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Characteristics of Unemployment Dynamics: The Chain Reaction Approach

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Abstract

The aim of this paper is to analyze and estimate salient characteristics of unemployment dynamics. Movements in unemployment are viewed as "chain reactions" of responses to labour market shocks, working their way through systems of interacting lagged adjustment processes. In the context of estimated labour market systems for Germany, the UK, and the US, we construct aggregate measures of unemployment responses to temporary and permanent shocks. These measures are temporal and quantitative. Furthermore, we estimate the contributions of individual lagged adjustments to these aggregate measures. Our empirical results indicate that lagged adjustment processes play an important part in explaining how temporary and permanent shocks affect unemployment, that temporary and permanent shocks can yield quite different inter-country comparisons of unemployment effects, and that the quantitative and temporal measures can also yield markedly different inter-country comparisons.

Keywords

Unemployment, natural rate hypothesis, labour markets, employment, adjustment costs

JEL Classification

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Comments

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1 Introduction

Much of the existing unemployment literature draws on two alternative, extreme views of how unemployment moves through time: the natural rate and hysteresis theories. The natural rate theory, in its simplest form, represents unemployment movements as random variations around a reasonably stable natural rate of unemployment; and in more sophisticated developments,[1] the natural rate is portrayed as moving in response to permanent shocks in the stationary, long-run equilibrium unemployment rate (e.g. changes in unemployment benefits, taxes, social security contributions, interest rates, and union density). The hysteresis theory, by contrast, asserts that unemployment tends to get stuck at wherever the previous labour market shocks have placed it. In the former view temporary labour market shocks have temporary unemployment effects; whereas in the latter view these shocks lead to permanent changes in unemployment.[2] Numerous studies attempt to bridge the gap between these extreme positions; they do so by supposing that unemployment depends on its lagged values but tends towards a stable natural rate, as in the dynamically stable variants of the equation[3] $u_t = \sum_{i=1}^{p} \alpha_i u_{t-i} + \beta x_t + \varepsilon_t$, $\varepsilon_t \sim i.i.d\,(0, \sigma^2)$. Then the natural rate is $u_t^n = \frac{\beta x_t}{1-\sum_{i=1}^{p} \alpha_i}$. In such hybrid models, temporary labour market shocks (ε_t) have persistent, but not permanent, effects on unemployment.

All the above approaches, however, tend ignore two interesting and potentially important dimensions of the unemployment problem:

1. Not only are current labour market decisions - such as employment, wage setting, and labour force participation decisions - characterised by lagged responses to past labour market activities,[4] but *these lagged responses interact with one another* in affecting unemployment. If the lagged responses are complementary to one another, it will take unemployment much longer to recover in the aftermath of a recession than the period spanned by any particular lag. For example, a current drop in labour demand can depress employment in the following period on

[1] See, for example, Phelps (1994).

[2] The simplest representation of the natural rate theory is $u_t = u^n + \varepsilon_t$, where u_t is the unemployment rate at time t, u^n is the natural rate, and ε_t is a strict white noise stochastic process. An analogous representation of the hysteresis theory is $u_t = u_{t-1} + \varepsilon_t$.

[3] The coefficients α_i are constants, βx_t is a linear combination of exogenous variables, and the roots of the characteristic equation $\lambda^p - \alpha_1 \lambda^{p-1} - ... - \alpha_p = 0$ lie inside the unit circle, so that the equation is dynamically stable.

[4] For instance, firms' current employment decisions commonly may depend on their past employment on account of labour turnover costs, and current wage decisions depend on past unemployment when search effort declines with people's duration of unemployment.

account of labour turnover costs; this raises the duration of people's unemployment spells, thereby reducing search intensity and depressing employment in the next period; and so on. Thus, unemployment movements that are commonly attributed to changes in the NAIRU or to hysteretic responses to temporary shocks may arise from a lengthy interaction among various lagged adjustment processes. This point is obvious, but the interaction of lagged adjustment processes has received little explicit attention in the unemployment literature thus far.

2. Furthermore *the resulting network of lagged labour market adjustment processes interacts with the dynamic structure of the labour market shocks.* Naturally, unemployment responds differently through time to a temporary shock than to a permanent one. In the context of a first-order unemployment autoregression it is well-known, for example, that the more long-lasting are the unemployment effects of a *temporary* shock, the longer it takes for unemployment to approach its long-run equilibrium in response to a *permanent* shock; in short, the degree of unemployment persistence in response to temporary shocks is positively related to the degree of inertia in response to permanent shocks. But for higher-order unemployment autoregressions, this correspondence breaks down: unemployment persistence may be positively or negatively related to unemployment inertia. Thus far little has been done to provide theoretical and empirical analyses of how lags in labour demand, wage setting, and labour force participation schedules influence the relation between the unemployment effects of temporary and permanent shocks.

The natural rate theory underplays the first dimension - the interactions among lagged labour market adjustment processes - by focusing attention on the long-run equilibrium unemployment rate that is reached once the adjustment processes have worked themselves out. The hysteresis theory avoids examining the lagged interactions by focusing on the unit root of the time-series unemployment process.[5] The hybrid models above, based on single-equation models of unemployment, do not do justice to the lagged interactions either. The empirical single-equation models are meant to be summaries of empirical multi-equation labour market systems,[6] but the lagged interactions in

[5]In general, there is of course no reason why the lagged interactions should imply a unit root; this could happen only by accident.

[6]In the systems approach unemployment is typically portrayed as the difference between labour supply and labour demand in a system containing employment, wage setting, and labour force participation equations.

the empirical systems do not aggregate to produce equivalent unemployment dynamics in the empirical single-equation models.[7] In particular, the statistically significant lags in the single-equation models characteristically imply far less unemployment persistence (in response to temporary shocks) than is implied by the statistically significant lags of estimated multi-equation labour market systems.

Many of the conventional unemployment models overlook the second dimension above, since they focus primarily on temporary labour market shocks. The random variations around the natural rate are temporary; so are the shocks that lead to the permanent unemployment effects in the hysteresis models and to the long-lasting unemployment effects in the hybrid models above. But in practice labour market shocks have both temporary and permanent components. Whereas some aggregate business cycle fluctuations are temporary, changes in productivity, exchange rates, raw material prices, taxes, and real interest rates are often permanent. The interplay between shock dynamics and labour market dynamics requires explicit attention.

The aim of this paper is to focus on the neglected dimensions above. We will consider labour market models where current decisions - regarding employment, wage setting, and labour force participation - depend on past decisions, and where these lagged adjustments interact. These interactions are the centerpiece of the *chain reaction theory of unemployment*, in which each labour market shock has a "chain reaction" of unemployment effects. The network of lagged adjustment processes is the propagation mechanism for this chain reaction.

In this context, we will construct aggregative summary measures of the dynamic unemployment responses to temporary and permanent shocks. We will be concerned with two important dynamic influences: (i) the persistent unemployment effects of temporary shocks, called *unemployment persistence*, and (ii) the delayed unemployment effects of permanent shocks, which we will call *imperfect unemployment responsiveness.* Our aggregative measures of unemployment persistence and imperfect unemployment responsiveness can perform a useful role in characterising the movement of unemployment through time, analogous to the way in which macroeconomic indices (such as GNP or inflation) are useful in characterising macroeconomic activity at each point in time.

Focusing on three countries - Germany, the United Kingdom, and the United States - we will identify significant lags in labour demand, wage setting, and labour force participation behaviour, and measure the degree to

[7] The econometric reasons for this failure are well-known, e.g. nonlinearities in t-statistics.

which these lags are responsible for unemployment persistence and imperfect responsiveness. Elsewhere it has been shown that lagged unemployment responses to temporary and persistent shocks - as captured through unemployment persistence and imperfect responsiveness - play an important role determining the evolution of unemployment in various industrialised economies.[8] Our analysis of the sources underlying persistence and responsiveness constitutes a first step toward providing an understanding the medium- and longer-term movements of unemployment.

The policy implications of our approach are striking. First, since different employment policies affect different lagged adjustments, examining the role of each lag within its network of lagged adjustments is important for policy formulation. Second, labour market shocks of different durations may require different policy responses. And finally, since lagged labour market adjustment processes tend to differ markedly from country to country, different countries may require different policies to deal with what looks superficially as a similar unemployment problem.

The rest of the paper is organised as follows. Section 2 presents a theoretical model of unemployment persistence and responsiveness and analyzes how the lagged adjustment processes contribute to these phenomena. Section 3 contains our empirical analysis, in which we examine how the movement of unemployment in Germany, the UK, and the US over the past four decades can be clarified through the concepts above. Finally Section 4 concludes.

2 A Model of Unemployment Persistence and Responsiveness

The theoretical model in this section provides a background for the empirical model of Section 3. In particular, the theoretical model illustrates how the interaction among lagged labour market adjustment processes generates persistence and imperfect responsiveness of unemployment. It indicates how these two phenomena are distinct from one another, describing different dynamic features of unemployment. These features, together, will provide insights into the way unemployment moves through time.

2.1 The Underlying Model

Our theoretical framework provides simple examples of lagged adjustment processes occurring in labour demand, wage setting, and labour force partic-

[8] See, for example, Karanassou and Snower (1998, 2000).

ipation decisions.[9] We consider a labour market containing a fixed number of identical firms with monopoly power in the product market. The i'th firm has a production function of the form

$$q_{i,t}^S = Ae_{i,t}^{\alpha}k_{i,t}^{\beta}, \tag{1a}$$

where $q_{i,t}^S$ is output supplied, $e_{i,t}$ is employment, $k_{i,t}$ is capital stock, A, α, β are positive constants, and $0 < \alpha < 1$. Each firm faces a product demand function of the form

$$q_{i,t}^D = \left(\frac{p_{i,t}}{p_t}\right)^{-\eta}\frac{y_t}{f}, \tag{1b}$$

where y_t stands for aggregate product demand, f is the number of firms, $p_{i,t}$ is the price charged by firm i, p_t is the aggregate price level, and η is the price elasticity of product demand (a positive constant). Note that the firms are assumed to face symmetric production and cost conditions.

To derive the firm's labour demand function, we observe that the firm sets its employment at the profit maximizing level, at which the marginal revenue from producing an extra unit of output is equal to the corresponding marginal cost (for a given capital stock). The marginal revenue is $MR_{i,t} = p_{i,t}\left(1 - \frac{1}{\eta}\right)$. Let the marginal cost be $MC_{i,t} = \omega_{i,t}\left(\frac{\partial e_{i,t}}{\partial q_{i,t}}\right)\xi_{i,t}$, where $\omega_{i,t}$ is the wage paid by the firm, $\frac{\partial e_{i,t}}{\partial q_{i,t}}$ is the marginal labour requirement, and $\xi_{i,t}$ is an employment adjustment parameter. The employment adjustment parameter is $\xi_{i,t} = \left(e_{i,t}/\sigma e_{i,t-1}\right)^{\delta}$, where δ is a positive constant and σ is the "survival rate," i.e. one minus the separation rate. For simplicity, we assume that the separation rate is sufficiently high (the survival rate is sufficiently low), so that $e_{i,t} > \sigma e_{i,t-1}$. The employment adjustment parameter may be interpreted in terms of training costs: $e_{i,t}/\sigma e_{i,t-1} = 1 + (h_{i,t}/\sigma e_{i,t-1})$, where $h_{i,t}$ is new hires. The training of new hires $(h_{i,t})$ in period t is done by the incumbent employees $(\sigma e_{i,t-1})$ in that period. The greater the ratio of new hires to incumbent employees, the greater the average training cost per employee $(\xi_{i,t})$. When $\delta = 0$ (so that $\xi_{i,t} = 1$), the employment adjustment cost is zero; and when $\delta > 0$ (so that $\xi_{i,t} > 1$), the adjustment cost is positive.

For the production function above, the marginal product of labour (the inverse of the marginal labour requirement) is $\frac{\partial q_{i,t}}{\partial e_{i,t}} = \alpha A e_{i,t}^{-(1-\alpha)}k_{i,t}^{\beta}$. Thus the marginal cost is $MC_{i,t} = \frac{\omega_{i,t}}{\alpha A}e_{i,t}^{1-\alpha}k_{i,t}^{-\beta}\xi_{i,t}$. Setting the marginal revenue equal

[9] Our analysis is in the spirit of recent theoretical models of aggregate labour market activity (e.g. Layard, Nickell and Jackman (1991), Lindbeck and Snower (1989), Nickell (1995)).

to the marginal cost, we obtain the firm's (implicit) labour demand function:

$$\frac{\omega_{i,t}}{\alpha A} e_{i,t}^{1-\alpha} k_{i,t}^{-\beta} \left(\frac{e_{i,t}}{\sigma e_{i,t-1}} \right)^{\delta} = p_{i,t} \left(1 - \frac{1}{\eta} \right). \tag{2}$$

In the labour market equilibrium, $p_{i,t} = p_t$ and $\omega_{i,t} = \omega_t$, due to symmetry. Aggregating the individual firms' labour demand functions, taking logarithms, so that $E_t = \log(f e_{i,t})$ $K_t = \log(f k_{i,t})$, and introducing an error term (ε_t) to capture technological shocks, we obtain the following aggregate employment equation:[10]

$$E_t = a^* + a_E^* E_{t-1} - a_w w_t + a_K^* K_t + \varepsilon_t, \tag{3}$$

where $w_t = \log(\omega_t / p_t)$ and $\varepsilon_t \sim i.i.d\,(0, \sigma_\varepsilon^2)$. The parameter a_E^* will be called the "employment inertia coefficient." When the employment adjustment cost is zero ($\delta = 0$), the employment inertia coefficient is zero; when the adjustment cost is positive ($\delta > 0$), the employment inertia coefficient is positive as well.

For simplicity, let the wage be equal to the reservation wage of the marginal employee. Suppose that the population of workers is heterogeneous in terms of the disutility of work and thus also in terms of the reservation wage. Moreover, suppose that this population can be ordered along a reservation wage continuum, from lowest to highest, so that when aggregate employment rises, the reservation of the marginal worker rises as well. Assuming this relation to be linear, our wage setting equation becomes

$$w_t = b + b_E E_t. \tag{4}$$

The labour force participation decision equates the marginal return from being in the labour force with the associated marginal cost being in the labour force. For simplicity, let the per capita return (in logs) from being in the labour force be positively related to the employment probability ($E_t - L_t$, where L_t is the size of the labour force, in logs) and to the wage (w_t), and negatively related to the inactivity rate ($L_t - Z_t$, where Z_t is the log of working age population). Specifically, let the return from being in the labour force be given by $d_1 + d_2(E_t - L_t) + d_3 w_t - d_4(L_t - Z_t)$, where d_1, d_2, d_3 and d_4 are positive constants.

Regarding the cost per capita of being in the labour force, suppose that there are costs of entry into the labour force and that these costs depend

[10] $a^* = \frac{\log\left(1 - \frac{1}{\eta}\right) + \log(\alpha A) + \delta \log \sigma + (1 - \alpha - \beta) \log f}{1 + \delta - \alpha}$, and $a_E^* = \frac{\delta}{1+\delta-\alpha}$, and $a_w = \frac{1}{1+\delta-\alpha}$, $a_K^* = \frac{\beta}{1+\delta-\alpha}$.

positively on the ratio of new labour force entrants to incumbent members of the labour force. Accordingly, let the cost per capita (in logs) be given by $c_1 + c_2 L_t - c_3 L_{t-1}$ (where the new labour force entrants are positively related to $L_t - L_{t-1}$, the number of incumbents are positively related to L_{t-1}, and $c_2 > c_3$). Setting the per capita return equal to the per capita marginal cost, we obtain the following labour force participation equation:[11]

$$L_t = c^* + c_L L_{t-1} + c_w w_t + c_E^* E_t + c_Z Z_t, \tag{5}$$

The coefficient c_L may be called the "labour force inertia coefficient."

Substitution of equation (4) into (3) and (5) yields:

$$E_t = a + a_E E_{t-1} + a_K K_t + \left(\frac{1}{1 + a_w b_E} \right) \varepsilon_t, \tag{6}$$

$$L_t = c + c_L L_{t-1} + c_E E_t + c_Z Z_t, \tag{7}$$

where $a = \frac{a^* - a_w b}{1 + a_w b_E}$, $a_E = \frac{a_{E^*}}{1 + a_w b_E}$, $a_K = \frac{a_K^*}{1 + a_w b_E}$, $c = c^* + c_w b$, $c_E = c_E^* + c_w b_E$. (Note that $0 < a_E < 1$, and $0 < c_L < 1$.)

