competitive banks they would be motivated to lend up to 20 and the expected outcome would be M. If prices are unbounded then there is no need to describe bankruptcy. If not, then we might wish to treat the lack of goods in the second period as leaving Trader 1 in debt to the bank by 20 and Trader 2 with 20 units of money and nothing to buy.

(4) The Pledge of Assets: Suppose that there were a third good, Z in the economy and that the first trader had a supply of Z. This good is durable and (as a good first approximation) yields a utility stream for several periods without wearing out (it might be a painting, for example).

The banker might require that the first trader "sterilize some of his assets" i.e. put them up as security (thus withdrawing them from possible trade during the length of the loan).

The full theoretical implications of this possibility cannot be explored until the meaning and need for bankruptcy laws is investigated.

(5) The Sale and Repurchase of Assets

Sometimes individuals in need actually sell an asset, pass title of ownership and buy it back later if they can afford to do so. They may even maintain an option for repurchase.

8.3.2 Efficiency, Ordinal or Cardinal Utility

All of the methods noted above do not make explicit use of the possibility of a cardinal measure of utility when considering projects involving risk.

If we believe that it is reasonable to use a cardinal measure for utility when there are prospects involving risk which must be evaluated, then we can describe the expected Pareto Optimal surface, as was done in Figure 7.

The expected Pareto Optimal surface is a cardinal utility construct and will change in location as risk reaction varies. Projects that are certain will be fixed points in any transformation reflecting different risk reactions. Thus in general there is no guarantee that market adjustments which do not explicitly utilize the cardinal properties of the utility scale will yield an outcome that is Pareto optimal in the sense of expectations, except by chance.

8.3.3 Strategies, Forward Contracts and Bankruptcy

Radner 12/ in his work previously noted, deals with general equilibrium under uncertainty by means of strategies for each player acting in a situation with less than perfect information. It is possible to calculate everyone of the myriads of states of economy that might exist and to calculate a price system for each one. A strategy (both in the sense of Radner and conventional game theory) is a set of instructions which a player could give to his agents, telling each agent how to act if the economy "went his way". This involves no buying or selling of future contracts. Transactions take place in current markets using current information.

Radner shows the existence of a general equilibrium point under uncertainty using this enormous inflation of price systems. This is certainly a step in the right direction inasmuch as it introduces the value of information into the system. However, as he notes, the computational requirements are unreasonably large.

A simple example of the value of information can be constructed using two traders and three time periods. Let Trader 1 have good X with exogenous supplies of x_1, x_2, x_3 and Trader 2 has Y with exogenous supplies of y_1, y_2, y_3 for periods 1, 2 and 3.

Each has the same utility function:

$$U_i = \min \left[\min (x_1^i, y_1^i),\ \min (x_2^i, y_2^i),\ \min (x_3^i, y_3^i) \right]$$

$$\begin{aligned} x_1 = y_1 &= 12 \\ x_2 = y_2 &= 12 \text{ with probability } 1/2 \\ &= 0 \text{ with probability } 1/2 \\ x_3 = y_3 &= 12 \text{ with probability } 1/2 \\ &= 0 \text{ with probability } 1/2 \end{aligned}$$

There are 4 possible states of nature characterized by:

12, 12, 12; 12, 12, 0; 12, 0, 12, and 12, 0, 0

If these states were known in advance, it is easy to observe that the expected payoff to each trader in equilibrium would be:

$$\tfrac{1}{4} \min (6, 6, 6) = 1\tfrac{1}{2}$$
$$\tfrac{1}{4} \min (4, 4, 4) = 1$$
$$\tfrac{1}{4} \min (3, 3, 6) = 3/4$$
$$\tfrac{1}{4} \min (2, 2, 2) = \underline{1/2}$$
$$3\ 3/4$$

If the states are not known in advance and X and Y can be carried forward in inventory then an optimum policy for each is to ship 4 units during the past period. If 12 appears the next period, ship 8; if 0 appears, ship 2. This is shown in Figure 8. The value of such a policy can be seen

