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RESEARCH ARTICLE

Initial shares can cause Pareto improvements when markets are incomplete

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Abstract The paper focuses on a single firm with constant returns to scale in a multi-period setting with incomplete markets and a single good per state. The firm can be organized as a partnership or as a corporation. In the case of a partnership, there are no initial shares and profits vanish. A corporation has initial shareholders and can sell its output at any market-clearing price. An example shows that the introduction of initial shares can cause a Pareto improvement. The firm sells its output below costs so that the net sellers of initial shares subsidize the net buyers. The initial shares are chosen such that, for each consumer, the benefits of the output expansion more than outweigh the cost increase.

Keywords Multi-period economies with incomplete markets · Partnerships and corporations · Marginal cost pricing · The role of initial shares · The objective of a firm · Efficiency and social welfare

JEL Classification D21 · D52 · D61

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1 Introduction

This paper focuses on a single firm with constant returns to scale in a setting with incomplete markets, more than two time periods and a single good per state. Two different types of firms, partnerships and corporations, are compared; see §31 and §32 of Magill and Quinzii (1996), henceforth referred to as MQ. In the case of a partnership, a group of consumers gets together to found a firm. Because of constant returns to scale, there are no incentives to exclude a consumer.

The main difference between partnerships and corporations is that the former have no initial owners, whereas the latter are initially owned by consumers. When the firm is organized as a partnership, the output price equals the cost and no profits accrue. In the case of a corporation, initial shares are exogenously allocated before markets open. Shares are traded at all non-terminal nodes and profits can have any sign.

In the particular case of finance economies with only two periods, the first-order condition for constrained efficiency requires marginal cost pricing and there is no need for initial shares. The paper addresses the following question: Shall the firm in the multi-period case always be organized as a partnership or can initial shares help to improve efficiency and welfare?

This question is studied from a purely normative perspective in a particularly simple and transparent model. The paper abstracts from all kinds of real life complications. In particular, there are no liability and bankruptcy problems, no competition and no strategic interaction. The only assets are shares in the firm. Moreover, every consumer participates in the firm and takes part in the firm's decision problem.

The objective of a firm used in this paper can be described most easily in the case of a corporation. In this case, social welfare maximization takes into account how the original shares impact market outcomes. When the initial shares are sold below costs, the net sellers of initial shares subsidize the net buyers. When the shares are sold above costs, the redistribution of wealth is reversed. In the case of a partnership, all market transactions leave the distribution of wealth unaltered.

Under the assumption that every consumer holds at least a tiny amount of initial shares, the welfare of the initial owners coincides with the welfare of the society. Otherwise, the group of initial owners could exploit the rest of the economy. A corporation chooses, as in any Cournot model, an output vector. All functions used to analyze the model depend directly or indirectly on the firm's output so that the independent variable can be dropped in the present introductory explanation. Consumers anticipate the market clearing prices correctly and determine their optimal trades on all markets. In equilibrium, all markets clear.

It is well known that even very weak welfare requirements can be out of reach because of second-order effects. Therefore, a first-order approach is adopted to select production plans that are candidates for social welfare maximization. More precisely, production plans are sorted out whenever first-order welfare improvements are possible.[1]

[1] There are Hicksian surplus concepts which incorporate higher order effects in different ways so as to obtain a cardinal social welfare measure or a cardinal efficiency measure. The referees suggested to leave

In line with the usual definition of state prices or stochastic discount factors, every utility function is normalized such that the marginal utility of good 0 equals 1 in equilibrium. The (indirect) social welfare function $\mathcal{W}$ is the sum of all normalized indirect utility functions. The corporation chooses its production plan such that the first-order condition for welfare maximization is satisfied. For a more extensive explanation, see Sect. 2.

In the case of a partnership, the basic principle is the same. However, the firm must take the pricing constraint into account. The partnership aims to satisfy the first-order condition for constrained welfare maximization. It is worth emphasizing that the degree of complexity of multi-period models of production economies with incomplete markets comes close to that of models of Cournot competition.

