FORECASTING VECTOR AUTOREGRESSIONS - THE INFLUENCE OF COINTEGRATION

A Monte Carlo Study

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ABSTRACT

This paper investigates the forecasting performance of cointegrated systems by simulation. It builds on the investigation by Engle and Yoo (1987) and extends their work in two important directions. First, we also consider a conditional maximum likelihood procedure due to Johansen (1988) and find improved forecasting performance relative to previous suggestions, but the gain remains small. Second, examination of a three-dimensional system shows that, even for long period forecasts, those VARs which overestimate the true number of common trends (overdifferenced systems) perform only slightly worse relative to the correct model but considerably better than VARs on which too much cointegration is imposed.

ZUSAMMENFASSUNG

In dieser Arbeit wird die Prognosefähigkeit kointegrierter VAR-Systeme durch Simulation überprüft. Die frühere Untersuchung von Engle und Yoo (1987) wird in zwei wichtigen Richtungen erweitert. Zum ersten schließen wir die bedingte Maximum-Likelihood-Schätzung nach Johansen (1988) in unsere Studie ein und finden verbesserte Prognosegüte im Vergleich zur zweistufigen Kleinstquadrate-Methode. Allerdings ist die Verbesserung nur gering. Zweitens ergibt die Analyse dreidimensionaler Systeme, daß eine Überschätzung der tatsächlichen Anzahl gemeinsamer Trends einen weit geringeren Verlust an Prognosegenauigkeit verursacht als eine Unterschätzung, also die Verwendung zu vieler kointegrierender Vektoren.

1. INTRODUCTION

Cointegrated processes have become an important issue in macroeconometrics, both for theoretical and practical reasons. If one is willing to accept a vector autoregression (VAR) as representation of the data-generating process of individually trending (integrated) series, cointegration is a generic feature. The theory of cointegration could finally settle the dispute about the use of differenced or level VARs in these cases. Error correction specifications, which emerge from cointegrated systems due to the Granger Representation Theorem (Engle and Granger (1987)), fall in between these extremes since they are modeled in stationary terms but nevertheless properly account for long-run information. Such EC-VARs are a multivariate generalization of a class of models first considered by Sargan (1964). Distinctive features like rapid convergence of the estimate of the cointegrating vector to the true values as well as the easy implementation of the required procedures add to the reasons for the surge of applied work in this field.

Research on cointegration primarily focuses on questions concerning identification and testing of cointegrated relations. As forecasting is one of the main purposes a model is designed for, however, the question of how relevant is cointegration in terms of forecasting performance arises. An important contribution in this field is the forecasting experiment carried out by Engle and Yoo (1987). In its restriction to the bivariate problem, it follows most of the empirical work.

As most econometric models used for projections contain more than two variables, it is interesting to look at whether the results by Engle and Yoo (1987) continue to hold in multivariate systems. In this case, also the gain in forecasting performance relative to specification effort has to be considered.

Let us first review the case of a two variable system. After testing and rejecting the hypothesis of no cointegration, an estimate for the long-run relationship can be obtained from a static regression and included in an error correction specification as if this parameter were known. This two-step procedure due to Engle and Granger (1987) relies on the $T-$consistency of the estimates from its first step. Engle and Yoo (1987) report substantial gains in prediction from applying this procedure relative to OLS estimation of an unrestricted VAR in levels. At the first glance this result does not seem surprising, because asymptotically the true restrictions are imposed.

Nevertheless, Banerjee, Dolado, Hendry, and Smith (1986) find in a Monte Carlo Study of a bivariate system that the finite sample bias of the first step can be large, being mainly a function of the R^2, and does not decline with the rapid rate suggested by asymptotic theory. Stock (1987) provides additional evidence for the poor finite sample properties. By simulation in a bivariate system he compares the two-step OLS procedure with a non-linear least squares estimator, which estimates the cointegrating relationship simultaneously with the other equation parameters.

Let us now turn to the multivariate framework. A higher-dimensional system in which cointegration among some variables could exist can be estimated and used for forecasting after identifying vectors that form a basis of the cointegrating space. In this paper, a multivariate extension of the Engle and Yoo (1987) two-step procedure is contrasted with a conditional maximum likelihood approach due to Johansen (1988). The latter one calculates a cointegrating basis by eigenvector analysis after estimating its dimensionality.

This paper is organized as follows: Section 2 briefly describes the procedures used for the estimation of the cointegrated models. Section 3 gives the setup for the Monte Carlo experiment, the results of which are summarized in Section 4.

2. PROCEDURES

Engle and Granger (1987) define the $n-$dimensional process $X_t = (X_{1t}, X_{2t}, \dots, X_{nt})$ to be cointegrated of order $(1,1)$ if

(1) X_t is integrated of order 1 which means that the individual series X_{it} are not stationary but the first differences $\Delta X_{it} = X_{it} - X_{it-1}$ are jointly (covariance-) stationary.

(2) a linear combination of the series $\beta_1 X_{1t} + \beta_2 X_{2t} + \cdots + \beta_n X_{nt}$ exists which generates a stationary process.

$(\beta_1, \beta_2, \dots, \beta_n)$ is called a cointegrating vector. At most $n-1$ linearly independent cointegrating vectors can exist. These are not uniquely defined but the linear space spanned by the vectors and its dimension are unique characteristics of the multivariate process. Several testing procedures exist in order to identify this cointegrating dimension (Stock and Watson (1988), Phillips and Ouliaris (1988), Johansen (1988), Phillips (1989)).

As shown e.g. in Engle and Granger (1987), there is a direct relationship between cointegrating models and so-called error-correcting models. Here and in the following, all processes will be assumed to follow finite-order autoregressive (AR) schemes. Two - in this case integrated - variables (X_{1t}) and (X_{2t}) are said to follow an error-correcting model if the VAR representation in differences depends on an error-correcting term

$$\Delta X_{1t} = \sum_{i=1}^{p_1} \Phi_{1i} \Delta X_{1t-i} + \sum_{i=1}^{p_2} \Phi_{2i} \Delta X_{2t-i} + \alpha (X_{1t-1} - \beta X_{2t-1}) + \epsilon_t \tag{2.1}$$

and similarly for ΔX_{2t}. The coefficient β in this equation corresponds to the coefficient in the cointegrating relation between X_{1t} and X_{2t}. Taken β as given, all right-hand side variables are stationary and the least-squares estimates for Φ_{ij} and α approach their true values asymptotically of order $T^{-1/2}$. If an estimate for β is taken from a primary regression of X_{1t} on X_{2t}, this property continues to hold since that estimate approaches its limit of order T^{-1}. These properties directly generalize to higher-dimensional systems. Take for example the $n-$dimensional system with m cointegrating vectors (now adopting the time series analytic convention with summation of the Φ_{ijk} running from $k = 0$ if $i = j$ and from $k = 1$ otherwise instead of the regression convention in (2.1)).

$$\begin{aligned}\Phi(L)\Delta X_t &= \begin{pmatrix} \sum \Phi_{11k}L_k & \cdots & \sum \Phi_{1nk}L_k \\ \sum \Phi_{21k}L_k & \cdots & \sum \Phi_{1nk}L_k \\ \vdots & \ddots & \vdots \\ \sum \Phi_{n1k}L_k & \cdots & \sum \Phi_{nnk}L_k \end{pmatrix} \begin{pmatrix} \Delta X_{2t} \\ \vdots \\ \Delta X_{nt} \end{pmatrix} \\ &= \begin{pmatrix} \alpha_{11} & \cdots & \alpha_{m1} \\ \vdots & \ddots & \vdots \\ \alpha_{1n} & \cdots & \alpha_{mn} \end{pmatrix} \begin{pmatrix} \beta_{11} & \cdots & \beta_{1n} \\ \vdots & \ddots & \vdots \\ \beta_{m1} & \cdots & \beta_{mn} \end{pmatrix} \begin{pmatrix} X_{1t-1} \\ \vdots \\ X_{nt-1} \end{pmatrix} + \epsilon_t \\ &= \alpha'\beta X_{t-1} + \epsilon_t \end{aligned} \tag{2.2}$$

where α and β are $(m \times n)$-matrices, the row vectors of β forming a basis for the cointegrating space. The long-run system properties can be seen directly from the eigenvalues of the matrix $I + \alpha'\beta$: $(n - m)$ eigenvalues are one and m eigenvalues are less than one. Increasing one or more of the non-unity eigenvalues amounts to approaching a model with a lower-dimensional cointegrating space, i.e. a model with more independent trends, more unit roots, and less cointegration.

Engle and Granger (1987) suggest to estimate the system parameters in (2.2) via the following "EC–two-step" procedure:

(1) Estimate the parameters of the cointegrating equations by regressing the original integrated variables on other integrated variables. (number of equations equal to dimension of cointegrating space m)

(2) Estimate the remaining parameters by regressing the first differences of a specific variable on lagged differences of all variables and on the residuals from all step one equations. (number of equations equal to system dimension n)

If m exceeds one, execution of the first step puts up difficulties since restrictions have to be imposed on the regressions to warrant independent basis vectors. If this is not done, there is a chance that all equations replicate the very same cointegrating basis vector. The solution to this problem is non-unique. One suggestion is to first regress the first variable on the remaining ones; then to regress the second variable on the third to last variable; the third on the fourth to last; and so on. In some non-generic cases, the method fails. In this paper, it was used to generate the EC–two-step forecasts (labeled *EC-i* with i the dimension of the cointegrating space).