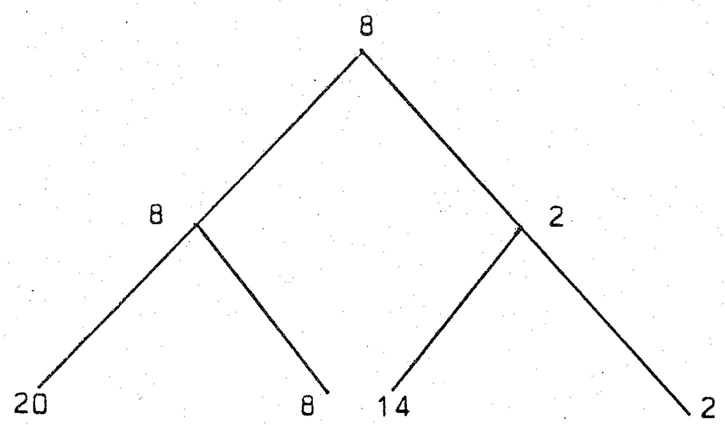

Figure 8

to be:

$$\tfrac{1}{4} (4 + 4 + 1 + 1) = 2\tfrac{1}{2}$$

This is 50 % less than that which can be guaranteed given the information in advance.

We note the nonsymmetry of time and its relationship to information. If we were able to ship goods backwards in the time this would be the equivalent of complete information and the two cases noted above would be the same.

We can see that information conditions are critical to the economy and that as soon as contingencies are considered the possibilities grow enormously and price systems proliferate with every contingency. Even so, the need to enlarge our model or to look for a different formulation does not arrive merely from computational considerations. The concept of trust once more becomes of importance.

Previously lack of trust was not accompanied with uncertainty, here it is. One way of dealing with lack of trust is by having individuals enter into legally binding contracts. The use of money is one such contract.

Fiat money is a form of a forward contract. There are several others. In particular in the economic life of the United States the following exist:

(1) "Futures": A contract to buy or sell a specific amount of a commodity for a specific price at some specific date in the future (also there are puts, calls and straddles).

(2) One can contract to buy an unspecified amount for a specific price at a specific future date. An offer to buy half of the crop before it has matured would involve this contingency.

(3) One can contract for a long term lease, where the service sold is fixed with a clause under which price is contingent.

Money is the least binding and most anonymous forward contract. There is no commodity, quantity, price or delivery date specified.

We have seen in 8.3.1 that if a price system is infinitely flexible the introduction of bankruptcy is not necessary. If the price system is not that flexible, it was argued that bankruptcy is needed. If we wish to include the possibility of the sale of futures contracts or of insurance then, in general, <u>bankruptcy conditions become a necessity</u>.

8.4 Brokers, Insurance and Numbers

Suppose that we replace the 2 traders of the example in 8.3.1 with 2,000,000 traders with 1,000,000 of each type. Furthermore let us consider that each trader of the first type will obtain 0 or 10 with a probability of 1/2 and that these probabilities are independent. In many realistic cases the probabilities would be correlated. Furthermore if they were highly correlated it would be hard for an individual to obtain insurance. Here however we take the simple example. The expected supply

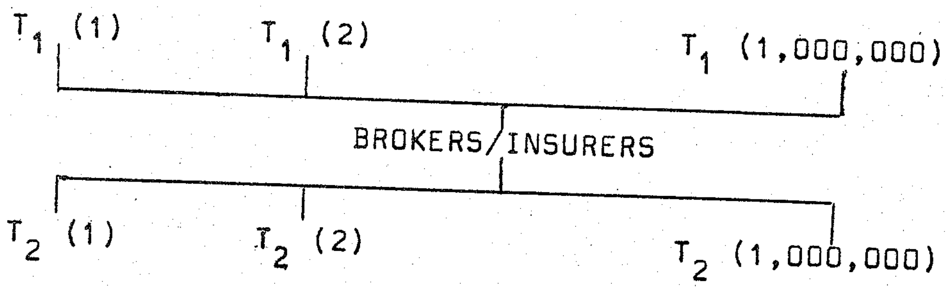

Figure 9

of the first commodity is = 5,000,000.

We simplify matters still further by considering one commodity each period so that utilities are:

$$U_1 = 10x_1 + x_2$$
$$U_2 = x_1 + 10x_2$$

where x_1 is the supply of the good in period 1 and x_2 in period 2.

There are several different institutional possibilities as are indicated in Figure 9. A broker-insurer might "buy the crop" from the traders of the first type before it exists. He might sell it before it exists or he might sell it only after it exists; or he might sell part before and part after. Suppose that after buying the future crop, the broker-insurer sells it in advance. We might assume that society fixes limits on that which cannot be sold without existing and on the penalties for failure to live up to contract.

For example there might be a law which says that given (μ, σ) for a future item, no more than $\mu - k\sigma$ future contracts may be sold. If ex post the amount q_s were sold and only the amount q comes into existence, then we may assume that a penalty of $k(q_s - q)$ where $q_s > q$ is levied against the seller. This is his out-of-stock or bankruptcy penalty.

