# COINTEGRATION IN A MACRO-ECONOMIC SYSTEM[1]

Robert M. KUNST

Forschungsbericht/
Research Memorandum Nr. 248
July 1988

1 This study makes part of a more extended working program of the author together with Klaus Neusser, University of Vienna.

## Zusammenfassung

Die vorliegende Arbeit versteht sich als explorative Studie, die ein System, welches aus sechs Haupt-Kenngrößen der österreichischen Volkswirtschaft besteht (BIP, privater Konsum, Investitionen, BIP-Deflator, Sekundärmarktzins, Lohnsatz), aufgreift und vektor-autoregressiv modelliert. Besonderes Augenmerk wird dabei dem Phänomen der Kointegration gewidmet, also der Bildung stationärer Variablen durch Linearkombination mehrerer instationärer (trendender) Variablen. Mittels der Prozedur von Johansen (1987) werden drei solcher Beziehungen identifiziert. Auch ein Versuch der ökonomischen Interpretation wird unternommen. Breiten Raum nimmt eine in dieser Form neuartige Sensitivitätsanalyse ein, die vor allem die Technik der kanonischen Korrelation zu Hilfe nimmt.

Eine kritische Bewertung der durch die Modellierung implizierten Prognosen (bis 1999) ergibt, daß ein gewisses Mißtrauen bezüglich der vorher identifizierten kointegrierenden Struktur angebracht ist bzw. daß höchstens eine oder zwei Beziehungen prognostisch - gegenüber der herkömmlichen Modellierung, die Kointegration überhaupt nicht berücksichtigt - effizienzsteigernd wirken könnten, während das volle kointegrierende System "excess stationarity" impliziert, die der Prognosebewertung nicht standhält.

## Abstract

This paper represents an exploratory study that investigates the vector autoregressive properties of a time series system containing six main indicators of the Austrian economy (gross domestic product GDP, private consumption, investment, GDP deflator, interest rate, wages). Interest focuses on cointegrating structures in the system, i.e. on linear combinations of some (trending) variates that generate stationary series. Johansen's (1987) procedure identifies three relations of this kind. It is also tried to give an economic interpretation to these relations. More extensively, a variety of sensitivity experiments are performed which make somehow original use of the technique of canonical correlations.

Contrary to the sensitivity experiments, a critical assessment of the identified structure by medium-term forecasting (until 1999) sheds considerable doubt on its correctness. At most one or two cointegrating relations may help to increase forecasting precision relative to traditional vector autoregression which do not use the idea of cointegration at all. On the other hand, using all three identified relations seems to impose "excess stationarity" on the system which is not replicated by actual data behavior.

CONTENTS

0. Introduction .......................................... 1

1. Procedures .............................................. 4

1.1 Estimating the cointegrated model according to Johansen

1.2 Canonical analysis according to Box and Tiao

2. Preliminary steps ....................................... 9

2.1 Data series

2.2 Order of integration

2.3 Seasonal adjustment

3. Sensitivity of the results .............................. 15

3.1 Sensitivity against seasonal adjustment

3.2 Sensitivity against lag order

3.3 Temporal sensitivity

3.4 Sensitivity against the Box/Tiao canonical analyis

4. Economic interpretation ................................. 19

5. Forecasting ............................................. 23

6. Tentative conclusions ................................... 25

Tables ..................................................... 26

References ................................................. 35

Figures

## 0. Introduction

Multivariate and, especially, vector autoregressive (VAR) time series models are now well on the way to establish themselves as an alternative to structural economic models. The principal problem of multivariate time series models, the inflation of parameters which restricts the possible dimensionality of the system, has already gained widespread attention and led to some interesting procedures to solve it. Only within the past few years, another aspect has shifted into the focus of research which is related to the inherent non-stationary nature of many economic variables. More specifically, most economic time series are well represented univariately as stationary after taking first differences, i.e. as "integrated of order one". To save the applicability of standard asymptotic econometric theory, it looks tempting to take first differences first and estimate the unrestricted vector autoregressive model for the transformed series. However, it can be shown that such models are only efficient in the special case that all linear combinations of the (trending) variables are also trending (i.e. integrated). For all other cases, the vector moving average representation of the first differences is not invertible. Therefore, a vector autoregressive representation does not even exist. The instrument to handle these difficulties is provided by the theory of cointegration introduced by Engle and Granger (1987).

Fundamental properties and definitions involved with cointegrated time series are well summarized in the seminal paper by Engle and Granger (1987) and therefore the problem will be explained only in short here. If there is a linear combination of the (integrated) variables which is (covariance-)stationary, the variables are said to be "cointegrated" (of order one-one) and the coefficients of the linear combination form a "cointegrating vector". Not only that cointegration is frequently observed in interdependent economic systems, economic theory suggests it to happen for specific sets of time series and provides an interpretation for the cointegrating vectors (see e.g. Campbell and Shiller (1987) and Juselius (1987)). The question whether cointegration may be viewed as a "generic" or "non-generic" event is unsettled. For two arbitrary integrated series, a stationary linear combination seems an unusual feature. However, in any non-stationary autoregressive system of m individually integrated series, the generic event would be exactly m-1 cointegrating vectors. Any other case (excluding the jointly stationary system) can be shown to correspond to imposing a rank deficiency on a matrix, which certainly is non-generic.[1]

This paper is explorative in nature. It takes up a set of six time series on Austria which describes a country more or less adequately. The set is quite similar to a core set for the Austrian economy which already has been exploited successfully for VAR forecasting both by models in levels and in differences (see Kunst and Neusser(1986)). Based on this data set, we shall try to answer the following questions: Are there cointegrating vectors in the system ? If yes, how many ? Can they be given an economic interpretation ?

First, the univariate properties of the time series are examined to see whether they are integrated of order one - a prerequisite for the analysis of cointegration. This is done by the tests introduced by Dickey and Fuller (1979) and Stock and Watson (1986). The tests are applied to seasonal series as well as to series adjusted by an MA filter that corresponds to seasonal differences. Seasonal differencing of some of the series is justified by additional tests.

While for any two series cointegration analysis may rely on straightforward least-squares regressions, in higher-dimensional systems it is a more difficult task. Several methods have been proposed in the literature to deal with this issue (Stock and Watson (1986), Phillips and Ouliaris (1987)). In this paper we explore the method developed by Johansen (1987) which, unlike previous approaches, provides a unified framework for testing and

1 Compare the description of Johansen's procedure in Section 1.1

estimation. We contrast this approach with one proposed by Box and Tiao (1977) which relies on "almost non-stationary" time series.

According to the author's knowledge, this study is the first one which (see Section 3) directly investigates into the sensitivity of the results with respect to several slight modifications of the original exercise: the share of de-seasonalized series in the data set; a change in the overall AR lag order hypothesized for the level series; the "classic" experiment of changing the sample length; finally, the similarities between the results from cointegration and from Box/Tiao canonical analyis. The interpretative value of the results and the applicability for forecasting purposes critically depends on these robustness features.

Section 4 tentatively tries to give the identified cointegrating relations an economic meaning, whereas section 5 performs selected forecasting experiments based on the estimated systems. The cointegrating dimension is changed between zero (the model in differences) and the estimated value of three.

## 1. Procedures

### 1.1. Estimating the cointegrated model according to Johansen

The question concerning the existence of cointegrating relationships in a multivariate system may be investigated into by several procedures, e.g. Stock and Watson (1986), Phillips and Ouliaris (1987), Johansen (1987). The former two articles are concerned solely with the problem of performing tests about the number of relationships, whereas Johansen's paper provides a unified approach for the testing problem as well as for the estimation of the cointegrating vectors or, rather, of an orthogonal basis for the r-dimensional cointegrating space if r denotes the number of independent relationships identified in the testing stage. This paper will be concerned primarily with Johansen's approach.