1.1 Relationship to the literature

Gabszewicz and Vial (1972) introduce a model that combines Cournot-Nash competition with Walrasian exchange of consumption goods under the assumption that markets are complete. The basic idea can be described as follows: The consumption goods are produced by firms who need non-marketable primary factors as inputs. Every firm chooses its production plan. The consumers possess preassigned shares of the firms, provide the primary factors in accordance with their shares, and receive their shares of the firms' output. Thereafter, Walrasian exchange of the consumption goods takes place at market clearing prices. The main difference between Gabszewicz and Vial (1972) and the present paper is that they focus on oligopolistic competition with complete markets, whereas this paper focuses on market incompleteness without oligopolistic competition.

Both papers have in common that they deal with preassigned, initial shares. First, the production plans are chosen. Thereafter, the output is distributed and the consumers obtain their intermediate endowments. Finally, Walrasian exchange takes place and the intermediate endowments are traded at their equilibrium prices. In multi-period models of corporations, the exchange occurs repeatedly. Both papers deal with the redistribution of initial wealth, however, from different perspectives. Gabszewicz and Vial focus on the profit motive of oligopolists, whereas this papers abstracts from that motive and uses the possibility to redistribute wealth to enhance efficiency and welfare.

Guesnerie (1975) points out that a redistribution of wealth can be needed to achieve a Pareto improvement when one leaves the classical Arrow–Debreu framework. In his paper, the aggregate production set fails to be convex and marginal cost pricing becomes a necessary requirement for Pareto efficiency. Several marginal cost pricing equilibria exist, however, none of them is Pareto efficient given the distribution of the firms' profits or losses. To obtain a Pareto-efficient marginal cost pricing equilibrium, the original distribution scheme needs to be changed. According to the fundamental

Footnote 1 continued
such issues out because the main point of the paper can be made on the basis of Pareto comparisons (rather than Kaldor–Hicks comparisons).

theorems of welfare economics, no such problem arises in the convex case. In the context of a standard GEI model with numéraire assets and a finite set of commodities at each of $S+1$ spot markets, the connection between endowment redistribution and Pareto improvements has been investigated by Mendolicchio and Pietra (2016).

The main goal of this paper is to present an example of a partnership equilibrium that is Pareto dominated by a corporation equilibrium of the same economy with suitably chosen initial shares.

2 Corporations, partnerships, and their objectives

It suffices to consider a three-period economy whose underlying date-event tree has the initial state $s = 0$ at $t = 0$ and states $s = 1, \ldots, S$ at $t > 0$. There is a single good per state and a single firm with constant returns to scale technology $\mathcal{Y} \subset \mathbb{R}_- \times \mathbb{R}^S_+$. A production vector is denoted $y = (y_0, y_+)$. The firm can be a corporation or a partnership. To define social welfare in either case, every (indirect) utility function is normalized such that the marginal utility of good 0 equals 1 at the equilibrium under consideration. One marginal unit of good 0 increases social welfare by one unit independently of who consumes the unit.[2]

Consumer i's normalized utility gradient π^i describes i's state price system or vector of stochastic discount factors. The social welfare of a group of consumers is the sum of the normalized indirect utility functions of its members. This paper focuses on the social welfare of all consumers.

Consider first the case of a corporation with initial shares $\delta^i \geq 0$ and $\sum_i \delta^i = 1$. Throughout the paper, it is assumed that consumers have smooth preferences in the sense of Debreu, see e.g., MQ, p. 50. There is a stock market at each non-terminal node. The implicit function theorem is used to express all functions directly or indirectly as functions of y_+. First one determines, for every consumer i and every non-terminal node s, the demand $\vartheta^i_s(y_+)$ for shares which determine i's consumption $x^i(y_+)$. Then one solves the system of market-clearing equations to obtain an equilibrium price vector. The equilibrium price at node s in denoted q_s.

In a corporation economy, the output stream y_+ is sold at the market-clearing price $q_0(y_+)$. The set of stock market equilibria is characterized by

$$\mathcal{Y}_{\text{corp}} = \left\{ y_+ \in \text{proj}_2 \mathcal{Y} \mid \sum_i \vartheta^i_s(y_+) = 1 \quad \text{for every non-terminal state } s \right\},$$

where proj_2 denotes the projection to $\mathbb{R}^S_+$.

