IHS Economics Series
Working Paper 283
February 2012

Capital Income Taxation and Risk Taking under Prospect Theory

Jaroslava Hlouskova
Panagiotis Tsigaris

Institut für Höhere Studien
Institute for Advanced Studies
Vienna

Impressum

Author(s):
Jaroslava Hlouskova, Panagiotis Tsigaris

Title:
Capital Income Taxation and Risk Taking under Prospect Theory

ISSN: Unspecified

2012 Institut für Höhere Studien - Institute for Advanced Studies (IHS)
Josefstädter Straße 39, A-1080 Wien
E-Mail: office@ihs.ac.at
Web: www.ihs.ac.at

283

Reihe Ökonomie

Economics Series

Capital Income Taxation and Risk Taking under Prospect Theory

Jaroslava Hlouskova, Panagiotis Tsigaris

Institut für Höhere Studien
Institute for Advanced Studies
Vienna

283

Reihe Ökonomie

Economics Series

Capital Income Taxation and Risk Taking under Prospect Theory

Jaroslava Hlouskova, Panagiotis Tsigaris

February 2012

Institut für Höhere Studien (IHS), Wien
Institute for Advanced Studies, Vienna

Contact:

Jaroslava Hlouskova
Department of Economics and Finance
Institute for Advanced Studies
Stumpergasse 56
1060 Vienna, Austria
☎: +43/1/599 91-142
email: jaroslava.hlouskova@ihs.ac.at
and
Department of Economics
Thompson Rivers University
Kamloops, BC, Canada

Panagiotis Tsigaris
Department of Economics
Thompson Rivers University
900 McGill Road
Kamloops, BC, Canada
V2C 0C8
email: ptsigaris@tru.ca

Abstract

This research examines capital income taxation for a loss averse investor under some acceptable in the literature reference levels relative to which are the changes in the level of wealth valued. Depending on the reference level, some results indicate that it could be possible for a capital income tax increase not to stimulate risk taking even if the tax code provides the attractive full loss offset provisions. However, risk taking can be stimulated if the investor interprets part of the tax as a loss instead as a reduced gain. Then investor becomes risk seeking and moves away from the discomfort zone of relative losses. This later response to taxation causes private risk taking to increase which is contrary to what evolves from assuming an expected utility model. Finally, a number of other reference standards are examined as well.

Keywords
Risk taking, portfolio choice, prospect theory, loss aversion, reference level, taxation

JEL Classification
G11, H2

Comments

An earlier version of the paper was presented at the 67th annual congress of the International Institute of Public Finance, University of Michigan, Ann Arbor, U.S.A., August 8th -11th, 2011. The authors acknowledge the thoughtful comments of Ines Fortin.

Contents

1 Introduction

If we were to ask investors how they would respond to a full loss offset capital income tax increase in terms of their asset allocation we would probably get many different answers. Some would say that they would not change their risky asset allocation. Others would say that they would increase their risky asset holdings because of the loss offset provisions provided by the tax code. Many would say that they would invest even more to recover the loss from the tax. Still some might say that they would reduce risk taking activity. The answer might well depend on the way the question is framed to the investor. The recent literature in behavioral economics indicates that inattention can cause some taxes not to be obvious or salient, complexity of the tax system can result in people making decision errors, as well as reference dependent preferences could impact how people respond to tax changes.[1]

Positive analysis of the effects of taxation on risk taking activity using the von-Newmann Morgenstern expected utility theory has been well documented in the literature. The general finding is a contradiction to the popular notion that higher taxes tend to discourage risk-taking. This notion of discouragement of investment in risky asset has not been supported by academic research starting from the seminal work of Domar and Musgrave (1944) particularly when full loss offset provisions are present in the tax code. The concept that risk-taking activity can be enhanced by taxation has continued to have support by considering more general expected utility models (Mossin, 1968; Stiglitz, 1969; Sandmo, 1985; Ahsan, 1974, 1989).[2]

However, the expected utility model cannot adequately explain human behavior under risk. An alternative that has been proposed to describe investors behavior is prospect theory (Kahneman and Tversky, 1979). Kahneman and Tversky (KT) experiments found a number of problems with the expected utility model to describe decisions under risk. Decision makers under risk seem to place value on the changes in the level of wealth coded relative to a reference point instead on terminal wealth. They also found that investors exhibit loss aversion which means that investors are more sensitive when they experience a loss in financial wealth than when experiencing a similar size gain. Even when there is no commonly accepted measure of loss aversion in the literature and there are many different alternatives introduced (Abdellaoui et al., 2007), the main characteristic seems to be that at the reference point the utility is steeper in the loss than the gain domain. The probabilities assigned to the utility of the outcomes are not objective but are subjective and weighted in a way to allow one to capture the tendency of people to under-react when faced with large probabilities and overreact to small probability events. Investors also display risk aversion in the domain of gains but become risk lovers when they deal with losses which is not observed with the expected utility model of the von-Newmann Morgenstern type. Finally, prospect theory provides an explanation to many of the anomalies observed in asset returns including the equity premium puzzle (see Benartzi and Thaler, 1995; Barberis et al., 2001).[3]

---

[1] Congdon et al. (2011) in chapter 7 provide a comprehensive literature review in the area of tax and behaviour.

[2] While Ahsan and Tsigaris (2009) demonstrated that discouragement of risky asset investment will occur even under full loss offset provisions if the government is no more efficient in handling risk than the private sector.

[3] Other alternatives to explain the behavior of asset returns include those of habit formation (Abel, 1990; Constantinides, 1990; Campbell and Cochrane, 1999), non-expected utility (Weil, 1990; Epstein and Zin, 1990), and market incompleteness due to uninsurable income risks (Weil, 1992; Heaton and Lucas, 1996;

The purpose of this research is to investigate the effects of capital income taxation on the demand for risky assets using prospect theory. To the best of our knowledge this has not been explored in the literature. The gap in the literature is surprising given that prospect theory has been in existence since the late seventies. In expected utility models the tax affects the investor's terminal wealth but in prospect theory the tax may or may not affect the coding of the reference level to measure gains and losses. This can alter the impact of the tax on the choice of risky asset holdings.[4]

The reference point is usually modelled as one where the investor is interested in maintaining the "status quo". A common benchmark level to code gains and losses is the investor's gross safe return from investing all of the initial wealth (Bernard and Ghossoub, 2010; He and Zhou, 2011; Gomes, 2005; Barberis et al, 2001).[5] Under such a benchmark, the investor experiences "severe" loss of utility when final wealth ends up below the reference level and moderate utility gain when above this level due to loss aversion. Berkelaar et al. (2004), Gomes (2005) and Barberis et al. (2001) use also a dynamic updating rule for the reference point. For example, Gomes (2005) used a weighted average between the gross return of investing all initial wealth in the safe asset and final wealth. He and Zhou (2011) used a reference point that deviates away from the gross risk free return arising from the investment of initial wealth. In general, the focal point of the reference point centers around the gross return from investing initial wealth in the safe asset. Kahneman and Tversky's (1979) experiments coded gains and losses relative to the status quo which they consider to be one's current asset position. However, KT indicate that one's current asset may not be the only coding used by people. On page 286 they state: "Although this is probably true for most choice problems, there are situations in which gains and losses are coded relative to an expectation or aspiration level that differs from the status quo. For example, an unexpected tax withdrawal from a monthly pay check is experienced as a loss, not as a reduced gain." Furthermore, goals could serve as reference levels (Heath et al., 1999). Reference points are assumed to be formed by expectations (Koszegi and Rabin, 2006). Recently, Abeler et al. (2011) using a real-effort experiment find that agents work longer and earn more money when expectations are high relative to those with low expectations.

In portfolio choice models, using the expected utility model, a capital income tax is modelled to affect terminal wealth as a reduced gain and also as a reduced loss due in principle to loss offset provisions of the tax code. But what happens if the investor also codes part of the tax as a loss because (s)he is interested in maintaining the reference level at the pre-tax position? What happens if the investor does not code the tax change into his reference level because of interest to code gains and losses in relation to initial wealth as in KT? What happens to the investor's asset allocation if the reference level is set according to another person's standard? The above questions bring into light that there is no easy answer as to how an investor experiences a tax change in terms of impacting the coding of gains and losses and hence risk taking activity. This research attempts to provide insights to the issue of capital income taxation changes and risk taking under prospect theory.

First, the solution of the optimal proportion invested in the risky asset when the reference level is set at relatively low levels is examined. Setting low reference levels is interpreted as the investor's choice to create a comfort level. A sufficiently loss averse investor follows a

Constantinides and Duffie, 1996).

[4]There are no experimental tests which examine how taxation affects the reference level. This would be a very interesting exercise in the future to pursue.

[5]Barberis et al. (2001) state on page 8: "the status quo scaled up by the risk free rate".

portfolio insurance strategy in order to avoid regions of relative losses as in Gomes (2005). Optimal allocation follows a simple rule. Namely, the amount invested in the risky asset is proportional to the exogenous comfort level. This research also explores the investor's optimal risky asset holding when the reference standard is set high enough to create discomfort levels. Again the solution is proportional to the discomfort level. Thus the response to taxation depends on how the comfort and discomfort levels adjust to tax changes in addition to the traditional effects of taxation on risk taking. Illustration of the impact of taxation is done by using a number of reasonable examples for the setting of the reference level. In particular, two reference standards are explored. When the investor sets the reference standard based on his own wealth level, and when the investor sets the standard relative to other people's reference levels.