Johansen (1987) starts from a vector-autoregressive process of order k+1 and dimension p

$$(1.1) \qquad X_t = \sum_{i=1}^{k+1} \pi_i X_{t-i} + e_t$$

with non-singular (not necessarily diagonal) covariance matrix $\Omega$. The process may be re-parameterized as a first-difference process

$$(1.2) \qquad \Delta X_t = \Sigma\, \Gamma_i \Delta X_{t-i} + \Gamma_{k+1} X_{t-k-1} + e_t \qquad (\Gamma_i = -I+\pi_1+\ldots+\pi_i)$$

If $\Gamma_{k+1}$ is zero, $(X_t)$ is autoregressive as a first-difference process. This coincides with the case of $X_t$ depending on p independent trends in the terminology of Stock and Watson (1986). On the other hand, if $\Gamma_{k+1}$ has full rank, $X_t$ is stationary. In all other cases, $\Gamma_{k+1}$ has rank r between 0 and p, i.e. it may be represented as

$$(1.3) \qquad -\Gamma_{k+1} = \alpha'\beta \quad (= I-\pi_1-\ldots-\pi_{k+1})$$

with $\alpha$ and $\beta$ dimensioned as $r \times p$ and of full rank $r$. The sign convention was chosen to recover the "impact matrix" in parentheses. The matrix of cointegrating vectors $\beta$ is not unique but the space spanned by its column vectors is and may be estimated together with the remaining parameters in the problem by the method of maximum likelihood assuming normally distributed errors. According to Johansen's derivation, this may be done by using moments and cross-moments matrices of the residuals from auxiliary regressions of $\Delta X$ on the lagged differences

$$(1.4) \qquad \Delta X_t = \sum_{i=1}^{k} a_i \Delta X_{t-i} + r_{0t}$$

and of $X_{t-k-1}$ on the same regressors

$$(1.5) \qquad X_{t-k-1} = \sum_{i=1}^{k} a_i \Delta X_{t-i} + r_{kt}$$

The moment matrices are called $S_{00}$, $S_{0k}$, $S_{kk}$. Now, the eigenvectors of

$$(1.6) \qquad S_{kk}^{-1} S_{0k}' S_{00}^{-1} S_{0k}$$

corresponding to the largest $r$ eigenvalues give the required $\beta$ basis and

$$(1.7) \qquad \alpha = -S_{0k}\beta$$

solves the corresponding $\alpha$ problem. Finally, the full model is given by

$$(1.8) \qquad \Delta X_t + \alpha'\beta X_{t-k-1} = \sum_{i=1}^{k} \Gamma_i \Delta X_{t-i}$$

from which regression the remaining $\Gamma_i$ may be estimated. While these calculations are rather straightforward, two problems will arise in macro-economic practice which have not been covered fully by present theory. First, seasonality remaining in the data might cause roots on the unit circle different from one which could impair or even destroy the procedure (see section 2.3). Second, the analysis is founded on polynomial matrices of order k while the optimal order k may differ between elements rendering the procedure inefficient. Moreover, multivariate MA terms may play a role but vector ARMA modeling does not look a promising task.

The number r which gives the number of cointegrating vectors or, likewise, the smaller dimension of $\alpha$ and $\beta$, is to be determined by a likelihood-ratio test of the null hypothesis of "at most r cointegrating vectors". The statistic to be used is

$$(1.9) \qquad -2 \log Q = -T \sum_{i=r+1}^{p} \log(1-\tau_i)$$

comprising the p-r smallest eigenvalues $\tau_i$ of the above problem. The asymptotic distribution of (1.9) is tractable but complicated. Selected fractiles are given in Johansen(1987).

## 1.2. Canonical analysis according to Box and Tiao

It seems worth contrasting Johansen's method with an older technique based on canonical correlation analysis. The concept of canonical correlations was introduced by Hotelling (1936) and applied to the vector-autoregressive setting by Box and Tiao (1977). Similar to Johansen's method, but relying on stationary theory with near-unit roots (sometimes called "almost non-stationary" processes ANS, see Tjøstheim and Paulsen (1982)), their procedure identifies orthogonal vectors which transform the time series at hand into stationary (far-from-unit-root) series.

As stated above, Box and Tiao's method relies on the assumption of jointly stationary processes which, however, includes almost non-stationary cases. It is well known that discrimination between highly dependent stationary processes and pure random walks is impossible at conventional significance levels for smaller samples like the one under investigation (e.g. compare the simulation results in Stock and Watson(1986) and section 2 of this paper). Conversely, the system which provided an example in Box and Tiao(1977) probably would be judged to be co-integrated by today's standards.

Given a correct forecast $Z_t$ for the vector $X_t$ from its past, ideally the conditional expectation, the eigenvalues and eigenvectors of the matrix

$$(X'X)^{-1}(Z'Z) \tag{1.10}$$

provide special information on the dynamic structure. In the univariate case, it is obvious that (1.10) is restricted to the interval [0,1] and that values close to one represent processes whose innovations variance is negligible as compared to the process variance. This condition is fulfilled for the near-unit root cases. Conversely, values close to zero represent random processes of low temporal dependence approaching white noise. In the multivariate case, the eigenvalues are restricted to the same interval. Additionally, the corresponding eigenvectors may be used to transform the vector $(X_t)$ into a component vector $(X_t^*)$ consisting of recursively dependent components corresponding to the respective location of the eigenvalues.

Although Box and Tiao only used the case of first-order autoregressive forecasts the method may be extended to higher-order processes. However, if the lag order is increased, all eigenvalues are taken towards one since no AIC-like term is involved impeding parameter inflation relative to prediction accuracy. An alternative way of generalization was followed by Velu et al.(1987) sticking to the interpretation of the AR(1) case as looking for the canonical correlations between $X_t$ and $X_{t-1}$. Consequently, they look for the canonical correlations between $X_t$ and $X_{t-p}$ in the AR(p) model, adjusted for $X_{t-1},\ldots,X_{t-p+1}$. Since these generalizations are not yet fully covered by theory, we shall restrict ourselves to the first-order case here.

What makes Box and Tiao's procedure interesting in the context of co-integration is that any component within $(X_t^*)$ corresponding to an eigenvalue far from unity, that is, a stationary component, necessarily gives a co-integrating relationship and makes the respective eigenvector a co-integrating vector. Thus, the Box-Tiao eigenvectors may be compared to the Johansen eigenvectors although, of course, only the latter ones are derived from genuine co-integration theory.

# 2. Preliminary steps

## 2.1. Data series

Taking the exposition in the previous sections into account, the following data set of quarterly Austrian time series (1964-1987) is investigated:

| | |
|---|---|
| Y | real gross domestic product |
| C | real private consumption |
| I | real gross fixed investment |
| P | deflator of GDP |
| R | interest rate on the secondary bond market |
| W | nominal gross wages per employee |

All series except R are used in logs to stabilize variances. Of course, this step could weaken dependencies which rather exist between original series - compare the definitional identity which connects output, investment, consumption and a remainder term which comprises public consumption, inventory changes, and exports minus imports - but this could not justify the use of data with their variance increasing over time. Note that the assumptions of stationary level differences and stationary log differences are mutually exclusive. Evidence seems to prefer the latter hypothesis.

## 2.2. Order of integration

A prerequisite for cointegration analysis is that all series of the system under investigation be integrated of order one, i.e. that their individual AR representations contain only roots outside the unit circle and exactly one root of unity. This implies that none of the series should be stationary or integrated of higher order.

To corroborate this assumption statistically, integration tests as introduced by Stock and Watson (1986), Dickey and Fuller (1979), and an integration test implied by the cointegration analysis of Johansen (1987) were performed on levels and differences of the original series as well as to levels and differences of series transformed by the seasonal moving average (SMA) filter $\frac{1}{4}(1+L+L^2+L^3)^2$. The SMA filter assures that no seasonal unit roots at -1, ±i enter into the investigation (see Section 2.3). In the following, data series adjusted by SMA will be labeled YMA, CMA, IMA, PMA, RMA, WMA whenever confusion could arise.

The power of the tests critically relies on the number of autoregressive corrections which should be equivalent to the true AR order generating the series. For the tests by Dickey and Fuller (labeled ADF) and by Stock and Watson (labeled SW), an order was selected which rendered the Ljung-Box Q statistic of the residuals insignificant at 10 %. For the Johansen test (J), it was set to four to ensure maximal compatibility with the Johansen (1987) cointegration analysis of Section 3 where a lag order of four was motivated by the above Q criterion on the multivariate differences model. SW is known to be more robust than ADF with respect to lag orders. Since an analysis of all series via the extended ACF approach (see Tsay and Tiao (1984)) gave indication of ARMA(p,q) models with p<q in most cases, ADF in particular could be of rather low power (compare Schwert (1987)).

2 The factor $\frac{1}{4}$ is used to retain original scales. It does not modify any dynamic properties.

The significance of the statistics is displayed in Table 1. If the integration hypothesis is correct, statistics should be insignificant for levels and significant for differences. If judgment relies on SW and J, this behavior is reproduced by the adjusted data with the exception of wages. SW and ADF render adjusted wages WMA to be integrated of order two while J rejects the second unit root at 2.5 %. Closer inspection of WMA increments by ARMA estimation resulted in an inverse root around 0.8 which is situated in an area that is impossible to discriminate from one by 80 data points (compare Stock and Watson (1986)). However, the root is much smaller than the roots of the level series and slight changes of the sample period generate significant SW. In this case, we decided to assume wages to be integrated of order one henceforth. During this sensitivity experiment, the interest rate series R and RMA sometimes indicated stationary behavior. Recent years enhance the integrated (random walk-like) nature of R and RMA.

## 2.3. Seasonal adjustment

An important implication of the integration requirement is that none of the series contain unit roots different from one which is a problem for seasonal data. A former investigation on the subject (see Kunst (1987)) used regression on seasonal dummies to extract the seasonal component, following a similar approach by Juselius (1987). This resulted in an estimated cointegrating dimension of two which, however, was rather sensitive towards changes of the sample length.

