UPDATING INPUT-OUTPUT MATRICES AND STRUCTURAL CHANGE

Klaus Ritzberger

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Abstract

The present paper develops an algorithm to update arbitrary Input-Output Tables from knowledge of row- and column-sums. The purpose is to generate unique Update-matrices, which are optimal in a specific sense. It turns out that the set of Lagrange multipliers used in this algorithm does possess economic significance.

Zusammenfassung

Die vorliegende Arbeit beschäftigt sich mit einem Fortschreibungsverfahren für beliebige Input-Output Tabellen mit Hilfe bekannter Randsummen. Es wird gezeigt, daß es ein Verfahren gibt, mit dem man eindeutig fortgeschriebene Matrizen erhält, die in einem bestimmten Sinne optimale Fortschreibungen sind. Darüber hinaus zeigt sich, daß die hierbei verwendeten Lagrange-Multiplikatoren auch eine ökonomische Deutung zulassen.

1. Introduction[+)]

Discussions on "structural change" frequently tend to focus on the quality of products and technology, on import and export values or even on income-distribution. Less attention seems to be paid to the "structure" of the economy, where the term "structure" is closer to what the layman would think it means. The latter notion of a "structure" of the economy relates to multisector-models introduced as an empirical device by Leontief (1951, 1966) and still widely applied to problems in studying indirect effects of, e.g., economic policy or growth (Carter, Brody, 1970, 1972; Ehret, 1970; Evers, 1974; Holub, Schnabl, 1982). These "Input-Output-models" or "linear-production models", however, are of definite interest to economics, since economic theory predicts - using the linear framework - that growth and the structure of an economy are fundamentally interlinked (von Neumann, 1945/46). Investigations of linear models have even generated a long tradition of distribution-theory-discussion (Sraffa, 1974; Harcourt, 1969). Turning to empirical applications in Austria, Input-Output-Tables have proved useful in studying sectorwise multiplier effects and other "structural" issues (Richter, 1981; Richter, Teufelsbauer, 1970, 1974; Schmoranz, 1983) and will prove useful in the future since new and reliable data is available (ÖStZ, 1985).

In an earlier paper it has been shown that Input-Output Tables can be used to study deviations of the economy from an optimal path (Ritzberger, 1984) both with respect to intensities and relative prices, by using properties of eigensystems. These investigations, however, refer to a given period in time - specifically to the year to which the Input-Output Table refers - and cannot generate conclusions on the path of the economy. To obtain the latter, one would need time series of Input-Output Tables which, however, are beyond the capabilities of data collection. Consequently, already quite early in the tradition of Input-Output Analysis, there have been studies devoted to using other information, than new tables each year, to generate statistical updates of Input-Output Tables (Bacharach, 1970; Evers, 1974; Stäglin, 1972). This has led to algorithms

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known as RAS or modified forms of biproportional RAS-methods. All of these algorithms use data on the row- and column-sums of the matrices to be updated. Most of them share the disadvantage that the results of the update-algorithm, i.e. the limit of the iteration process, needs not be unique.

The present paper aims at developing a method of updating arbitrary matrices from knowledge of time-series data on row- and column-sums. This method is designed to result in unique updates under very weak assumptions which, moreover, are optimal in the sense that a weighted sum of the distances between the original matrix-entries and the updated ones is minimized. Throughout it is assumed that an initial matrix is given, e.g. an Input-Output Table from a certain year, and that data on the historical development of row- and column-sums is also given. There is, however, no assumption on the magnitude of the entries in the initial matrix, i.e. they may be either coefficients or absolute values, and no assumption on whether the initial matrix is rectangular or quadratic.

