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Panel VAR Models with Spatial Dependence

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Abstract

I consider a panel vector-autoregressive model with cross-sectional dependence of the disturbances characterized by a spatial autoregressive process. I propose a three-step estimation procedure. Its first step is an instrumental variable estimation that ignores the spatial correlation. In the second step, the estimated disturbances are used in a multivariate spatial generalized moments estimation to infer the degree of spatial correlation. The final step of the procedure uses transformed data and applies standard techniques for estimation of panel vector-autoregressive models. I compare the small-sample performance of various estimation strategies in a Monte Carlo study.

Keywords

Spatial PVAR, multivariate dynamic panel data model, spatial GM, spatial Cochrane-Orcutt transformation, constrained maximum likelihood estimation

JEL Classification

C13, C31, C33

Comments
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Contents

1 Introduction

Vector autoregressive (VAR) models are extensively used in econometric applications in a wide variety of fields. The extension to panel data represents an interesting challenge due to the likely presence of cross-sectional heterogeneity. In this paper I tackle the issue by considering a panel VAR model with a particular class of dependence structure in the disturbances. I consider the situation where the time dimension T is fixed. As a result, the correlations across cross-sectional units have to be parsimoniously parameterized in order to avoid the incidental parameters problem.[1] I follow the spatial econometrics literature and study a first order spatial autocorrelation model with a known spatial weighting matrix. Of course there are other ways to specify the spatial dependence in the model and prominent alternatives include a non-parametric specification generalizing Conley (1991) and Chen and Conley (2001), or common factor specification inspired by work of Spearman (1904); see e.g. Mulaik, 1972, or Ng and Bai, 2008, for a more recent treatment and overview.

The panel spatial autocorrelation model considered in this paper is a generalization of spatial econometric models that include single equation models, e.g., Cliff and Ord (1973, 1981), and simultaneous equation models, such as Whittle (1954), Anselin (1988) or Kelejian and Prucha (1998, 1999 and 2004). Lee (2004) provides formal large sample results for a (quasi) maximum likelihood estimator of a static single cross-section model. Extensions to panel data with single equation include Lee and Yu (2008) who extend the formal results for the maximum likelihood estimation for static panel data models, and Kapoor et al. (2007) who introduce and derive formal large sample results for the spatial generalized moments method.

On the other hand, the current paper extends the panel VAR literature to allow for cross-sectional dependence of the model disturbances; for models with independent disturbances see, e.g., Binder et al. (2005) for the quasi maximum likelihood (QML) and minimum distance (MD) estimators, or Arellano and Bond (1991), Ahn and Schmidt (1995) and Arellano and Bover (1995) for the generalized method of moments (GMM) approach in a single equation framework.

The next section will specify the model and state the assumptions maintained throughout the paper. Section 3 describes the various estimation procedures, while Section 4 presents the results from a Monte Carlo study comparing small-sample performance of these estimators. Section 5 then concludes.

2 The Panel VAR Model

In this section I specify the model and discuss the main assumptions that will be maintained throughout. The specification adopted here uses the spatial autoregressive framework with known spatial weighting matrix to capture the heteroscedasticity in the data. Hence it replaces the assumption that the disturbances of the model are independent among units and as such can be viewed as an alternative to other approaches such a principle component models.

[1] This would be, for example, the case in the seemingly unrelated regressions model with fixed time dimension. There, the number of parameters grows quadratically with the sample size.

The model under consideration can be expressed as a first order panel VAR model:

$$\begin{aligned} \mathbf{y}_{it} &= \mathbf{\Phi}\mathbf{y}_{i,t-1} + \mathbf{u}_{it}, \\ \mathbf{u}_{it} &= \lambda \sum_{j=1}^{N} \mathbf{w}_{ij}\mathbf{u}_{jt} + (\mathbf{I}_m - \mathbf{\Phi})\,\boldsymbol{\mu}_i + \boldsymbol{\varepsilon}_{it} \end{aligned} \tag{1}$$

where the first subscript $i \in \{1,..,N\}$ refers to the cross-sectional dimension and the second subscript $t \in \{1,..,T\}$ refers to the time dimension of the panel of observations $\{\mathbf{y}_{it}\}_{1\leq t\leq T}^{1\leq i\leq N}$. I also allow the model to contain more than one equation and so the observations $\mathbf{y}_{it}$, the individual-specific effects $\boldsymbol{\mu}_i$ and the disturbances $\mathbf{u}_{it}$ and $\boldsymbol{\varepsilon}_{it}$ are $m \times 1$ vectors and the known weighting parameters $\mathbf{w}_{ij}$, the unknown model parameters $\mathbf{\Phi}$ and the identity matrix $\mathbf{I}_m$ are all $m \times m$ matrices. The degree of spatial autocorrelation is captured by the scalar parameter λ. Note that I restrict the individual effects to be of the form $(\mathbf{I}_m - \mathbf{\Phi})\,\boldsymbol{\mu}_i$ so that when the model contains units roots (for example when $\mathbf{\Phi} = \mathbf{I}_m$), the trending behavior remains the same as in the stationary case.

Stacking across individuals we obtain

$$\begin{aligned} \mathbf{y}_t &= (\mathbf{I}_N \otimes \mathbf{\Phi})\,\mathbf{y}_{t-1} + \mathbf{u}_t, \\ \mathbf{u}_t &= \lambda \mathbf{W}\mathbf{u}_t + [\mathbf{I}_N \otimes (\mathbf{I}_m - \mathbf{\Phi})]\,\boldsymbol{\mu} + \boldsymbol{\varepsilon}_t, \end{aligned} \tag{2}$$

where

$$\begin{aligned} \mathbf{y}_t &= \underset{mN\times 1}{(\mathbf{y}_{1t}', .., \mathbf{y}_{Nt}')'}, \quad \boldsymbol{\mu} = \underset{mN\times 1}{(\boldsymbol{\mu}_1', .., \boldsymbol{\mu}_N')'} \\ \mathbf{u}_t &= \underset{mN\times 1}{(\mathbf{u}_{1t}', .., \mathbf{u}_{Nt}')'}, \quad \boldsymbol{\varepsilon}_t = \underset{mN\times 1}{(\boldsymbol{\varepsilon}_{1t}', .., \boldsymbol{\varepsilon}_{Nt}')'} \end{aligned} \tag{3}$$

and the $mN \times mN$ weighting matrix $\mathbf{W}$ is

$$\mathbf{W} = \begin{pmatrix} \mathbf{w}_{11} & \cdots & \mathbf{w}_{1N} \\ \vdots & \ddots & \vdots \\ \mathbf{w}_{N1} & \cdots & \mathbf{w}_{NN} \end{pmatrix}_{mN\times mN} \tag{4}$$

Solving for the disturbance terms yields[2]

$$\mathbf{u}_t = (\mathbf{I}_{mN} - \lambda\mathbf{W})^{-1}\left([\mathbf{I}_N \otimes (\mathbf{I}_m - \mathbf{\Phi})]\,\boldsymbol{\mu} + \boldsymbol{\varepsilon}_t\right). \tag{5}$$

Observe that the solution to the disturbance process depends on the sample size N and hence the disturbances as well as the observed process $\mathbf{y}_{it}$ form triangular arrays. However, I omit the indexation by N in order to maintain legible notation.

I will next motivate and then formulate the basic set of maintained assumptions. These will consist of an assumption on the innovations ε_{it} (e.g. that they are independently distributed), an optional assumption on the individual effects μ_i, restrictions on the spatial weights $\mathbf{W}$ and the parameter space (λ) that guarantee stability in the spatial dimension, and finaly an assumption on how the initial observation of the process was generated.

[2]Note that this assumes that the inverse of $(\mathbf{I}_{mN} - \lambda\mathbf{W})$ exists. This will indeed be the case under the assumptions maintained in the paper (see Section 2.3).

2.1 Random vs. Fixed Effects Specification

Allowing for individual effects without any additional restrictions, leads to an incidental parameters problem. As the time dimension of the panel is fixed, one cannot consistently estimate a general form of the individual-specific effects with a finite number of observations per parameter. To resolve this problem, there are two options. Either to assume that there is a well-behaved distribution (e.g. with finite fourth moments) from which the individual-specific effects are generated (the random effects specification), or transform the data to obtain specification that does not contain the individual-specific effects (the fixed effect specification). The usual approach in the fixed effect specification is to first-difference the data; see the argument in Hsiao, Pesaran and Tahmiscioglu (2002) who show in a univariate context that the QML estimator is invariant to the choice of the transformation matrix that eliminates the individual-specific effects. The argument is readily extended to the multivariate setting in this paper. However, the fixed effect specification and first-differencing does not eliminate the incidental parameter problem unless we assume that the spatial weighting matrices are constant over time. Hence the choice between fixed and random effects specification depends on which of the two assumptions (constant weighting matrix or existence of the distribution that generates the individual-specific effects) is more appropriate.

In this paper I use the transformed likelihood approach (e.g. fixed effects specification). Nevertheless, the initial estimation procedure I suggest in this paper, can incorporate the random effects assumption. In particular, in the second step of the procedure is a spatial generalized moments method that uses estimated disturbances from the first step. The spatial GM estimation provides estimates of the degree of spatial autocorrelation in the disturbances, as well as an estimates of the variance covariance matrices of both the independent innovations and the individual effects.[3] Hence an extension of the likelihood approach to include individual effects would be straightforward and it is easily implemented using the procedure discussed in this paper.

2.2 Initial Disturbances Specification

Instead of conditioning on initial observations, I explicitly treat the initial conditions when defining the likelihood function. There are several assumptions one can make. Since the data is not observed beyond the time period 0, the initial observations $\mathbf{y}_0$ are just equal to the initial disturbances (which now potentially include all the lagged effects), i.e.

$$\mathbf{y}_0 = \mathbf{u}_0. \tag{6}$$

I assume that $\mathbf{u}_0$ is spatially correlated and is generated by

$$\mathbf{u}_0 = \lambda \mathbf{W} \mathbf{u}_0 + \boldsymbol{\mu} + \boldsymbol{\xi}, \tag{7}$$

where $\boldsymbol{\xi} = (\boldsymbol{\xi}_1', .., \boldsymbol{\xi}_N')'$ with each $\boldsymbol{\xi}_i$ being an $m \times 1$ vector of independently (of $\boldsymbol{\mu}_j$ and ε_{jt}, $t > 0$) and identically distributed initial random disturbances with a constant (over i) variance covariance matrix $\boldsymbol{\Omega}_\xi$.

[3] However, the unweighted spatial GM procedure does not utilize the random effects assumption and hence is suitable for the fixed effects specification.

Hence the initial observations in first differences are

$$\begin{aligned}\Delta \mathbf{y}_1 &= (\mathbf{I}_N \otimes \boldsymbol{\Phi})\, \mathbf{y}_0 + \mathbf{u}_1 - \mathbf{y}_0 \\ &= \mathbf{u}_1 - [\mathbf{I}_N \otimes (\mathbf{I}_m - \boldsymbol{\Phi})]\, \mathbf{u}_0 \\ &= (\mathbf{I}_{mN} - \lambda \mathbf{W})^{-1} (\boldsymbol{\varepsilon}_1 - [\mathbf{I}_N \otimes (\mathbf{I}_m - \boldsymbol{\Phi})]\, \boldsymbol{\xi})\,.\end{aligned} \tag{8}$$

I denote by $\boldsymbol{\Psi}$ the variance covariance matrix of the initial observation in first differences $(\Delta \mathbf{y}_1)$ after the spatial autocorrelation is removed, i.e.

$$\boldsymbol{\Psi} = VC\left[(\mathbf{I}_{mN} - \lambda \mathbf{W})\, \Delta \mathbf{y}_1\right], \tag{9}$$

where the notation $VC(.)$ stands for variance covariance matrix. Given that $\boldsymbol{\Phi} \neq \mathbf{I}_m$, we have that

$$\boldsymbol{\Psi} = \boldsymbol{\Omega}_\varepsilon + (\mathbf{I}_m - \boldsymbol{\Phi})\, \boldsymbol{\Omega}_\xi\, (\mathbf{I}_m - \boldsymbol{\Phi}')\,, \tag{10}$$

and hence $\boldsymbol{\Psi}$ is unconstrained and the entries in it will enter as additional parameters into the likelihood function. In the pure unit root case $(\boldsymbol{\Phi} = \mathbf{I}_m)$, the variance of the initial innovations is constrained to be $\boldsymbol{\Psi} = \boldsymbol{\Omega}_\varepsilon$.

In general, if the eigenvalues of $\boldsymbol{\Phi}$ are inside the unit circle, one could make further assumptions on the $\boldsymbol{\xi}$ disturbances and express $\boldsymbol{\Psi}$ in terms of $\boldsymbol{\Phi}$ and $\boldsymbol{\Omega}_\varepsilon$. In particular, since in this case the data generating process is dynamically stable and, therefore, one could assume that it has started in an infinite past. This would imply that the initial observations $\mathbf{y}_0$ are drawn from the limiting stationary distribution of the process, e.g. that:

$$\mathbf{y}_0 = (\mathbf{I}_{mN} - \lambda \mathbf{W})^{-1} \sum_{j=0}^{\infty} (\mathbf{I}_N \otimes \boldsymbol{\Phi})^{j-1} \left([\mathbf{I}_N \otimes (\mathbf{I}_m - \boldsymbol{\Phi})]\, \boldsymbol{\mu} + \boldsymbol{\varepsilon}_{-j}\right) \tag{11}$$

Therefore, the initial observations in first differences are

$$\Delta \mathbf{y}_1 = (\mathbf{I}_{mN} - \lambda \mathbf{W})^{-1} \sum_{j=0}^{\infty} (\mathbf{I}_N \otimes \boldsymbol{\Phi})^{j-1}\, \Delta \boldsymbol{\varepsilon}_{1-j} \tag{12}$$

and

$$\begin{aligned}VC\left[(\mathbf{I}_{mN} - \lambda \mathbf{W})\, \Delta \mathbf{y}\right] &= \mathbf{I}_N \otimes VC\left(\sum_{j=0}^{\infty} \boldsymbol{\Phi}^{j-1} \Delta \boldsymbol{\varepsilon}_{i,-j}\right) \\ &= \mathbf{I}_N \otimes VC\left(\boldsymbol{\varepsilon}_{i0} + (\mathbf{I}_m - \boldsymbol{\Phi}) \sum_{j=0}^{\infty} \boldsymbol{\Phi}^j \boldsymbol{\varepsilon}_{i,-j-1}\right) \\ &= \mathbf{I}_N \otimes \left[\boldsymbol{\Omega}_\varepsilon + (\mathbf{I}_m - \boldsymbol{\Phi}) \left(\sum_{j=0}^{\infty} \boldsymbol{\Phi}^j \boldsymbol{\Omega}_\varepsilon \boldsymbol{\Phi}'^j\right) (\mathbf{I}_m - \boldsymbol{\Phi}')\right],\end{aligned} \tag{13}$$

or given the definition of $\boldsymbol{\Psi}$, we have

$$\boldsymbol{\Psi} = \left[\boldsymbol{\Omega}_\varepsilon + (\mathbf{I}_m - \boldsymbol{\Phi}) \left(\sum_{j=0}^{\infty} \boldsymbol{\Phi}^j \boldsymbol{\Omega}_\varepsilon \boldsymbol{\Phi}'^j\right) (\mathbf{I}_m - \boldsymbol{\Phi}')\right]. \tag{14}$$

Hence, I distinguish three assumptions on how the elements of $\boldsymbol{\Psi}$ are determined:

1. (UR) In the pure unit root case ($\mathbf{\Phi} = \mathbf{I}_m$), we have to set $\mathbf{\Psi} = \mathbf{\Omega}_\varepsilon$.
2. (IOR) When all of the eigenvalues of $\mathbf{\Phi}$ are inside the unit circle, we could impose an additional assumption and restrict the elements of $\mathbf{\Psi}$ to be a function of $\mathbf{\Omega}_\varepsilon$ and $\mathbf{\Phi}$, i.e. or rewriting the expression in the equation above:

$$vec\mathbf{\Psi} = \mathbb{D}vech\mathbf{\Omega}_\varepsilon + \left[(\mathbf{I}_m - \mathbf{\Phi}) \otimes (\mathbf{I}_m - \mathbf{\Phi})\right]\left[\mathbf{I} - (\mathbf{\Phi} \otimes \mathbf{\Phi})\right]^{-1} \mathbb{D}vech\mathbf{\Omega}_\varepsilon, \tag{15}$$

   where $\mathbb{D}$ is a duplication matrix such that $vec\mathbf{\Omega}_\varepsilon = \mathbb{D}vech\mathbf{\Omega}_\varepsilon$.
3. (NR) No restrictions are placed on elements of $\mathbf{\Psi}$ (other then imposing that $\mathbf{\Psi}$ is symmetric and strictly positive definite matrix).

In all of the cases, we have that the variance covariance matrix of the first difference of the initial observations is

$$E\left(\Delta\mathbf{y}_1\Delta\mathbf{y}_1'\right) = \left(\mathbf{I}_{mN} - \lambda\mathbf{W}\right)^{-1}\mathbf{\Psi}\left(\mathbf{I}_{mN} - \lambda\mathbf{W}'\right)^{-1}. \tag{16}$$

2.3 Maintained Assumptions

In order to guarantee that the model and the estimation procedure is well defined, I maintain the following assumptions about the disturbances and the spatial weighting matrices.

Assumption 1 *The disturbance vectors $\boldsymbol{\varepsilon}_{it}$ are identically and independently (of $\boldsymbol{\varepsilon}_{js}$ for $j \neq i$) distributed with zero mean, and finite absolute $4+\delta$ moments for some $\delta > 0$. Furthermore, the vector $\boldsymbol{\varepsilon}_{it}$ has a finite positive-definite variance matrix $\mathbf{\Omega}_\varepsilon$.*

The above assumption is needed to ensure that the observable data, which is a transformation of the $\boldsymbol{\varepsilon}_{it}$ process, has a well-defined asymptotic properties.

The next two assumptions ensure that the weighting matrices do not 'explode' as the sample size increases.

Assumption 2 *The matrices $(\mathbf{I}_{mN} - \gamma\mathbf{W})$ are nonsingular for all $|\gamma| < 1/\rho(\mathbf{W})$, where $\rho(.)$ denotes the spectral radius of a matrix. Furthermore, the parameter λ also satisfies $|\lambda| < 1/\rho(\mathbf{W})$.*

Assumption 3 *The row and column sums of the matrices $\mathbf{W}$ and $(\mathbf{I}_{mN} - \lambda\mathbf{W})^{-1}$ are uniformly bounded in absolute value.*

Finally, I formalize the discussion in the preceding section into an assumption on the generation of the initial observation in first differences:

Assumption 4 *The initial observations $\Delta\mathbf{y}_{i0}$ are identically and independently (of $\boldsymbol{\varepsilon}_{jt}$ for $t > 0$) distributed with zero mean, and finite absolute $4+\delta$ moments for some $\delta > 0$. Furthermore, the vector $\Delta\mathbf{y}_{i0}$ has a finite positive-definite variance matrix $\mathbf{\Psi}$, given by one of the following:*

(UR) $\mathbf{\Psi} = \mathbf{\Omega}_\varepsilon$, *and* $\mathbf{\Phi} = \mathbf{I}_m$,

(IOR) $vec\mathbf{\Psi} = \mathbb{D}vech\mathbf{\Omega}_\varepsilon + \left[(\mathbf{I}_m - \mathbf{\Phi}) \otimes (\mathbf{I}_m - \mathbf{\Phi})\right]\left[\mathbf{I} - (\mathbf{\Phi} \otimes \mathbf{\Phi})\right]^{-1} \mathbb{D}vech\mathbf{\Omega}_\varepsilon$, *and* $\mathbf{\Phi} \neq \mathbf{I}_m$,

(NR) no restrictions are placed on $\mathbf{\Psi}$, *and* $\mathbf{\Phi} \neq \mathbf{I}_m$,.

3 Estimation

The model can be estimated using a variety of approaches. Straightforward least squares estimation of the first differences of the observations on its lagged values is not consistent because the error term $\Delta \mathbf{u}_t$ is correlated with the explanatory variable $\Delta \mathbf{y}_{t-1}$. However, based on results in Mutl (2006), there is an alternative instrumental variable (IV) estimation that leads to a consistent estimates of the spatially correlated disturbances. Given an initial estimator of the slope coefficients, we can then use a spatial generalized moments estimation (spatial GM) to obtain a consistent estimator of the spatial parameter λ; e.g. use the moment conditions based on the estimated disturbances:

$$\widehat{\mathbf{u}}_t = \mathbf{y}_t - \left(\mathbf{I}_N \otimes \widehat{\mathbf{\Phi}}_{IV} \right) \mathbf{y}_{t-1} \tag{17}$$

where $\widehat{\mathbf{\Phi}}_{IV}$ is the IV estimator of $\mathbf{\Phi}$. Generalizing the univariate results in Kapoor et al. (2007), it then follows that this two stage procedure leads to a consistent estimator of λ.

Finally, in the last step of the proposed estimation procedure, we can use the spatial Cochrane-Orcutt transformation and write the model as[4]

$$(\mathbf{I}_{mN} - \lambda \mathbf{W}) \Delta \mathbf{y}_t = (\mathbf{I}_{mN} - \lambda \mathbf{W}) (\mathbf{I}_N \otimes \mathbf{\Phi}) \Delta \mathbf{y}_{t-1} + \Delta \boldsymbol{\varepsilon}_t. \tag{18}$$

If λ is known, the transformed model can be estimated with standard techniques, such as the quasi-maximum likelihood (QML) method in Binder, et al. (2005) or a multivariate extension of the generalized method of moments (GMM) approach of Arellano and Bond (1991), Ahn and Schmidt (1995), or Arellano and Bover (1995).

An alternative to the above procedure is to use the maximum likelihood function of the entire model and obtain a QML estimate. Given the computational complexity of this approach, is important to have reasonable initial estimates. Hence even if one ultimately employs the full likelihood approach, it is of interest to study the properties of the initial estimators. In the following, I first define the IV estimator and then discuss the spatial GM estimator of the spatial parameter. Finally, I define the full as well as the constrained QML procedures.

3.1 Initial Estimation

Unlike the transformed likelihood approach, the initial estimators are based on moment conditions that involve (lagged) levels of the endogenous variable. Therefore, their large (as well as small) sample properties are not independent of the distribution of the individual effects. Similarly, the spatial GM procedure is based on estimated levels of the disturbances and directly uses a random effects assumption. Hence I maintain the following assumption:

Assumption 5 *The disturbance vectors $\boldsymbol{\mu}_i$ are identically and independently (of $\boldsymbol{\varepsilon}_{jt}$, $\boldsymbol{\mu}_j$, and $\Delta \mathbf{y}_{j0}$) distributed with zero mean, and finite absolute $4+\delta$ moments for some $\delta > 0$. Furthermore, the vector $\boldsymbol{\mu}_{it}$ has a finite positive-definite variance matrix $\mathbf{\Omega}_\mu$.*

[4]It would also be possible to use the full spatial panel GLS tranfromation since the spatial GM procedure also provides estimates of $\mathbf{\Omega}_\varepsilon$ and $\mathbf{\Omega}_\mu$. Nevertheless, the tranformed likelihood approach on a model after the spatial Cochrane-Orcutt transformation does not depend on the variance of the inidividual effects.

3.1.1 Instrumental Variable Estimator of the Slope Coefficients

To be able to define the IV estimator, it turns out to be convenient to stack the model differently. The model is:

$$\Delta \mathbf{y}_{it} = \mathbf{\Phi} \Delta \mathbf{y}_{i,t-1} + \Delta \mathbf{u}_{it} \tag{19}$$

where $\Delta \mathbf{y}_{it}$ and $\Delta \mathbf{u}_{it}$ are $m \times 1$ vectors. After taking transpose and staking the observations at different times for a given cross-section, we have

$$\begin{pmatrix} \Delta \mathbf{y}'_{i1} \\ \vdots \\ \Delta \mathbf{y}'_{iT} \end{pmatrix}_{T\times m} = \begin{pmatrix} \Delta \mathbf{y}'_{i0} \\ \vdots \\ \Delta \mathbf{y}'_{i,T-1} \end{pmatrix}_{T\times m} \mathbf{\Phi}'_{m\times m} + \begin{pmatrix} \Delta \mathbf{u}'_{i1} \\ \vdots \\ \Delta \mathbf{u}'_{iT} \end{pmatrix}_{T\times m} \tag{20}$$

or with the obvious notation

$$\Delta \mathbf{Y}_i = \Delta \mathbf{Y}_{i,-1} \mathbf{\Phi}' + \Delta \mathbf{U}_i \tag{21}$$

Stacking the cross-sections yields

$$\Delta \mathbf{Y} = \Delta \mathbf{Y}_{-1} \mathbf{\Phi}' + \Delta \mathbf{U} \tag{22}$$

where $\Delta \mathbf{Y} = (\Delta \mathbf{Y}'_1, ..., \Delta \mathbf{Y}'_N)'$, $\Delta \mathbf{Y}_{-1} = (\Delta \mathbf{Y}'_{1,-1}, ..., \Delta \mathbf{Y}'_{N,-1})'$ and $\Delta \mathbf{U} = (\Delta \mathbf{U}'_1, ..., \Delta \mathbf{U}'_N)'$.