For several reasons, seasonal adjustment by dummy regressions was not satisfactory. First, *visual inspection* showed remaining seasonal patterns in the adjusted series. In particular, the dummy regression method seems to extract seasonality in the center of the sample interval but even to add some seasonality at the beginning and at the end. This adding of noise might distort the results in two different directions: on the one hand, it reduces significance of actual interdependence between the underlying processes; on the other hand, it increases spurious interdependence between deterministic seasonal components.

Second, subjecting the original and the adjusted (!) series to definite *tests* like the likelihood-ratio approximation of Kunst (1988) or the modified Dickey-Fuller test of Dickey, Hasza & Fuller (1984) - in both cases with a null hypothesis of a factor $1-L^4$ making part of the AR polynomial of a time series model - resulted in insignificant values at least for Y,C,I. This means that seasonal differencing might be needed to produce stationary series. Dummy regression would be unable to de-seasonalize such a series.

Third, *not all series* seem to show seasonal patterns. In particular, R and P do not show any sign of seasonality, according to visual inspection as well as to the tests mentioned above. Seasonally adjusting these series could, at best, prove worthless, at worst it could even add spurious patterns or smooth out important characteristics.

The conclusion that dummy regression is inappropriate does not help much in finding the appropriate method. Several adjustment procedures are available, based either on additive or multiplicative modeling of seasonality, the most popular being Census X-11. Application of Census X-11 is less inviting here, as it does not imply a linear, data-independent transfer function. The risk of destroying interdependence patterns is too high. A good alternative seems to be looking for a fixed filter whose properties are known. Since tests cannot reject $1-L^4$ as an AR factor and $1-L^4=(1-L)(1+L+L^2+L^3)$, the fourth-order MA transform

$$X_t^S = \tfrac{1}{4}(1+L+L^2+L^3)X_t$$

suggests itself as a good procedure[2]. It is perfectly compatible with the usual Box/Jenkins (1976) approach of time series modeling.

Since tests reported Y,C,I as seasonally infested and W as a borderline case (significance between 5 and 15 %), while visual inspection renders Y,C,I,W as seasonal cases, several versions were investigated:

a) all six series adjusted by moving averages
b) Y,C,I,W adjusted; R and P not adjusted
c) Y,C,I adjusted; W,R,P not adjusted
d) original series system

As outlined in Section 1, the AR lag order has to be determined for all models. The use of multivariate information criteria is not very reliable for typical sample sizes and in general underestimates orders (see Nickelsburg (1985)). Alternatively, model order in a first-difference AR model was increased until an insignificant Q statistic was found. This rendered order 4 for all models except for (d) where order 5 was indicated. Note that the optimal order could be smaller if Q is calculated on the final equation of the Johansen (1987) setup. Intuitively, too high orders should bias the results less than too low ones, as is usual in time series problems.

Table 2 comprises the principal results from subjecting versions a) to d) to Johansen's method: the eigenvalues, the significance of cointegrating relationships, and the first three eigenvectors.

## 3. Sensitivity of the results

### 3.1. Sensitivity against seasonal adjustment

According to Table 2, the detected dimension of the cointegrating space is influenced by the share of deseasonalized series within the system. It seems that more seasonal adjustment generates more cointegrating vectors. Since it was motivated why we tend to believe in model b), furthermore it seems that mistakes can be made in both directions: on the one hand, seasonal patterns may mask existing cointegrating relations; on the other hand, oversmoothing could produce spurious relations if there is high correlation between the added seasonal noise components in two or more series. Overall evidence leaves us with three cointegrating vectors, i.e. the system may be seen as being generated by three independent random walks. This dimension of 3 is detected at the 2.5 % level of significance for all models but d), in which case dimension 2 is preferred. This dimension of 2 coincides with the cointegrating dimension reported in Kunst (1987) where all variables were de-seasonalized by regression on dummies.

Although the estimated dimension of the cointegrating space is affected by the model selected, the space itself, with its dimension fixed at three, is more or less reproduced in all cases. A first test on this subject may be performed by simply regressing the basis vectors of the respective models on those of model b). This gives the results summarized in Table 3a. Not only the three vectors sets are closely related but this also holds for certain subsets containing one or two vectors. In particular, the first and second a) vector span almost the same space as the first two b) vectors, and the same holds for the third vectors in each set. Similarly, the first c) vector is approximately the same as the first b) vector, and the remaining two vectors of both b) and c) sets correspond. The connection with the d) set is less obvious.

A mathematically more sustainable test may be done by calculating the canonical correlations between any two sets of basis vectors.

The root of the separate $R^2$ corresponds to the cosine of the angle between the regressand vectors and the regressors hyperplane which may significantly exceed the cosine of the angles between the two hyperplanes, i.e. the canonical correlations. The larger two canonical correlations (not shown) indicate that a two-dimensional subspace is shared almost completely by the hyperplanes, and the smallest correlations given in Table 3b show that even the third axes almost coincide.

### 3.2. Sensitivity against lag order

In the Johansen estimating procedure, the number of autoregressive lags included plays an important role. It was motivated above why we used the criterion of an insignificant Q statistic to determine a common lag order. For some series, this almost inevitably leads to an overfitting of the process. For model b), the sensitivity of the results against a reduction in lag order was investigated by calculating the canonical correlations between the estimated three-dimensional cointegrating spaces based on lag orders varying from one to four. The results are displayed in Table 4. It is seen that the presumably misspecified models with lag orders one to three generate very similar results while increasing the lag order to four changes the results slightly. This could indicate that a lag order of four is necessary to capture the remaining seasonality in the data.

### 3.3. Temporal sensitivity

To investigate into the question whether the results are robust towards changes of the time interval, the end of the sample was reduced gradually back to 1985:4 (1985, 4th quarter) and the resulting eigenvalues and eigenvectors of model b) were evaluated. Note that the seasonal adjustment is not time-dependent or data-dependent and that therefore the data does not need to be revised for this experiment.

With regard to the eigenvalues, the results are very stable (see Table 5a). The first root shows its first major change in 1986:1; inclusion of the 1986:2 and succeeding observations reduce it from around .51 to around .45. The third root shows a similar decline from .32 to .28 and once more moves down slightly from .29 to .26 in 1987:1. The remaining roots are even more stable. There is slight evidence on a tendency of the third cointegrating component to have become "less important" or "less stationary" in recent quarters. No indication of seasonal cycles in the results can be found.

More fluctuations show up in the respective cointegrating spaces (see Table 5b). Throughout all variants, a common 2-dimensional hyperplane is shared, but the angle between the third principal axes changes. While we should expect this angle to open gradually with a backward movement of the sample end, in fact it opens to around 25° in the samples ending on 1987:3 and and 1987:2 and then closes again to 9°, afterwards opening slowly and more monotonously. This means that, had we ended the sample at 1987:2 or 1987:3, we should have got different cointegrating vectors. Future observations will decide whether these two time points may be regarded as outliers or whether the data rather is beginning to switch towards a new regime and the return to the old one in 1987:4 is transitory. Again, there is no indication for seasonality in the results.

### 3.4 Sensitivity against the Box/Tiao canonical analysis

As outlined in section 1.2, the results of the cointegration analysis may be compared to results based on a canonical analysis developed by Box and Tiao (1977) which relies on the assumption of joint stationary but maybe "almost non-stationary processes" and on the canonical correlations between the observations and their AR(1) forecasts.

Contrary to the Johansen method, now the eigenvectors corresponding to the *smallest* eigenvalues are the most interesting ones since they generate processes which are poorly explained by their own past and are, therefore, the "most stationary" ones. Eigenvalues and the three interesting eigenvectors are given in Table 6a for models a) through d). In Table 6b, these eigenvectors are compared to the cointegrating spaces detected by the Johansen analysis by smallest canonical correlations and the resulting third axes angles. The spaces are not equal but again share 2-dimensional hyperplanes and look very similar if it is taken into account that AR(1) forecasts are far from the optimal forecasts theoretically needed for the analysis. Remember that the Johansen models include up to six lags of the time series.

## 4. Economic interpretation

One of the main perspectives of cointegration modeling is the extraction of long-run relationships from the data which reflect equilibrium conditions known from macroeconomic theory. The other one is improved forecasting performance. The former point will be treated in this section, the latter one is deferred to section 5.

Economists would be far from unanimous as far as the long-run or steady-state properties of the present model are concerned. A tentative specification, however, could include the following three relations:

a) C-Y is stationary, i.e. the consumption quota is, since all variables enter in logs;
b) I-Y is stationary, i.e. the investment quota is.
c) Y-(W-P) is stationary, i.e. real national income grows proportional to wage income keeping the income distribution constant in the long run.