If the seller has other assets the creditors can collect otherwise they are out of luck. Suppose in our example there were two broker-insurers. They have no assets, but as society is rather conservative they are not permitted to sell more than $\mu - 10\sigma$. Competition will force each to offer 10 for the whole crop of any T_1 and they will offer a future of 4.975 units priced at 10 to T_2. They will (to a 10σ safety level) almost always be able to deliver. An individual T_1 would find it difficult to sell a futures contract to an individual T_2 owing to the enormous variance, but via the broker-insurer owing to the law of large numbers this becomes a reasonable transaction. The brokers have an expected profit (in terms of some slight surplus of the good in period 2). This profit could be made arbitrarily small by requirements on the holding of reserve assets and limits on the contracts of the brokers.

8.5 Optimal Laws for Future Contracts, Reserves and Bankruptcy

The example in 8.4 was extremely simple and special. A point almost on the expected Pareto optimal surface of T_1 and T_2 was obtained. It is conjectured that for noncorrelated risks in a society with sufficient assets relative to the risks that it should be possible to select a combination of laws, governing future contracts, reserve requirements for insuring agents and bankruptcy laws such that trading can yield outcomes arbitrarily close to an expected Pareto optimal value in general.

8.6 The Evaluation of Risk and the Financial Infrastructure

It is difficult to lay enough emphasis upon the role of the institutions making up the financial infrastructure as brokers, agents of institutionalized trust, information processors and above all, evaluators of risk.

Given lack of trust but no uncertainty among the traders, financial institutions may act as brokers or trusted intermediaries. Their full role appears, however, in the presence of uncertainty. Without uncertainty there is no difference between a bond and common stock. With uncertainty, information and its evaluation play a central role. Fiat money, bonds, common stock, promissory notes etc... all take on meaning as different types of forward contracts reflecting different conditions of risk sharing, trust and participation in information evaluation and decision-making.

If individuals are risk averse then the presence of large numbers causes <u>increasing returns to scale</u> in an economy. Furthermore if individuals have different information then their fates become strategically interlinked via <u>externalities</u> in information. For example a lack of information by one trader can cause him to act in a way that hurts both himself and another. The example in 8.3.3 can be modified to show this. Consider that the first trader knows what the future states will be for both and the second trader does not. The latter'saction will hurt the former unless he can be informed.

9. CONCLUSIONS

The implications of the use of money are many. There are highly different properties of "moneyness" and some of them can be studied separately.

A money can be modeled exhibiting only the property of transaction convenience. The work of Foley and Hahn has centered on this. Stress can be laid on the trust aspects of money. The analysis in the first seven sections of this paper stresses this feature of money.

When uncertainty is considered one can avoid the use of money in the manner adopted by Radner. If, however, lack of trust and uncertainty are both present then money or some other form of forward contract is needed.

A disposition towards risk aversion causes increasing returns to scale as numbers increase in an economy with noncorrelated (or imperfectly correlated) risks. This implies that for efficiency, the fewer the insuring agents the better.

If a semblance of a competitive market is to be attained in an economy involving lack of trust and uncertainty the rules for forward contracts, bankruptcy laws and the processing of information must be specified. The definition of optimality will depend upon the information state, thus any model of an economy which ignores the trading in information misses a key aspect of economic life.

The man on the street is in general a saver and not an investor (or is a foolish investor) because in contrast with banks, insurance companies, industrial corporations, factors, investment trusts, etc., if they are any good, they know considerably more than he does.

The above remarks do not include the role of money as an instrument of goverment policy. Nor do they cover the important game theoretic properties of money which would call for the study of cooperative and noncooperative solutions to markets with uncertainty.

Fiat money is:

(1) A measure of value (when one price is fixed)
(2) A means of exchange (by law, by custom and because it avoids added transaction costs caused by barter) and
(3) A symbol of trust (by law and custom).

Together with other instruments and bankruptcy laws money is:
(4) A forward contract
(5) A means for providing an extra degree of freedom in transacting business (i.e. accounts need not balance instantaneously).

Together with taxation laws, public finance and international trade it is:

(6) A means of paying taxes
(7) An instrument of goverment internal policy and
(8) An instrument of foreign policy.

APPENDIX

THE FORMAL DESCRIPTION OF A MONEY GAME

A money game is an n+b person cooperative game without side-payments given in characterizing function form. It is a c-game or orthogonal coalition game (i.e. once coalitions have been chosen, their strategies are independent, they do not influence each other).