When the investor maintains the reference level at the status quo, or one's current asset position, or at the gross after tax risk free return from investing one's initial wealth, a capital income tax change has no impact on stimulating risky asset choice. This result is different from the expected utility model. For example, Stiglitz (1969) found that "A proportional capital income tax leads to an increase in the demand for the risky asset if absolute risk aversion is decreasing and relative risk aversion is increasing or constant" (proposition 2, p. 274). Stimulus of risky asset choice arising from taxation can occur even under prospect theory, but has a different spin. Stimulus can occur if the investor has no reference level as in expected utility models. Encouragement of risk taking with a tax increase can also occur if the investor considers the capital income tax as a loss. This can happen if the investor maintains his reference level at the pre-tax gross return from investing the initial wealth in the safe asset. This high standard causes the investor to become risk seeking in order to recover part of the loss from the tax. We also explore the impact of the tax on private and public sector volatility of final wealth and tax revenues respectively and find that the results differ from the expected utility models when tax imposition is seen as a loss by the investor.

Furthermore, the investor could set reference standards equal to another investor's level in order to code gains and losses. People compare themselves to others all the time and some set their reference levels in relation to others (Falk and Knell, 2004). According to the psychology literature people are governed by the self-enhancement and self-improvement motives. The former motive occurs when people want to make themselves feel better by setting their reference level at low levels possibly reflecting the wealth of poorer people. In terms of our research these sufficiently loss averse investors create comfort zones and become active market participants but follow a conservative generalized portfolio insurance strategy. However, people also place importance on the self-improvement motive. Here people compare themselves with others who are more successful (i.e., the Warren Buffet, Soros, etc.) and as a result set their reference level high, creating a discomfort zone in our terminology, and become risk seeking. Taxation encourages risk taking if the investor is driven by the self-enhancement motive coding gains and losses relative to a poorer or less wealthy investor but it also stimulates risk taking if driven by the self-improvement motive of coding gains and losses relative to more wealthy investors. However, the effect is of a smaller magnitude than an investor who does not have a reference level. In addition, the reasons behind the stimulus under these motives are different.

The paper proceeds as follows. In section 2 the basic model is introduced. Section 3 investigates the effects of a capital income tax that evolve from the basic model when the investor has a comfort level to allocate to the assets. Section 4 examines capital income taxation when the investor has a discomfort level created by having a high reference level.

Section 5 concludes and recommends future directions.

2 The model

The investor is assumed to maximize expectation of the following loss averse utility function[6]

$$U_{LA}(W_2-\Gamma_2)=\left\{\begin{array}{lcll} U_G(W_2-\Gamma_2) & = & \frac{(W_2-\Gamma_2)^{1-\gamma}}{1-\gamma}, & W_2\geq\Gamma_2 \\ \\ \lambda U_L(W_2-\Gamma_2) & = & -\lambda\frac{(\Gamma_2-W_2)^{1-\gamma}}{1-\gamma}, & W_2<\Gamma_2 \end{array}\right\} \tag{1}$$

The loss aversion utility has the KT characteristics. First, the investor is assumed to derive utility from terminal wealth, W_2, relative to a reference level, Γ_2. Second, the individual's reduction in utility arising from a loss in relative financial wealth is greater (in absolute terms) than the marginal utility from a financial gain. The λ parameter captures loss aversion and is greater than unity ($\lambda > 1$).[7] This is consistent with the fact that investors are more sensitive when they experience an infinitesimal loss in financial wealth than when experiencing a similar size relative gain. The individual is risk averse in the domain of relative gains and risk loving in the domain of losses. In the domain $W_2 \geq \Gamma_2$ the marginal utility of wealth is positive $d\frac{U_G}{dW_2} > 0$ and diminishing $d^2\frac{U_G}{dW_2^2} < 0$ which implies risk aversion. In the domain of losses the marginal utility of wealth is $d\frac{\lambda U_L}{dW_2} > 0$ but the marginal utility of wealth increases as wealth increases $d^2\frac{\lambda U_L}{dW_2^2} > 0$ implying that in the region of losses the investor is a risk lover. Finally, the γ parameter determines the curvature of the utility function for relative gains and losses.[8] It is assumed that $\gamma \in (0,1)$ in order to be consistent with the experimental findings of Tversky and Kahneman (1992).[9]

The investor allocates initial wealth, $W_1 > 0$, towards a riskless investment in the amount of m_1 and a risky investment of a_1. The safe asset yields an after tax return $(1-\tau)r > 0$ net of the dollar invested, where we assume a positive before tax return of the safe asset, $r > 0$, and income tax being $\tau \in (0,1)$.[10] Two states of nature i, bad and good, determine the after tax return of the risky asset, $(1-\tau)x_2$. In the good state of nature, the risky asset yields a net of the dollar after tax return of $(1-\tau)x_{2g} > 0$ with probability p and in the bad state of nature the risky asset yields $(1-\tau)x_{2b} < 0$ with probability $(1-p)$.[11] Note that in the bad state of nature the investor gets a tax rebate due to loss offsets in the amount of $-\tau x_{2b}$.

[6] This is the typical Tversky and Kahneman (1992) specification. See Berkelaar et al. (2004), Gomes (2005), He and Zhou (2011), Hwang and Satchell (2010), and Bernard and Ghossoub (2010) for similar loss averse specifications.

[7] Empirical evidence indicate that λ is statistically significantly greater than unity (Booij and van de Kuilen, 2009).

[8] Some research has used different curvature parameter, γ, in the domain of gains than in losses ($\gamma_1 > \gamma_2$). See for example Hwang and Satchell (2010). However, empirical estimates indicate that one cannot reject the null hypothesis of non difference in the value of γ in the domain of gains and losses (Booij and van de Kuilen, 2009). In addition, such preferences ($\gamma_1 > \gamma_2$) cause the risky asset to be an inferior good which is not empirically or theoretically supported.

[9] Booij and van de Kuilen (2009) find the γ parameter to be (statistically) significantly less than unity.

[10] We also examine a capital income subsidy, $\tau < 0$.

[11] In order to keep the analysis simple we do not use subjective weight functions for the probabilities (Tversky and Kahneman, 1992). Future research should examine this area.

Furthermore, the rates of returns of the two assets are assumed to be such that $x_{2b} < r < x_{2g}$ and $x_{2b} > -1$.

Terminal wealth net of the reference level, $W_{2i} - \Gamma_2$, is uncertain and equals

$$W_2 - \Gamma_2 = \left\{ \begin{array}{ll} (1-\tau)(x_{2g} - r)W_1\alpha + (1 + r(1-\tau))W_1 - \Gamma_2, & \text{for} \quad x_2 = x_{2g} \\ \\ (1-\tau)(x_{2b} - r)W_1\alpha + (1 + r(1-\tau))W_1 - \Gamma_2, & \text{for} \quad x_2 = x_{2b} \end{array} \right\} \tag{2}$$

where $\alpha = \frac{a_1}{W_1}$ is the proportion of initial wealth invested in the risky asset and $W_2 = (1-\tau)(x_2 - r)W_1\alpha + (1 + r(1-\tau))W_1$ is the (after tax) final wealth. Thus the investor is assumed to choose the proportion of the risky asset α by solving

$$\left. \begin{array}{rl} \text{Max}_\alpha : & \mathbb{E}(U_{LA}(W_2 - \Gamma_2)) \\ \text{such that} : & W_{2i} - \Gamma_2 = \ (1-\tau)(x_{2i} - r)W_1\alpha + \Omega \end{array} \right\} \tag{3}$$

where Ω is defined as $(1+r(1-\tau))W_1 - \Gamma_2$ and is the residual of the relative wealth level around zero risky investment $W_{2i} - \Gamma_2 \mid_{\alpha=0} = \Omega$.[12] Assuming $\Omega \leq (1+r(1-\tau))W_1$ yields $\Gamma_2 \geq 0$. The Ω term is certain and known to the investor. For reasons that will become obvious shortly we call $\Omega > 0$ the investor's comfort level and the discomfort level when $\Omega < 0$. It is set by the investor prior to his investment decision at the same time as the reference level is selected to code gains and losses. Setting low reference levels results in the investor having a high comfort level and vice versa. A non-negative Ω setting provides a comfort zone to the investor to avoid experiencing relative losses in either state of nature, good or bad, as long as investment or short selling stays within the boundaries $\frac{-\Omega}{(1-\tau)W_1(x_{2g}-r)} \leq \alpha \leq \frac{\Omega}{(1-\tau)W_1(r-x_{2b})}$.[13] The region where $\frac{-\Omega}{(1-\tau)W_1(x_{2g}-r)} \leq \alpha \leq \frac{\Omega}{(1-\tau)W_1(r-x_{2b})}$ is labeled the comfort zone of a loss averse investor. Investment beyond these boundaries leads to experiencing losses. Thus the choice of Ω reflects the loss averse investor's preference to create a zone within his opportunity set and thus provide some comfort so that relative losses in either state of nature will not be observed. The higher the value of Ω the higher the comfort level and thus the comfort zone of a loss averse investor. Setting very high goals (high Γ_2) can lead to $\Omega < 0$. Negative Ω creates a discomfort zone $\frac{\Omega}{(1-\tau)W_1(r-x_{2b})} < \alpha < \frac{-\Omega}{(1-\tau)W_1(x_{2g}-r)}$ where the investor experiences relative losses, irrespective of the state of nature.