I define the IV estimator of $\mathbf{\Phi}$ as

$$\widehat{\mathbf{\Phi}}_{IV} = \left[\widehat{\mathbf{Z}}' \widehat{\mathbf{Z}} \right]^{-1} \widehat{\mathbf{Z}}' \Delta \mathbf{Y} \tag{23}$$

where $\widehat{\mathbf{Z}} = \mathbf{P}_H \Delta \mathbf{Y}$ with $\mathbf{P}_H = \mathbf{H}(\mathbf{H}'\mathbf{H})^{-1}\mathbf{H}'$ where $\mathbf{H}$ is vector of instruments used for $\Delta \mathbf{Y}_{-1}$. I suggest the use of the instruments $\mathbf{H} = \mathbf{Y}_{-2} = (\mathbf{Y}'_{1,-2}, ..., \mathbf{Y}'_{N,-2})'$ where $\mathbf{Y}_{i,-2} = (\mathbf{y}_{i,-1}, ..., \mathbf{y}_{i,T-2})'$. However, any instruments that satisfy the following conditions lead to consistent estimates of the spatially correlated disturbances.

Assumption 6 *The instrument matrix* $\mathbf{H}$ *has a full column rank.*

Assumption 7 *The instruments satisfy the following:*

1. $p\lim \frac{1}{N}\mathbf{H}'\mathbf{H} = \mathbf{Q}_{HH}$ *where* $\mathbf{Q}_{HH}$ *is finite and nonsingular;*
2. $p\lim \frac{1}{N}\mathbf{H}'\Delta \mathbf{Y} = \mathbf{Q}_{HY}$ *where* $\mathbf{Q}_{HY}$ *is finite and has a full column rank;*
3. *The instruments* $\mathbf{H}$ *can be expressed as* $\mathbf{H} = \mathbf{F}(\varsigma_1, .., \varsigma_m)$ *where each* ς_i *is a* $NT \times 1$ *vector of identically and independently distributed random variables and* $\mathbf{F}$ *is an* $N \times N$ *nonstochastic absolutely summable matrix. Furthermore, each* ς_i *is independent of* ε_{it}.

The first two assumptions guarantee that the instruments are not degenerate and that they are asymptotically correlated with the variables they replace. The last assumption implies that the instruments are not correlated with the error terms and that a central limit theorem for triangular arrays of quadratic forms can be applied. Given these additional assumptions, the IV estimation produces $N^{-1/2}$ consistent estimates. Note that the rate of convergence is important for consistency of estimation λ (the degree of spatial correlation in the residuals) in the the second step of the procedure.

Observe that our suggested instruments meet the required conditions. By backward substitution we can eliminate the lagged dependent variables and express the instruments as a function of lagged disturbance terms and lagged explanatory variables. It is easily verified that

$$\Delta \mathbf{y}_t = (\mathbf{I}_{mN} - \lambda \mathbf{W})^{-1} \left(\sum_{j=0}^{t-2} (\mathbf{I}_N \otimes \mathbf{\Phi})^{j-1} \Delta \boldsymbol{\varepsilon}_{t-j} + (\mathbf{I}_N \otimes \mathbf{\Phi})^{t-1} [\boldsymbol{\varepsilon}_1 - (\mathbf{I}_m - \mathbf{\Phi}) \boldsymbol{\xi}] \right) \quad (24)$$

and hence we have that

$$\begin{aligned} \mathbf{H} &= (\mathbf{Y}'_{1,-2}, ..., \mathbf{Y}'_{N,-2})' & (25) \\ &= \mathbf{F} \cdot [\boldsymbol{\varepsilon}_1 - (\mathbf{I} - \mathbf{\Phi}) \boldsymbol{\xi}, \Delta \boldsymbol{\varepsilon}_2, ..., \Delta \boldsymbol{\varepsilon}_{T-2}]' & (26) \end{aligned}$$

where

$$\mathbf{F} = \left[\mathbf{I}_T \otimes (\mathbf{I}_{mN} - \lambda \mathbf{W}')^{-1} \right] \left[\mathbf{I}_T \otimes (\mathbf{I}_{mN} - \lambda \mathbf{W})^{-1} \right]$$

Our assumptions on the spatial weighting matrices imply that the $mNT \times mNT$ matrix $\mathbf{F}$ is absolutely summable.

3.2 Estimation of the Degree of Spatial Autocorrelation

The second step in the proposed estimation procedure is to use moment conditions based on the estimated disturbances:

$$\widehat{\mathbf{u}}_t = \mathbf{y}_t - \left(\mathbf{I}_N \otimes \widehat{\mathbf{\Phi}}_{IV} \right) \mathbf{y}_{t-1} \quad (27)$$

where $\widehat{\mathbf{\Phi}}_{IV}$ is the IV estimators of $\mathbf{\Phi}$. Kelejian and Prucha (1999) show consistency of a similar two stage procedure for univariate single cross-section model with spatial lags in both the dependent variable as well as the error term. Kapoor et al. (2007) extend the results for a univariate static panel model. Note that both of these papers consider nonstochastic exogenous variables and hence their results are not directly applicable to the panel VAR model considered here. However, Mutl (2006) contains a straightforward extension of their proofs for univariate panel autoregressive models. Hence I conjecture that the spatial GM procedure will also be consistent in a multivariate setting (under an appropriate set of assumptions).

To be able to describe the multivariate version of the spatial GM estimation procedure, it proves to be convenient to stack the model differently. It is also possible to impose more structure on the innovations $\boldsymbol{\varepsilon}_{it}$ and, in particular, consider that they are generated from a two-way error component model. Recall that the disturbances of the model are generated from

$$\mathbf{u}_{it} = \lambda \sum_{j=1}^{N} \mathbf{w}_{ij} \mathbf{u}_{jt} + \boldsymbol{\nu}_{it}. \quad (28)$$

I now assume that the innovations have a two-way error components structure

$$\boldsymbol{\nu}_{it} = (\mathbf{I}_m - \mathbf{\Phi}) \boldsymbol{\mu}_i + \boldsymbol{\varepsilon}_{it}, \quad (29)$$

where the elements of the $m \times 1$ vectors $\boldsymbol{\mu}_i$ and $\boldsymbol{\varepsilon}_{it}$ are independent with

$$\begin{aligned} E\left[\left(\mathbf{I}_m - \boldsymbol{\Phi}\right)\boldsymbol{\mu}_i\boldsymbol{\mu}_i'\left(\mathbf{I}_m - \boldsymbol{\Phi}'\right)\right] &= \boldsymbol{\Omega}_\mu, \\ E\left(\boldsymbol{\varepsilon}_{it}\boldsymbol{\varepsilon}_{it}'\right) &= \boldsymbol{\Omega}_\varepsilon. \end{aligned} \tag{30}$$

We can now stack the disturbances and innovations over the different cross-sections. In contrast to Section 2, I define

$$\begin{aligned} \underset{N\times m}{\widetilde{\mathbf{u}}_t} &= \left(\mathbf{u}_{1t},..,\mathbf{u}_{Nt}\right)', & \underset{N\times m}{\widetilde{\boldsymbol{\nu}}_t} &= \left(\boldsymbol{\nu}_{1t},..,\boldsymbol{\nu}_{Nt}\right)', \\ \underset{NT\times m}{\widetilde{\mathbf{u}}} &= \left(\widetilde{\mathbf{u}}_1,..,\widetilde{\mathbf{u}}_T\right)', & \underset{NT\times m}{\widetilde{\boldsymbol{\nu}}} &= \left(\widetilde{\boldsymbol{\nu}}_1,..,\widetilde{\boldsymbol{\nu}}_T\right)'. \end{aligned} \tag{31}$$

I additionally define the notation for the spatial lag as $\overline{\mathbf{u}}_{it} = \sum_{j=1}^N \mathbf{w}_{ij}\mathbf{u}_{jt}$ and $\overline{\boldsymbol{\nu}}_{it} = \sum_{j=1}^N \mathbf{w}_{ij}\boldsymbol{\nu}_{jt}$. The stacked spatially lagged disturbances and innovations hence become

$$\begin{aligned} \overline{\widetilde{\mathbf{u}}}_t &= \left(\overline{\mathbf{u}}_{1t},..,\overline{\mathbf{u}}_{Nt}\right)', & \overline{\widetilde{\boldsymbol{\nu}}}_t &= \left(\overline{\boldsymbol{\nu}}_{1t},..,\overline{\boldsymbol{\nu}}_{Nt}\right)', \\ \overline{\widetilde{\mathbf{u}}} &= \left(\overline{\widetilde{\mathbf{u}}}_1,..,\overline{\widetilde{\mathbf{u}}}_T\right)', & \overline{\widetilde{\boldsymbol{\nu}}} &= \left(\overline{\widetilde{\boldsymbol{\nu}}}_1,..,\overline{\widetilde{\boldsymbol{\nu}}}_T\right)'. \end{aligned} \tag{32}$$

The multivariate version of the spatial GM estimation is based on the following moment conditions (see the Appendix B for their derivation):

$$\begin{aligned} E\frac{1}{N(T-1)}\widetilde{\boldsymbol{\nu}}'\mathbf{Q}_0\widetilde{\boldsymbol{\nu}} &= \boldsymbol{\Omega}_\varepsilon, \\ E\frac{1}{N(T-1)}\overline{\widetilde{\boldsymbol{\nu}}}'\mathbf{Q}_0\overline{\widetilde{\boldsymbol{\nu}}} &= N^{-1}\sum_{i=1}^N\sum_{j=1}^N \mathbf{w}_{ij}\boldsymbol{\Omega}_\varepsilon\mathbf{w}_{ij}', \\ E\frac{1}{N(T-1)}\overline{\widetilde{\boldsymbol{\nu}}}'\mathbf{Q}_0\widetilde{\boldsymbol{\nu}} &= N^{-1}\sum_{i=1}^N \mathbf{w}_{ii}\boldsymbol{\Omega}_\varepsilon, \end{aligned} \tag{33}$$

$$\begin{aligned} E\frac{1}{N}\widetilde{\boldsymbol{\nu}}'\mathbf{Q}_1\widetilde{\boldsymbol{\nu}} &= \boldsymbol{\Omega}_1, \\ E\frac{1}{N}\overline{\widetilde{\boldsymbol{\nu}}}'\mathbf{Q}_1\overline{\widetilde{\boldsymbol{\nu}}} &= N^{-1}\sum_{i=1}^N\sum_{j=1}^N \mathbf{w}_{ij}\boldsymbol{\Omega}_1\mathbf{w}_{ij}', \\ E\frac{1}{N}\overline{\widetilde{\boldsymbol{\nu}}}'\mathbf{Q}_1\widetilde{\boldsymbol{\nu}} &= N^{-1}\sum_{i=1}^N \mathbf{w}_{ii}\boldsymbol{\Omega}_1, \end{aligned}$$

where $\mathbf{Q}_0$ and $\mathbf{Q}_1$ are the within and between transformation matrices defined as

$$\mathbf{Q}_0 = \left(\mathbf{I}_T - \tfrac{1}{T}\mathbf{J}_T\right)\otimes\mathbf{I}_N, \qquad \mathbf{Q}_1 = \tfrac{1}{T}\mathbf{J}_T\otimes\mathbf{I}_N, \tag{34}$$

and $\boldsymbol{\Omega}_1 = \boldsymbol{\Omega}_\varepsilon + T\cdot\boldsymbol{\Omega}_\mu$.

To express the above moment conditions in terms of the disturbances $\widetilde{\mathbf{u}}$ I note that

$$\widetilde{\boldsymbol{\nu}} = \widetilde{\mathbf{u}} - \lambda\overline{\widetilde{\mathbf{u}}}, \qquad \text{and} \qquad \overline{\widetilde{\boldsymbol{\nu}}} = \overline{\widetilde{\mathbf{u}}} - \lambda\overline{\overline{\widetilde{\mathbf{u}}}}, \tag{35}$$

where $\overline{\overline{\widetilde{\mathbf{u}}}} = \left(\overline{\overline{\widetilde{\mathbf{u}}}}_1', .., \overline{\overline{\widetilde{\mathbf{u}}}}_T'\right)'$ with $\overline{\overline{\widetilde{\mathbf{u}}}}_t = \left(\overline{\overline{\mathbf{u}}}_{1t}, .., \overline{\overline{\mathbf{u}}}_{Nt}\right)'$ and $\overline{\overline{\mathbf{u}}}_{it} = \sum_{j=1}^N \mathbf{w}_{ij}\overline{\mathbf{u}}_{jt}$. The six moment conditions then can be written as

$$\mathbf{\Gamma} \cdot \left[\lambda, \lambda^2, vec\left(\mathbf{\Omega}_\varepsilon\right)', vec\left(\mathbf{\Omega}_1\right)'\right]' - \boldsymbol{\gamma} = \mathbf{0}, \tag{36}$$

where

$$\mathbf{\Gamma} = \begin{bmatrix} \boldsymbol{\gamma}_{11}^0 & \boldsymbol{\gamma}_{12}^0 & \boldsymbol{\gamma}_{13}^0 & 0 \\ \boldsymbol{\gamma}_{21}^0 & \boldsymbol{\gamma}_{22}^0 & \boldsymbol{\gamma}_{23}^0 & 0 \\ \boldsymbol{\gamma}_{31}^0 & \boldsymbol{\gamma}_{32}^0 & \boldsymbol{\gamma}_{33}^0 & 0 \\ \boldsymbol{\gamma}_{11}^1 & \boldsymbol{\gamma}_{12}^1 & 0 & \boldsymbol{\gamma}_{13}^1 \\ \boldsymbol{\gamma}_{21}^1 & \boldsymbol{\gamma}_{22}^1 & 0 & \boldsymbol{\gamma}_{23}^1 \\ \boldsymbol{\gamma}_{31}^1 & \boldsymbol{\gamma}_{32}^1 & 0 & \boldsymbol{\gamma}_{33}^1 \end{bmatrix}, \ \boldsymbol{\gamma} = \begin{bmatrix} \boldsymbol{\gamma}_1^0 \\ \boldsymbol{\gamma}_2^0 \\ \boldsymbol{\gamma}_3^0 \\ \boldsymbol{\gamma}_1^1 \\ \boldsymbol{\gamma}_2^1 \\ \boldsymbol{\gamma}_3^1 \end{bmatrix}, \tag{37}$$

with $(i = 0, 1)$

$$\begin{aligned}
\boldsymbol{\gamma}_{11}^i &= \frac{1}{N\left(T-1\right)^{1-i}} vec\left(E\widetilde{\mathbf{u}}'\mathbf{Q}_i\overline{\widetilde{\mathbf{u}}} + E\overline{\widetilde{\mathbf{u}}}'\mathbf{Q}_{i,N}\widetilde{\mathbf{u}}\right), \ \boldsymbol{\gamma}_{12}^i = \frac{-1}{N\left(T-1\right)^{1-i}} vec\left(E\overline{\widetilde{\mathbf{u}}}'\mathbf{Q}_i\overline{\widetilde{\mathbf{u}}}\right), \\
\boldsymbol{\gamma}_{21}^i &= \frac{1}{N\left(T-1\right)^{1-i}} vec\left(E\overline{\overline{\widetilde{\mathbf{u}}}}'\mathbf{Q}_i\overline{\widetilde{\mathbf{u}}} + E\overline{\widetilde{\mathbf{u}}}_N'\mathbf{Q}_i\overline{\overline{\widetilde{\mathbf{u}}}}\right), \ \boldsymbol{\gamma}_{22}^i = \frac{-1}{N\left(T-1\right)^{1-i}} vec\left(E\overline{\overline{\widetilde{\mathbf{u}}}}'\mathbf{Q}_i\overline{\overline{\widetilde{\mathbf{u}}}}\right), \\
\boldsymbol{\gamma}_{31}^i &= \frac{1}{N\left(T-1\right)^{1-i}} vec\left(E\widetilde{\mathbf{u}}'\mathbf{Q}_i\overline{\overline{\widetilde{\mathbf{u}}}} + E\overline{\widetilde{\mathbf{u}}}'\mathbf{Q}_i\overline{\widetilde{\mathbf{u}}}\right), \ \boldsymbol{\gamma}_{32}^i = \frac{-1}{N\left(T-1\right)^{1-i}} vec\left(E\overline{\widetilde{\mathbf{u}}}'\mathbf{Q}_i\overline{\overline{\widetilde{\mathbf{u}}}}\right),
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{\gamma}_{13}^i &= \mathbf{I}_{m^2}, \ \boldsymbol{\gamma}_1^i = \frac{1}{N\left(T-1\right)^{1-i}} vec\left(E\widetilde{\mathbf{u}}'\mathbf{Q}_i\widetilde{\mathbf{u}}\right), \\
\boldsymbol{\gamma}_{23}^i &= \frac{1}{N}\sum_{i=1}^N\sum_{j=1}^N \left(\mathbf{w}_{ij}' \otimes \mathbf{w}_{ij}\right), \ \boldsymbol{\gamma}_2^i = \frac{1}{N\left(T-1\right)^{1-i}} vec\left(E\overline{\widetilde{\mathbf{u}}}'\mathbf{Q}_i\overline{\widetilde{\mathbf{u}}}\right), \\
\boldsymbol{\gamma}_{33}^i &= \frac{1}{N}\sum_{i=1}^N \left(\mathbf{I}_m \otimes \mathbf{w}_{ii}\right), \ \boldsymbol{\gamma}_3^i = \frac{1}{N\left(T-1\right)^{1-i}} vec\left(E\widetilde{\mathbf{u}}'\mathbf{Q}_i\overline{\widetilde{\mathbf{u}}}\right).
\end{aligned} \tag{38}$$

The multivariate spatial GM procedure is based on the sample counterpart of the six moment conditions above. In particular, given an initial estimate, say $\widehat{\mathbf{\Phi}}$, of the slope coefficients, we calculate the projected disturbances $(t = p, .., T)$

$$\begin{aligned}
\widehat{\mathbf{u}}_{it} &= \mathbf{y}_{it} - \widehat{\mathbf{\Phi}}\mathbf{y}_{i,t-1}, \\
\widehat{\overline{\mathbf{u}}}_{it} &= \sum_{j=1}^N \mathbf{W}_{ij}\mathbf{y}\widehat{\mathbf{u}}_{jt}, \\
\widehat{\overline{\overline{\mathbf{u}}}}_{it} &= \sum_{j=1}^N \mathbf{W}_{ij}\mathbf{y}\widehat{\overline{\mathbf{u}}}_{jt}.
\end{aligned} \tag{39}$$

Thus the estimated vectors $\widehat{\widetilde{\mathbf{u}}}$, $\widehat{\overline{\widetilde{\mathbf{u}}}}$, and $\widehat{\overline{\overline{\widetilde{\mathbf{u}}}}}$ have dimensions $N\left(T-p+1\right) \times m$ where p is the number of lags in the model (in contrast to e.g. $\widetilde{\mathbf{u}}$ which has dimensions $NT \times m$). However, when the PVAR model only has one lag $(p = 1)$, as it is for the case considered in this paper, the dimensions do not change.

The sample analogue of the moment conditions is then based on

$$\mathbf{G}\cdot\left[\lambda,\lambda^{2},vec\left(\mathbf{\Omega}_{\varepsilon}\right)',vec\left(\mathbf{\Omega}_{1}\right)'\right]'-\mathbf{g}=\boldsymbol{\varsigma}, \tag{40}$$

where the vector $\boldsymbol{\varsigma}$ depends on the parameters $\left[\lambda,vec\left(\mathbf{\Omega}_{\varepsilon}\right)',vec\left(\mathbf{\Omega}_{1}\right)'\right]$ and:

$$\mathbf{G}=\begin{bmatrix} \mathbf{g}_{11}^{0} & \mathbf{g}_{12}^{0} & \mathbf{g}_{13}^{0} & 0 \\ \mathbf{g}_{21}^{0} & \mathbf{g}_{22}^{0} & \mathbf{g}_{23}^{0} & 0 \\ \mathbf{g}_{31}^{0} & \mathbf{g}_{32}^{0} & \mathbf{g}_{33}^{0} & 0 \\ \mathbf{g}_{11}^{1} & \mathbf{g}_{12}^{1} & 0 & \mathbf{g}_{13}^{1} \\ \mathbf{g}_{21}^{1} & \mathbf{g}_{22}^{1} & 0 & \mathbf{g}_{23}^{1} \\ \mathbf{g}_{31}^{1} & \mathbf{g}_{32}^{1} & 0 & \mathbf{g}_{33}^{1} \end{bmatrix},\ \mathbf{g}=\begin{bmatrix} \mathbf{g}_{1}^{0} \\ \mathbf{g}_{2}^{0} \\ \mathbf{g}_{3}^{0} \\ \mathbf{g}_{1}^{1} \\ \mathbf{g}_{2}^{1} \\ \mathbf{g}_{3}^{1} \end{bmatrix}, \tag{41}$$

with $(i=0,1)$

$$\begin{aligned}
\mathbf{g}_{11}^{i} &= \frac{1}{N\left(T-1\right)^{1-i}}vec\left(\widehat{\widetilde{\mathbf{u}}}'\mathbf{Q}_{i}\widehat{\overline{\widetilde{\mathbf{u}}}}+\widehat{\overline{\widetilde{\mathbf{u}}}}'\mathbf{Q}_{i,N}\widehat{\widetilde{\mathbf{u}}}\right),\ \mathbf{g}_{12}^{i}=\frac{-1}{N\left(T-1\right)^{1-i}}vec\left(\widehat{\overline{\widetilde{\mathbf{u}}}}'\mathbf{Q}_{i}\widehat{\overline{\widetilde{\mathbf{u}}}}\right), \\
\mathbf{g}_{21}^{i} &= \frac{1}{N\left(T-1\right)^{1-i}}vec\left(\widehat{\overline{\overline{\widetilde{\mathbf{u}}}}}'\mathbf{Q}_{i}\widehat{\overline{\widetilde{\mathbf{u}}}}+\widehat{\overline{\widetilde{\mathbf{u}}}}'_{N}\mathbf{Q}_{i}\widehat{\overline{\overline{\widetilde{\mathbf{u}}}}}\right),\ \mathbf{g}_{22}^{i}=\frac{-1}{N\left(T-1\right)^{1-i}}vec\left(\widehat{\overline{\overline{\widetilde{\mathbf{u}}}}}'\mathbf{Q}_{i}\widehat{\overline{\overline{\widetilde{\mathbf{u}}}}}\right), \\
\mathbf{g}_{31}^{i} &= \frac{1}{N\left(T-1\right)^{1-i}}vec\left(\widehat{\widetilde{\mathbf{u}}}'\mathbf{Q}_{i}\widehat{\overline{\overline{\widetilde{\mathbf{u}}}}}+\widehat{\overline{\widetilde{\mathbf{u}}}}'\mathbf{Q}_{i}\widehat{\overline{\widetilde{\mathbf{u}}}}\right),\ \mathbf{g}_{32}^{i}=\frac{-1}{N\left(T-1\right)^{1-i}}vec\left(\widehat{\overline{\widetilde{\mathbf{u}}}}'\mathbf{Q}_{i}\widehat{\overline{\overline{\widetilde{\mathbf{u}}}}}\right),
\end{aligned}$$

$$\begin{aligned}
\mathbf{g}_{13}^{i} &= \mathbf{I}_{m^{2}},\ \mathbf{g}_{1}^{i}=\frac{1}{N\left(T-1\right)^{1-i}}vec\left(\widehat{\widetilde{\mathbf{u}}}'\mathbf{Q}_{i}\widehat{\widetilde{\mathbf{u}}}\right), \\
\mathbf{g}_{23}^{i} &= \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\left(\mathbf{w}_{ij}'\otimes\mathbf{w}_{ij}\right),\ \mathbf{g}_{2}^{i}=\frac{1}{N\left(T-1\right)^{1-i}}vec\left(\widehat{\overline{\widetilde{\mathbf{u}}}}'\mathbf{Q}_{i}\widehat{\overline{\widetilde{\mathbf{u}}}}\right), \\
\mathbf{g}_{33}^{i} &= \frac{1}{N}\sum_{i=1}^{N}\left(\mathbf{I}_{m}\otimes\mathbf{w}_{ii}\right),\ \mathbf{g}_{3}^{i}=\frac{1}{N\left(T-1\right)^{1-i}}vec\left(\widehat{\widetilde{\mathbf{u}}}'\mathbf{Q}_{i}\widehat{\overline{\widetilde{\mathbf{u}}}}\right).
\end{aligned} \tag{42}$$

The spatial GM procedure can be based on either the first three sets of (unweighted) moment conditions, or on the full set of (weighted) moment conditions. Define the vector $\boldsymbol{\varsigma}_0$ and as function of the parameters (analogically to $\boldsymbol{\varsigma}$):

$$\boldsymbol{\varsigma}_{0}\left[\lambda,vec\left(\mathbf{\Omega}_{\varepsilon}\right)'\right]=\mathbf{G}_{0}\cdot\left[\lambda,\lambda^{2},vec\left(\mathbf{\Omega}_{\varepsilon}\right)'\right]'-\mathbf{g}_{0}, \tag{43}$$

where

$$\mathbf{G}_{0}=\left(g_{ij}^{0}\right)_{ij=1,2,3},\text{ and }\mathbf{g}_{0}=\left(g_{i}^{0}\right)_{i=1,2,3}.$$

In the first case, the (unweighted) spatial GM procedure maximizes the objective function

$$\left(\boldsymbol{\varsigma}_{0}\left[\lambda,vec\left(\mathbf{\Omega}_{\varepsilon}\right)'\right]\right)'\left(\boldsymbol{\varsigma}_{0}\left[\lambda,vec\left(\mathbf{\Omega}_{\varepsilon}\right)'\right]\right), \tag{44}$$

subject to the fact that the matrix $\mathbf{\Omega}_{\varepsilon}$ has to be strictly positive definite. This is easily implemented by replacing the variance covariance matrices with their Cholesky decompositions and maximizing only with respect to the $m\left(m+1\right)/2$ free parameters in each of the two $m\times m$ variance covariance matrices.

