COINTEGRATION IN MACROECONOMIC SYSTEMS: SEASONALITY AND EXPLOSIVE ROOTS*

Robert M. KUNST

Forschungsbericht/
Research Memorandum Nr. 255
October 1989

* Paper presented at the Econometric Society European Meeting, Munich, September 1989

Abstract.

This paper deals with some of the problems evolving from application of cointegration analysis to VAR models of economic time series.

Unless data have been seasonally adjusted by popular but frequently criticized routines such as Census X-11, they often exhibit seasonal patterns which are sometimes treated by including dummies in the VAR system. Sometimes models of additional unit roots at seasonal frequencies are suggested. The latter ones lead to the conception of seasonal cointegration.

There is no guarantee that the system identified by the analysis is indeed first-order integrated as implied by standard assumptions. Two ways are suggested to detect unexpected explosive and non-stationary behavior: prediction and eigenvalue analysis of the state-space form.

Macroeconomic systems of Austrian, Danish, German, and U.K. data illustrate the phenomena. In one case, explosive cycles are obtained.

Zusammenfassung

Dieses Papier beschäftigt sich mit einigen Problemen, die bei der Anwendung der Kointegrationsanalyse auf VAR-Modelle ökonomischer Zeitreihen auftreten.

Außer wenn Daten durch übliche, jedoch häufig kritisierte Routinen saisonbereinigt wurden - etwa durch Census X-11 -, zeigen sie oft Saisonmuster, welche bisweilen durch das Einfügen von Dummies in das VAR-System behandelt werden. Fallweise werden Modelle mit zusätzlichen Einheitswurzeln bei saisonalen Frequenzen vorgeschlagen. Letztere führen zum Konzept der saisonalen Kointegration.

Es gibt keine Garantie, daß das in der Analyse identifizierte System tatsächlich integriert von Ordnung 1 ist, wie dies die Standardannahmen unterstellen. Zwei Pfade können beschritten werden, die zur Aufdeckung unerwarteten explosiven Verhaltens und anderer Instationaritäten führen: Prognose und Eigenwertanalyse in der Zustandsraum-Darstellung.

Makroökonomische Systeme österreichischer, dänischer, bundesdeutscher und großbritischer Zeitreihen illustrieren die Phänomene. In einem Falle treten explosive Zyklen auf.

Contents

1. Introduction

For some time, vector autoregressions (VAR) have been in use now as the basic mechanical method for description and prediction of multivariate economic time series. A rather lengthy dispute between those who advocate the use of differenced data and those who prefer modeling original ("level") series has been all but reconciled by the theory of cointegration [Engle and Granger (1987)]. Detailed and readily applicable results have been published for first-order integrated systems only but extensions for higher-order systems and the seasonal case are to be expected soon [e.g. see Engle, Granger, and Hallman (1989)].

Here, a first-order integrated system will be defined as a VAR with roots outside the unit disc or at one only. Additionally, we will assume that all individual series can be made stationary by first-order differences. The solution of the Gaussian maximum-likelihood problem for this case has been analyzed by Johansen (1988) and Johansen and Juselius (1988, 1989). The main results will be discussed in Section 2.

Raw (non-adjusted) monthly or quarterly economic time series frequently show seasonal patterns which shed some doubt on the assumption of stationary first differences. The question whether these seasonal patterns should be eliminated by regression on seasonal dummies (the "deterministic" model) or by treating them by seasonal differencing, thereby assuming additional unit roots on the unit circle (the "stochastic" model), reminds of the discussion of deterministic and stochastic trend models. These features are revisited in Section 3.

Interestingly, most time series analysts choose to either ignore the possibility of unit roots within (!) the unit discs or rule it out by theoretical arguments. If such "explosive" roots should appear in practice, they will quite probably invalidate most of the statistical properties valid for the first-order system. The point is that such explosive roots may be hidden by the estimation procedure and do not even show necessarily in prediction experiments. The state space form of the VAR system can be used to recover the roots. These questions are the subject of Section 4.

Section 5 presents evidence on near-unit roots close to -1 or the seasonal frequencies at ±j in a macroeconomic core system which has been estimated for Austrian, Danish, German, and U.K. data. In one case, explosive roots are encountered. It will be tried to give explanations for these unexpected near-non-stationary modes in the systems. Section 6 concludes.

2. The Maximum Likelihood Estimator

The following solution of the problem of estimating a VAR system with cointegrating restrictions is due to Johansen (1988). The relevant statistical foundations are found in Tso (1981). Only the main results will be reviewed here.

Assume that the VAR system is given in the following form

$$X_t = \pi_1 X_{t-1} + \pi_2 X_{t-2} + \dots + \pi_k X_{t-k} + \epsilon_t$$

Then, without any assumptions on its stability, it can be rewritten in differences ($\Delta X_t = X_t - X_{t-1} = (1-B)X_t$) [1] but with lag order reduced by 1 plus a matrix which takes care of the fact that the unit factor 1-B possibly is not a factor of all polynomial elements:

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \dots + \Gamma_{k-1} \Delta X_{t-k+1} + \Gamma_k X_{t-k} + \epsilon_t$$

Generally, 1-B will not be a factor at all and Γ_k will be a full-rank matrix. If we restrict attention to systems of first-order integrated variables and assume ΔX_t to be stationary, the rank of Γ_k necessarily is less than the system dimension, say, p. It can be shown that, in this case, Γ_k can be represented in the form $\alpha\beta'$ with the factor matrices α and β being pxr-matrices with full smaller rank r where $r = \text{rank } \Gamma_k$. This representation is unique except for a transformation by a non-singular rxr-matrix.

β has a straightforward interpretation as its columns contain linearly independent cointegrating vectors. The matrix α distributes the influence of the implied stationary error-correction variables $\beta' X_t$ to the components of ΔX_t.

The solution to the problem of estimating the parameters in the Γ_i matrices under the restriction rank $\Gamma_k = r$ on the basis of Gaussian white noise errors is obtained by the following steps:

1) Regress ΔX_t on $\Delta X_{t-1}, \dots, \Delta X_{t-k+1}$ (least squares equation by equation)

---

1 B will denote the backshift or lag operator in the following.

2) Regress X_{t-k} on $\Delta X_{t-1},\ldots,\Delta X_{t-k+1}$
3) Calculate the canonical correlations between the residuals from steps 1 and 2. The eigenvectors corresponding to the non-zero correlations are the columns of β.
4) An estimate for α is obtained from $S_{0k}\beta$ with S_{0k} being the cross-moments matrix of the residuals from steps 1 and 2.
5) Retrieve estimates for the $\Gamma_1,\ldots,\Gamma_{k-1}$ from regressing $\Delta X_t-\alpha\beta' X_{t-k}$ (using the estimates for α and β) on $\Delta X_{t-1},\ldots,\Delta X_{t-k+1}$.

These estimates can be shown to be the maximum likelihood estimates and to be consistent of different order (β consistent of order T, the remainder of order $T^{\frac{1}{2}}$). The variance matrix of the ϵ_t can also be estimated by the same procedure.

The canonical correlations or roots calculated in step 3 are important. Decisions about whether they are zero or not can be based on the likelihood-ratio (LR) statistic

$$-T\ \Sigma\ \log(1-r_i)$$

with summation running over the smallest roots r_i. Some fractiles of the distribution of the LR statistic have been tabulated by Johansen (1988).

Note that the LR statistic relies on correlations between differences and level series, conditional on lagged differences and is therefore a direct multivariate generalization of the popular Dickey-Fuller statistic for univariate series. The column vectors of β transform X_t into different variable coordinates which have non-zero correlation to their differences. This property is taken as an indicator for stationarity.