Finally, the unemployment rate u_t may be approximated as the difference between the log of the labour force L_t and the log of employment E_t:

$$u_t = L_t - E_t. \tag{8}$$

The model contains two lagged adjustment effects: (i) current employment depends on past employment, (ii) the current labour force depends on the past labour force. For ease of exposition, we will call these two effects the *employment adjustment effect*,[12] and the *labour force adjustment effect*, respectively. It is important to emphasize that these names are merely heuristic devices that help us refer the individual lagged effect.[13] The employment adjustment, and labour force adjustment effects are usually taken to be positive: $a_E, c_L > 0$ and we will maintain this assumption here.

[11] $c^* = \frac{-c_1 + d_1}{c_2 + d_2 + d_4}$, $c_w = \frac{d_3}{c_2 + d_2 + d_4}$, $c_E^* = \frac{d_2}{c_2 + d_2 + d_4}$, $c_L = \frac{c_3}{c_2 + d_2 + d_4}$, and $c_Z = \frac{d_4}{c_2 + d_2 + d_4}$.

[12] For instance, firms' current employment decisions commonly may depend on their past employment on account of costs of labour turnover costs (e.g. Lindbeck and Snower (1988) and Nickell (1978)).

[13] We naturally do not wish to imply that the employment adjustment effect arises only account of employment adjustment costs, and that the labour force adjustment effect arises only account of labour force adjustment costs . Clearly, when agents optimise their objectives intertemporally, each of the individual lagged effects will, in general, arise from a variety of sources.

Equations (6)-(8) yield the following reduced form unemployment rate equation:[14]

$$\begin{aligned} u_t &= (a_E + c_L)\, u_{t-1} - a_E c_L u_{t-2} \\ &\quad - \left(\frac{1-c_E}{1+a_w b_E}\right) \varepsilon_t + \left(\frac{c_L}{1+a_w b_E}\right) \varepsilon_{t-1} - a_K (1-c_E) K_t + a_K c_L K_{t-1} \\ &\quad + a c_E + (1-a_E)\, c - (1-c_L)\, a + c_Z Z_t - c_Z a_E Z_{t-1}. \end{aligned} \tag{9}$$

As noted, the labour market system (6)-(8) is merely illustrative of interacting lagged adjustment processes in the labour market. The microfoundations of these and other lagged adjustments have been explored extensively in the theoretical literature.[15] Whereas the equations above have been derived in particularly simple ways, in general these equations are the outcomes of complex intertemporal optimisation problems. For instance, a labour demand equation is usually derived from the maximisation of the firms' present value of profits (under perfect or imperfect competition) subject to sequences of production function constraints; a wage setting equation is generally be derived on the basis of bargaining between firms and their employees, efficiency wage setting by firms, or labour union wage setting; and a labour force participation equation is often derived from workers' intertemporal utility maximisation subject to sequences of budget constraints. In these intertemporal contexts, employment, wage, and labour supply decisions depend on the agents' rational expectations of future economic variables. These expectations may be expressed in terms of the present and past values of these economic variables, which are then substituted into the relevant first-order conditions of agents' optimisation problems. Consequently, the labour demand, wage setting, and labour force participation equations may be expressed in terms of present and past variables, as illustrate in the model above.

The focus of attention in this paper is not, however, the microeconomic sources of the lagged labour market adjustments; rather, we are interested in how these lagged adjustments (whatever their sources) interact with one another and with labour market shocks to generate an unemployment trajectory. For this purpose, we suppose that the participants in the labour market face known distributions of labour market shocks. These shocks may take the form of white noise variations in the labour demand, wage setting,

[14]See Appendix 1, eq.(A1.4) and (A1.4').

[15]For example, Taylor (1979) rationalise the dependence of current wages on past wages, Lindbeck and Snower (1987) rationalise the relation between current wages and past employment, and so on.

and labour supply equations (that are temporary shocks) and variations in the future realizations of the exogenous variables (that could be permanent shocks).[16] On this basis, they make their labour market decisions, yielding an equation system with lagged adjustments (such as the illustrative one above), which we take as the starting point of our analysis. We then consider how realizations of the temporary and permanent shocks interact with the system's network of lagged adjustments to generate unemployment persistence and imperfect unemployment responsiveness. It is this interaction that occupies centre-stage in our analysis.[17]

With this in mind, we now examine how temporary and permanent shocks give rise to chain reactions of unemployment effects.

2.2 Unemployment Persistence

Suppose that in period t there is a temporary (one-period) unit fall in labour demand: $d\varepsilon_t = -1$. The immediate impact is of course to reduce employment and thereby raise unemployment by a unit. Thereafter (in periods $t+i$, $i > 0$) the labour demand shock $d\varepsilon_t$ has the following two effects.

- (i) Via the *employment adjustment effect* (whose magnitude is given by the coefficient a_E), the shock $d\varepsilon_t$ reduces employment E_{t+i} (by $a_E d\varepsilon_t$) below what it would have been in the absence of the shock (and thus raises unemployment u_{t+i}).
- (ii) Via the *labour force adjustment* effect (whose magnitude is given by the coefficient c_L), the shock reduces the labour force L_{t+i} (by $c_E d\varepsilon_t$) below what it would otherwise have been (and thereby reducing unemployment u_{t+i}).

The movement of unemployment in response to the temporary shock may be explained wholly through the interactions of these three effects.

It can be shown that the unemployment response j periods after the shock is[18]

$$du_{t+j} = \frac{a_E^j \left[(1 - c_E)\, a_E - c_L \right] + c_E c_L^{j+1}}{(1 + a_w b_E)\,(a_E - c_L)}, \tag{10}$$

[16] An example is capital stock in the model above.

[17] It is important that the shocks be realizations from the known distributions generating the error terms and exogenous variables, for otherwise the occurrence of a new shock may be expected to affect the agents' decision making, leading to revised employment, wage setting, and labour supply equations (with revised lag structures).

[18] The chain reaction of unemployment movements, period by period, is described in Appendix 1.

where du_{t+j} is the difference between unemployment in the presence and absence of the shock.[19] This indicates how the unemployment effects of the temporary shock persist through time.

2.2.1 Quantitative and Temporal Persistence

The phenomenon of "unemployment persistence" has two interesting features, which may be measured by two separate statistics:

(1) *Quantitative unemployment persistence* measures the degree to which unemployment is affected by the temporary shock after that shock has disappeared. Specifically, for a unit shock occurring in period t, it is the sum of the unemployment effects for all periods $t+j, j \geq 1$:

$$\pi^Q = \sum_{j=1}^{\infty} du_{t+j}. \tag{11a}$$

In the absence of lagged labour market adjustment processes, unemployment would not be affected after the temporary shock has disappeared and thus quantitative unemployment persistence π^Q would be zero. At the opposite extreme of hysteresis, the temporary shock would have a permanent effect on unemployment and thus π^Q would be infinite.

(2) *Temporal unemployment persistence* measures how long it takes for the unemployment effect of the shock to shrink to a fraction κ of its initial value. Specifically, it measures the maximum number of periods, after the occurrence of the unit shock, over which the unemployment effect exceeds the fraction κ of the initial effect:

$$\pi^T = \underset{j>0}{\arg\max} \left(|du_{t+j}| > \kappa \, |du_t| \right). \tag{11b}$$

Once again, in the absence of lagged adjustment processes, π^T would be zero; whereas in the presence of hysteresis, π^T would be infinite.

The two statistics are concerned with different economic phenomena. Whereas *quantitative* persistence is concerned with the cumulative amount of unused labour resources over time generated in the aftermath of the temporary shock, *temporal* persistence deals with the time required for the influence of the shock to disappear.[20]

[19] Thus the operator d describes a comparative static difference (rather than a change through time).

[20] In other words, temporal persistence measures how long it takes for unemployment to return to a specified neighbourhood of the time path it would have followed in the absence of the shock.

For the unemployment equation (9), the degree of quantitative unemployment persistence is [21]:

$$\pi^Q = \frac{a_E (1 - c_L)(1 - c_E) - c_E c_L}{(1 + a_w b_E)(1 - a_E)(1 - c_L)}, \tag{12}$$

whereas the degree of temporal unemployment persistence cannot be derived explicitly in general terms.

2.2.2 Sources of Unemployment Persistence

With a view to the empirical analysis later, we examine the sources of unemployment persistence by showing how each of the lagged adjustment processes contributes to unemployment persistence. For brevity, we focus on quantitative persistence π^Q.

To measure the influence of the employment adjustment effect (EA) on quantitative persistence, we compute the difference between π^Q in the presence and absence of the EA effect, given that the labour force adjustment effect is in operation. In the absence of the employment adjustment effect, the employment equation (3) becomes $E_t = a^* + a_E^* E_t - a_w w_t + a_K^* K_t + \varepsilon_t$. It can be shown[22] that the associated degree of persistence is

$$\pi^Q_{\sim EA} = \frac{-c_E c_L}{(1 + a_w b_E)(1 - a_E)(1 - c_L)} < 0,$$

where "$\sim EA$" stands for the "absence of the employment adjustment effect." Thus our measure of the degree of quantitative unemployment persistence attributable to the employment adjustment effect is total persistence minus persistence in the absence of the EA effect:[23]

$$\pi^Q_{EA} = \pi^Q - \pi^Q_{\sim EA} = \frac{a_E (1 - c_E)}{(1 + a_w b_E)(1 - a_E)}. \tag{13a}$$

[21] See Appendix 1.

[22] The derivation of this and other sources of persistence is given in Appendix 1.

[23] Note that the influence of the employment adjustment effect on quantitative persistence cannot be measured by taking the derivative of π^Q with respect to a_E, since a change in a_E alters not only the magnitude of the employment adjustment process towards a given labour market equilibrium, but also alters the equilibrium itself. After all, unemployment persistence is about protracted adjustment to equilibrium rather than a shift of the equilibrium. To measure the contribution of a small change in the adjustment process on persistence, it would be necessary to specify the coefficients of the lagged variables in our labour market system in such a way that coefficient changes would leave the labour market equilibrium unchanged.

Similarly, the influence of the labour force adjustment effect on quantitative persistence may be evaluated by computing the difference between persistence in the presence and absence of this effect, in the presence of the employment adjustment process. In the absence of the labour force adjustment effect, the labour force equation (5) becomes $L_t = c^* + c_L L_t + c_w w_t + c_E^* E_t + c_Z Z_t$ and the associated degree of persistence is

$$\pi^Q_{\sim LF} = \frac{a_E \left(1 - c_E - c_L\right)}{\left(1 + a_w b_E\right)\left(1 - a_E\right)\left(1 - c_L\right)}.$$

Hence the degree of quantitative unemployment persistence attributable to the labour force adjustment effect is

$$\pi^Q_{LF} = \pi^Q - \pi^Q_{\sim LF} = \frac{-c_E c_L}{\left(1 + a_w b_E\right)\left(1 - c_L\right)} < 0. \tag{13b}$$

It is interesting to observe that the role of the adjustment processes is to determine how the unemployment effects of a temporary shock are split between the present and the future. To see this, let m (the "$m$ultiplier") stand for the current effect of the shock on unemployment; f stand for the sum of the future effects; and $\tau = m + f$ stand for the total effect (over the present and future). For the labour market system (6)-(8),

$$m \equiv du_t = \frac{1 - c_E}{1 + a_w b_E}; \quad f \equiv \pi^Q; \quad \tau = \frac{\left(1 - c_L - c_E\right)}{\left(1 + a_w b_E\right)\left(1 - a_E\right)\left(1 - c_L\right)}. \tag{14a}$$

However, in the absence of the employment adjustment effect:

$$m_{\sim EA} = \frac{1 - c_E}{\left(1 + a_w b_E\right)\left(1 - a_E\right)}; \quad f_{\sim EA} = \pi^Q_{\sim EA}; \quad m_{\sim EA} + f_{\sim EA} = \tau; \tag{14b}$$

and in the absence of the labour force adjustment effect:

$$m_{\sim LF} = \frac{1 - c_E - c_L}{\left(1 + a_w b_E\right)\left(1 - c_L\right)}; \quad f_{\sim LF} = \pi^Q_{\sim LF}; \quad m_{\sim LF} + f_{\sim LF} = \tau. \tag{14c}$$

Equations (14a-c) show that the presence of the lagged adjustment processes influences the distribution of unemployment effects through time (the relative magnitude of m and τ, the current and future effects), but not the total effect τ.

Finally, observe that adding the sources of persistence ($\pi^Q_{EA} + \pi^Q_{LF}$) does *not* yield the aggregate measure of persistence (π^Q). The reason of course is that each source of persistence is measured by taking the difference between persistence in the presence and absence of that sources, *assuming that the other source is operative.* Since the different sources interact with one another, the various sources cannot be added to yield aggregate persistence.[24]

[24]In fact, it can be shown that $\pi^Q_{EA} + \pi^Q_{LF} > \pi^Q$, i.e. the employment and labour force

2.3 Imperfect Unemployment Responsiveness

Next consider the chain reaction of unemployment changes in response to a permanent labour demand shock. Assuming that the capital stock follows a random walk, $K_t = K_{t-1} + v_t$, we let the permanent shock be represented by a realization of the white noise error term v_t in this stochastic process.[25] Specifically, suppose that in period t there is a temporary (one-period) unit fall in v_t, which has a permanent negative influence on the capital stock K_t. In the initial period t, the employment effect of this fall in the capital stock is $dE_t = -a_K$. In subsequent periods the shock has the following effects: (i) it continues to have a direct effect on employment by $dE_{t+j} = -a_K$ in each period $j > 0$ (since the influence on the capital stock is permanent); (ii) due to the employment adjustment effect, the fall in employment E_{t+j} reduces employment E_{t+j+1} below what it would have been in the absence of the shock; and (iii) due to the labour force adjustment effect, the fall in employment E_{t+j} reduces the labour force L_{t+j+1} below what it would otherwise have been.

These three effects all interact with one another, producing a chain reaction of unemployment movements, so that the unemployment response j periods after the period-t shock is

$$du_{t+j} = \left(\frac{a_K}{a_E - c_L}\right)\left[(1-c_E)\sum_{i=0}^{j}\left(a_E^{i+1} - c_L^{i+1}\right) - c_L\sum_{i=0}^{j}\left(a_E^{i} - c_L^{i}\right)\right], \quad (15)$$

where du_{t+j} is now defined as the difference between unemployment in the presence and absence of the permanent labour demand shock.[26]

2.3.1 Quantitative and Temporal Responsiveness

Analogously to our analysis of unemployment persistence, we assess imperfect unemployment responsiveness from two vantage points:

(1) *Quantitative imperfect responsiveness* measures the cumulative unemployment effect of the permanent shock that arises because unemployment does not adjust immediately to the new long-run equilibrium. In particular, for a unit shock beginning in period t, quantitative imperfect responsiveness is the sum of the differences through time between (a) the disparity between

adjustment effects are substitutes in this model (i.e. the joint effects are less than the sum of the individual effects).

[25] For simplicity, we also assume that the error terms ε_t and v_t are independent of one another.

[26] The theoretical results on imperfect responsiveness are derived in Appendix 2.

actual and long-run unemployment in the presence of the shock and (b) this disparity in the absence of the shock:[27]

$$\rho^Q = \sum_{j=0}^{\infty} \left(du_{t+j} - du^* \right). \tag{16a}$$

By equation (15), the effect of the permanent shock on long-run employment is

$$du^* \equiv \lim_{j \to \infty} du_{t+j} = \frac{a_K \left(1 - c_E - c_L\right)}{\left(1 - a_E\right)\left(1 - c_L\right)}.$$

In the absence of lagged labour market adjustment processes, unemployment would be "perfectly responsive," i.e. it would adjust immediately to its new long-run expected rate, and thus ρ^Q would be zero. If however the full effects of the permanent labour demand shock emerge only gradually, so that the short-run unemployment effects of the shock are less than the long-run effect, then unemployment is "under-responsive": $\rho^Q < 0$, i.e. unemployment displays *inertia.* On the other hand, if unemployment *overshoots* its long-run equilibrium, then our measure may be positive, making unemployment "over-responsive": $\rho^Q > 0$. As we approach hysteresis, ρ^Q approaches infinity.

For the unemployment rate equation (9), the degree of quantitative imperfect unemployment responsiveness may be derived explicitly:[28]

$$\rho^Q = \frac{a_K \left[-a_E \left(1 - c_E - c_L\right)\left(1 - c_L\right) + c_L c_E \left(1 - a_E\right) \right]}{\left[\left(1 - a_E\right)\left(1 - c_L\right) \right]^2}. \tag{17}$$

(2) *Temporal imperfect responsiveness* measures how long its takes for unemployment to reach a particular neighbourhood of its new long-run equilibrium. Specifically, it measures the maximum number of periods j over which the difference $du_{t+j} - du^*$ (i.e. the difference between the actual and long-run expected unemployment rates in the presence and absence of the shock in any period $t + j$) exceeds the fraction κ of the initial difference $du_t - du^*$:

$$\rho^T = \arg\max_{j>0} \left(\left| du_{t+j} - du^* \right| > \kappa \left| du_t - du^* \right| \right). \tag{16b}$$

As above, ρ^T would be zero in the absence of labour market lags, and it approaches infinity as we approach hysteresis.