Note that the unweighted spatial GM procedure only uses within-transformed data (using the matrix $\mathbf{Q}_0$) and hence does not rely on the random effects assumption. This then produces suitable initial estimate of λ for a fixed effeects estimation procedure discussed below. If an initial estimate of $\mathbf{\Omega}_1$ is desired, it can be obtained by substitution into the fourth moment condition.

In the second case, the spatial GM objective function is

$$\left(\boldsymbol{\varsigma}\left[\lambda, vec\left(\mathbf{\Omega}_\varepsilon\right)', vec\left(\mathbf{\Omega}_1\right)'\right]\right)' \mathbf{\Xi}^{-1}\left(\boldsymbol{\varsigma}\left[\lambda, vec\left(\mathbf{\Omega}_\varepsilon\right)', vec\left(\mathbf{\Omega}_1\right)'\right]\right), \tag{45}$$

where $\mathbf{\Xi}$ is a variance covariance matrix of the moment conditions. I follow Kapoor et al. and note that under the normality assumptions $\mathbf{\Xi}$ becomes

$$\mathbf{\Xi} = \begin{bmatrix} \frac{1}{T-1}\left(\mathbf{\Omega}_\varepsilon \otimes \mathbf{\Omega}_\varepsilon\right)\mathbf{T}_W & \mathbf{0} \\ \mathbf{0} & \left(\mathbf{\Omega}_1 \otimes \mathbf{\Omega}_1\right)\mathbf{T}_W \end{bmatrix}, \tag{46}$$

where $\mathbf{T}_W = \frac{1}{N}\left[\mathbf{T}_1, \mathbf{T}_2, \mathbf{T}_3\right]$, with

$$\mathbf{T}_1 = \begin{bmatrix} 2N\mathbf{I}_{m^2} \\ 2\sum_{i=1}^N\sum_{j=1}^N\left(\mathbf{w}_{ij}' \otimes \mathbf{w}_{ij}\right) \\ \sum_{i=1}^N\left[\left(\mathbf{I}_m \otimes \mathbf{w}_{ii}\right) + \left(\mathbf{I}_m \otimes \mathbf{w}_{ii}'\right)\right] \end{bmatrix}, \tag{47}$$

$$\mathbf{T}_2 = \begin{bmatrix} 2\sum_{i=1}^N\sum_{j=1}^N\left(\mathbf{w}_{ij}' \otimes \mathbf{w}_{ij}\right) \\ 2\sum_{i=1}^N\sum_{j=1}^N\left(\mathbf{w}_{ij}'\mathbf{w}_{ij}' \otimes \mathbf{w}_{ij}\mathbf{w}_{ij}\right) \\ \sum_{i=1}^N\sum_{j=1}^N\left(\mathbf{w}_{ij}' \otimes \mathbf{w}_{ij}\right)\left[\left(\mathbf{I}_m \otimes \mathbf{w}_{ii}\right) + \left(\mathbf{I}_m \otimes \mathbf{w}_{ii}'\right)\right] \end{bmatrix},$$

$$\mathbf{T}_3 = \begin{bmatrix} \sum_{i=1}^N\left[\left(\mathbf{I}_m \otimes \mathbf{w}_{ii}\right) + \left(\mathbf{I}_m \otimes \mathbf{w}_{ii}'\right)\right] \\ \sum_{i=1}^N\sum_{j=1}^N\left(\mathbf{w}_{ij}' \otimes \mathbf{w}_{ij}\right)\left[\left(\mathbf{I}_m \otimes \mathbf{w}_{ii}\right) + \left(\mathbf{I}_m \otimes \mathbf{w}_{ii}'\right)\right] \\ \sum_{i=1}^N\sum_{j=1}^N\left[\left(\mathbf{w}_{ij} \otimes \mathbf{w}_{ij}\right) + \left(\mathbf{w}_{ij}' \otimes \mathbf{w}_{ij}\right)\right] \end{bmatrix}.$$

The matrix $\mathbf{\Xi}$ depends on unknown parameters ($\mathbf{\Omega}_\varepsilon$ and $\mathbf{\Omega}_1$) and hence these have to be replaced by initial consistent estimators in order to make the weighted spatial GM procedure operational. These can be, for example, based on the unweighted spatial GM procedure.

3.3 Quasi Maximum Likelihood (QML) Estimation

The likelihood function for the panel VAR model is easily derived under the assumption that $\boldsymbol{\varepsilon}_{it} \sim N(0, \mathbf{\Omega}_\varepsilon)$ where $\mathbf{\Omega}_\varepsilon$ is the $m \times m$ variance-covariance matrix of $\boldsymbol{\varepsilon}_{ti}$. I specify the exact distribution of the initial observations as in Binder et al. (2001) and derive the QML function taking this into account. We can define the $mNT \times 1$ vector

$$\Delta\boldsymbol{\eta} = \left(\Delta\mathbf{y}_1', \Delta\mathbf{u}_2', .., \Delta\mathbf{u}_T'\right)'. \tag{48}$$

Recall that the variance of the initial observations is given by

$$E\left(\Delta\mathbf{y}_1\Delta\mathbf{y}_1'\right) = \left(\mathbf{I}_{mN} - \lambda\mathbf{W}\right)^{-1}\left(\mathbf{I}_N \otimes \mathbf{\Psi}\right)\left(\mathbf{I}_{mN} - \lambda\mathbf{W}'\right)^{-1}. \tag{49}$$

Similarly, we also have that

$$\begin{aligned} E\left(\Delta\mathbf{u}_t\Delta\mathbf{u}_t'\right) &= 2\left(\mathbf{I}_{mN} - \lambda\mathbf{W}\right)^{-1}\left(\mathbf{I}_N \otimes \mathbf{\Omega}_\varepsilon\right)\left(\mathbf{I}_{mN} - \lambda\mathbf{W}'\right)^{-1}, \\ E\left(\Delta\mathbf{u}_t\Delta\mathbf{u}_{t-1}'\right) &= -\left(\mathbf{I}_{mN} - \lambda\mathbf{W}\right)^{-1}\left(\mathbf{I}_N \otimes \mathbf{\Omega}_\varepsilon\right)\left(\mathbf{I}_{mN} - \lambda\mathbf{W}'\right)^{-1}. \end{aligned} \tag{50}$$

Thus, we then have that $E(\Delta\boldsymbol{\eta}) = \mathbf{0}$ and

$$VC(\Delta\boldsymbol{\eta}) = (\mathbf{I}_{mNT} - \lambda\mathtt{W})^{-1}(\mathbf{I}_N \otimes \boldsymbol{\Sigma})(\mathbf{I}_{mNT} - \lambda\mathtt{W}')^{-1}, \tag{51}$$

where $\mathtt{W} = \mathbf{I}_T \otimes \mathbf{W}$, and

$$\boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Psi} & -\boldsymbol{\Omega}_\varepsilon & & 0 \\ -\boldsymbol{\Omega}_\varepsilon & 2\boldsymbol{\Omega}_\varepsilon & & \\ & & \ddots & -\boldsymbol{\Omega}_\varepsilon \\ 0 & & -\boldsymbol{\Omega}_\varepsilon & 2\boldsymbol{\Omega}_\varepsilon \end{pmatrix}, \tag{52}$$

with $\boldsymbol{\Psi}$ being a $m\times m$ symmetric matrix of parameters (under Assumption NR). The (NR) specification leaves the variance-covariance matrix of the initial observations unrestricted - e.g. there are $m(m+1)/2$ free additional parameters.

The likelihood function for the entire sample is then

$$\begin{aligned} L_N(\boldsymbol{\theta}) &= -\frac{mNT}{2}\log(2\pi) - \frac{N}{2}\ln|\boldsymbol{\Sigma}| + \ln|\mathbf{I}_{mNT} - \lambda\mathtt{W}| \\ & -\frac{1}{2}tr\left[(\mathbf{I}_N \otimes \mathbf{R}')(\mathbf{I}_{mNT} - \lambda\mathtt{W})\left(\mathbf{I}_N \otimes \boldsymbol{\Sigma}^{-1}\right)(\mathbf{I}_{mNT} - \lambda\mathtt{W}')(\mathbf{I}_N \otimes \mathbf{R})\,\mathbf{S}\right], \end{aligned} \tag{53}$$

where $\boldsymbol{\theta} = (vech\boldsymbol{\Psi}', vech\boldsymbol{\Omega}_\varepsilon', vec\boldsymbol{\Phi}', \lambda)$ is the vector of parameters. The $mT \times mT$ matrix $\mathbf{R}$ is defined as

$$\mathbf{R} = \begin{pmatrix} \mathbf{I}_m & & & 0 \\ -\boldsymbol{\Phi} & \mathbf{I}_m & & \\ & & \ddots & \\ 0 & & -\boldsymbol{\Phi} & \mathbf{I}_m \end{pmatrix} \tag{54}$$

and the matrix $\mathbf{S}$ is

$$\mathbf{S} = (\Delta\mathbf{y}_1', ..., \Delta\mathbf{y}_T') \cdot (\Delta\mathbf{y}_1', ..., \Delta\mathbf{y}_T')'. \tag{55}$$

Under the (IOR) or the (UR) conditions, the vector of parameters is composed of only $\boldsymbol{\vartheta} = (vech\boldsymbol{\Omega}_\varepsilon', vec\boldsymbol{\Phi}', \lambda)$ and the likelihood function is as above but with $\boldsymbol{\Psi}$ being a function of of $\boldsymbol{\Omega}_\varepsilon$ and $\boldsymbol{\Phi}$, as described in Section 2.2.

3.3.1 Computational Issues

The computation of the likelihood function should exploit the structure of the $[\mathbf{I}_{mT} \otimes (\mathbf{I}_N - \lambda\mathbf{W})]$ and $\boldsymbol{\Sigma}$ matrices when evaluating their determinants and inverses. In particular, we can express $\boldsymbol{\Sigma}$ as

$$\boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Psi} & (\mathbf{A}_1 \otimes \boldsymbol{\Omega}_\varepsilon) \\ (\mathbf{A}_1' \otimes \boldsymbol{\Omega}_\varepsilon) & (\mathbf{A}_2 \otimes \boldsymbol{\Omega}_\varepsilon) \end{pmatrix}, \tag{56}$$

where $\mathbf{A}_1$ and $\mathbf{A}_2$ are matrices of constants. The inverse of $\boldsymbol{\Sigma}_{\Delta\eta}$ is then

$$\boldsymbol{\Sigma}^{-1} = \begin{pmatrix} \mathbf{D}^{-1} & -\mathbf{D}^{-1}(\mathbf{A}_1\mathbf{A}_2^{-1} \otimes \boldsymbol{\Omega}_\varepsilon) \\ (\mathbf{A}_2^{-1}\mathbf{A}_1' \otimes \boldsymbol{\Omega}_\varepsilon)\mathbf{D}^{-1} & \mathbf{D}^{-1} - (\mathbf{A}_2^{-1}\mathbf{A}_1' \otimes \boldsymbol{\Omega}_\varepsilon)\mathbf{D}^{-1}(\mathbf{A}_1\mathbf{A}_2^{-1} \otimes \boldsymbol{\Omega}_\varepsilon) \end{pmatrix}, \tag{57}$$

where $\mathbf{D} = \boldsymbol{\Psi} - \left(\mathbf{A}_1\mathbf{A}_2^{-1}\mathbf{A}_1 \otimes \boldsymbol{\Omega}_\varepsilon\right)$.

It order to practically implement the likelihood procedure, it is important to specify the analytical gradients of the likelihood function. These are provided in the Appendix.

Note that the analytical derivatives speed up the optimization of the maximum likelihood function substantially, especially in the case where the variance covariance matrix of the initial observations is itself function of the remaining parameters of the model (the IOR case).

3.4 Constrained QML Estimation

Although the QML estimation based on the likelihood function (53) is feasible,[5] it might become extremely computationally intensive. In this paper, I propose an alternative approach that takes a consistent estimator of the spatial correlation parameter λ and maximizes a constrained likelihood function. That is, maximize

$$\begin{aligned} Q_N\left(\widetilde{\boldsymbol{\theta}}\right) &= -\frac{mNT}{2}\log(2\pi) - \frac{N}{2}\ln|\boldsymbol{\Sigma}| + \ln\left|\mathbf{I}_{mNT} - \widehat{\lambda}\mathtt{W}\right| \\ &\quad -\frac{1}{2}tr\left[\mathbf{R}'\left(\mathbf{I}_{mNT} - \widehat{\lambda}\mathtt{W}\right)\left(\mathbf{I}_N \otimes \boldsymbol{\Sigma}^{-1}\right)\left(\mathbf{I}_{mNT} - \widehat{\lambda}\mathtt{W}'\right)\mathbf{R}\mathbf{S}_N\right], \end{aligned} \tag{58}$$

with respect to $\widetilde{\boldsymbol{\theta}} = (vech\boldsymbol{\Psi}', vech\boldsymbol{\Omega}_\varepsilon', vec\boldsymbol{\Phi}')'$, taking the consistent estimator $\widehat{\lambda}$ of λ as given. The consistent estimator of the spatial correlation be based on the two-step procedure proposed above. Note that the constrained likelihood estimator is equivalent to using the spatial Cochrane-Orcutt transformation $(\mathbf{I}_{mN} - \lambda\mathbf{W})$ and then maximizing the QML function derived under the assumption that the disturbances are independent, i.e. the same as in Binder et al. (2005).

4 Monte Carlo Simulations

I now turn to the small sample performance of the different estimators. To this end I replicate the simulations in Binder et al. (2005) who consider the same PVAR model but with independent disturbances. I modify their generation of the disturbances and, in particular, consider the spatial autoregressive specification with a spatial weight matrix $\mathbf{W}$ and parameter λ. The specification of the spatial weights corresponds to the designs used in Kapoor et al. (2007). In particular, I consider three specifications of $\mathbf{W}$ which differ in their degree of sparseness. Each matrix uses a rook design with $J = 2, 6$ or 10 non-zero off-diagonal elements. The parameter λ takes values in the set $\{0, .25, .5, .9\}$. I thus have the three different weights matrices and four different values of λ for each of the five simulation designs considered in Binder et al. (2005), i.e. 60 different simulation designs in total. For each parameter design, I consider four different sample sizes given by combinations of $T \in \{3, 10\}$ and $N \in \{50, 250\}$. In each simulation design and sample size, I use the same VC matrix for the innovations and the individual effects, i.e. set $\boldsymbol{\Omega}_\varepsilon = \boldsymbol{\Omega}_\mu$ and draw the random variables from a normal distribution.[6] As a robustness check, results are available upon request that use different ratio of the two variances (denoted by τ) as well as alternative distributions (chi-square and student-t).

[5] The QML estimator is likely to be computationally expensive due to the necessity to calculate eigenvalues of a sparse matrix $(\mathbf{I}_{mN} - \lambda\mathbf{W})$ which is of the dimension mN. With large N this becomes a very demanding problem.

[6] Note that it has now been well documented that the performance of the GMM estimators deteriorates with increasing the ratio of variance of the individual effects to the variance of the innovations, while the QML procedure is invariant to this ratio. The setup in this paper sets this ratio to one and hence sets the odds in favor of the GMM procedures.

Each simulation design and sample size is replicated 1,000 times and the resulting estimates are saved. Tables 1 provides the biases and root-mean-square errors of the different estimators. The estimators considered in the experiments are the same estimators as considered in Binder et al. (2005), i.e. the four initial GMM estimators as well as the FE-QML derived under the assumption that the disturbances are independent. Additionally I report results for the three-step procedures described in this paper, i.e. the constrained likelihood approach based on spatial GM estimator of the parameter λ, which is in turn based on an initial GMM estimation.

In particular, the estimator labeled GMMs uses the standard moment conditions as suggested by Arellano and Bond (1991); estimator labeled GMMe1 appends these moment conditions by initialization restrictions as proposed by Arellano and Bover (1995) and Blundell and Bond (1998); while estimator GMMe2 uses the standard Arellano and Bond (1991) orthogonality conditions appended by homoscedasticity conditions suggested in Ahn and Schmidt (1995, 1997); and finally the estimator labeled GMMe3 uses the standard orthogonality conditions appended by both the initialization and homoscedasticity restrictions. Please see Binder et al. (2005) for definition and more detailed discussion of these estimators in a multivariate context. Next estimator, labeled QMLiid, uses the incorrectly specified likelihood function under the assumption of independent disturbances, while the estimator labeled QMLco, uses the same likelihood estimation but with data transformed by the spatial Cochrane-Orcutt transformation based on estimates from spatial GM estimation which is in turn based in estimated disturbances from the GMMe1 procedure.

To save space only results for one particular spatial weights matrix are reported ($J = 6$, meaning that the matrix $\mathbf{W}$ has 6 non-zero off-diagonal elements). The designs are as follows:

Design 1: Stationary PVAR with maximum eigenvalue of 0.6

$$\boldsymbol{\Phi} = \begin{pmatrix} 0.4 & 0.2 \\ 0.2 & 0.4 \end{pmatrix}, \qquad \boldsymbol{\Omega}_{\varepsilon} = \begin{pmatrix} 0.07 & 0.05 \\ 0.05 & 0.07 \end{pmatrix}.$$

Design 2: Stationary PVAR with maximum eigenvalue of 0.8

$$\boldsymbol{\Phi} = \begin{pmatrix} 0.6 & 0.2 \\ 0.2 & 0.6 \end{pmatrix}, \qquad \boldsymbol{\Omega}_{\varepsilon} = \begin{pmatrix} 0.07 & -0.02 \\ -0.02 & 0.07 \end{pmatrix}.$$

Design 3: Stationary PVAR with maximum eigenvalue of 0.95

$$\boldsymbol{\Phi} = \begin{pmatrix} 0.7 & 0.25 \\ 0.25 & 0.7 \end{pmatrix}, \qquad \boldsymbol{\Omega}_{\varepsilon} = \begin{pmatrix} 0.08 & -0.05 \\ -0.05 & 0.08 \end{pmatrix}.$$

Design 4: PVAR with unit roots (but not cointegrated)

$$\boldsymbol{\Phi} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \boldsymbol{\Omega}_{\varepsilon} = \begin{pmatrix} 0.08 & -0.05 \\ -0.05 & 0.08 \end{pmatrix}.$$

Design 5: Cointegrated PVAR

$$\boldsymbol{\Phi} = \begin{pmatrix} 0.5 & 0.1 \\ -0.5 & 1.1 \end{pmatrix}, \qquad \boldsymbol{\Omega}_{\varepsilon} = \begin{pmatrix} 0.05 & 0.03 \\ 0.03 & 0.05 \end{pmatrix}.$$

Table 1: Bias and RMSE of Phi_11

Bias | RMSE

J=2 Design 1

N	250				50				250				50			
lambda	0	0.2	0.5	0.9	0	0.2	0.5	0.9	0	0.2	0.5	0.9	0	0.2	0.5	0.9
T=5; GMMs	0.020	0.021	0.029	0.159	0.113	0.114	0.134	0.289	0.084	0.086	0.103	0.276	0.211	0.213	0.243	0.475
GMMe1	-0.008	-0.008	-0.011	-0.053	-0.037	-0.039	-0.054	-0.147	0.063	0.064	0.076	0.184	0.148	0.149	0.167	0.300
GMMe2	0.007	0.007	0.011	0.056	0.041	0.041	0.045	-0.016	0.072	0.073	0.087	0.196	0.166	0.167	0.183	0.310
GMMe3	-0.008	-0.008	-0.011	-0.065	-0.045	-0.048	-0.065	-0.156	0.063	0.064	0.075	0.188	0.151	0.152	0.171	0.303
QMLiid	0.000	0.000	0.001	0.015	0.005	0.004	0.003	0.026	0.056	0.057	0.067	0.164	0.117	0.118	0.138	0.335
QMLco	0.000	0.000	0.000	0.000	0.004	0.004	0.004	0.002	0.056	0.056	0.056	0.056	0.118	0.118	0.118	0.119
T=10; GMMs	0.018	0.019	0.026	0.110	0.096	0.097	0.107	0.165	0.047	0.048	0.058	0.162	0.139	0.140	0.154	0.271
GMMe1	-0.011	-0.012	-0.017	-0.107	-0.069	-0.071	-0.089	-0.181	0.037	0.038	0.047	0.150	0.112	0.114	0.132	0.255
GMMe2	0.007	0.008	0.010	0.037	0.054	0.054	0.051	-0.085	0.040	0.041	0.048	0.116	0.107	0.108	0.114	0.216
GMMe3	-0.011	-0.011	-0.017	-0.112	-0.074	-0.076	-0.096	-0.183	0.037	0.038	0.047	0.155	0.119	0.120	0.138	0.257
QMLiid	-0.001	-0.001	-0.002	-0.003	0.004	0.004	0.004	0.020	0.030	0.031	0.038	0.097	0.072	0.073	0.087	0.216
QMLco	-0.001	-0.001	-0.001	-0.001	0.004	0.004	0.003	0.003	0.031	0.031	0.031	0.031	0.072	0.072	0.072	0.072

J=2 Design 2

N	250				50				250				50			
lambda	0	0.2	0.5	0.9	0	0.2	0.5	0.9	0	0.2	0.5	0.9	0	0.2	0.5	0.9
T=5; GMMs	0.033	0.034	0.048	0.245	0.177	0.183	0.223	0.421	0.084	0.085	0.102	0.331	0.256	0.262	0.304	0.528
GMMe1	-0.005	-0.005	-0.005	-0.022	-0.012	-0.012	-0.016	-0.051	0.049	0.049	0.057	0.124	0.106	0.106	0.119	0.189
GMMe2	0.014	0.014	0.019	0.089	0.068	0.071	0.082	0.068	0.064	0.065	0.076	0.183	0.151	0.153	0.173	0.240
GMMe3	-0.005	-0.005	-0.006	-0.032	-0.015	-0.016	-0.022	-0.058	0.049	0.049	0.057	0.126	0.107	0.107	0.121	0.191
QMLiid	0.001	0.001	0.002	0.017	0.010	0.010	0.014	0.071	0.043	0.043	0.051	0.126	0.098	0.100	0.117	0.260
QMLco	0.001	0.001	0.001	0.001	0.008	0.008	0.008	0.008	0.043	0.043	0.043	0.043	0.098	0.098	0.098	0.099
T=10; GMMs	0.030	0.030	0.040	0.148	0.131	0.132	0.145	0.211	0.046	0.046	0.057	0.175	0.158	0.159	0.171	0.260
GMMe1	-0.006	-0.007	-0.011	-0.066	-0.042	-0.043	-0.054	-0.097	0.027	0.027	0.033	0.093	0.076	0.076	0.086	0.151
GMMe2	0.013	0.013	0.017	0.061	0.070	0.070	0.072	-0.015	0.033	0.033	0.039	0.099	0.100	0.101	0.105	0.138
GMMe3	-0.006	-0.007	-0.010	-0.068	-0.044	-0.045	-0.057	-0.098	0.027	0.027	0.033	0.096	0.080	0.081	0.090	0.152
QMLiid	0.001	0.001	0.000	0.002	0.005	0.005	0.005	0.030	0.021	0.022	0.026	0.066	0.050	0.050	0.058	0.147
QMLco	0.001	0.001	0.001	0.001	0.005	0.005	0.005	0.004	0.021	0.021	0.021	0.021	0.050	0.050	0.050	0.053