The estimation and testing procedures suggested by Johansen (1987) include an approximative likelihood-ratio test on whether the theoretical vectors are in the estimated cointegrating space. Since this test is rather cumbersome and all such tests are subject to some arbitrariness concerning the level of significance and the validity of asymptotics, we preferred the insight given by directly calculating the canonical correlations between the space spanned by the theoretical vectors and the empirical cointegrating space. The two spaces almost share a two-dimensional hyperplane but it is seen from Table 7 that the third coordinate axes form a non-zero angle of about arccos(.88) (28 degrees) for b) and c) models and an even larger angle for the extreme models a) and d). This means that, at least, one of the above restrictions is not corroborated by the data.

It is still possible that one or two of the restrictions are valid in the long run. Since optical evidence renders the investment quota as non-stationary, the canonical analysis was repeated on the basis of the theoretical vectors C-Y and Y-W+P and the three-dimensional cointegrating spaces. Here, one of the roots is necessarily zero and evidence relies on the second root. From Table 6b, we see that these roots are close to one and that, keeping in mind that we do not know any stochastic properties of this kind of "testing", our investigation should concentrate on identifying the third, missing, vector.

A different way to obtain information is to look at the estimated cointegrating vectors first and to base theoretical vectors on relations which are easy to interpret. Since Table 3 tells us that the four different sets of basic vectors listed in Table 2 span about the same space, any combination of relations of Table 2 can be used as a starting point for such investigations. After some trial and error, the following three relations were found, coefficients obtained by appropriate least-squares regression on the model b) variates:

a) I-W = 1.8(Y-P) - 0.5C

had a slightly better fit than regressing Y-P on I-W and C. The inclusion of C was necessary to warrant stationarity. The equation may tell that, if real income grows at the same rate as prices, wages still grow faster than investment unless consumption stagnates. This relation is reflected in the first component of model b) and definitely seems to hold over the whole sample.

b) Y = .56C + .22I + .15P

replicates the average consumption and investment quota by the first two respective coefficients. The third coefficient is difficult to interpret. It seems that P approximates the behavior of the sum of public consumption and net foreign trade.

c) R = -15(C-Y) + 2.4(W-P)

relates the consumption quota, real wages and the interest rate. Taken as an equation for C-Y, it reflects the dependence of the consumption quota on real wages (positive) and on R (negative).

By the results of the canonical analysis in Table 7, we see that these vectors give a far better fit than the "theory" vectors, i.e. that the estimated space almost satisfies the imposed identity and zero restrictions on the coefficients. Further on, we see that the fit may be improved by parameter modifications relative to the least-squares solution, in particular for the relevant models b) and c).

From the second equation, we deduce that, in the long run, assuming an average current balance of zero - a usual external equilibrium condition - expansions of the government sector occur in phases of high inflation while contractive policy usually accompanies low inflation phases.

Returning to the missing vector problem, the 2-dimensional system generated by Y-C and Y-W+P was augmented by each of the "optimal restricted" basis vectors. The canonical correlations between these augmented systems and the estimated model b) system indicate whether the respective additional vector could be the missing third one. The results are summarized in Table 8: the 2-dimensional theoretical space together with any of the first two "optimal restricted" basis vectors approximately spans the same space as the estimated b) vectors. On the other hand, the third vector fails to do so. Since the second vector contains more zero restrictions and is somehow easier to interpret than the first one, let us focus on the cointegrating basis vectors Y-C, Y-W+P, and Y-.55C-.27I-.18P, to be labeled henceforth $c_1, c_2, c_3$. Note that R does not appear in the basis and now does not seem to provide any important long-run information whatsoever.

The explanation for $c_3$ given above may be corroborated by actually calculating the difference between the output exp(Y) and the sum of the principal demand aggregates exp(C)+exp(I). This difference consists of public consumption, exports minus imports, and inventory changes and statistical differences. Its logarithm may be regressed on P. This was done for the seasonally adjusted and non-adjusted series. In both cases, although residual autocorrelation was strong, visual inspection of the residuals gave strong indication of their stationary nature. This means that the difference between output and principal domestic demand on the one hand and the output deflator on the other are cointegrated.

A different interpretation is obtained if C in $c_3$ is eliminated by using $c_1$ and the resulting vector is normalized in Y. This gives

$$Y = .6I+.4P$$

which reminds of a production function with the factors capital (expressed by I) and labor including technical progress, this latter factor expressed by P. The approximation of the factor labor by the GDP deflator may be viewed as a variant of the Phillips curve. The Phillips curve is the best known example for an interdependence between prices and real aggregates (even though recent economic theory has reduced it to a transitory short-run phenomenon). The cointegration analysis shows that at least one such (long-run) transmission exists in the data.

## 5. Forecasting

According to the study by Engle and Yoo (1987), which is based on a two-dimensional system, the inclusion of cointegrating restrictions helps to outperform models in differences with respect to predictive accuracy from around ten steps on even if the cointegrating vectors have to be estimated from the data. Brandner and Kunst (1988) apply cointegration analysis to three-dimensional systems with two cointegrating vectors and conclude that forecasts using estimated vectors dominate those from modeling in differences even at less steps but that this benefit is lost on the average if the cointegrating dimension is unknown since any over-estimation of this number leads to "excess stationarity" and to inferior forecasting performance.

For this paper, ex-ante forecasts from model b) samples ending at the last quarters of 1984, 1985, 1986, 1987 were generated until the end of the present century. In all experiments but the last one, the outcomes may be compared to actual realizations, at least for some observations. We believe that a graphic display of these experiments is more telling than the use of conventional summary statistics.

The results from the 1985, 1986, and 1987 experiments are displayed in Figures 1, 2, and 3, respectively. The different curves correspond to different assumptions with respect to the cointegrating dimension which was changed between zero and three, the former number representing a model in differences generated by six independent random walks, the latter number corresponding to the estimated dimension reported in Section 3.

In summary, the strongest reaction to cointegrating vectors happens in the series I and R. For three or two vectors, R more or less approaches a stationary series and tends to revert towards its historic average. This mean-reverting behavior has not been corroborated by actual data within the last years, although it cannot be rejected easily for the most part of the sample. One may compare the results from the unit root tests for R which cannot reject the unit root hypothesis now but would have done so some years ago.

I negatively reacts to R which means that less cointegrating vectors generate continuous though somewhat slow growth in I while higher dimensions entail cycles at a stagnating level. Note that, for the sample ending 1985, a cointegrating dimension of two was enough to produce such cycles while, for the whole sample, dimensions of one and two generate the same slow upward trend.

Y and C almost simultaneously react to the business cycles created by R and I. The pure differences model is linked to the most expansionary path while the three-dimensional cointegrating space heads for a persistent recession. The nominal indicators P and W parallel these scenarios.

Although all models slightly overpredicted the actual observations for 1986 and 1987, a visual summary seems to prefer cointegrating dimensions of one or two but rejects the zero and three models. Note that the expansionary growth path predicted by the zero model is not overwhelming. It corresponds to an annual growth rate of around one percent.

The results of the 1984 experiment are not reported since during 1983-84 a legislative change of consumer taxes and other transitory phenomena disturbed all time series forecasts. For the 1984 experiment, the relative predictive performance of the pure differences VAR model was best while all cointegrating models drew all variables towards zero in the long run.

Since Kunst and Neusser (1987) reported VAR models in differences restricted by subset searches as dominating all other models (including Doan/Litterman/Sims Bayesian VAR) their suggested subset search was performed on the differences model. The results are not reported in detail here as they more or less coincide with those from the differences VAR without subset modeling.

## 6. Tentative conclusions

It would be foolhardy to draw general conclusions from any work on a single system with individual features. Some conjectures, however, inevitably arise. Future work with different data and additional information by mathematical analysis will show whether these conjectures are justified.

First, we note that the robustness of the cointegrating dimension and the cointegrating space is of satisfactory quality with respect to slight modifications of the data set, such as shortening of the sample and seasonal adjustment. This may be contrasted with the estimated individual cointegrating vectors which are highly sensitive and unstable. Of course, more research is needed to determine the sensitivity of forecasts via cointegrating vectors with respect to the "distance" between underlying vector spaces.

Second, the size of Johansen(1987)'s cointegration test does not seem appropriate in typical economic samples. The null hypothesis of no (or less) cointegration is rejected too often even if 2.5 %-fractiles are used. Together with the results of Brandner and Kunst (1988) concerning the high upward risk of estimating cointegrating dimensions with respect to forecasting performance, this would imply the recommendation to use very low levels of significance for cointegration analyis by the Johansen (1987) procedure, e.g. 1 % or less. Regrettably, necessary fractiles have not been published and would have to be simulated.

Finally - but this has already been evident from other empirical studies - the step from the estimated cointegrating space to economically reasonable relationships is a difficult one, even if simple relations are contained in the estimated space. To find the relations, it sometimes pays to fit regressions with zero restrictions and use some trial and error. Of course, sometimes the resulting relations do not satisfy the economic theorist.