Johansen (1988) based his alternative suggestion on the conditional maximum likelihood (ML) estimation of the system under the restriction that all lag orders are equal. In that case, the canonical correlations between the differenced variables and past level variables, adjusted for lagged differences, form the foundation of a unified strategy for testing on the dimension of the cointegrating space (via likelihood-ratio test) and estimating all system parameters. In detail, the canonical problem

$$(\lambda_i S_{pp} - S_{p0} S_{00}^{-1} S_{0p}) v_i = 0 \tag{2.3}$$

is solved. In this equation, S_{00} denotes the $(n \times n)$ - matrix of empirical second moments of the vector (ΔX_t), conditional on the first $p-1$ lags of (ΔX_t). S_{pp} denotes the matrix of moments of (X_{t-p}), again conditional on the lagged differences. S_{0p}, S_{p0} denote the corresponding matrices of cross-moments. Technically, conditioning is attained by regressing the coordinate variables in the vectors (ΔX_t) and (X_{t-p}) on $(\Delta X_{t-1}, \ldots, \Delta X_{t-p+1})$ and calculating S_{00} etc. from the regression residuals.

The cointegrating rank then is given by the number of non-zero λ_i. Assume that the λ_i are ranked in descending order. Then the likelihood-ratio test statistic on this rank is given by

$$-T \sum_{i=j}^{n} \log(1 - \lambda_i)$$

which uses the $n - j + 1$ smallest roots. Under the null hypothesis that λ_j is zero, its distribution only depends on $n - j + 1$ and has been tabulated by Johansen (1988). If conditioning has been performed on a constant, additional to the lagged differences, the distribution changes to the ones tabulated by Johansen and Juselius (1989, Tables VI and VII). Two different distributions arise, depending on whether there is a multivariate drift or not. One can start testing with only the smallest root λ_n in the sum. If the test rejects, there are n cointegrating vectors and the system is stationary. If not, one may try the statistic for $j = n - 1$. If then rejection is obtained, there are $n - 1$ cointegrating vectors. One can continue in this way and if finally the test does not reject for $j = 1$, we can state that there is no evidence on cointegration in the data.

The cointegrating vectors, i.e. a basis for the cointegrating space, also evolve from (2.3) as those v_i which correspond to the non-zero λ_i. The estimate for the matrix β contains these v_i' as row vectors. It follows that the problem can be seen as a search for that coordinate transformation (rotation) of the integrated data which yields a vector with the maximum number of stationary entries.

An estimate of α is obtained as a byproduct from the canonical analysis. The error-correcting term $\alpha' \beta X_{t-i}$ (compare (2.2)) may then be subtracted from the level variates and the resulting variable regressed on the lagged differences to retrieve the

Φ_{ij} parameters. Here, least-squares is identical to conditional maximum-likelihood because of the restriction of identical lag orders.

In the simulations, this procedure will be denoted *JOH-i* for fixed cointegrating dimension and *JOH-x* if estimation is based on cointegrating dimension obtained from the likelihood-ratio test.

3. MONTE CARLO

The simulations of this paper follow six different data generating processes. The designs are extensions of the bivariate forecasting experiment of Engle and Yoo (1987) to the trivariate case. Three of them incorporate a second cointegrating vector, two use only one vector, and one experiment models a system without cointegration. For the first three models (model-x, model-y, model-z), the following basic design was kept constant[1]:

$$\Delta X_t = \begin{pmatrix} \alpha_1 & -.1 \\ .1 & 0 \\ \alpha_3 & \alpha_2 \end{pmatrix} \begin{pmatrix} 1 & -2 & 0 \\ 1 & 1 & -1 \end{pmatrix} X_{t-1} + \epsilon_t$$

$$\text{var}(\epsilon_t) = \begin{pmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 100 \end{pmatrix}, \qquad x_0 = y_0 = 0,$$

where $X_t = (X_{1t}, X_{2t}, X_{3t})'$. The cointegrating vector $X_{1t} = 2X_{2t}$ corresponds to the Engle and Yoo (1987) model. While keeping the cointegrating relationships, we allow for variations in the α' s to control the size of the roots μ_i of $(I + \alpha'\beta)$. These roots characterize the amount of the dependence of the system on the cointegrating vectors. In particular we have selected the following models:

	α_1	α_2	α_3	μ_1	μ_2	μ_3
model-x	-.4	.2	.2	.281	.819	1.000
model-y	.1	.2	.2	.700	.900	1.000
model-z	-.4	.8	-.4	.362	.138	1.000

For example, model-z with its two rather small roots describes a system that heavily depends on its cointegrating structure while model-y with two roots close to unity approximates a system generated by three independent random walks.

[1] Introducing instantaneous correlations among the innovations variates does not change the overall results.

For the two models which contain one cointegrating vector (model-a, model-b) a similar design was chosen:

$$\Delta X_t = \begin{pmatrix} \alpha_1 \\ .1 \\ .4 \end{pmatrix} (1 \quad 1 \quad -1) X_{t-1} + \epsilon_t$$

$$\text{var}(\epsilon_t) = \begin{pmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 100 \end{pmatrix}, \qquad x_0 = y_0 = 0,$$

The following table gives the specification for α_1 and the implied roots corresponding to model-a and model-b:

	α_1	μ_1	μ_2	μ_3
model-a	.1	.8	1.000	1.000
model-b	-.5	.2	1.000	1.000

Finally, we generated the data for model-n by three independent random walks with the same error process as in the preceding systems. As in the simulation experiment of Engle and Yoo (1987), 100 replications of each model were computed. Also, the same sample size of 100 observations was used for fitting the models, but the forecasting horizon was considerably increased to 50 periods. This seems to be important because the comparison of the results from 20–step ahead forecasts can lead to wrong conclusions concerning the long run forecasting performance.

4. SUMMARY AND CONCLUSIONS

A graphical display of the main results from the Monte Carlo experiment is given in Figures 1 to 6. The same information is contained in Tables III to VIII. The mean square forecast errors, which are defined as the trace of the sample covariance matrix of the forecast errors, are plotted against the forecast horizon.

The main findings are as follows. The relative performance of the different procedures is remarkably similar for all models. As long as the true number of cointegrating relations is known, the conditional ML procedure dominates the two–step procedure in all models. Both methods perform slightly better than the VAR in differences and substantially better than the unrestricted VAR in levels. This is at variance with the conjecture by Engle and Yoo (1987) (p.152) who presume that the misspecified differenced VAR should be of inferior forecasting performance.

On the other hand, if the true cointegrating dimension is unknown and has to be estimated, the conditional ML procedure in some cases gives rather poor results because of a low but nevertheless substantial probability of imposing too much cointegration on the system. Table I shows that the probability of overestimating the true cointegrating dimension more or less coincides with the 5 % significance level used if the true dimension is two but exceeds it substantially for systems with less cointegration. This points to a simultaneous testing problem. The deterioration of forecasting performance caused by this "excess stationarity" is felt for each of our generated models. On the other hand, the a priori misspecification of setting the number of cointegrating vectors at a too low value, i.e. imposing too little cointegration, results in only a slight loss of forecasting accuracy. Unfortunately, if the cointegrating dimension is tested for, its under-estimation appears to be less frequent than the reverse error (compare Table I). This outcome would suggest keeping the significance level of the testing stage low (e.g. 1 %).

The asymmetric loss of accuracy resulting from different misspecifications may be explained by the different order of integration of the missing regressor. In models with too few cointegrating vectors, a stationary error correction variable is missing among the explanatory variables. If too much cointegration and therefore spurious error-correction terms are imposed, additional integrated variables enter into the equations. Asymptotically, the coefficients on these variables converge to zero and their influence vanishes but, for finite samples, this effect can be quite harmful as our simulations show.

The results concerning the differences in forecasting performance between the conditional ML and the two–step procedure are corroborated by the average smallest canonical correlations between the estimated and the true cointegrating space. The correlations and their standard deviations are given in Table IIa and Table IIb. The warnings by Banerjee et al. (1986) concerning the high small sample bias of cointegrating vectors are not confirmed by these results.

Let us conclude our exercise with two tentative recommendations. First, the conditional ML procedure due to Johansen (1988) should be preferred but, unless the true number of cointegrating relationships is known a priori, a low significance level should be used in the testing stage. Second, it should be kept in mind that the final estimates of all multi–step procedures - including conditional ML - rely on preliminary estimates and are, therefore, sensitive to misspecification errors. Our results suggest that, in any case of doubt, forecasting should be accomplished via a model with less cointegration or a simple VAR in differences.

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Table I

ESTIMATED COINTEGRATING DIMENSION

	percentage of			
	dim0 (Δ-VAR)	dim1	dim2	dim3 (VAR)
model-x	0	1	93	6
model-y	0	3	94	3
model-z	0	0	94	6
model-a	0	92	8	0
model-b	0	90	10	0
model-n	84	14	2	0

Table IIa

AVERAGE SMALLER CANONICAL CORRELATIONS BETWEEN THE ESTIMATED AND THE TRUE COINTEGRATING SPACE

	model-x	model-y	model-z
JO	.9966879	.9997879	.9998174
	(.0110387)	(.0004511)	(.0007673)
EC	.9932024	.9995217	.9995572
	(.0173741)	(.0007248)	(.0012203)

Numbers in parentheses refer to standard deviations.