Table 1 (cont.): Bias and RMSE of Phi_11

Bias | RMSE

J=2 Design 3

N	250				50				250				50			
lambda	0	0.2	0.5	0.9	0	0.2	0.5	0.9	0	0.2	0.5	0.9	0	0.2	0.5	0.9
T=5; GMMs	0.090	0.092	0.123	0.363	0.321	0.324	0.358	0.475	0.184	0.187	0.239	0.540	0.496	0.498	0.537	0.644
GMMe1	0.003	0.003	0.002	-0.011	-0.007	-0.007	-0.010	-0.037	0.059	0.059	0.068	0.126	0.115	0.116	0.125	0.168
GMMe2	0.019	0.019	0.023	0.086	0.076	0.077	0.084	0.068	0.080	0.081	0.095	0.203	0.182	0.181	0.189	0.227
GMMe3	0.002	0.002	0.001	-0.018	-0.011	-0.012	-0.017	-0.041	0.058	0.058	0.066	0.122	0.112	0.113	0.121	0.165
QMLiid	0.004	0.004	0.003	0.017	0.012	0.012	0.016	0.072	0.050	0.051	0.060	0.143	0.109	0.110	0.128	0.294
QMLco	0.004	0.004	0.004	0.004	0.011	0.011	0.011	0.006	0.050	0.050	0.050	0.050	0.110	0.110	0.110	0.117
T=10; GMMs	0.058	0.060	0.079	0.215	0.199	0.200	0.213	0.263	0.087	0.090	0.112	0.261	0.245	0.247	0.261	0.330
GMMe1	-0.003	-0.003	-0.005	-0.043	-0.028	-0.029	-0.037	-0.065	0.033	0.034	0.039	0.076	0.066	0.066	0.073	0.121
GMMe2	0.016	0.017	0.022	0.071	0.077	0.078	0.079	0.008	0.044	0.044	0.053	0.117	0.115	0.116	0.120	0.132
GMMe3	-0.003	-0.003	-0.005	-0.046	-0.031	-0.032	-0.040	-0.066	0.033	0.034	0.039	0.078	0.068	0.069	0.075	0.123
QMLiid	0.002	0.002	0.002	0.008	0.010	0.010	0.013	0.053	0.025	0.025	0.030	0.075	0.057	0.058	0.068	0.166
QMLco	0.002	0.002	0.002	0.002	0.010	0.010	0.010	0.010	0.025	0.025	0.025	0.025	0.057	0.057	0.057	0.057

J=2 Design 4

N	250				50				250				50			
lambda	0	0.2	0.5	0.9	0	0.2	0.5	0.9	0	0.2	0.5	0.9	0	0.2	0.5	0.9
T=5; GMMs	0.790	0.793	0.802	0.777	0.810	0.806	0.778	0.694	0.915	0.916	0.917	0.884	0.917	0.916	0.894	0.819
GMMe1	0.000	0.001	0.002	0.014	0.007	0.007	0.009	0.024	0.025	0.026	0.030	0.071	0.056	0.055	0.062	0.091
GMMe2	0.008	0.008	0.013	0.072	0.051	0.052	0.067	0.086	0.045	0.046	0.059	0.166	0.130	0.131	0.153	0.186
GMMe3	0.000	0.000	0.001	0.009	0.008	0.007	0.010	0.024	0.025	0.025	0.030	0.060	0.052	0.053	0.060	0.087
QMLiid	0.006	0.005	0.008	0.032	0.021	0.019	0.025	0.092	0.048	0.050	0.050	0.119	0.091	0.094	0.103	0.242
QMLco	0.004	0.005	0.003	0.004	0.019	0.019	0.022	0.018	0.072	0.058	0.103	0.064	0.103	0.093	0.092	0.101
T=10; GMMs	0.528	0.529	0.521	0.437	0.498	0.495	0.476	0.396	0.569	0.569	0.560	0.471	0.535	0.532	0.514	0.452
GMMe1	0.000	0.000	0.000	0.003	0.004	0.004	0.004	0.011	0.012	0.012	0.014	0.026	0.024	0.024	0.025	0.044
GMMe2	0.003	0.003	0.004	0.044	0.041	0.041	0.046	0.041	0.019	0.020	0.024	0.078	0.073	0.074	0.078	0.083
GMMe3	0.000	0.000	0.000	0.004	0.005	0.005	0.005	0.011	0.012	0.012	0.013	0.025	0.024	0.024	0.025	0.044
QMLiid	0.002	0.002	0.002	0.013	0.012	0.012	0.015	0.058	0.020	0.025	0.031	0.058	0.048	0.051	0.056	0.138
QMLco	0.003	0.002	0.003	-0.012	0.012	0.010	0.012	0.009	0.020	0.026	0.030	0.072	0.048	0.062	0.052	0.066

Table 1 (cont.): Bias and RMSE of Phi_11

Bias | RMSE

J=2 Design 5

N	250				50				250				50			
lambda	0	0.2	0.5	0.9	0	0.2	0.5	0.9	0	0.2	0.5	0.9	0	0.2	0.5	0.9
T=5; GMMs	0.423	0.423	0.422	0.443	0.436	0.431	0.424	0.439	0.505	0.503	0.500	0.515	0.511	0.509	0.507	0.528
GMMe1	0.000	0.000	0.000	0.007	0.002	0.002	0.003	0.021	0.012	0.013	0.015	0.033	0.026	0.026	0.029	0.054
GMMe2	0.002	0.002	0.002	0.016	0.008	0.009	0.014	0.034	0.019	0.019	0.023	0.058	0.047	0.048	0.056	0.084
GMMe3	0.001	0.001	0.001	0.008	0.004	0.004	0.006	0.022	0.012	0.013	0.015	0.031	0.025	0.026	0.029	0.053
QMLiid	0.001	0.001	0.001	0.011	0.004	0.005	0.006	0.037	0.022	0.022	0.026	0.068	0.048	0.049	0.058	0.157
QMLco	0.001	0.001	0.001	-0.001	0.004	0.004	0.003	-0.007	0.022	0.022	0.022	0.029	0.048	0.049	0.051	0.070
T=10; GMMs	0.271	0.271	0.266	0.232	0.254	0.253	0.247	0.225	0.295	0.295	0.290	0.253	0.277	0.277	0.272	0.260
GMMe1	0.001	0.001	0.001	0.008	0.007	0.007	0.009	0.022	0.007	0.007	0.009	0.019	0.016	0.016	0.018	0.036
GMMe2	0.000	0.000	0.000	0.011	0.009	0.009	0.012	0.026	0.009	0.009	0.011	0.029	0.026	0.026	0.028	0.046
GMMe3	0.001	0.001	0.001	0.009	0.008	0.008	0.010	0.023	0.007	0.007	0.008	0.019	0.016	0.016	0.018	0.037
QMLiid	0.001	0.001	0.000	0.003	0.004	0.004	0.006	0.024	0.009	0.009	0.011	0.029	0.022	0.023	0.028	0.074
QMLco	0.001	0.001	0.001	0.001	0.004	0.004	0.004	0.004	0.009	0.009	0.009	0.009	0.022	0.022	0.022	0.022

The performance of all estimators deteriorates with increased degree of spatial autocorrelation. This is due to the fact that as λ increases the expected R^2 of the regression decreases. However, not all estimators are sensitive in the same way. First, note that the QML procedure using the Cochrane-Orcutt transformed data (QMLco) is overall the best performer (in terms of RMSE) when spatial autocorrelation is present. However, the QMLco also shows almost no loss of efficiency relative to the QMLiid procedure even for the cases where there is no spatial autocorrelation ($\lambda = 0$). Furthermore, note that the performance of the QMLco estimator is sensitive to the degree of spatial autocorrelation only in the presence of unit roots and/or cointegration. The unit root and cointegrated designs (design 4 and 5) are also the only cases where the extended GMM procedures perform better than the QML procedure, provided that there is no or minimal amount of spatial autocorrelation.

Next, note that the extended GMM and the QMLiid estimators perform reasonably well even in the presence of spatial autocorrelation. Their performance in terms of RMSE is about the same and deteriorates as λ increases in absolute value. Additional results with higher τ show that the QMLiid procedure starts dominating the extended GMM estimators, analogically to the results for $\lambda = 0$ in Binder et al. (2005). Finally, I note that our simulations document that the performance of the standard GMM procedure deteriorates rapidly with presence of autocorrelation in either dimension (time or space). The GMMs estimator breaks down in presence of unit roots and/or cointegration (as was documented in other studies) but also in presence of high degree of spatial autocorrelation.

Therefore, it is interesting to note that the performance of the QMLiid estimator is no worse and often better than that of the extended GMM estimators. This is somewhat surprising because the presence of spatial autocorrelation invalidates the independence assumption on which the QMLiid estimator relies. On the other hand, the moment conditions of the GMM estimators remain valid even if spatial autocorrelation is present.

Table 2 gives the size and power properties of hypothesis tests based on the different estimation procedures. To save space I only report hypotheses tests concerning the first element of $vec\mathbf{\Phi}$. Results for the remaining elements are similar and are available upon request. The results show that despite satisfactory performance in terms of RMSE, the GMM estimators fail to provide adequate confidence intervals even in the absence of spatial autocorrelation. The nominal size of the hypothesis tests for the extended GMM estimators is between 8 and 11 percent instead of the correct size of 5 percent even in the largest sample ($N = 250$, $T = 10$) whereas the nominal size of the QML estimators remains between 5 to 7 percent. An exception is the pure unit root case (design 4) where there is a tendency for overrejection and the nominal size is 8-9 percent for the QML estimators, while the extended GMM estimators have nominal size of 6 to 10 percent. These results replicate the findings in Binder et al. (2005).

When spatial autocorrelation is present, all except the QMLco estimator stop providing reasonable confidence intervals and the nominal sizes increase to above 50 percent for $\lambda = .9$ even in the largest sample. In contrast, the QMLco estimator remains correctly sized at 5 to 6 percent in this case. With small degree of spatial autocorrelation ($\lambda = .2$), the extended GMM estimators have a nominal size of about 8-11 percent in the largest sample, while both QML estimators have the sizes 5-7 percent (again with the exception of the pure unit root design).

Table 2: Size and Power Properties of Tests for Φ11

Estimator	T	N	lambda	J	design	true - 0.3	true - 0.2	true - 0.1	Φ11 = true	true + 0.1	true + 0.2	true + 0.3
GMMs	5	250	0	2	1	0.951	0.615	0.168	**0.067**	0.389	0.819	0.979
GMMe1	5	250	0	2	1	0.998	0.932	0.439	**0.081**	0.377	0.895	0.998
GMMe2	5	250	0	2	1	0.995	0.799	0.272	**0.054**	0.379	0.847	0.987
GMMe3	5	250	0	2	1	0.999	0.943	0.467	**0.076**	0.389	0.905	1.000
QMLiid	5	250	0	2	1	1.000	0.956	0.468	**0.054**	0.462	0.957	1.000
QMLco	5	250	0	2	1	1.000	0.956	0.470	**0.052**	0.459	0.955	1.000
GMMs	5	250	0	2	2	0.928	0.593	0.153	**0.099**	0.469	0.872	0.985
GMMe1	5	250	0	2	2	1.000	0.979	0.618	**0.073**	0.549	0.995	1.000
GMMe2	5	250	0	2	2	0.999	0.891	0.346	**0.083**	0.538	0.938	0.999
GMMe3	5	250	0	2	2	1.000	0.980	0.634	**0.081**	0.555	0.995	1.000
QMLiid	5	250	0	2	2	1.000	0.994	0.635	**0.049**	0.651	0.998	1.000
QMLco	5	250	0	2	2	1.000	0.995	0.633	**0.048**	0.652	0.997	1.000
GMMs	5	250	0	2	3	0.335	0.144	0.069	**0.112**	0.250	0.507	0.741
GMMe1	5	250	0	2	3	0.992	0.904	0.472	**0.047**	0.401	0.950	0.998
GMMe2	5	250	0	2	3	0.948	0.679	0.224	**0.074**	0.383	0.821	0.980
GMMe3	5	250	0	2	3	0.996	0.920	0.494	**0.061**	0.447	0.957	0.998
QMLiid	5	250	0	2	3	1.000	0.974	0.510	**0.050**	0.567	0.991	1.000
QMLco	5	250	0	2	3	1.000	0.974	0.512	**0.051**	0.565	0.991	1.000
GMMs	5	250	0	2	4	0.420	0.523	0.622	**0.711**	0.774	0.839	0.879
GMMe1	5	250	0	2	4	1.000	0.998	0.945	**0.047**	0.968	1.000	1.000
GMMe2	5	250	0	2	4	0.997	0.976	0.674	**0.083**	0.728	0.992	1.000
GMMe3	5	250	0	2	4	1.000	1.000	0.954	**0.058**	0.969	1.000	1.000
QMLiid	5	250	0	2	4	0.848	0.820	0.537	**0.114**	0.675	0.839	0.848
QMLco	5	250	0	2	4	0.853	0.829	0.541	**0.121**	0.685	0.845	0.852
GMMs	5	250	0	2	5	0.218	0.313	0.439	**0.580**	0.737	0.851	0.926
GMMe1	5	250	0	2	5	1.000	0.999	0.826	**0.060**	0.832	1.000	1.000
GMMe2	5	250	0	2	5	1.000	0.997	0.621	**0.077**	0.728	0.998	1.000
GMMe3	5	250	0	2	5	1.000	0.999	0.841	**0.059**	0.838	1.000	1.000
QMLiid	5	250	0	2	5	1.000	0.998	0.718	**0.044**	0.825	1.000	1.000
QMLco	5	250	0	2	5	1.000	0.998	0.719	**0.044**	0.823	1.000	1.000
GMMs	5	250	0.2	2	1	0.945	0.610	0.174	**0.085**	0.396	0.816	0.978
GMMe1	5	250	0.2	2	1	0.999	0.931	0.449	**0.083**	0.379	0.892	0.998
GMMe2	5	250	0.2	2	1	0.991	0.802	0.273	**0.067**	0.390	0.846	0.989
GMMe3	5	250	0.2	2	1	0.999	0.939	0.476	**0.085**	0.396	0.899	0.999
QMLiid	5	250	0.2	2	1	1.000	0.954	0.459	**0.062**	0.463	0.953	1.000
QMLco	5	250	0.2	2	1	1.000	0.956	0.472	**0.053**	0.461	0.954	1.000
GMMs	5	250	0.2	2	2	0.930	0.588	0.155	**0.102**	0.488	0.861	0.988
GMMe1	5	250	0.2	2	2	1.000	0.982	0.623	**0.080**	0.548	0.992	1.000
GMMe2	5	250	0.2	2	2	0.998	0.893	0.350	**0.091**	0.545	0.939	0.999
GMMe3	5	250	0.2	2	2	1.000	0.984	0.628	**0.085**	0.559	0.988	1.000
QMLiid	5	250	0.2	2	2	1.000	0.996	0.626	**0.050**	0.656	0.998	1.000
QMLco	5	250	0.2	2	2	1.000	0.995	0.633	**0.048**	0.653	0.998	1.000
GMMs	5	250	0.2	2	3	0.346	0.136	0.077	**0.118**	0.274	0.531	0.737
GMMe1	5	250	0.2	2	3	0.992	0.905	0.478	**0.052**	0.411	0.942	0.998
GMMe2	5	250	0.2	2	3	0.942	0.675	0.232	**0.076**	0.387	0.819	0.980
GMMe3	5	250	0.2	2	3	0.995	0.924	0.501	**0.065**	0.455	0.957	0.998
QMLiid	5	250	0.2	2	3	1.000	0.976	0.520	**0.056**	0.550	0.992	1.000
QMLco	5	250	0.2	2	3	1.000	0.974	0.512	**0.053**	0.563	0.991	1.000
GMMs	5	250	0.2	2	4	0.434	0.530	0.623	**0.704**	0.789	0.848	0.885
GMMe1	5	250	0.2	2	4	1.000	0.996	0.946	**0.050**	0.970	1.000	1.000
GMMe2	5	250	0.2	2	4	0.995	0.971	0.670	**0.086**	0.728	0.990	1.000
GMMe3	5	250	0.2	2	4	1.000	1.000	0.955	**0.064**	0.970	1.000	1.000
QMLiid	5	250	0.2	2	4	0.850	0.820	0.514	**0.124**	0.676	0.842	0.848
QMLco	5	250	0.2	2	4	0.863	0.841	0.547	**0.131**	0.694	0.857	0.862
GMMs	5	250	0.2	2	5	0.219	0.323	0.438	**0.596**	0.746	0.864	0.935
GMMe1	5	250	0.2	2	5	1.000	0.999	0.828	**0.058**	0.810	1.000	1.000
GMMe2	5	250	0.2	2	5	1.000	0.995	0.612	**0.084**	0.731	0.997	1.000
GMMe3	5	250	0.2	2	5	1.000	0.999	0.838	**0.062**	0.826	1.000	1.000
QMLiid	5	250	0.2	2	5	1.000	0.998	0.714	**0.048**	0.819	1.000	1.000
QMLco	5	250	0.2	2	5	1.000	0.998	0.722	**0.044**	0.824	1.000	1.000

Table 2. (cont.): Size and Power Properties of Tests for Φ11

Estimator	T	N	lambda	J	design	true - 0.3	true - 0.2	true - 0.1	Φ11 = true	true + 0.1	true + 0.2	true + 0.3
GMMs	5	250	0.5	2	1	0.895	0.551	0.186	**0.150**	0.473	0.825	0.968
GMMe1	5	250	0.5	2	1	0.998	0.905	0.483	**0.145**	0.398	0.853	0.993
GMMe2	5	250	0.5	2	1	0.972	0.765	0.290	**0.135**	0.416	0.816	0.973
GMMe3	5	250	0.5	2	1	0.999	0.917	0.496	**0.157**	0.404	0.852	0.991
QMLiid	5	250	0.5	2	1	0.998	0.927	0.443	**0.115**	0.495	0.919	0.996
QMLco	5	250	0.5	2	1	1.000	0.957	0.472	**0.053**	0.462	0.953	1.000
GMMs	5	250	0.5	2	2	0.863	0.523	0.180	**0.180**	0.565	0.871	0.976
GMMe1	5	250	0.5	2	2	1.000	0.963	0.607	**0.149**	0.554	0.980	1.000
GMMe2	5	250	0.5	2	2	0.995	0.845	0.350	**0.156**	0.569	0.911	0.996
GMMe3	5	250	0.5	2	2	1.000	0.978	0.634	**0.166**	0.563	0.978	1.000
QMLiid	5	250	0.5	2	2	1.000	0.991	0.609	**0.104**	0.632	0.994	1.000
QMLco	5	250	0.5	2	2	1.000	0.995	0.633	**0.047**	0.653	0.998	1.000
GMMs	5	250	0.5	2	3	0.354	0.194	0.147	**0.227**	0.416	0.600	0.762
GMMe1	5	250	0.5	2	3	0.989	0.899	0.507	**0.115**	0.460	0.929	0.998
GMMe2	5	250	0.5	2	3	0.918	0.636	0.258	**0.152**	0.463	0.837	0.966
GMMe3	5	250	0.5	2	3	0.993	0.918	0.545	**0.134**	0.500	0.933	0.999
QMLiid	5	250	0.5	2	3	1.000	0.952	0.500	**0.110**	0.546	0.976	1.000
QMLco	5	250	0.5	2	3	1.000	0.974	0.511	**0.053**	0.563	0.991	1.000
GMMs	5	250	0.5	2	4	0.506	0.604	0.718	**0.784**	0.836	0.881	0.919
GMMe1	5	250	0.5	2	4	0.998	0.998	0.926	**0.092**	0.940	1.000	1.000
GMMe2	5	250	0.5	2	4	0.994	0.934	0.634	**0.174**	0.734	0.985	0.999
GMMe3	5	250	0.5	2	4	1.000	0.997	0.932	**0.117**	0.957	1.000	1.000
QMLiid	5	250	0.5	2	4	0.835	0.793	0.514	**0.158**	0.680	0.820	0.836
QMLco	5	250	0.5	2	4	0.837	0.819	0.523	**0.148**	0.689	0.830	0.837
GMMs	5	250	0.5	2	5	0.297	0.386	0.545	**0.706**	0.831	0.913	0.960
GMMe1	5	250	0.5	2	5	1.000	0.998	0.802	**0.133**	0.780	1.000	1.000
GMMe2	5	250	0.5	2	5	1.000	0.981	0.588	**0.145**	0.731	0.996	1.000
GMMe3	5	250	0.5	2	5	1.000	0.998	0.813	**0.148**	0.790	1.000	1.000
QMLiid	5	250	0.5	2	5	1.000	0.989	0.671	**0.104**	0.774	1.000	1.000
QMLco	5	250	0.5	2	5	1.000	0.998	0.720	**0.045**	0.823	1.000	1.000
GMMs	5	250	0.9	2	1	0.652	0.593	0.596	**0.687**	0.792	0.877	0.939
GMMe1	5	250	0.9	2	1	0.941	0.852	0.745	**0.647**	0.631	0.694	0.837
GMMe2	5	250	0.9	2	1	0.815	0.672	0.595	**0.621**	0.702	0.810	0.916
GMMe3	5	250	0.9	2	1	0.949	0.875	0.769	**0.670**	0.660	0.719	0.830
QMLiid	5	250	0.9	2	1	0.892	0.750	0.586	**0.511**	0.602	0.763	0.893
QMLco	5	250	0.9	2	1	1.000	0.957	0.470	**0.052**	0.458	0.952	1.000
GMMs	5	250	0.9	2	2	0.633	0.610	0.675	**0.777**	0.872	0.933	0.980
GMMe1	5	250	0.9	2	2	0.978	0.908	0.764	**0.633**	0.645	0.842	0.971
GMMe2	5	250	0.9	2	2	0.829	0.689	0.610	**0.661**	0.793	0.912	0.979
GMMe3	5	250	0.9	2	2	0.972	0.916	0.808	**0.663**	0.659	0.850	0.976
QMLiid	5	250	0.9	2	2	0.940	0.801	0.619	**0.515**	0.638	0.849	0.968
QMLco	5	250	0.9	2	2	1.000	0.994	0.638	**0.045**	0.650	0.998	1.000
GMMs	5	250	0.9	2	3	0.656	0.687	0.718	**0.768**	0.812	0.877	0.924
GMMe1	5	250	0.9	2	3	0.965	0.910	0.734	**0.620**	0.663	0.854	0.978
GMMe2	5	250	0.9	2	3	0.808	0.675	0.637	**0.657**	0.762	0.886	0.960
GMMe3	5	250	0.9	2	3	0.972	0.916	0.769	**0.640**	0.695	0.884	0.986
QMLiid	5	250	0.9	2	3	0.908	0.762	0.548	**0.490**	0.654	0.852	0.970
QMLco	5	250	0.9	2	3	1.000	0.973	0.511	**0.051**	0.566	0.991	1.000
GMMs	5	250	0.9	2	4	0.812	0.857	0.878	**0.917**	0.943	0.968	0.977
GMMe1	5	250	0.9	2	4	0.990	0.963	0.852	**0.573**	0.911	0.996	1.000
GMMe2	5	250	0.9	2	4	0.902	0.788	0.700	**0.675**	0.829	0.955	0.991
GMMe3	5	250	0.9	2	4	0.994	0.973	0.887	**0.584**	0.952	0.998	1.000
QMLiid	5	250	0.9	2	4	0.730	0.627	0.527	**0.467**	0.658	0.766	0.797
QMLco	5	250	0.9	2	4	0.785	0.758	0.495	**0.198**	0.686	0.771	0.783
GMMs	5	250	0.9	2	5	0.688	0.757	0.853	**0.922**	0.972	0.992	0.995
GMMe1	5	250	0.9	2	5	0.985	0.924	0.777	**0.615**	0.700	0.936	0.997
GMMe2	5	250	0.9	2	5	0.910	0.768	0.613	**0.649**	0.825	0.948	0.999
GMMe3	5	250	0.9	2	5	0.988	0.928	0.809	**0.639**	0.712	0.940	0.997
QMLiid	5	250	0.9	2	5	0.947	0.816	0.610	**0.533**	0.720	0.940	0.990
QMLco	5	250	0.9	2	5	0.995	0.993	0.719	**0.055**	0.821	0.993	0.995