[27]This is equivalent to the differences through time between (a) the disparity between the actual unemployment rate in the presence and absence of the shock (du_{t+j}), and (b) the disparity between the long-run unemployment rate in the presence and absence of the shock (du^*).

[28]See Appendix 2.

2.3.2 Sources of Imperfect Responsiveness

We now turn to the sources of imperfect responsiveness. In the absence of the employment adjustment effect, it can be shown that the degree of imperfect responsiveness is[29]

$$\rho^Q_{\sim EA} = \frac{a_K c_L c_E}{(1 - a_E)(1 - c_L)^2} > 0.$$

If $c_E + c_L < 1$ then $\rho^Q_{\sim EA} > \rho^Q$, i.e. the employment adjustment effect magnifies the inertia of unemployment, making unemployment to respond more slowly to a permanent labour demand shock than it would otherwise have done.[30] Thus the degree of quantitative responsiveness attributable to the employment adjustment effect is

$$\rho^Q_{EA} = \rho^Q - \rho^Q_{\sim EA} = -\frac{a_K a_E (1 - c_E - c_L)}{(1 - a_E)^2 (1 - c_L)}, \tag{18a}$$

which is negative if $c_E + c_L < 1$, i.e. the employment adjustment effect makes unemployment more under-responsive than it would otherwise have been ($\rho^Q_{EA} < 0$).

Along the same lines, the degree of quantitative responsiveness attributable to the labour force adjustment effect is

$$\rho^Q_{LF} = \frac{a_K c_L c_E}{(1 - a_E)(1 - c_L)^2} > 0. \tag{18b}$$

Since the labour force adjustment effect causes the labour force to fall in the future (when $c_L > 0$), in tandem with the fall in employment, this effect thereby reduces the inertia of unemployment, making unemployment less under-responsive that it would otherwise have been ($\rho^Q_{LF} > 0$).

Observe that in this model there are no complementarities or substitutabilities between the two adjustment effects since $\rho^Q_{EA} + \rho^Q_{LF} = \rho^Q$.[31]

2.4 The Relation between Persistence and Imperfect Responsiveness

Persistence and imperfect responsiveness are the outcome of the same constellation of lagged labour market adjustment processes; the only difference

[29] See Appendix 2.

[30] In other words, unemployment is more over-responsive in the absence of the employment adjustment effect than in its presence.

[31] This result is specific to our model. In other models, of course, sources of imperfect responsiveness may be complements or substitutes, in the sense that the joint effects may be greater or less than the sum of the individual effects, respectively.

between these phenomena lies in the nature of the labour market shock initiating these processes. By exploring the relation between persistence and imperfect responsiveness, we can gain insight into how the interaction between the shocks and the adjustment processes depends on the durability of the shocks.

The simplest - and, unfortunately, the most misleading - way of thinking about the relation between persistence and imperfect responsiveness is in the context of a first-order difference equation in unemployment. For instance, in our model above, suppose that there were no labour force adjustment effect ($c_L = 0$). Thus the only remaining adjustment process is the employment adjustment effect ($a_E > 0$). The unemployment equation (9') then reduces to

$$\begin{aligned} u_t &= a_E u_{t-1} - \left(\frac{1 - c_E}{1 + a_w b_E} \right) \varepsilon_t + -a_K (1 - c_E) K_t \\ &\quad + a c_E + (1 - a_E) c + c_Z Z_t - c_Z a_E Z_{t-1}, \end{aligned}$$

In this simple context, it is easy to see that quantitative persistence and quantitative imperfect responsiveness are inversely related to one another. Both depend on the magnitude of the autoregressive coefficient (a_E). The greater is this coefficient, the greater is quantitative persistence $\left(\pi^Q = \frac{a_E(1-c_E)}{(1-a_E)(1+a_w b_E)} \right)$ and the smaller is quantitative imperfect responsiveness$\left(\rho^Q = -\frac{a_E a_K}{(1-a_E)^2} \right)$. In other words, the greater is the autoregressive coefficient, the greater is the sum of the unemployment after-effects from a temporary shock and the smaller the sum of the unemployment responses to a permanent shock. Since persistence and imperfect responsiveness are tied to one another in this way, it is clearly unnecessary to view them as separate phenomena.

However, as noted, this account of the relation between persistence and imperfect responsiveness is misleading, since it invariably occurs only in first-order unemployment equations. For higher-order equations - the sort we are overwhelmingly likely to encounter in empirical labour market systems, where a variety of lagged adjustment processes are operative - the above relation is only one of various possibilities. Then, of course, persistence and imperfect responsiveness are indeed separate phenomena. Reintroducing the labour force adjustment effect into our model indicates why this is so.

In this expanded model (containing an employment adjustment effect ($a_E > 0$) and a labour force adjustment effect ($c_L > 0$)), it turns out that, when $c_E + c_L < 1$, the relation between persistence and imperfect responsiveness depends critically on the autoregressive coefficient a_E, measuring the employment adjustment effect. There are three scenarios:[32]

1. When the employment adjustment effect is "high," $a_E > \frac{c_E c_L}{(1-c_L)(1-c_E-c_L)+c_E c_L}$, then quantitative persistence is positive $\left(\pi^Q > 0\right)$ and quantitative imperfect responsiveness is negative $\rho^Q < 0$ (viz., "under-responsiveness" to permanent shocks). This scenario exhibits inertia in response to both temporary and permanent shocks: (a) an unemployment-increasing *temporary* shock leads cumulatively to more unemployment after the shock has disappeared; whereas (b) an unemployment-increasing *permanent* shock leads cumulatively to less unemployment than would have occurred under instantaneous adjustment. Furthermore, at the lower bound of this scenario, when $a_E = \frac{c_E c_L}{(1-c_L)(1-c_E-c_L)+c_E c_L}$, unemployment becomes perfectly responsive ($\rho^Q = 0$).

2. When the employment adjustment effect falls within an "intermediate" range, $\frac{c_E c_L}{(1-c_L)(1-c_E)} < a_E < \frac{c_E c_L}{(1-c_L)(1-c_E-c_L)+c_E c_L}$, both persistence and imperfect responsiveness are positive ($\pi^Q > 0$ and $\rho^Q > 0$). Here we find inertia with respect to temporary shocks, but overshooting with respect to permanent shocks. The unemployment-increasing temporary shock still generates more unemployment, cumulatively, after the shock has disappeared. But the unemployment-increasing permanent shock leads the unemployment rate to overshoot its long-run equilibrium by such a large amount that, cumulatively, there is more unemployment than would have occurred under instantaneous adjustment. Moreover, at the lower bound of this scenario, when $a_E = \frac{c_E c_L}{(1-c_L)(1-c_E)}$, unemployment persistence falls to zero ($\pi^Q = 0$).

3. Finally, when the employment adjustment effect is "low," $a_E < \frac{c_E c_L}{(1-c_L)(1-c_E)}$, persistence is negative and imperfect responsiveness is positive ($\pi^Q < 0$ and $\rho^Q > 0$). In this scenario there is overshooting in response to both temporary and permanent shocks. The unemployment-increasing permanent shock leads to overshooting along the same lines as in the "intermediate" scenario. But now the unemployment-increasing temporary shock also leads to overshooting and, as result, it leads to cumulatively less unemployment after the shock has disappeared.

In short, as the employment adjustment effect gradually rises, the degree of persistence rises (progressively larger quantitative after-effects of a temporary shock on unemployment) and the degree of imperfect responsiveness

[32]For details see equations (A1.9') and (A2.6') in Appendices 1 and 2, respectively.

falls (progressively smaller quantitative after-effects of a permanent shock on unemployment).

3 Empirical Analysis

3.1 Estimating a Labour Market Model

We now explore how the movement of unemployment in three countries - Germany, the UK, and the US - may be explained through the phenomena above. For this purpose we estimate a labour market model for the UK and US using annual data over the period 1964-1992, and for Germany over the period 1964-1990 . The model is a straightforward extension of the illustrative theoretical model in Section 2; it has the following general form:

$$a_0(B)\,E_t = a_1(B)\,w_t + a_2(L)\,K_t + a_3(L)\,X_{1t} + \varepsilon_{1t}, \quad \text{(14a)}$$
$$b_0(B)\,w_t = b_1(B)\,E_t + b_2(B)\,K_t + b_3(B)\,X_{2t} + \varepsilon_{2t}, \quad \text{(14b)}$$
$$c_0(B)\,L_t = c_1(B)\,w_t + c_2(B)\,u_t + c_3(B)\,Z_t + c_4(B)\,X_{3t} + \varepsilon_{3t}, \quad \text{(14c)}$$
$$u_t = L_t - E_t, \quad \text{(14d)}$$

where $a_i(B)$, $b_i(B)$, and $c_i(B)$ are polynomials in the backshift operator and X_{it} are vectors of exogenous variables. The three equations stand for a labour demand function (representing the aggregate employment level, for a given real wage, capital stock, and other exogenous variables), a wage setting function (representing the equilibrium real wage, for a given employment level, capital stock, and other exogenous variables), a labour supply function (representing the equilibrium labour force, for a given real wage, unemployment rate, population, and other exogenous variables), and a definition of the unemployment rate. The underlying definitions and sources of our variables are given in Table 1.

Our estimation is based on the "autoregressive distributed lag (ARDL) modelling approach to cointegration analysis" proposed by Pesaran and Shin (1995)[33]. The reason for adopting the ARDL modelling approach, instead of the popular "cointegration/error-correction" one, is twofold: first, since the ARDL approach is applicable irrespective of whether the regressors are $I(0)$

[33] According to Pesaran and Shin (1995) "...the traditional ARDL approach justified in the case of trend-stationary regressors, is in fact equally valid even if the regressors are first-difference stationary".

See also Pesaran (1997), and Pesaran, Shin and Smith (1996).

or $I(1)$, the pre-testing problems that surround the cointegration analysis do not arise, and second, the estimated coefficients can be given a straightforward economic interpretations, e.g. the coefficients of the lagged employment terms in the labour demand equation may be interpreted in terms of the costs of employment adjustment.[34]

The equations for our labour market model were selected on the basis of either the Akaike Information Criterion or the Schwartz Bayesian Criterion (equations [T1]-[T9], in Tables 2-4, for the UK, US, and German labour markets), and they all pass the CUSUM and CUSUMSQ tests for structural stability. For the UK and US equations neither endogeneity nor cross-equation correlation were detected. However, since this was not the case for Germany, we estimated the German labour market equations as a system using 3SLS (equations [T10]-[T12] in Table 5). Table 7 presents a full range of misspecification tests for equations [T1]-[T12] and Table 6 reports Sargan's test for overidentifying restrictions. Observe that our selected equations consist of stationary, well-specified linear combinations of the variables involved.

Two features of our equations deserve special mention. First, whereas employment depends inversely on the real wage in the German employment equation, it depends positively on the real wage in the UK and US employment equations. These results are readily interpreted in terms of the recent theoretical literature showing that whereas the labour demand curve is generally downward-sloping under full capacity and diminishing returns to labour, it may be flat or upward-sloping under excess capital capacity.[35] The reason is that, in the presence of unused capital, a rise in employment is generally accompanied by a rise in the amount of capital used, and thereby returns to scale - rather than returns to labour - come to play a dominant role in determining the slope of the labour demand curve. Second, the insider membership effect is negative in the US wage equation, but positive in the German wage equation. These results are interpretable through the insider-outsider literature (see footnote 6).

[34]See Karanassou and Snower (1997) for further details on the choice of the empirical model and estimation methodology.

[35]See, for example, Lindbeck and Snower (1994).

3.2 Estimating Persistence and Imperfect Responsiveness

3.2.1 Deriving the Measures of Persistence and Imperfect Responsiveness

To derive our measures of unemployment persistence, we conduct the following post-sample simulation exercises on each country's labour market system. We generate the time series for unemployment in the absence of the shock (u_t) by solving each system forwards until the unemployment rate attains its long-run equilibrium level (u^*), holding the exogenous variables constant.[36] We create the time series for unemployment in the presence of the temporary shock ($u_t^{'}$) by reducing the constant term in the labour demand equation by 1.0 in the first year only of the simulation period and solving the resulting system forwards.[37] The sum of the differences, $du_t = u_t^{'} - u_t$, from year 2 onwards, yields our measure of quantitative unemployment persistence (π^Q), of eq. (7a).[38] The measure of temporal unemployment persistence (π^T), of eq. (7b), is given by the maximum number of years it takes for the unemployment effect of the temporary shock to remain above 10% of the initial impact.

To derive our measures of imperfect unemployment responsiveness, we assume that the log of the capital stock (K_t) follows a random walk process, and we impose a permanent shock whereby the constant term in the capital stock equation is reduced by 1.0 in the first year only of the simulation period.[39] The resulting system is solved forwards until unemployment

[36]Since each labour market system consists of equations which satisfy the stability conditions, u_t will converge to its long-run value, regardless of the initial conditions, once the exogenous variables are held constant. In addition, for the purposes of computing unemployment persistence and imperfect responsiveness, it does not matter at what levels the exogenous variables are held constant for the following reason. Since each system is linear, the values of the exogenous variables do not affect the difference between unemployment in the presence and absence of the shock or the difference between long-run unemployment in the presence and absence of a permanent shock.

[37]Since the German labour market system exhibits cross-equation correlation, a shock to the employment equation will be accompanied by a shock to both the wage-setting and labour force equations. To incorporate the latter in our simulation exercises, we assume that $\widehat{\varepsilon}_{2t} = b_1\widehat{\varepsilon}_{1t} + v_{2t}$, $\widehat{\varepsilon}_{3t} = b_2\widehat{\varepsilon}_{1t} + b_3\widehat{\varepsilon}_{2t} + v_{3t}$, where $\widehat{\varepsilon}_{1t}, \widehat{\varepsilon}_{2t}, \widehat{\varepsilon}_{3t}$ are the residuals from the labour demand, wage-setting and labour supply equations. So we estimate the above equations by OLS and we then reduce the constants in the German wage setting and labour supply equations by b_1 and $(b_2 + b_3b_1)$, respectively.

[38]In Appendix 3 we present an analytical method for the computation of our measures of quantitative unemployment persistence.

[39]Since K_t is assumed to follow a random walk, a one-period unit shock to the capital

attains its new long-run equilibrium $\widehat{u}^*$, holding the other exogenous variables constant. Letting $\widehat{u}_t$ and u_t be the predicted unemployment rates in the presence and absence of the permanent shock (respectively) and letting $\widehat{u}^*$and u^* be the long-run unemployment rates in the presence and absence of the shock (respectively), we obtain our measure of quantitative imperfect responsiveness (ρ^Q) through equation (12a). We measure temporal imperfect responsiveness (ρ^T), of equation (12b), as the maximum number of years it takes for $(du_t - du^*)$, where $du_t = (\widehat{u}_t - u_t)$ and $du^* = (\widehat{u}^* - u^*)$, to remain above 10% of the initial difference between these disparities.

Our measures of persistence and responsiveness are reported in Table 8. The standard errors and 95% confidence intervals for these measures were obtained from Monte Carlo simulations. Each Monte Carlo experiment consists of $R = 400$ replications. In each replication (i), a vector of error terms $\varepsilon_t^{(i)} = \left(\varepsilon_{1t}^{(i)}, \varepsilon_{2t}^{(i)}, \varepsilon_{3t}^{(i)}\right)'$(of the labour demand, wage setting, and labour supply equations, respectively) was drawn from the normal distribution,[40] $N(0, \sum)$. The vector $\varepsilon_t^{(i)}$ was then added to the labour market system to generate a new vector of endogenous variables $y_t^{(i)} = \left(E_t^{(i)}, w_t^{(i)}, L_t^{(i)}, u_t^{(i)} = L_t^{(i)} - E_t^{(i)}\right)$. Next, the labour market system was estimated using the new vector endogenous variables $y_t^{(i)}$, and the set of exogenous variables. Finally, the above simulation exercises were conducted on the newly estimated labour market system. In this way, each replication (i) yielded a set of persistence and responsiveness measures: $x_i = \left\{\pi_i^Q, \pi_i^T, \rho_i^Q, \rho_i^T\right\}$. The standard errors, reported in Table 8, were computed as $\left(\sum_{i=1}^R x_i^2 - R\overline{x}^2\right) / (R-1)$, where $\overline{x} = \left(\sum_{i=1}^R x_i\right) / R$. Furthermore, since each generated series x_i, $i = 1, ...R$, does not necessarily follow the normal distribution, we also computed the corresponding 95% confidence intervals by ranking the values of each element of x_i in ascending order and cutting off the bottom 10 and top 10 observations.