Assume that there is a planner who can choose the production plan and make infinitesimal transfers of good 0. However, the use of the transfers is severely restricted because this paper uses the concept of minimal efficiency which prevents any change

[2] This social welfare concept differs from the one used in utilitarian welfare theory. The latter relies on cardinal utility measures and cardinal unit comparability across consumers whereas the present approach is based on the comparison of marginal utility units.

of consumption at $t > 0$.[3] Can this planner find a first-order Pareto improvement over the allocation of the reference equilibrium induced by y^*_+? To answer this question, define social welfare as:

$$\mathcal{W}_{y^*}(y_+) = \sum_i \frac{U^i(x^i(y_+))}{\partial_0 U^i(x^i(y^*_+))}. \tag{1}$$

Whenever $D\mathcal{W}_{y^*}(y^*_+)$ does not vanish, a first-order welfare improvement exists. To avoid such equilibria, corporations are required to satisfy the first-order condition $D\mathcal{W}_{y^*}(y^*_+) = 0$ for welfare maximization. When one differentiates $\mathcal{W}_{y^*}(y_+)$ with respect to y_s, $s = 1, \ldots, S$, one obtains, dropping the arguments, the first-order condition

$$\partial_s y_0 + \sum_{i=1}^{I} \sum_{\sigma=1}^{S} \pi^i_\sigma \, \partial_s x^i_\sigma = 0 \quad \text{for } s = 1, \ldots, S. \tag{2}$$

The objective of the corporation is to satisfy condition (2). A stock market equilibrium is a corporation equilibrium iff $D\mathcal{W}_{y^*}(y^*_+) = 0$. Observe that equation (2) is significantly more complex than a convex combination of utility gradients π^i. In contrast to the two-period case, no envelope theorem applies and $\pi^i_\sigma \, \partial_s x^i_\sigma$ does typically not vanish when $s \neq \sigma$.

One may feel tempted to require the corporation to fulfill more than the first-order condition for welfare maximization. However, the following problem arises already in the two-period case. In that particular setting, the first-order condition for welfare maximization coincides with the first-order condition for constrained efficiency. Dierker and Dierker (2010) consider two-period economies and present robust examples that show that a unique Drèze equilibrium can maximize welfare although it is not minimally efficient. The Drèze equilibrium can also minimize welfare although it is constrained efficient.

Turn now to the case of partnership economies. At $t = 0$, every consumer i can become a partner by obtaining the share $\vartheta^i_0 > 0$ of the output y_+ in exchange for the cost share $\vartheta^i_0 C$ where $C = |y_0|$. The partnership operates at a scale that is determined by the condition $\sum_i \vartheta^i_0(y_+) = 1$.

Apart from $t = 0$, there is no difference between the description of a partnership or a corporation. Loosely speaking, a partnership is a corporation with constant returns to scale, a missing stock market at $t = 0$, and price-taking behavior.

When one wants to convert a corporation with constant returns to scale into a partnership one has to abolish the initial shares δ^i. This is achieved by the pricing rule $q_0 = C$. In a corporation, i's consumption at $t = 0$ is $x^i_0 = e^i_0 - \delta^i C + (\delta^i - \vartheta^i_0) q_0 = e^i_0 + \delta^i (q_0 - C) - \vartheta^i_0 q_0$ where e^i_0 is i's initial endowment at $t = 0$. If $q_0 = C$ the initial shares δ^i vanish so that $x^i_0 = e^i_0 - \vartheta^i_0 C$ as in a partnership.

Consider the case in which $\delta^i = \vartheta^i_0$ for every i. Then i's demand for good 0 is independent of whether the firm is a corporation or a partnership. However, unless

[3] A planner associated with constrained efficiency is much stronger because he can allocate all shares.

i's utility is quasilinear, δ^i will typically impact i's demand for shares at subsequent stock markets.

At $t = 1$, the partnership goes public. There is a stock market at every non-terminal node $s \geq 1$ on which the shares $\vartheta_s^i(y_+)$ are sold at the market-clearing price $q_s(y_+)$. In equilibrium, all stock markets clear, that is to say, $\sum_i \vartheta_s^i(y_+) = 1$. In the case of a partnership economy, the set of stock market equilibria is characterized by

$$\mathcal{Y}_{\text{part}} = \left\{ y_+ \in \text{proj}_2\mathcal{Y} \mid q_0(y_+) = C(y_+) \text{ and } \sum_i \vartheta_s^i(y_+) = 1 \text{ for all markets} \right\}.$$

A partnership equilibrium is a stock market equilibrium with the property that first-order welfare gains on $\mathcal{Y}_{\text{part}}$ are impossible.