In practice, it is sometimes difficult to fix the lag order k. We suggest to increase k gradually until the residuals from step 5 are white noise according to a portmanteau statistic like the Q by Ljung and Box which is displayed automatically by the RATS software package. Note, however, that this decision depends on the cointegrating dimension r. Therefore, some users prefer using the Q of the regressions in step 1. In most cases, this procedure over-estimates the lag order as the error correction terms should help to whiten the residuals.

Contrary to widespread belief, an over-estimation of the lag order is not innocuous. Of course, high lag orders decrease the degrees of freedom but there is a more important point to this. For an example, take a white noise series. The correlation between the series and its differences is 0.5. If a spurious lag is taken into account, the conditional correlation is reduced to 0.33.[2] In the language of our likelihood problem, inserting spurious lags decreases the chance of identifying cointegrating relations and imposes more integratedness on the system. This is particularly important in the presence of small samples and of inhomogeneous lag structures with the Γ_i matrices showing more zero elements [3] with increasing i. Both properties are met in this paper's examples.

2 More generally, it is easily shown that k spurious additional lags reduce the correlation between levels and differences for the process $y_t = \alpha y_{t-1} + e_t$ to $(1-\alpha)/[k(1-\alpha)+2]$. Compare Kunst (1989).

3 That is, insignificant elements by their t-value. If these are really zero, the outlined procedure is not maximum likelihood and the zero restrictions would have to be imposed on estimation.

3. Cointegration and Seasonality

For the results summarized in Section 2 to be valid, a basic assumption is that the vector ΔX_t is stationary. Therefore, it can only contain stationary and first-order integrated components. This excludes from further investigation all series that are suspected of being integrated of higher order, like price series or nominal wages. In principle, these series could be treated by a second-order difference representation like [4]

$$\Delta^2 X_t = \Gamma_1 \Delta^2 X_{t-1} + \dots + \Gamma_{k-2} \Delta^2 X_{t-k+2} + \Gamma_{k-1} \Delta X_{t-k+1} + \Gamma_k X_{t-k} + \epsilon_t$$

The analogue to the estimation procedure of the last section has not been elaborated yet for this case, and it is doubtful whether it can be worked out in the same straightforward fashion. Moreover, in most cases such higher-order difference representations contain over-differenced components which tend to inflate the overall lag order needed to capture a moving-average by a VAR process.

Note that second-order integrated (I(2)) processes can play havoc with the procedure set out in Section 2. The real problem is that conditioning on the differenced lags of the I(2) variate which are still I(1) reduces the order of integration in the residuals. Therefore, any conclusions about cointegrating structures between the level variates from the conditioned values tend to be wrong. Misspecifications of this type are an important problem in practice and could be responsible for the feature that some of the estimated β column vectors in this very investigation have turned out as being unable to cointegrate the corresponding level variates (compare Section 5.1).

With economic sub-annual data, another phenomenon seems somehow more important. Most of these series contain seasonal cycles that contradict the stationarity assumption of time-constant means. As set out in the introduction, the correct method to handle seasonal patterns is subject to ongoing discussions. If seasonal dummies are introduced, these can be inserted into the system as well as

4 The notation Γ_i has different meaning in different formulae. If this is recognized, it should not confuse the reader too much.

into all regression (steps 1, 2, 5) in a straightforward manner

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \dots + \Gamma_{k-1} \Delta X_{t-k+1} + \alpha\beta' X_{t-k} + \Sigma\, a_i S_{it} + \epsilon_t$$

where summation runs over four or twelve dummy series, depending on the seasonal frequency. Johansen and Juselius (1989) have shown that the distribution of the LR statistic in this case slightly deviates from that in the original setting but is exactly the same as in the original model taking intercepts into account. Most economic series not only are well represented by integrated processes but also show non-negligible "drifts" and, therefore, this corrected distribution is the more relevant one for most empirical problems. A third and also slightly different distribution comes up if intercepts are imposed in the level regression (steps 2 and 5) only or sample averages are subtracted from the individual series before the analysis.

An alternative conception of seasonality has been suggested e.g. by Hylleberg, Engle, Granger, Yoo (1988). A model like

$$(1-B^4)X_t = (1-B)(1+B)(1+B^2)X_t = \epsilon_t$$

with white noise or general stationary errors ϵ_t is known to generate stochastic cycles of semi-annual and annual periodicity (for quarterly data). As unit roots are imposed, neither X_t nor its differences are stationary. Hylleberg et al. suggest a test on the factors for univariate series. According to the outcomes from that test, most economic time series (unless seasonally adjusted before analysis e.g. by Census X-11) contain seasonal unit roots at -1 and ±i. The basic model has been criticized frequently as it allows the seasonal pattern to change, in other words it allows "summer to become winter". This issue is less important if drift terms are included and actual series often exhibit substantial changes of their seasonal pattern.

A process demanding for filtering by $1-B^4$ in order to become stationary can be named a "seasonally integrated process" [5]. The naïve procedure to handle such a process consists in application

5 This expression does not have exactly equivalent meaning if it is used by different authors or in different articles.

of the filter $1+B+B^2+B^3$ which removes the cycles but leaves stochastic trends in the data. The so constructed series is integrated of order one and standard cointegration analysis can be applied to a vector consisting of these filtered series.

The loss in efficiency and information implied by this strategy parallels the loss by analyzing systems in differences instead of using cointegration analysis. Suppose the VAR system is written in its seasonal differences representation ($\Delta_4 = 1-B^4$)

$$\Delta_4 X_t = \Gamma_1 \Delta_4 X_{t-1} + \dots + \Gamma_{k-4} \Delta_4 X_{t-k+4} + \Sigma\, \Gamma_{k-i} X_{t-k+i} + \epsilon_t$$

with summation running over $i=0,\dots,3$. Strictly speaking, application of the seasonal filter is only allowed if $(1+B)(1+B^2)$ cancels from the lag polynomial $\Gamma_{k-3}+\Gamma_{k-2}B+\Gamma_{k-1}B^2+\Gamma_k B^3$. This amounts to imposing $3p^2$ restrictions on the general representation. This becomes more obvious from the decomposition used by Hylleberg et al. (1988)

$$\Delta_4 X_t = \Gamma_1 \Delta_4 X_{t-1} + \dots + \Gamma_{k-4} \Delta_4 X_{t-k+4} + \Sigma\, A_i Y_{it-k+3} + \epsilon_t$$

where summation runs over $i=1,\dots,4$. The Y_{it} $(i=1,\dots,4)$ are obtained from $(X_t, X_{t-1}, X_{t-2}, X_{t-3})$ via a one-to-one transformation by applying the filter factors $(1+B)(1+B^2)$, $(1-B)(1+B^2)$, $(1-B)(1+B)$ and $B(1-B)(1+B)$ to X_t. If $A_2 = A_3 = A_4 = 0$, the model immediately reduces to the first-order cointegration model of Section 2. A_1 then corresponds to the "impact matrix" $\alpha\beta'$.

An elegant solution to estimating the above model is not yet available. Following the lines by Tso (1981), estimates can be obtained, even taking the possibly reduced rank of the full matrix

$$[A_1\ A_2\ A_3\ A_4]$$

into account. Empirical experience tells that this matrix frequently has full rank as cointegrating and "seasonally cointegrating" [in the sense of Engle et al. (1989)] relationships are only found from the component matrices. Correlation between the Y_{it} should be low, however, and a decomposition of the individual A_i can be a useful exercise.