The characterizing function is obtained by solving a closed static general equilibrium market defined for n traders, mK(T+1) commodities and T time periods, selecting a price system (in a manner described below) and using this price system in the manner that is now given. A set R of players is selected. There are 2^{n+b} such sets. Associated with each set R is a set S^R. The members of this set are distinguished by different loan conditions.(The specific meaning of a loan is given below). The set R forms an independent community constrained to buy and sell at the price system specified, but in a position to achieve different outcomes by adjusting loans among themselves. They are furthermore constrained to trade among themselves using their own resources.

The n-person Trading Market

There is a market with n traders. They possess at most mK(T+1) different commodities. There are m different classes of commodity such as "automobile". After one "period" of use the commodity is deemed to be a different commodity: "a used automobile, one period old". Although the general equilibrium system at this point is being treated statically, we introduce T+1 time periods. Every commodity must now bear a date, thus we have: " a used automobile, one period old in 1969".

Each individual i has a utility function of the form:

(1) $$U_i = \Psi_i(y^i_{j,k,t};\ \hat{y}^i_{j,k,T+1})$$

Where $y^i_{j,k,t}$ is the amount used by Trader i of the class of commodity j, of age k during time t.

The term $\hat{y}^i_{j,k,T+1}$ is the asset evaluation attached to the individual's remaining assets at the end of trade. The numbers n,m,K and T are all finite and the functions Ψ_i have all the conditions needed for the existence of a competitive equilibrium.

At any period t, t=1,2,...,T the traders may obtain new endowments not only from their possession of classes of goods which are now one period older, but also exogenously. Let the amount of exogenous endowment be given by: $A^i_{j,k,t}$ then we may write:

(2) $$\hat{y}^i_{j,k,t} = A^i_{j,k,t} + \hat{y}^i_{j,k-1,t-1} + d^i_{j,k-1,t-1} - s^i_{j,k-1,t-1},$$

which states that the amount held at the start of time t equals the amount held at the start of previous period plus the amounts bought minus the amounts sold plus the new endowments at the start of the current period. Where d stands for his buying and s, selling.

(3) $$y^i_{j,k,t} = \hat{y}^i_{j,k,t} - s^i_{j,k,t} + d^i_{j,k,t}$$

Equation (3) states that Trader i during a period t utilizes the amount he starts with plus the amount he buys minus the amount he sells. This uses the convention that he can utilize the goods he buys for the period during which he buy them, but not the goods he sells.

Using Rader's theorem 13/, this trading economy is the equivalent to one with production and trade. It is desirable that the utility functions have an extra limitation, that they should not be the sum of utility functions defined for each period, or otherwise we need to introduce at least one inventory process which makes it possible to keep at least one commodity in a state "as good as new" for at least one time period. These conditions guarantee that the price levels at each period are interlinked.

A Single Equilibrium Point per system

The n trader general equilibrium system is solved for its set of competitive equilibrium points. A numeraire is selected at random (actually in most societies the choice is far from random, it may be the price of gold, the lowest wage rate in the civil service and so forth). The price of one unit of the physical numeraire is fixed at one unit of money which is a legal fiction that has its manifestation in pieces of paper or numbers in accounting systems and which constitute the only means of payment in this society. We now select the price system which minimizes the use of money. We can evaluate the money worth of all trades leading to an equilibrium point. This set will have a lowest bound associated with some price system. If there is a tie select one at random (actually we could take all of them, however for the purposes of specifying a game that could at least conceptually, be played let us consider one).

The motivation for picking this rule is that even in the mind of the most urbane of "social man" there lurks as suspicion of fiat money and its "veil", hence in some sense the less it is needed, the better.

Trading in Money: The Money Supply

We now have an absolute price system. A set of pieces of paper in some sort of institutional form such as fiat bills or demand deposits or reserve levels or federal reserve deposits are handed out to the n+b players. For simplicity we make no institutional distinctions at this point and call all of the stuff, "money". Each player receives an amount M_i where $i=1,2,\ldots,n+b$. A further rule of the game requires that at the end of the game (after the close of business after the end of the T th period - or more simply at settlement day at period T+1) the players return to the referee exactly the amount that they were handed out to start with.

Trading in Money: The Order of Trade and Cash Flows

Each period t is subdivided into n subperiods. An order of trade is selected at random among the n! orders possible for the traders. The bankers, if they enter at all, enter as buyers hence do not need to be part of this "lineup of sellers".

During each period a seller sells once and appears as a buyer n-1 times. The bankers appear as buyers n times. A transaction involves the exchange of goods for money. All traders and the bankers (as buyers) function in competitive markets. The price system is given by the computations specified above.