2. A weighted least-squares method

Let the given initial matrix be denoted A_o, which is an (nxm)matrix. Denote the given sequence of row-sums by u_t, t=1,2,..., the sequence of column-sums by v_t, t=1,2,..., $u_t \varepsilon R^n$, $v_t \varepsilon R^m$, and denote the row- and column-sums of the initial matrix A_o by $u_o \varepsilon R^n$, $v_o \varepsilon R^m$. An (nxm)matrix $G=[g_{ij}]$, $g_{ij}>0$, will denote the matrix of weights and the (nxm)matrix $G^+=[g_{ij}^{-1}]$ will denote the matrix containing the inverses of the weights as its elements. Let $e' := [1,1,\ldots,1]$, with the appropriate dimension, and write the elements of A_o as $a_{ij}(0)$, i=1,...,n, j=1,...,m. The updating method aims at generating a matrix $A_t=[a_{ij}(t)]$, t=1,2,..., which solves the following problem:

$$(1) \quad \min_{\{a_{ij}(t)\}} \sum_{i=1}^{n} \sum_{j=1}^{m} g_{ij}(a_{ij}(t)-a_{ij}(0))^2$$

$$\text{s.t.} \quad A_t e = u_t, \; e'A_t = v_t.$$

For reference note the following definitions:

$A_o := [a_{ij}(0)]$, $A_t := [a_{ij}(t)]$, $G := [g_{ij}]$, $g_{ij}>0$,

$G^+ := [g_{ij}^{-1}]$, $e' := [1,1,\dots,1]$;

$A_o e =: u_o$, $e'A_o =: v_o$, $A_t e =: u_t$, $e'A_t =: v_t$,

$G^+ e =: u$, $e'G^+ =: v$,

$\Lambda_t :=$ diag λ_t, $\lambda_t \varepsilon R^n$, $U_t :=$ diag μ_t, $\mu_t \varepsilon R^m$

where $\lambda_t \varepsilon R^n$ and $\mu_t \varepsilon R^m$ will denote Lagrange multipliers.

Writing down the Lagrangian for problem (1)

$$L = \sum_i \sum_j g_{ij}\,(a_{ij}(t)-a_{ij}(0))^2 + \sum_i \lambda_i(t)(u_i(t) - \sum_j a_{ij}(t)) +$$

$$+ \sum_j \mu_j(t)(v_j(t) - \sum_i a_{ij}(t))$$

it is straightforward to obtain the necessary condition for an optimum

(2) $2g_{ij}(a_{ij}(t)-a_{ij}(0)) - \lambda_i(t) - \mu_j(t) = 0$,

which clearly also satisfies the second order condition. Suppressing the t-notation on the Lagrange multipliers one may rewrite (2) as

(2') $A_t = A_o + 1/2\, \Lambda\, G^+ + 1/2\, G^+\, U$

which implies

(3) $u_t = u_o + 1/2(\text{diag } u)\lambda + 1/2\, G^+\, '$,

(4) $v_t = v_o + 1/2\, \lambda' G^+ + 1/2\mu\, (\text{diag } v)$,

where $\lambda \varepsilon R^n$ has been defined as a column vector and $\mu \varepsilon R^m$ as a row vector. Substitution now yields

(5) $\mu = [2(v_t - v_o) - \lambda' G^+]\,(\text{diag } v)^{-1}$

and

$$u_t - u_o = 1/2(\text{diag } u)\lambda + 1/2\; G^+(\text{diag } v)^{-1}\;[2(v_t' - v_o') - G^{+\prime}\lambda] =$$

$$= 1/2(\text{diag } u)\lambda - 1/2\; G^+(\text{diag } v)^{-1} G^{+\prime}\lambda + G^+(\text{diag } v)^{-1}(v_t' - v_o').$$

Defining now

$$B := [(\text{diag } u) - G^+\,(\text{diag } v)^{-1}\,G^{+\prime}]$$

$$\varepsilon_t := \;2(u_t - u_o) - 2G^+(\text{diag } v)^{-1}\,G^{+\prime}(v_t' - v_o')$$

one obtains

(6) $[(\text{diag } u) - G^+(\text{diag } v)^{-1} G^{+\prime}]\lambda \;= 2(u_t - u_o) - 2G^+(\text{diag } v)^{-1} G^{+\prime}(v_t' - v_o')$

from (5) which may now be rewritten, using the above definitions, as

(6') $B\lambda = \varepsilon_t$.