3 Capital income taxation with a comfort zone

This section explores comfort levels $\Omega \geq 0$. To proceed with the analysis let's introduce the following notation

$$K_\gamma = \frac{(1-p)(r - x_{2b})^{1-\gamma}}{p(x_{2g} - r)^{1-\gamma}} \tag{4}$$

where K_γ shows the attractiveness of short selling the risky asset, while the inverse $1/K_\gamma$ shows the attractiveness of investing in the risky asset and coincides with thresholds used in He and Zhou (2011). The following proposition states the solution to the problem for an investor who is modest in setting goals and hence does not create a discomfort level; i.e.,

[12] Future research should explore how to make Ω or Γ_2 endogenous and hence a choice variable.

[13] The boundaries are easily obtained from requiring that $W_{2i} - \Gamma_2 \geq 0$.

$\Omega \geq 0$. The case when the investor's reference setting is high and hence $\Omega < 0$ is discussed separately in section 4.

Proposition 1 *Assuming $\Omega \geq 0$ and $\tau < 1$ the problem (3) is well-posed and it is optimal for a loss averse investor to follow a generalized portfolio insurance strategy and invest in the risky asset to protect relative losses to occur provided that $\mathbb{E}(x_2 - r) > 0$ and that the investor is sufficiently loss averse, namely, $\lambda \geq max\{K_\gamma, 1/K_\gamma\}$. The optimal proportion invested in the risky asset is*

$$\begin{aligned} \alpha^* = & \quad \beta\tfrac{\Omega}{(1-\tau)W_1} > 0 \qquad for \quad \Omega > 0 \\ \alpha^* = & \quad 0 \qquad\qquad\qquad for \quad \Omega = 0 \end{aligned} \tag{5}$$

where $\beta = \frac{1-K_0^{1/\gamma}}{r-x_{2b}+K_0^{1/\gamma}(x_{2g}-r)}$.

Proof. See Appendix 1. ■

The above problem is well-posed for a sufficiently loss averse investor, $\lambda \geq \max\{K_\gamma, 1/K_\gamma\}$. Well-posed solution means that the household does not demand infinite leverage to invest (short sell) the risky asset.[14] When $\lambda > 1/K_\gamma$ the investor's loss aversion is strong enough to offset the continuous attractiveness of investing in the risky asset as indicated by $1/K_\gamma$. On the other hand, when $\lambda > K_\gamma$ loss aversion is sufficient to offset the continuous attractiveness of short selling the risky asset as indicated by the value of K_γ. The investor will demand a finite positive amount of the risky asset provided that $\Omega > 0$. If the investor is "sufficiently" loss averse and is compensated for the risk ($\mathbb{E}(x_2 - r) > 0$) then the amount invested in the risky asset will be proportional to the comfort level Ω.[15] (S)he will invest a positive amount in the risky asset by following a generalized portfolio insurance (GPI) strategy. A GPI strategy is the one where "the investor tries to protect current wealth from falling below the reference point" (Gomes 2005, p. 685).[16] The risky asset choice selection fully protects the investor from relative losses.[17] The choice is kept within the comfort zone so that wealth in the bad state of nature is still above the reference level. Simple comparative statics indicate that a lower reference setting, hence a higher comfort level, will lead to an increase in the proportion invested in the risky asset, ceteris paribus.

Figure 1 shows the investor's relative wealth levels in all possible risky investment opportunities given that the reference level is set equal to the current asset position ($\Gamma_2 = W_1$) and hence the comfort level is $\Omega = (1-\tau)rW_1$. In regions 1 (short selling) and 3 (investment) the investor faces possible losses if the good or bad state of nature occurs. Hence a sufficiently loss averse investor, $\lambda > \max\{K_\gamma, 1/K_\gamma\}$, will avoid these areas of investment. The investor

[14]It is possible for the investor to become aggressive towards investing in the risky asset if $\lambda < 1/K_\gamma$ or alternatively to aggressively short sell the risky asset when $\lambda < K_\gamma$. By aggressive we mean demanding infinite leverage. See Appendix 2 for the proof. As stated by He and Zhou (2011) this is an ill-posed problem as the investor will demand to buy or short sell an infinite amount of the risky asset. In this paper we do not examine the ill-posed problems as they would require arbitrary limits to borrowing or risk aversion in the domain of losses.

[15]The optimal solution could also occur in the short selling side provided the investor is sufficiently loss averse and $\mathbb{E}(x_2 - r) < 0$.

[16]Leland (1980) introduced the concept of portfolio insurance strategy.

[17]This can be seen substituting the optimal proportion of the risky asset into the constraint of relative wealth and seeing that it is positive in both states of nature.

will attempt to find an investment level within region 2 which we name the comfort zone of the sufficiently loss averse investor. In this region the investor is shield of relative losses occurring no matter what state of nature evolves. It is easy to show that in region 2 is the expected loss averse utility concave and the optimal investment in the risky asset is given by Proposition 1 and occurs if $\mathbb{E}(x_2 - r) > 0$ and $\lambda > \max\{K_\gamma, 1/K_\gamma\}$.

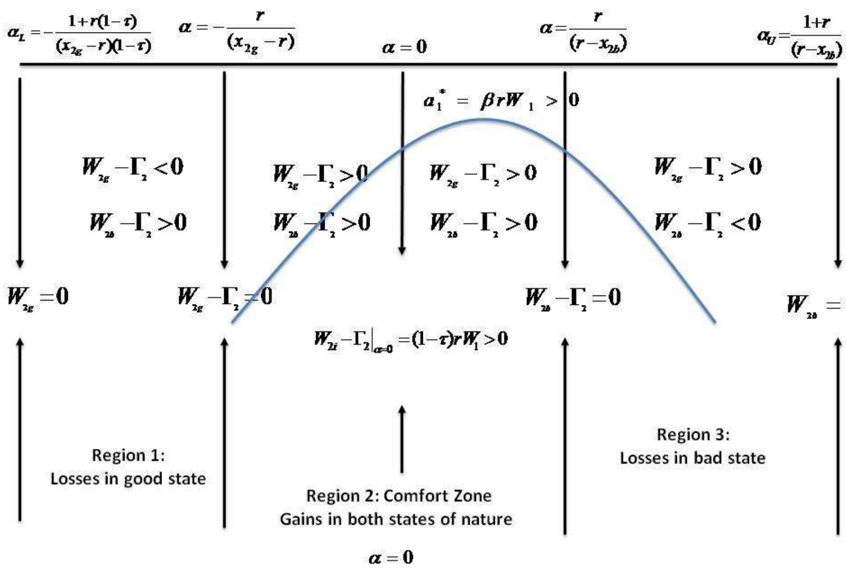

Figure 1: $\Gamma_2 = W_1$

3.1 Analysis of risk taking

The impact of a change in the capital income tax depends not only on income and substitution effects as in the traditional expected utility models but also on the setting of the reference level or the comfort level and the changes of these levels to tax changes.

Under Proposition 1 is the impact of a change of capital income tax on risky asset choice given as

$$\frac{\partial \alpha^*}{\partial \tau} = \frac{\beta}{(1-\tau)^2} \left(\frac{\Omega + (1-\tau)\frac{\partial \Omega}{\partial \tau}}{W_1} \right)$$

The term outside the brackets is similar to the traditional expected utility stimulus on risk taking due to the loss offset provisions of the tax code and arises under the assumption of a constant relative risk aversion and an investor who does not use a reference. In the traditional expected utility model there are two opposite effects operating on risky asset choice. A substitution effect which stimulates risk taking due to the loss-offset provision and an income effect which reduces risk taking because the investor experiences a reduction in real income. Under constant relative risk aversion the substitution effect of the tax is stronger than the income (endowment) effect resulting in an increase in risk taking (see Stiglitz, 1969, Proposition 2, p. 274).

Under prospect theory the total effect is influenced by additional terms which depend on the reference level or the comfort level of the investor and the after tax adjustment of expectations or aspirations (comfort) to the tax change. Below is a number of reasonable reference level illustrations.

Example 1 *Setting* $\Gamma_2 = 0$. *When the investor does not code gains and losses relative to a reference level (or a goal), as in expected utility models, then the comfort level becomes the largest possible* $\Omega = (1 + r(1 - \tau))W_1$ *and the comfort zone the widest possible* $\frac{-(1+r(1-\tau))}{(1-\tau)(x_{2g}-r)} \leq \alpha \leq \frac{1+r(1-\tau)}{(1-\tau)(r-x_{2b})}$ *so that there are no losses when investing or short selling since within this zone* $W_{2i} \geq 0$. *Investment beyond these boundaries leads to experiencing losses,* $W_{2i} < 0$, *and a sufficiently loss averse investor will avoid that as will a risk averse investor. The comfort zone expands as the capital income tax increases. Risk taking is stimulated with a tax because the investor's comfort level drops proportionately less than the drop in the after tax excess return earned by investing in the risky asset,* $\frac{\partial \alpha^*}{\partial \tau} = \frac{\beta}{(1-\tau)^2}$.