Table IIb

AVERAGE CORRELATIONS BETWEEN THE ESTIMATED AND THE TRUE COINTEGRATING SPACE

	model-a	model-b
JO	.9987363	.9980208
	(.0024997)	(.0039175)
EC	.9576241	.9558999
	(.0806911)	(.0882144)

Numbers in parentheses refer to standard deviations.

Table III

MEAN SQUARE FORECAST ERRORS OF MODEL-X

step	joh-2	joh-1	joh-x	ec-2	ec-1	uvar	noco
1	312.86	351.68	314.99	308.89	349.96	301.82	367.87
2	587.14	682.21	598.57	596.44	677.47	603.34	780.19
3	862.26	1027.2	874.43	869.20	1029.4	866.67	1122.2
4	1136.4	1346.4	1160.7	1173.0	1347.5	1145.4	1434.1
5	1337.3	1691.1	1381.1	1444.0	1672.7	1363.6	1759.4
6	1590.7	2051.6	1668.0	1727.6	2031.7	1666.9	2125.3
7	1765.3	2284.7	1859.8	1908.3	2272.1	1838.8	2373.1
8	2166.9	2871.1	2293.9	2354.5	2830.5	2234.5	2989.2
9	2548.0	3283.1	2714.6	2754.4	3248.2	2622.7	3419.7
10	2814.6	3603.1	3005.8	3040.7	3567.8	2954.7	3746.1
11	3163.5	3984.7	3392.1	3397.4	3972.4	3302.0	4139.2
12	3515.5	4400.7	3765.1	3808.7	4373.3	3651.0	4580.9
13	3856.3	4890.4	4146.9	4182.2	4829.7	4015.5	5054.6
14	3957.1	4903.7	4314.5	4314.6	4809.1	4130.5	4998.9
15	4287.6	5224.8	4695.5	4676.7	5134.3	4528.4	5335.7
16	4607.6	5689.9	5032.8	5022.2	5584.2	4804.4	5751.0
17	5002.9	6075.6	5571.5	5402.4	5943.7	5345.3	6175.8
18	5455.6	6690.7	6119.5	5907.3	6549.0	5948.5	6769.0
19	5734.2	7049.5	6489.7	6176.7	6913.4	6335.8	7117.2
20	5716.9	6984.8	6621.0	6164.6	6849.4	6446.4	7034.7
21	6006.0	7378.0	7003.0	6457.3	7208.7	6837.3	7451.8
22	6268.3	7730.9	7439.9	6745.5	7577.9	7352.1	7785.7
23	6754.6	8078.7	7980.6	7215.6	7948.7	7919.2	8150.0
24	7172.2	8488.0	8625.6	7665.9	8360.4	8601.3	8557.4
25	7434.8	8752.5	9013.8	7952.4	8615.4	9039.8	8884.7
26	7585.1	8818.0	9355.4	8161.7	8689.7	9282.5	8958.5
27	8098.2	9266.6	10125.	8686.1	9139.0	10265.	9358.3
28	8311.9	9549.4	10455.	8866.3	9399.2	10660.	9633.0
29	8444.3	9693.2	10713.	9009.4	9523.9	10995.	9760.5
30	8794.3	10179.	11343.	9340.2	10059.	11672.	10269.
31	8774.2	10220.	11511.	9298.4	10103.	11998.	10296.
32	8865.8	10489.	11597.	9306.5	10360.	12408.	10563.
33	8833.8	10482.	11715.	9304.5	10383.	12550.	10593.
34	8755.5	10413.	11738.	9228.5	10306.	12673.	10525.
35	9204.8	10843.	12389.	9685.1	10767.	13394.	11006.
36	9449.0	11011.	12783.	9929.9	10953.	13867.	11184.
37	9876.4	11543.	13312.	10349.	11483.	14616.	11708.
38	10529.	12144.	14303.	10971.	12091.	15721.	12265.
39	10990.	12762.	15141.	11494.	12702.	16460.	12920.
40	11084.	13008.	15442.	11598.	12919.	16700.	13142.
41	11310.	13175.	15842.	11790.	13051.	17497.	13299.
42	11756.	13436.	16128.	12250.	13327.	17858.	13548.
43	11725.	13399.	16154.	12232.	13278.	17964.	13571.
44	11980.	13703.	16659.	12589.	13595.	18629.	13863.
45	12155.	13880.	17030.	12773.	13803.	19274.	14019.
46	12230.	14050.	17516.	12808.	13964.	20048.	14201.
47	12423.	14180.	18134.	12932.	14054.	20929.	14303.
48	12669.	14255.	18967.	13160.	14150.	21926.	14374.
49	13082.	14631.	19874.	13520.	14505.	23353.	14734.
50	13458.	14918.	20939.	13855.	14767.	24825.	15001.

Table IV

MEAN SQUARE FORECAST ERRORS OF MODEL-Y

step	joh-2	joh-1	joh-x	ec-2	ec-1	uvar	noco
1	289.47	339.28	290.78	305.48	313.29	274.53	333.32
2	490.17	726.43	493.36	559.60	741.96	489.86	827.35
3	751.98	1174.6	753.97	908.80	1274.4	760.32	1452.9
4	1152.6	1675.2	1153.9	1438.2	2081.1	1219.6	2342.2
5	1639.2	2473.3	1648.2	2045.3	2957.4	1743.6	3407.5
6	2398.9	3192.2	2423.1	2881.4	4245.1	2605.5	4841.0
7	3304.8	4027.7	3333.7	4001.9	5673.6	3622.3	6389.7
8	4207.6	5177.8	4214.8	5048.6	7139.8	4710.4	7930.3
9	5413.3	6270.8	5428.5	6433.7	8925.7	6185.4	9838.6
10	6512.8	7832.4	6533.6	7385.4	10416.	7590.6	11416.
11	7942.7	9032.9	7974.7	8932.4	12364.	9332.5	13508.
12	9584.6	10249.	9624.0	10645.	14562.	11380.	15788.
13	11066.	11624.	11124.	12212.	16460.	13274.	17742.
14	12535.	13245.	12582.	13918.	18478.	15116.	19777.
15	14556.	14971.	14591.	16010.	21047.	17641.	22420.
16	15909.	16649.	15959.	17499.	22600.	19649.	23950.
17	17637.	18420.	17677.	19337.	24745.	22131.	26057.
18	19076.	20540.	19092.	20891.	26425.	24250..	27764.
19	20669.	22760.	20665.	22475.	28451.	26583.	29876.
20	21767.	25154.	21772.	23984.	30413.	28458.	31809.
21	23532.	27226.	23490.	25868.	32603.	31029.	34081.
22	24907.	29882.	24868.	27389.	34420.	32982.	35900.
23	26452.	32294.	26369.	29049.	36279.	35510.	37810.
24	27887.	34673.	27797.	30602.	37991.	38318.	39628.
25	29760.	37592.	29693.	32353.	39945.	41557.	41662.
26	31967.	39904.	31920.	34640.	42377.	45372.	44111.
27	33590.	42482.	33556.	36336.	43706.	49033.	45544.
28	35887.	44844.	35911.	38515.	45652.	53305.	47536.
29	38199.	47895.	38260.	40754.	47215.	58023.	49138.
30	40660.	49969.	40723.	43358.	49492.	62531.	51432.
31	43507.	52374.	43564.	46421.	52383.	67834.	54391.
32	46265.	54925.	46363.	49575.	55326.	72435.	57287.
33	48719.	57183.	48839.	52234.	57527.	76946.	59516.
34	51027.	59497.	51163.	54725.	59669.	82213.	61694.
35	52969.	61901.	53159.	56688.	61330.	87109.	63306.
36	55170.	65288.	55357.	58893.	63435.	92628.	65390.
37	57109.	68602.	57354.	61344.	65236.	97311.	67201.
38	59894.	70995.	60189.	64376.	67844.	.10275E+6	69871.
39	63179.	73403.	63455.	67557.	70570.	.10914E+6	72619.
40	66171.	76612.	66522.	70506.	73557.	.11582E+6	75548.
41	68970.	80276.	69371.	73626.	76491.	.12193E+6	78627.
42	72434.	82993.	72915.	77357.	79509.	.12871E+6	81737.
43	75023.	87840.	75525.	79988.	81599.	.13490E+6	83768.
44	76950.	91949.	77513.	82440.	83983.	.14045E+6	86133.
45	79686.	94980.	80253.	85592.	86410.	.14688E+6	88592.
46	82988.	98241.	83588.	88843.	89095.	.15493E+6	91387.
47	86007.	.10262E+6	86599.	91587.	91754.	.16317E+6	94144.
48	89535.	.10682E+6	90146.	95273.	95464.	.17024E+6	97909.
49	92210.	.11105E+6	92785.	97746.	98407.	.17802E+6	.10095E+6
50	95410.	.11508E+6	95982.	.10089E+6	.10101E+6	.18571E+6	.10363E+6