Since μ can be calculated from knowledge of λ equation (6') is the fundamental system of equations to be solved in order to obtain the Lagrange multipliers needed to generate the updated matrix A_t.

Lemma 1: (i) $|B| = 0$,
(ii) $\text{rk}\,(B) = n-1$,
(iii) $\inf_i u_i \leq \xi\,(G^+(\text{diag } v)^{-1} G^{+\prime}) \leq \sup_i u_i$, where
$\xi(.)$ denotes the dominant eigenvalue of the matrix,
(iv) $\sum_i \varepsilon_{it} = 0$.

Proof: (i) $Be = B'e = [(\text{diag } u) - G^+(\text{diag } v)^{-1} G^{+\prime}]\;e =$

$$= \;u - G^+ e = 0,$$

such that the homogeneous system $Bx = 0$, $x \varepsilon R^n$, has a solution, and therefore $|B| = 0$.

(ii) From the definition of B it follows that all its off-diagonal elements b_{ij}, $i \neq j$, are negative, and $Be = 0$ implies

$$b_{jj} = \sum_{\substack{i \\ i \neq j}} |b_{ij}| \, .$$

By taking out the k-th column from B one obtains

$$b_{jj} > \sum_{\substack{i \\ i \neq j \\ i \neq k}} |b_{ij}|$$

such that the matrix B reduced by one dimension has a dominant diagonal and is consequently regular. Thus it follows that $rk(B) = n-1$.

(iii) $G^+(\text{diag } v)^{-1}G^{+'}$ is a (symmetric) positive square matrix which is consequently irreducible, such that (Gantmacher, II, pp.55)

$$\inf_i u_i \leq \xi(G^+(\text{diag } v)^{-1}G^{+'}) \leq \sup_i u_i$$

as required.

(iv) $e'[2(u_t-u_o) - 2\, G^+(\text{diag } v)^{-1}(v_t'-v_o')] = e'\varepsilon_t =$

$$= 2\, e'A_te - 2e'A_te - 2e'A_oe + 2e'A_oe = 0.$$

Lemma 2: (i) $B\lambda = \varepsilon_t \iff B(\lambda+he) = \varepsilon_t, \quad h \in R$

(ii) $\exists\, \lambda \in R^n : B\lambda = \varepsilon_t$.

Proof: (i) $B\lambda = \varepsilon_t \iff B(\lambda+he) = B\lambda + h\, Be = B\lambda = \varepsilon_t$.

(ii) $Be = 0 \iff b_j^{(i)} + B_{(ij)}e = 0,$

where $b_j^{(i)}$ denotes the j-th column of B with the i-th element deleted and $B_{(ij)}$ denotes B with the i-th row and the j-th column deleted. By Lemma 1, (ii), $B_{(ij)}$ is nonsingular and possesses therefore a unique inverse; thus

$$B_{(ij)}^{-1}\, b_j^{(i)} = -e$$

and analogously $b_i^{(j)}B_{(ij)}^{-1} = -e'$, where $b_i^{(j)}$ denotes the i-th row of B with the j-th element deleted. Now reduce (6') by the i-th equation, such that

(7) $\lambda_j b_j^{(i)} + B_{(ij)}\lambda^{(j)} = \varepsilon_t^{(i)}$,

where $\lambda_j \varepsilon R$, $\lambda^{(j)}$ denotes $\lambda \varepsilon R^n$ with the j-th component deleted, $\lambda^{(j)} \varepsilon R^{n-1}$, and $\varepsilon_t^{(i)} \varepsilon R^{n-1}$ denotes ε_t with the i-th component deleted. Now (7) will be satisfied by $\lambda^{(j)}$ if and only if

(8) $\lambda^{(j)} = B_{(ij)}^{-1}\ \varepsilon_t^{(i)} + \lambda_j e$

using $B_{(ij)}^{-1}\ b_j^{(i)} = -e$. Substituting into the i-th equation of (6') yields

$$\lambda_j b_{ij} + b_i^{(j)}\ B_{(ij)}^{-1}\ \varepsilon_t^{(i)} + \lambda_j b_i^{(j)} e = -\ e'\varepsilon_t^{(i)} = \varepsilon_{it}$$

by Lemma 1, (iv).