TABLE 1: Approximate significance of unit root test statistics[3]

| | ADF | SW | J | ADF▲ | SW▲ | J▲ |
|---|---|---|---|---|---|---|
| Y | *** | * | - | - | *** | *** |
| C | * | ** | - | *** | *** | *** |
| I | * | *** | - | ** | *** | *** |
| P | - | - | - | - | *** | *** |
| R | ** | - | - | ** | *** | *** |
| W | - | - | - | - | *** | *** |
| YMA | - | - | - | ** | *** | *** |
| CMA | * | - | - | * | *** | *** |
| IMA | - | - | * | ** | *** | *** |
| PMA | - | - | - | - | *** | *** |
| RMA | - | - | - | ** | ** | *** |
| WMA | - | - | - | - | - | *** |

3 For explanation of ADF, SW, J see text; ▲ indicates statistics for differenced series. * indicates significance at 10 %, ** at 5 %, *** at 1 % (2.5 % for J and J▲)

TABLE 2: Primary results of cointegration analysis[4]

Model a)

| | | | | | | |
|---|---|---|---|---|---|---|
| eigenvalues | .469 | .442 | .267 | .198 | .057 | .001 |
| (significance) | *** | *** | *** | ** | | |

eigenvectors

| Y | C | I | P | R | W |
|---|---|---|---|---|---|
| 81.1 | -27.1 | -6.21 | 22.12 | 1.27 | -31.6 |
| 84.3 | 16.6 | -57.8 | -99.7 | -1.59 | 47.5 |
| 51.0 | -173.4 | 8.47 | -61.0 | -1.35 | 85.26 |

Model b)

| | | | | | | |
|---|---|---|---|---|---|---|
| eigenvalues | .435 | .307 | .249 | .157 | .071 | .015 |
| (significance) | *** | *** | *** | * | | |

eigenvectors

| Y | C | I | P | R | W |
|---|---|---|---|---|---|
| 59.0 | -10.2 | -36.8 | -83.5 | -1.66 | 48.1 |
| 102.8 | -15.1 | -32.3 | -31.5 | .26 | -1.35 |
| -42.3 | 47.3 | -7.19 | 21.6 | 1.97 | -15.51 |

Model c)

| | | | | | | |
|---|---|---|---|---|---|---|
| eigenvalues | .436 | .268 | .252 | .153 | .064 | .003 |
| (significance) | *** | *** | *** | | | |

eigenvectors

| Y | C | I | P | R | W |
|---|---|---|---|---|---|
| -83.3 | 8.16 | 49.4 | 94.9 | 1.65 | -50.3 |
| 78.5 | -43.4 | -12.0 | -20.1 | -.28 | 6.06 |
| 14.7 | -44.0 | 12.5 | -18.5 | -2.07 | 20.6 |

Model d)

| | | | | | | |
|---|---|---|---|---|---|---|
| eigenvalues | .430 | .329 | .189 | .148 | .041 | .000 |
| (significance) | *** | *** | * | | | |

eigenvectors

| Y | C | I | P | R | W |
|---|---|---|---|---|---|
| 36.0 | 78.4 | -37.1 | -47.0 | -1.32 | -.08 |
| -64.3 | -.48 | 12.4 | -9.55 | -.98 | 26.00 |
| -84.8 | 112.1 | 10.63 | 73.0 | 2.34 | -64.3 |

4 * indicates significance at 10 %, ** at 5 %, *** at 2.5 %

TABLE 3a: Relations between the three-dimensional cointegrating spaces from models a) to d) and respective $R^2$

$$V_{a)} = \begin{bmatrix} -.76 & 1.08 & -.37 \\ 1.21 & .39 & .68 \\ .44 & -1.12 & -3.53 \end{bmatrix} V_{b)} \qquad \begin{matrix} .999 \\ .999 \\ .948 \end{matrix}$$

$$V_{c)} = \begin{bmatrix} -1.08 & -.25 & -.14 \\ -.12 & .54 & -.73 \\ .09 & -.32 & -1.00 \end{bmatrix} V_{b)} \qquad \begin{matrix} 1.00 \\ .996 \\ .999 \end{matrix}$$

$$V_{d)} = \begin{bmatrix} .67 & .64 & 1.77 \\ .44 & -.95 & -.18 \\ -.54 & .34 & 2.17 \end{bmatrix} V_{b)} \qquad \begin{matrix} .935 \\ .997 \\ .983 \end{matrix}$$

TABLE 3b: Smallest canonical correlations between the estimated cointegrating vector sets from models a) to d)

| | a) | b) | c) | d) |
|---|---|---|---|---|
| a) | 1 | .9355 | .9454 | .9941 |
| b) | - | 1 | .9932 | .9251 |
| c) | - | - | 1 | .9388 |

TABLE 4: Smallest canonical correlations between b) type models based on changing autoregressive lag order (4 lags retrieve the original b) model)

| lag orders | 2 | 3 | 4 |
|---|---|---|---|
| 1 | .9809 | .9970 | .9212 |
| 2 | | .9909 | .8812 |
| 3 | | | .9134 |

TABLE 5a: Sensitivity of Johansen roots against shortening of the sample

| sample end | $r_1$ | $r_2$ | $r_3$ | $r_4$ | $r_5$ | $r_6$ |
|---|---|---|---|---|---|---|
| 1987:4 | .435 | .307 | .249 | .157 | .071 | .015 |
| 1987:3 | .435 | .331 | .233 | .169 | .074 | .011 |
| 1987:2 | .435 | .346 | .242 | .169 | .052 | .019 |
| 1987:1 | .456 | .343 | .259 | .172 | .056 | .019 |
| 1986:4 | .457 | .334 | .292 | .170 | .074 | .006 |
| 1986:3 | .456 | .345 | .288 | .172 | .070 | .004 |
| 1986:2 | .452 | .341 | .283 | .174 | .081 | .002 |
| 1986:1 | .518 | .336 | .323 | .176 | .089 | .006 |
| 1985:4 | .514 | .331 | .322 | .177 | .084 | .001 |

TABLE 5b: Sensitivity of 3-dimensional cointegrating space: smallest canonical correlations and corresponding angles between model b) space of sample ending at 1987:4 and reduced samples

| sample end | can.corr. | angle |
|---|---|---|
| 1987:3 | .840 | 24° |
| 1987:2 | .811 | 26° |
| 1987:1 | .977 | 9° |
| 1986:4 | .972 | 10° |
| 1986:3 | .961 | 11° |
| 1986:2 | .963 | 11° |
| 1986:1 | .945 | 14° |
| 1985:4 | .945 | 14° |

TABLE 6a: Results of the Box/Tiao canonical analysis relying on first-order autoregressive forecasts

Model a)

| eigenvalues | .999 | .994 | .964 | .941 | .906 | .613 |
|---|---|---|---|---|---|---|

| Y | C | I | P | R | W |
|---|---|---|---|---|---|
| -8.990 | 1.972 | 2.898 | 1.562 | -.010 | .788 |
| -2.007 | 1.547 | -1.029 | -6.127 | -.123 | 4.705 |
| .393 | -12.983 | 1.405 | -4.145 | -.088 | 7.339 |

Model b)

| eigenvalues | .999 | .987 | .948 | .910 | .695 | .254 |
|---|---|---|---|---|---|---|

| Y | C | I | P | R | W |
|---|---|---|---|---|---|
| -4.709 | 1.528 | 1.674 | -.107 | -.112 | .860 |
| -.160 | 11.391 | -2.274 | -.480 | .036 | -3.389 |
| 1.771 | -4.600 | -.559 | -5.754 | -.056 | 5.167 |

Model c)

| eigenvalues | .998 | .986 | .927 | .890 | .615 | .483 |
|---|---|---|---|---|---|---|

| Y | C | I | P | R | W |
|---|---|---|---|---|---|
| 8.193 | -5.887 | -1.672 | -.665 | .051 | .039 |
| -2.055 | -6.132 | 2.409 | 3.707 | .004 | -.030 |
| -.830 | -3.260 | .555 | -1.086 | -.031 | 2.202 |

Model d)

| eigenvalues | .996 | .937 | .796 | .695 | .553 | .304 |
|---|---|---|---|---|---|---|

| Y | C | I | P | R | W |
|---|---|---|---|---|---|
| -3.200 | .046 | 1.203 | 2.206 | .038 | -.665 |
| 3.926 | -3.766 | -.246 | -2.436 | -.041 | 1.712 |
| -3.016 | .403 | .698 | -1.767 | -.031 | 2.016 |

TABLE 6b: Smallest canonical correlations[5] and corresponding third axes angles between spaces spanned by Box and Tiao eigenvectors and cointegrating spaces

| | corresponding estimated spaces | | [Y-C,Y-W+P] | optimal restricted vectors | |
|---|---|---|---|---|---|
| model | can.corr. | angle | can.corr. | can.corr. | angle |
| a) | .9876 | 6° | .9918 | .9123 | 17° |
| b) | .8814 | 20° | .9674 | .9211 | 16° |
| c) | .9168 | 17° | .9910 | .8961 | 19° |
| d) | .9786 | 8° | .9806 | .9416 | 14° |