An empirical example is provided by a preliminary estimation of the Austrian macroeconomic system to be revisited in Section 5. It consists of 6 quarterly series on : real gross domestic product (Y), real consumption expenditures (C), real gross investment (I), real interest rate (deflated bond rate R), real goods exports (X), real wages (deflated per capita wages W). All series are first-order integrated - this has been corroborated by several tests, see Kunst and Neusser (1988) - and all but exports show seasonal patterns. The presumption that the β matrices obtained from decomposing A_1 and from the "naïve" approach are close to one another in the sense of the angle between the column spaces is corroborated empirically. Such a comparison between similar r-dimensional vector spaces can be performed by means of canonical correlations. If the spaces are close to one another, they share an r-1-dimensional subspace almost completely and therefore r-1 canonical correlations are 1. The smallest root then gives the cosine of the angle between the r^{th} axes.

Furthermore, the fifth unit vector, which corresponds to the exports series, is contained in the column spaces of the β decompositions of the other three matrices which shows once more that exports do not contain seasonal unit roots. Note that the β obtained by decomposing A_2 to A_4 more or less accurately describe vectors that transform the seasonally integrated series into processes where certain parts of the seasonality (either the "jump root" or the complex pair ±j) are absent. In our case, the fifth unit vector describes such a seasonal cointegration structure. As all A_i matrices seem to be of rank 3, two more seasonal cointegrating vectors can exist. One of them links the seasonality in Y to those in C and I, which allows for an easy interpretation since the remaining demand components usually do not exhibit seasonal patterns. A related analysis (there of U.S. energy series) has been given by Engle et al. (1989).

4. Explosive roots

Autoregressive processes with zeros on the wrong side of the unit disc are generally excluded from empirical analysis. One of the reasons is that their trajectories tend to follow exponentially increasing trends or cycles like those displayed in the Figure contained in section 5. These pictures are rarely found in actual data. On the other hand, within the limits of a finite sample, such an exponential growth can be a good representation of that sample as long as the explosive roots remain close to the unit circle.

Empirical findings of explosive behavior are not so rare if slight misspecifications are present. For example, if a drifting random walk is estimated as an AR(1) process without a drift term, the estimated coefficient will usually exceed one. Even more frequently, explosive roots are identified implicitly - without ever showing - in multivariate VAR systems. A full representation of a VAR system is obtained from its state space form

$$\begin{bmatrix} x_t \\ x_{t-1} \\ \cdots \\ x_{t-k+1} \end{bmatrix} = \begin{bmatrix} \pi_1 & \pi_2 & \cdots & \pi_{k-1} & \pi_k \\ I & 0 & \cdots & 0 & 0 \\ & & \cdots & & \\ 0 & 0 & \cdots & I & 0 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ x_{t-2} \\ \cdots \\ x_{t-k} \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ 0 \\ \cdot\cdot \\ 0 \end{bmatrix}$$

It is known from the literature [e.g. Hannan (1970)] that the eigenvalues of the square asymmetric matrix above are the inverse roots of the determinantal equation $|I-\pi_1 z-\ldots-\pi_k z^k| = 0$ which, in turn, give the stationary ($|z|>1$) and non-stationary ($|z|\leq 1$) modes of the VAR system. There are kp roots and some of them may be greater than one by pure chance or, more exactly, by the properties of small sample distributions.

In all preceding results, only the eigenvalues of the impact matrix were used. The publication by Velu, Wichern, Reinsel (1987) treats some of the difficulties in making conclusions on the state space modes from the impact matrix. An exact correspondence is provided only for the AR(1) case where it is straightforward. For higher-order processes, only conjectures are given.

While many explosive roots are left unnoticed - estimation procedures usually do not restrict parameters to lie in the stationarity area although that property is frequently needed to establish results on the estimation procedure - they almost necessarily show up in forecasting experiments. Unless the state of the system in the immediate pre-period of forecasting is exactly orthogonal to the eigenvector space corresponding to the explosive roots, the behavior implied by the inherently explosive nature of the system will dominate in the longer run. A joint attack by prediction experiments and eigenvalue calculation will detect all hidden explosiveness safely.

Sometimes, deletion of the first few or last few observations makes the VAR system explosive by shifting one of the roots over the boundary. This happens e.g. in the Austrian model 4 given in section 5. Even if this feature is not observed, unexpected roots close to the unit circle, though still on the right side, give warning that the whole system runs the risk of becoming unstable after moderate changes, e.g. updating or data revision.

Recently, Phillips (1988) conceded in a side remark the possibility of mildly explosive systems. Anyway, here these will be viewed as unwanted features stemming from small sample effects and misspecification, which point of view is certainly enhanced if a different treatment of "design specification parameters" like the implied modeling of deterministics makes them into well-behaved integrated VAR processes. Of course, the argument is valid that, if one accepts a root of 1 as a plausible choice, one should also take the possibility of roots in an ϵ-vicinity into account. Within the VAR systems of this paper, it should be kept in mind that all data has been logged and therefore an explosive root does not describe exponential growth in the original series but double exponential growth.

5. Empirical evidence

5.1. The Austrian system

The Austrian system already has been the subject of some empirical studies of the author [Kunst and Neusser(1988), Kunst(1988)] where its properties have been presented and analyzed in detail. So only the main results will be summarized in short.

The system consist of one output series (gross domestic product) and three demand aggregates (consumption, investment, goods exports), all in real terms. A deflated interest rate was added to represent the influence of the monetary system and real wages were also inserted. Neoclassical growth theory would imply stationary quotas of consumption to output and of investment to output, moreover proportionality between wages and output and a stationary real interest rate. This makes a system of 6 series of quarterly data, available from 1964 on. All series have been logged, except for the interest rate.

According to appropriate tests [see Hylleberg et al. (1988)] on the unit factors, four of the six series contain the factor $1-B^4$. Exports seem to contain 1-B but there is no evidence on seasonal unit roots, and for the interest rate the evidence is inconclusive. Experiments with the unfiltered interest rate were unsatisfactory, and so our main version performed the cointegration analysis on five series adjusted by the filter $(1+B)(1+B^2)$ and on the original export series.

The number of cointegrating vectors and the lag length depend on the treatment of the intercept. Not including any drift at all, 4 lags whiten the residuals (5 lags in the VAR). Adjusting the series to their sample means (a popular strategy in time series analysis) also gives 4(5) lags. In these cases, evidence prefers 2 cointegrating vectors but a third vector seems possible. If drifts are properly taken into account, the lag length is reduced to 3(4) and any evidence on a third vector disappears. This is not changed if exports are also subjected to pre-filtering. The vector spaces and roots are only moderately sensitive to these changes.

Replacing the pre-filtering by seasonal dummies in all regressions, the lag length is again reduced to 2(3) lags and now 3 cointegrating vectors are found again (1 % significance). According to the statistics, a fully stationary system is possible but we will rule out this possibility in the following. The error correction components implied by the versions using seasonal dummies do not look stationary even if corrected for deterministic seasonals. In particular, the component corresponding to the largest root, which is supposed to be the "most stationary" one, shows strong negative first-order correlation, an indication for both the inability of dummy extraction to remove seasonality and for the inability of the testing procedure to discriminate between stationary and non-stationary processes when the latter ones do not contain the unit root at 1.