A trader has available to spend during the period t the amount he had at the last settlement date. This he may diminish by purchases until he reaches his turn to sell. After he has sold, his monetary resources are immediately increased by the revenues from the sale. These, along with any other money remaining can be spent during the rest of period t, or carried over to t+1.

Let $M_{i,t}$ be the amount of funds available to a trader i at the start of period t. Let $F_{i,t}$ be the payment made to the bank (this can be positive or negative). This payment is made at the start of period t. One useful way of viewing the process is that all periods are split into three. First there are the financial preliminaries. Having settled with the banks, the traders have their accounts in order for the opening of trade, there are then n subperiods of trade followed by closing the books for the end of the period prior to dealing with the bankers.

(4) $$M_{i,1} = M_i + F_{i,1}$$

This states that the available funds at the start of trade during period 1 for Trader i equals his initial amount plus the payment from the bank.

(5) $$M_{i,t} = M_{i,t-1} + F_{i,t} + \sum_{j=1}^{m} \sum_{k=1}^{k} p_{j,k,t}\, s^i_{j,k,t} - \sum_{j=1}^{m} \sum_{k=1}^{k} p_{j,k,t}\, d^i_{j,k,t}$$

This states that the amount of funds available to trader i at the start of period t is the amount at the start of trading at the previous period plus the current bank payment plus the income from sales minus the expenditures on purchases.

A <u>financial policy</u> of trader i is described by $(F_{i,1}, F_{i,2}, \ldots F_{i,n})$.

We require that:

(6) $$M_{i,t} \geq 0 \quad \text{and} \quad M_{i,T+1} \geq M_i$$

No immediate indebtedness is permitted beyond whatever is reflected in the formal arrangements with a bank. These are described in the financial policy. Note that $\sum_{t=1}^{T} F_{i,t}$ can be positive, negative or zero. There is no assumption in the game that a rate of interest need be positive.

In summary, in this "pseudo-dynamic version" traders can deposit in or borrow from the bank (if the bank agrees) however there are no informal credit arrangements among the traders. The description is given now.

The Bankers and Banking System

Each banker has a utility function defined on the same goods as the traders. For case we will assume that the bankers form a single class i.e. their utility functions and their initial endowments are the same. This although unnecessarily special has the property of making subsequent proofs trivial.

Bankers do not borrow from or lend to each other; they deal in both these ways with traders. A banking policy for bank i is a set of n vectors of length T which may be described as $(-F^i_{1,1}, -F^i_{1,2}, \ldots, F^i_{n,T})$ where $F^i_{j,t}$ is the amount borrowed by trader j at time t from bank i.

A banker may loan up to the limit of his loanable funds. These include the initial endowment from the referee, any deposits on hand and any profits from banking. If he has not lent them a banker is permitted to spend his profits. This is to some extent a definitional problem as any tax lawyer defining that which is a return of capital and that which is some form of gain will point out. For our purposes any definition that meets the property that by final settlement time the amount of repaid capital equals the loan will do.

The Calculation of the Characterizing Function and Blocking Coalitions

A coalition of N_1 traders and B_1 bankers can obtain any point obeying the conditions that they trade only among themselves in a general equilibrium market with money, constraints and with a numeraire fixed. This trade takes place after they have agreed upon a joint financial and banking policy among themselves.

Consider a money game $\Gamma(N \cup B)$. It is a market game and can be regarded as an n+b person trading game in T commodities where the T commodities are money at every time period. A different way of showing that the game Γ is a market game 14/ is as follows:

Theorem: The game Γ is a market game.

Proof: Suppose that fewer than b bankers are needed to supply enough funds for the traders to achieve the competitive equilibrium in the n-person unconstrained trading game where the equilibrium is denoted by E, then E is in the core of this game. If any banker demands more than zero he can be excluded by a coalition of all traders and the remaining bankers. If the bankers obtain zero in the core then the traders alone cannot dominate E hence it is in the core of the money game.

Suppose that all bankers were needed to finance trade. Select the banking and financial policies which give them all an equal amount where this amount equals the marginal value productivity of a banker to the economy as a whole. This point is undominated hence the game has a core. But any subgame of a money game is also a money game with the same structure hence it will also have a core, hence a money game is a market game.

Given that a money game is a market game then we know the limiting behavior of the core. In particular if the bankers have an amount of money equal to or greater than M^*, the amount needed to attain E, the core in the limit will be E. If it is less than M^* the core will be a point in the full n+b dimensional space where each banker obtains (his nonzero) marginal value productivity.

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