3.2.2 Empirical Results on Aggregate Persistence

Our empirical results concerning these aggregate measures of *quantitative persistence*, summarised in the π^Q and π^T rows of Table 8, are striking. Ob-

stock equation will have a permanent impact on K_t. Consequently, employment and the unemployment rate are permanently affected.

[40]We used the normal distribution because the assumption of normality is valid in all three estimated labour market systems. Thus $\varepsilon_t \sim N(0, \sum)$, where $\sum$ is the variance-covariance matrix of the estimated labour market model. For the UK and US, the off-diagonal elements of $\sum$ were set to zero, since for these countries the cross-equation correlations were found to be statistically insignificant.

serve that our quantitative and temporal persistence measures yield quite different inter-country comparisons. Although this contrast is not surprising - since quantitative and temporal persistence are quite different phenomena[41] - it has not received attention in the literature thus far. Quantitative persistence (π^Q in Table 8) is both positive and significant (in terms of standard errors) in both the UK and German labour market systems, but insignificant in the US system. By contrast, all three countries feature significant temporal persistence (π^T in Table 8).

Quantitative persistence of UK is about three times as large as that of Germany. In particular, a temporary unit fall in labour demand leads to a cumulative future rise in unemployment by 2.17 in the UK and 0.72 in Germany.

On the other hand, *temporal persistence* in the UK is about double that in Germany; and whereas the US system displays no positive persistence, its temporal persistence is roughly of the same magnitude as that of Germany. As shown in Table 8, it takes about 6 years for 90% of the unemployment effect of a temporary shock to work itself out in the UK, but only about 3 years to do so in Germany and the US.

Our estimates of the German, UK, and US systems suggest that the labour market lag structures of these countries are quite different and that these differences have important consequences for the temporal distribution of unemployment effects. As Table 8 shows, the *total* quantitative unemployment effect (τ) of a labour demand shock differs markedly among the three countries, viz., it is larger in the UK than in Germany and larger in Germany than in the US. Interestingly, these differences are not significantly due to disparities in the *current* unemployment effect of a temporary shock, since all three countries have roughly comparable unemployment multipliers (m). Rather, these differences arise primarily from disparities in the *future* unemployment effect of such a shock (since the countries differ sharply in terms of quantitative persistence (π^Q). For example, in the UK the future unemployment effect (π^Q) of a temporary shock is nearly three times as large as the current effect (m); whereas in Germany the current and future unemployment effects are about equal.

3.2.3 Empirical Results on Imperfect Responsiveness

Now turn to our empirical results concerning aggregate measures of *quantitative imperfect responsiveness*, summarised in the ρ^Q and ρ^T rows of Table

[41] Recall that quantitative persistence captures the magnitude of the future unemployment effects of a current shock, whereas and temporal persistence measures the time it takes to return to equilibrium.

8. It is interesting to compare these results with those on persistence. As with persistence, both the UK and German systems are characterised by significant, positive quantitative imperfect responsiveness, whereas imperfect responsiveness in the US system is insignificant.

Both the UK and Germany are over-responsive: A permanent unit shock leads to overshooting, so that unemployment is cumulatively higher than it would have been under instantaneous adjustment. Over-responsiveness is about three times as large as that of Germany. (In the UK cumulative unemployment is 4.88 times higher than under instantaneous adjustment, whereas in Germany it is 1.65 times higher.)

By contrast, all the countries exhibit roughly comparable degrees of *temporal imperfect responsiveness*. Specifically, it takes about 3 years for 90% of the unemployment effect of a permanent shock to work itself out in all three countries.

The overall picture that emerges from these results is that (i) lagged labour market adjustment processes play an important role in explaining how temporary and permanent shocks affect unemployment, (ii) the inter-country comparisons of unemployment effects due to temporary shocks (as measured by persistence) differ markedly from those due to permanent shocks (as measured by imperfect responsiveness), and (iii) the inter-country comparisons of quantitative measures of dynamic processes (regarding both persistence and imperfect responsiveness) are quite distinct from those of temporal measures.

3.3 Estimating the Sources of Persistence and Imperfect Responsiveness

We now examine the contributions of individual labour market lags to unemployment persistence and imperfect responsiveness. We focus attention on the following lagged effects:[42]

- the lagged employment terms in the employment equation: the *employment adjustment effect* (EA);
- the lagged employment terms in the wage setting equation: the *insider membership effect* (IM);
- the lagged real wage terms in the wage setting equation: the *wage staggering effect* (WS);

[42] As above, the names assigned to these effects are merely a heuristic device; they do not imply, of course, that the lagged terms could not arise for reasons other than the named ones.

- the lagged unemployment terms in the wage setting equation are the *long-term unemployment effect* (LU);[43]

- The lagged labour force terms in the labour force equation are the *labour force adjustment effect* (LF).[44]

(It is important to recall that these names are just heuristic devices; they are not meant to identify unique sources of the lagged effects.)

To derive our measures of persistence[45] and imperfect responsiveness in the absence of each of these lagged effects, we repeated the post-sample simulation exercises above, with the following amendments:

- to derive unemployment in the absence of the employment adjustment effect, we set $E_{t-2} = E_{t-1} = E_t$ in the employment equation;

- to examine unemployment in the absence of the wage staggering effect, we set $w_{t-2} = w_{t-1} = w_t$ in the wage equation;

- to find unemployment in the absence of the long-term unemployment effect, we set $u_{t-1} = u_t$ in the wage equation;

- to examine unemployment in the absence of the insider membership effect, we set $E_{t-1} = E_t$ in the wage equation; and

- to derive unemployment in the absence of the labour force adjustment effect, we set $L_{t-2} = L_{t-1} = L_t$ in the labour force equation.

Tables 9a and 9b give the measures of quantitative unemployment persistence and imperfect responsiveness in the absence of the individual lagged effects, while Tables 10a and 10b report the corresponding measures of the contributions of the individual effects (EA, WS, LU, IM, and LF) to persistence and imperfect responsiveness.

Observe that the employment adjustment effect (π^Q_{EA}) augments quantitative persistence by approximately the same amount in all three countries. This effect, however, diminishes quantitative imperfect responsiveness in these countries, with the negative contribution in the UK exceeding that of the US which in turn exceeds that of Germany.

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[43]This name is appropriate since the long-term unemployed tend to search less intensively for jobs and thus have less influence on the wage setting process than the short-term unemployed (e.g. Layard and Bean (1989).

[44]This name is appropriate since costs of entry to and exit from the labour force make the current labour force depend on its past magnitudes.

[45]The measures of quantitative persistence in the absence of the above adjustment effects are derived analytically in Appendix 3.

The labour force adjustment effect (π^Q_{LF}), by contrast, reduces quantitative persistence in all three countries, whereas it augments quantitative imperfect responsiveness in the US and Germany. The insider membership effect (π^Q_{IM}) reduces quantitative persistence and augments imperfect responsiveness in the US and Germany. In addition, the wage staggering effect and the long-term unemployment effect contribute positively to quantitative persistence and negatively to quantitative imperfect responsiveness in Germany.

4 Concluding Thoughts

Although the analytical building blocks of our analysis are quite standard - there is nothing surprising in the recognition that labour market behaviour is characterised by significant lags, or that these lags interact with one another - the thrust of our analysis is quite distinct from the natural rate and hysteresis literatures. Rather than concentrating primarily on the determinants of the long-run equilibrium unemployment rate (as the natural theory commonly does) or on the history of labour market shocks (as the hysteresis theory does), our analysis focuses on the interplay among lagged adjustment processes and the interplay between a network of lagged adjustments (on the one hand) and temporary versus permanent shocks (on the other). Instead of seeing labour market behaviour in essentially static terms, the chain reaction theory views unemployment as the dynamic response to an unending sequence of labour market shocks with various dynamic characteristics. Since the shocks never stop, the adjustment processes never have the opportunity to work themselves out fully, and consequently the adjustment dynamics play a central role determining the movement of unemployment.

This paper has sought to capture some salient features of unemployment adjustment dynamics: persistence and imperfect responsiveness. We have constructed two aggregate measures of these features: quantitative and temporal measures. And finally, we have estimated the contributions of individual lagged effects to these aggregate measures.

In the context of estimated systems describing the German, UK, and US labour markets, we have shown that lagged adjustment processes play an important part in explaining how temporary and permanent shocks affect unemployment. We have seen that temporary and permanent shocks can yield quite different inter-country comparisons of unemployment effects. For example, temporal persistence (measuring unemployment responses to temporary shocks) is greater in the UK system than in the German system which, in turn, is greater than that in the US system; but temporal responsiveness (measuring unemployment responses to permanent shocks) is roughly the same in all these systems.

Furthermore, we have seen that our quantitative and temporal measures also can yield markedly different inter-country comparisons. For example, our empirical results suggest that while the German, UK, and US systems are all characterised by significant temporal persistence, only the German and UK systems have significant quantitative persistence.

The chain reaction theory has quite different policy implications from those of the natural rate and hysteresis theories. The natural rate theory focuses attention on policy measures that affect the long-term structure of the labour market (such as tax changes, interest rate changes, and unemployment benefit reform), while the hysteresis theory focuses on employment-creating policy shocks and stabilisation policies to avoid the effects of adverse exogenous shocks. By contrast, the chain reaction theory focuses on measures that influence the *flexibility* of labour markets and their *resilience* in the aftermath of recessions, by affecting the underlying lagged adjustment processes. For instance, changes in job security legislation may affect how current employment decisions depend on past employment (by influencing labour turnover costs); job counseling for the unemployed may influence how current wage decisions depend on past unemployment (by influencing the search intensity of the long-term unemployed); and wage indexation may affect the degree to which these wage decisions depend on past wages. Since different policy variables affect different lagged adjustment processes, the identification and measurement of behavioural labour market lags is important for policy purposes.

Although our analysis has not been explicitly concerned with the design of unemployment policy, it is easy to see that the empirical results above may have some potentially significant policy implications. First, our empirical analysis indicates that, within a particular country, different labour market lags have quite different effects on unemployment persistence and imperfect responsiveness. This is significant for policy formulation since, as noted, different labour market policies affect different lags. Thus *it is important to assess the empirical importance of the various lags before unemployment policy can be formulated.*

Second, our analysis has shown that countries displaying a comparatively high degree of unemployment persistence need not necessarily display a comparatively high degree of unemployment under-responsiveness as well. In other words, the fact that temporary shocks have prolonged effects on unemployment, does not mean that the full effects of permanent shocks will be slow to manifest themselves. This result suggests that different policies may be required to deal with temporary and permanent shocks. In short, *assessments of the durability of labour market shocks have an important role to play in the appropriate design of labour market policies.*

Finally, our analysis showed that a particular lagged effect can have quite different implications for unemployment dynamics in different countries. For example, although the lags representing the employment adjustment effect are quite similar in the UK and German systems, the contributions of this effect to unemployment persistence and imperfect responsiveness are quite different as between the two countries. The reason, of course, is that the unemployment effect of any particular lag depends crucially on its interactions with the other lags in the labour market system, and the latter vary from country to country. This result, too, is potentially important for policy, since it implies that *different countries may require quite different policies to deal with what may be similar unemployment trajectories.*

The upshot of all these implications is that the appropriate design of unemployment policy is a complex matter. We need to make judgements on the durability of the shocks, relative importance of different labour market lags, and their interaction before we can formulate the appropriate policies. This is a tall order. But if it were easy, the European unemployment problem would probably have been mastered by now.

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Tables

Table 1: Definitions of variables		
E_t	:	log of employment
L_t	:	log of labour force
u_t	:	unemployment rate ($u_t = L_t - E_t$)
w_t	:	log of real wage(=average earnings)
K_t	:	log of real capital stock
b_t	:	log of real social security benefits
c_t	:	log of real social security contributions
τ_t^I	:	indirect taxes as a % of GDP
τ_t^D	:	direct taxes as a % of GDP
p_t^{oil}	:	log of real oil price
Z_t	:	log of working age population
i_t	:	real long-term interest rate
r_t	:	log of competitiveness $\left[r_t = \log\left(\frac{\text{price of imports}}{\text{GDP deflator}}\right)\right]$
Nominal variables were deflated using the GDP deflator		
Sources: OECD, IFS, Datastream		

Table 2: UK, OLS, 1964-1992						
[T1]	$\Delta E_t =$	3.21	$-0.34E_{t-2}$	$+0.06w_t$	$+4.40K_t$	
		(0.72)	(0.06)	(0.03)	(0.52)	
		$-6.70K_{t-1}$	$+2.44K_{t-2}$	$-0.39\tau_t^I$	$-0.07c_t$,	$R^2 = 0.91$
		(0.91)	(0.47)	(0.13)	(0.03)	
[T2]	$\Delta w_t =$	-1.53	$-0.28w_{t-2}$	$+0.16b_t$	$-0.03p_t^{oil}$	
		(0.43)	(0.08)	(0.05)	(0.01)	
		$-1.27\tau_t^I$	$+0.86\tau_{t-1}^I$,			$R^2 = 0.56$
		(0.50)	(0.44)			
[T3]	$\Delta L_t =$	-0.87	$-0.49L_{t-2}$	$-0.23\Delta u_t$	$-0.09w_t$	
		(0.78)	(0.08)	(0.07)	(0.05)	
		$+0.12w_{t-1}$	$+0.56Z_t$,			$R^2 = 0.71$
		(0.05)	(0.11)			
(standard errors in parentheses)						

Table 3: US, OLS, 1964-1992						
[T4]	$E_t =$	3.45	$+0.56E_{t-1}$	$+0.23w_t$	$+1.20K_t$	
		(0.80)	(0.12)	(0.10)	(0.37)	
		$-2.37K_{t-1}$	$+1.39K_{t-2}$	$-2.43\tau_t^I$	$-0.02p_t^{oil}$,	$R^2 = 0.99$
		(0.53)	(0.28)	(0.46)	(0.003)	
[T5]	$\Delta w_t =$	1.86	$-0.28w_{t-2}$	$-0.20E_{t-1}$	$+0.07b_t$	
		(0.29)	(0.07)	(0.03)	(0.01)	
		$+0.37\tau_t^D$	$-0.85\tau_t^I$	$+0.20i_t$,		$R^2 = 0.79$
		(0.16)	(0.32)	(0.05)		
[T6]	$L_t =$	-2.09	$+0.64L_{t-1}$	$-0.18u_t$	$+0.53Z_t$,	$R^2 = 0.99$
		(0.28)	(0.04)	(0.05)	(0.06)	
(standard errors in parentheses)						

Table 4: GE, OLS, 1964-1990						
[T7]	$\Delta E_t =$	2.11	$-0.33E_{t-2}$	$-0.12w_t$	$+2.37K_t$	
		(0.89)	(0.10)	(0.06)	(0.33)	
		$-3.23K_{t-1}$	$+1.00K_{t-2}$	$+0.05\Delta r_t$,		$R^2 = 0.85$
		(0.66)	(0.36)	(0.02)		
[T8]	$w_t =$	-4.15	$+0.37w_{t-1}$	$-0.60u_t$	$+0.44u_{t-1}$	
		(0.94)	(0.10)	(0.19)	(0.17)	
		$+0.24E_{t-1}$	$+0.29c_t$,			$R^2 = 0.99$
		(0.08)	(0.05)			
[T9]	$\Delta L_t =$	-2.69	$-0.47L_{t-2}$	$-0.29u_t$	$-0.03w_{t-1}$	
		(0.45)	(0.07)	(0.06)	(0.01)	
		$+1.27Z_t$	$-1.57Z_{t-1}$	$+1.00Z_{t-2}$,		$R^2 = 0.94$
		(0.13)	(0.25)	(0.13)		
(standard errors in parentheses)						

Table 5: GE, 3SLS, 1964-1990						
[T10]	$\Delta E_t =$	2.51	$-0.37E_{t-2}$	$-0.14w_t$	$+2.29K_t$	
		(0.68)	(0.07)	(0.05)	(0.25)	
		$-3.03K_{t-1}$	$+0.89K_{t-2}$	$+0.05\Delta r_t,$		$R^2 = 0.84$
		(0.49)	(0.27)	(0.01)		
[T11]	$w_t =$	-4.15	$+0.41w_{t-1}$	$-0.89u_t$	$+0.68u_{t-1}$	
		(0.79)	(0.08)	(0.16)	(0.15)	
		$+0.25E_{t-1}$	$+0.27c_t,$			$R^2 = 0.99$
		(0.07)	(0.04)			
[T12]	$\Delta L_t =$	-2.75	$-0.49L_{t-2}$	$-0.30u_t$	$-0.03w_{t-1}$	
		(0.39)	(0.06)	(0.05)	(0.01)	
		$+1.21Z_t$	$-1.40Z_{t-1}$	$+0.93Z_{t-2},$		$R^2 = 0.94$
		(0.11)	(0.21)	(0.11)		
Instruments: constant, $E_{t-1}, E_{t-2}, w_{t-1}, w_{t-2}, L_{t-1}, L_{t-2}, K_t, K_{t-1}, K_{t-2}, \Delta r_t, r_{t-2}, c_t, Z_t, Z_{t-1}, Z_{t-2}$.						
(asymptotic standard errors in parentheses)						