Following the discussion in Section 4, the following models will be checked for stability of the roots of the state space form:

1. 5 series pre-filtered, 4(5) lags, 2 cointegrating vectors
2. 5 series pre-filtered, drift, 3(4) lags, 2 vectors
3. 5 series pre-filtered, adjusted to means, 4(5) lags, 2 vectors
4. 5 series pre-filtered, adjusted to means, 4(5) lags, 3 vectors
5. 6 series pre-filtered, drift, 3(4) lags, 2 vectors
6. dummies included, 2(3) lags, 3 vectors

The largest roots implied by the state space representation of these models are given in the following table:

Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
1.00 (*)	1.00 (*)	1.00 (*)	1.00 (*)	1.00 (*)	1.00 (*)
1.00 (*)	1.00 (*)	1.00 (*)	1.00 (*)	1.00 (*)	1.00 (*)
1.00 (*)	1.00 (*)	1.00 (*)	1.00 (*)	1.00 (*)	1.00 (*)
1.00 (*)	1.00 (*)	1.00 (*)	.98	1.00 (*)	.95
.98	-.92	.98	-.95	$.83\pm.36j$	$-.02\pm.93j$
-.95	$.90\pm.05j$	-.95	$.89\pm.13j$	$.89\pm.03j$	$.01\pm.79j$
$.82\pm.42j$	$.81\pm.32j$	$.86\pm.15j$	$-.67\pm.56j$	$-.56\pm.56j$	-.71
$-.53\pm.71j$	$.05\pm.78j$	$-.77\pm.55j$	$-.54\pm.67j$	$.59\pm.51j$	-.55

It is seen that, additional to the unit roots imposed by the identification procedure marked by asterisks, all models contain near-unit roots. In particular, Models 1, 3, 4 imply one positive root of .98 which disappears if eventual linear trends are properly accounted for by drift terms as in Models 2 and 5. One further root close to -1 is caused by different seasonal adjustment of individual series. It is even possible that the exports series itself follows semi-annual fluctuations as the corresponding eigenvector strongly relies on oscillating exports. The near-unit complex roots in the seasonal dummies model indicate that dummy extraction is insufficient to stationarize the relations. The eigenvector corresponding to the ±.93j root has its largest entries at the interest rate and investment variables. This could indicate that fluctuations in these variables are the "most stochastic" ones. It was already mentioned that Model 4 produced divergent cycles for shortened samples (e.g. 1964 to 1985). In stable cases, the system variables quickly approach a linear trend lines if drift terms are included but only slowly approach horizontal lines if they are excluded.

A further interesting phenomenon is illustrated by Figures 1 to 3. These are graphs of the autocorrelation function (ACF) estimates of each of the six components of $\beta_e'X$ where β_e simply extends the estimated β (the cointegrating vectors) by the remaining eigenvectors which correspond to the roots which are statistically zero. One would expect that the first two or three components are stationary and the remaining ones are integrated and, moreover, that the last component is the "most non-stationary". In all cases where seasonality has remained in the data, one would also expect to see seasonal cycles in the ACF of at least some of the components. Figures 1 and 2 which correspond to Models 2 and 6 show that the first expectation is not necessarily corroborated. In fact, the second component looks extremely non-stationary which raises severe doubts on whether the second β column really cointegrates. On the other hand, Figure 3 based on the fully de-seasonalized model 5 fulfils the expectations. Even though ex-post prediction experiments see Models 2 and 6 in the lead relative to Model 5, which might be oversmoothed in its turn, this kind of visual evidence can only be recommended in cointegration analysis. It has been pointed out in Sections 2 and 3 that, in the presence of ignored unit roots, the β vectors may cointegrate the conditioned residuals on which they were constructed but not the data series. In both Models 2 and 6, near-unit root seasonality has remained in the system, in model 2 due to the seasonal fluctuation in exports and in model 6 due to the stochastic seasonal patterns which cannot be extracted by dummy regression and this might have given rise to the counter-intuitive Figures 1 and 2.

5.2. The German system

The German system differs from the Austrian one with respect to the interest rate where we replaced the average yield of long-run emissions by an average bond rate and used total exports in stead of goods exports. Tourism plays a minor role in the Federal Republic of Germany as compared to Austria, and therefore this second modification should not have tremendous effects. Start time of the German quarterly series is 1960.

Again, all series except for exports show significant seasonal patterns. Here, the real interest rate is among the most obviously seasonal series because of the wild seasonal fluctuations in the implicit price deflator. Contrary to the Austrian case, seasonal dummies were unable to reduce lag order and a lag order of 4(5) was needed in order to obtain white noise errors. Inclusion of seasonal dummies together with seasonal differencing was also tried but again failed to decrease lag order.

The statistical cointegration tests according to Johansen and Juselius (1989) indicate two cointegrating vectors for most specifications (the word "specification" refers to the assumptions concerning lag order, drifts, and seasonal behavior). Interestingly, the statistically preferred specification of 4(5) lags and seasonal differencing with drift casts doubts on the existence of the second vector. Therefore, the following models have been submitted to the state space eigenvalue analysis:

1. 5 series pre-filtered, drift included, 4(5) lags, 2 cointegrating vectors
2. As in 1. but only 1 cointegrating vector
3. Seasonal dummies, 4(5) lags, 2 cointegrating vectors

None of the models resulted in explosive unit roots. In the following list the largest roots for each of the models are displayed:

Model 1	Model 2	Model 3
1.00 (*)	1.00 (*)	1.00 (*)
1.00 (*)	1.00 (*)	1.00 (*)
1.00 (*)	1.00 (*)	1.00 (*)
1.00 (*)	1.00 (*)	1.00 (*)
-.96	1.00 (*)	±.97j
.96	-.96	-.97
.88±.26j	.95	.04±.89j
.78±.42j	.86±27j	-.88±.05j

It is seen that the dynamics implied by model 3 are less satisfactory. Three seasonal roots are clearly there, quite close to the values -1,±j implied by seasonal differencing. This means that seasonal dummies are unable to extract the seasonal structure. The corresponding eigenvectors show that the real interest rate is the principal source of the possible oscillations. Model 1 and 2 have remarkably similar roots, remarkable insofar as the additional root of 1 imposed by Model 2 does not reduce the amount of remaining near-unit roots. Complex roots and consequent four-quarters oscillations are not found but one of the roots is close to the "jump root" of -1 implying semi-annual cycles. These seem to stem from the export series which, unlike the Austrian one, exhibits certain signs of seasonality.

These findings are corroborated by prediction experiments. Cycles in exports and interest rate are quite persistent but usually decay after around 20 steps. Some artificial states generate negative trends in investment but, on the whole, a long-run growth in investment of 1.6 % and in consumption and total output of around 3 % is projected.

5.3: The Danish system

The Danish data starts at 1974, with the beginning of quarterly national accounting in Denmark. This restricts the number of usable observations to around 50, which makes statistical decisions based on asymptotic results extremely difficult. Note that the tables provided by Johansen and Juselius (1989) demand for a value of around .5 for the third or fourth root (counted from the smallest) to warrant a decision with a significance of 5 % on whether or not the fourth or third cointegrating vectors are present. This value, however, is already the ideal value representing the correlation between levels and differences of a white noise process. In other words, the procedure is unable to discriminate white noise from a random walk. Consequently, the absolute size of the roots should be rather used as a guideline. Similar caveats apply to the portmanteau statistics of residual correlation.

If seasonal dummies are used, 1(2) lags already suffice to whiten the residual series, according to the Q-statistic by Ljung and Box. Two cointegrating vectors are found but this decision is extremely sensitive to specification changes and therefore a model with 3 vectors is also investigated. Among these vectors we find a relation between output and the demand components, with almost equal coefficients with respect to I and X and with a slightly higher one with respect to C. This vector is also found in most cointegrating spaces in the German and in the Austrian model. A second vector seems to involve the consumption quota and the interest rate whereas wages do not play a role in cointegration.