Table 2. (cont.): Size and Power Properties of Tests for Φ11

Estimator	T	N	lambda	J	design	true - 0.3	true - 0.2	true - 0.1	Φ11 = true	true + 0.1	true + 0.2	true + 0.3
GMMs	5	50	0	2	1	0.287	0.134	0.087	**0.174**	0.325	0.512	0.712
GMMe1	5	50	0	2	1	0.776	0.554	0.298	**0.154**	0.171	0.352	0.622
GMMe2	5	50	0	2	1	0.499	0.264	0.104	**0.109**	0.250	0.472	0.697
GMMe3	5	50	0	2	1	0.808	0.593	0.346	**0.159**	0.175	0.376	0.628
QMLiid	5	50	0	2	1	0.699	0.356	0.114	**0.053**	0.152	0.404	0.737
QMLco	5	50	0	2	1	0.708	0.369	0.117	**0.055**	0.154	0.399	0.730
GMMs	5	50	0	2	2	0.225	0.108	0.114	**0.253**	0.469	0.698	0.858
GMMe1	5	50	0	2	2	0.900	0.670	0.363	**0.146**	0.213	0.583	0.900
GMMe2	5	50	0	2	2	0.609	0.284	0.111	**0.144**	0.379	0.671	0.891
GMMe3	5	50	0	2	2	0.924	0.730	0.420	**0.160**	0.227	0.590	0.910
QMLiid	5	50	0	2	2	0.877	0.520	0.163	**0.052**	0.208	0.593	0.906
QMLco	5	50	0	2	2	0.881	0.526	0.166	**0.050**	0.210	0.593	0.903
GMMs	5	50	0	2	3	0.176	0.179	0.237	**0.306**	0.414	0.529	0.662
GMMe1	5	50	0	2	3	0.808	0.580	0.313	**0.111**	0.239	0.613	0.888
GMMe2	5	50	0	2	3	0.466	0.260	0.143	**0.176**	0.334	0.580	0.786
GMMe3	5	50	0	2	3	0.863	0.645	0.364	**0.148**	0.266	0.650	0.924
QMLiid	5	50	0	2	3	0.746	0.406	0.139	**0.070**	0.180	0.495	0.842
QMLco	5	50	0	2	3	0.740	0.413	0.144	**0.067**	0.185	0.492	0.834
GMMs	5	50	0	2	4	0.424	0.518	0.628	**0.716**	0.797	0.846	0.900
GMMe1	5	50	0	2	4	0.977	0.931	0.604	**0.056**	0.654	0.970	0.998
GMMe2	5	50	0	2	4	0.748	0.543	0.270	**0.165**	0.468	0.830	0.955
GMMe3	5	50	0	2	4	0.989	0.942	0.671	**0.073**	0.727	0.987	1.000
QMLiid	5	50	0	2	4	0.634	0.470	0.272	**0.107**	0.353	0.674	0.776
QMLco	5	50	0	2	4	0.655	0.490	0.280	**0.109**	0.357	0.690	0.794
GMMs	5	50	0	2	5	0.243	0.335	0.492	**0.657**	0.803	0.883	0.944
GMMe1	5	50	0	2	5	0.960	0.791	0.418	**0.135**	0.304	0.811	0.991
GMMe2	5	50	0	2	5	0.818	0.497	0.167	**0.130**	0.426	0.808	0.973
GMMe3	5	50	0	2	5	0.978	0.826	0.439	**0.142**	0.330	0.826	0.992
QMLiid	5	50	0	2	5	0.909	0.589	0.183	**0.075**	0.316	0.792	0.981
QMLco	5	50	0	2	5	0.914	0.596	0.182	**0.072**	0.314	0.784	0.984
GMMs	5	50	0.2	2	1	0.279	0.142	0.088	**0.185**	0.341	0.526	0.716
GMMe1	5	50	0.2	2	1	0.777	0.556	0.289	**0.157**	0.163	0.352	0.639
GMMe2	5	50	0.2	2	1	0.496	0.259	0.110	**0.115**	0.269	0.482	0.703
GMMe3	5	50	0.2	2	1	0.819	0.586	0.350	**0.171**	0.180	0.375	0.631
QMLiid	5	50	0.2	2	1	0.696	0.352	0.117	**0.058**	0.165	0.409	0.731
QMLco	5	50	0.2	2	1	0.708	0.370	0.116	**0.055**	0.154	0.401	0.727
GMMs	5	50	0.2	2	2	0.228	0.106	0.127	**0.259**	0.482	0.709	0.854
GMMe1	5	50	0.2	2	2	0.894	0.667	0.380	**0.165**	0.226	0.574	0.899
GMMe2	5	50	0.2	2	2	0.598	0.289	0.121	**0.148**	0.387	0.674	0.888
GMMe3	5	50	0.2	2	2	0.928	0.729	0.439	**0.180**	0.236	0.587	0.908
QMLiid	5	50	0.2	2	2	0.866	0.519	0.172	**0.062**	0.223	0.595	0.901
QMLco	5	50	0.2	2	2	0.879	0.527	0.166	**0.049**	0.209	0.593	0.903
GMMs	5	50	0.2	2	3	0.187	0.201	0.242	**0.321**	0.427	0.547	0.681
GMMe1	5	50	0.2	2	3	0.822	0.590	0.318	**0.117**	0.245	0.612	0.894
GMMe2	5	50	0.2	2	3	0.484	0.268	0.139	**0.172**	0.342	0.602	0.779
GMMe3	5	50	0.2	2	3	0.870	0.657	0.375	**0.157**	0.271	0.648	0.922
QMLiid	5	50	0.2	2	3	0.740	0.411	0.141	**0.071**	0.196	0.496	0.833
QMLco	5	50	0.2	2	3	0.741	0.411	0.146	**0.070**	0.184	0.488	0.835
GMMs	5	50	0.2	2	4	0.435	0.517	0.632	**0.724**	0.802	0.857	0.903
GMMe1	5	50	0.2	2	4	0.981	0.923	0.607	**0.058**	0.661	0.965	0.998
GMMe2	5	50	0.2	2	4	0.753	0.558	0.264	**0.179**	0.470	0.831	0.955
GMMe3	5	50	0.2	2	4	0.987	0.941	0.678	**0.082**	0.740	0.989	0.999
QMLiid	5	50	0.2	2	4	0.636	0.460	0.270	**0.114**	0.334	0.654	0.773
QMLco	5	50	0.2	2	4	0.644	0.484	0.290	**0.129**	0.353	0.666	0.786
GMMs	5	50	0.2	2	5	0.249	0.352	0.507	**0.676**	0.803	0.896	0.951
GMMe1	5	50	0.2	2	5	0.962	0.797	0.428	**0.135**	0.317	0.813	0.997
GMMe2	5	50	0.2	2	5	0.820	0.488	0.176	**0.145**	0.426	0.813	0.977
GMMe3	5	50	0.2	2	5	0.975	0.825	0.455	**0.162**	0.331	0.818	0.994
QMLiid	5	50	0.2	2	5	0.904	0.586	0.184	**0.069**	0.316	0.791	0.981
QMLco	5	50	0.2	2	5	0.911	0.594	0.182	**0.069**	0.313	0.786	0.979

Table 2. (cont.): Size and Power Properties of Tests for Φ11

Estimator	T	N	lambda	J	design	true - 0.3	true - 0.2	true - 0.1	Φ11 = true	true + 0.1	true + 0.2	true + 0.3
GMMs	5	50	0.5	2	1	0.288	0.177	0.171	**0.275**	0.441	0.622	0.781
GMMe1	5	50	0.5	2	1	0.798	0.609	0.363	**0.210**	0.226	0.377	0.640
GMMe2	5	50	0.5	2	1	0.515	0.313	0.175	**0.206**	0.323	0.528	0.730
GMMe3	5	50	0.5	2	1	0.845	0.651	0.420	**0.254**	0.242	0.397	0.638
QMLiid	5	50	0.5	2	1	0.644	0.359	0.143	**0.107**	0.229	0.439	0.710
QMLco	5	50	0.5	2	1	0.705	0.363	0.118	**0.054**	0.153	0.402	0.727
GMMs	5	50	0.5	2	2	0.254	0.177	0.226	**0.380**	0.581	0.765	0.882
GMMe1	5	50	0.5	2	2	0.896	0.711	0.435	**0.252**	0.296	0.595	0.885
GMMe2	5	50	0.5	2	2	0.602	0.329	0.219	**0.251**	0.455	0.723	0.888
GMMe3	5	50	0.5	2	2	0.933	0.759	0.495	**0.271**	0.310	0.601	0.898
QMLiid	5	50	0.5	2	2	0.811	0.506	0.207	**0.111**	0.292	0.603	0.868
QMLco	5	50	0.5	2	2	0.878	0.526	0.167	**0.049**	0.210	0.593	0.903
GMMs	5	50	0.5	2	3	0.285	0.296	0.347	**0.427**	0.543	0.645	0.742
GMMe1	5	50	0.5	2	3	0.834	0.641	0.383	**0.186**	0.317	0.656	0.892
GMMe2	5	50	0.5	2	3	0.513	0.334	0.227	**0.261**	0.428	0.661	0.836
GMMe3	5	50	0.5	2	3	0.875	0.690	0.445	**0.241**	0.344	0.683	0.913
QMLiid	5	50	0.5	2	3	0.693	0.407	0.177	**0.132**	0.254	0.534	0.813
QMLco	5	50	0.5	2	3	0.743	0.415	0.145	**0.068**	0.185	0.486	0.836
GMMs	5	50	0.5	2	4	0.517	0.606	0.696	**0.784**	0.853	0.892	0.921
GMMe1	5	50	0.5	2	4	0.979	0.913	0.617	**0.117**	0.717	0.971	0.996
GMMe2	5	50	0.5	2	4	0.740	0.557	0.322	**0.253**	0.552	0.858	0.969
GMMe3	5	50	0.5	2	4	0.989	0.942	0.695	**0.157**	0.770	0.980	0.998
QMLiid	5	50	0.5	2	4	0.599	0.453	0.294	**0.138**	0.376	0.660	0.765
QMLco	5	50	0.5	2	4	0.636	0.482	0.317	**0.162**	0.372	0.682	0.776
GMMs	5	50	0.5	2	5	0.308	0.426	0.592	**0.732**	0.850	0.924	0.967
GMMe1	5	50	0.5	2	5	0.947	0.784	0.478	**0.219**	0.364	0.780	0.990
GMMe2	5	50	0.5	2	5	0.771	0.473	0.226	**0.233**	0.518	0.840	0.980
GMMe3	5	50	0.5	2	5	0.960	0.807	0.501	**0.244**	0.376	0.792	0.993
QMLiid	5	50	0.5	2	5	0.860	0.538	0.194	**0.121**	0.361	0.782	0.972
QMLco	5	50	0.5	2	5	0.912	0.595	0.186	**0.072**	0.315	0.786	0.982
GMMs	5	50	0.9	2	1	0.680	0.700	0.722	**0.795**	0.829	0.890	0.924
GMMe1	5	50	0.9	2	1	0.894	0.834	0.794	**0.755**	0.730	0.747	0.782
GMMe2	5	50	0.9	2	1	0.822	0.774	0.728	**0.706**	0.724	0.763	0.812
GMMe3	5	50	0.9	2	1	0.914	0.851	0.806	**0.790**	0.756	0.758	0.806
QMLiid	5	50	0.9	2	1	0.605	0.540	0.495	**0.494**	0.541	0.610	0.696
QMLco	5	50	0.9	2	1	0.701	0.358	0.115	**0.056**	0.155	0.402	0.726
GMMs	5	50	0.9	2	2	0.678	0.742	0.808	**0.863**	0.921	0.959	0.977
GMMe1	5	50	0.9	2	2	0.925	0.880	0.796	**0.735**	0.723	0.796	0.895
GMMe2	5	50	0.9	2	2	0.841	0.772	0.724	**0.727**	0.769	0.853	0.929
GMMe3	5	50	0.9	2	2	0.938	0.901	0.829	**0.750**	0.740	0.816	0.883
QMLiid	5	50	0.9	2	2	0.639	0.564	0.517	**0.516**	0.580	0.681	0.785
QMLco	5	50	0.9	2	2	0.873	0.526	0.170	**0.053**	0.212	0.594	0.899
GMMs	5	50	0.9	2	3	0.737	0.762	0.795	**0.845**	0.877	0.914	0.951
GMMe1	5	50	0.9	2	3	0.920	0.864	0.789	**0.716**	0.701	0.825	0.944
GMMe2	5	50	0.9	2	3	0.835	0.750	0.701	**0.711**	0.756	0.859	0.953
GMMe3	5	50	0.9	2	3	0.936	0.880	0.810	**0.731**	0.730	0.844	0.955
QMLiid	5	50	0.9	2	3	0.576	0.500	0.495	**0.503**	0.570	0.666	0.767
QMLco	5	50	0.9	2	3	0.729	0.420	0.148	**0.077**	0.200	0.491	0.825
GMMs	5	50	0.9	2	4	0.814	0.866	0.898	**0.915**	0.934	0.953	0.968
GMMe1	5	50	0.9	2	4	0.980	0.932	0.791	**0.648**	0.891	0.987	0.997
GMMe2	5	50	0.9	2	4	0.892	0.806	0.700	**0.706**	0.876	0.956	0.990
GMMe3	5	50	0.9	2	4	0.988	0.944	0.820	**0.683**	0.909	0.991	0.998
QMLiid	5	50	0.9	2	4	0.538	0.490	0.450	**0.459**	0.551	0.668	0.735
QMLco	5	50	0.9	2	4	0.585	0.441	0.289	**0.192**	0.361	0.660	0.745
GMMs	5	50	0.9	2	5	0.752	0.799	0.868	**0.921**	0.960	0.980	0.992
GMMe1	5	50	0.9	2	5	0.914	0.876	0.775	**0.696**	0.721	0.846	0.955
GMMe2	5	50	0.9	2	5	0.825	0.764	0.713	**0.723**	0.797	0.912	0.971
GMMe3	5	50	0.9	2	5	0.930	0.873	0.805	**0.717**	0.742	0.850	0.956
QMLiid	5	50	0.9	2	5	0.605	0.516	0.485	**0.521**	0.632	0.756	0.860
QMLco	5	50	0.9	2	5	0.858	0.552	0.196	**0.105**	0.340	0.752	0.924

Table 2. (cont.): Size and Power Properties of Tests for Φ11

Estimator	T	N	lambda	J	design	true - 0.3	true - 0.2	true - 0.1	Φ11 = true	true + 0.1	true + 0.2	true + 0.3
GMMs	10	250	0	2	1	1.000	0.984	0.491	**0.105**	0.833	0.998	1.000
GMMe1	10	250	0	2	1	1.000	1.000	0.842	**0.075**	0.772	0.999	1.000
GMMe2	10	250	0	2	1	1.000	0.999	0.635	**0.087**	0.816	1.000	1.000
GMMe3	10	250	0	2	1	1.000	1.000	0.843	**0.076**	0.772	1.000	1.000
QMLiid	10	250	0	2	1	1.000	1.000	0.850	**0.052**	0.891	1.000	1.000
QMLco	10	250	0	2	1	1.000	1.000	0.851	**0.050**	0.890	1.000	1.000
GMMs	10	250	0	2	2	1.000	0.997	0.569	**0.157**	0.970	1.000	1.000
GMMe1	10	250	0	2	2	1.000	1.000	0.980	**0.091**	0.961	1.000	1.000
GMMe2	10	250	0	2	2	1.000	1.000	0.845	**0.101**	0.965	1.000	1.000
GMMe3	10	250	0	2	2	1.000	1.000	0.979	**0.098**	0.963	1.000	1.000
QMLiid	10	250	0	2	2	1.000	1.000	0.989	**0.069**	0.996	1.000	1.000
QMLco	10	250	0	2	2	1.000	1.000	0.989	**0.067**	0.996	1.000	1.000
GMMs	10	250	0	2	3	0.961	0.712	0.192	**0.217**	0.790	0.992	1.000
GMMe1	10	250	0	2	3	1.000	1.000	0.899	**0.098**	0.881	1.000	1.000
GMMe2	10	250	0	2	3	1.000	0.997	0.662	**0.102**	0.870	1.000	1.000
GMMe3	10	250	0	2	3	1.000	1.000	0.906	**0.105**	0.892	1.000	1.000
QMLiid	10	250	0	2	3	1.000	1.000	0.979	**0.051**	0.977	1.000	1.000
QMLco	10	250	0	2	3	1.000	1.000	0.980	**0.052**	0.976	1.000	1.000
GMMs	10	250	0	2	4	0.481	0.661	0.824	**0.935**	0.976	0.994	0.998
GMMe1	10	250	0	2	4	1.000	1.000	1.000	**0.061**	1.000	1.000	1.000
GMMe2	10	250	0	2	4	1.000	1.000	1.000	**0.095**	0.999	1.000	1.000
GMMe3	10	250	0	2	4	1.000	1.000	1.000	**0.061**	1.000	1.000	1.000
QMLiid	10	250	0	2	4	0.851	0.850	0.834	**0.081**	0.847	0.849	0.851
QMLco	10	250	0	2	4	0.860	0.860	0.847	**0.089**	0.859	0.860	0.860
GMMs	10	250	0	2	5	0.307	0.354	0.622	**0.907**	0.995	0.999	1.000
GMMe1	10	250	0	2	5	1.000	1.000	0.999	**0.076**	0.998	1.000	1.000
GMMe2	10	250	0	2	5	1.000	1.000	0.974	**0.095**	0.997	1.000	1.000
GMMe3	10	250	0	2	5	1.000	1.000	0.999	**0.069**	0.999	1.000	1.000
QMLiid	10	250	0	2	5	1.000	1.000	0.999	**0.058**	1.000	1.000	1.000
QMLco	10	250	0	2	5	1.000	1.000	0.998	**0.057**	1.000	1.000	1.000
GMMs	10	250	0.2	2	1	1.000	0.982	0.500	**0.123**	0.835	0.999	1.000
GMMe1	10	250	0.2	2	1	1.000	1.000	0.835	**0.083**	0.767	0.999	1.000
GMMe2	10	250	0.2	2	1	1.000	0.997	0.625	**0.094**	0.820	1.000	1.000
GMMe3	10	250	0.2	2	1	1.000	1.000	0.832	**0.085**	0.774	1.000	1.000
QMLiid	10	250	0.2	2	1	1.000	1.000	0.842	**0.057**	0.892	1.000	1.000
QMLco	10	250	0.2	2	1	1.000	1.000	0.851	**0.050**	0.890	1.000	1.000
GMMs	10	250	0.2	2	2	1.000	1.000	0.569	**0.175**	0.965	1.000	1.000
GMMe1	10	250	0.2	2	2	1.000	1.000	0.982	**0.095**	0.959	1.000	1.000
GMMe2	10	250	0.2	2	2	1.000	1.000	0.843	**0.109**	0.963	1.000	1.000
GMMe3	10	250	0.2	2	2	1.000	1.000	0.980	**0.107**	0.959	1.000	1.000
QMLiid	10	250	0.2	2	2	1.000	1.000	0.988	**0.063**	0.993	1.000	1.000
QMLco	10	250	0.2	2	2	1.000	1.000	0.989	**0.068**	0.996	1.000	1.000
GMMs	10	250	0.2	2	3	0.955	0.708	0.196	**0.228**	0.793	0.991	1.000
GMMe1	10	250	0.2	2	3	1.000	1.000	0.907	**0.092**	0.882	1.000	1.000
GMMe2	10	250	0.2	2	3	1.000	0.997	0.662	**0.104**	0.877	1.000	1.000
GMMe3	10	250	0.2	2	3	1.000	1.000	0.908	**0.114**	0.893	1.000	1.000
QMLiid	10	250	0.2	2	3	1.000	1.000	0.977	**0.059**	0.977	1.000	1.000
QMLco	10	250	0.2	2	3	1.000	1.000	0.980	**0.052**	0.978	1.000	1.000
GMMs	10	250	0.2	2	4	0.502	0.675	0.834	**0.929**	0.976	0.992	0.998
GMMe1	10	250	0.2	2	4	1.000	1.000	1.000	**0.066**	1.000	1.000	1.000
GMMe2	10	250	0.2	2	4	1.000	1.000	0.998	**0.103**	1.000	1.000	1.000
GMMe3	10	250	0.2	2	4	1.000	1.000	1.000	**0.070**	1.000	1.000	1.000
QMLiid	10	250	0.2	2	4	0.848	0.846	0.833	**0.093**	0.843	0.846	0.848
QMLco	10	250	0.2	2	4	0.860	0.858	0.843	**0.112**	0.856	0.858	0.860
GMMs	10	250	0.2	2	5	0.315	0.342	0.640	**0.909**	0.995	1.000	1.000
GMMe1	10	250	0.2	2	5	1.000	1.000	0.998	**0.089**	0.997	1.000	1.000
GMMe2	10	250	0.2	2	5	1.000	1.000	0.973	**0.103**	0.997	1.000	1.000
GMMe3	10	250	0.2	2	5	1.000	1.000	0.998	**0.084**	0.998	1.000	1.000
QMLiid	10	250	0.2	2	5	1.000	1.000	0.998	**0.053**	1.000	1.000	1.000
QMLco	10	250	0.2	2	5	1.000	1.000	0.998	**0.055**	1.000	1.000	1.000

Table 2. (cont.): Size and Power Properties of Tests for Φ11

Estimator	T	N	lambda	J	design	true - 0.3	true - 0.2	true - 0.1	Φ11 = true	true + 0.1	true + 0.2	true + 0.3
GMMs	10	250	0.5	2	1	1.000	0.958	0.466	**0.208**	0.827	0.998	1.000
GMMe1	10	250	0.5	2	1	1.000	0.999	0.841	**0.158**	0.700	0.998	1.000
GMMe2	10	250	0.5	2	1	1.000	0.987	0.592	**0.153**	0.795	0.999	1.000
GMMe3	10	250	0.5	2	1	1.000	0.999	0.832	**0.169**	0.709	0.996	1.000
QMLiid	10	250	0.5	2	1	1.000	1.000	0.798	**0.116**	0.848	1.000	1.000
QMLco	10	250	0.5	2	1	1.000	1.000	0.851	**0.050**	0.890	1.000	1.000
GMMs	10	250	0.5	2	2	1.000	0.988	0.476	**0.304**	0.974	1.000	1.000
GMMe1	10	250	0.5	2	2	1.000	1.000	0.980	**0.153**	0.928	1.000	1.000
GMMe2	10	250	0.5	2	2	1.000	1.000	0.772	**0.205**	0.957	1.000	1.000
GMMe3	10	250	0.5	2	2	1.000	1.000	0.975	**0.156**	0.928	1.000	1.000
QMLiid	10	250	0.5	2	2	1.000	1.000	0.977	**0.101**	0.984	1.000	1.000
QMLco	10	250	0.5	2	2	1.000	1.000	0.989	**0.067**	0.996	1.000	1.000
GMMs	10	250	0.5	2	3	0.921	0.616	0.211	**0.381**	0.846	0.990	1.000
GMMe1	10	250	0.5	2	3	1.000	1.000	0.910	**0.175**	0.863	1.000	1.000
GMMe2	10	250	0.5	2	3	1.000	0.988	0.616	**0.180**	0.893	1.000	1.000
GMMe3	10	250	0.5	2	3	1.000	1.000	0.917	**0.187**	0.869	1.000	1.000
QMLiid	10	250	0.5	2	3	1.000	1.000	0.948	**0.108**	0.956	1.000	1.000
QMLco	10	250	0.5	2	3	1.000	1.000	0.980	**0.052**	0.978	1.000	1.000
GMMs	10	250	0.5	2	4	0.578	0.749	0.870	**0.945**	0.977	0.995	1.000
GMMe1	10	250	0.5	2	4	1.000	1.000	1.000	**0.115**	1.000	1.000	1.000
GMMe2	10	250	0.5	2	4	1.000	1.000	0.983	**0.167**	1.000	1.000	1.000
GMMe3	10	250	0.5	2	4	1.000	1.000	1.000	**0.121**	1.000	1.000	1.000
QMLiid	10	250	0.5	2	4	0.854	0.852	0.817	**0.142**	0.847	0.853	0.854
QMLco	10	250	0.5	2	4	0.853	0.854	0.841	**0.103**	0.850	0.854	0.854
GMMs	10	250	0.5	2	5	0.391	0.413	0.708	**0.946**	0.998	1.000	1.000
GMMe1	10	250	0.5	2	5	1.000	1.000	0.995	**0.151**	0.995	1.000	1.000
GMMe2	10	250	0.5	2	5	1.000	1.000	0.953	**0.161**	0.992	1.000	1.000
GMMe3	10	250	0.5	2	5	1.000	1.000	0.997	**0.152**	0.996	1.000	1.000
QMLiid	10	250	0.5	2	5	1.000	1.000	0.991	**0.117**	1.000	1.000	1.000
QMLco	10	250	0.5	2	5	1.000	1.000	0.998	**0.056**	1.000	1.000	1.000
GMMs	10	250	0.9	2	1	0.894	0.696	0.624	**0.747**	0.921	0.981	0.999
GMMe1	10	250	0.9	2	1	1.000	0.993	0.943	**0.809**	0.644	0.779	0.950
GMMe2	10	250	0.9	2	1	0.970	0.855	0.693	**0.651**	0.848	0.961	0.993
GMMe3	10	250	0.9	2	1	1.000	0.994	0.945	**0.831**	0.662	0.777	0.942
QMLiid	10	250	0.9	2	1	0.988	0.916	0.677	**0.514**	0.713	0.942	0.994
QMLco	10	250	0.9	2	1	1.000	1.000	0.852	**0.050**	0.888	1.000	1.000
GMMs	10	250	0.9	2	2	0.896	0.679	0.679	**0.903**	0.994	0.999	1.000
GMMe1	10	250	0.9	2	2	1.000	1.000	0.976	**0.757**	0.710	0.969	0.999
GMMe2	10	250	0.9	2	2	0.992	0.907	0.671	**0.760**	0.962	0.999	1.000
GMMe3	10	250	0.9	2	2	1.000	1.000	0.973	**0.778**	0.713	0.966	0.999
QMLiid	10	250	0.9	2	2	1.000	0.982	0.789	**0.523**	0.843	0.993	1.000
QMLco	10	250	0.9	2	2	1.000	1.000	0.989	**0.067**	0.995	1.000	1.000
GMMs	10	250	0.9	2	3	0.739	0.665	0.753	**0.907**	0.974	0.995	1.000
GMMe1	10	250	0.9	2	3	1.000	0.999	0.959	**0.746**	0.749	0.990	1.000
GMMe2	10	250	0.9	2	3	0.973	0.881	0.686	**0.742**	0.941	0.995	1.000
GMMe3	10	250	0.9	2	3	1.000	0.999	0.958	**0.750**	0.758	0.994	1.000
QMLiid	10	250	0.9	2	3	0.998	0.979	0.740	**0.498**	0.817	0.988	1.000
QMLco	10	250	0.9	2	3	1.000	1.000	0.980	**0.053**	0.978	1.000	1.000
GMMs	10	250	0.9	2	4	0.752	0.878	0.961	**0.994**	0.998	1.000	1.000
GMMe1	10	250	0.9	2	4	1.000	1.000	0.994	**0.590**	1.000	1.000	1.000
GMMe2	10	250	0.9	2	4	0.995	0.961	0.842	**0.736**	0.993	1.000	1.000
GMMe3	10	250	0.9	2	4	1.000	1.000	0.995	**0.603**	1.000	1.000	1.000
QMLiid	10	250	0.9	2	4	0.780	0.755	0.619	**0.434**	0.750	0.780	0.780
QMLco	10	250	0.9	2	4	0.709	0.707	0.692	**0.187**	0.696	0.708	0.709
GMMs	10	250	0.9	2	5	0.698	0.729	0.895	**0.988**	1.000	1.000	1.000
GMMe1	10	250	0.9	2	5	1.000	1.000	0.977	**0.736**	0.784	0.998	1.000
GMMe2	10	250	0.9	2	5	0.999	0.973	0.775	**0.718**	0.970	1.000	1.000
GMMe3	10	250	0.9	2	5	1.000	1.000	0.982	**0.749**	0.781	0.998	1.000
QMLiid	10	250	0.9	2	5	1.000	0.989	0.811	**0.521**	0.915	0.999	1.000
QMLco	10	250	0.9	2	5	1.000	1.000	0.998	**0.055**	1.000	1.000	1.000