5 For case 2: smallest non-zero eigenvalue

TABLE 7: Smallest canonical correlations[6] between the estimated cointegration spaces of

| models | a) | b) | c) | d) | and |
|---|---|---|---|---|---|
| 1. "theory" vectors (1,-1,0,0,0,0),(1,0,-1,0,0,0),(1,0,0,1,0,-1) | | | | | |
| | .6595 | .7774 | .7746 | .6033 | |
| 2. only two theory vectors (1,-1,0,0,0,0),(1,0,0,1,0,-1) | | | | | |
| | .9964 | .9718 | .9803 | .9988 | |
| 3. restricted regression vectors (1.8,-.5,-1,-1.8,0,1), (1,-.56,-.22,-.15,0,0),(15,-15,0,-2.4,-1,2.4) | | | | | |
| | .9266 | .8950 | .8998 | .9240 | |
| 4. optimal restricted vectors (1.8,-.6,-1,-1.8,0,1), (1,-.55,-.27,-.18,0,0),(34,-34,0,-2.3,-1,2.3) | | | | | |
| | .9312 | .9721 | .9848 | .9236 | |

6 For case 2: smallest non-zero eigenvalue

TABLE 8: Canonical correlations between augmented vector space [(1,-1,0,0,0,0),(1,0,0,1,0,-1),V] and estimated model b) space

| vector V | canonical correlations | | |
|---|---|---|---|
| (1.8,-.6,-1,-1.8,0,1) | 1.0000 | .9994 | .9664 |
| (1,-.55,-.27,-.18,0,0) | .9995 | .9979 | .9704 |
| (-34,34,0,-2.3,-1,2.3) | 1.0000 | .9839 | .4017 |

## References


BOX, G.E.P. and G.C. TIAO (1977): A canonical analysis of multiple time series; *Biometrika* 64,2,355-365.

BRANDNER, P. and R.M. KUNST (1988): The effects of cointegration on forecasting: A Monte Carlo study; mimeo, Institute of Advanced Studies, Vienna.

CAMPBELL, J.Y. and R. SHILLER (1987): Cointegration and Tests of Present Value Models; *Journal of Political Economy* 95,5,1062-1088.

DICKEY, D.A., D.P. HASZA, W.A. FULLER (1984): Testing for Unit Roots in Seasonal Time Series; *Journal of the American Statistical Association* 79,386,355-367.

DICKEY, D.A. and W.A. FULLER (1979): Distribution of the estimator for autoregressive time series with a unit root; *Journal of the American Statistical Association* 74,427-431.

ENGLE, R.F. and C.W.J. GRANGER (1987): Cointegration and error correction: Representation, estimation and testing; *Econometrica* 55,251-276.

ENGLE, R.F. and B.S. YOO (1987): Forecasting in co-integrated systems; *Journal of Econometrics* 35,143-159.

HOTELLING, H. (1936): Relations between two sets of variables; *Biometrika* 28,321-377.

JOHANSEN, S. (1987): Statistical Analysis of Cointegration Vectors; Paper presented at the Econometric Society European Meeting, Copenhagen.

JUSELIUS, K. (1987): Cointegration and Identification in a Vector Time Series Model. An Application to the Demand for Money in Denmark; Paper presented at the Econometric Society European Meeting, Copenhagen.

KUNST, R.M. (1987): Cointegration in a Macroeconomic System; in: Time Series Modeling in Economics, Workshop Proceedings No.1, Institute for Advanced Studies, Vienna.

KUNST, R.M. (1988): A Canonical Correlation Test for Seasonal Unit Roots; mimeo, Institute for Advanced Studies, Vienna.

KUNST, R.M. and K. NEUSSER (1986): A Forecasting Comparison of Some VAR Techniques; *International Journal of Forecasting* 2,447-456.

NICKELSBURG, G. (1985): Small-Sample Properties of Dimensionality Statistics for Fitting VAR Models to Aggregate Economic Data; A Monte Carlo Study; *Journal of Econometrics* 28,183-192.

PHILLIPS, P.C.B. and S. OULIARIS (1987): Testing for cointegration; Cowles Foundation discussion paper No. 809.

SCHWERT, G.W. (1987): Effects of Model Specification on Tests for Unit Roots in Macroeconomic Data; *Journal of Monetary Economics* 20,73-105.

STOCK, J.H. and M.W. WATSON (1986): Testing for Common Trends; Harvard Institute of Economic Research, Harvard University, Discussion Paper No. 1222, March 1986.

TJØSTHEIM, D. and J. PAULSEN (1982): Empirical Identification of Multiple Time Series; *Journal of Time Series Analysis* 3,4,265-281.

TSAY, R.S. and G.C. TIAO (1984): Consistent Estimates of Autoregressive Parameters and Extended Sample Autocorrelation Function for Stationary and Non-Stationary ARMA Models; *Journal of the American Statistical Association* 79,385,84-96.

VELU, R.P., D.W. WICHERN and G.C. REINSEL(1987): A Note on Non-Stationarity and Canonical Analysis of Multiple Time Series Models; *Journal of Time Series Analysis* 8,4,479-487.