Table 6: Sargan's test		
GE	Labour demand equation:	$\chi^2\,(9) = 16.63\,[0.06]$
	Wage-setting equation:	$\chi^2\,(10) = 12.14\,[0.28]$
	Labour supply equation:	$\chi^2\,(9) = 11.10\,[0.27]$
	Instruments: constant, $E_{t-1}, E_{t-2}, w_{t-1}, w_{t-2}, L_{t-1}, L_{t-2}, K_t, K_{t-1}, K_{t-2}, \Delta r_t, r_{t-2}, c_t, Z_t, Z_{t-1}, Z_{t-2}$	
[probabilities in square brackets]		

Table 7: Misspecification tests

Equation	**[T1]**	**[T2]**	**[T3]**	**[T4]**	**[T5]**	**[T6]**
SC[$\chi^2(1)$]	3.79	0.39	2.37	0.54	0.01	0.64
LIN[$\chi^2(1)$]	1.63	0.21	1.19	2.22	3.54	2.26
NOR[$\chi^2(2)$]	0.16	2.40	1.82	0.74	1.86	0.01
HET[$\chi^2(1)$]	0.76	0.50	2.03	1.24	2.87	0.28
ARCH[$\chi^2(1)$]	1.54	0.81	1.05	0.55	0.09	0.10
Equation	**[T7]**	**[T8]**	**[T9]**	**[T10]**	**[T11]**	**[T12]**
SC[$\chi^2(1)$]	1.08	0.88	2.80	1.82	4.68	4.16
SC[F-test]					$F(1,19)$ =2.26	$F(1,18)$ =3.18
LIN[$\chi^2(1)$]	2.28	0.02	4.10	2.97	2.70	4.59
LIN[$F(1,19)$]			3.40			3.76
NOR[$\chi^2(1)$]	0.01	3.35	0.47	0.06	1.34	0.63
HET[$\chi^2(1)$]	0.16	0.61	0.63	0.03	0.30	0.70
ARCH[$\chi^2(1)$]	1.73	1.05	1.39	1.04	2.26	0.24

5% critcal values: $\chi^2(1) = 3.84$, $\chi^2(2) = 5.99$,
$F(1,18) = 4.41$, $F(1,19) = 4.38$.

Table 8: Aggregate measures of persistence and imperfect responsiveness

	UK	**US**	**GE**
π^Q	2.17	0.27	0.72
(s.e)	(0.10)	(0.18)	(0.27)
[95% c.i.]	$[1.93, 2.33]$	$[-0.04, 0.66]$	$[0.27, 1.22]$
m	0.81	0.85	0.64
(s.e)	(0.03)	(0.03)	(0.07)
[95% c.i.]	$[0.79, 0.92]$	$[0.82, 0.94]$	$[0.51, 0.77]$
τ	2.98	1.12	1.36
(s.e)	(0.10)	(0.19)	(0.29)
[95% c.i.]	$[2.78, 3.18]$	$[0.81, 1.56]$	$[0.84, 1.90]$
π^T	6	3	3
(s.e)	(0.45)	(0.53)	(1.14)
[95% c.i.]	$[5, 6]$	$[2, 3]$	$[2, 6]$
ρ^Q	4.88	-0.36	1.65
(s.e)	(1.04)	(0.42)	(0.33)
[95% c.i.]	$[2.69, 6.88]$	$[-1.18, 0.43]$	$[1.12, 2.39]$
ρ^T	3	4	2
(s.e)	(1.41)	(0.67)	(1.18)
[95% c.i.]	$[2, 8]$	$[2, 4]$	$[2, 5]$

Table 9a: Quantitative unemployment persistence in the absence of individual lagged effects

UK : $\tau = 2.98$, $m = 0.81$, $\pi^Q = 2.17$

$m_{\sim EA} = 2.43, \quad m_{\sim WS} = 0.81, \quad m_{\sim LF} = 0.68$
$\pi^Q_{\sim EA} = 0.55, \quad \pi^Q_{\sim WS} = 2.17, \quad \pi^Q_{\sim LF} = 2.30$

US : $\tau = 1.12$, $m = 0.85$, $\pi^Q = 0.27$

$m_{\sim EA} = 1.94, \quad m_{\sim WS} = 0.85, \quad m_{\sim IM} = 0.81, \quad m_{\sim LF} = 0.67$
$\pi^Q_{\sim EA} = -0.82, \quad \pi^Q_{\sim WS} = 0.27, \quad \pi^Q_{\sim IM} = 0.31, \quad \pi^Q_{\sim LF} = 0.45$

GE : $\tau = 1.36$, $m = 0.64$, $\pi^Q = 0.72$

$m_{\sim EA} = 1.70, \quad m_{\sim WS} = 0.65, \quad m_{\sim LU} = 0.69, \quad m_{\sim IM} = 0.62, \quad m_{\sim LF} = 0.42$
$\pi^Q_{\sim EA} = -0.34, \quad \pi^Q_{\sim WS} = 0.71, \quad \pi^Q_{\sim LU} = 0.67, \quad \pi^Q_{\sim IM} = 0.74, \quad \pi^Q_{\sim LF} = 0.94$

Table 9b: Quantitative imperfect unemployment responsiveness in the absence of individual lagged effects

	ρ^Q	$\rho^Q_{\sim EA}$	$\rho^Q_{\sim WS}$	$\rho^Q_{\sim LU}$	$\rho^Q_{\sim IM}$	$\rho^Q_{\sim LF}$
UK	4.88	5.27	4.88			4.88
US	-0.36	-0.13	-0.46		-0.42	-0.50
GE	1.65	1.76	1.63	1.68	1.63	1.64

Table 10a: Sources of unemployment persistence

	π^Q	π^Q_{EA}	π^Q_{LF}	π^Q_{WS}	π^Q_{LU}	π^Q_{IM}
UK	2.17	1.62	-0.13	0		
US	0.27	1.09	-0.18	0		-0.04
GE	0.72	1.06	-0.22	0.01	0.05	-0.02

Table 10b: Sources of imperfect unemployment responsiveness

	ρ^Q	ρ^Q_{EA}	ρ^Q_{LF}	ρ^Q_{WS}	ρ^Q_{LU}	ρ^Q_{IM}
UK	4.88	-0.39	0	0		
US	-0.36	-0.23	0.14	0.10		0.06
GE	1.65	-0.11	0.01	0.02	-0.03	0.02

APPENDIX 1

Consider our labour market equations (6)-(8):

$$
\begin{aligned}
E_t &= a + a_E E_{t-1} + a_K K_t + \left(\frac{1}{1 + a_w b_E}\right) \varepsilon_t, && \text{(A1.1)} \\
L_t &= c + c_L L_{t-1} + c_E E_t + c_Z Z_t, && \text{(A1.2)} \\
u_t &= L_t - E_t, && \text{(A1.3)}
\end{aligned}
$$

where $a, a_K, a_w, b_E, c, c_E, c_Z > 0$, $0 < a_E < 1$, and $0 < c_L < 1$. Using the backshift operator B we can express (A1.1)-(A1.2) as

$$
\begin{aligned}
(1 - a_E B)\, E_t &= a + a_K K_t + \left(\frac{1}{1 + a_w b_E}\right) \varepsilon_t, && \text{(A1.1')} \\
(1 - c_L B)\, L_t &= c + c_E E_t + c_Z Z_t && \text{(A1.2')}
\end{aligned}
$$

Algebraic manipulation of (A1.1'), (A1.2'), and (A1.3) yields the following reduced form unemployment rate equation:

$$
\begin{aligned}
(1 - a_E B)(1 - c_L B)\, u_t &= -\theta (1 - c_E - c_L B)\, \varepsilon_t && \text{(A1.4)} \\
&\quad - a_K (1 - c_E - c_L B)\, K_t \\
&\quad + a c_E + (1 - a_E)\, c - (1 - c_L)\, a + c_Z (1 - a_E B)\, Z_t,
\end{aligned}
$$

where $\theta = \left(\frac{1}{1+a_w b_E}\right)$. Observe that (A1.4) is dynamically stable since $0 < a_E, c_L < 1$.

Consider a negative one-off unit shock to the labour demand equation (A1.1), occuring at period t, i.e $d\varepsilon_t = -1$, $d\varepsilon_{t+j} = 0$, $j = 1, 2, 3, ...$ Because of the stability of the unemployment rate equation, the impact of the temporary shock on unemployment will dissipate with the passage of time. Note that (A1.4) is an ARMA(2,1) process in terms of ε_t. So the effects of the shock on the unemployment rate, through time, are easily seen from its infinite moving average (I.M.A.) representation[46]:

$$
u_t = \frac{1}{a_E - c_L} \sum_{j=0}^{\infty} \left(a_E^{1+j} - c_L^{1+j}\right) \left[-\theta (1 - c_E)\, \varepsilon_{t-j} + \theta c_L \varepsilon_{t-1-j} + \xi_{t-j}\right], \tag{A1.5}
$$

where

$$
\xi_t = -a_K (1 - c_E - c_L B)\, K_t + a c_E + (1 - a_E)\, c - (1 - c_L)\, a + c_Z (1 - a_E B)\, Z_t.
$$

[46]See Sargent (1987), pp.184,191.

Therefore, the resulting change in the unemployment rate, j periods after the occurence of the shock, du_{t+j}, is given by

$$\begin{aligned} du_{t+j} &= \frac{\theta\left[a_E^j\left[(1-c_E)\,a_E-c_L\right]-c_L^j\left[(1-c_E)\,c_L-c_L\right]\right]}{(a_E-c_L)}, \\ &= \frac{a_E^j\left[(1-c_E)\,a_E-c_L\right]+c_E c_L^{j+1}}{(1+a_w b_E)\,(a_E-c_L)}, j \geq 0. \end{aligned} \tag{A1.6}$$

The degree of *quantitative unemployment persistence*, π^Q, is measured by

$$\pi^Q = \sum_{j=1}^{\infty} du_{t+j}. \tag{A1.7}$$

Substitution of eq.(A1.6) into eq.(A1.7) leads, after some algebraic manipulation, to

$$\pi^Q = \frac{a_E\,(1-c_L)-c_E\left[a_E\,(1-c_L)+c_L\right]}{(1+a_w b_E)\,(1-a_E)\,(1-c_L)}, \text{ or} \tag{A1.8}$$

$$\pi^Q = \frac{a_E\,(1-c_L)\,(1-c_E)-c_E c_L}{(1+a_w b_E)\,(1-a_E)\,(1-c_L)}, \text{ or} \tag{A1.8a}$$

$$\pi^Q = \frac{a_E\,(1-c_L-c_E)-c_E c_L\,(1-a_E)}{(1+a_w b_E)\,(1-a_E)\,(1-c_L)}. \tag{A1.8b}$$

Observe that the denominator of the above equations is positive, so the sign of π^Q depends on the sign of the numerator of the above expressions. Inspection of (A1.8a) and (A1.8b) shows that:

$$\text{if } \left\{ \begin{array}{lll} c_E+c_L>1 & & \\ c_E+c_L<1 & \text{and} & a_E<\frac{c_E c_L}{(1-c_L)(1-c_E)} \\ c_E+c_L<1 & \text{and} & a_E=\frac{c_E c_L}{(1-c_L)(1-c_E)} \\ c_E+c_L<1 & \text{and} & a_E>\frac{c_E c_L}{(1-c_L)(1-c_E)} \end{array} \right\} \text{ then } \left\{ \begin{array}{l} \pi^Q<0 \\ \pi^Q<0 \\ \pi^Q=0 \\ \pi^Q>0 \end{array} \right\}. \tag{A1.9}$$

Furthermore, the total impact (τ) of the above temporary shock on the unemployment rate can be expressed as a sum of "present",m, and "future",π^Q, effects:

$$\tau = m+\pi^Q = \frac{(1-c_L-c_E)}{(1+a_w b_E)\,(1-a_E)\,(1-c_L)}, \text{ where} \tag{A1.10}$$

$$m \equiv du_t = \theta \equiv \frac{1}{(1+a_w b_E)}. \tag{A1.11}$$

To derive the unemployment rate equation in the **absence of the employment adjustment effect** ($\sim EA$), we set the backshift operator in the $(1-a_E B)$ expression of eq.(A1.4) equal to one.[47] So we get that:

$$\begin{aligned}(1-c_L B)\, u_t &= -\frac{\theta}{(1-a_E)}(1-c_E-c_L B)\,\varepsilon_t \\ &\quad -\frac{a_K}{(1-a_E)}(1-c_E-c_L B)\,K_t \\ &\quad +\frac{a c_E}{(1-a_E)}+c-\frac{(1-c_L)\,a}{(1-a_E)}+c_Z Z_t.\end{aligned} \tag{A1.12}$$

Since $0<c_L<1$, the above equation is stable. Note that (A1.12) is an ARMA(1,1) process in terms of ε_t. So the effects of the temporary labour demand shock on the unemployment rate, through time, can be seen from the infinite moving average (I.M.A.) representation of (A1.12):

$$u_t = \frac{-\theta\,(1-c_E)}{(1-a_E)}\varepsilon_t + \frac{\theta c_E}{(1-a_E)}\sum_{j=1}^{\infty} c_L^j \varepsilon_{t-j} + \sum_{j=0}^{\infty} c_L^j \xi_{t-j}, \tag{A1.13}$$

where

$$\xi_t = -\frac{a_K}{(1-a_E)}(1-c_E-c_L B)\,K_t + \frac{a c_E}{(1-a_E)} + c - \frac{(1-c_L)\,a}{(1-a_E)} + c_Z Z_t.$$

So we have that

$$du_{t+j} = -\frac{\theta c_E c_L^j}{(1-a_E)},\ j\geq 1, \tag{A1.14}$$

$$m_{\sim EA} \equiv du_t = \frac{\theta\,(1-c_E)}{(1-a_E)}. \tag{A1.15}$$

To measure the degree of quantitative unemployment persistence in the absence of the employment adjustment effect we insert (A1.14) into eq.(A1.7), and with some algebraic manipulation, we get

$$\pi^Q_{\sim EA} = -\frac{\theta c_E c_L}{(1-a_E)\,(1-c_L)} < 0. \tag{A1.16}$$

Observe that

$$m + \pi^Q = m_{\sim EA} + \pi^Q_{\sim EA} = \tau. \tag{A1.17}$$

[47] Alternatively, we can set $E_t = E_{t-1}$ in the labour demand equation (A1.1) and then derive the corresponding reduced form unemployment rate equation.

Furthermore, the degree of *quantitative unemployment persistence attributable to the employment adjustment effect* is measured as follows:

$$\pi_{EA}^{Q} = \pi^{Q} - \pi_{\sim EA}^{Q} = \frac{\theta a_E \left(1 - c_E\right)}{\left(1 - a_E\right)}. \tag{A1.18}$$

Note that if $c_E < 1$ then $\pi_{EA}^{Q} > 0$.

To derive the unemployment rate in the **absence of the labour force adjustment effect** ($\sim LF$), we set the backshift operator in the $(1 - c_L B)$ and $(1 - c_E - c_L B)$ polynomials of eq.(A1.4) equal to one,[48] and we get

$$\begin{aligned} \left(1 - a_E B\right) u_t &= -\frac{\theta}{\left(1 - c_L\right)} \left(1 - c_E - c_L\right) \varepsilon_t \\ &\quad -\frac{a_K}{\left(1 - c_L\right)} \left(1 - c_E - c_L\right) K_t \\ &\quad +\frac{a c_E}{\left(1 - c_L\right)} + \frac{\left(1 - a_E\right) c}{\left(1 - c_L\right)} - a + \frac{c_Z \left(1 - a_E B\right) Z_t}{\left(1 - c_L\right)}. \end{aligned} \tag{A1.19}$$

Since $0 < a_E < 1$, the above equation is stable. Note that (A1.19) is an AR(1) process in terms of ε_t. So the effects of the temporary labour demand shock on the unemployment rate, through time, can be seen from the infinite moving average (I.M.A.) representation of the above equation:

$$u_t = \frac{-\theta \left(1 - c_E - c_L\right)}{\left(1 - c_L\right)} \sum_{j=0}^{\infty} a_E^j \varepsilon_{t-j} + \sum_{j=0}^{\infty} a_E^j \xi_{t-j}, \tag{A1.20}$$

where

$$\xi_t = -\frac{a_K}{\left(1 - c_L\right)} \left(1 - c_E - c_L\right) K_t + \frac{a c_E}{\left(1 - c_L\right)} + \frac{\left(1 - a_E\right) c}{\left(1 - c_L\right)} - a + \frac{c_Z \left(1 - a_E B\right) Z_t}{\left(1 - c_L\right)}.$$

So the unemployment responses to a negative one-off unit labour demand shock ($d\varepsilon_t = -1$) are given by

$$du_{t+j} = \frac{\theta \left(1 - c_E - c_L\right) a_E^j}{\left(1 - c_L\right)}, \; j \geq 0, \tag{A1.21}$$

$$m_{\sim LF} \equiv du_t = \frac{\theta \left(1 - c_E - c_L\right)}{\left(1 - c_L\right)}. \tag{A1.21'}$$

To measure the degree of quantitative unemployment persistence in the absence of the labour force adjustment effect, we insert (A1.21) into eq.(A1.7) to obtain

$$\pi_{\sim LF}^{Q} = \frac{\theta a_E \left(1 - c_E - c_L\right)}{\left(1 - a_E\right) \left(1 - c_L\right)}. \tag{A1.22}$$

[48] Alternatively, we can set $L_t = L_{t-1}$ in the labour force equation (A1.2) and then derive the corresponding reduced form unemployment rate equation.