Table 2. (cont.): Size and Power Properties of Tests for Φ11

Estimator	T	N	lambda	J	design	true - 0.3	true - 0.2	true - 0.1	Φ11 = true	true + 0.1	true + 0.2	true + 0.3
GMMs	10	50	0	2	1	0.728	0.361	0.131	**0.300**	0.682	0.935	0.993
GMMe1	10	50	0	2	1	1.000	0.948	0.727	**0.340**	0.241	0.545	0.897
GMMe2	10	50	0	2	1	0.890	0.587	0.197	**0.218**	0.575	0.901	0.993
GMMe3	10	50	0	2	1	1.000	0.955	0.753	**0.373**	0.252	0.562	0.883
QMLiid	10	50	0	2	1	0.983	0.811	0.290	**0.068**	0.292	0.805	0.988
QMLco	10	50	0	2	1	0.984	0.815	0.293	**0.071**	0.282	0.804	0.988
GMMs	10	50	0	2	2	0.754	0.347	0.197	**0.561**	0.921	0.996	1.000
GMMe1	10	50	0	2	2	1.000	0.994	0.861	**0.331**	0.367	0.921	1.000
GMMe2	10	50	0	2	2	0.968	0.697	0.214	**0.379**	0.860	0.993	1.000
GMMe3	10	50	0	2	2	1.000	0.994	0.872	**0.365**	0.393	0.915	1.000
QMLiid	10	50	0	2	2	1.000	0.969	0.482	**0.057**	0.542	0.985	1.000
QMLco	10	50	0	2	2	1.000	0.968	0.484	**0.057**	0.538	0.983	1.000
GMMs	10	50	0	2	3	0.406	0.248	0.332	**0.583**	0.831	0.963	0.998
GMMe1	10	50	0	2	3	1.000	0.983	0.787	**0.284**	0.450	0.947	0.999
GMMe2	10	50	0	2	3	0.925	0.586	0.244	**0.356**	0.754	0.974	1.000
GMMe3	10	50	0	2	3	0.997	0.985	0.822	**0.325**	0.465	0.949	1.000
QMLiid	10	50	0	2	3	0.999	0.937	0.441	**0.057**	0.456	0.962	1.000
QMLco	10	50	0	2	3	0.999	0.936	0.445	**0.053**	0.448	0.963	1.000
GMMs	10	50	0	2	4	0.478	0.677	0.850	**0.943**	0.984	0.996	1.000
GMMe1	10	50	0	2	4	1.000	0.999	0.976	**0.123**	0.997	1.000	1.000
GMMe2	10	50	0	2	4	0.978	0.918	0.601	**0.304**	0.939	0.997	1.000
GMMe3	10	50	0	2	4	1.000	0.999	0.984	**0.153**	0.998	1.000	1.000
QMLiid	10	50	0	2	4	0.821	0.758	0.499	**0.126**	0.696	0.816	0.820
QMLco	10	50	0	2	4	0.809	0.752	0.484	**0.126**	0.683	0.806	0.810
GMMs	10	50	0	2	5	0.277	0.364	0.680	**0.945**	0.994	1.000	1.000
GMMe1	10	50	0	2	5	1.000	0.998	0.892	**0.264**	0.535	0.993	1.000
GMMe2	10	50	0	2	5	1.000	0.917	0.403	**0.244**	0.860	1.000	1.000
GMMe3	10	50	0	2	5	1.000	0.998	0.897	**0.303**	0.551	0.991	1.000
QMLiid	10	50	0	2	5	0.999	0.973	0.537	**0.076**	0.706	0.999	1.000
QMLco	10	50	0	2	5	0.999	0.973	0.543	**0.075**	0.703	0.999	1.000
GMMs	10	50	0.2	2	1	0.722	0.364	0.144	**0.315**	0.688	0.936	0.993
GMMe1	10	50	0.2	2	1	0.999	0.952	0.729	**0.366**	0.238	0.527	0.894
GMMe2	10	50	0.2	2	1	0.891	0.583	0.209	**0.221**	0.576	0.904	0.990
GMMe3	10	50	0.2	2	1	1.000	0.961	0.759	**0.402**	0.262	0.554	0.885
QMLiid	10	50	0.2	2	1	0.976	0.797	0.292	**0.069**	0.290	0.792	0.989
QMLco	10	50	0.2	2	1	0.984	0.814	0.292	**0.071**	0.284	0.805	0.988
GMMs	10	50	0.2	2	2	0.753	0.340	0.201	**0.568**	0.919	0.996	1.000
GMMe1	10	50	0.2	2	2	1.000	0.993	0.868	**0.336**	0.351	0.922	1.000
GMMe2	10	50	0.2	2	2	0.966	0.692	0.220	**0.374**	0.877	0.993	1.000
GMMe3	10	50	0.2	2	2	1.000	0.995	0.864	**0.373**	0.385	0.916	0.999
QMLiid	10	50	0.2	2	2	1.000	0.961	0.487	**0.067**	0.541	0.979	1.000
QMLco	10	50	0.2	2	2	1.000	0.968	0.485	**0.057**	0.538	0.983	1.000
GMMs	10	50	0.2	2	3	0.400	0.261	0.343	**0.594**	0.838	0.970	0.999
GMMe1	10	50	0.2	2	3	1.000	0.984	0.795	**0.296**	0.459	0.947	0.999
GMMe2	10	50	0.2	2	3	0.917	0.596	0.244	**0.350**	0.773	0.974	1.000
GMMe3	10	50	0.2	2	3	0.998	0.987	0.818	**0.336**	0.461	0.953	1.000
QMLiid	10	50	0.2	2	3	0.999	0.933	0.430	**0.058**	0.458	0.964	1.000
QMLco	10	50	0.2	2	3	0.999	0.936	0.445	**0.053**	0.448	0.963	1.000
GMMs	10	50	0.2	2	4	0.481	0.682	0.846	**0.942**	0.983	0.996	0.998
GMMe1	10	50	0.2	2	4	1.000	0.997	0.976	**0.131**	0.998	1.000	1.000
GMMe2	10	50	0.2	2	4	0.981	0.917	0.589	**0.307**	0.934	0.996	1.000
GMMe3	10	50	0.2	2	4	1.000	0.998	0.981	**0.155**	0.997	1.000	1.000
QMLiid	10	50	0.2	2	4	0.824	0.768	0.483	**0.125**	0.693	0.816	0.823
QMLco	10	50	0.2	2	4	0.829	0.774	0.491	**0.131**	0.705	0.822	0.830
GMMs	10	50	0.2	2	5	0.279	0.360	0.687	**0.948**	0.995	1.000	1.000
GMMe1	10	50	0.2	2	5	1.000	0.999	0.887	**0.283**	0.547	0.989	1.000
GMMe2	10	50	0.2	2	5	1.000	0.905	0.411	**0.260**	0.866	1.000	1.000
GMMe3	10	50	0.2	2	5	1.000	0.998	0.891	**0.315**	0.560	0.992	1.000
QMLiid	10	50	0.2	2	5	0.999	0.973	0.541	**0.081**	0.707	0.999	1.000
QMLco	10	50	0.2	2	5	0.999	0.973	0.541	**0.076**	0.701	0.999	1.000

Table 2. (cont.): Size and Power Properties of Tests for Φ11

Estimator	T	N	lambda	J	design	true - 0.3	true - 0.2	true - 0.1	Φ11 = true	true + 0.1	true + 0.2	true + 0.3
GMMs	10	50	0.5	2	1	0.699	0.363	0.204	**0.406**	0.741	0.944	0.997
GMMe1	10	50	0.5	2	1	0.997	0.965	0.798	**0.483**	0.302	0.508	0.851
GMMe2	10	50	0.5	2	1	0.895	0.621	0.297	**0.286**	0.611	0.887	0.985
GMMe3	10	50	0.5	2	1	0.999	0.965	0.826	**0.520**	0.337	0.524	0.831
QMLiid	10	50	0.5	2	1	0.958	0.743	0.329	**0.121**	0.340	0.744	0.974
QMLco	10	50	0.5	2	1	0.983	0.814	0.295	**0.071**	0.285	0.803	0.988
GMMs	10	50	0.5	2	2	0.717	0.349	0.304	**0.688**	0.947	0.998	1.000
GMMe1	10	50	0.5	2	2	1.000	1.000	0.885	**0.457**	0.381	0.900	0.998
GMMe2	10	50	0.5	2	2	0.956	0.701	0.286	**0.461**	0.869	0.993	1.000
GMMe3	10	50	0.5	2	2	1.000	0.995	0.881	**0.506**	0.416	0.882	0.996
QMLiid	10	50	0.5	2	2	0.997	0.936	0.459	**0.108**	0.558	0.965	0.999
QMLco	10	50	0.5	2	2	1.000	0.968	0.482	**0.057**	0.539	0.983	1.000
GMMs	10	50	0.5	2	3	0.411	0.312	0.440	**0.695**	0.900	0.977	0.997
GMMe1	10	50	0.5	2	3	1.000	0.988	0.834	**0.384**	0.502	0.956	1.000
GMMe2	10	50	0.5	2	3	0.901	0.629	0.290	**0.428**	0.827	0.980	1.000
GMMe3	10	50	0.5	2	3	1.000	0.988	0.852	**0.427**	0.493	0.952	1.000
QMLiid	10	50	0.5	2	3	0.996	0.897	0.406	**0.107**	0.505	0.947	1.000
QMLco	10	50	0.5	2	3	0.999	0.938	0.446	**0.053**	0.451	0.963	1.000
GMMs	10	50	0.5	2	4	0.531	0.701	0.854	**0.945**	0.983	0.995	1.000
GMMe1	10	50	0.5	2	4	1.000	0.998	0.978	**0.203**	0.999	1.000	1.000
GMMe2	10	50	0.5	2	4	0.979	0.910	0.578	**0.394**	0.959	0.998	1.000
GMMe3	10	50	0.5	2	4	1.000	0.998	0.981	**0.214**	0.998	1.000	1.000
QMLiid	10	50	0.5	2	4	0.816	0.738	0.447	**0.147**	0.694	0.812	0.822
QMLco	10	50	0.5	2	4	0.831	0.770	0.463	**0.134**	0.717	0.825	0.833
GMMs	10	50	0.5	2	5	0.354	0.418	0.721	**0.950**	0.997	1.000	1.000
GMMe1	10	50	0.5	2	5	1.000	0.998	0.899	**0.408**	0.531	0.985	1.000
GMMe2	10	50	0.5	2	5	0.995	0.885	0.448	**0.360**	0.872	0.999	1.000
GMMe3	10	50	0.5	2	5	1.000	0.998	0.890	**0.435**	0.549	0.988	1.000
QMLiid	10	50	0.5	2	5	0.996	0.949	0.512	**0.124**	0.697	0.994	1.000
QMLco	10	50	0.5	2	5	0.999	0.973	0.541	**0.075**	0.705	0.999	1.000
GMMs	10	50	0.9	2	1	0.838	0.799	0.803	**0.855**	0.891	0.945	0.978
GMMe1	10	50	0.9	2	1	0.990	0.982	0.944	**0.882**	0.828	0.830	0.856
GMMe2	10	50	0.9	2	1	0.974	0.917	0.868	**0.814**	0.831	0.846	0.890
GMMe3	10	50	0.9	2	1	0.991	0.983	0.936	**0.894**	0.848	0.831	0.853
QMLiid	10	50	0.9	2	1	0.765	0.638	0.559	**0.525**	0.603	0.695	0.803
QMLco	10	50	0.9	2	1	0.980	0.811	0.299	**0.068**	0.289	0.802	0.988
GMMs	10	50	0.9	2	2	0.815	0.799	0.837	**0.935**	0.984	0.996	1.000
GMMe1	10	50	0.9	2	2	1.000	0.990	0.939	**0.847**	0.818	0.898	0.971
GMMe2	10	50	0.9	2	2	0.990	0.926	0.852	**0.829**	0.868	0.939	0.988
GMMe3	10	50	0.9	2	2	0.999	0.990	0.945	**0.854**	0.825	0.902	0.969
QMLiid	10	50	0.9	2	2	0.859	0.713	0.549	**0.534**	0.675	0.847	0.951
QMLco	10	50	0.9	2	2	0.999	0.965	0.488	**0.059**	0.541	0.982	0.999
GMMs	10	50	0.9	2	3	0.777	0.808	0.862	**0.931**	0.958	0.987	0.996
GMMe1	10	50	0.9	2	3	0.997	0.985	0.936	**0.826**	0.823	0.929	0.998
GMMe2	10	50	0.9	2	3	0.977	0.924	0.853	**0.811**	0.856	0.956	0.999
GMMe3	10	50	0.9	2	3	0.997	0.987	0.932	**0.832**	0.819	0.929	0.999
QMLiid	10	50	0.9	2	3	0.833	0.685	0.511	**0.516**	0.654	0.810	0.949
QMLco	10	50	0.9	2	3	0.999	0.937	0.445	**0.055**	0.450	0.962	1.000
GMMs	10	50	0.9	2	4	0.836	0.880	0.926	**0.973**	0.984	0.994	0.997
GMMe1	10	50	0.9	2	4	1.000	0.995	0.938	**0.776**	0.995	1.000	1.000
GMMe2	10	50	0.9	2	4	0.991	0.951	0.858	**0.812**	0.982	1.000	1.000
GMMe3	10	50	0.9	2	4	1.000	0.996	0.945	**0.801**	0.996	1.000	1.000
QMLiid	10	50	0.9	2	4	0.647	0.556	0.484	**0.485**	0.696	0.783	0.803
QMLco	10	50	0.9	2	4	0.769	0.718	0.436	**0.178**	0.690	0.768	0.772
GMMs	10	50	0.9	2	5	0.797	0.800	0.882	**0.964**	0.992	0.998	1.000
GMMe1	10	50	0.9	2	5	0.997	0.989	0.930	**0.838**	0.847	0.917	0.993
GMMe2	10	50	0.9	2	5	0.987	0.925	0.856	**0.825**	0.889	0.955	0.998
GMMe3	10	50	0.9	2	5	0.998	0.989	0.944	**0.837**	0.848	0.927	0.994
QMLiid	10	50	0.9	2	5	0.877	0.693	0.569	**0.555**	0.733	0.915	0.989
QMLco	10	50	0.9	2	5	0.998	0.973	0.539	**0.073**	0.704	0.998	0.999

Table 3: Bias and RMSE of Lambda

T=5

		Bias								RMSE							
	N	250				N=50				N=250				N=50			
	Lambda	0	0.2	0.5	0.9	0	0.2	0.5	0.9	0	0.2	0.5	0.9	0	0.2	0.5	0.9
Design	J																
1	1	0.000	0.001	0.002	0.001	0.000	0.003	0.006	-0.014	0.001	0.001	0.001	0.000	0.005	0.004	0.003	0.006
	2	0.004	0.004	0.005	0.000	0.011	0.013	0.009	-0.021	0.003	0.002	0.001	0.001	0.014	0.011	0.016	0.010
	3	0.007	0.008	0.009	-0.006	0.018	0.020	0.010	-0.018	0.005	0.004	0.002	0.003	0.024	0.018	0.024	0.011
2	1	0.000	0.001	0.001	0.001	0.001	0.003	0.005	-0.006	0.001	0.001	0.000	0.000	0.003	0.003	0.002	0.003
	2	-0.002	-0.001	0.001	-0.002	-0.004	-0.001	0.003	-0.020	0.002	0.002	0.001	0.001	0.010	0.008	0.006	0.008
	3	-0.002	0.000	0.002	-0.008	-0.004	-0.001	0.000	-0.029	0.003	0.002	0.001	0.003	0.017	0.012	0.013	0.012
3	1	0.001	0.002	0.002	0.001	0.000	0.002	0.004	-0.015	0.001	0.001	0.001	0.000	0.004	0.004	0.003	0.006
	2	0.004	0.004	0.005	-0.001	0.009	0.010	0.011	-0.020	0.003	0.002	0.001	0.001	0.014	0.011	0.008	0.009
	3	0.004	0.005	0.006	-0.003	0.016	0.016	0.010	-0.032	0.005	0.003	0.002	0.002	0.023	0.018	0.018	0.014
4	1	0.000	0.001	0.001	0.000	0.001	0.002	0.002	-0.014	0.001	0.001	0.001	0.000	0.005	0.004	0.003	0.005
	2	0.002	0.001	0.002	-0.002	0.006	0.007	0.005	-0.022	0.003	0.002	0.001	0.001	0.014	0.011	0.008	0.008
	3	0.002	0.003	0.004	-0.005	0.012	0.012	0.011	-0.030	0.005	0.004	0.002	0.002	0.024	0.017	0.010	0.012
5	1	0.000	0.000	0.000	0.000	0.000	0.001	0.001	-0.006	0.001	0.001	0.000	0.000	0.003	0.002	0.002	0.002
	2	-0.003	-0.003	-0.001	-0.003	-0.009	-0.007	-0.005	-0.027	0.002	0.001	0.001	0.001	0.008	0.006	0.005	0.009
	3	-0.005	-0.004	-0.002	-0.013	-0.015	-0.012	-0.011	-0.032	0.003	0.002	0.001	0.004	0.014	0.011	0.011	0.011

T=10

		Bias								RMSE							
	N	250				N=50				N=250				N=50			
	Lambda	0	0.2	0.5	0.9	0	0.2	0.5	0.9	0	0.2	0.5	0.9	0	0.2	0.5	0.9
Design	J																
1	1	0.001	0.001	0.002	0.001	-0.002	0.000	0.001	-0.003	0.000	0.000	0.000	0.000	0.002	0.002	0.001	0.002
	2	0.003	0.003	0.003	0.001	0.002	0.003	0.005	-0.007	0.001	0.001	0.001	0.000	0.007	0.005	0.003	0.003
	3	0.004	0.005	0.005	0.001	0.008	0.008	0.010	-0.018	0.002	0.002	0.001	0.000	0.012	0.009	0.005	0.008
2	1	0.000	0.000	0.001	0.001	-0.002	-0.001	0.000	-0.001	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.001
	2	-0.001	0.000	0.001	0.001	-0.005	-0.003	0.000	-0.012	0.001	0.001	0.000	0.000	0.004	0.003	0.002	0.004
	3	-0.001	0.001	0.002	0.000	-0.006	-0.003	0.001	-0.023	0.001	0.001	0.001	0.000	0.007	0.005	0.003	0.008
3	1	-0.001	0.000	0.000	0.000	-0.002	-0.001	0.001	-0.005	0.000	0.000	0.000	0.000	0.002	0.002	0.001	0.002
	2	-0.001	-0.001	0.000	0.000	0.003	0.003	0.004	-0.009	0.001	0.001	0.001	0.000	0.006	0.005	0.003	0.004
	3	-0.002	0.000	0.002	0.000	0.009	0.009	0.009	-0.016	0.002	0.002	0.001	0.000	0.011	0.008	0.004	0.007
4	1	0.000	0.000	0.000	0.000	-0.001	-0.001	0.000	-0.004	0.000	0.000	0.000	0.000	0.002	0.002	0.001	0.001
	2	0.001	0.000	0.001	0.000	0.000	0.001	0.001	-0.009	0.001	0.001	0.001	0.000	0.006	0.005	0.003	0.003
	3	0.000	0.001	0.002	-0.001	0.003	0.004	0.005	-0.019	0.002	0.001	0.001	0.000	0.011	0.008	0.004	0.007
5	1	0.000	0.000	0.000	0.001	-0.002	-0.002	-0.001	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.000
	2	-0.001	-0.001	0.000	0.000	-0.008	-0.007	-0.004	-0.015	0.001	0.001	0.000	0.000	0.004	0.003	0.002	0.005
	3	-0.002	-0.001	0.000	-0.003	-0.011	-0.009	-0.006	-0.032	0.001	0.001	0.000	0.001	0.006	0.005	0.002	0.010

Overall, I conclude that the QMLco estimator provides good small sample guidance regardless of the degree of spatial and temporal correlation in the data. The other estimators break down when the degree of spatial autocorrelation is high but, however, the QMLiid estimator is robust to small amounts of spatial autocorrelation.

Since this paper extends the spatial GM procedure to the multivariate context, I also report in Table 3 the performance of the spatial GM estimation. The results show that spatial GM procedure works well in all sample sizes and data generation designs under consideration. The highest RMSE is .024 when $N = 50$ and $T = 5$ and it drops to below 0.002 in the largest sample size ($N = 250$, $T = 10$).

5 Conclusion

This paper develops an estimation approach for a panel VAR model with spatial dependence. I extend the literature in several aspects. First, it is studied how cross-sectional dependence of a particular form affects the various panel VAR estimators. Secondly, I generalize the spatial GM procedure to the multivariate context.

This paper proposes a three-step estimation procedure. In the first step, instrumental variables procedure is used to consistently estimate the spatially correlated disturbances. In the second step, a method of moments estimation is used to obtain a consistent estimate of the spatial parameter. The final step of the procedure is either a constrained maximum likelihood procedure or moments estimation based on a model transformed by a spatial Cochrane-Orcutt transformation.

Finally, the small sample properties of the different estimation procedures are studied in a Monte Carlo study. The results show that the constrained likelihood procedure works well in small samples. They also document that the QML estimation based on the independence assumption is robust to small amount of spatial autocorrelation in the data.

In future research, it would also be of interest to prove asymptotic normality of the proposed estimator as well as to derive the asymptotic properties of the QML estimator under some reasonable set of assumptions. An interesting complement to the approach of this paper would be to develop estimation procedures for panel VAR models with alternative specifications of cross-sectional correlation (e.g. nonparametric, or factor models) and compare the their relative performance under different data generating designs.