Example 2 *Setting* $\Gamma_2 = W_1$. *If the investor uses the current asset position as a reference level to measure relative gains and losses*[18] *then the capital income tax change will not affect the risky asset choice. As the reference level remains unchanged to the tax, the investor's comfort level becomes a function of the after tax safe interest income from initial wealth,* $\Omega = (1 - \tau)rW_1$. *As a result the boundaries of the comfort zone become* $\frac{-r}{x_{2g}-r} \leq \alpha \leq \frac{r}{r-x_{2b}}$ *and are thus unaffected by the tax. When the tax increases, the comfort level* Ω *of the investor falls at the same rate as the reduction in the excess return earned by investing in the risky asset, thus canceling out completely the later positive impact on decision to invest in the risky asset due to loss offset provisions. The investor gains nothing from the loss offset provisions because the tax has reduced his comfort level by the same magnitude, namely by* $(1-\tau)$. *Thus, the investor has no incentive to alter his portfolio allocation towards the risky asset when the reference level is static at the current asset position. This result is different from the expected utility model where, as Stiglitz showed, the tax stimulates investment in the risky asset.*

Example 3 *Setting* $\Gamma_2 = (1 + r(1 - \tau))W_1$. *When the investor sets as a reference level the gross after tax return from investing all of his initial wealth into the safe asset then he does not create a comfort level or a comfort zone for possible investment in the risky asset; i.e.,* $\Omega = 0$. *The investor experiences losses around zero risky investment.*[19] *Setting the reference level to this level can provide one rationale why people do not hold stocks (Haliassos and Bertaut, 1995).*[20] *Risk taking will be unaffected by the tax.*

Example 4 *Setting* $\Gamma_2 = (1+r)W_1$ *when* $\tau < 0$. *An interesting case occurs when the investor keeps the reference level at the pre-tax level of investing initial wealth in the safe asset under a subsidy to capital income (subsidizing gains and taxing losses). The investor interprets the*

[18] KT used one's current asset position as a reference level to code gains and losses which can be interpreted as the investor's initial wealth.

[19] This does not imply that the investor will not undertake risky investment. The investor can still undertake investment but the compensation required to induce investment will be higher than only a positive risk premium.

[20] There are many other explanations as to why many households do not own stocks (i.e., liquidity constraints).

subsidy as providing a comfort level in the amount of tax savings equal to $\Omega = -r\tau W_1 > 0$. *The investor will invest the comfort level* Ω *to the risky asset proportionately if compensated with a positive risk premium and is sufficiently loss averse. The proportion invested in the risky asset will increase as the subsidy increases by* $\frac{\partial\alpha^*}{\partial\tau} = -\frac{\beta}{(1-\tau)^2}r < 0$.

Example 5 *Setting* $\Gamma_2 = (1 + r(1 - \tau))W_{1,O}$, *where* $W_{1,O}$ *is the initial wealth of another investor. When social comparison are done the comfort level becomes* $\Omega = (1+r(1-\tau))(W_1 - W_{1,O})$ *and depends on differences in the initial asset positions of the two investors compared. A loss averse investor creates a comfort level to invest in risky asset if his initial wealth is greater than the one of the comparison investor* $W_1 > W_{1,O}$. *This occurs when the investor is governed by the self-enhancement motive and compares her initial wealth with others that are at lower levels, thus* $W_1 > W_{1,O}$. *Risk taking will increase with the tax increase but by a fraction of what it would had there been no reference level as can be seen from* $\frac{\partial\alpha^*}{\partial\tau} = \frac{\beta}{(1-\tau)^2}\frac{W_1 - W_{1,O}}{W_1} > 0$. *Note that the case of* $W_1 = W_{1,O}$ *is already dealt in Example 3.*

Table 1 below provides a summary of the influence of capital income taxation under different comfort levels $\Omega \geq 0$ for sufficiently loss averse investors.

Reference level: Γ_2	Goal:	Comfort level: Ω	Response: $\frac{\partial\alpha^*}{\partial\tau}$
0	None	$(1 + r(1 - \tau))W_1$	$\frac{\beta}{(1-\tau)^2}$
W_1	Modest	$(1 - \tau)rW_1$	0
$(1 + r(1 - \tau))W_1$	Very high	0	0
$(1 + r)W_1$	Very high	$-r\tau W_1 > 0$, $\tau < 0$	$-\frac{\beta}{(1-\tau)^2}r$
$(1 + r(1 - \tau))W_{1,O}$	$W_1 > W_{1,O}$	$(1 + r(1 - \tau))(W_1 - W_{1,O})$	$\frac{\beta}{(1-\tau)^2}\frac{(W_1 - W_{1,O})}{W_1}$

Table 1: A sufficiently loss averse investor with $\mathbb{E}(x_2 - r) > 0$ and $\Omega \geq 0$

3.2 Private and social risk taking

Another interesting comparative analysis is to examine the impact of the tax on private and social risk taking. Private risk taking was defined by Stiglitz to be the standard deviation of terminal wealth, namely $S(W_2) = (1 - \tau)W_1 |\alpha^*| \sigma$, where σ is the standard deviation of the risky asset's return.[21] Private risk taking is affected directly by taxation via the $(1 - \tau)$ term in $S(W_2)$ and indirectly via changes in the risky asset holdings, namely changes in $|\alpha^*|$ as τ changes. This can be seen also by differentiating $\frac{\partial S(W_2)}{\partial\tau} = -W_1 |\alpha^*| \sigma + (1 - \tau)W_1 \frac{\partial|\alpha^*|}{\partial\tau}\sigma = \sigma\beta\frac{\partial\Omega}{\partial\tau}$, and observing that the first term is the direct effect while the second term is the indirect effect. When the risky asset choice is stimulated this adds to the standard deviation of terminal wealth and offsets some of the direct reduction arising from the increased taxation. Meanwhile public sector risk is defined as the standard deviation of taxes, $S(T) = \tau W_1 |\alpha^*| \sigma$, and is affected by taxes as $\frac{\partial S(T)}{\partial\tau} = W_1|\alpha^*|\sigma + \tau W_1\frac{\partial|\alpha^*|}{\partial\tau}\sigma = \frac{\sigma\beta}{1-\tau}\left(\frac{\Omega}{1-\tau} + \tau\frac{\partial\Omega}{\partial\tau}\right)$. Total risk taking is $S(W_2) + S(T) = W_1 |\alpha^*| \sigma$ and hence total risk taking is proportional to the absolute

[21]This can be seen easily by using the definition of variance, namely it is $\text{Var}(W_2) = (1 - \tau)^2 (\alpha^*)^2 \sigma^2 W_1^2$, where $W_{2i} = [1 + r(1 - \tau) + (1 - \tau)(x_{2i} - r)\alpha]W_1$.

value of the fraction of initial wealth invested in the risky asset. Hence if the investor does not respond to taxation as in some of the above examples, social risk taking remains unaffected. Public sector risk offsets exactly private risk taking. When the reference level is set at zero, social risk taking increases. This later effect was observed in the traditional expected utility models under constant or increasing relative risk aversion (and decreasing absolute risk aversion).[22]

The next section examines capital income taxation when a discomfort level is the case; i.e., $\Omega < 0$.

4 Capital income taxation with a discomfort zone

It could be possible for an investor to set a very high reference or expectation/aspiration level (a high goal) above that of the gross after tax return from investing all of the initial wealth in the safe asset, creating a discomfort zone $\Omega < 0$. When operating in the discomfort zone, $\Omega < 0$, the investor experiences losses under both states of nature. Given that the preferences are risk seeking in the domain of losses, a sufficiently loss averse investor will avoid these areas of relative losses. The following Proposition 2 indicates the conditions when is it optimal to invest in the risky asset, while Proposition 3 describes the conditions under which it is optimal to short sell the risky asset. Utility in these two regions is concave and hence the two solutions are local maxima. The decision as to whether the investor short sells or invests in the risky asset depends again on the value of loss averse parameter relative to the attractiveness of the risky asset.

Proposition 2 *If $\tau \in (0,1)$ and assuming that the investor is sufficiently loss averse, $\lambda > 1/K_\gamma$, then a positive amount of risk taking will happen and a local maximum occurs at*

$$\alpha^{*,+} = \beta_1 \frac{-\Omega}{(1-\tau)W_1} > 0 \tag{6}$$

where $\beta_1 = \dfrac{1+\left(\frac{1}{\lambda K_0}\right)^{\frac{1}{\gamma}}}{x_{2g}-r-\left(\frac{1}{\lambda K_0}\right)^{\frac{1}{\gamma}}(r-x_{2b})} = \dfrac{\left(\frac{1}{\lambda K_\gamma}\right)^{1/\gamma}+\frac{r-x_{2b}}{x_{2g}-r}}{(r-x_{2b})\left[1-\left(\frac{1}{\lambda K_\gamma}\right)^{1/\gamma}\right]}.$

Proof. See Appendix 3. ■

Proposition 3 *If $\tau \in (0,1)$ and assuming that the investor is sufficiently loss averse, $\lambda > K_\gamma$, then a local maximum occurs at*

$$\alpha^{*,-} = \beta_2 \frac{\Omega}{(1-\tau)W_1} < 0 \tag{7}$$

where $\beta_2 = \dfrac{1+\left(\frac{\lambda}{K_0}\right)^{\frac{1}{\gamma}}}{\left(\frac{\lambda}{K_0}\right)^{\frac{1}{\gamma}}(r-x_{2b})-(x_{2g}-r)} = \dfrac{\left(\frac{K_\gamma}{\lambda}\right)^{1/\gamma}+\frac{x_{2g}-r}{r-x_{2b}}}{(x_{2g}-r)\left[1-\left(\frac{K_\gamma}{\lambda}\right)^{1/\gamma}\right]}.$

[22] See Stiglitz (1969).