Figure 1a:  Forecasts from a sample ending at 1985:4

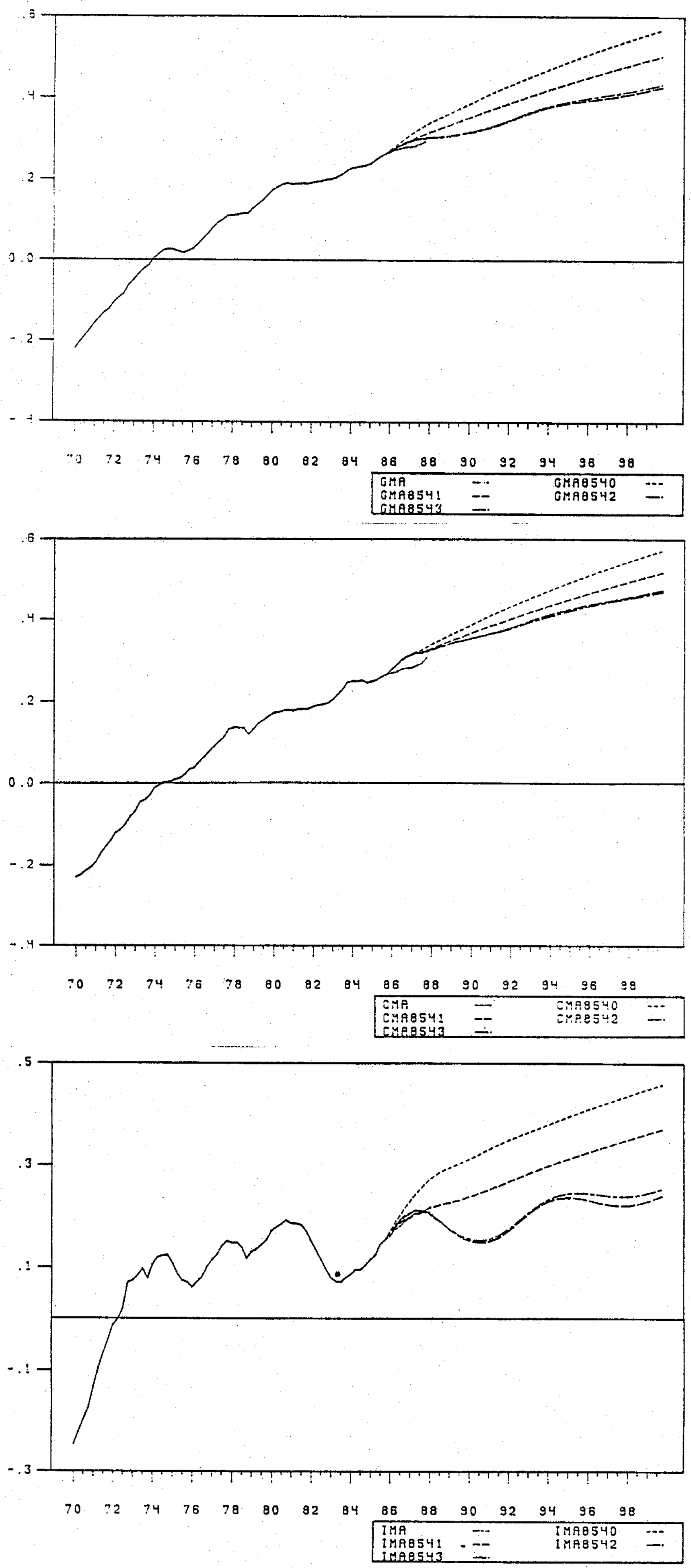


Figure 1b: Forecasts from a sample ending at 1985:4

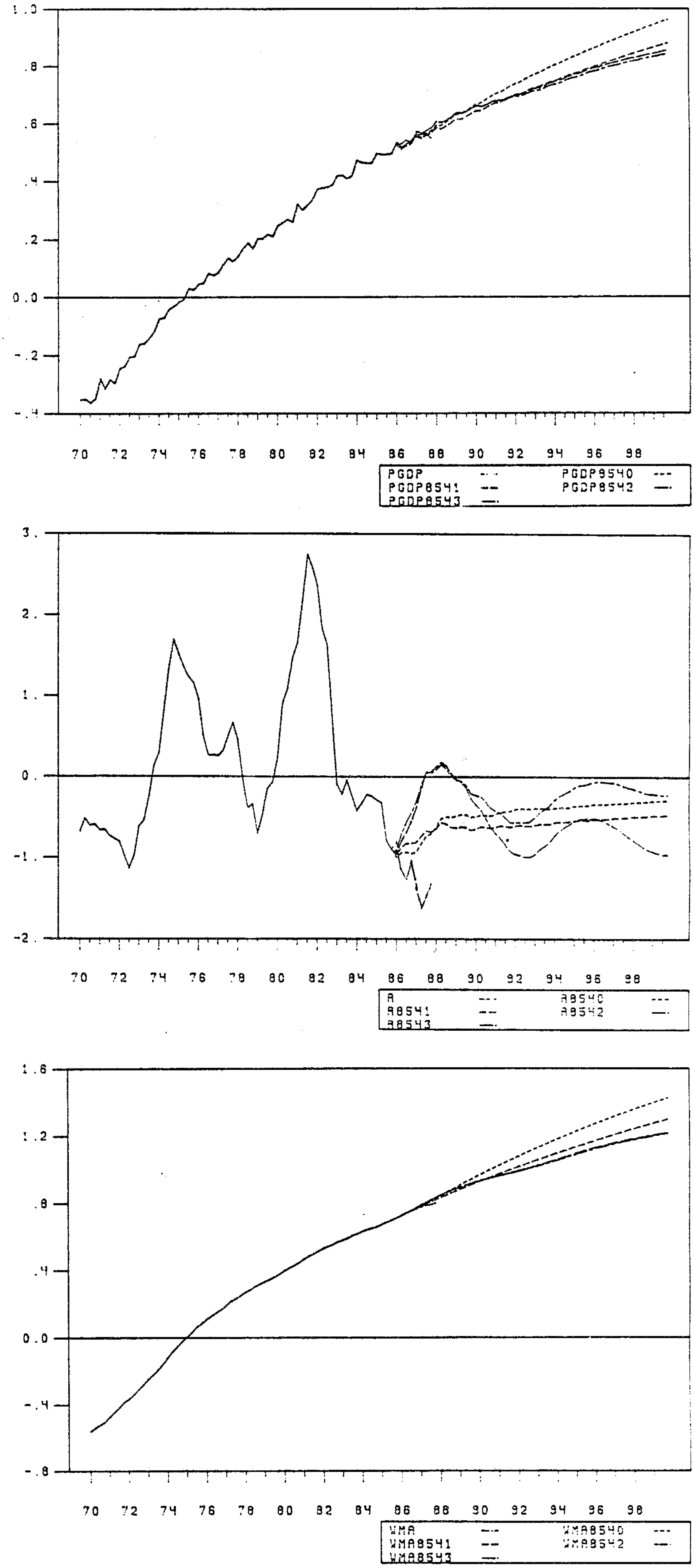


Figure 2a: Forecasts from a sample ending at 1986:4

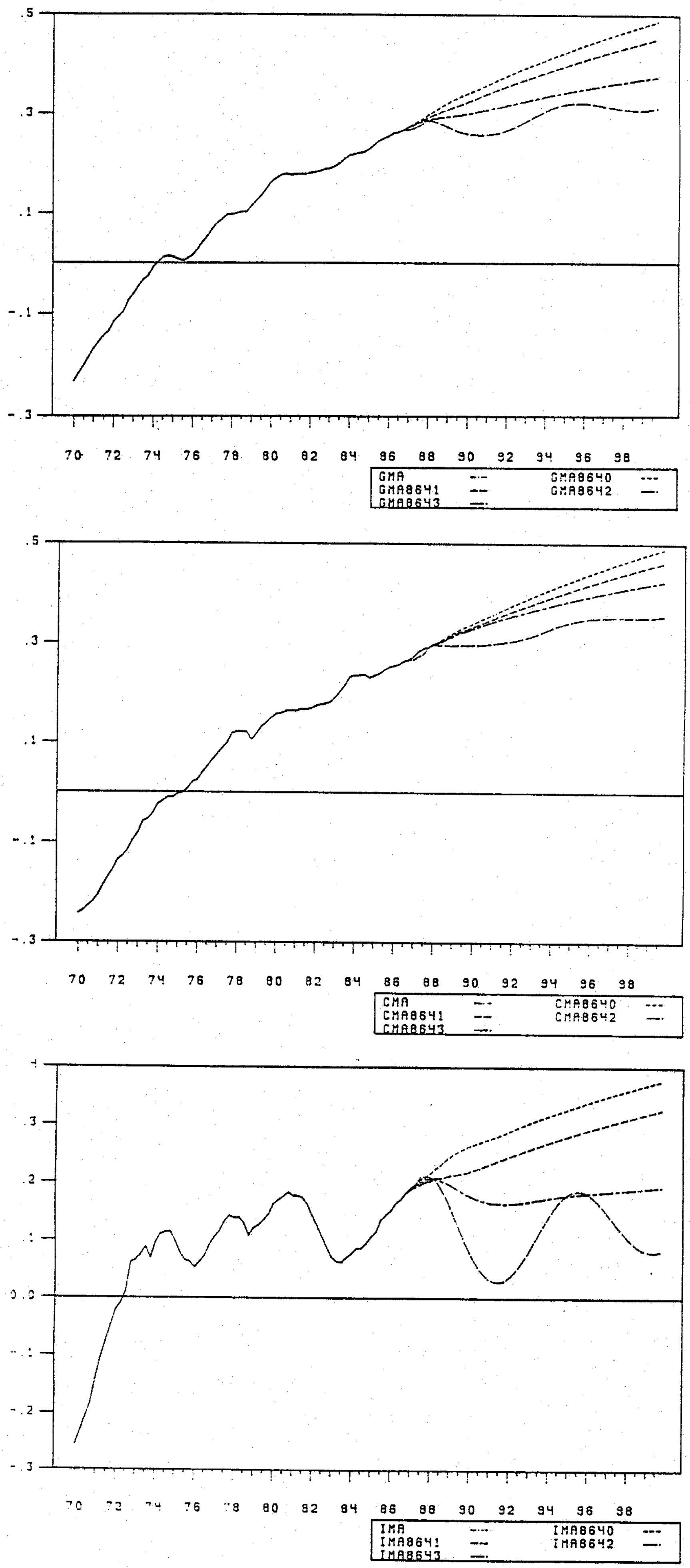
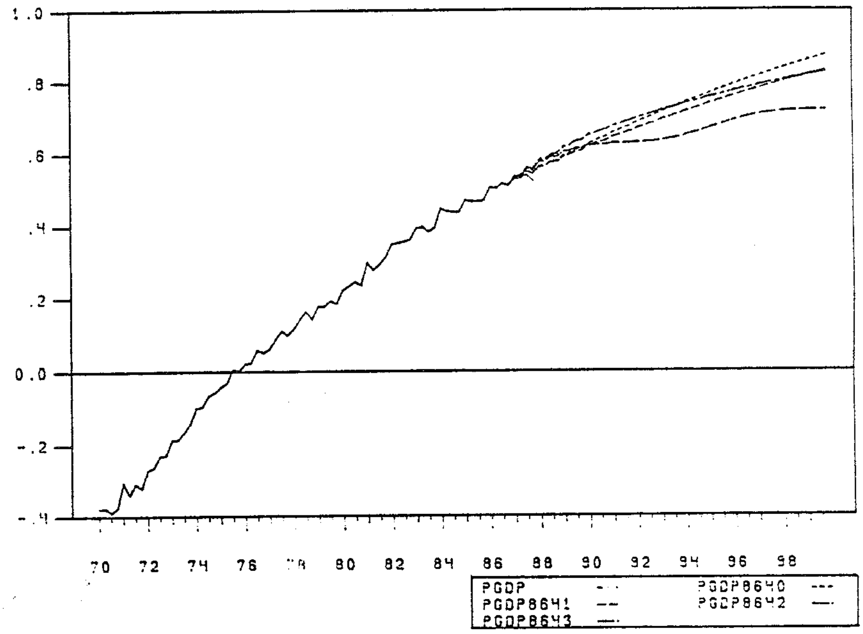
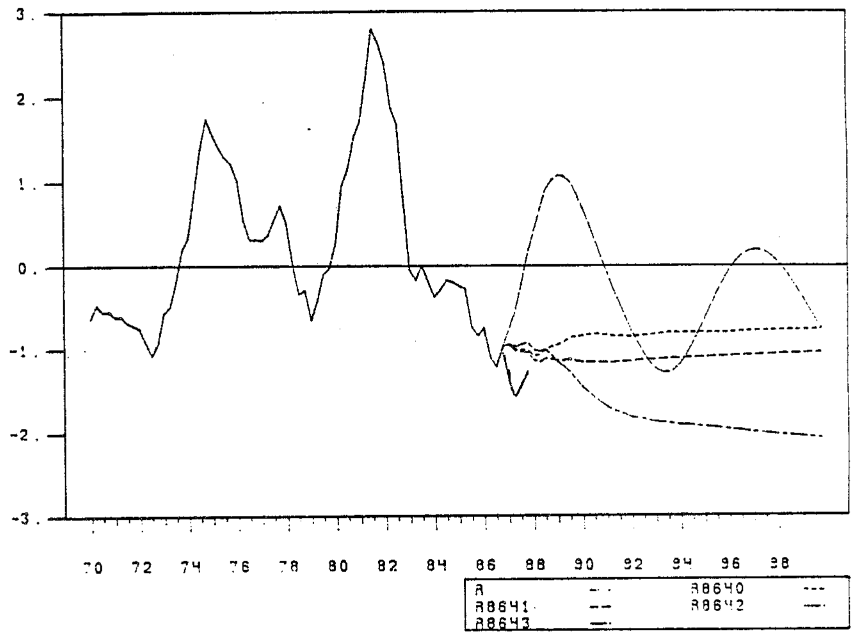