If $c_E + c_L < 1$ then $\pi^Q_{\sim LF} > 0$. Also note that

$$m + \pi^Q = m_{\sim EA} + \pi^Q_{\sim EA} = m_{\sim LF} + \pi^Q_{\sim LF} = \tau. \tag{A1.23}$$

Finally, the degree of *quantitative unemployment persistence attributable to the labour force adjustment effect* is measured as follows:

$$\pi^Q_{LF} = \pi^Q - \pi^Q_{\sim LF} = -\frac{\theta c_E c_L}{(1 - c_L)} < 0. \tag{A1.24}$$

Note that $\pi^Q_{EA} + \pi^Q_{LF} = \pi^Q$, i.e. the employment and labour force adjustment effects interact as substitutes in generating the aggregate measure of quantitative responsiveness.

APPENDIX 2

Within the framework of Appendix 1, let the capital stock (K_t) follow a random walk stochastic process, given by $K_t = K_{t-1} + v_t$, $v_t \sim i.i.d\left(0, \sigma_v^2\right)$, and consider a negative one-off unit shock to the capital stock equation, occuring at period t, i.e. $dv_t = -1$, $dv_{t+j} = 0$, $j = 1, 2, 3, ...$ As a result, the capital stock is permanently reduced by one unit from period t onwards. In particular, the response of the capital stock to the above shock, j periods after its occurence, is $dK_{t+j} = -1$, $j \geq 0$.

Since the unemployment rate depends on capital stock, a permanent change in the latter will permanently affect the former. The resulting change in the unemployment rate will depend on its dynamic structure. Below, we examine three different possibilities:

(I) In the **presence of all the lagged effects** the unemployment rate equation (A1.4) can be written in the following ARMA(2,1) form:

$$u_t = (a_E + c_L)\, u_{t-1} - a_E c_L u_{t-2} - a_K (1 - c_E)\, K_t + a_K c_L K_{t-1} + \zeta_t, \quad \text{(A2.1)}$$

where

$$\zeta_t = -\theta\,(1 - c_E - c_L B)\,\varepsilon_t + a c_E + (1 - a_E)\, c - (1 - c_L)\, a + c_Z\,(1 - a_E B)\, Z_t.$$

The change in the unemployment rate, j periods after the occurence of the above shock, is given by the following second order difference equation:

$$(1 - a_E B)(1 - c_L B)\, du_{t+j} = -a_K (1 - c_E)\, dK_{t+j} + a_K c_L dK_{t+j-1}, \; j \geq 0, \quad \text{(A2.2)}$$

$$(1 - a_E B)(1 - c_L B)\, du_{t+j} = a_K (1 - c_E) - a_K c_L, \; j \geq 1. \quad \text{(A2.2')}$$

In the long-run, the change in the unemployment rate is

$$du^* = \frac{a_K (1 - c_E - c_L)}{(1 - a_E)(1 - c_L)}. \quad \text{(A2.3)}$$

The infinite moving average (I.M.A) representation of u_t is given byf

$$u_t = \frac{1}{a_E - c_L} \sum_{j=0}^{\infty} \left(a_E^{1+j} - c_L^{1+j}\right) \left[-a_K (1 - c_E)\, K_{t-j} + a_K c_L K_{t-1-j} + \zeta_{t-j}\right]. \quad \text{(A2.4)}$$

Therefore, the response of the unemployment rate to a permanent unit decrease in the capital stock is

$$du_{t+j} = \frac{a_K}{a_E - c_L} \left[(1 - c_E) \sum_{i=0}^{j} \left(a_E^{i+1} - c_L^{i+1}\right) - c_L \sum_{i=0}^{j} \left(a_E^{i} - c_L^{i}\right)\right], \; j \geq 0, \quad \text{(A2.5)}$$

The degree of *imperfect responsiveness* is measured by

$$\rho^{Q}=\sum_{j=0}^{\infty}\left(du_{t+j}-du^{*}\right). \tag{A2.6}$$

Since $\sum_{i=0}^{j}\lambda^{i+1}=\frac{\lambda\left(1-\lambda^{j+1}\right)}{1-\lambda}$, $\sum_{i=0}^{j}\lambda^{i}=\frac{\left(1-\lambda^{j+1}\right)}{1-\lambda}$, where $|\lambda|<1$, it can be shown that substitution of (A2.3) and (A2.5) into (A2.6) leads to

$$\rho^{Q}=\frac{a_{K}\left[-a_{E}\left(1-c_{E}-c_{L}\right)\left(1-c_{L}\right)+c_{L}c_{E}\left(1-a_{E}\right)\right]}{\left[\left(1-a_{E}\right)\left(1-c_{L}\right)\right]^{2}} \tag{A2.7}$$

Since the denominator of the above equation is positive, the sign of ρ^{Q} depends on the sign of the numerator. Inspection of (A2.7) gives that:

$$\text{if}\left\{\begin{array}{lll} c_{E}+c_{L}>1 & & \\ c_{E}+c_{L}<1 & \text{and} & a_{E}<\frac{c_{E}c_{L}}{(1-c_{L})(1-c_{E}-c_{L})+c_{E}c_{L}} \\ c_{E}+c_{L}<1 & \text{and} & a_{E}=\frac{c_{E}c_{L}}{(1-c_{L})(1-c_{E}-c_{L})+c_{E}c_{L}} \\ c_{E}+c_{L}<1 & \text{and} & a_{E}>\frac{c_{E}c_{L}}{(1-c_{L})(1-c_{E}-c_{L})+c_{E}c_{L}} \end{array}\right\}\text{then}\left\{\begin{array}{l} \rho^{Q}>0 \\ \rho^{Q}>0 \\ \rho^{Q}=0 \\ \rho^{Q}<0 \end{array}\right\}. \tag{A2.8}$$

Observe that when $c_{E}+c_{L}<1$ then $\frac{c_{E}c_{L}}{(1-c_{L})(1-c_{E}-c_{L})+c_{E}c_{L}}=\frac{c_{E}c_{L}}{(1-c_{L})(1-c_{E})-c_{L}(1-c_{L}-c_{E})}>\frac{c_{E}c_{L}}{(1-c_{L})(1-c_{E})}$ (ref. Figure 1).

(II) In the **absence of the employment adjustment effect** the unemployment rate is given by equation (A1.12) which can be written in the following ARMA(1,1) form:

$$\left(1-c_{L}B\right)u_{t}=-\frac{a_{K}}{\left(1-a_{E}\right)}\left(1-c_{E}-c_{L}B\right)K_{t}+\zeta_{t}, \tag{A2.9}$$

where

$$\zeta_{t}=-\frac{\theta}{\left(1-a_{E}\right)}\left(1-c_{E}-c_{L}B\right)\varepsilon_{t}+\frac{ac_{E}}{\left(1-a_{E}\right)}+c-\frac{\left(1-c_{L}\right)a}{\left(1-a_{E}\right)}+c_{Z}Z_{t}.$$

Its I.M.A. representation is given by

$$u_{t}=\frac{-a_{K}\left(1-c_{E}\right)}{\left(1-a_{E}\right)}K_{t}+\frac{a_{K}c_{E}}{\left(1-a_{E}\right)}\sum_{j=1}^{\infty}c_{L}^{j}K_{t-j}+\sum_{j=0}^{\infty}c_{L}^{j}\zeta_{t-j}. \tag{A2.10}$$

Thus the response of the unemployment rate to a permanent unit decrease in capital stock is

$$du_{t}=\frac{a_{K}\left(1-c_{E}\right)}{1-a_{E}},\ du_{t+j}=\frac{a_{K}}{1-a_{E}}\left[\left(1-c_{E}\right)-c_{E}\sum_{i=1}^{j}c_{L}^{i}\right],\ j\geq 1. \tag{A2.11}$$

To derive our measure of the degree of quantitative imperfect unemployment responsiveness in the absence of the employment adjustment effect $\left(\rho^{Q}_{\sim EA}\right)$ we substitute (A2.11) and (A2.3) into (A2.6) and, with simple algebraic manipulation, we get

$$\rho^{Q}_{\sim EA} = \frac{a_K c_E c_L}{(1-a_E)(1-c_L)^2} > 0. \tag{A2.12}$$

Furthermore, the degree of *quantitative imperfect unemployment responsiveness attributable to the employment adjustment effect* is given by

$$\rho^{Q}_{EA} = \rho^{Q} - \rho^{Q}_{\sim EA} = \frac{-a_K a_E (1-c_E-c_L)}{(1-a_E)^2(1-c_L)}. \tag{A2.13}$$

Observe that the above measure is negative when $c_E + c_L < 1$.

(III) In the **absence of the labour force adjustment effect** the unemployment rate is given by equation (A1.19) which we can rewrite in the following AR(1) form:

$$(1-a_E B)\, u_t = -\frac{a_K}{(1-c_L)}(1-c_E-c_L) K_t + \zeta_t, \tag{A2.14}$$

where

$$\zeta_t = -\frac{\theta}{(1-c_L)}(1-c_E-c_L)\varepsilon_t + \frac{a c_E}{(1-c_L)} + \frac{(1-a_E)c}{(1-c_L)} - a + \frac{c_Z(1-a_E B) Z_t}{(1-c_L)}.$$

The I.M.A. representation of (A2.14) is

$$u_t = \frac{-a_K(1-c_E-c_L)}{(1-c_L)} \sum_{j=0}^{\infty} a_E^j K_{t-j} + \sum_{j=0}^{\infty} a_E^j \zeta_{t-j}. \tag{A2.13}$$

In this case the response of the unemployment rate to a permanent unit decrease in capital stock is given by

$$du_{t+j} = \frac{-a_K(1-c_E-c_L)}{(1-c_L)} \sum_{i=0}^{j} a_E^i, \; j \geq 0. \tag{A2.14}$$

Substitution of (A2.14) and (A2.3) into (A2.6) gives our measure of the degree of quantitative imperfect unemployment responsiveness in the absence of the labour force adjustment effect:

$$\rho^{Q}_{\sim LF} = \frac{-a_K a_E (1-c_E-c_L)}{(1-a_E)^2(1-c_L)}. \tag{A2.15}$$

So the degree of *quantitative imperfect unemployment responsiveness attributable to the labour force adjustment effect* is given by

$$\rho_{LF}^{Q} = \rho^{Q} - \rho_{\sim LF}^{Q} = \frac{a_K c_E c_L}{(1 - a_E)(1 - c_L)^2} > 0. \tag{A2.16}$$

Note that $\rho_{EA}^{Q} + \rho_{LF}^{Q} = \rho^{Q}$, i.e. the employment and labour force adjustment effects are neither complementary nor substitutes when generating the aggregate measure of quantitative responsiveness.

APPENDIX 3

In what follows we present an analytical method for the computation of our measures of quantitative unemployment persistence which are given in Table 9a.

(I) The estimated **UK labour market system** (see Table 2) is

$$\begin{aligned}\Delta E_t &= \beta_1 + \beta_2 E_{t-2} + \beta_3 w_t + \beta_4 K_t + \beta_5 K_{t-1} + \beta_6 K_{t-2} + \beta_7 \tau_t^I + \beta_8 c_t, \qquad \text{(A3.1)}\\ \Delta w_t &= \beta_9 + \beta_{10} w_{t-2} + \beta_{11} b_t + \beta_{12} p_t^{oil} + \beta_{13} \tau_t^I + \beta_{14} \tau_{t-1}^I, \qquad \text{(A3.2)}\\ \Delta L_t &= \beta_{15} + \beta_{16} L_{t-2} + \beta_{17} \Delta u_t + \beta_{18} w_t + \beta_{19} w_{t-1} + \beta_{20} Z_t, \qquad \text{(A3.3)}\end{aligned}$$

where Δ is the difference operator, the β's are the esimated parameters, and the capital stock (K_t) is assumed to follow a random walk. Note that a one-off unit reduction in the constant of eq.(A3.1) gives rise to our temporary labour demand shock, whereas, a once and for all unit reduction in capital stock generates our permanent labour demand shock. An alternative way to express the above equations is as follows:

$$\begin{aligned}\left(1 - B - \beta_2 B^2\right) E_t &= C_t^E + \beta_3 w_t, \qquad \text{(A3.1')}\\ \left(1 - B - \beta_{10} B^2\right) w_t &= C_t^w, \qquad \text{(A3.2')}\\ \left(1 - B - \beta_{16} B^2\right) L_t &= C_t^L + \beta_{17}(1-B) u_t + (\beta_{18} + \beta_{19} B) w_t, \qquad \text{(A3.3')}\end{aligned}$$

where B is the backshift operator, and

$$\begin{aligned}C_t^E &= \beta_1 + \beta_4 K_t + \beta_5 K_{t-1} + \beta_6 K_{t-2} + \beta_7 \tau_t^I + \beta_8 c_t, \qquad \text{(A3.1")}\\ C_t^w &= \beta_9 + \beta_{11} b_t + \beta_{12} p_t^{oil} + \beta_{13} \tau_t^I + \beta_{14} \tau_{t-1}^I, \qquad \text{(A3.2")}\\ C_t^L &= \beta_{15} + \beta_{20} Z_t. \qquad \text{(A3.3")}\end{aligned}$$

Algebraic manipulation of equations (A3.1')-(A3.3') together with the unemployment rate definition, $u_t = L_t - E_t$, give the *reduced form equation* of the unemployment rate:

$$\begin{aligned}&\left[\left(1 - B - \beta_{16} B^2\right) - \beta_{17}(1-B)\right]\left(1 - B - \beta_2 B^2\right)\left(1 - B - \beta_{10} B^2\right) u_t\\ =\ & -\left(1 - B - \beta_{10} B^2\right)\left(1 - B - \beta_{16} B^2\right) C_t^E\\ &+ \left[(\beta_{18} + \beta_{19} B)\left(1 - B - \beta_2 B^2\right) - \beta_3\left(1 - B - \beta_{16} B^2\right)\right] C_t^w\\ &+ \left(1 - B - \beta_2 B^2\right)\left(1 - B - \beta_{10} B^2\right) C_t^L. \qquad \text{(A3.4)}\end{aligned}$$

The steady-state solution of the above is obtained by setting the backshift operator equal to one, and is given by

$$u_t = \frac{C_t^E}{\beta_2} + \frac{\left[(\beta_{18} + \beta_{19})(-\beta_2) + \beta_3 \beta_{16}\right] C_t^w}{-\beta_2 \beta_{10} \beta_{16}} + \frac{C_t^L}{-\beta_{16}}. \qquad \text{(A3.4')}$$