If seasonal adjustment is performed via moving average filtering, 2 to 3 lags are needed to whiten the residuals in the preliminary regressions of differenced data on lagged differences. Because of the small sample, residuals sometimes show substantial autocorrelation if error correction terms are included (this is contrary to all experience from the Austrian and the German system and a tentative explanation is given in the concluding section). If the asymptotic tables are used, 2 lags generate a stationary (!) system with 6 cointegrating vectors. 3 lags entail 4 vectors but the resulting model does not look satisfactory. Below the

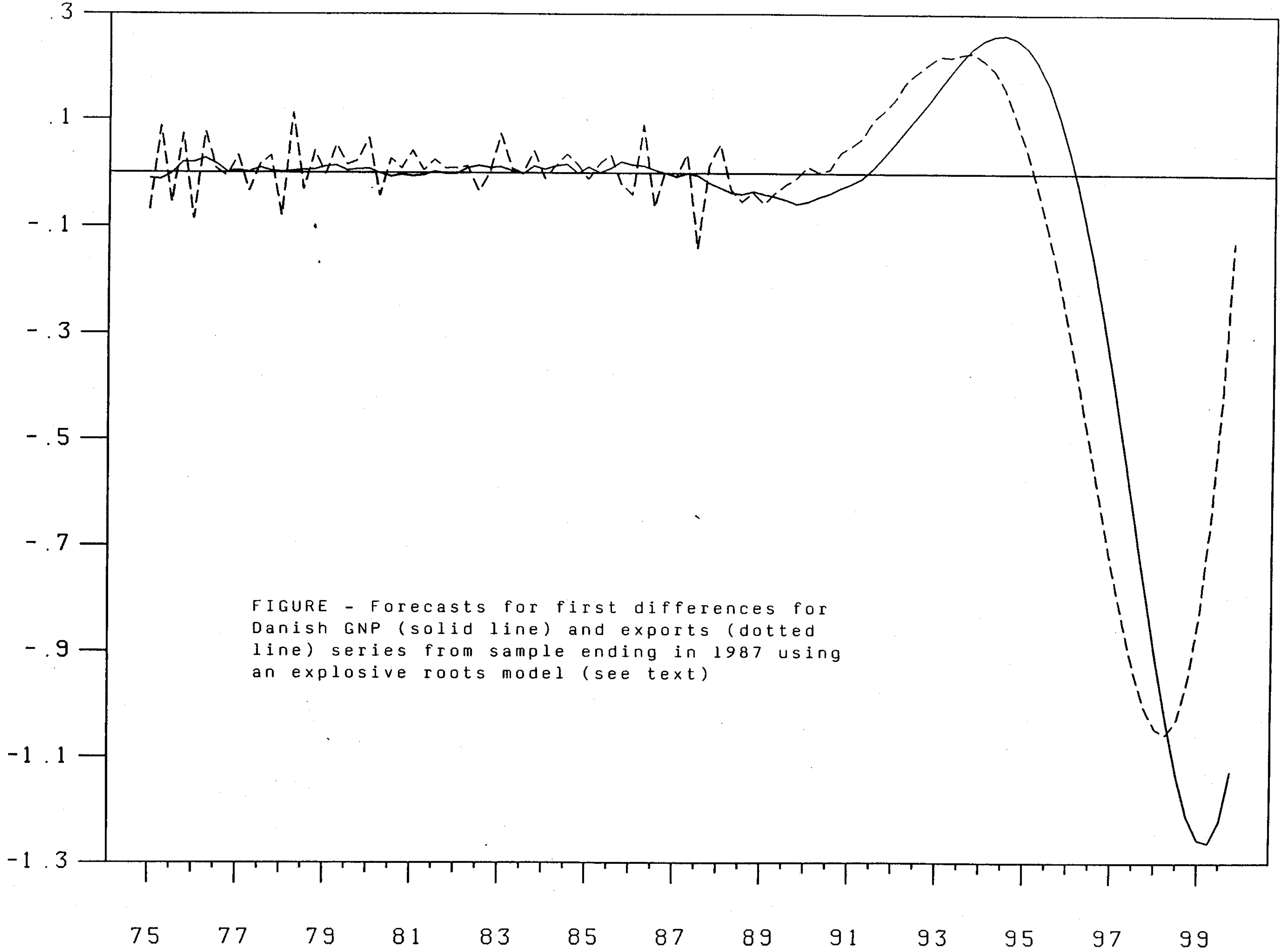


FIGURE - Forecasts for first differences for Danish GNP (solid line) and exports (dotted line) series from sample ending in 1987 using an explosive roots model (see text) largest roots of the state representation of three selected models are given: first, dummies with 2 vectors; second, dummies with 3 vectors; third, pre-filtering with 4 vectors:

Model 1	Model 2	Model 3
1.00 (*)	1.00 (*)	$1.07\pm.18j$
1.00 (*)	1.00 (*)	1.00 (*)
1.00 (*)	1.00 (*)	1.00 (*)
1.00 (*)	$-.12\pm.68j$	$.51\pm.79j$
$-.14\pm.69j$	.59	$.67\pm.64j$
$-.54\pm.32j$	$-.51\pm.29j$	$-.53\pm.76j$

Model 3 obviously produces explosive behavior. The corresponding state vector describes four successive quarters of expansion with particularly high investment, rapidly falling interest rate and low wages. Contrary to Austrian and German evidence, seasonal dummies seem to be a better description of seasonal patterns in the data. Model 1 and 2 show similar roots, with model 1 obviously restricting a root of .6 to unity which would recommend the use of Model 2. Anyway, the small sample caveats should be kept in mind.

To illustrate explosive behavior, a forecast from Model 3 for the (presumably stationary) series ΔY and ΔX (i.e. the first differences of the output and the exports series) until the end of the century is displayed graphically. The years 1988-1991 show unusual behavior - relative to past performance - but do not obviously contradict stationarity. After those years, prediction degenerates into nonsensical cycles with increasing amplitude and an uncertain frequency which should correspond to the detected pair of complex roots.

5.4. The British system

For the British system, six series equivalent to the three other countries were used. As for the Federal Republic of Germany, exports included goods as well as service exports. Contrary to the other countries, the real interest shows no seasonal patterns. The "not seasonally adjusted" real series accounts published by the Central Statistical Office differ from the respective nominal series by non-seasonal deflators. And the GDP deflator has been used for all countries to deflate the non-seasonal nominal interest rate. Quarterly series used here start in 1963.

According to statistical tests on the factor $1-B^4$, all series except interest rate and exports show significant seasonality. Since data clearly exhibit drifting behavior, the following three specifications were used for further investigation:

1. Dummies included, 5(6) lags, 2 cointegrating vectors.
2. All series filtered by $(1+B)(1+B^2)$, 5(6) lags, 2 cointegrating vectors.
3. Four series filtered by $(1+B)(1+B^2)$, 5(6) lags, 2 cointegrating vectors.

Note the long lags enforced on the system by the consumption series. According to some integration tests, British private consumption could be I(2) but we shall not accept this possibility here. The following table shows the largest roots implied by the three specifications.

Model 1	Model 2	Model 3	
1.00 (*)	1.00 (*)	1.00 (*)	(4 times)
$-.06 \pm .90j$	$.92 \pm .23j$	$.92 \pm .22j$	
$-.89 \pm .06j$	$.81 \pm .49j$	$.77 \pm .48j$	
$.71 \pm .46j$	$-.62 \pm .67j$	$-.64 \pm .65j$	
$.47 \pm .70j$	$.59 \pm .69j$	$-.89$	

Taking some summary statistics and forecasting performance into account, Model 3 seems to be the best model of the three. Contrary to Austrian and German evidence, both seasonal adjustment methods generate qualitatively similar results as none of them shifts seasonal roots too close to the unit circle.