Table V

MEAN SQUARE FORECAST ERRORS OF MODEL-Z

step	joh-2	joh-1	joh-x	ec-2	ec-1	uvar	noco
1	286.27	360.16	286.17	291.24	368.41	272.90	395.63
2	565.14	891.87	562.34	589.64	703.70	566.49	749.70
3	1033.7	1368.7	1007.9	1066.6	1251.3	984.42	1288.9
4	1364.4	2173.6	1364.0	1427.4	1608.6	1276.5	1642.1
5	2074.2	2351.5	2113.1	2138.6	2332.7	2027.2	2460.8
6	2496.1	3193.3	2567.3	2577.4	2708.4	2489.7	2828.2
7	3229.2	3817.6	3349.3	3294.3	3475.2	3330.9	3563.9
8	4039.0	4604.6	4136.3	4099.9	4201.3	4120.1	4268.1
9	4012.3	5439.4	4132.9	4066.2	4210.8	4201.3	4283.0
10	4523.8	6362.2	4690.9	4597.3	4653.1	4719.3	4758.3
11	4980.6	7174.8	5333.0	5039.9	5148.0	5403.8	5265.3
12	5700.6	7717.2	6213.3	5754.0	5780.7	6388.4	5875.0
13	6412.8	8380.5	7075.3	6470.1	6415.1	7257.8	6510.7
14	7203.2	8209.7	7972.0	7258.5	7240.5	8195.3	7332.6
15	7291.3	8805.1	8139.9	7355.4	7314.6	8455.2	7476.5
16	7369.8	9571.6	8231.6	7447.6	7395.8	8512.9	7561.5
17	7205.8	10113.	8294.2	7305.7	7207.0	8519.6	7371.6
18	7308.6	10411.	8483.1	7405.5	7403.9	8716.3	7526.2
19	8240.0	10536.	9523.2	8329.4	8384.6	9986.8	8526.5
20	8767.9	10606.	10150.	8869.8	8841.8	10710.	8943.3
21	8767.1	11157.	10409.	8867.0	8848.5	11069.	8987.2
22	9076.1	12016.	10806.	9180.3	9131.9	11640.	9260.4
23	9795.7	13019.	11792.	9886.1	9840.4	12664.	9935.3
24	10135.	13582.	12462.	10239.	10090.	13244.	10184.
25	11092.	14154.	13835.	11201.	11090.	14565.	11218.
26	11930.	14702.	14760.	12064.	11861.	15324.	11949.
27	12204.	15402.	15276.	12344.	12150.	15865.	12305.
28	13135.	15944.	16587.	13281.	13075.	16967.	13231.
29	14457.	15959.	18217.	14632.	14432.	18283.	14612.
30	15864.	16934.	19934.	16003.	15751.	20444.	15931.
31	16956.	16760.	21708.	17086.	16808.	22389.	16963.
32	17657.	16572.	22924.	17784.	17530.	24001.	17664.
33	17229.	16838.	22715.	17338.	17120.	24545.	17199.
34	16969.	17355.	22887.	17069.	16767.	25350.	16916.
35	17818.	18359.	23694.	17929.	17587.	26005.	17713.
36	19200.	18354.	25336.	19317.	18971.	27588.	19087.
37	19606.	19406.	26401.	19746.	19344.	28230.	19523.
38	19834.	20168.	26323.	20003.	19495.	27981.	19613.
39	20699.	21185.	27470.	20893.	20477.	28871.	20582.
40	20726.	21425.	27972.	20912.	20429.	29633.	20512.
41	20836.	21120.	28202.	21026.	20534.	29838.	20643.
42	21260.	22207.	29131.	21465.	20892.	30950.	20974.
43	21647.	22644.	30301.	21850.	21239.	32347.	21302.
44	22023.	23433.	31864.	22241.	21655.	33799.	21735.
45	23229.	24180.	33911.	23443.	22872.	36542.	22968.
46	23222.	25050.	34409.	23428.	22878.	38257.	22999.
47	24760.	25147.	36852.	24949.	24405.	41864.	24536.
48	24771.	25230.	38453.	24971.	24435.	44029.	24553.
49	25768.	26965.	41117.	25955.	25463.	47104.	25559.
50	26303.	27626.	42589.	26519.	25973.	48511.	26047.

Table VI

MEAN SQUARE FORECAST ERRORS OF MODEL-A

step	joh-2	joh-1	joh-x	ec-2	ec-1	uvar	noco
1	307.25	302.67	302.81	302.15	327.29	296.22	375.21
2	740.34	726.13	721.51	762.77	828.46	724.89	912.93
3	1218.5	1193.3	1176.8	1289.3	1449.9	1185.7	1570.5
4	1898.4	1854.9	1806.2	2085.7	2253.7	1845.6	2415.5
5	2545.6	2484.8	2391.5	2892.8	3042.0	2475.8	3486.4
6	3563.1	3482.8	3321.3	4118.7	4091.2	3413.8	4785.3
7	4404.4	4270.9	4057.2	5161.0	5074.8	4159.3	5787.2
8	5857.4	5724.6	5425.2	7034.2	6790.7	5516.7	7481.9
9	7239.3	6941.3	6591.0	8618.4	7982.6	6808.0	8639.7
10	9024.8	8612.4	8134.9	10525.	9567.0	8417.3	10340.
11	10277.	9666.5	9175.6	11990.	10637.	9583.2	11267.
12	12005.	11370.	10814.	13963.	12361.	11219.	12788.
13	13631.	12941.	12281.	15811.	13972.	12739.	14266.
14	14859.	14096.	13371.	17167.	15152.	13779.	15264.
15	16723.	15701.	14901.	19125.	16649.	15443.	17049.
16	18464.	17180.	16349.	20990.	18045.	17002.	18776.
17	19834.	18420.	17516.	22399.	19281.	18305.	20180.
18	21434.	19818.	18767.	23962.	20533.	19461.	21752.
19	22662.	20606.	19478.	24891.	21295.	20479.	22754.
20	23780.	21142.	20062.	25807.	21768.	21541.	23127.
21	24962.	22140.	21071.	27139.	22989.	22536.	24429.
22	26093.	22932.	21967.	28372.	23864.	23583.	25276.
23	28001.	24385.	23380.	30523.	25244.	25497.	26620.
24	29541.	25487.	24552.	31857.	26290.	26899.	27749.
25	31123.	26974.	26080.	33559.	27760.	28651.	28989.
26	32632.	28198.	27329.	34787.	28627.	30417.	29941.
27	34117.	29239.	28503.	36084.	29705.	32104.	31021.
28	35259.	30070.	29352.	37175.	30694.	33662.	32090.
29	35613.	29974.	29439.	37446.	30824.	34706.	31942.
30	36871.	30682.	30194.	38476.	31744.	36172.	32947.
31	38300.	31785.	31339.	39984.	32817.	37909.	33934.
32	39839.	33282.	32726.	41229.	34162.	39674.	35769.
33	40641.	33759.	32950.	41538.	34302.	40360.	36661.
34	41308.	34434.	33607.	42114.	34811.	41780.	37376.
35	43285.	36143.	35378.	44008.	36136.	44584.	39135.
36	43886.	36850.	36316.	44731.	36811.	45584.	39844.
37	45585.	38474.	38059.	47039.	38468.	48415.	41662.
38	47131.	40133.	40007.	48762.	40289.	50596.	43409.
39	47914.	41092.	41006.	49998.	41228.	51851.	44502.
40	48032.	41242.	41301.	50452.	41635.	52767.	44621.
41	49422.	42011.	42284.	51792.	42418.	54638.	45060.
42	50073.	42525.	43002.	52549.	43289.	56063.	45292.
43	50996.	43375.	43918.	53920.	44271.	57258.	46349.
44	52051.	44403.	44824.	54850.	45368.	58863.	47397.
45	53729.	45396.	45927.	56110.	46050.	60823.	48087.
46	54661.	45707.	46440.	56414.	46080.	62198.	48571.
47	55433.	46507.	47149.	57526.	46594.	63796.	48886.
48	58054.	48090.	48873.	59650.	47915.	66786.	49998.
49	60525.	49890.	50891.	61514.	49633.	70053.	51451.
50	63716.	51839.	53149.	63586.	51601.	73776.	53026.