Proof. See Appendix 3. ■

The optimal solution is proportional to the discomfort level relative to the initial wealth of the investor. The proportionality factor, $\frac{\beta_i}{1-\tau}$, depends on the safe asset return r, the distribution of the risky asset return, and the λ, γ preference parameters of the loss averse investor. Appendix 3 shows the conditions that are required to yield a global maximum. It can be shown that see Proposition 4 in Appendix 3 $\mathbb{E}(U_{LA}(\alpha^{*,+})) > \mathbb{E}(U_{LA}(\alpha^{*,-}))$ occurs if the investor is sufficiently loss averse, and the risky asset is attractive to demand, namely $\lambda > 1/K_\gamma > 1$.

To illustrate the outcome consider the following two reference standards when people see taxes as a loss instead of as reduced gain.

Example 6 *Setting* $\Gamma_2 = (1+r)W_1$ *when* $\tau > 0$. *Consider an investor who wants to keep the reference level at the pre-tax standard as a code for measuring gains and losses either because the tax is unexpected or because the person wants to maintain the pre-tax high reference level. As a result a component of the tax,* $-\tau r W_1$, *is regarded as a loss instead of a reduced gain. The investor's discomfort level is set at the tax loss from safe interest income on initial wealth* $\Omega = -\tau r W_1$.[23]

Figure 2 illustrates Example 6 with the investment opportunities and the corresponding relative wealth levels the investor can achieve under the capital income tax rates within the region $0 < \tau < 1$. Region 1 occurs in the short selling region below the level where the investor short sells enough to neutralize the relative welfare loss if a bad state of nature happens, namely where $W_{2b} - \Gamma_2 = 0$. In region 1, a region with a concave utility, the investor gains if the outcome is a bad state of nature and loses if the good state of nature occurs as expected given short selling activity. Alternatively in region 3, also concave in α, to the far right of Figure 2, the experience is a relative gain if the good state of nature occurs and a loss if the bad state of nature occurs as the investor holds a positive amount of the risky assets. Region 2, the middle region of Figure 2, is the discomfort zone, and is such that $-\frac{r\tau}{(r-x_{2b})(1-\tau)} < \alpha < \frac{r\tau}{(x_{2g}-r)(1-\tau)}$. Here the investor experiences relative wealth losses in the bad and in the good state of nature. A marginal investor, one that is not investing in the risky asset, perceives the capital income tax as a loss in terms of relative wealth levels. When operating in the domain of losses the investor is characterized by risk loving attributes and will move away from region 2. As region 2 is convex in α the maximum will be reached at one of the end points of the region. Note that the discomfort zone increases as the tax increases.

Examining the effect of an increase in capital income taxation under $\Omega = -r\tau W_1$ and when $\lambda > 1/K_\gamma$, demand for the risky asset is stimulated from increased taxation $\frac{\partial \alpha^{*,+}}{\partial \tau} = \frac{1}{\tau(1-\tau)}\alpha^{*,+} > 0$. This increase happens because the increased tax increases the losses which the investor wants to recover leading to more risky investment and such a response re-enforces the stimulus achieved by allowing loss offset provisions. This additional stimulus due to risk seeking activity causes private risk taking also to increase, $\frac{\partial S(W_2)}{\partial \tau} = -W_1\alpha^{*,+}\sigma + (1-\tau)W_1\frac{\partial \alpha^{*,+}}{\partial \tau}\sigma = \frac{1-\tau}{\tau}W_1\alpha^{*,+}\sigma > 0$. In other words, the investor experiences an increase in the standard deviation of final wealth, $S(W_2) = (1-\tau)W_1\alpha^{*,+}\sigma$, with the increased capital income taxation. This result is new and indicates that the government's capital income

[23] Note that here the condition for $\Omega < 0$ is that $\tau r > 0$.

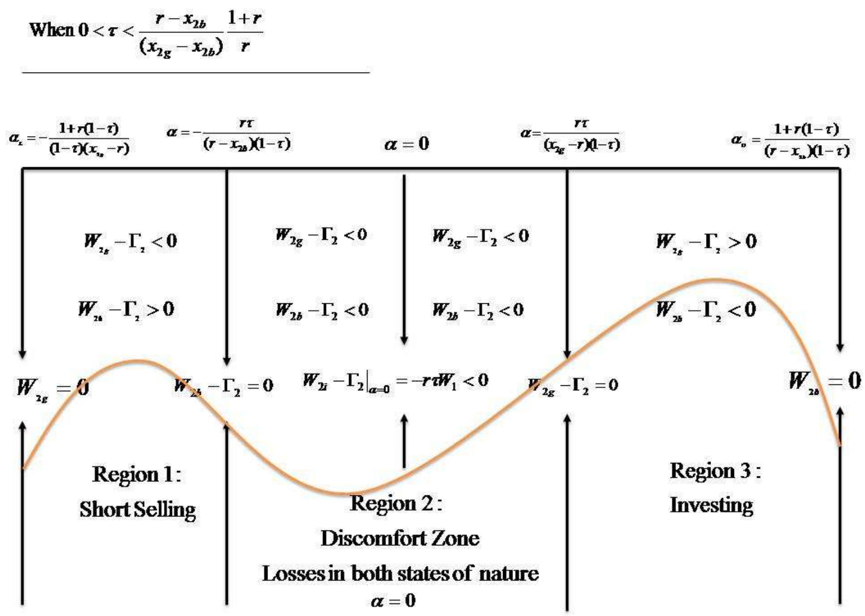

Figure 2: $\Gamma_2 = (1 + r)W_1$

taxation cannot reduce private sector risk even if taxation has a direct reduction effect. The indirect effect of increased risky activity, namely $\frac{\partial \alpha^{*,+}}{\partial \tau} > 0$, on the standard deviation of final wealth, more than offsets the direct reduction in private risk arising from the tax increase. Not only the government has an increased risk in terms of tax revenue fluctuations but the private investor experiences an increase in volatility as measured by standard deviation of final wealth.

Example 7 *Setting* $\Gamma_2 = (1 + r(1 - \tau))W_{1,O}$. *This example is the social comparison which creates a discomfort level for the investor because of the reference level set with respect to rich investors. The investor is motivated by the self-improvement motive and invests in the risky asset. The sufficiently loss averse investor sets the discomfort level to* $\Omega = (1 + r(1 - \tau))(W_1 - W_{1,O}) < 0$ *given* $W_1 < W_{1,O}$ *and invests a proportional amount towards the risky asset provided it is attractive enough* ($K_\gamma < 1$). *The effect of an increase in the capital income tax is to stimulate risk taking,* $\frac{\partial \alpha^{*,+}}{\partial \tau} = -\frac{\beta_1}{(1-\tau)^2} \frac{W_1 - W_{1,O}}{W_1} > 0$ *as was the case with the self-enhancement motive.*

Table 2 below provides a summary of the influence of capital income taxation under two different discomfort levels $\Omega < 0$ for sufficiently loss averse investors and when the risky asset is attractive to demand.

5 Concluding remarks and future directions

We examine capital income taxation under prospect theory and some acceptable in the literature reference levels relative to which are the changes in the level of wealth valued. Response

Reference level: Γ_2	Goal:	Comfort level: Ω	Response: $\frac{\partial \alpha^*}{\partial \tau}$
$(1+r)W_1$	Very high	$-\tau r W_1$	$\frac{\alpha^{*,+}}{\tau(1-\tau)}$
$(1+r(1-\tau))W_{1,O}$	$W_1 < W_{1,O}$	$(1+r(1-\tau))(W_1 - W_{1,0})$	$-\frac{\beta_1}{(1-\tau)^2}\frac{W_1 - W_{1,O}}{W_1}$

Table 2: A sufficiently loss averse investor with $\lambda > 1/K_\gamma > 1$ and $\Omega < 0$

to taxation on risk allocation critically depends on the investor's setting of the reference level. The reference level can be seen as a goal that the loss averse investor sets to make investment decisions (Heath et al., 1999). The setting of a goal serves as a reference level to measure the investor's performance. This setting of the reference point creates a comfort level for the loss averse investor to invest or short sell the risky asset. The effects of taxation are illustrated using some reasonable reference levels such as one's current asset position, $\Gamma_2 = W_1$, or reference levels set at the gross after tax safe return from investing initial wealth, $\Gamma_2 = (1 + r(1-\tau))W_1$. Under these cases, a capital income tax has no effect on risk taking even if it offers the attractive loss offset provisions in the tax code. This result is different from the expected utility models where taxation stimulates risk taking activity. Risk taking activity can also be stimulated under a loss averse investor when the investor either does not set a reference level or alternatively views part of the tax as a loss and maintains the goal at the pre-tax position arising from the gross risk free return of initial wealth. The investor becomes aggressive and either invest or short sells the risky asset in order to move away from the discomfort zone of relative losses. This later case causes private risk taking or volatility to increase which has not been observed in expected utility models. Furthermore, taxation encourages risk taking if the investor is driven by the self-enhancement or the self-improvement motive. The effects of taxation on risk taking are of a smaller magnitude than those for an investor without a reference level.

One major criticism of the above research is the exogenous setting of the reference level and hence the comfort level of a loss averse investor. Future research should attempt and make the reference point a choice variable for an investor. A promising approach is the research by Falk and Knell (2004) who develop a model where people choose their reference level in order to serve the motives of self-enhancement and self-improvement.