The three above models tacitly assume two cointegrating vectors. Evidence on the second vector is, however, not clear and it is possible that only one cointegrating vector is present in the British system. The real interest rate already is "almost" stationary by itself and much more so if linked to some of the other variables. A link between wages and consumption and the accounts growth vector already encountered in the other systems are also candidates for cointegrating relations.

TABLE 1: First two β column vectors (cointegrating vectors) for selected country models

	Y	C	I	R	X	W[6]
Austria (model 2)						
	240.16	-54.44	-67.61	-3.98	-59.63	16.65
	-6.37	-56.29	3.69	70.09	18.83	12.93
Denmark (model 1)						
	-54.86	58.60	-3.39	.34	14.68	1.96
	-127.35	32.06	14.74	-.14	20.31	2.75
Germany, Federal Republic (model 1)						
	68.15	-14.72	-12.22	.88	-19.45	-.22
	172.79	-90.55	-26.24	-1.10	-33.22	11.32
United Kingdom (model 3)						
	-27.86	-45.40	24.64	.38	13.99	25.20
	135.71	-122.50	-34.24	.16	-35.38	88.82

6 The numbers beneath the labels denote the coefficients of: gross national (or domestic) product; private consumption; gross fixed investment; real interest rate; exports; real wages.

TABLE 2: Squared canonical correlations of 2-vector spaces corresponding to the largest eigenvalues [7]

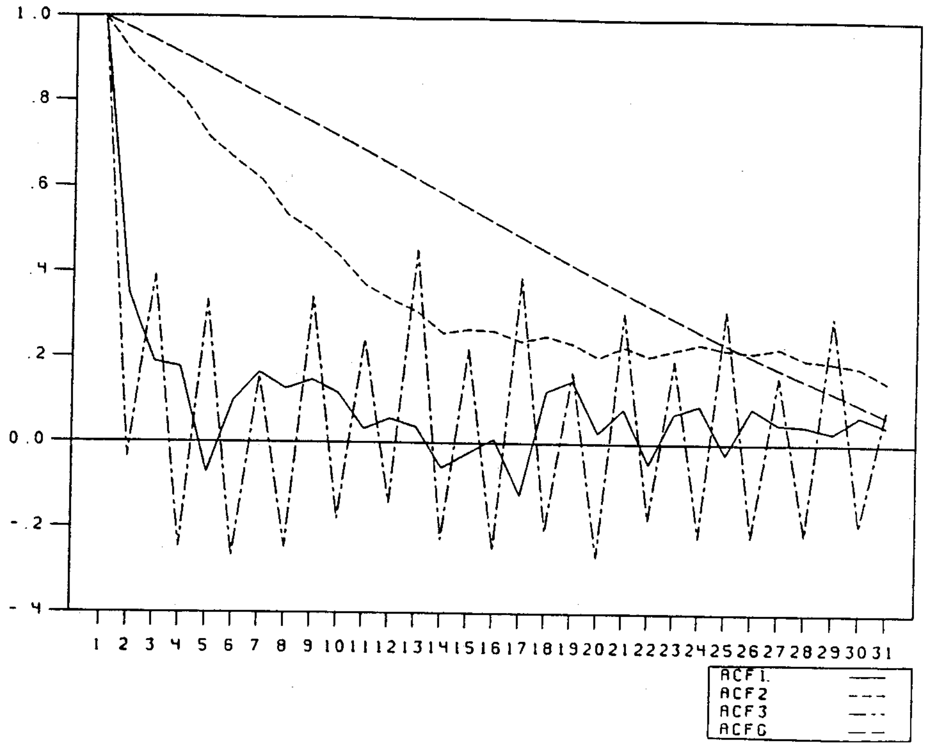
	D	DK	UK
A	.986	.966	.965
	.373	.321	.369
D		.990	.929
		.760	.841
DK			.892
			.684

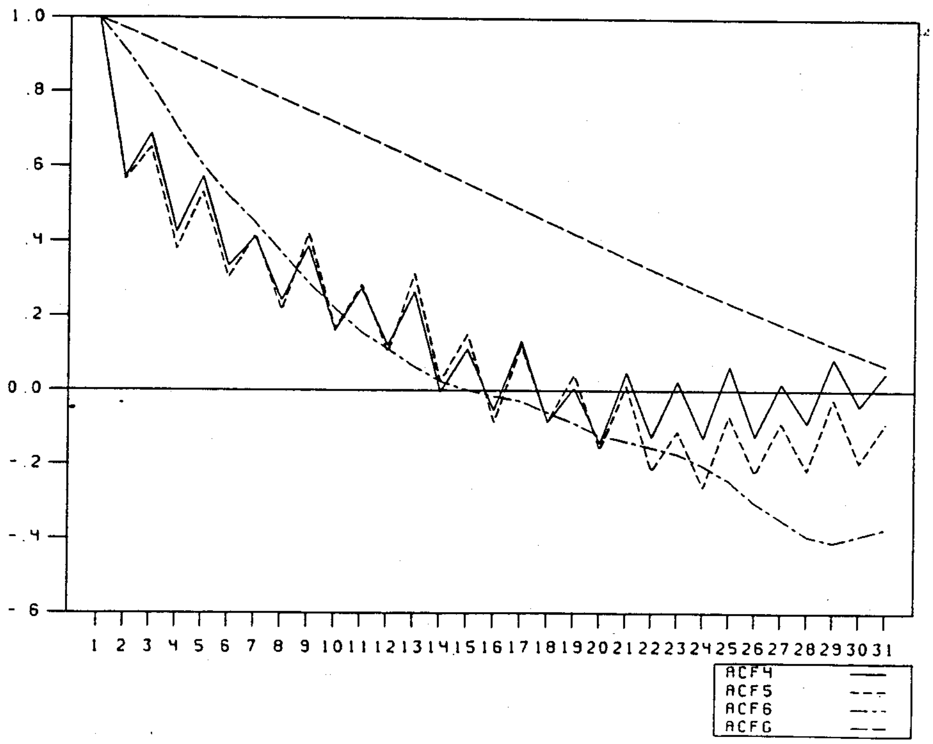
7 Country labels are defined as follows: A = Austria, D = Federal Republic of Germany, DK = Denmark, UK = United Kingdom of Great Britain

TABLE 3: Squared canonical correlations of estimated spaces and prescribed spaces [$Y-\alpha C-\alpha I-\alpha X$, v_2] where $\alpha=1/3$

v_2	C-Y		I-Y		W-Y		R	
A	.985	.354	.985	.039	.985	.048	.985	.567
D	.994	.825	.985	.104	.981	.105	.979	.004
DK	.986	.652	.957	.372	.956	.047	.943	.000
UK	.983	.654	.960	.244	.990	.298	.946	.000

FIGURE 1: ACF of transformed components of Austrian model 2 (all series except X filtered by $1+B+B^2+B^3$) [a]