Table VII

MEAN SQUARE FORECAST ERRORS OF MODEL-B

step	joh-2	joh-1	joh-x	ec-2	ec-1	uvar	noco
1	308.58	306.10	305.49	311.72	310.15	292.55	369.19
2	646.33	630.67	633.92	646.61	641.46	630.71	711.17
3	1029.4	976.09	995.86	1023.7	991.93	1021.2	1057.1
4	1419.4	1337.7	1358.9	1387.0	1350.3	1427.7	1382.8
5	1721.3	1612.5	1649.3	1668.8	1629.6	1744.9	1700.3
6	2244.1	2022.6	2107.1	2117.8	2041.3	2270.8	2112.6
7	2416.3	2152.7	2243.8	2271.2	2201.3	2448.7	2269.0
8	2914.9	2603.3	2697.4	2758.7	2649.2	2999.6	2718.0
9	3158.5	2768.0	2871.1	2976.1	2801.3	3270.8	2864.2
10	3755.6	3303.0	3427.3	3606.1	3311.1	3922.2	3411.0
11	3809.7	3257.1	3389.4	3662.0	3276.5	4086.4	3287.8
12	4126.8	3549.6	3684.5	3960.7	3539.5	4461.7	3578.0
13	4295.6	3674.5	3807.1	4145.8	3662.3	4623.0	3716.3
14	4603.4	3943.7	4104.8	4372.1	3918.6	4890.4	3956.0
15	4923.3	4238.9	4383.0	4683.0	4242.7	5175.6	4296.7
16	5384.0	4544.8	4696.6	5119.1	4580.8	5685.2	4630.6
17	5834.2	4813.8	4937.0	5478.6	4861.6	6230.2	4904.1
18	6137.0	4961.8	5119.2	5670.4	4986.8	6583.5	5081.0
19	6583.1	5247.8	5447.8	6064.7	5272.2	7115.8	5358.2
20	6756.4	5344.1	5566.9	6327.5	5383.4	7374.8	5426.8
21	7350.0	5841.8	6059.5	6911.8	5912.6	8127.5	5948.0
22	7890.7	6180.6	6418.8	7433.4	6267.6	8713.9	6282.2
23	8395.7	6476.2	6749.9	7713.7	6582.4	9274.5	6564.2
24	9172.7	6934.7	7253.4	8251.1	6999.5	10120.	7042.9
25	9480.9	7160.9	7447.8	8574.1	7216.3	10543.	7256.1
26	10029.	7388.9	7661.7	8929.0	7441.2	11356.	7436.1
27	10777.	7731.5	8081.3	9491.8	7769.6	12248.	7742.0
28	10804.	7790.9	8211.1	9821.2	7856.8	12393.	7868.4
29	10807.	7673.9	8069.1	9766.5	7745.6	12561.	7752.0
30	10820.	7509.5	7951.5	9645.0	7594.9	12675.	7594.3
31	11356.	7761.0	8281.7	10063.	7841.9	13330.	7804.0
32	11712.	7975.5	8468.1	10410.	8041.3	13885.	8044.4
33	12089.	8174.6	8691.8	10790.	8243.1	14378.	8268.6
34	12044.	8100.3	8569.4	10786.	8146.3	14575.	8182.5
35	12530.	8510.2	8993.1	11280.	8558.5	15287.	8580.2
36	12915.	8755.0	9178.7	11519.	8815.9	15703.	8837.0
37	13320.	8788.3	9268.7	11984.	8846.9	16377.	8879.1
38	14220.	9458.5	9936.5	12795.	9487.1	17682.	9497.6
39	14499.	9572.1	9985.0	12901.	9582.2	18157.	9666.7
40	14688.	9481.6	9888.4	12930.	9492.3	18536.	9555.5
41	15030.	9589.6	10150.	13186.	9582.3	19088.	9667.9
42	15146.	9638.6	10157.	13192.	9654.5	19384.	9663.0
43	16310.	10400.	11041.	14283.	10401.	20891.	10491.
44	17149.	11046.	11620.	15087.	11039.	21919.	11149.
45	17726.	11526.	12114.	15643.	11525.	22805.	11630.
46	18293.	11648.	12208.	15894.	11637.	23753.	11752.
47	19164.	11858.	12589.	16464.	11819.	24873.	11966.
48	20255.	12538.	13355.	17454.	12474.	26285.	12603.
49	20703.	12631.	13527.	17780.	12554.	27235.	12669.
50	21502.	13095.	13921.	18228.	13019.	28295.	13126.

Table VIII

MEAN SQUARE FORECAST ERRORS OF MODEL-N

step	joh-2	joh-1	joh-x	ec-2	ec-1	uvar	noco
1	303.38	309.21	303.58	305.46	300.71	290.85	298.76
2	682.81	711.81	665.48	692.33	660.84	695.29	634.95
3	1099.0	1147.4	1047.1	1129.6	1066.6	1132.2	995.04
4	1444.6	1492.4	1378.6	1495.0	1403.2	1495.4	1300.3
5	1828.5	1872.8	1737.6	1873.3	1793.3	1877.1	1654.2
6	2270.6	2318.8	2114.9	2272.1	2112.8	2308.0	1964.7
7	2512.1	2541.7	2319.4	2506.8	2359.8	2536.9	2148.8
8	3009.8	3067.3	2734.4	3006.7	2775.7	3087.7	2518.2
9	3385.6	3509.6	3107.3	3469.8	3164.3	3555.6	2837.8
10	3770.0	3937.5	3424.3	3885.7	3487.6	4026.8	3128.0
11	4090.4	4293.7	3705.2	4255.6	3761.1	4439.3	3401.4
12	4337.5	4665.3	3977.0	4671.9	4083.6	4833.5	3639.4
13	4551.7	4861.4	4172.6	4941.6	4326.1	5082.4	3806.6
14	4814.6	5125.2	4438.3	5202.0	4599.1	5377.6	4050.6
15	5087.8	5346.9	4645.7	5434.2	4793.0	5673.8	4237.1
16	5465.8	5862.5	4975.9	5853.6	5092.5	6192.6	4549.7
17	5855.9	6331.7	5316.5	6324.1	5415.2	6747.4	4864.8
18	6308.6	6862.1	5670.2	6708.9	5716.8	7248.3	5184.2
19	6777.1	7306.8	6066.6	7163.3	5993.2	7745.3	5492.6
20	6906.6	7475.1	6116.6	7479.8	6135.6	8009.0	5547.9
21	7380.8	7992.0	6622.1	8099.2	6592.8	8585.9	6000.6
22	7907.7	8556.8	7076.5	8679.7	6982.8	9204.2	6389.0
23	8076.4	8894.0	7283.3	9067.2	7317.1	9641.1	6571.5
24	8312.3	9241.3	7515.5	9417.6	7489.6	10008.	6747.1
25	8390.0	9434.7	7715.3	9595.8	7611.8	10302.	6964.6
26	8601.3	9790.7	7927.2	9993.2	7991.5	10824.	7165.2
27	9219.5	10423.	8520.0	10776.	8445.5	11657.	7622.1
28	9169.9	10674.	8509.9	10986.	8653.3	12005.	7647.4
29	9558.8	11037.	8779.0	11339.	8840.1	12475.	7883.0
30	9544.6	11111.	8678.2	11381.	8731.4	12720.	7760.8
31	9658.6	11312.	8781.5	11580.	8711.5	12987.	7765.7
32	9767.9	11686.	8874.0	12100.	9005.1	13476.	7887.4
33	9853.8	11842.	8990.1	12426.	9052.1	13827.	7973.4
34	9882.2	12234.	9045.7	12722.	9145.0	14306.	8014.4
35	10068.	12689.	9257.1	13149.	9275.2	14801.	8159.7
36	10642.	13310.	9877.8	13933.	9926.8	15578.	8778.5
37	11118.	14255.	10184.	14818.	10282.	16652.	9041.3
38	11489.	14845.	10610.	15584.	10459.	17446.	9362.8
39	11535.	15104.	10801.	15898.	10583.	17690.	9462.4
40	11645.	15373.	10830.	16328.	10601.	18237.	9441.6
41	12146.	16057.	11231.	16963.	11036.	19223.	9701.3
42	12179.	16222.	11206.	17204.	11311.	19507.	9713.5
43	12689.	16964.	11547.	17829.	11488.	20626.	10014.
44	13319.	17808.	12259.	18709.	12139.	21528.	10651.
45	13648.	18494.	12595.	19318.	12408.	22493.	10905.
46	13896.	19129.	12853.	19745.	12706.	23284.	11115.
47	14544.	20152.	13373.	20787.	13336.	24590.	11512.
48	15191.	20879.	13935.	21610.	13833.	25552.	12027.
49	15515.	21501.	14095.	22188.	14046.	26701.	12165.
50	16029.	22769.	14466.	23354.	14454.	27863.	12594.