A Appendix - Derivatives of the QML Function

To speed up computation, I derive analytical expressions for the partial derivatives of the likelihood function $L_N(\boldsymbol{\theta})$. The first differential is

$$
\begin{aligned}
dL_N = & -\frac{N}{2}tr\left(\boldsymbol{\Sigma}^{-1}d\boldsymbol{\Sigma}\right) - tr\left[(\mathbf{I}_{mNT}-\lambda\mathtt{W})^{-1}\cdot d\lambda\mathtt{W}\right] \\
& +\frac{1}{2}tr\left[(\mathbf{I}_N\otimes\mathbf{R}')(\mathbf{I}_{mNT}-\lambda\mathtt{W})\left(\mathbf{I}_N\otimes\boldsymbol{\Sigma}^{-1}d\boldsymbol{\Sigma}\cdot\boldsymbol{\Sigma}^{-1}\right)(\mathbf{I}_{mNT}-\lambda\mathtt{W}')(\mathbf{I}_N\otimes\mathbf{R})\mathbf{S}\right] \\
& -\frac{1}{2}tr\left[(\mathbf{I}_N\otimes d\mathbf{R}')(\mathbf{I}_{mNT}-\lambda\mathtt{W})\left(\mathbf{I}_N\otimes\boldsymbol{\Sigma}^{-1}\right)(\mathbf{I}_{mNT}-\lambda\mathtt{W}')(\mathbf{I}_N\otimes\mathbf{R})\mathbf{S}\right] \\
& -\frac{1}{2}tr\left[(\mathbf{I}_N\otimes\mathbf{R}')(\mathbf{I}_{mNT}-\lambda\mathtt{W})\left(\mathbf{I}_N\otimes\boldsymbol{\Sigma}^{-1}\right)(\mathbf{I}_{mNT}-\lambda\mathtt{W}')(\mathbf{I}_N\otimes d\mathbf{R})\mathbf{S}\right] \\
& -\frac{1}{2}tr\left[(\mathbf{I}_N\otimes\mathbf{R}')(-d\lambda\mathtt{W})\left(\mathbf{I}_N\otimes\boldsymbol{\Sigma}^{-1}\right)(\mathbf{I}_{mNT}-\lambda\mathtt{W}')(\mathbf{I}_N\otimes\mathbf{R})\mathbf{S}\right] \\
& -\frac{1}{2}tr\left[(\mathbf{I}_N\otimes\mathbf{R}')(\mathbf{I}_{mNT}-\lambda\mathtt{W})\left(\mathbf{I}_N\otimes\boldsymbol{\Sigma}^{-1}\right)(-d\lambda\mathtt{W}')(\mathbf{I}_N\otimes\mathbf{R})\mathbf{S}\right]
\end{aligned} \tag{A.1}
$$

$$
\begin{aligned}
= & -\frac{N}{2}vec\boldsymbol{\Sigma}^{-1}\mathbb{D}_{mT}dvech\boldsymbol{\Sigma} - vec\left(\mathbf{I}_{mNT}-\lambda\mathtt{W}'\right)^{-1}vec\mathtt{W}\cdot d\lambda \\
& +\frac{1}{2}\left(vec\left[\left(\mathbf{I}_N\otimes\boldsymbol{\Sigma}^{-1}\right)(\mathbf{I}_{mNT}-\lambda\mathtt{W}')(\mathbf{I}_N\otimes\mathbf{R})\mathbf{S}(\mathbf{I}_N\otimes\mathbf{R}')(\mathbf{I}_{mNT}-\lambda\mathtt{W})\left(\mathbf{I}_N\otimes\boldsymbol{\Sigma}^{-1}\right)\right]\right)' \\
& \cdot vec\left(\mathbf{I}_N\otimes d\boldsymbol{\Sigma}\right) \\
& -\frac{1}{2}\left(vec\left[\mathbf{S}(\mathbf{I}_N\otimes\mathbf{R}')(\mathbf{I}_{mNT}-\lambda\mathtt{W})\left(\mathbf{I}_N\otimes\boldsymbol{\Sigma}^{-1}\right)(\mathbf{I}_{mNT}-\lambda\mathtt{W}')\right]\right)'\cdot vec\left(\mathbf{I}_N\otimes d\mathbf{R}'\right) \\
& -\frac{1}{2}\left(vec\left[(\mathbf{I}_{mNT}-\lambda\mathtt{W})\left(\mathbf{I}_N\otimes\boldsymbol{\Sigma}^{-1}\right)(\mathbf{I}_{mNT}-\lambda\mathtt{W}')(\mathbf{I}_N\otimes\mathbf{R})\mathbf{S}\right]\right)'\cdot vec\left(\mathbf{I}_N\otimes d\mathbf{R}\right) \\
& +\frac{1}{2}tr\left[(\mathbf{I}_N\otimes\mathbf{R}')\mathtt{W}\left(\mathbf{I}_N\otimes\boldsymbol{\Sigma}^{-1}\right)(\mathbf{I}_{mNT}-\lambda\mathtt{W}')(\mathbf{I}_N\otimes\mathbf{R})\mathbf{S}\right]\cdot d\lambda \\
& +\frac{1}{2}tr\left[(\mathbf{I}_N\otimes\mathbf{R}')(\mathbf{I}_{mNT}-\lambda\mathtt{W})\left(\mathbf{I}_N\otimes\boldsymbol{\Sigma}^{-1}\right)\mathtt{W}'(\mathbf{I}_N\otimes\mathbf{R})\mathbf{S}\right]\cdot d\lambda,
\end{aligned}
$$

where $\mathbb{D}_{mT}$ is a duplication matrix (such as that $\mathbb{D}_k vech(\mathbf{X}) = vec(\mathbf{X})$ for any $k\times k$ matrix $\mathbf{X}$), $\mathbb{K}_{sq}$ is a commutation matrix (such that $\mathbb{K}_{sq}vec(\mathbf{X}) = vec(\mathbf{X}')$ for any $s\times q$ matrix $\mathbf{X}$). Hence

$$
\begin{aligned}
dL_N = & -\frac{1}{2}vec\left(\mathbf{I}_N\otimes\boldsymbol{\Sigma}^{-1}\right)\mathbf{H}\mathbb{D}_{mT}dvech\boldsymbol{\Sigma} - tr\left[(\mathbf{I}_{mNT}-\lambda\mathtt{W})^{-1}\mathtt{W}\right]\cdot d\lambda \\
& +\frac{1}{2}\left(vec\left[\left(\mathbf{I}_N\otimes\boldsymbol{\Sigma}^{-1}\right)(\mathbf{I}_{mNT}-\lambda\mathtt{W}')(\mathbf{I}_N\otimes\mathbf{R})\mathbf{S}(\mathbf{I}_N\otimes\mathbf{R}')(\mathbf{I}_{mNT}-\lambda\mathtt{W})\left(\mathbf{I}_N\otimes\boldsymbol{\Sigma}^{-1}\right)\right]\right)' \\
& \cdot\mathbf{H}\mathbb{D}_{mT}\cdot vech\boldsymbol{\Sigma} \\
& -\frac{1}{2}\left(vec\left[\mathbf{S}(\mathbf{I}_N\otimes\mathbf{R}')(\mathbf{I}_{mNT}-\lambda\mathtt{W})\left(\mathbf{I}_N\otimes\boldsymbol{\Sigma}^{-1}\right)(\mathbf{I}_{mNT}-\lambda\mathtt{W}')\right]\right)'\mathbf{H}\mathbb{K}_{mT,mT}\cdot dvec\mathbf{R} \\
& -\frac{1}{2}\left(vec\left[(\mathbf{I}_{mNT}-\lambda\mathtt{W})\left(\mathbf{I}_N\otimes\boldsymbol{\Sigma}^{-1}\right)(\mathbf{I}_{mNT}-\lambda\mathtt{W}')(\mathbf{I}_N\otimes\mathbf{R})\mathbf{S}\right]\right)'\mathbf{H}\cdot dvec\mathbf{R} \\
& +tr\left[(\mathbf{I}_N\otimes\mathbf{R}')\mathtt{W}\left(\mathbf{I}_N\otimes\boldsymbol{\Sigma}^{-1}\right)(\mathbf{I}_{mNT}-\lambda\mathtt{W}')(\mathbf{I}_N\otimes\mathbf{R})\mathbf{S}\right]\cdot d\lambda,
\end{aligned} \tag{A.2}
$$

where the matrix of constants $\mathbf{H}$ is given by

$$
\mathbf{H} = \left[(\mathbf{I}_N\otimes\mathbb{K}_{mT,N})(vec\mathbf{I}_N)\otimes\mathbf{I}_{mT}\right]\otimes\mathbf{I}_{mT}. \tag{A.3}
$$

The differential of the inverse of the variance covariance matrix under condition (IOR) is

$$\begin{aligned} dvech\boldsymbol{\Sigma} &= vech\left[(\mathbf{A}_1 \otimes d\boldsymbol{\Psi}) + (\mathbf{A}_2 \otimes d\boldsymbol{\Omega})\right] \\ &= \mathbb{D}_{mT}^{-1}(\mathbf{I}_T \otimes \mathbb{K}_{m,T} \otimes \mathbf{I}_m)(vec\mathbf{A}_1 \otimes \mathbf{I}_{m^2})\mathbb{D}_m dvech\boldsymbol{\Psi} + \\ &\quad + \mathbb{D}_{mT}^{-1}(\mathbf{I}_T \otimes \mathbb{K}_{m,T} \otimes \mathbf{I}_m)(vec\mathbf{A}_2 \otimes \mathbf{I}_{m^2})\mathbb{D}_m dvech\boldsymbol{\Omega} \\ &= \mathbb{D}_{mT}^{-1}\mathbf{B}_1\mathbb{D}_m dvech\boldsymbol{\Psi} + \mathbb{D}_{mT}^{-1}\mathbf{B}_2\mathbb{D}_m dvech\boldsymbol{\Omega} \end{aligned} \tag{A.4}$$

and the differential of the matrix $\mathbf{R}$ containing the slope coefficients is

$$\begin{aligned} dvec\mathbf{R} &= vec(\mathbf{A}_3 \otimes d\boldsymbol{\Phi}) \\ &= (\mathbf{I}_T \otimes \mathbb{K}_{m,T} \otimes \mathbf{I}_m)(vec\mathbf{A}_3 \otimes \mathbf{I}_{m^2})dvec\boldsymbol{\Phi} \\ &= \mathbf{B}_3 dvec\boldsymbol{\Phi} \end{aligned} \tag{A.5}$$

with $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{A}_3$, $\mathbf{B}_1$, $\mathbf{B}_2$ and $\mathbf{B}_3$ being matrices of constants reflecting the structure of $\boldsymbol{\Sigma}^{-1}$ and $\mathbf{R}$.

In particular,

$$\mathbf{A}_1 = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & & 0 \end{pmatrix} \tag{A.6}$$

$$\mathbf{A}_2 = \mathbf{I}_T - \mathbf{A}_1 \tag{A.7}$$

and

$$\mathbf{A}_3 = \begin{pmatrix} 0 & & & \cdots & 0 \\ -1 & 0 & & & \vdots \\ 0 & -1 & \ddots & & \\ \vdots & \ddots & \ddots & 0 & \\ 0 & \ldots & 0 & -1 & 0 \end{pmatrix} \tag{A.8}$$

Defining

$$\begin{aligned} \mathbf{M}_1 &= -\left(\mathbf{I}_N \otimes \boldsymbol{\Sigma}^{-1}\right) \\ &\quad + \left[\left(\mathbf{I}_N \otimes \boldsymbol{\Sigma}^{-1}\right)(\mathbf{I}_{mNT} - \lambda \mathtt{W}')(\mathbf{I}_N \otimes \mathbf{R})\mathbf{S}(\mathbf{I}_N \otimes \mathbf{R}')(\mathbf{I}_{mNT} - \lambda \mathtt{W})\left(\mathbf{I}_N \otimes \boldsymbol{\Sigma}^{-1}\right)\right], \\ \mathbf{M}_2 &= -(\mathbf{I}_{mNT} - \lambda \mathtt{W})\left(\mathbf{I}_N \otimes \boldsymbol{\Sigma}^{-1}\right)(\mathbf{I}_{mNT} - \lambda \mathtt{W}')(\mathbf{I}_N \otimes \mathbf{R})\mathbf{S}, \\ \mathbf{M}_3 &= (\mathbf{I}_N \otimes \mathbf{R}')\mathtt{W}\left(\mathbf{I}_N \otimes \boldsymbol{\Sigma}^{-1}\right)(\mathbf{I}_{mNT} - \lambda \mathtt{W}')(\mathbf{I}_N \otimes \mathbf{R}) - (\mathbf{I}_{mNT} - \lambda \mathtt{W})^{-1}\mathtt{W}, \end{aligned} \tag{A.9}$$

we can write the Jacobian of $L_N(\boldsymbol{\theta})$ in a partitioned form as

$$DL_N(\boldsymbol{\theta}) = \frac{1}{2}\begin{bmatrix} [vec(\mathbf{M}_1)]'\mathbf{H}\mathbf{B}_1\mathbb{D}_m : \\ [vec(\mathbf{M}_1)]'\mathbf{H}\mathbf{B}_2\mathbb{D}_m : \\ [vec(\mathbf{M}_2)]'\mathbf{H}(\mathbf{I} + \mathbb{K}_{mT,mT})\mathbf{B}_3 : \\ 2tr(\mathbf{M}_3) \end{bmatrix}, \tag{A.10}$$

where : denotes horizontal stacking.

Under the (IOR) condition the differential of the variance covariance matrix of the initial observations becomes

$$
\begin{aligned}
dvech\boldsymbol{\Psi} &= dvech\boldsymbol{\Omega}_{\varepsilon} &\text{(A.11)}\\
&+\mathbb{D}_m^{-1}\left[(\mathbf{I}_m-\boldsymbol{\Phi})\otimes(\mathbf{I}_m-\boldsymbol{\Phi})\right]\left[\mathbf{I}_{m^2}-(\boldsymbol{\Phi}\otimes\boldsymbol{\Phi})\right]^{-1}\mathbb{D}_m dvech\boldsymbol{\Omega}_{\varepsilon}\\
&-\mathbb{D}_m^{-1}\left[d\boldsymbol{\Phi}\otimes(\mathbf{I}_m-\boldsymbol{\Phi})\right]\left[\mathbf{I}_{m^2}-(\boldsymbol{\Phi}\otimes\boldsymbol{\Phi})\right]^{-1}vec\boldsymbol{\Omega}_{\varepsilon}\\
&-\mathbb{D}_m^{-1}\left[(\mathbf{I}_m-\boldsymbol{\Phi})\otimes d\boldsymbol{\Phi}\right]\left[\mathbf{I}_{m^2}-(\boldsymbol{\Phi}\otimes\boldsymbol{\Phi})\right]^{-1}vec\boldsymbol{\Omega}_{\varepsilon}\\
&-\mathbb{D}_m^{-1}\left[(\mathbf{I}_m-\boldsymbol{\Phi})\otimes(\mathbf{I}_m-\boldsymbol{\Phi})\right]\left[\mathbf{I}_{m^2}-(\boldsymbol{\Phi}\otimes\boldsymbol{\Phi})\right]^{-1}\\
&\cdot\left[(d\boldsymbol{\Phi}\otimes\boldsymbol{\Phi})+(\boldsymbol{\Phi}\otimes d\boldsymbol{\Phi})\right]\left[\mathbf{I}_{m^2}-(\boldsymbol{\Phi}\otimes\boldsymbol{\Phi})\right]^{-1}vec\boldsymbol{\Omega}_{\varepsilon}
\end{aligned}
$$

$$
\begin{aligned}
= \;& dvech\boldsymbol{\Omega}_{\varepsilon}+\mathbb{D}_m^{-1}\left[(\mathbf{I}_m-\boldsymbol{\Phi})\otimes(\mathbf{I}_m-\boldsymbol{\Phi})\right]\left[\mathbf{I}_{m^2}-(\boldsymbol{\Phi}\otimes\boldsymbol{\Phi})\right]^{-1}\mathbb{D}_m dvech\boldsymbol{\Omega}_{\varepsilon}\\
&-\left[(vec\boldsymbol{\Omega}_{\varepsilon})'\left[\mathbf{I}_{m^2}-(\boldsymbol{\Phi}'\otimes\boldsymbol{\Phi}')\right]^{-1}\otimes\mathbb{D}_m^{-1}\right]\cdot vec\left[d\boldsymbol{\Phi}\otimes(\mathbf{I}_m-\boldsymbol{\Phi})\right]\\
&-\left[(vec\boldsymbol{\Omega}_{\varepsilon})'\left[\mathbf{I}_{m^2}-(\boldsymbol{\Phi}'\otimes\boldsymbol{\Phi}')\right]^{-1}\otimes\mathbb{D}_m^{-1}\right]\cdot vec\left[(\mathbf{I}_m-\boldsymbol{\Phi})\otimes d\boldsymbol{\Phi}\right]\\
&-\left[(vec\boldsymbol{\Omega}_{\varepsilon})'\left[\mathbf{I}_{m^2}-(\boldsymbol{\Phi}'\otimes\boldsymbol{\Phi}')\right]^{-1}\otimes\mathbb{D}_m^{-1}\left[(\mathbf{I}_m-\boldsymbol{\Phi})\otimes(\mathbf{I}_m-\boldsymbol{\Phi})\right]\left[\mathbf{I}_{m^2}-(\boldsymbol{\Phi}\otimes\boldsymbol{\Phi})\right]^{-1}\right]\\
&\cdot vec\left[(d\boldsymbol{\Phi}\otimes\boldsymbol{\Phi})+(\boldsymbol{\Phi}\otimes d\boldsymbol{\Phi})\right]
\end{aligned}
$$

$$
\begin{aligned}
= \;& dvech\boldsymbol{\Omega}_{\varepsilon}+\mathbb{D}_m^{-1}\left[(\mathbf{I}_m-\boldsymbol{\Phi})\otimes(\mathbf{I}_m-\boldsymbol{\Phi})\right]\left[\mathbf{I}_{m^2}-(\boldsymbol{\Phi}\otimes\boldsymbol{\Phi})\right]^{-1}\mathbb{D}_m dvech\boldsymbol{\Omega}_{\varepsilon}\\
&-\left[(vec\boldsymbol{\Omega}_{\varepsilon})'\left[\mathbf{I}_{m^2}-(\boldsymbol{\Phi}'\otimes\boldsymbol{\Phi}')\right]^{-1}\otimes\mathbb{D}_m^{-1}\right](\mathbf{I}_m\otimes\mathbb{K}_m\otimes\mathbf{I}_m)\left[\mathbf{I}_{m^2}\otimes vec(\mathbf{I}_m-\boldsymbol{\Phi})\right]\cdot dvec\boldsymbol{\Phi}\\
&-\left[(vec\boldsymbol{\Omega}_{\varepsilon})'\left[\mathbf{I}_{m^2}-(\boldsymbol{\Phi}'\otimes\boldsymbol{\Phi}')\right]^{-1}\otimes\mathbb{D}_m^{-1}\right](\mathbf{I}_m\otimes\mathbb{K}_m\otimes\mathbf{I}_m)\left[vec(\mathbf{I}_m-\boldsymbol{\Phi})\otimes\mathbf{I}_{m^2}\right]dvec\boldsymbol{\Phi}\\
&-\left[(vec\boldsymbol{\Omega}_{\varepsilon})'\left[\mathbf{I}_{m^2}-(\boldsymbol{\Phi}'\otimes\boldsymbol{\Phi}')\right]^{-1}\otimes\mathbb{D}_m^{-1}\left[(\mathbf{I}_m-\boldsymbol{\Phi})\otimes(\mathbf{I}_m-\boldsymbol{\Phi})\right]\left[\mathbf{I}_{m^2}-(\boldsymbol{\Phi}\otimes\boldsymbol{\Phi})\right]^{-1}\right]\\
&\cdot(\mathbf{I}_m\otimes\mathbb{K}_m\otimes\mathbf{I}_m)\left[(\mathbf{I}_{m^2}\otimes\boldsymbol{\Phi})+(\boldsymbol{\Phi}\otimes\mathbf{I}_{m^2})\right]\cdot dvec\boldsymbol{\Phi}
\end{aligned}
$$

Jacobian of then becomes

$$
DL_N(\boldsymbol{\vartheta})=\frac{1}{2}\left[\begin{array}{c}
\left[vec(\mathbf{M}_1)\right]'\mathbf{H}(\mathbf{B}_1\mathbf{C}_1+\mathbf{B}_2)\mathbb{D}_m:\\
\left[vec(\mathbf{M}_1)\right]'\mathbf{H}\mathbf{B}_1\mathbf{C}_2+\left[vec(\mathbf{M}_2)\right]'\mathbf{H}(\mathbf{I}+\mathbb{K}_{mT,mT})\mathbf{B}_3:\\
2tr(\mathbf{M}_3)
\end{array}\right], \qquad \text{(A.12)}
$$

where

$$
\begin{aligned}
\mathbf{C}_1 &= \mathbf{I}_{m^2}+\left[(\mathbf{I}_m-\boldsymbol{\Phi})\otimes(\mathbf{I}_m-\boldsymbol{\Phi})\right]\left[\mathbf{I}_{m^2}-(\boldsymbol{\Phi}\otimes\boldsymbol{\Phi})\right]^{-1} &\text{(A.13)}\\
\mathbf{C}_2 &= -\left[(vec\boldsymbol{\Omega}_{\varepsilon})'\left[\mathbf{I}_{m^2}-(\boldsymbol{\Phi}'\otimes\boldsymbol{\Phi}')\right]^{-1}\otimes\mathbb{D}_m^{-1}\right](\mathbf{I}_m\otimes\mathbb{K}_m\otimes\mathbf{I}_m)\left[\mathbf{I}_{m^2}\otimes vec(\mathbf{I}_m-\boldsymbol{\Phi})\right]\\
&\quad-\left[(vec\boldsymbol{\Omega}_{\varepsilon})'\left[\mathbf{I}_{m^2}-(\boldsymbol{\Phi}'\otimes\boldsymbol{\Phi}')\right]^{-1}\otimes\mathbb{D}_m^{-1}\right](\mathbf{I}_m\otimes\mathbb{K}_m\otimes\mathbf{I}_m)\left[vec(\mathbf{I}_m-\boldsymbol{\Phi})\otimes\mathbf{I}_{m^2}\right]\\
&\quad-\left[(vec\boldsymbol{\Omega}_{\varepsilon})'\left[\mathbf{I}_{m^2}-(\boldsymbol{\Phi}'\otimes\boldsymbol{\Phi}')\right]^{-1}\otimes\mathbb{D}_m^{-1}\left[(\mathbf{I}_m-\boldsymbol{\Phi})\otimes(\mathbf{I}_m-\boldsymbol{\Phi})\right]\left[\mathbf{I}_{m^2}-(\boldsymbol{\Phi}\otimes\boldsymbol{\Phi})\right]^{-1}\right]\\
&\quad\cdot(\mathbf{I}_m\otimes\mathbb{K}_m\otimes\mathbf{I}_m)\left[(\mathbf{I}_{m^2}\otimes\boldsymbol{\Phi})+(\boldsymbol{\Phi}\otimes\mathbf{I}_{m^2})\right]
\end{aligned}
$$

B Appendix - Derivation of the Moment Conditions

Observe first that

$$\begin{aligned}\mathbf{Q}_0\widetilde{\boldsymbol{\nu}} &= \left[\left(\mathbf{I}_T - \tfrac{1}{T}\mathbf{J}_T\right)\otimes \mathbf{I}_N\right]\left[(\boldsymbol{\iota}_T \otimes \widetilde{\boldsymbol{\mu}}) + \widetilde{\boldsymbol{\varepsilon}}\right] \\ &= \mathbf{Q}_0\widetilde{\boldsymbol{\varepsilon}} + \left[\left(\mathbf{I}_T - \tfrac{1}{T}\mathbf{J}_T\right)\otimes \mathbf{I}_N\right](\boldsymbol{\iota}_T \otimes \widetilde{\boldsymbol{\mu}}) \\ &= \mathbf{Q}_0\widetilde{\boldsymbol{\varepsilon}} + \left[\left(\mathbf{I}_T - \tfrac{1}{T}\mathbf{J}_T\right)\boldsymbol{\iota}_T \otimes \mathbf{I}_N\widetilde{\boldsymbol{\mu}}\right] \\ &= \mathbf{Q}_0\widetilde{\boldsymbol{\varepsilon}} + \left[\left(\mathbf{I}_T\boldsymbol{\iota}_T - \tfrac{1}{T}\mathbf{J}_T\boldsymbol{\iota}_T\right)\otimes \widetilde{\boldsymbol{\mu}}\right] \\ &= \mathbf{Q}_0\widetilde{\boldsymbol{\varepsilon}} + \left[\left(\boldsymbol{\iota}_T - \tfrac{T}{T}\boldsymbol{\iota}_T\right)\otimes \widetilde{\boldsymbol{\mu}}\right] = \mathbf{Q}_0\widetilde{\boldsymbol{\varepsilon}}.\end{aligned} \tag{B.1}$$