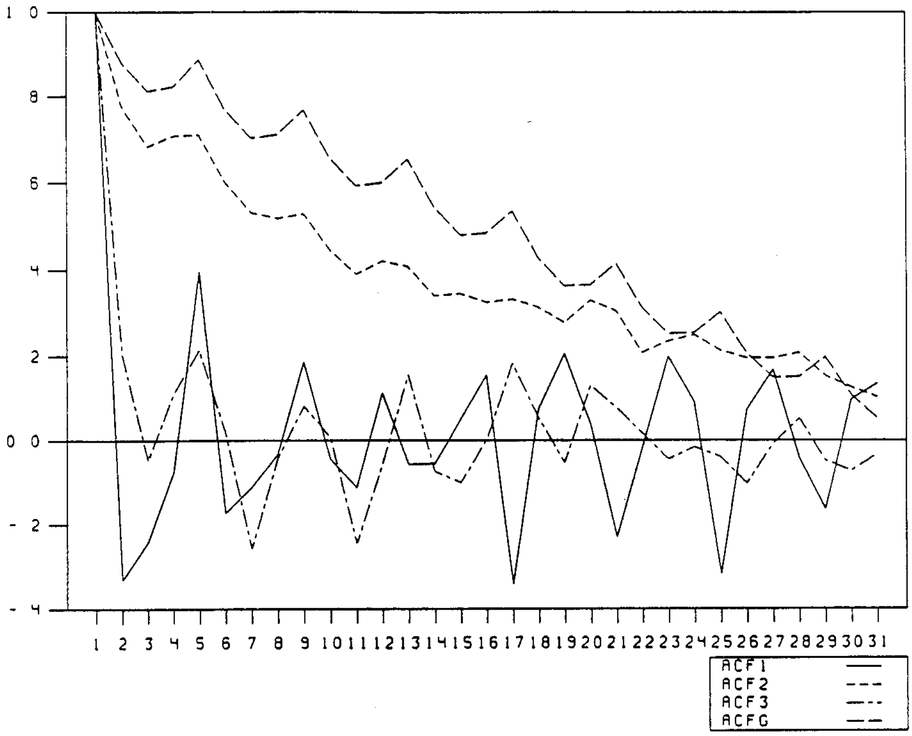


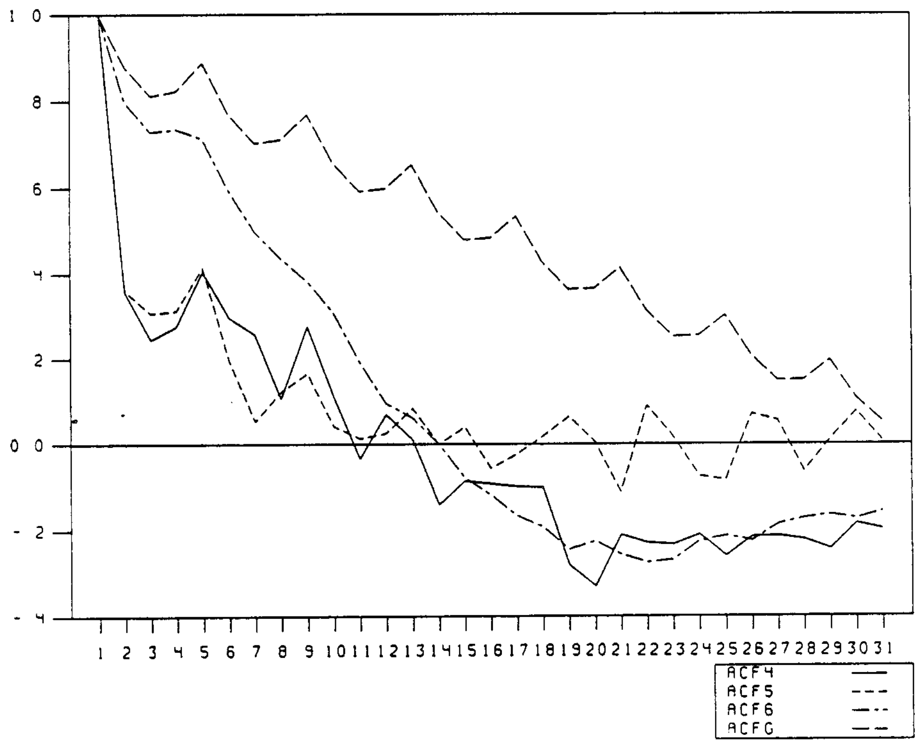


---

a ACFi corresponds to ith component. Long dashes depict ACF of output series as a means for comparison.

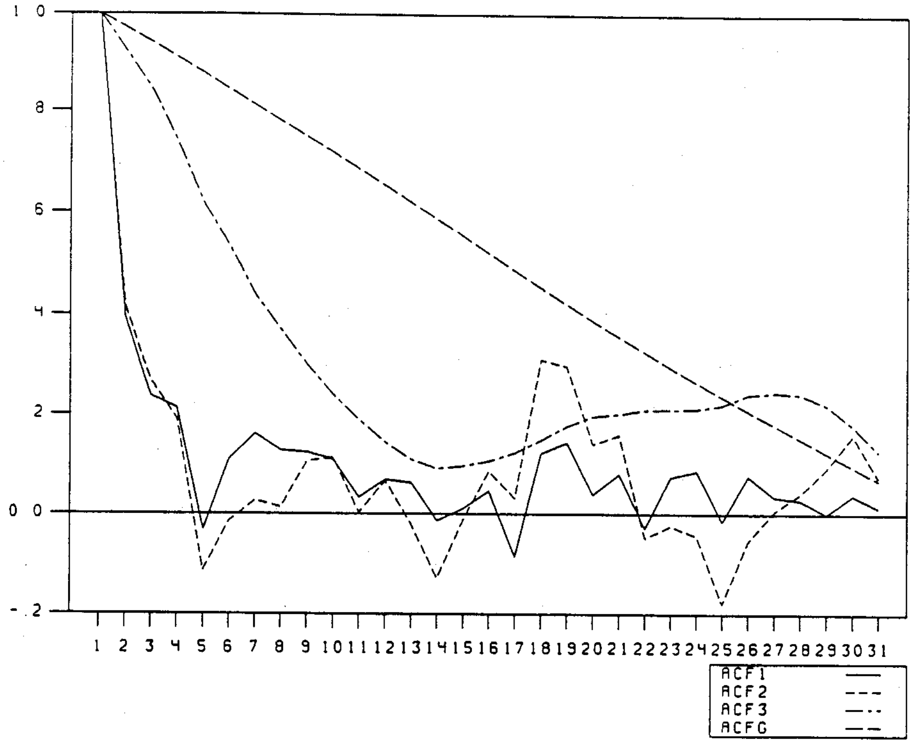
FIGURE 2: ACF of transformed components of Austrian model 6 (seasonal dummies used in all regressions) [a]

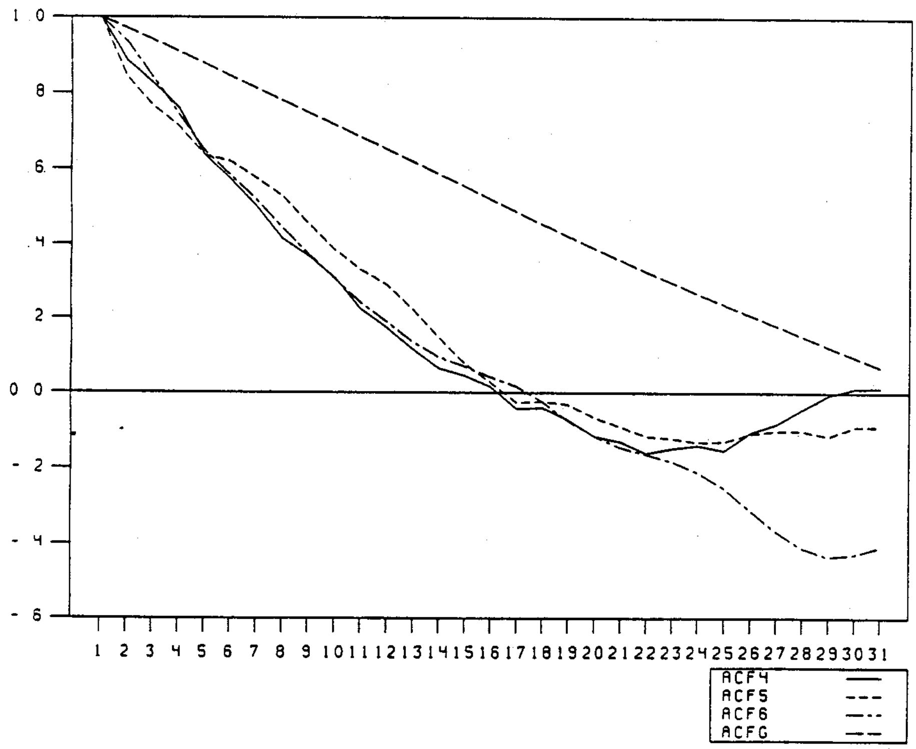




a ACFi corresponds to i[th] component. Long dashes depict ACF of output series as a means for comparison.