Figure 1

MODEL - A

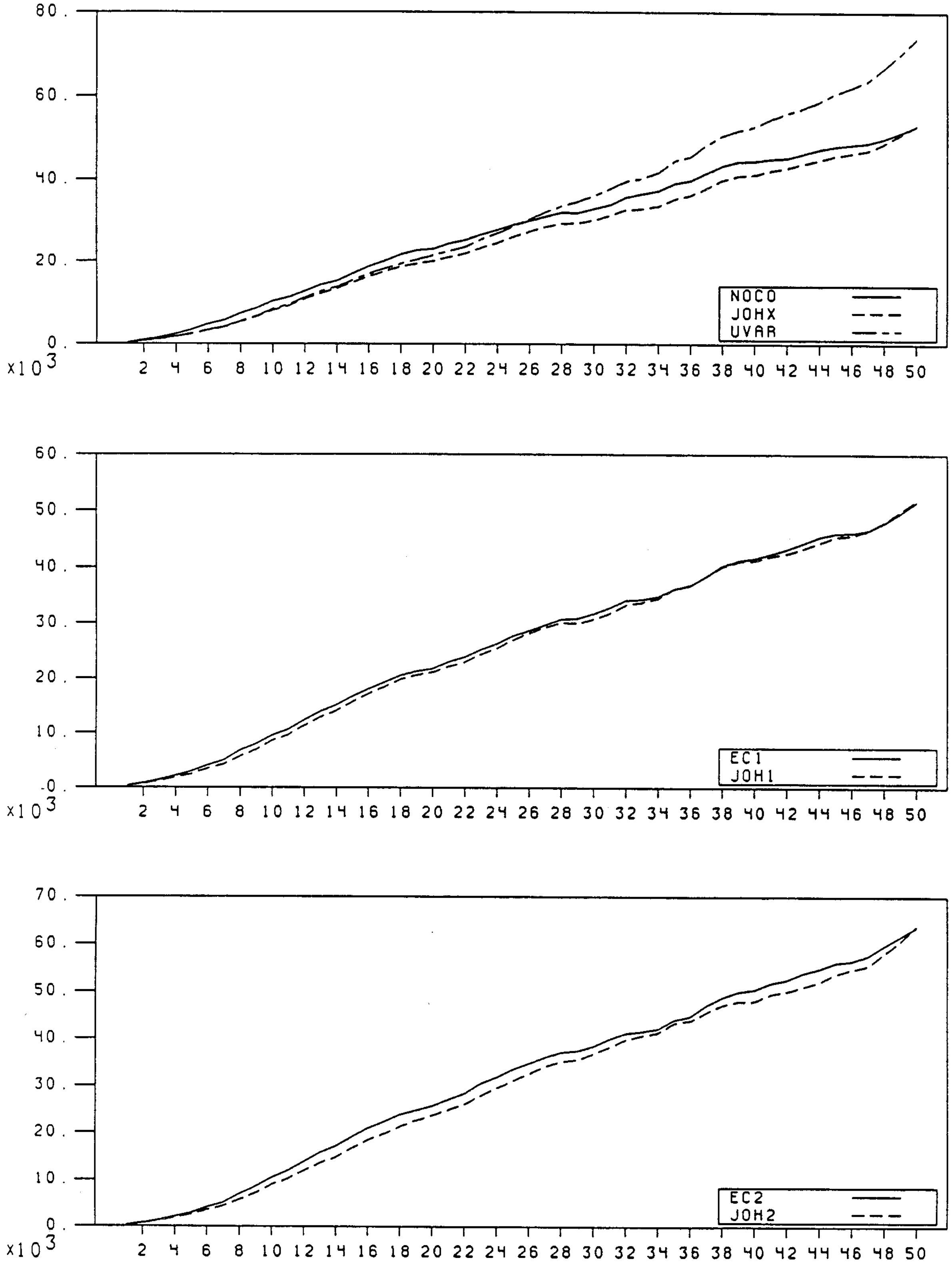
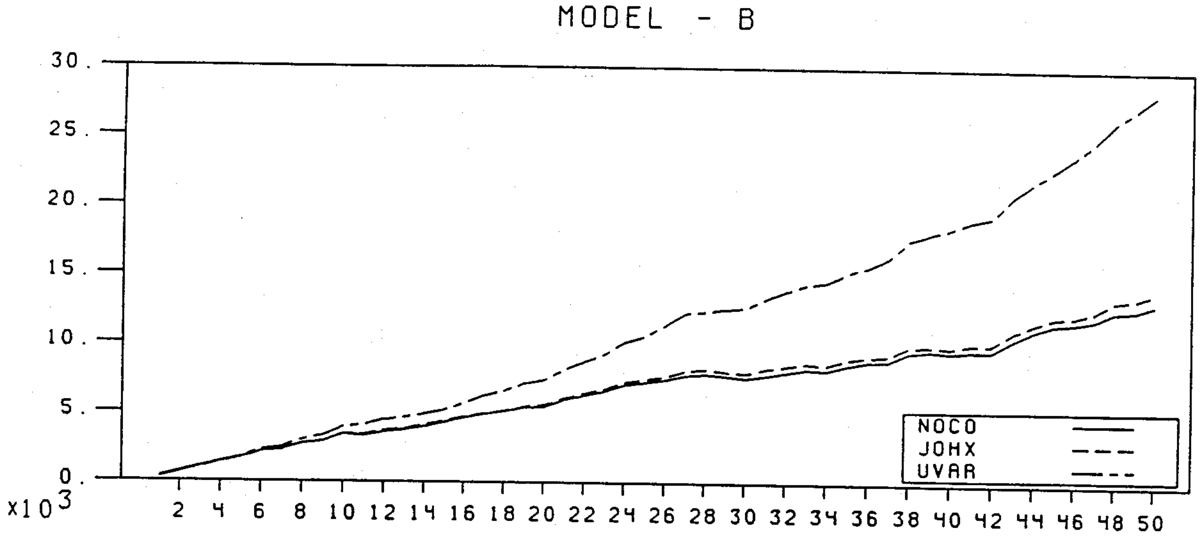
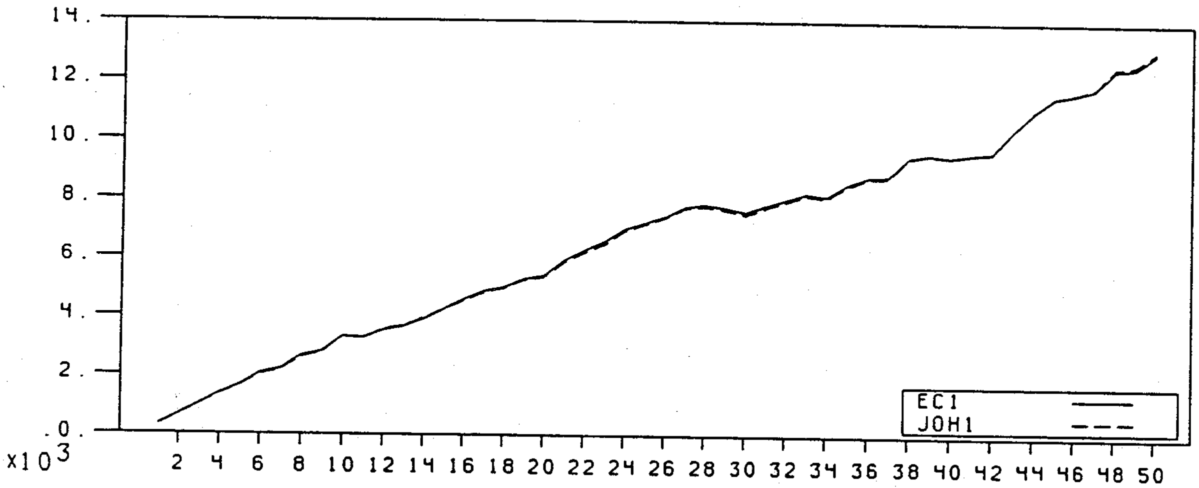


Figure 2


MODEL - B
30.
25.
20.
15.
10.
5.
0.
x10 3
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50
NOCO
JOHX
UVAR


14.
12.
10.
8.
6.
4.
2.
.0.
x10 3
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50
EC1
JOH1

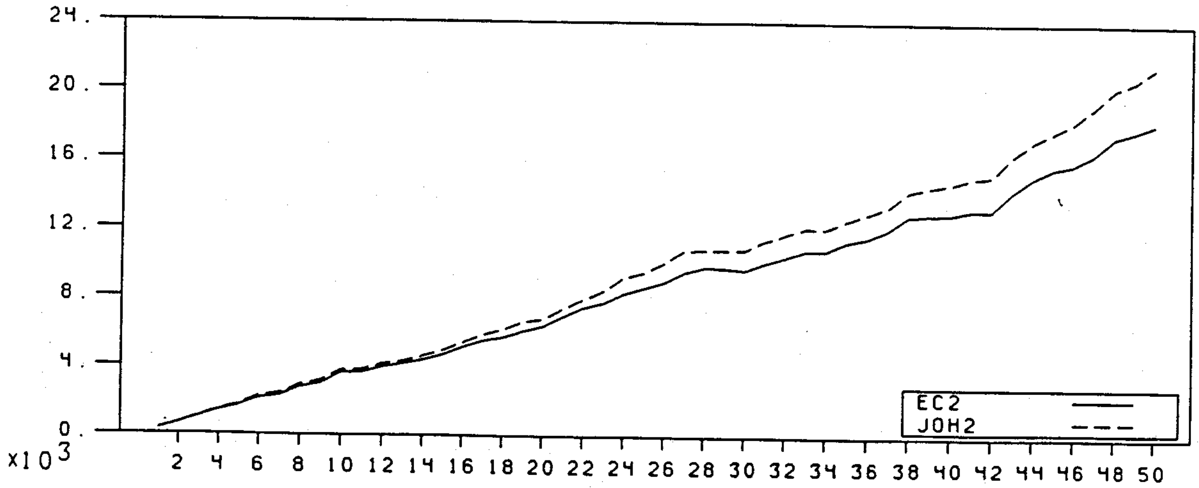

24.
20.
16.
12.
8.
4.
0.
x10 3
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50
EC2
JOH2