Another drawback is that the behavior of a not "sufficiently" loss averse investor is not examined in this framework. It is possible for the investor to become aggressive towards investing in the risky asset if $\lambda < 1/K_\gamma$ or alternatively to aggressively short sell the risky asset when $\lambda < K_\gamma$ (see Appendix 2). When $\lambda < 1/K_\gamma$ the investor's degree of loss aversion is not sufficiently high to offset the continuous attractiveness of investing in the risky asset as indicated by $1/K_\gamma$. On the other hand, when $\lambda < K_\gamma$ loss aversion is not sufficient to offset the attractiveness of short selling the risky asset as indicated by a higher value of K_γ. As a result the investor demands infinite leverage to invest or short sell risky assets. In order to overcome the demand for infinite leverage other constraints or alternative preference structure need to be imposed into the model to limit the desire for infinite leverage. Demand for infinite leverage does not allow us to examine behavior from a tax policy change for such not sufficient loss averse investors. This is a drawback but future research should explore such investors with relatively low loss aversion. Gomes (2005) offers a possible solution by imposing risk aversion again in the domain of losses after a certain threshold level for terminal wealth.

The research has not found theoretical support for the discouragement of risk taking due to the strong influence of loss offset provisions which are maintained. Behavioral responses to taxation have either reduced the responsiveness for change in risky holdings as seen under various comfort levels and social motivational factors or increased the responsiveness as observed in the case of the discomfort level taking the role of tax losses. In order for discouragement to occur the behavioural investor would have to discount fully the benefits associated with loss offset provisions which does not happen in our setup. However, this might occur if the investor believes that the public sector is not better in handling risk than the private. The framework which has been setup in this research allows future exploration for such alternative hypothesis.

We have developed a portfolio choice framework which yields predictions on how taxation affects risk taking based on the exogenous choice of the reference level of the investor. The framwork can be used to examine other reference levels which we have not explored in this paper. However, there are virtually no experiments that examine taxation and risk taking to confirm the predictions of the model. The predictions of the model seem reasonable and consistent with recent evidence (Abeler et al., 2011). Future research should examine how the reference level is determined in portfolio choice models as well as how taxation affects such reference level.

In terms of taxation extensions there are numerous that can be examined within the above framework. One can also analyze a wealth tax and compare it to a capital income tax. The impact of imperfect loss offset provisions in the tax code can be an alternative research agenda. To study the effects of a tax imposed only to the safe asset and thus exempting capital gains (the excess return) from the tax base is another possibility. Finally, this work is a positive analysis and has not indulged into welfare analysis. A welfare analysis would also be of importance. For example, the question as to whether loss offset provisions are necessarily a good tax policy when such provisions might increase aggressive investment behavior and market volatility is of interest to public policy makers.

References

[1] Abdellaoui, M., H. Bleichrodt, and C. Paraschiv, 2007. Loss aversion under prospect theory: A parameter-free measurement, *Management Science,* 53, 1659-1674.

[2] Abel, A., 1990. Asset prices under habit formation and catching up with the Joneses, *American Economic Review,* 80, 38-42.

[3] Abeler, J., A. Falk, L., Goette, and D. Huffman, 2011. Reference points and effort provision, *American Economic Review,* 101, 470-492.

[4] Ahsan, S.M., 1974. Progression and risk-taking, *Oxford Economic Papers,* 26, 318-328.

[5] Ahsan, S.M., 1989. Choice of tax base under uncertainty: Consumption or income?, *Journal of Public Economics,* 40, 99-134.

[6] Ahsan, S.M. and P. Tsigaris, 2009. The efficiency cost of capital income taxation under imperfect loss offset provisions, *Public Finance Review,* 37, 710-731.

[7] Barberis, N., M. Huang, and T. Santos, 2001. Prospect theory and asset prices, *Quarterly Journal of Economics*, 116, 1-53.

[8] Benartzi, S. and R.H. Thaler, 1995. Myopic loss aversion and the equity premium puzzle, *Quarterly Journal of Economics,* 110, 73-92.

[9] Berkelaar, A., R., Kouwenberg, and T., Post, 2004. Optimal portfolio choice under loss aversion, *The Review of Economics and Statistics,* 86, 973–987.

[10] Bernard C. and M. Ghossoub, 2010. Static portfolio choice under cumulative prospect theory, *Mathematics and Financial Economics*, 2, 277-306.

[11] Booij, A.S. and G. van de Kuilen, 2009. A parameter-free analysis of the utility of money for the general population under prospect theory, *Journal of Economic Psychology*, 30, 651-666.

[12] Campbell, J. and J.H. Cochrane, 1999. By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy*, 107, 205-251.

[13] Congdon, W.J., J.R. Kling, and S. Mullainathan, 2011. *Policy and Choice: Public Finance through the Lens of Behavioral Economics*, Brookings Institute Press, *Washington, D.C.*

[14] Constantinides, G.M., 1990. Habit formation: A resolution of the equity premium puzzle, *Journal of Political Economy,* 98, 531-552.

[15] Constantinides G.M. and D. Duffie, 1996. Asset pricing with heterogeneous consumers, *The Journal of Political Economy*, 104, 219-240.

[16] Domar, E.D. and R.A. Musgrave, 1944. Proportional income taxation and risk taking, *Quarterly Journal of Economics*, 58, 388-422.

[17] Epstein, L.G. and S.E. Zin, 1990. First-order risk aversion and the equity premium puzzle, *Journal of Monetary Economics,* 26, 387-407

[18] Falk A. and M. Knell, 2004. Choosing the Joneses: Endogenous goals and reference standards, *Scandinavian Journal of Economics*, 106, 417-435.

[19] Gomes F.J., 2005. Portfolio choice and trading volume with loss-averse investors, *Journal of Business*, 78, 675-706.

[20] Haliassos, M. and C.C. Bertaut, 1995. Why do so few hold stocks?, *The Economic Journal*, 105, 1110-1129.

[21] He, X.D. and Zhou, X.Y., 2011. Portfolio choice under cumulative prospect theory: An analytical treatment. *Management Science*, 57, 315-331.

[22] Heath, C., R.P. Larrick, and G. Wu, 1999. Goals as reference points, *Cognitive Psychology*, 38, 79-109

[23] Heaton J. and D.J. Lucas, 1996. Evaluating the effects of incomplete markets on risk sharing and asset pricing, *Journal of Political Economy,* 104, 443-487.

[24] Hwang, S. and S.E. Satchell, 2010. How loss averse are investors in financial markets?, *Journal of Banking and Finance*, 34, 2425-2438.

[25] Kahneman, D. and A. Tversky, 1979. Prospect theory: An analysis of decision under risk, *Econometrica*, 47, 363-391.

[26] Koszegi, B., and M. Rabin, 2006. A model of reference-dependent preferences, *Quarterly Journal of Economics*, 121, 1133-1165.

[27] Leland, H.E., 1980. Who should buy portfolio insurance?, *Journal of Finance*, 35, 581-590.

[28] Mossin, J., 1968. Taxation and risk-taking: An expected utility approach, *Economica*, 35, 74-82.

[29] Sandmo, A., 1985. The effect of taxation on savings and risk taking. in: Alan J. Auerbach and Martin Feldstein, eds, *Handbook of Public Economics*, Vol. I (Amsterdam, North Holland).

[30] Stiglitz, J.E., 1969. The effects of income, wealth and capital gains taxation on risk taking, *Quarterly Journal of Economics*, 83, 263-283.

[31] Tversky, A. and D. Kahneman, 1992. Advances in prospect theory: Cumulative representation of uncertainty, *Journal of Risk and Uncertainty,* 5, 297-323.

[32] Weil, P., 1990. Non-expected utility in macroeconomics, *Quarterly Journal of Economics*, 105, 29-42.

[33] Weil, P., 1992. Equilibrium asset prices with undiversiable labor income risk, *Journal of Economic Dynamics and Control,* 16, 769-790.

Appendix 1: When $\Omega \geq 0$

Proof of Proposition 1: Let's assume at first that $\Omega > 0$. Based on this and the domain of α, there are three cases that can occur

(P1): $W_{2b} \geq \Gamma_{2b},\ W_{2g} \geq \Gamma_{2g}$ for $-\frac{\Omega}{(1-\tau)W_1(x_{2g}-r)} \leq \alpha \leq \frac{\Omega}{(1-\tau)W_1(r-x_{2b})}$

(P2): $W_{2b} < \Gamma_{2b},\ W_{2g} \geq \Gamma_{2g}$ for $\frac{\Omega}{(1-\tau)W_1(r-x_{2b})} < \alpha \leq +\infty$

(P3): $W_{2b} \geq \Gamma_{2b},\ W_{2g} < \Gamma_{2g}$ for $-\infty \leq \alpha < -\frac{\Omega}{(1-\tau)W_1(x_{2g}-r)}$