3 Numerical example

There are three time periods, $t = 0, 1, 2$, and seven states. State 0 at $t = 0$ is followed by states 1 and 2 at $t = 1$. At $t = 2$, states 3 and 4 follow state 1 and states 5 and 6 follow state 2. There is a single good per state and a single firm.

Consider three types of consumers, A, B and Q, with additively separable, concave utility functions. The utility function of type Q is quasilinear. Define

$$\begin{aligned} U^A(x_0, x_1, \ldots, x_6) &= 10\log(x_0) + 1\log(x_1) + 2\log(x_2) + 3\log(x_3) \\ &\quad + 4\log(x_4) + 5\log(x_5) + 6\log(x_6), \\ U^B(x_0, x_1, \ldots, x_6) &= 10\log(x_0) + 3\log(x_1) + 2\log(x_2) + 1\log(x_3) \\ &\quad + 1\log(x_4) + 2\log(x_5) + 3\log(x_6), \\ U^Q(x_0, x_1, \ldots, x_6) &= x_0 + \log(x_1) + \log(x_2) + \log(x_3) \\ &\quad + \log(x_4) + \log(x_5) + \log(x_6), \end{aligned} \tag{3}$$

respectively. There are no initial endowments except at $t = 0$ where every consumer is endowed with $e_0^A = e_0^B = e_0^Q = 30$. Ten consumers are of type A, ten of type B and fifty of type Q. A production plan is denoted $y = (y_0, y_+) \in \mathbb{R}_- \times \mathbb{R}_+^6$ where $y_+ = (y_1, \ldots, y_6)$. The cost is

$$C(y_+) = y_1 + y_2 + \cdots, y_6. \tag{4}$$

Sections 3.1 and 3.2 contain the computation of the partnership equilibrium and of the corporation equilibria, respectively. Section 3.3 explains how a corporation equilibrium manages to Pareto dominate the partnership equilibrium.

3.1 Partnership equilibrium

In the partnership, consumer i consumes $e_0^i + \vartheta_0^i y_0$ at $t = 0$. The consumption at an intermediate node ξ_s is $x_s^i = q_s(\vartheta_{s_-}^i - \vartheta_s^i) + \vartheta_{s_-}^i y_s$ at $t = 1$, where ξ_{s_-} is the immediate predecessor of ξ_s. If ξ_s is a terminal node, then i consumes $x_s^i = \vartheta_{s\_}^i\, y_s$. The size of the partnership is such that $\sum_i \vartheta_0^i = 1$.

The initial investment of a consumer of type A is $\vartheta_0^A = 630/(31\; C)$, $\vartheta_1^A = 2205(q_1 + y_1)/(124\; q_1\; C)$, and $\vartheta_2^A = 6930(q_2 + y_2)/(403\; q_2\; C)$ where the variable y_+ has been dropped. For consumers of type B, one obtains $\vartheta_0^B = 180/(11\; C)$, $\vartheta_1^B = 72(q_1 + y_1)/(11\; q_1\; C)$, and $\vartheta_2^B = 900(q_2 + y_2)/(77\; q_2\; C)$. A consumer of type Q demands $\vartheta_0^Q = 6/C$, $\vartheta_1^Q = 4(q_1 + y_1)/(q_1\; C)$, and $\vartheta_2^Q = 4(q_2 + y_2)/(q_2\; C)$. When the shares ϑ_0^i add up to 1 then $C = 227400/341$. Solving the market-clearing equations for markets 1 and 2 leads to $q_1 = (60463/30497)\; y_1$ and $q_2 = (151693/55241)\; y_2$.

Let $\hat{y} = (y_1, \ldots, y_5)$. The cost function (4) is used to eliminate the last component y_6 of y_+ by defining $y_6 = g(\hat{y}) = 227400/341 - y_1 - \cdots - y_5$. Let $y(\hat{y}) = (\hat{y}, g(\hat{y}))$. Then every function of y is indirectly a function of $\hat{y}$.