Figure 3

MODEL - N

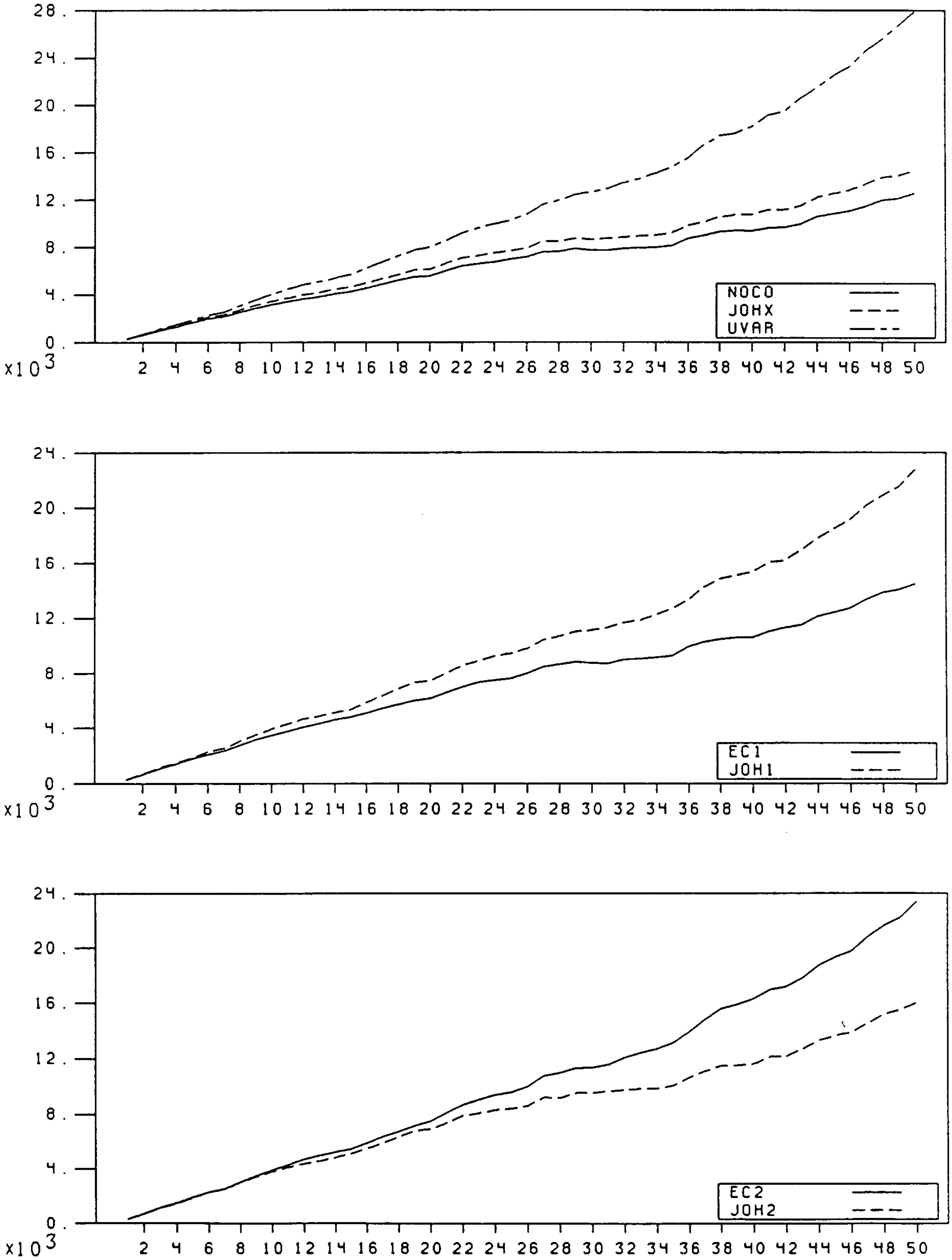
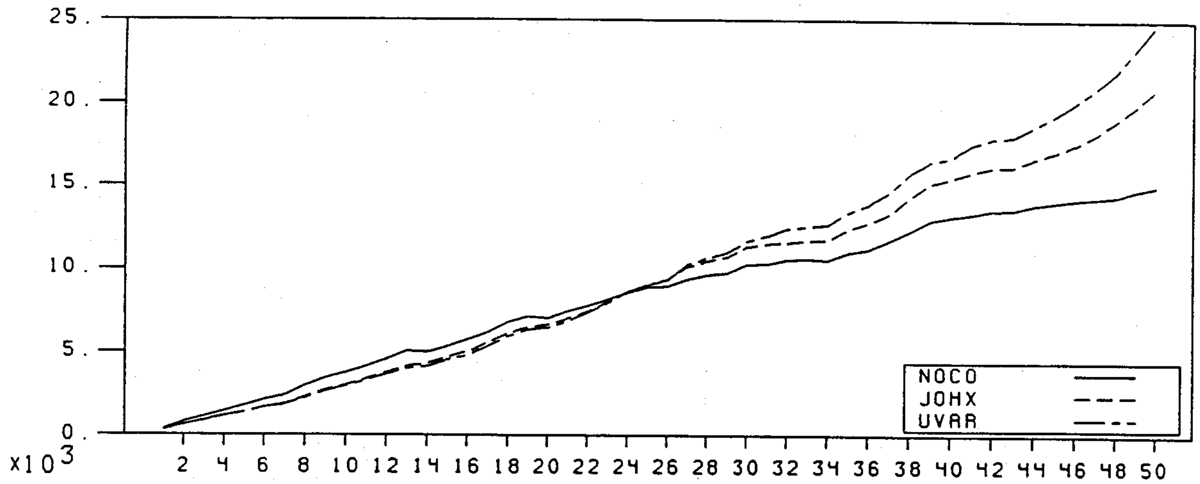
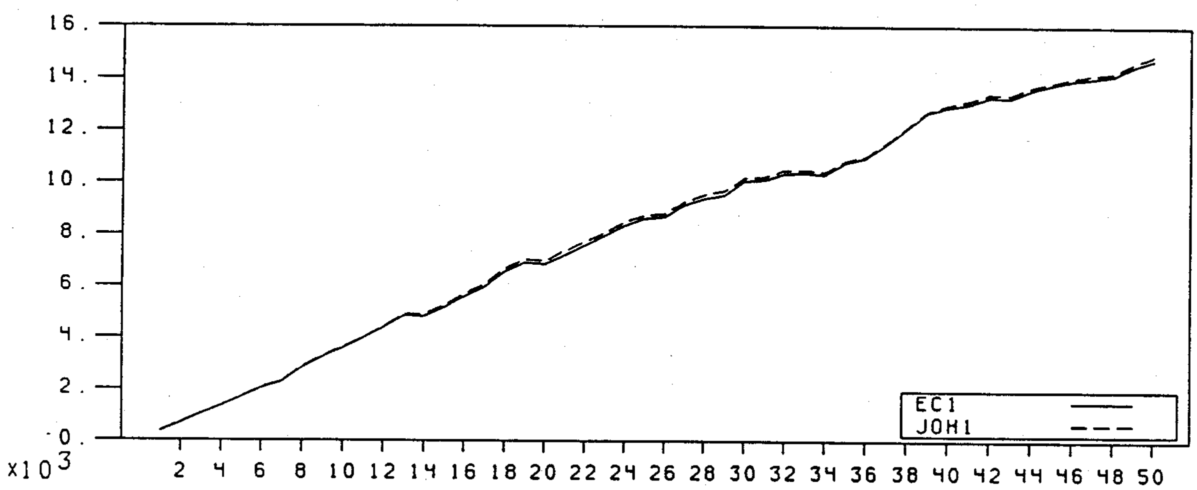