Further algebraic manipulation of (A3.4) yields:

$$\begin{aligned} u_t &= \phi_1 u_{t-1} + \phi_2 u_{t-2} + \phi_{t-3} u_{t-3} + \phi_4 u_{t-4} + \phi_5 u_{t-5} + \phi_6 u_{t-6} \\ &\quad + \theta_{10} C_t^E + \theta_{11} C_{t-1}^E + \theta_{12} C_{t-2}^E + \theta_{13} C_{t-3}^E + \theta_{14} C_{t-4}^E \\ &\quad + \theta_{20} C_t^w + \theta_{21} C_{t-1}^w + \theta_{22} C_{t-2}^w + \theta_{23} C_{t-3}^w \\ &\quad + \theta_{30} C_t^L + \theta_{31} C_{t-1}^L + \theta_{32} C_{t-2}^L + \theta_{33} C_{t-3}^L + \theta_{34} C_{t-4}^L, \end{aligned} \tag{A3.5}$$

where

$$\begin{aligned} \phi_1 &= 3,\ \phi_2 = -3 + \beta_2 + \beta_{10} + \frac{\beta_{16}}{1-\beta_{17}}, \\ \phi_3 &= 1 - 2(\beta_2 + \beta_{10}) - \frac{2\beta_{16}}{1-\beta_{17}}, \\ \phi_4 &= \beta_2 + \beta_{10} - \beta_2\beta_{10} + \frac{\beta_{16}(1-\beta_2-\beta_{10})}{1-\beta_{17}}, \\ \phi_5 &= \beta_2\beta_{10} + \frac{\beta_{16}(\beta_2+\beta_{10})}{1-\beta_{17}},\ \phi_6 = \frac{\beta_2\beta_{10}\beta_{16}}{1-\beta_{17}}, \\ \theta_{10} &= \frac{-1}{1-\beta_{17}},\ \theta_{11} = \frac{2}{1-\beta_{17}},\ \theta_{12} = \frac{-1+\beta_{16}+\beta_{10}}{1-\beta_{17}}, \\ \theta_{13} &= \frac{-\beta_{16}-\beta_{10}}{1-\beta_{17}},\ \theta_{14} = \frac{-\beta_{16}\beta_{10}}{1-\beta_{17}}, \\ \theta_{20} &= \frac{\beta_{18}-\beta_3}{1-\beta_{17}},\ \theta_{21} = \frac{\beta_3+\beta_{19}-\beta_{18}}{1-\beta_{17}}, \\ \theta_{22} &= \frac{\beta_3\beta_{16}-\beta_{19}-\beta_{18}\beta_2}{1-\beta_{17}},\ \theta_{23} = \frac{-\beta_{19}\beta_2}{1-\beta_{17}}, \\ \theta_{30} &= \frac{1}{1-\beta_{17}},\ \theta_{31} = \frac{-2}{1-\beta_{17}},\ \theta_{32} = \frac{1-\beta_2-\beta_{10}}{1-\beta_{17}}, \\ \theta_{33} &= \frac{\beta_2+\beta_{10}}{1-\beta_{17}},\ \theta_{34} = \frac{\beta_2\beta_{10}}{1-\beta_{17}}. \end{aligned}$$

From equation (A3.5) it is clear that the immediate impact of a temporary (permanent) negative unit labour demand shock on the unemployment rate is $-\theta_{10}\ (-\theta_{10}\beta_4)$, i.e. $m = \frac{1}{1-\beta_{17}} = 0.81\,(m\beta_4 = 3.58)$. Furthermore, the total impact (τ) of a temporary negative unit labour demand shock on the unemployment rate is given by minus the coefficient of C_t^E in equation (A3.4'), i.e. $\tau = \frac{-1}{\beta_2} = 2.98$.[49] Therefore, our measure of quantitative persistence (π^Q) can be obtained by subtracting the immediate impact of the shock from its total impact: $\pi^Q = \tau - m = 2.17$. In addition, the long-run change in the

[49]See Hamilton (1994), ch.1, pp7.

unemployment rate, du^*, due to a permanent negative unit labour demand shock, can be computed as: $du^* = \frac{\beta_4+\beta_5+\beta_6}{-\beta_2} = \tau\left(\beta_4+\beta_5+\beta_6\right) = 0.39$.

In the *absence of the employment adjustment effect* $(\sim EA)$, the reduced form unemployment rate equation is obtained by setting the backshift operator in the $(1-B-\beta_2 B^2)$ expression of eq.(A3.4) equal to one[50]:

$$
\begin{aligned}
(\sim EA) \quad : \quad & -\beta_2\left[\left(1-B-\beta_{16}B^2\right)-\beta_{17}\left(1-B\right)\right]\left(1-B-\beta_{10}B^2\right)u_t \\
= \quad & -\left(1-B-\beta_{10}B^2\right)\left(1-B-\beta_{16}B^2\right)C_t^E \\
& +\left[-\beta_2\left(\beta_{18}+\beta_{19}B\right)-\beta_3\left(1-B-\beta_{16}B^2\right)\right]C_t^w \\
& -\beta_2\left(1-B-\beta_{10}B^2\right)C_t^L. \qquad \text{(A3.6)}
\end{aligned}
$$

Observe that $m_{\sim EA} = \frac{1}{-\beta_2(1-\beta_{17})} = 2.43$, $\pi^Q_{\sim EA} = \tau - m_{\sim EA} = 0.55$.

In the *absence of the wage staggering effect* $(\sim WS)$, the reduced form unemployment rate equation is obtained by setting the backshift operator in the $(1-B-\beta_{10}B^2)$ expression of eq.(A3.4) equal to one:

$$
\begin{aligned}
(\sim WS) \quad : \quad & -\beta_{10}\left[\left(1-B-\beta_{16}B^2\right)-\beta_{17}\left(1-B\right)\right]\left(1-B-\beta_2B^2\right)u_t \\
= \quad & \beta_{10}\left(1-B-\beta_{16}B^2\right)C_t^E \\
& +\left[\left(\beta_{18}+\beta_{19}B\right)\left(1-B-\beta_2B^2\right)-\beta_3\left(1-B-\beta_{16}B^2\right)\right]C_t^w \\
& -\beta_{10}\left(1-B-\beta_2B^2\right)C_t^L. \qquad \text{(A3.7)}
\end{aligned}
$$

Observe that $m_{\sim WS} = m$, and so $\pi^Q_{\sim WS} = \pi$. Since the wage-setting equation (A3.2) does not include, in its right-hand side, any employment or unemployment terms, the wage staggering effect influences neither our measure of quantitative persistence nor the imperfect responsiveness one.

In the *absence of the labour force adjustment effect* $(\sim LF)$, the reduced form unemployment rate equation is obtained by setting the backshift operator in the $(1-B-\beta_{16}B^2)$ expression of eq.(A3.4) equal to one:

$$
\begin{aligned}
(\sim LF) \quad : \quad & \left[-\beta_{16}-\beta_{17}\left(1-B\right)\right]\left(1-B-\beta_2B^2\right)\left(1-B-\beta_{10}B^2\right)u_t \\
= \quad & \beta_{16}\left(1-B-\beta_{10}B^2\right)C_t^E \\
& +\left[\left(\beta_{18}+\beta_{19}B\right)\left(1-B-\beta_2B^2\right)+\beta_3\beta_{16}\right]C_t^w \\
& +\left(1-B-\beta_2B^2\right)\left(1-B-\beta_{10}B^2\right)C_t^L. \qquad \text{(A3.8)}
\end{aligned}
$$

Observe that $m_{\sim LF} = \frac{\beta_{16}}{\beta_{16}+\beta_{17}} = 0.68$, $\pi^Q_{\sim LF} = \tau - m_{\sim LF} = 2.30$.

[50] By setting the backshift operator equal to one $(B=1)$ we preserve the long-run solution of the unemployment rate equation. This implies that the total impact (τ) of a temporary labour demand shock on unemployment, and the long-run unemployment change (du^*) due to a permanent labour demand shock, are not affected by the absence of any of the individual lagged effects.

(II) The estimated **US labour market system** (see Table 3) is

$$\begin{aligned}
E_t &= \beta_1 + \beta_2 E_{t-1} + \beta_3 w_t + \beta_4 K_t + \beta_5 K_{t-1} + \beta_6 K_{t-2} + \beta_7 p_t^{oil} + \beta_8 \tau_t^I, && \text{(A3.9)}\\
\Delta w_t &= \beta_9 + \beta_{10} w_{t-2} + \beta_{11} E_{t-1} + \beta_{12} b_t + \beta_{13} \tau_t^D + \beta_{14} \tau_t^I + \beta_{15} i_t, && \text{(A3.10)}\\
L_t &= \beta_{16} + \beta_{17} L_{t-1} + \beta_{18} u_t + \beta_{19} Z_t && \text{(A3.11)}
\end{aligned}$$

Alternatively, we can write:

$$\begin{aligned}
(1 - \beta_2 B) E_t &= C_t^E + \beta_3 w_t, && \text{(A3.9')}\\
\left(1 - B - \beta_{10} B^2\right) w_t &= C_t^w + \beta_{11} B E_t, && \text{(A3.10')}\\
(1 - \beta_{17} B) L_t &= C_t^L + \beta_{18} u_t, && \text{(A3.11')}
\end{aligned}$$

where

$$\begin{aligned}
C_t^E &= \beta_1 + \beta_4 K_t + \beta_5 K_{t-1} + \beta_6 K_{t-2} + \beta_7 p_t^{oil} + \beta_8 \tau_t^I, && \text{(A3.9")}\\
C_t^w &= \beta_9 + \beta_{12} b_t + \beta_{13} \tau_t^D + \beta_{14} \tau_t^I + \beta_{15} i_t, && \text{(A3.10")}\\
C_t^L &= \beta_{16} + \beta_{19} Z_t. && \text{(A3.11")}
\end{aligned}$$

Algebraic manipulation of equations (A3.9')-(A3.11') together with the unemployment rate definition, $u_t = L_t - E_t$, give the *reduced form equation* of the unemployment rate:

$$\begin{aligned}
& [(1 - \beta_{17} B) - \beta_{18}] \left[(1 - \beta_2 B) \left(1 - B - \beta_{10} B^2\right) - \beta_3 \beta_{11} B\right] u_t \\
= & - \left(1 - B - \beta_{10} B^2\right) (1 - \beta_{17} B) C_t^E - \beta_3 (1 - \beta_{17} B) C_t^w \\
& + \left[(1 - \beta_2 B) \left(1 - B - \beta_{10} B^2\right) - \beta_3 \beta_{11} B\right] C_t^L. \qquad \text{(A3.12)}
\end{aligned}$$

The steady-state solution of eq.(A3.12) is obtained by setting the backshift operator equal to one, and is given by

$$u_t = \frac{\beta_{10} (1 - \beta_{17}) C_t^E - \beta_3 (1 - \beta_{17}) C_t^w}{(1 - \beta_{17} - \beta_{18}) (\beta_2 \beta_{10} - \beta_{10} - \beta_3 \beta_{11})} + \frac{C_t^L}{(1 - \beta_{17} - \beta_{18})}. \qquad \text{(A3.12')}$$

Further algebraic manipulation of (A3.12) yields:

$$\begin{aligned}
u_t = & \ \phi_1 u_{t-1} + \phi_2 u_{t-2} + \phi_{t-3} u_{t-3} + \phi_4 u_{t-4} \\
& + \theta_{10} C_t^E + \theta_{11} C_{t-1}^E + \theta_{12} C_{t-2}^E + \theta_{13} C_{t-3}^E \\
& + \theta_{20} C_t^w + \theta_{21} C_{t-1}^w \\
& + \theta_{30} C_t^L + \theta_{31} C_{t-1}^L + \theta_{32} C_{t-2}^L + \theta_{33} C_{t-3}^L, \qquad \text{(A3.13)}
\end{aligned}$$

where

$$
\begin{aligned}
\phi_1 &= 1+\beta_2+\beta_3\beta_{11}+\frac{\beta_{17}}{1-\beta_{18}}, \\
\phi_2 &= \beta_{10}-\beta_2-\frac{\beta_{17}\left(1+\beta_2+\beta_3\beta_{11}\right)}{1-\beta_{18}}, \\
\phi_3 &= -\beta_2\beta_{10}-\frac{\beta_{17}\left(\beta_{10}-\beta_2\right)}{1-\beta_{18}},\ \phi_4=\frac{\beta_{17}\beta_2\beta_{10}}{1-\beta_{18}}, \\
\theta_{10} &= \frac{-1}{1-\beta_{18}},\ \theta_{11}=\frac{1+\beta_{17}}{1-\beta_{18}},\ \theta_{12}=\frac{\beta_{10}-\beta_{17}}{1-\beta_{18}},\ \theta_{13}=\frac{-\beta_{10}\beta_{17}}{1-\beta_{18}}, \\
\theta_{20} &= \frac{-\beta_3}{1-\beta_{18}},\ \theta_{21}=\frac{\beta_3\beta_{17}}{1-\beta_{18}}, \\
\theta_{30} &= \frac{1}{1-\beta_{18}},\ \theta_{31}=\frac{-\left(1+\beta_2+\beta_3\beta_{11}\right)}{1-\beta_{18}},\ \theta_{32}=\frac{\beta_2-\beta_{10}}{1-\beta_{18}},\ \theta_{33}=\frac{\beta_2\beta_{10}}{1-\beta_{18}}.
\end{aligned}
$$

Inspection of equations (A3.12)-(A3.13) gives: $m=\frac{1}{1-\beta_{18}}=0.85$,

$$
\tau=\frac{-\beta_{10}\left(1-\beta_{17}\right)}{\left(1-\beta_{17}-\beta_{18}\right)\left(\beta_2\beta_{10}-\beta_{10}-\beta_3\beta_{11}\right)}=1.12,
$$

$\pi^Q=\tau-m=0.27$, $du^*=\tau\left(\beta_4+\beta_5+\beta_6\right)=0.24$, and immediate impact of the permanent shock is $m\beta_4=1.02$.

In the *absence of the employment adjustment effect* $(\sim EA)$, the reduced form unemployment rate equation is obtained by setting the backshift operator in the $(1-\beta_2 B)$ expression of eq.(A3.12) equal to one:

$$
\begin{aligned}
(\sim EA) &: \left[(1-\beta_{17}B)-\beta_{18}\right]\left[(1-\beta_2)\left(1-B-\beta_{10}B^2\right)-\beta_3\beta_{11}B\right]u_t \\
&= -\left(1-B-\beta_{10}B^2\right)(1-\beta_{17}B)\,C_t^E-\beta_3\left(1-\beta_{17}B\right)C_t^w \\
&\quad +\left[(1-\beta_2)\left(1-B-\beta_{10}B^2\right)-\beta_3\beta_{11}B\right]C_t^L. \qquad \text{(A3.14)}
\end{aligned}
$$

Observe that $m_{\sim EA}=\frac{1}{(1-\beta_2)(1-\beta_{18})}=0.94$, $\pi^Q_{\sim EA}=\tau-\ m_{\sim EA}=-0.82$.

In the *absence of the wage staggering effect* $(\sim WS)$, the reduced form unemployment rate equation is obtained by setting the backshift operator in the $(1-B-\beta_{10}B^2)$ expression of eq.(A3.12) equal to one:

$$
\begin{aligned}
(\sim WS) &: \left[(1-\beta_{17}B)-\beta_{18}\right]\left[-\beta_{10}\left(1-\beta_2 B\right)-\beta_3\beta_{11}B\right]u_t \\
&= +\beta_{10}\left(1-\beta_{17}B\right)C_t^E-\beta_3\left(1-\beta_{17}B\right)C_t^w \\
&\quad +\left[-\beta_{10}\left(1-\beta_2 B\right)-\beta_3\beta_{11}B\right]C_t^L. \qquad \text{(A3.15)}
\end{aligned}
$$

Observe that $m_{\sim WS}=m=\frac{1}{1-\beta_{18}}$, so $\pi^Q_{\sim WS}=\pi$.

In the *absence of the insider membership effect* $(\sim IM)$, the reduced form unemployment rate equation is obtained by setting the backshift operator in the $\beta_{11}B$ expression of eq.(A3.12) equal to one:

$$\begin{aligned}(\sim IM) \quad &: \quad [(1-\beta_{17}B)-\beta_{18}]\left[(1-\beta_2 B)\left(1-B-\beta_{10}B^2\right)-\beta_3\beta_{11}\right]u_t \\ &= \quad -\left(1-B-\beta_{10}B^2\right)(1-\beta_{17}B)\,C_t^E-\beta_3\,(1-\beta_{17}B)\,C_t^w \\ &\quad +\left[(1-\beta_2 B)\left(1-B-\beta_{10}B^2\right)-\beta_3\beta_{11}\right]C_t^L. \qquad \text{(A3.16)}\end{aligned}$$

Observe that $m_{\sim IM}=\frac{1}{(1-\beta_3\beta_{11})(1-\beta_{18})}=0.81$, $\pi^Q_{\sim IM}=\tau-\ m_{\sim IM}=0.31$.

In the *absence of the labour force adjustment effect* $(\sim LF)$, the reduced form unemployment rate equation is obtained by setting the backshift operator in the $(1-\beta_{17}B)$ expression of eq.(A3.12) equal to one:

$$\begin{aligned}(\sim LF) \quad &: \quad [(1-\beta_{17})-\beta_{18}]\left[(1-\beta_2 B)\left(1-B-\beta_{10}B^2\right)-\beta_3\beta_{11}B\right]u_t \\ &= \quad -\left(1-B-\beta_{10}B^2\right)(1-\beta_{17})\,C_t^E-\beta_3\,(1-\beta_{17})\,C_t^w \\ &\quad +\left[(1-\beta_2 B)\left(1-B-\beta_{10}B^2\right)-\beta_3\beta_{11}B\right]C_t^L. \qquad \text{(A3.17)}\end{aligned}$$

Observe that $m_{\sim LF}=\frac{1-\beta_{17}}{1-\beta_{17}-\beta_{18}}=0.67$, $\pi^Q_{\sim LF}=\tau-\ m_{\sim LF}=0.45$.