Dropping the variable $\hat{y}$, i's consumption equals

$$x^i = (e_0^i - \vartheta_0^i C,\; q_1(\vartheta_0^i - \vartheta_1^i) + \vartheta_0^i y_1,\; q_2(\vartheta_0^i - \vartheta_2^i) + \vartheta_0^i y_2,\; \vartheta_1^i y_3,\; \vartheta_1^i y_4,\; \vartheta_2^i y_5,\; \vartheta_2^i g).$$

Let $u^i(\hat{y}) = U^i(x^i(\hat{y}))$ be the utility i obtains when $\hat{y}$ is chosen.

All consumers are partners so that the firm acts on behalf of the whole society. Because $x_0^A = 30 - 630/31 = 300/31$, A's marginal utility of good 0 equals 31/30. Similarly, B's marginal utility equals 11/15. Thus, both normalization factors, $\alpha = 30/31$ and $\beta = 15/11$, are independent of the allocation. Since there are 10 consumers of type A, 10 of type B, and 50 of type Q social welfare $\hat{\mathcal{W}}$ in the partnership is given by

$$\hat{\mathcal{W}}_{y^*}(\hat{y}) = 10\,\alpha\; U^A(x^A(\hat{y})) + 10\,\beta\; U^B(x^B(\hat{y})) + 50\; U^Q(x^Q(\hat{y})). \tag{5}$$

The first-order condition $D\hat{\mathcal{W}}_{y^*}(\hat{y}) = 0$ can be solved algebraically. For simplicity, numerical approximations are used to replace fractions and one obtains $\hat{y}^* \approx (100.5865,\; 96.6276, 92.6686,\; 102.346,\; 125.6598)$. The cost is $C \approx 666.8622$ and the last coordinate of the production plan y^* is $y_6^* \approx 148.9736$. The stock prices are $q_1 \approx 1.9826\; y_1$ and $q_2 \approx 2.746\; y_2$.

A consumer of type A, B, Q consumes, respectively,

$$\begin{aligned}
x^A(\hat{y}^*) &\approx (9.67742,\; 1.14284,\; 1.69707,\; 3.71744,\; 4.10566,\; 4.42033,\; 5.24044)\\
x^B(\hat{y}^*) &\approx (13.6364,\; 4.41701,\; 2.53774,\; 1.36835,\; 1.51125,\; 3.00454,\; 3.56198)\\
x^Q(\hat{y}^*) &\approx (24.0000,\; 0.89976,\; 1.08559,\; 0.83621,\; 0.92354,\; 1.02822,\; 1.21899).
\end{aligned}$$

This entails the utility profile $(u^A, u^B, u^Q) \approx (50.8474,\; 39.1841,\; 23.9439)$. At $t = 0$, the consumers choose $(\vartheta_0^A, \vartheta_0^B, \vartheta_0^Q) \approx (0.030475, 0.024538, 0.008997)$.

Consider a weak planner who can change the allocation of shares at $t = 0$ but not the production plan. After a reallocation of the shares ϑ_0^i, the prices of the stock markets at $t = 1$ adjust and give rise to the weak planner's optimal allocation.

In the example, the weak planner redistributes shares from A and B to Q and selects

$$(\vartheta_0^A, \vartheta_0^B, \vartheta_0^Q) \approx (0.030470, 0.024507, 0.009005).$$

A redistribution of shares at $t = 0$ can have welfare implications.

3.2 Corporation equilibria

Let δ^τ denote the amount of original shares owned by an individual consumer of type $\tau = A, B, Q$. At $s = 0$, a consumer of type τ consumes the amount $x_0^\tau = 30 + \delta^\tau(q_0 - C) - \vartheta_0^\tau\, q_0$. The original shares δ^τ change the consumption by $\Delta x_0^\tau = \delta^\tau(q_0 - C)$. When $\tau = A$ or $\tau = B$, there is an indirect impact on the demand for final shares caused by an income effect. This leads to

$$\vartheta_0^A = \frac{21(30 + \delta^A(q_0 - C))}{31 q_0},\ \vartheta_1^A = \vartheta_0^A\,\frac{7(q_1 + y_1)}{8 q_1},\ \vartheta_2^A = \vartheta_0^A\,\frac{11(q_2 + y_2)}{13 q_2} \tag{6}$$

$$\vartheta_0^B = \frac{6\,(30 + \delta^B(q_0 - C))}{11 q_0},\ \vartheta_1^B = \vartheta_0^B\,\frac{2(q_1 + y_1)}{5 q_1},\ \vartheta_2^B = \vartheta_0^B\,\frac{5(q_2 + y_2)}{7 q_2}. \tag{7}$$