Figure 4

MODEL - X





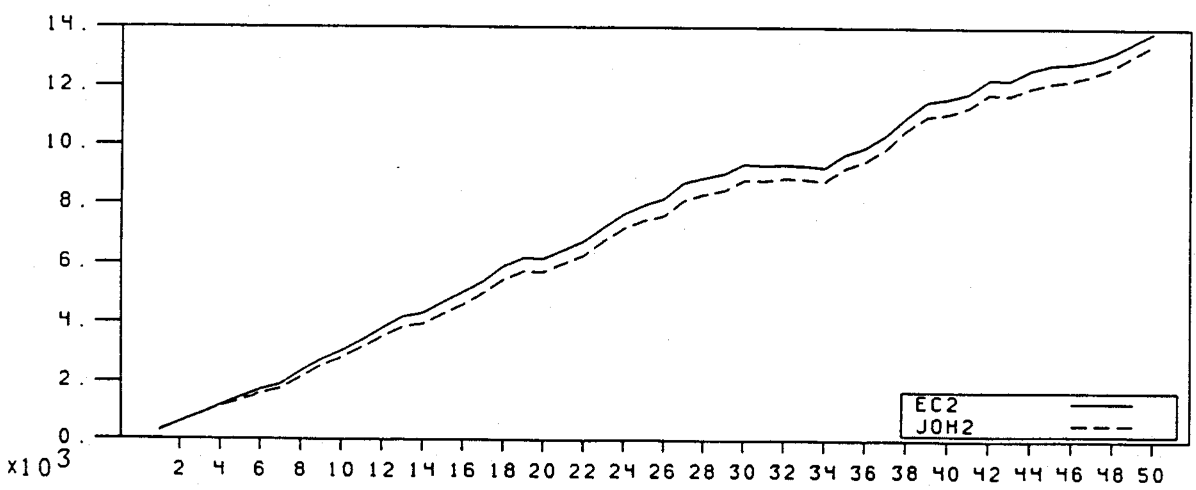
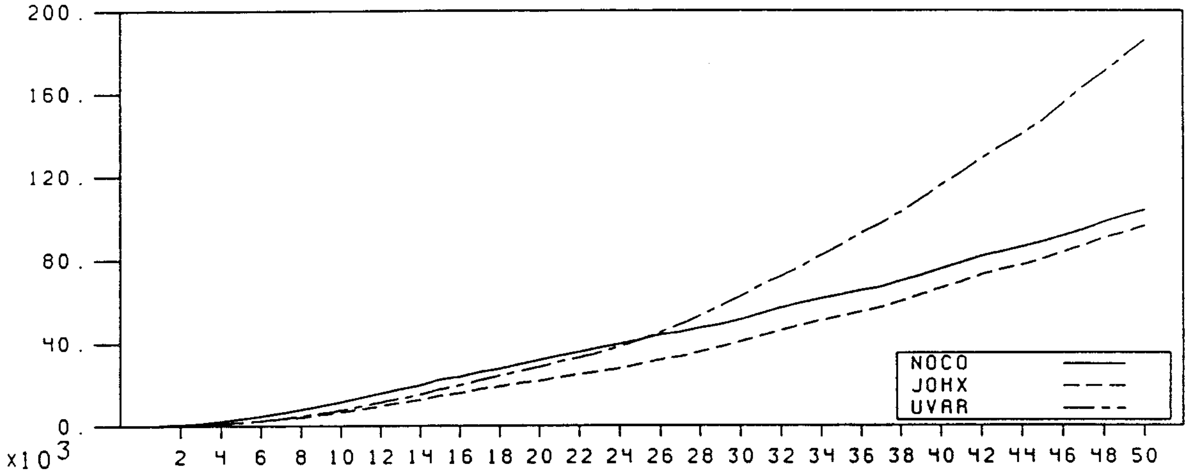
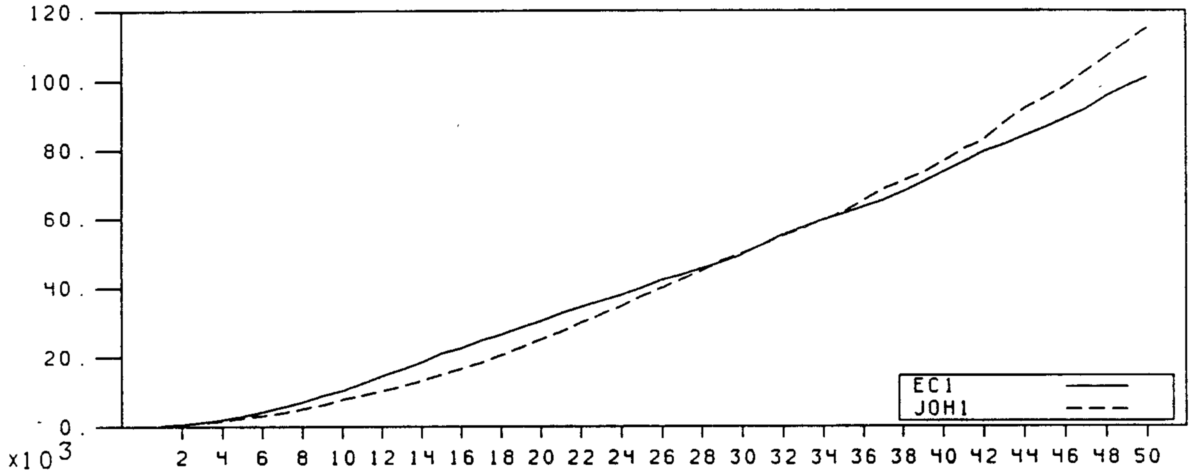


Figure 5

MODEL - Y





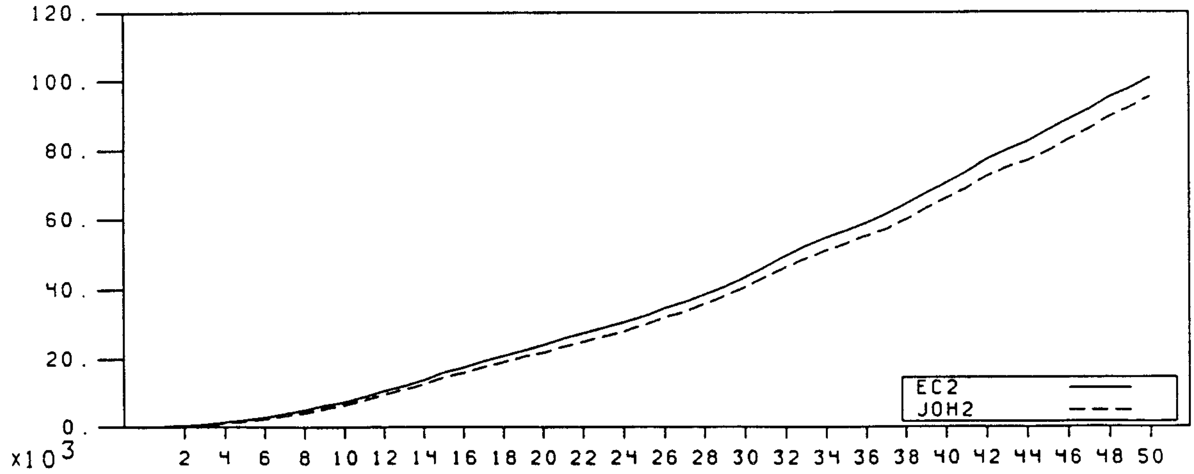
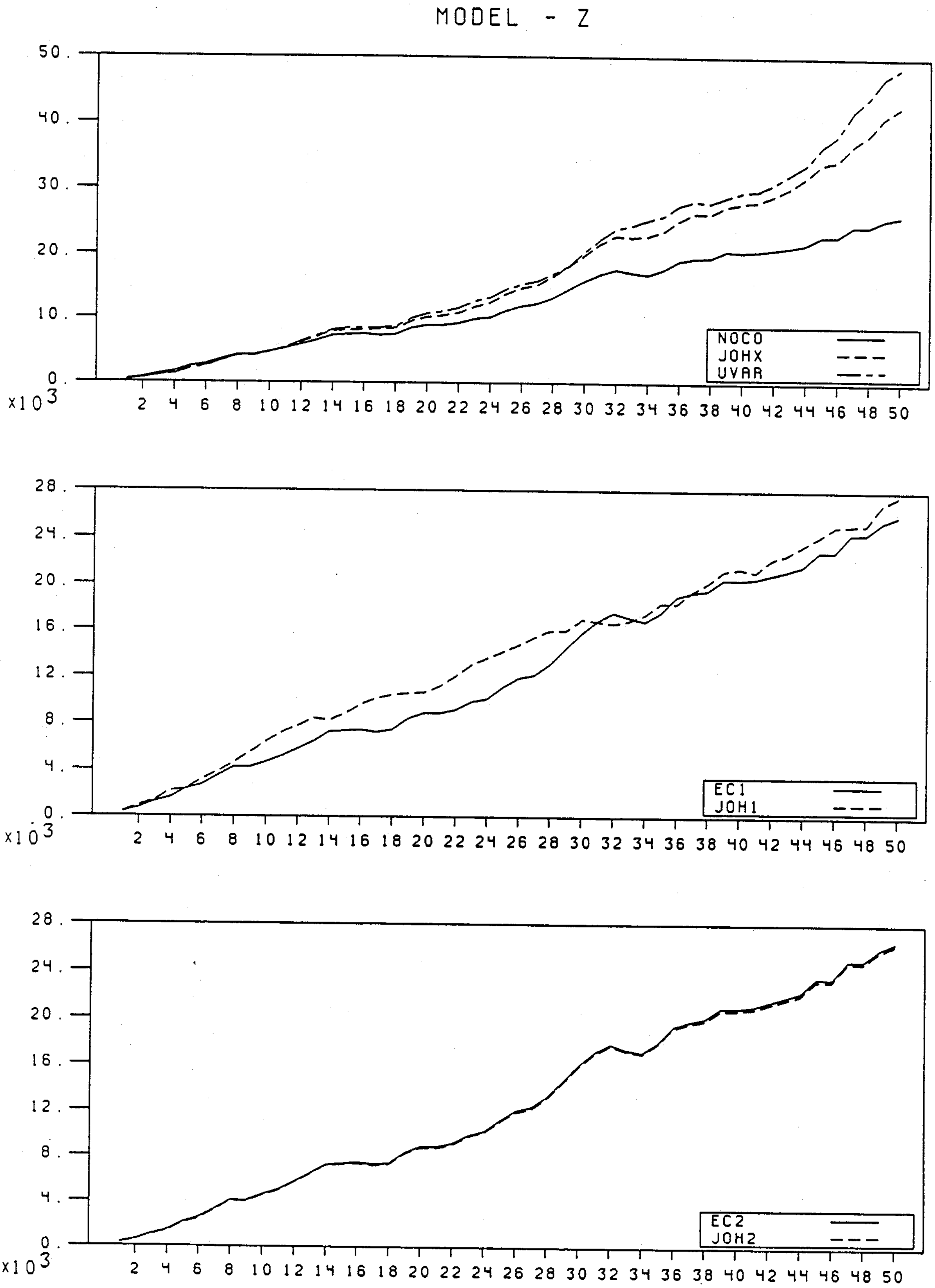


Figure 6


MODEL - Z
50.
40.
30.
20.
10.
0.
x10^3
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50
NOCO
JOHX
UVAR
28.
24.
20.
16.
12.
8.
4.
0.
x10^3
EC1
JOH1
28.
24.
20.
16.
12.
8.
4.
0.
x10^3
EC2
JOH2