The corresponding problems are

$$\left.\begin{array}{rl} \text{Max}_\alpha: & pU_G(W_{2g}-\Gamma_2) + (1-p)U_G(W_{2b}-\Gamma_2) = \\ & \frac{1}{1-\gamma}\left\{p[W_1(1-\tau)(x_{2g}-r)\alpha+\Omega]^{1-\gamma} + (1-p)[-W_1(1-\tau)(r-x_{2b})\alpha+\Omega]^{1-\gamma}\right\} \\ \text{such that}: & -\frac{\Omega}{W_1(1-\tau)(x_{2g}-r)} \leq \alpha \leq \frac{\Omega}{W_1(1-\tau)(r-x_{2b})} \end{array}\right\} \tag{P1}$$

$$\left.\begin{array}{rl} \text{Max}_\alpha: & pU_G(W_{2g}-\Gamma_2) + (1-p)\lambda U_L(W_{2b}-\Gamma_2) = \\ & \frac{1}{1-\gamma}\left\{p[W_1(1-\tau)(x_{2g}-r)\alpha+\Omega]^{1-\gamma} - \lambda(1-p)[W_1(1-\tau)(r-x_{2b})\alpha-\Omega]^{1-\gamma}\right\} \\ \text{such that}: & \frac{\Omega}{W_1(1-\tau)(r-x_{2b})} < \alpha \leq +\infty \end{array}\right\} \tag{P2}$$

$$\left.\begin{array}{rl} \text{Max}_\alpha: & p\lambda U_L(W_{2g}-\Gamma_2) + (1-p)U_G(W_{2b}-\Gamma_2) = \\ & -\frac{\lambda p}{1-\gamma}[-W_1(1-\tau)(x_{2g}-r)\alpha-\Omega]^{1-\gamma} \\ & +\frac{1-p}{1-\gamma}[-W_1(1-\tau)(r-x_{2b})\alpha+\Omega]^{1-\gamma} \\ \text{such that}: & -\infty \leq \alpha < -\frac{\Omega}{W_1(1-\tau)(x_{2g}-r)} \end{array}\right\} \tag{P3}$$

The idea of the proof is to show that (P1) is a concave programming problem with the global maximum being α^* as defined by (5) which is such that $-\frac{\Omega}{(1-\tau)W_1(x_{2g}-r)} \leq \alpha^* \leq \frac{\Omega}{(1-\tau)W_1(r-x_{2b})}$, the utility of (P2) is decreasing for $\lambda > 1/K_\gamma$ and the utility of (P3) is increasing for $\lambda > K_\gamma$.

First order conditions (FOC) for (P1) are

$$p[W_1(1-\tau)(x_{2g}-r)\alpha+\Omega]^{-\gamma}(x_{2g}-r) - (1-p)[-W_1(1-\tau)(r-x_{2b})\alpha+\Omega]^{-\gamma}(r-x_{2b}) = 0$$

and the second order conditions are

$$\begin{aligned} - &\quad \gamma W_1^2 p\,[W_1(1-\tau)(x_{2g}-r)\alpha+\Omega]^{-1-\gamma}\,[(1-\tau)(x_{2g}-r)]^2 - \\ - &\quad \gamma W_1^2(1-p)\,[-W_1(1-\tau)(r-x_{2b})\alpha+\Omega]^{-1-\gamma}\,[(1-\tau)(r-x_{2b})]^2 < 0 \end{aligned}$$

which implies that (P1) is a concave problem. In addition, it can be easily seen that

$$\begin{aligned} \alpha^* &= \frac{\Omega\left[(p(x_{2g}-r))^{1/\gamma} - ((1-p)(r-x_{2b}))^{1/\gamma}\right]}{(1-\tau)W_1\left[(r-x_{2b})(p(x_{2g}-r))^{1/\gamma} + (x_{2g}-r)((1-p)(r-x_{2b}))^{1/\gamma}\right]} \\ &= \frac{\Omega\left(1-K_0^{1/\gamma}\right)}{(1-\tau)W_1\left[r-x_{2b}+K_0^{1/\gamma}(x_{2g}-r)\right]} \end{aligned}$$

satisfies the FOC and that $-\frac{\Omega}{W_1(1-\tau)(x_{2g}-r)} \leq \alpha^* \leq \frac{\Omega}{W_1(1-\tau)(r-x_{2b})}$. As $\mathbb{E}(x_2 - r) > 0$ implies $K_0 < 1$ then $\alpha^* > 0$.

Regarding problem (P2), note that

$$\begin{aligned} 1/K_\gamma &= \frac{p(x_{2g}-r)^{1-\gamma}}{(1-p)(r-x_{2b})^{1-\gamma}} = \frac{p[W_1(1-\tau)(r-x_{2b})\alpha]^\gamma (x_{2g}-r)}{(1-p)[W_1(1-\tau)(x_{2g}-r)\alpha]^\gamma (r-x_{2b})} \\ &> \frac{p[W_1(1-\tau)(r-x_{2b})\alpha - \Omega]^\gamma (x_{2g}-r)}{(1-p)[W_1(1-\tau)(x_{2g}-r)\alpha + \Omega]^\gamma (r-x_{2b})} \end{aligned} \tag{8}$$

Thus, based on (8) and $\lambda > 1/K_\gamma$ it follows that

$$\lambda > \frac{p[W_1(1-\tau)(r-x_{2b})\alpha - \Omega]^\gamma (x_{2g}-r)}{(1-p)[W_1(1-\tau)(x_{2g}-r)\alpha + \Omega]^\gamma (r-x_{2b})}$$

and consequently

$$0 > \frac{d\mathbb{E}(U_{LA}(W_2 - \Gamma_2))}{d\alpha}$$

implying that the utility of (P2) is a decreasing function in α.

The property of the utility of (P3) being an increasing function in α for $\lambda > K_\gamma$ can be shown in a similar way.

Note that for $\Omega = 0$ the set of feasible solutions of (P1) consists only from $\alpha = 0$. As the rest is the same; i.e., utility increases for $\alpha < 0$ and decreases for $\alpha \geq 0$, then the maximum is reached at $\alpha^* = 0$. This concludes the proof. ■

Appendix 2: Aggressive Investor

It is possible for the investor to become aggressive towards investing in the risky asset or alternatively to aggressively short sell the risky asset when the person is not sufficiently loss averse. In this case, the problem is ill-posed and the person will demand to buy or short sell an infinite amount of the risky asset.

Given that $\lambda > 1$ (otherwise investor is not loss-averse). Note that if bounds on α are not imposed then $1/K_\gamma < \lambda < K_\gamma$ implies $\lim_{\alpha\to-\infty} \mathbb{E}(U_{LA}(W_2-\Gamma_2)) = +\infty$, $\lim_{\alpha\to+\infty} \mathbb{E}(U_{LA}(W_2-\Gamma_2)) = -\infty$ and this and continuity of $\mathbb{E}(U_{LA}(W_2-\Gamma_2))$ in α imply that $\alpha^* = -\infty$. Similarly, $K_\gamma < \lambda < 1/K_\gamma$ implies $\lim_{\alpha\to-\infty} \mathbb{E}(U_{LA}(W_2-\Gamma_2)) = -\infty$, $\lim_{\alpha\to+\infty} \mathbb{E}(U_{LA}(W_2-\Gamma_2)) = +\infty$ and this and continuity of $\mathbb{E}(U_{LA}(W_2-\Gamma_2))$ in α imply that $\alpha^* = +\infty$. This concludes the proof. ■

Appendix 3: When $\Omega < 0$

Proof of Propositions 2 and 3:

(P4) $W_{2b} < \Gamma_{2b}$, $W_{2g} < \Gamma_{2g}$ for $\frac{\Omega}{(1-\tau)W_1(r-x_{2b})} < \alpha < \frac{-\Omega}{(1-\tau)W_1(x_{2g}-r)}$

(P5) $W_{2b} < \Gamma_{2b},\ W_{2g} \geq \Gamma_{2g}$ for $\alpha \geq \frac{-\Omega}{(1-\tau)W_1(x_{2g}-r)}$

(P6) $W_{2b} \geq \Gamma_{2b},\ W_{2g} < \Gamma_{2g}$ for $\alpha < \frac{\Omega}{(1-\tau)W_1(r-x_{2b})}$

Thus, the corresponding problems are

$$\left.\begin{array}{rl} \text{Max}_\alpha: & p\lambda U_L(W_{2g}) + (1-p)\lambda U_L(W_{2b}) = \\ & -\frac{1}{1-\gamma}\lambda p[-(x_{2g}-r)(1-\tau)W_1\alpha - \Omega]^{1-\gamma} + \\ & -\frac{1}{1-\gamma}\lambda(1-p)[(r-x_{2b})(1-\tau)W_1\alpha - \Omega]^{1-\gamma} \\ \text{such that}: & \frac{\Omega}{(1-\tau)W_1(r-x_{2b})} < \alpha < \frac{-\Omega}{(1-\tau)W_1(x_{2g}-r)} \end{array}\right\} \tag{P4}$$

$$\left.\begin{array}{rl} \text{Max}_\alpha: & pU_G(W_{2g}) + (1-p)\lambda U_L(W_{2b}) = \\ & \frac{1}{1-\gamma}p[(x_{2g}-r)(1-\tau)W_1\alpha + \Omega]^{1-\gamma} \\ & -\frac{1}{1-\gamma}\lambda(1-p)[(r-x_{2b})(1-\tau)W_1\alpha - \Omega]^{1-\gamma} \\ \text{such that}: & \alpha \geq \frac{-\Omega}{(1-\tau)W_1(x_{2g}-r)} \end{array}\right\} \tag{P5}$$

$$\left.\begin{array}{rl} \text{Max}_\alpha: & p\lambda U_L(W_{2g}) + (1-p)U_G(W_{2b}) = \\ & -\frac{1}{1-\gamma}\lambda p[-(x_{2g}-r)(1-\tau)W_1\alpha - \Omega]^{1-\gamma} \\ & +\frac{1}{1-\gamma}(1-p)[-(r-x_{2b})(1-\tau)W_1\alpha + \Omega]^{1-\gamma} \\ \text{such that}: & \alpha < \frac{\Omega}{(1-\tau)W_1(r-x_{2b})} \end{array}\right\} \tag{P6}$$

Problem (P4) is a convex programming problem (in α) and thus its maximum will be reached at one of the end points.