Observe that, δ^A and δ^B enters into ϑ_0^i and thereby also into ϑ_1^i and ϑ_2^i.

For the quasilinear type Q, there is no income effect and

$$\vartheta_0^Q = \frac{6}{q_0},\quad \vartheta_1^Q = \vartheta_0^Q\,\frac{2(q_1 + y_1)}{3 q_1},\quad \vartheta_2^Q = \vartheta_0^Q\,\frac{2(q_2 + y_2)}{3 q_2}. \tag{8}$$

The original shares δ^A and δ^B of the two non-quasilinear consumers impact all market-clearing prices. The prices are

$$q_0 = \frac{30\,(77\,\delta^A\,C + 62\,\delta^B\,C - 7580)}{2310\,\delta^A + 1860\,\delta^B - 341}$$

$$q_1 = \frac{105\,(77\,C - 12340)\,\delta^A + 24\,(124\,C + 54725)\,\delta^B - 604630}{105\,(11\,C + 12340)\,\delta^A + 24\,(186\,C - 54725)\,\delta^B - 304970}\,y_1$$

$$q_2 = \frac{21\,(847\,C - 75500)\,\delta^A + 30\,(403\,C + 7060)\,\delta^B - 1516930}{42\,(77\,C + 37750)\,\delta^A + 12\,(403\,C - 17650)\,\delta^B - 552410}\,y_2.$$

For $\tau = A, B$, the consumption change $\Delta x_0^\tau = \delta^\tau(q_0 - C)$ appears in the normalization factors of τ's utility function. These factors are equal to the equilibrium values α and β of

$$(30 + \delta^A(q_0 - C))/31 \quad \text{and} \quad (30 + \delta^B(q_0 - C))/22,$$

respectively. These normalization factors are not constant and must be determined together with the optimal allocation. This completes the description of the Cournot–Walras model of the corporation apart from its objective.

Consider the welfare function of the corporation given by

$$\mathcal{W}_{y^*}(y_+) = 10\,\alpha\,U^A(x^A(y_+)) + 10\,\beta\,U^B(x^B(y_+)) + 50\,U^Q(x^Q(y_+)). \tag{9}$$

The main difference between (5) and (9) is that the welfare function $\mathcal{W}_{y^*}$ in (9) depends on the S-dimensional output vector y_+, whereas the welfare function $\hat{\mathcal{W}}_{y^*}$ in (5) depends on the $(S-1)$-dimensional vector $\hat{y}$ due to the constraint $q_0 = C$.

When does q_0 equal C in the example? Because

$$q_0 - C = \frac{341\, C - 227400}{2310\, \delta^A + 1860\, \delta^B - 341}$$

the price q_0 equals C if and only if $C = 227400/341$. Thus, the constraint $q_0 = C$ is satisfied if and only if C is equal to the cost in the partnership equilibrium of the previous subsection. This is the case if all original shares are owned by the quasilinear type Q.

3.3 A Pareto-dominating corporation equilibrium

When a corporation equilibrium Pareto dominates the partnership equilibrium slightly, the utility profiles must be nearly proportional. To obtain a Pareto domination, choose $\delta^A = 0.03035$ and $\delta^B = 0.02445$ so that $\delta^Q = 0.00904$. Then the equilibrium output of the corporation becomes $y_+ \approx (100.6208, 96.6684, 92.7, 102.3789, 125.6972, 149.0154)$ and exceeds the equilibrium output of the partnership. The size of the output expansion is $\Delta y_+ \approx (0.034, 0.033, 0.031, 0.033, 0.037, 0.042)$. The equilibrium prices are $(q_0, q_1, q_2) \approx (666.7542, 199.4863, 265.4182)$ and the allocation is

$$\begin{aligned}
x^A &\approx (9.6743, 1.14303, 1.69732, 3.71813, 4.10634, 4.421, 5.24114)\\
x^B &\approx (13.6328, 4.41803, 2.53826, 1.36869, 1.51159, 3.00518, 3.56268)\\
x^Q &\approx (23.9971, 0.900203, 1.08609, 0.836637, 0.923992, 1.02871, 1.21954).
\end{aligned}$$