Lemma 2, (i), shows that all solutions to (6') ly on a manifold in n-dimensional space, which is defined by adding a scalar multiple of the one-vector, e, to any solution of (6'). Lemma 2, (ii), demonstrates the existence of such a solution. Now consider the problem

$$\min_{\{h\}} \left[(\lambda-he)'(\lambda-he) + (\mu+he)(\mu+he)'\right], \quad h \varepsilon R$$

with the first order conditions

$$\sum_{i=1}^{n} 2(\lambda_i-h) + \sum_{j=1}^{m} 2(\mu_j+h) = 0$$

and the second order condition $2(m-n) > 0 \iff m>n$. (For the case $m<n$ one defines $\tilde{h} = -h$ and minimizes with respect to $\tilde{h}$). This particular choise of a vector on the solution manifold of (6') is motivated by

$$[2(v_t - v_o) - (\lambda' - \hbar e)\, G^+]\,(\text{diag } v)^{-1} = \mu + \hbar e \qquad \text{from} \quad (5)$$

and results in

$$(9) \qquad \hbar = \frac{1}{n-m}\,[e'\lambda + \mu e]\ .$$

This choice of $\hbar \varepsilon R$, therefore, leaves $(\lambda_i + \mu_j)$ unaffected but minimizes the Euclidian distance of Lagrange multipliers to the origin.

Proposition 1: There exists a unique solution to the problem

$$\min\,(\lambda'\lambda + \mu\mu'),\ \text{s.t. } B\lambda = \varepsilon_t.$$

Proof: Existence of $\lambda \varepsilon R^n$ is ensured by Lemma 2, (ii). One now has

$$\lambda'\lambda + \mu\mu' = \lambda'\lambda + 4\, v_t(\text{diag } v)^{-2}\, v_t' + 4 v_o(\text{diag } v)^{-2}\, v_o' -$$

$$- 8 v_t(\text{diag } v)^{-2} v_o' - 4\, v_t(\text{diag } v)^{-2} G^{+\prime}\lambda +$$

$$+ 4 v_o(\text{diag } v)^{-2} G^{+\prime}\lambda + \lambda' G^+(\text{diag } v)^{-2} G^{+\prime}\lambda$$

using (5). Letting $\bar{e}_i$ denote a vector containing only zeros except for the i-th element, where it contains 1, the first order condition for the appropriate Lagrangian (with Lagrange multiplier vector z) reads

$$2\lambda_j - 4(v_t - v_o)(\text{diag } v)^{-2} G^{+\prime}\bar{e}_j + 2\lambda' G^+(\text{diag } v)^{-2} G^{+\prime}\bar{e}_j - z\, B\, \bar{e}_j = 0$$

with the second order condition

$$2\, \bar{e}_j'\, G^+(\text{diag } v)^{-2} G^{+\prime}\bar{e}_j + 2 = 2 + 2 \sum_{k=1}^{m} \frac{g_{jk}^2}{v_k^2} > 0\ .$$

Since the objective function in the problem is quadratic and the second derivative, consequently, a constant function with respect to λ_j and positive, the convexity of the objective function and the linearity of the constraints imply a unique global minimum for the problem.

Within the set of vectors solving (6'), there is therefore a unique element which has minimal Euclidian distance to the origin. The proof of this conjecture has not directly been constructive, but it is easy to see that (9) gives the rule to find these "minimal" Lagrange multupliers. The proof of existence of λ satisfying (6') has, however, been constructive. Hence it suffices to proceed as in Lemma 2, (ii), then calculate h as given in (9) and shift the solution to (6') by this h. This will determine the Lagrange multipliers $\{\lambda^*, \mu^*\}$ which are closest to the origin, and will thereby generate a unique update-matrix A_t from (2').