Figure 2b: Forecasts from a sample ending at 1986:4



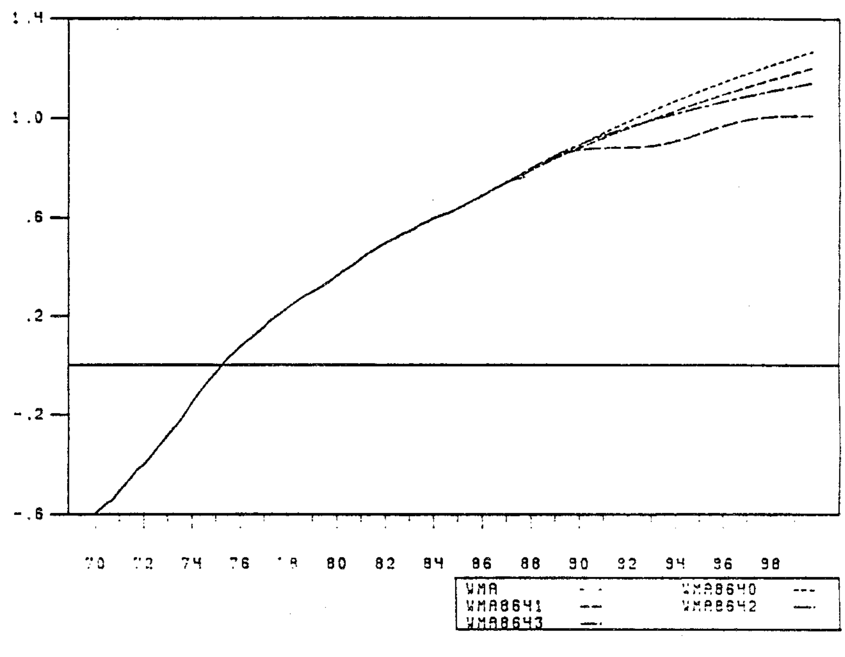
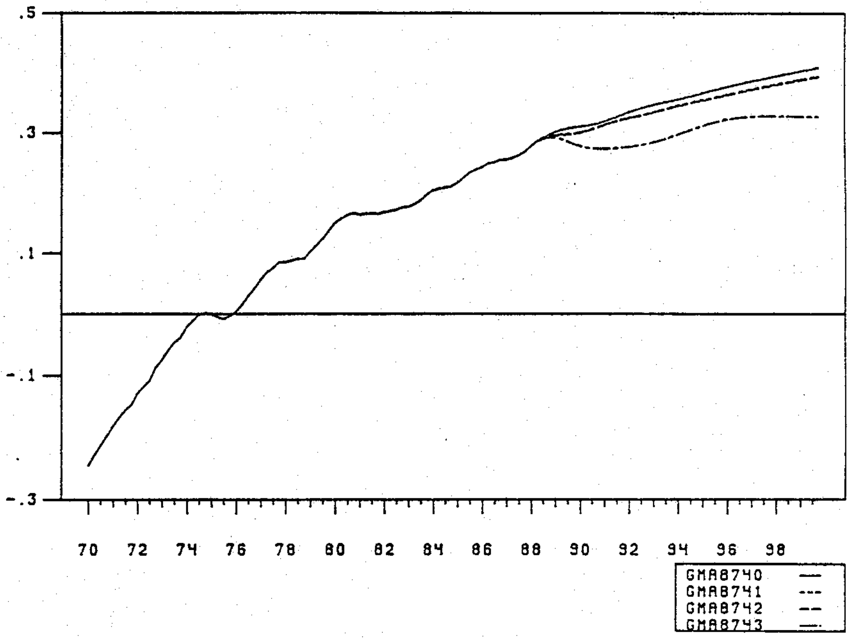
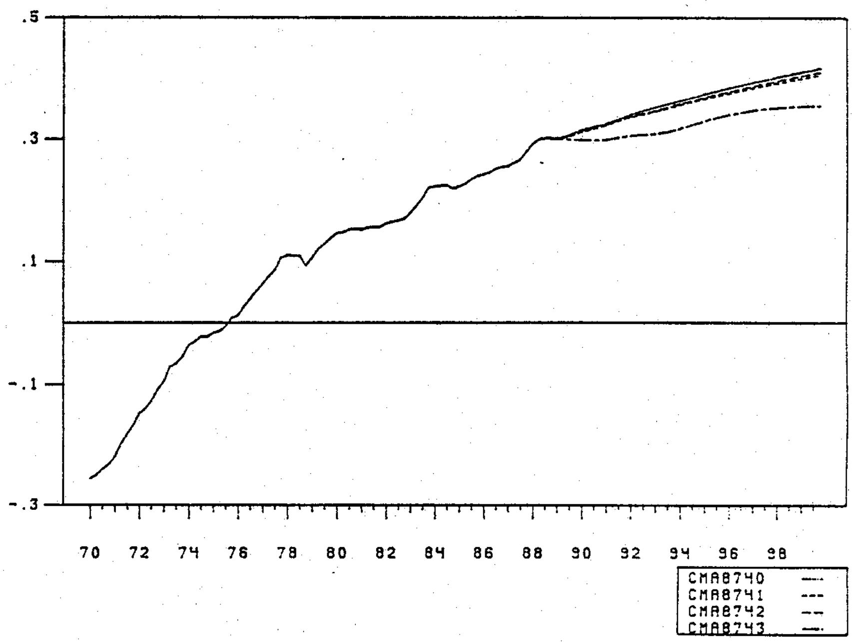



Figure 3a: Forecasts from a sample ending at 1987:4



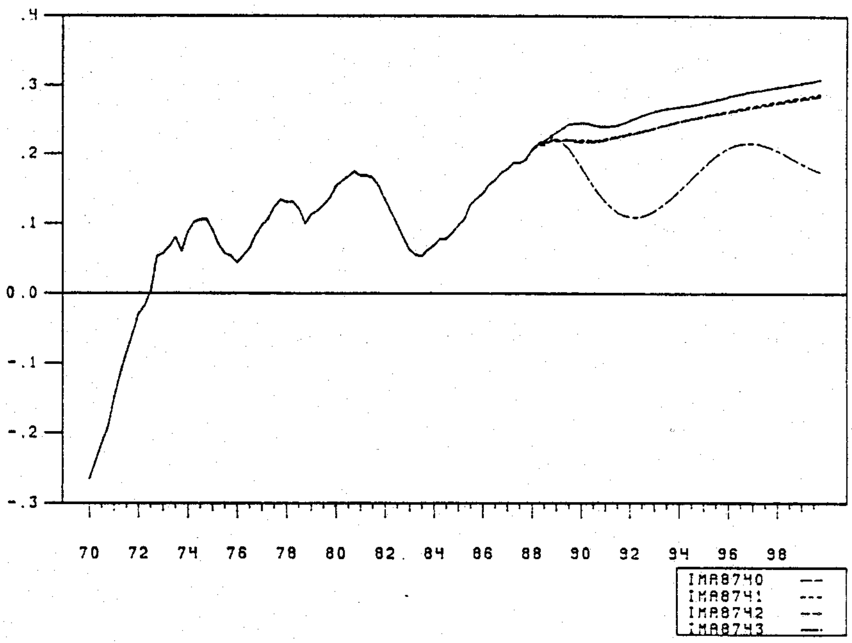



Figure 3b: Forecasts from a sample ending at 1987:4