The utility profile $(u^A, u^B, u^Q) \approx (50.8474, 39.1841, 23.9439)$ exceeds that of the partnership equilibrium by about $(3 \cdot 10^{-6}, 6 \cdot 10^{-7}, 3 \cdot 10^{-7})$. Furthermore, $C \approx 667.073$ exceeds $q_0 \approx 666.754$.

Clearly, a strong planner, who can choose the production plan and the individual shares, can do at least as well as the Pareto-dominating corporation. However, no such planner is needed for the corporation equilibrium.

To understand how the corporation steers the market with the aid of initial shares to obtain the Pareto improvement, consider the underlying redistribution across types. First, observe that consumers of type A and B are net buyers and Q is a net seller at $t = 0$ because $(\vartheta_0^A - \delta^A,\ \vartheta_0^B - \delta^B, \vartheta_0^Q - \delta^Q) \approx (0.00012,\ 0.000086, -\ 0.000041)$ in equilibrium. A's and B's utility functions place more weight on future goods than Q's, cf. (3). This phenomenon is more pronounced for A than for B.

Type Q needs types A and B to increase the output because A and B exhibit, in contrast to Q, income effects at $t = 0$ that raise their demand for shares when they become richer. The transfer of wealth from Q to A and B increases the demand for future goods and thereby the output to an extent that turns out to be close to the increase caused by the strong planner.

The output expansion is accompanied by a cost increase. The transition from the partnership to the corporation makes all consumers worse off at $t = 0$ because costs increase. All become better off at $t > 0$ due to the output expansion. We know from a numerical calculation that the total result is a Pareto improvement. What makes the net effect beneficial?

Here, the definition of the objective of a corporation comes into play. At the partnership equilibrium, the pricing rule $q_0 = C$ is binding and there is underproduction. In principle, a corporation can achieve a first-order welfare gain by selling above or below production costs. In the present example, infinitesimal welfare gains are achieved by an output expansion until $D\mathcal{W}_{y^*} = 0$, that is to say, until the firm has reached its objective.

Roughly speaking, the situation in the example is as follows. When Q holds all initial shares, there is no difference between the corporation and the partnership. When a small amount of initial shares is assigned to types A and B, then A and B become better off and Q becomes worse off than at the partnership. The output is expanded and A and B are subsidized by Q. Near the Pareto-improving corporation, the situation becomes volatile. If one raises the value of $\delta^A = 0.03025$ and $\delta^B = 0.02445$ slightly to 0.03026 and 0.02446, respectively, then A and B are both worse off than in the partnership. High values of δ^A and δ^B become good for Q and bad for A and B.

4 Conclusions

The paper investigates the role of initial shares in multi-period production economies with incomplete markets and a single corporation. A suitable allocation of initial shares can help to correct inefficient consumption decisions due to a wedge between the output price q_0 and the production cost C.

Depending on the example under consideration, the initial output price q_0 can be higher or lower than the production cost C. Prices $q_0 < C$ can be needed to expand the output to a socially desirable level.

In contrast to corporations, partnerships are too rigid to react appropriately to future needs. At $t = 0$, the corporation foresees the danger of an underproduction and sells its output below cost. The partnership cannot do so because q_0 must equal C. The output expansion is limited by the degree of substitution incorporated in the preferences but it suffices in the example to generate a Pareto improvement.

The window of opportunity to obtain a Pareto-efficient allocation is typically small because it is difficult to distribute the individual changes so as to keep every agent above the utility level reached in the partnership.

Forward-looking behavior of economic agents can improve welfare if it is not prevented by a pricing rule. By definition, a corporation is guided by the gradients of a family of welfare functions. The individual decisions take all market interactions into account and, in the example, the corporation induces a socially beneficial redistribution of wealth due to income effects.

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