By definition the vectors $\{\lambda_t^*, \mu_t^*\}$ give the partial derivatives with respect to the restrictions in problem (1). Thus they can be interpreted as measures of sensitivity with respect to the restriction of problem (1). Since they are chosen to stay in a neighbourhood of the origin, $\{\lambda_t^*, \mu_t^*\}$ may, loosly speaking, be regarded as measures of the minimal sensitivity of the matrix elements with respect to the restrictions imposed by given row- and column-sums. Hence the values of these vectors may give an indication on the possible location of "structural change". The relative change of their minimal distances to the origin

$$\Delta s_t^* := \frac{(\lambda_t^{*\prime}\lambda_t^*)^{1/2} + (\mu_t^*\mu_t^{*\prime})^{1/2} - (\lambda_{t-1}^{*\prime}\lambda_{t-1}^*)^{1/2} - (\mu_{t-1}^*\mu_{t-1}^{*\prime})^{1/2}}{(\lambda_{t-1}^{*\prime}\lambda_{t-1}^*)^{1/2} + (\mu_{t-1}^*\mu_{t-1}^{*\prime})^{1/2}}$$

may be used as a measure of the speed of "structural change".

The latter interpretation, however, depends crucially on how one defines "structural change" and indirectly, therefore, on the choice of the weights g_{ij}, $i = 1,\dots,n$, $j=1,\dots,m$. The question of how to choose the weights remains to be investigated.

<u>Proposition 2:</u> If $G^+ = A_o \geq 0$, then

$$u_t = (1+g)^t u_o \text{ and } v_t = (1+g)^t v_o \text{ implies}$$

$$\Rightarrow A_t = (1+g)^t A_o.$$

<u>Proof:</u> $G^+ = A_o \geq 0 \Rightarrow v_o = v$, $u_o = u$. By hypothesis $u_t = (1+g)^t u_o$, $v_t = (1+g)^t v_o$, $g \in R$, and consequently

$$\varepsilon_t = [2\ (1+g)^t u - u]\ - 2G^+(\text{diag } v)^{-1}[(1+g)^t v' - v'] =$$

$$= 2[(1+g)^t - 1]\ (u-u) = 0.$$

Therefore (6') has the solution $\lambda = re$, $r \in R$, since

$$Be = 0 \quad \Rightarrow \quad rBe = 0 = \varepsilon_t;$$

moreover from (5) one obtains

$$\mu = 2\ [(1+g)^t - 1]\ v(\text{diag } v)^{-1} - r\ e'G^+(\text{diag } v)^{-1} =$$

$$= 2\ [(1+g)^t - 1]\ e' - re'$$

$$\Leftrightarrow \quad \mu_j = 2[(1+g)^t - 1] \ - r,\ j = 1,\ldots,m$$

$$\Rightarrow \quad 1/2(\lambda_i + \mu_j) = (1+g)^t - 1.$$

From $g_{ij} = 1/a_{ij}(0)$ one obtains

$$a_{ij}(t) = a_{ij}(0) + \frac{1}{2g_{ij}}(\lambda_i + \mu_j) = (1+g)^t\ a_{ij}(0).$$

The conclusion of Proposition 2 turns out also to hold, if A_o is a non-negative matrix except for the j-th column, which contains only non-positive entries. In this case $g_{ij} = 1/|a_{ij}(0)|$ yields the same result as in Proposition 2, except that for the negative $a_{ij}(0)$'s the last equation in the proofs reads

$$a_{ij}(t) = -\ (1+g)^t |a_{ij}(0)|\ , \text{ as required.}$$

Using $g_{ij} = 1/a_{ij}(0)$ one may consider (9) for this case:

$$h = \frac{1}{(n-m)} [nr + 2m(1+g)^t - 1) - mr] =$$

$$= r + \frac{2m}{n-m} (1+g)^t - \frac{2m}{n-m}$$

and consequently

$$\lambda_t^* = re - re - \frac{2m}{n-m} (1+g)^t e + \frac{2m}{n-m} e = \frac{2m}{n-m} [(1+g)^t - 1] e$$