FIGURE 3: ACF of transformed components of Austrian model 5 (all series filtered by $1+B+B^2+B^3$) [a]





a ACFi corresponds to i[th] component. Long dashes depict ACF of output series as a means for comparison.

6. Summary and conclusions

In summary, findings of explosive roots have proved to be infrequent but still a non-negligible nuisance in empirical VAR systems. It is true that the displayed behavior has its sources in a thorough misspecification of the system but similar misspecifications could easily evolve in similar situations. The inclusion of four cointegrating vectors in the Danish system is based on asymptotic fractiles which are incorrect in a sample of 50. This misspecification is seen from the deteriorating error structure in the final models relative to the preliminary regression (step 5 relative to step 1 in section 2). The spurious cointegrating vectors produce integrated instead of stationary components which are added to the original differences and the whole procedure breaks down. Note, however, that in principle this mistake can be reduced to ignoring a rank restriction. Lots of rank restrictions on the short-run coefficient matrices Γ_i are usually not imposed.

The final answer on the correct treatment of seasonality cannot be given here. Two systems seem to prefer stochastic patterns though the Austrian one can be modeled more parsimoniously (but probably less correctly) by dummies. The Danish system not only prefers the deterministic model but stochastic modeling was a crucial factor in attaining explosive behavior. Different treatment of individual series is often suggested by statistics like those of Hylleberg et al. (1988) but it might destroy information about relations between original variables of the non-adjusted and the adjusted set. This "naive" procedure is responsible for some of the near-unit roots encountered. Estimation of the "full" model set out in section 3 is unbacked yet by statistical theory.

Apart from the explosive behavior encountered in two specifications only out of fifteen (and in one of them only for a reduced sample), there are more interesting findings evolving from the economic nature of our comparison of economies of different countries. A full analysis of these cannot be given at the moment (and would be beyond the scope of this paper) but it can be stated that the detected cointegrating relations are remarkably similar. To illustrate this remark, the spaces spanned by the first two columns in the four best individual country models were compared by calculating the mutual canonical correlations (see Table 2). Interestingly, the German and the British vectors are the "closest" ones. The Danish system is slightly more different whereas the Austrian two-dimensional system only shares one axis with the other countries in the sample.

These coincidences are reflected in the structure of the cointegrating relations. One (usually the one corresponding to the largest root) links output to consumption, investment, and exports in the form

$$Y = \alpha C + \beta I + \beta X$$

The equality restriction of the coefficients also seems to hold for all three economies, and generally α is greater than β. Kunst and Neusser (1988) used model 4 for the Austrian economy and were unable to reject a vector with $\alpha=\beta$. Their result could rely on a misspecification of drift terms.

The remaining relations (usually only one) are more difficult to interpret. It always contains the interest rate and links it to different variables. In Denmark, the consumption quota (C-Y) seems to complete the relation optimally, leaving wages outside of the cointegrating space. In the large-country economy of Germany, exports seem to play a key role in this second relation whereas in Austria wages and investment enter in a way that is difficult to interpret. For none of the countries, there is evidence on the "classical" growth assumptions of stationary consumption and investment quota to hold and only the UK system could allow for a stationary real interest rate. This may be due partly, however, to the manner of deflating the interest rate and to the income variable selected.

Just for the sake of an experiment, two-vector systems containing the output growth vector with $\alpha=\beta=1/3$ and one out of the four "theoretical" vectors assumed by neoclassical growth theory (consumption quota; investment quota; real wage to output ratio; real interest rate, i.e. fourth unit vector) were compared to all fifteen country models and mutual canonical correlations with the spaces given by the first two β columns were evaluated. The specified systems share at least one axis with all models which seems to be generated by the output growth vector. The system including the stationary real interest rate coincides best with the Austrian specifications. For the three other countries, inclusion of the consumption quota yields the best fit (compare Table 3 for selected results). Note, however, that all of these two-vector specifications are rejected by statistical tests as suggested by Johansen and Juselius (1989), though by iterating the coefficients in the output growth vector the null region could be possibly reached in some cases.

Acknowledgments

Credits to Sören Johansen for pointing out some key questions to me which are partly treated in the paper. Credits to both Sören Johansen and Katarina Juselius for helpful comments on my work and for the Danish data. Credits to Klaus Neusser for the German data and the basic selection of the economic variables. All errors are mine.

Literature:

Engle, R.F. and C.W.J. Granger (1987): Co-integration and error correction : Representation, estimation and testing; Econometrica 55, 251-276.

Engle, R.F., C.W.J. Granger and J.J. Hallman (1989): Merging Short- and Long-Run Forecasts: An Application of Seasonal Cointegration to Monthly Electricity Sales Forecasting; Journal of Econometrics 40, 45-62.

Engle, R.F. and B.S. Yoo (1987): Forecasting and Testing in Co-Integrated Systems; Journal of Econometrics 35, 143-159.

Hannan, E.J. (1970): Multiple Time Series; New York, Wiley.

Hylleberg, S., R.F. Engle, C.W.J. Granger and B.S. Yoo (1988): Seasonal Integration and Cointegration; University of California, San Diego, Discussion Paper.

Johansen, S. (1988): Statistical analysis of cointegration vectors; Journal of Economic Dynamics and Control 12,2/3,231-254.

Johansen, S. and K. Juselius (1988): Hypothesis Testing for Cointegration Vectors with an Application to the Demand for Money in Denmark and Finland; Preprint 1988/2, Institute of Mathematical Statistics, University of Copenhagen.

Johansen, S. and K. Juselius (1989): Hypothesis Testing for Cointegration Vectors for Gaussian Vector Autoregressive Processes with Constant and Seasonal Dummies; Working paper, University of Copenhagen.

Kunst, R.M. (1988): Cointegration in a Macro-Economic System: Sensitivity and Prediction; paper presented at the Basel Cointegration Day, January 1989.

Kunst, R.M. and K. Neusser (1988): Cointegration in a Macro-Economic System; Discussion paper, University of Vienna.

Kunst, R.M. (1989): A Note on the Influence of Order Selection on Correlation Analysis in Autoregressive Processes; mimeo, Institute for Advanced Studies, Vienna.

Phillips, P.C.B. (1988): Regression Theory for Near-Integrated Time Series; Econometrica 56, 1021-1044.

Tso, M.K.-S. (1981): Reduced-rank Regression and Canonical Analysis; Journal of the Royal Statistical Society B 43, 183-189.

Velu, R.P., D.W. Wichern and G.C. Reinsel (1987): A Note on Non-Stationarity and Canonical Analysis of Multiple Time Series Models; Journal of Time Series Analysis 8, 479-487.