First order conditions for (P5) are

$$(1-\tau)W_1 p[(x_{2g}-r)(1-\tau)W_1\alpha + \Omega]^{-\gamma}(x_{2g}-r) - \\ -(1-\tau)W_1\lambda(1-p)[(r-x_{2b})(1-\tau)W_1\alpha - \Omega]^{-\gamma}(r-x_{2b}) = 0$$

and it can be easily verified that $\alpha^{*,+}$ given by (6) satisfies them. The second order conditions for (P5) indicate that if $\lambda > 1/K_\gamma$ then (P5) is concave for $\alpha < \hat{\alpha}_U$ and convex for $\alpha > \hat{\alpha}_U$, where

$$\hat{\alpha}_U = \frac{-\Omega\left[\left(\frac{1}{\lambda K_\gamma}\right)^{1/(1+\gamma)} + \frac{r-x_{2b}}{x_{2g}-r}\right]}{(1-\tau)W_1(r-x_{2b})\left[1-\left(\frac{1}{\lambda K_\gamma}\right)^{1/(1+\gamma)}\right]}$$

It can be easily seen from (6) that $(\alpha^*)^+ > \frac{-\Omega}{(1-\tau)W_1(x_{2g}-r)}$ and also that $(\alpha^*)^+ < \hat{\alpha}_U$ as $\lambda > 1/K_\gamma$ and $\lim_{\alpha\to+\infty}\mathbb{E}(U_{LA}(W_2-\Gamma_2)) = -\infty$ as $\lambda > 1/K_\gamma$ and thus the maximum is reached at $(\alpha^*)^+$.

Problem (P5) is concave when

$$-((1-\tau)W_1)^2\gamma p[(x_{2g}-r)(1-\tau)W_1\alpha+\Omega]^{-1-\gamma}(x_{2g}-r)^2+ \\ +((1-\tau)W_1)^2\gamma\lambda(1-p)[(r-x_{2b})(1-\tau)W_1\alpha-\Omega]^{-1-\gamma}(r-x_{2b})^2<0$$

and thus

$$\left[(p(x_{2g}-r)^2)^{1/(1+\gamma)}(r-x_{2b})-(\lambda(1-p)(r-x_{2b})^2)^{1/(1+\gamma)}(x_{2g}-r)\right](1-\tau)W_1\alpha$$

$$>\Omega\left[(p(x_{2g}-r)^2)^{1/(1+\gamma)}+(\lambda(1-p)(r-x_{2b})^2)^{1/(1+\gamma)}\right]$$

As $\lambda>\frac{1}{K_\gamma}$ then

$$\alpha<\frac{\Omega\left[(p(x_{2g}-r)^2)^{1/(1+\gamma)}+(\lambda(1-p)(r-x_{2b})^2)^{1/(1+\gamma)}\right]}{(1-\tau)W_1\left[(p(x_{2g}-r)^2)^{1/(1+\gamma)}(r-x_{2b})-(\lambda(1-p)(r-x_{2b})^2)^{1/(1+\gamma)}(x_{2g}-r)\right]}=\hat{\alpha}_U$$

First order conditions for (P6) are

$$(1-\tau)W_1\lambda p[-(x_{2g}-r)(1-\tau)W_1\alpha-\Omega]^{-\gamma}(x_{2g}-r)- \\ -(1-\tau)W_1(1-p)[-(r-x_{2b})(1-\tau)W_1\alpha+\Omega]^{-\gamma}(r-x_{2b})=0$$

and it can be easily verified that $(\alpha^*)^-$ given by (7) satisfies them. The second order conditions for (P3) indicated that if $\lambda>K_\gamma$ then (P6) is concave for $\alpha>\hat{\alpha}_L$ and convex for $\alpha<\hat{\alpha}_L$, where

$$\hat{\alpha}_L=\frac{\Omega\left[\left(\frac{K_\gamma}{\lambda}\right)^{1/(1+\gamma)}+\frac{x_{2g}-r}{r-x_{2b}}\right]}{(1-\tau)W_1(x_{2g}-r)\left[1-\left(\frac{K_\gamma}{\lambda}\right)^{1/(1+\gamma)}\right]}$$

It can be easily seen from (7) that $(\alpha^*)^-<\frac{\Omega}{(1-\tau)W_1(r-x_{2b})}$ and also that $(\alpha^*)^->\hat{\alpha}_L$ as $\lambda>K_\gamma$ and $\lim_{\alpha\to-\infty}\mathbb{E}(U_{LA}(W_2-\Gamma_2))=-\infty$ as $\lambda>K_\gamma$ and thus the maximum is reached at $(\alpha^*)^-$. This concludes the proof. ■

Proposition 4 *Let $\lambda>\max\{K_\gamma,1/K_\gamma\}$. Then the following holds: (i) $\mathbb{E}(U_{LA}(\alpha^{*,+}))>\mathbb{E}(U_{LA}(\alpha^{*,-}))$ if $K_\gamma<1$, (ii) $\mathbb{E}(U_{LA}(\alpha^{*,+}))<\mathbb{E}(U_{LA}(\alpha^{*,-}))$ if $K_\gamma>1$, and (iii) $\mathbb{E}(U_{LA}(\alpha^{*,+}))=\mathbb{E}(U_{LA}(\alpha^{*,-}))$ if $K_\gamma=1$.*

Proof: Case (i). It can be shown that

$$\begin{aligned}\mathbb{E}(U_{LA}(\alpha^{*,+})) &= \frac{[(x_{2g}-x_{2b})(-\Omega)]^{1-\gamma}}{1-\gamma}\frac{p\left(\frac{1}{\lambda K_0}\right)^{\frac{1-\gamma}{\gamma}}-\lambda(1-p)}{\left[x_{2g}-r-\left(\frac{1}{\lambda K_0}\right)^{\frac{1}{\gamma}}(r-x_{2b})\right]^{1-\gamma}} \\ &= \frac{[(x_{2g}-x_{2b})(-\Omega)]^{1-\gamma}}{1-\gamma}\frac{p\left(\frac{1}{K_0}\right)^{\frac{1-\gamma}{\gamma}}-\lambda^{\frac{1}{\gamma}}(1-p)}{\left[\lambda^{\frac{1}{\gamma}}(x_{2g}-r)-\left(\frac{1}{K_0}\right)^{\frac{1}{\gamma}}(r-x_{2b})\right]^{1-\gamma}}\end{aligned}$$

and

$$\begin{aligned}
\mathbb{E}(U_{LA}(\alpha^{*,-})) &= \frac{[(x_{2g}-x_{2b})(-\Omega)]^{1-\gamma}}{1-\gamma} \frac{-\lambda p\left(\frac{\lambda}{K_0}\right)^{\frac{1-\gamma}{\gamma}}+1-p}{\left[\left(\frac{\lambda}{K_0}\right)^{\frac{1}{\gamma}}(r-x_{2b})-(x_{2g}-r)\right]^{1-\gamma}} \\
&= \frac{[(x_{2g}-x_{2b})(-\Omega)]^{1-\gamma}}{1-\gamma} \frac{-\lambda^{\frac{1}{\gamma}} p\left(\frac{1}{K_0}\right)^{\frac{1-\gamma}{\gamma}}+1-p}{\left[\left(\frac{\lambda}{K_0}\right)^{\frac{1}{\gamma}}(r-x_{2b})-(x_{2g}-r)\right]^{1-\gamma}}
\end{aligned}$$

Thus, showing $\mathbb{E}(U_{LA}(\alpha^{*,+})) > \mathbb{E}(U_{LA}(\alpha^{*,-}))$ (in case (i)) boils down to proving

$$\frac{\lambda^{\frac{1}{\gamma}} p\left(\frac{1}{K_0}\right)^{\frac{1-\gamma}{\gamma}}-1+p}{\lambda^{\frac{1}{\gamma}}(1-p)-p\left(\frac{1}{K_0}\right)^{\frac{1-\gamma}{\gamma}}} > \left[\frac{\left(\frac{\lambda}{K_0}\right)^{\frac{1}{\gamma}}(r-x_{2b})-(x_{2g}-r)}{\lambda^{\frac{1}{\gamma}}(x_{2g}-r)-\left(\frac{1}{K_0}\right)^{\frac{1}{\gamma}}(r-x_{2b})}\right]^{1-\gamma}$$

which follows from the fact that

$$\frac{\lambda^{\frac{1}{\gamma}} p\left(\frac{1}{K_0}\right)^{\frac{1-\gamma}{\gamma}}-1+p}{\lambda^{\frac{1}{\gamma}}(1-p)-p\left(\frac{1}{K_0}\right)^{\frac{1-\gamma}{\gamma}}} = \frac{\left(\frac{\lambda}{K_0}\right)^{\frac{1}{\gamma}}(r-x_{2b})-(x_{2g}-r)}{\lambda^{\frac{1}{\gamma}}(x_{2g}-r)-\left(\frac{1}{K_0}\right)^{\frac{1}{\gamma}}(r-x_{2b})} > 1 \tag{9}$$

where the last inequality is implied by $K_\gamma < 1$. Cases (ii) and (iii) directly follow. This concludes the proof. ■

Authors: Jaroslava Hlouskova, Panagiotis Tsigaris

Title: Capital Income Taxation and Risk Taking under Prospect Theory

Reihe Ökonomie / Economics Series 283

ISSN: 1605-7996

Stumpergasse 56, A-1060 Vienna • ☎ +43 1 59991-0 • Fax +43 1 59991-555 • http://www.ihs.ac.at

ISSN: 1605-7996