$$\mu_t^* = \frac{2n}{m-n} [1-(1+g)^t] e'.$$

Thus

$$\frac{\lambda_{it}^* + \mu_{jt}^*}{2} |a_{ij}(0)| = \begin{cases} [(1+g)^t - 1] \; |a_{ij}(0)| & \text{if } a_{ij}(0) \geq 0 \\ \\ -[(1+g)^t - 1] |a_{ij}(0)| & \text{if } a_{ij}(0) < 0 \end{cases}$$

i.e. the sum of Lagrange multipliers in the balanced growth case - which could sensibly be regarded as a case, where "structural change" is absent - is

$$\frac{\lambda_{it}^* + \mu_{jt}^*}{2} = (1+g)^t - 1.$$

In empirical application one may therefore take

$$(1+g) := \frac{1}{T} \sum_{t=1}^{T} \frac{e'u_t}{e'u_{t-1}} \xrightarrow[T\to\infty]{} E(1+g)$$

as an estimate for the balanced growth rate. Using this estimate it is possible to analyse the deviations from the balanced growth path

$$\lambda_{it}^* - \frac{2m}{(m-n)} [(1+g)^t - 1] \quad \text{and} \quad \mu_{jt}^* + \frac{2n}{(m-n)} [(1+g)^t - 1]$$

Of course, one could also use the inverse of the dominant eigenvalue of A_o as the growth-factor $(1+g)$ instead of the proposed estimate.

Note that an Input-Output Table can be written in the form

$$A_t = (\text{diag } p_t)\ C_t\ (\text{diag } x_t),$$

where p_t denotes (relative) prices and x_t denotes intensities, while C_t denotes the matrix of technical coefficients. Working empirically "in real terms" amounts to holding prices constant, $p_t = p_o$, such that "structural change" can be split into two parts:

"structural change"

/ \

"sectoral shifts" "technical changes"

The first part, "sectoral shifts", may be identified with variations in x_t, i.e. with shifts in the relative weights of the columns of A_t, while, with constant prices, the "technical change"-part should be reflected in shifts of the relative weights of the rows of A_t. Splitting "structural change" in this way, when working with constant prices, allows the following interpretation of Lagrange multipliers:

(i) The deviation $(\lambda^*_{it} - \frac{2m}{(m-n)}[(1+g)^t - 1])$ indicates "technical changes", i.e. changes in the coefficients of the matrix C_t - although in a rather vague way by summarizing coefficient-changes along rows.

(ii) The deviation $(\mu^*_{jt} + \frac{2n}{(m-n)}[(1+g)^t - 1])$ indicates "structural shifts", i.e. changes in activity levels of the sectors.

If both deviations do not significantly differ from zero, then this is a clearcut notion of the absence of "structural change".

These last remarks demonstrate that the updating-method developed above cannot only be used to generate unique update-matrices, but does also allow for an analysis which may answer questions, more directly relevant to economic policy.

3. Summary

The present paper develops a method to generate update-matrices from a given initial matrix using time-series data on row- and column-sums. The method has two advantages in comparison with the well known RAS-algorithm: It generates unique solutions for the update-matrices and, second, these solutions are optimal in the sense that a weighted sum of squared deviations from the initial matrix-entries is minimized. The idea behing the approach is to calculate appropriate Lagrange-multiplier-vectors. These multipliers, moreover, do possess an interpretation as measures of "structural change" suitable defined. Furthermore, time-series of the optimal Lagrange-multiplier-vectors could be analysed in various ways to gain insight into the connection between the development of row- and column-sums and the evolution of a conservative estimate of the corresponding Input-Output-Tables. Forecasts of the Lagrange-multiplier-vestors could also be used to generate projections of Input-Output-Tables as an extra source of information besides data on the forecasts of economic aggregates.

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