(III) The estimated **German labour market system** (see Table 5) is

$$\begin{aligned}\Delta E_t &= \beta_1+\beta_2 E_{t-2}+\beta_3 w_t+\beta_4 K_t+\beta_5 K_{t-1}+\beta_6 K_{t-2}+\beta_7\Delta r_t, && \text{(A3.18)}\\ w_t &= \beta_8+\beta_9 w_{t-1}+\beta_{10}u_t+\beta_{11}u_{t-1}+\beta_{12}E_{t-1}+\beta_{13}c_t, && \text{(A3.19)}\\ \Delta L_t &= \beta_{14}+\beta_{15}L_{t-2}+\beta_{16}u_t+\beta_{17}w_{t-1}+\beta_{18}Z_t+\beta_{19}Z_{t-1}+\beta_{20}Z_{t-2}. && \text{(A3.20)}\end{aligned}$$

Alternatively, we can write:

$$\begin{aligned}\left(1-B-\beta_2 B^2\right)E_t &= C_t^E+\beta_3 w_t, && \text{(A3.18')}\\ (1-\beta_9 B)\,w_t &= C_t^w+(\beta_{10}+\beta_{11}B)\,u_t+\beta_{12}BE_t, && \text{(A3.19')}\\ \left(1-B-\beta_{15}B^2\right)L_t &= C_t^L+\beta_{16}u_t+\beta_{17}Bw_t, && \text{(A3.20')}\end{aligned}$$

where

$$\begin{aligned}C_t^E &= \beta_1+\beta_4 K_t+\beta_5 K_{t-1}+\beta_6 K_{t-2}+\beta_7\Delta r_t, && \text{(A3.18")}\\ C_t^w &= \beta_8+\beta_{13}c_t, && \text{(A3.19")}\\ C_t^L &= \beta_{14}+\beta_{18}Z_t+\beta_{19}Z_{t-1}+\beta_{20}Z_{t-2}. && \text{(A3.20")}\end{aligned}$$

Algebraic manipulation of equations (A3.18')-(A3.20') together with the unemployment rate definition, $u_t=L_t-E_t$, give the *reduced form equation* of the unemployment rate:

$$
\begin{aligned}
&\left\{ \begin{array}{l} \left[(1-B-\beta_{15}B^2)-\beta_{16}\right]\left[(1-\beta_9 B)\left(1-B-\beta_2 B^2\right)-\beta_3\beta_{12}B\right] \\ +\left(\beta_{10}+\beta_{11}B\right)\left[\beta_3\left(1-B-\beta_{15}B^2\right)-\beta_{17}B\left(1-B-\beta_2 B^2\right)\right] \end{array} \right\} u_t \\
= \;& \left[\beta_{12}B\beta_{17}B-\left(1-B-\beta_{15}B^2\right)\left(1-\beta_9 B\right)\right] C_t^E \\
& -\left[\beta_3\left(1-B-\beta_{15}B^2\right)-\beta_{17}B\left(1-B-\beta_2 B^2\right)\right] C_t^w \\
& +\left[\left(1-\beta_9 B\right)\left(1-B-\beta_2 B^2\right)-\beta_3\beta_{12}B\right] C_t^L. \qquad \text{(A3.21)}
\end{aligned}
$$

The steady-state solution of the above equation is obtained by setting $B = 1$, and is given by

$$
u_t = \frac{\left[\beta_{12}\beta_{17}+\beta_{15}(1-\beta_9)\right]C_t^E+(\beta_3\beta_{15}+\beta_2\beta_{17})C_t^w-\left[\beta_2(1-\beta_9)+\beta_3\beta_{12}\right]C_t^L}{(\beta_{15}+\beta_{16})\left[\beta_2(1-\beta_9)+\beta_3\beta_{12}\right]+(\beta_{10}+\beta_{11})(\beta_2\beta_{17}-\beta_3\beta_{15})} \qquad \text{(A3.21')}
$$

Further algebraic manipulation of (A3.21) gives

$$
\begin{aligned}
u_t \;=\; & \phi_1 u_{t-1}+\phi_2 u_{t-2}+\phi_{t-3}u_{t-3}+\phi_4 u_{t-4}+\phi_5 u_{t-5} \\
& +\Theta_1(B)\,C_t^E+\Theta_2(B)\,C_t^w+\Theta_3(B)\,C_t^L, \qquad \text{(A3.22)}
\end{aligned}
$$

where

$$
\begin{aligned}
\phi_1 &= \frac{2+\beta_9+\beta_3\beta_{12}+\beta_{10}\beta_3-\beta_{11}\beta_3+\beta_{10}\beta_{17}-\beta_{16}\left(1+\beta_9+\beta_3\beta_{12}\right)}{1+\beta_{10}\beta_3-\beta_{16}}, \\
\phi_2 &= \frac{\beta_{10}\beta_3\beta_{15}+\beta_{11}\beta_3-\beta_{16}\left(\beta_2-\beta_9\right)+\beta_{11}\beta_{17}}{1+\beta_{10}\beta_3-\beta_{16}} \\
&\quad +\frac{\beta_2+\beta_{15}-\beta_3\beta_{12}-\beta_{10}\beta_{17}-1-2\beta_9}{1+\beta_{10}\beta_3-\beta_{16}}, \\
\phi_3 &= \frac{\beta_{11}\beta_3\beta_{15}+\beta_9-\beta_{10}\beta_{17}\beta_2-\beta_{11}\beta_{17}-\beta_9\beta_2}{1+\beta_{10}\beta_3-\beta_{16}} \\
&\quad +\frac{-\beta_2-\beta_{15}\beta_3\beta_{12}-\beta_{15}+\beta_{16}\beta_9\beta_2-\beta_{15}\beta_9}{1+\beta_{10}\beta_3-\beta_{16}}, \\
\phi_4 &= \frac{\beta_9\beta_2-\beta_{15}\left(\beta_2-\beta_9\right)-\beta_{11}\beta_{17}\beta_2}{1+\beta_{10}\beta_3-\beta_{16}},\; \phi_5 = \frac{\beta_2\beta_9\beta_{15}}{1+\beta_{10}\beta_3-\beta_{16}}, \\
\Theta_1(B) &= \frac{\left[-1+\left(1+\beta_9\right)B+\left(\beta_{15}+\beta_{17}\beta_{12}-\beta_9\right)B^2-\beta_9\beta_{15}B^3\right]}{1+\beta_{10}\beta_3-\beta_{16}}, \\
\Theta_2(B) &= \frac{\left[-\beta_3+\left(\beta_3+\beta_{17}\right)B+\left(\beta_3\beta_{15}-\beta_{17}\right)B^2-\beta_{17}\beta_2 B^3\right]}{1+\beta_{10}\beta_3-\beta_{16}}, \\
\Theta_3(B) &= \frac{\left[1-\left(1+\beta_3\beta_{12}+\beta_9\right)B-\left(\beta_2-\beta_9\right)B^2+\beta_9\beta_2 B^3\right]}{1+\beta_{10}\beta_3-\beta_{16}}.
\end{aligned}
$$

Inspection of eq.(A3.21') gives

$$
du^* = \frac{-\left[\beta_{12}\beta_{17}+\beta_{15}(1-\beta_9)\right](\beta_4+\beta_5+\beta_6)}{(\beta_{15}+\beta_{16})\left[\beta_2(1-\beta_9)+\beta_3\beta_{12}\right]+(\beta_{10}+\beta_{11})(\beta_2\beta_{17}-\beta_3\beta_{15})} = 0.21.
$$

From eq.(A3.21) we can see that the immediate impact of the permanent shock on unemployment is equal to $\frac{\beta_4}{1+\beta_{10}\beta_3-\beta_{16}} = 1.61$. In contrast to the UK and US models, the German one is characterised by cross equation correlation. In this case, as it was explained in Section *, when we compute the unemployment persistence measures we assume that $C_t^w = b_1 C_t^E$ and $C_t^L = b_2 C_t^E + b_3 C_t^w$, where $b_1 = -0.72$, $b_2 = 0.28$, $b_3 = 0.14$ [51]. Therefore, we have that: $m = \frac{1+\beta_3 b_1 - b_2 - b_1 b_3}{1+\beta_{10}\beta_3-\beta_{16}} = 0.64$, and

$$\tau = -\frac{[\beta_{12}\beta_{17}+\beta_{15}(1-\beta_9)]+(\beta_3\beta_{15}+\beta_2\beta_{17})b_1-[\beta_2(1-\beta_9)+\beta_3\beta_{12}](b_2+b_1b_3)}{(\beta_{15}+\beta_{16})[\beta_2(1-\beta_9)+\beta_3\beta_{12}]+(\beta_{10}+\beta_{11})(\beta_2\beta_{17}-\beta_3\beta_{15})} = 1.36.$$

In the *absence of the employment adjustment effect* $(\sim EA)$, the reduced form unemployment rate equation is obtained by setting the backshift operator in the $(1 - B - \beta_2 B^2)$ expression of eq.(A3.21) equal to one:

$$\begin{aligned} (\sim EA) \quad : \quad & \left\{ \begin{array}{l} [(1 - B - \beta_{15}B^2) - \beta_{16}]\,[-\beta_2\,(1 - \beta_9 B) - \beta_3\beta_{12}B] \\ +\,(\beta_{10} + \beta_{11}B)\,[\beta_3\,(1 - B - \beta_{15}B^2) - \beta_{17}B\,(1 - B - \beta_2 B^2)] \end{array} \right\} u_t \\ = \quad & \left[\beta_{12}B\beta_{17}B - \left(1 - B - \beta_{15}B^2\right)(1 - \beta_9 B)\right] C_t^E \\ & - \left[\beta_3\left(1 - B - \beta_{15}B^2\right) + \beta_2\beta_{17}B\right] C_t^w \\ & + [-\beta_2\,(1 - \beta_9 B) - \beta_3\beta_{12}B]\, C_t^L. \end{aligned} \tag{A3.23}$$

Note that $m_{\sim EA} = \frac{1+\beta_3 b_1+\beta_2(b_2+b_1b_3)}{-\beta_2(1-\beta_{16})+\beta_3\beta_{10}} = 1.70$, $\pi^Q_{\sim EA} = \tau - m_{\sim EA} = -0.34$.

In the *absence of the wage staggering effect* $(\sim WS)$, the reduced form unemployment rate equation is obtained by setting the backshift operator in the $(1 - \beta_9 B)$ expression of eq.(A3.21) equal to one:

$$\begin{aligned} (\sim WS) \quad : \quad & \left\{ \begin{array}{l} [(1 - B - \beta_{15}B^2) - \beta_{16}]\,[(1 - \beta_9)\,(1 - B - \beta_2 B^2) - \beta_3\beta_{12}B] \\ +\,(\beta_{10} + \beta_{11}B)\,[\beta_3\,(1 - B - \beta_{15}B^2) - \beta_{17}B\,(1 - B - \beta_2 B^2)] \end{array} \right\} u_t \\ = \quad & \left[\beta_{12}B\beta_{17}B - \left(1 - B - \beta_{15}B^2\right)(1 - \beta_9)\right] C_t^E \\ & - \left[\beta_3\left(1 - B - \beta_{15}B^2\right) - \beta_{17}B\left(1 - B - \beta_2 B^2\right)\right] C_t^w \\ & + \left[(1 - \beta_9)\left(1 - B - \beta_2 B^2\right) - \beta_3\beta_{12}B\right] C_t^L. \end{aligned} \tag{A3.24}$$

Observe that $m_{\sim WS} = \frac{(1-\beta_9)+\beta_3 b_1-(1-\beta_9)(b_2+b_1b_3)}{(1-\beta_9)(1-\beta_{16})+\beta_3\beta_{10}} = 0.65$, so $\pi^Q_{\sim WS} = \tau - m_{\sim WS} = 0.71$.

In the *absence of the long-term unemployment effect* $(\sim LU)$, the reduced form unemployment rate equation is obtained by setting the backshift

[51] Recall that b_1 is the coefficient obtained by regressing the residuals of (A3.19) on those of (A3.18); to obtain b_2, b_3 we regress the residuals of (A3.20) on the ones of (A3.18) and (A3.19).

operator in the $\beta_{11}B$ expression of eq.(A3.21) equal to one:

$$\begin{aligned}
(\sim LU) \quad &: \quad \left\{ \begin{array}{l} [(1-B-\beta_{15}B^2)-\beta_{16}]\,[(1-\beta_9 B)\,(1-B-\beta_2 B^2)-\beta_3\beta_{12}B] \\ +(\beta_{10}+\beta_{11})\,[\beta_3\,(1-B-\beta_{15}B^2)-\beta_{17}B\,(1-B-\beta_2 B^2)] \end{array} \right\} u_t \\
&= \left[\beta_{12}B\beta_{17}B-\left(1-B-\beta_{15}B^2\right)(1-\beta_9 B)\right] C_t^E \\
&\quad -\left[\beta_3\left(1-B-\beta_{15}B^2\right)-\beta_{17}B\left(1-B-\beta_2 B^2\right)\right] C_t^w \\
&\quad +\left[(1-\beta_9 B)\left(1-B-\beta_2 B^2\right)-\beta_3\beta_{12}B\right] C_t^L. \qquad \text{(A3.25)}
\end{aligned}$$

Observe that $m_{\sim LU}=\frac{1+\beta_3 b_1-b_2-b_1 b_3}{1-\beta_{16}+\beta_3(\beta_{10}+\beta_{11})}=0.69$, so $\pi^Q_{\sim WS}=\tau-m_{\sim LU}=0.67$.

In the *absence of the insider membership effect* $(\sim IM)$, the reduced form unemployment rate equation is obtained by setting the backshift operator in the $\beta_{12}B$ expression of eq.(A3.21) equal to one:

$$\begin{aligned}
(\sim IM) \quad &: \quad \left\{ \begin{array}{l} [(1-B-\beta_{15}B^2)-\beta_{16}]\,[(1-\beta_9 B)\,(1-B-\beta_2 B^2)-\beta_3\beta_{12}] \\ +(\beta_{10}+\beta_{11}B)\,[\beta_3\,(1-B-\beta_{15}B^2)-\beta_{17}B\,(1-B-\beta_2 B^2)] \end{array} \right\} u_t \\
&= \left[\beta_{12}\beta_{17}B-\left(1-B-\beta_{15}B^2\right)(1-\beta_9 B)\right] C_t^E \\
&\quad -\left[\beta_3\left(1-B-\beta_{15}B^2\right)-\beta_{17}B\left(1-B-\beta_2 B^2\right)\right] C_t^w \\
&\quad +\left[(1-\beta_9 B)\left(1-B-\beta_2 B^2\right)-\beta_3\beta_{12}\right] C_t^L. \qquad \text{(A3.26)}
\end{aligned}$$

Observe that $m_{\sim IM}=\frac{1+\beta_3 b_1-(1-\beta_3\beta_{12})(b_2+b_3 b_1)}{(1-\beta_3\beta_{12})(1-\beta_{16})+\beta_3\beta_{10}}=0.62$, $\pi^Q_{\sim IM}=\tau-\ m_{\sim IM}=0.74$.

In the *absence of the labour force adjustment effect* $(\sim LF)$, the reduced form unemployment rate equation is obtained by setting the backshift operator in the $(1-B-\beta_{15}B^2)$ expression of eq.(A3.21) equal to one:

$$\begin{aligned}
(\sim LF) \quad &: \quad \left\{ \begin{array}{l} [-\beta_{15}-\beta_{16}]\,[(1-\beta_9 B)\,(1-B-\beta_2 B^2)-\beta_3\beta_{12}B] \\ +(\beta_{10}+\beta_{11}B)\,[-\beta_3\beta_{15}-\beta_{17}B\,(1-B-\beta_2 B^2)] \end{array} \right\} u_t \\
&= \left[\beta_{12}B\beta_{17}B+\beta_{15}\,(1-\beta_9 B)\right] C_t^E \\
&\quad -\left[-\beta_3\beta_{15}-\beta_{17}B\left(1-B-\beta_2 B^2\right)\right] C_t^w \\
&\quad +\left[(1-\beta_9 B)\left(1-B-\beta_2 B^2\right)-\beta_3\beta_{12}B\right] C_t^L. \qquad \text{(A3.27)}
\end{aligned}$$

Note that $m_{\sim LF}=\frac{-\beta_{15}-\beta_3\beta_{15}b_1-b_2-b_3 b_1}{1-\beta_{16}-\beta_3\beta_{10}\beta_{15}}=0.42$, $\pi^Q_{\sim LF}=\tau-\ m_{\sim LF}=0.94$.

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