The first moment condition is based on the following observation:

$$\begin{aligned}E\widetilde{\boldsymbol{\nu}}'\mathbf{Q}_0\widetilde{\boldsymbol{\nu}} &= E\widetilde{\boldsymbol{\nu}}'\mathbf{Q}_0'\mathbf{Q}_0\widetilde{\boldsymbol{\nu}} = E\widetilde{\boldsymbol{\varepsilon}}'\mathbf{Q}_0\widetilde{\boldsymbol{\varepsilon}} = E\widetilde{\boldsymbol{\varepsilon}}'\left[\left(\mathbf{I}_T - \tfrac{1}{T}\mathbf{J}_T\right)\otimes \mathbf{I}_N\right]\widetilde{\boldsymbol{\varepsilon}} \\ &= E\widetilde{\boldsymbol{\varepsilon}}'\widetilde{\boldsymbol{\varepsilon}} - \tfrac{1}{T}E\left(\widetilde{\boldsymbol{\varepsilon}}_1',..,\widetilde{\boldsymbol{\varepsilon}}_T'\right)(\mathbf{J}_T\otimes\mathbf{I}_N)\left(\widetilde{\boldsymbol{\varepsilon}}_1',..,\widetilde{\boldsymbol{\varepsilon}}_T'\right)' \\ &= \sum_{t=1}^{T}E\widetilde{\boldsymbol{\varepsilon}}_t'\widetilde{\boldsymbol{\varepsilon}}_t - \tfrac{1}{T}E\left(\sum_{t=1}^{T}\widetilde{\boldsymbol{\varepsilon}}_t',..,\sum_{t=1}^{T}\widetilde{\boldsymbol{\varepsilon}}_t'\right)\left(\widetilde{\boldsymbol{\varepsilon}}_1',..,\widetilde{\boldsymbol{\varepsilon}}_T'\right)' \\ &= T\sum_{i=1}^{N}E\widetilde{\boldsymbol{\varepsilon}}_{it}\widetilde{\boldsymbol{\varepsilon}}_{it}' - \frac{1}{T}\sum_{t=1}^{T}\sum_{s=1}^{T}E\widetilde{\boldsymbol{\varepsilon}}_t'\widetilde{\boldsymbol{\varepsilon}}_s \\ &= NT\cdot\boldsymbol{\Omega}_\varepsilon - \frac{1}{T}E\sum_{t=1}^{T}\sum_{s=1}^{T}\widetilde{\boldsymbol{\varepsilon}}_t'\widetilde{\boldsymbol{\varepsilon}}_s = \left(NT - \frac{TN}{T}\right)\boldsymbol{\Omega}_\varepsilon = N(T-1)\cdot\boldsymbol{\Omega}_\varepsilon\end{aligned} \tag{B.2}$$

The second moment conditions follows from

$$\begin{aligned}E\overline{\widetilde{\boldsymbol{\nu}}}'\mathbf{Q}_0\overline{\widetilde{\boldsymbol{\nu}}} &= E\overline{\widetilde{\boldsymbol{\varepsilon}}}'\mathbf{Q}_0\overline{\widetilde{\boldsymbol{\varepsilon}}} = E\overline{\widetilde{\boldsymbol{\varepsilon}}}'\left[\left(\mathbf{I}_T - \tfrac{1}{T}\mathbf{J}_T\right)\otimes\mathbf{I}_N\right]\overline{\widetilde{\boldsymbol{\varepsilon}}} \\ &= E\overline{\widetilde{\boldsymbol{\varepsilon}}}'\overline{\widetilde{\boldsymbol{\varepsilon}}} - \tfrac{1}{T}E\left(\overline{\widetilde{\boldsymbol{\varepsilon}}}_1',..,\overline{\widetilde{\boldsymbol{\varepsilon}}}_T'\right)(\mathbf{J}_T\otimes\mathbf{I}_N)\left(\overline{\widetilde{\boldsymbol{\varepsilon}}}_1',..,\overline{\widetilde{\boldsymbol{\varepsilon}}}_T'\right)' \\ &= \sum_{t=1}^{T}E\overline{\widetilde{\boldsymbol{\varepsilon}}}_t'\overline{\widetilde{\boldsymbol{\varepsilon}}}_t - \frac{1}{T}\sum_{t=1}^{T}\sum_{s=1}^{T}E\overline{\widetilde{\boldsymbol{\varepsilon}}}_t'\overline{\widetilde{\boldsymbol{\varepsilon}}}_s \\ &= \sum_{t=1}^{T}\sum_{i=1}^{N}E\overline{\boldsymbol{\varepsilon}}_{it}\overline{\boldsymbol{\varepsilon}}_{it}' - \frac{1}{T}\sum_{t=1}^{T}\sum_{s=1}^{T}\sum_{i=1}^{N}E\overline{\boldsymbol{\varepsilon}}_{it}\overline{\boldsymbol{\varepsilon}}_{is}' = (T-1)\sum_{i=1}^{N}E\overline{\boldsymbol{\varepsilon}}_{it}\overline{\boldsymbol{\varepsilon}}_{it}' \\ &= (T-1)\sum_{i=1}^{N}(\mathbf{w}_{i1},..,\mathbf{w}_{iN})\,E\left(\boldsymbol{\varepsilon}_{1t}',..,\boldsymbol{\varepsilon}_{Nt}'\right)'\left(\boldsymbol{\varepsilon}_{1t}',..,\boldsymbol{\varepsilon}_{Nt}'\right)(\mathbf{w}_{i1},..,\mathbf{w}_{iN})' \\ &= (T-1)\sum_{i=1}^{N}(\mathbf{w}_{i1},..,\mathbf{w}_{iN})(\mathbf{I}_N\otimes\boldsymbol{\Omega}_\varepsilon)(\mathbf{w}_{i1},..,\mathbf{w}_{iN})' \\ &= (T-1)\sum_{i=1}^{N}\sum_{j=1}^{N}\mathbf{w}_{ij}\boldsymbol{\Omega}_\varepsilon\mathbf{w}_{ij}'.\end{aligned} \tag{B.3}$$

The third moment condition is based on

$$\begin{aligned}
E\overline{\widetilde{\boldsymbol{\nu}}}'\mathbf{Q}_0\widetilde{\boldsymbol{\nu}} &= E\overline{\widetilde{\boldsymbol{\varepsilon}}}'\mathbf{Q}_0\widetilde{\boldsymbol{\varepsilon}} = E\overline{\widetilde{\boldsymbol{\varepsilon}}}'\left[\left(\mathbf{I}_T - \tfrac{1}{T}\mathbf{J}_T\right)\otimes\mathbf{I}_N\right]\widetilde{\boldsymbol{\varepsilon}} \qquad \text{(B.4)}\\
&= E\overline{\widetilde{\boldsymbol{\varepsilon}}}'\widetilde{\boldsymbol{\varepsilon}} - \tfrac{1}{T}E\left(\overline{\widetilde{\boldsymbol{\varepsilon}}}_1',..,\overline{\widetilde{\boldsymbol{\varepsilon}}}_T'\right)\left(\mathbf{J}_T\otimes\mathbf{I}_N\right)\left(\widetilde{\boldsymbol{\varepsilon}}_1',..,\widetilde{\boldsymbol{\varepsilon}}_T'\right)'\\
&= \sum_{t=1}^{T}E\overline{\widetilde{\boldsymbol{\varepsilon}}}_t'\widetilde{\boldsymbol{\varepsilon}}_t - \frac{1}{T}\sum_{t=1}^{T}\sum_{s=1}^{T}E\overline{\widetilde{\boldsymbol{\varepsilon}}}_t'\widetilde{\boldsymbol{\varepsilon}}_s\\
&= \sum_{t=1}^{T}\sum_{i=1}^{N}E\overline{\boldsymbol{\varepsilon}}_{it}\boldsymbol{\varepsilon}_{it}' - \frac{1}{T}\sum_{t=1}^{T}\sum_{s=1}^{T}\sum_{i=1}^{N}E\overline{\boldsymbol{\varepsilon}}_{it}\boldsymbol{\varepsilon}_{is}' = (T-1)\sum_{i=1}^{N}E\overline{\boldsymbol{\varepsilon}}_{it}\boldsymbol{\varepsilon}_{it}'\\
&= (T-1)\sum_{i=1}^{N}\left(\mathbf{w}_{i1},..,\mathbf{w}_{iN}\right)E\left(\boldsymbol{\varepsilon}_{1t}',..,\boldsymbol{\varepsilon}_{Nt}'\right)'\boldsymbol{\varepsilon}_{it}'\\
&= (T-1)\sum_{i=1}^{N}\left(\mathbf{w}_{i1},..,\mathbf{w}_{iN}\right)\left(\mathbf{e}_{i,N}\otimes\boldsymbol{\Omega}_\varepsilon\right) = (T-1)\sum_{i=1}^{N}\mathbf{w}_{ii}\boldsymbol{\Omega}_\varepsilon,
\end{aligned}$$

where we use $\mathbf{e}_{i,N}$ to denote an $N\times 1$ vector of zeros with an entry of one on the $i-th$ position.

To derive the next set of moment conditions involving $\mathbf{Q}_1$, we note that

$$\begin{aligned}
\mathbf{Q}_1\widetilde{\boldsymbol{\nu}} &= \left(\tfrac{1}{T}\mathbf{J}_T\otimes\mathbf{I}_N\right)\left[\left(\boldsymbol{\iota}_T\otimes\widetilde{\boldsymbol{\mu}}\right)+\widetilde{\boldsymbol{\varepsilon}}\right] \qquad \text{(B.5)}\\
&= \mathbf{Q}_1\widetilde{\boldsymbol{\varepsilon}} + \left(\tfrac{1}{T}\mathbf{J}_T\otimes\mathbf{I}_N\right)\left(\boldsymbol{\iota}_T\otimes\widetilde{\boldsymbol{\mu}}\right)\\
&= \mathbf{Q}_1\widetilde{\boldsymbol{\varepsilon}} + \left(\tfrac{1}{T}\mathbf{J}_T\boldsymbol{\iota}_T\otimes\widetilde{\boldsymbol{\mu}}\right)\\
&= \mathbf{Q}_1\widetilde{\boldsymbol{\varepsilon}} + \left(\boldsymbol{\iota}_T\otimes\mathbf{I}_N\right)\widetilde{\boldsymbol{\mu}}.
\end{aligned}$$

Furthermore, denoting

$$\widetilde{\mathbf{W}} = \left[\left(\mathbf{w}_{11},..,\mathbf{w}_{1N}\right),..,\left(\mathbf{w}_{1N},..,\mathbf{w}_{NN}\right)\right]', \qquad \text{(B.6)}$$

we have that

$$\begin{aligned}
\mathbf{Q}_1\overline{\widetilde{\boldsymbol{\nu}}} &= \mathbf{Q}_1\overline{\widetilde{\boldsymbol{\varepsilon}}} + \left(\tfrac{1}{T}\mathbf{J}_T\otimes\mathbf{I}_N\right)\begin{bmatrix}\left(\mathbf{I}_N\otimes\boldsymbol{\mu}_1'\right),..,\left(\mathbf{I}_N\otimes\boldsymbol{\mu}_N'\right)\\ \vdots \\ \left(\mathbf{I}_N\otimes\boldsymbol{\mu}_1'\right),..,\left(\mathbf{I}_N\otimes\boldsymbol{\mu}_N'\right)\end{bmatrix}\widetilde{\mathbf{W}} \qquad \text{(B.7)}\\
&= \mathbf{Q}_1\overline{\widetilde{\boldsymbol{\varepsilon}}} + \left(\tfrac{1}{T}\mathbf{J}_T\otimes\mathbf{I}_N\right)\left(\boldsymbol{\iota}_T\otimes\left[\left(\mathbf{I}_N\otimes\boldsymbol{\mu}_1'\right),..,\left(\mathbf{I}_N\otimes\boldsymbol{\mu}_N'\right)\right]\widetilde{\mathbf{W}}\right)\\
&= \mathbf{Q}_1\overline{\widetilde{\boldsymbol{\varepsilon}}} + \left(\tfrac{1}{T}\mathbf{J}_T\boldsymbol{\iota}_T\otimes\left[\left(\mathbf{I}_N\otimes\boldsymbol{\mu}_1'\right),..,\left(\mathbf{I}_N\otimes\boldsymbol{\mu}_N'\right)\right]\widetilde{\mathbf{W}}\right)\\
&= \mathbf{Q}_1\overline{\widetilde{\boldsymbol{\varepsilon}}} + \left(\boldsymbol{\iota}_T\otimes\left[\left(\mathbf{I}_N\otimes\boldsymbol{\mu}_1'\right),..,\left(\mathbf{I}_N\otimes\boldsymbol{\mu}_N'\right)\right]\widetilde{\mathbf{W}}\right)\\
&= \mathbf{Q}_1\overline{\widetilde{\boldsymbol{\varepsilon}}} + \left(\boldsymbol{\iota}_T\otimes\overline{\widetilde{\boldsymbol{\mu}}}\right).
\end{aligned}$$

Thus we have

$$
\begin{aligned}
E\widetilde{\boldsymbol{\nu}}'\mathbf{Q}_1\widetilde{\boldsymbol{\nu}} &= E\left[\widetilde{\boldsymbol{\varepsilon}}+(\boldsymbol{\iota}_T\otimes\mathbf{I}_N)\widetilde{\boldsymbol{\mu}}\right]'\left(\tfrac{1}{T}\mathbf{J}_T\otimes\mathbf{I}_N\right)\left[\widetilde{\boldsymbol{\varepsilon}}+(\boldsymbol{\iota}_T\otimes\mathbf{I}_N)\widetilde{\boldsymbol{\mu}}\right] \qquad \text{(B.8)}\\
&= E\widetilde{\boldsymbol{\varepsilon}}'\left(\tfrac{1}{T}\mathbf{J}_T\otimes\mathbf{I}_N\right)\widetilde{\boldsymbol{\varepsilon}}+E\widetilde{\boldsymbol{\mu}}'\left(\boldsymbol{\iota}_T'\otimes\mathbf{I}_N\right)\left(\tfrac{1}{T}\mathbf{J}_T\otimes\mathbf{I}_N\right)(\boldsymbol{\iota}_T\otimes\mathbf{I}_N)\widetilde{\boldsymbol{\mu}}\\
&= \tfrac{1}{T}E\left(\widetilde{\boldsymbol{\varepsilon}}_1',..,\widetilde{\boldsymbol{\varepsilon}}_T'\right)(\mathbf{J}_T\otimes\mathbf{I}_N)\left(\widetilde{\boldsymbol{\varepsilon}}_1',..,\widetilde{\boldsymbol{\varepsilon}}_T'\right)'+E\widetilde{\boldsymbol{\mu}}'\left(\tfrac{1}{T}\boldsymbol{\iota}_T'\mathbf{J}_T\boldsymbol{\iota}_T\otimes\mathbf{I}_N\right)\widetilde{\boldsymbol{\mu}}\\
&= \tfrac{1}{T}E\left(\sum_{t=1}^{T}\widetilde{\boldsymbol{\varepsilon}}_t',..,\sum_{t=1}^{T}\widetilde{\boldsymbol{\varepsilon}}_t'\right)\left(\widetilde{\boldsymbol{\varepsilon}}_1',..,\widetilde{\boldsymbol{\varepsilon}}_T'\right)'+T\cdot E\widetilde{\boldsymbol{\mu}}'\widetilde{\boldsymbol{\mu}}\\
&= \frac{1}{T}\sum_{t=1}^{T}\sum_{s=1}^{T}E\widetilde{\boldsymbol{\varepsilon}}_t'\widetilde{\boldsymbol{\varepsilon}}_s+T\cdot E\widetilde{\boldsymbol{\mu}}'\widetilde{\boldsymbol{\mu}}=\frac{1}{T}\sum_{t=1}^{T}\sum_{s=1}^{T}\sum_{i=1}^{N}E\widetilde{\boldsymbol{\varepsilon}}_{it}\widetilde{\boldsymbol{\varepsilon}}_{is}'+T\sum_{i=1}^{N}E\widetilde{\boldsymbol{\mu}}_i\widetilde{\boldsymbol{\mu}}_i'\\
&= \left(\frac{TN}{T}\right)\boldsymbol{\Omega}_\varepsilon+TN\cdot\boldsymbol{\Omega}_\mu=N\cdot\boldsymbol{\Omega}_1.
\end{aligned}
$$

Furthermore,

$$
\begin{aligned}
E\overline{\overline{\boldsymbol{\nu}}}'\mathbf{Q}_1\overline{\overline{\boldsymbol{\nu}}} &= E\left[\overline{\overline{\boldsymbol{\varepsilon}}}+(\boldsymbol{\iota}_T\otimes\mathbf{I}_N)\overline{\overline{\boldsymbol{\mu}}}\right]'\left(\tfrac{1}{T}\mathbf{J}_T\otimes\mathbf{I}_N\right)\left[\overline{\overline{\boldsymbol{\varepsilon}}}+(\boldsymbol{\iota}_T\otimes\mathbf{I}_N)\overline{\overline{\boldsymbol{\mu}}}\right] \qquad \text{(B.9)}\\
&= E\overline{\overline{\boldsymbol{\varepsilon}}}'\left(\tfrac{1}{T}\mathbf{J}_T\otimes\mathbf{I}_N\right)\overline{\overline{\boldsymbol{\varepsilon}}}+E\overline{\overline{\boldsymbol{\mu}}}'\left(\boldsymbol{\iota}_T'\otimes\mathbf{I}_N\right)\left(\tfrac{1}{T}\mathbf{J}_T\otimes\mathbf{I}_N\right)(\boldsymbol{\iota}_T\otimes\mathbf{I}_N)\overline{\overline{\boldsymbol{\mu}}}\\
&= \tfrac{1}{T}E\left(\overline{\overline{\boldsymbol{\varepsilon}}}_1',..,\overline{\overline{\boldsymbol{\varepsilon}}}_T'\right)(\mathbf{J}_T\otimes\mathbf{I}_N)\left(\overline{\overline{\boldsymbol{\varepsilon}}}_1',..,\overline{\overline{\boldsymbol{\varepsilon}}}_T'\right)'+E\overline{\overline{\boldsymbol{\mu}}}'\left(\tfrac{1}{T}\boldsymbol{\iota}_T'\mathbf{J}_T\boldsymbol{\iota}_T\otimes\mathbf{I}_N\right)\overline{\overline{\boldsymbol{\mu}}}\\
&= \tfrac{1}{T}E\left(\sum_{t=1}^{T}\overline{\overline{\boldsymbol{\varepsilon}}}_t',..,\sum_{t=1}^{T}\overline{\overline{\boldsymbol{\varepsilon}}}_t'\right)\left(\overline{\overline{\boldsymbol{\varepsilon}}}_1',..,\overline{\overline{\boldsymbol{\varepsilon}}}_T'\right)'+T\cdot E\overline{\overline{\boldsymbol{\mu}}}'\overline{\overline{\boldsymbol{\mu}}}\\
&= \frac{1}{T}\sum_{t=1}^{T}\sum_{s=1}^{T}E\overline{\overline{\boldsymbol{\varepsilon}}}_t'\overline{\overline{\boldsymbol{\varepsilon}}}_s+T\cdot E\overline{\overline{\boldsymbol{\mu}}}'\overline{\overline{\boldsymbol{\mu}}}=\frac{1}{T}\sum_{t=1}^{T}\sum_{s=1}^{T}\sum_{i=1}^{N}E\overline{\overline{\boldsymbol{\varepsilon}}}_{it}\overline{\overline{\boldsymbol{\varepsilon}}}_{is}'+T\sum_{i=1}^{N}E\overline{\overline{\boldsymbol{\mu}}}_i\overline{\overline{\boldsymbol{\mu}}}_i'\\
&= \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N}(\mathbf{w}_{i1},..,\mathbf{w}_{iN})E\left(\boldsymbol{\varepsilon}_{1t}',..,\boldsymbol{\varepsilon}_{Nt}'\right)'\left(\boldsymbol{\varepsilon}_{1t}',..,\boldsymbol{\varepsilon}_{Nt}'\right)(\mathbf{w}_{i1},..,\mathbf{w}_{iN})'\\
&\quad +T\sum_{i=1}^{N}(\mathbf{w}_{i1},..,\mathbf{w}_{iN})E\left(\boldsymbol{\mu}_1',..,\boldsymbol{\mu}_N'\right)'\left(\boldsymbol{\mu}_1',..,\boldsymbol{\mu}_N'\right)(\mathbf{w}_{i1},..,\mathbf{w}_{iN})'\\
&= \sum_{i=1}^{N}(\mathbf{w}_{i1},..,\mathbf{w}_{iN})(\mathbf{I}_N\otimes\boldsymbol{\Omega}_\varepsilon)(\mathbf{w}_{i1},..,\mathbf{w}_{iN})'\\
&\quad +T\sum_{i=1}^{N}(\mathbf{w}_{i1},..,\mathbf{w}_{iN})(\mathbf{I}_N\otimes\boldsymbol{\Omega}_\mu)(\mathbf{w}_{i1},..,\mathbf{w}_{iN})'\\
&= \sum_{i=1}^{N}\sum_{j=1}^{N}\mathbf{w}_{ij}\boldsymbol{\Omega}_\varepsilon\mathbf{w}_{ij}'+T\sum_{i=1}^{N}\sum_{j=1}^{N}\mathbf{w}_{ij}\boldsymbol{\Omega}_\mu\mathbf{w}_{ij}'=\sum_{i=1}^{N}\sum_{j=1}^{N}\mathbf{w}_{ij}\boldsymbol{\Omega}_1\mathbf{w}_{ij}',
\end{aligned}
$$

and

$$
\begin{aligned}
E\overline{\widetilde{\boldsymbol{\nu}}}'\mathbf{Q}_1\widetilde{\boldsymbol{\nu}} &= E\left[\overline{\widetilde{\boldsymbol{\varepsilon}}} + (\boldsymbol{\iota}_T \otimes \mathbf{I}_N)\,\overline{\widetilde{\boldsymbol{\mu}}}\right]'\left(\tfrac{1}{T}\mathbf{J}_T \otimes \mathbf{I}_N\right)\left[\widetilde{\boldsymbol{\varepsilon}} + (\boldsymbol{\iota}_T \otimes \mathbf{I}_N)\,\widetilde{\boldsymbol{\mu}}\right] \qquad \text{(B.10)} \\
&= E\overline{\widetilde{\boldsymbol{\varepsilon}}}'\left(\tfrac{1}{T}\mathbf{J}_T \otimes \mathbf{I}_N\right)\widetilde{\boldsymbol{\varepsilon}} + E\overline{\widetilde{\boldsymbol{\mu}}}'\left(\boldsymbol{\iota}_T' \otimes \mathbf{I}_N\right)\left(\tfrac{1}{T}\mathbf{J}_T \otimes \mathbf{I}_N\right)(\boldsymbol{\iota}_T \otimes \mathbf{I}_N)\,\widetilde{\boldsymbol{\mu}} \\
&= \tfrac{1}{T}E\left(\overline{\widetilde{\boldsymbol{\varepsilon}}}_1', .., \overline{\widetilde{\boldsymbol{\varepsilon}}}_T'\right)(\mathbf{J}_T \otimes \mathbf{I}_N)\left(\widetilde{\boldsymbol{\varepsilon}}_1', .., \widetilde{\boldsymbol{\varepsilon}}_T'\right)' + E\overline{\widetilde{\boldsymbol{\mu}}}'\left(\tfrac{1}{T}\boldsymbol{\iota}_T'\mathbf{J}_T\boldsymbol{\iota}_T \otimes \mathbf{I}_N\right)\widetilde{\boldsymbol{\mu}} \\
&= \tfrac{1}{T}E\left(\sum_{t=1}^{T}\overline{\widetilde{\boldsymbol{\varepsilon}}}_t', .., \sum_{t=1}^{T}\overline{\widetilde{\boldsymbol{\varepsilon}}}_t'\right)\left(\widetilde{\boldsymbol{\varepsilon}}_1', .., \widetilde{\boldsymbol{\varepsilon}}_T'\right)' + T \cdot E\overline{\widetilde{\boldsymbol{\mu}}}'\widetilde{\boldsymbol{\mu}} \\
&= \frac{1}{T}\sum_{t=1}^{T}\sum_{s=1}^{T}E\overline{\widetilde{\boldsymbol{\varepsilon}}}_t'\widetilde{\boldsymbol{\varepsilon}}_s + T \cdot E\overline{\widetilde{\boldsymbol{\mu}}}'\widetilde{\boldsymbol{\mu}} = \frac{1}{T}\sum_{t=1}^{T}\sum_{s=1}^{T}\sum_{i=1}^{N}E\overline{\widetilde{\boldsymbol{\varepsilon}}}_{it}\widetilde{\boldsymbol{\varepsilon}}_{is}' + T\sum_{i=1}^{N}E\overline{\widetilde{\boldsymbol{\mu}}}_i\widetilde{\boldsymbol{\mu}}_i' \\
&= \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N}\sum_{i=1}^{N}E\left(\boldsymbol{\varepsilon}_{1t}', .., \boldsymbol{\varepsilon}_{Nt}'\right)'\boldsymbol{\varepsilon}_{it}' \\
&\quad + \sum_{i=1}^{N}\sum_{i=1}^{N}E\left(\boldsymbol{\mu}_1', .., \boldsymbol{\mu}_N'\right)'\widetilde{\boldsymbol{\mu}}_i' \\
&= \sum_{i=1}^{N}\mathbf{w}_{ii}\boldsymbol{\Omega}_{\varepsilon} + T\sum_{i=1}^{N}\mathbf{w}_{ii}\boldsymbol{\Omega}_{\mu} = \sum_{i=1}^{N}\mathbf{w}_{ii}\boldsymbol{\Omega}_1.
\end{aligned}
$$

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