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Growth Effects of Consumption Jealousy in a Two-Sector Model

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Founded in 1963 by two prominent Austrians living in exile – the sociologist Paul F. Lazarsfeld and the economist Oskar Morgenstern – with the financial support from the Ford Foundation, the Austrian Federal Ministry of Education and the City of Vienna, the Institute for Advanced Studies (IHS) is the first institution for postgraduate education and research in economics and the social sciences in Austria. The **Economics Series** presents research done at the Department of Economics and Finance and aims to share “work in progress” in a timely way before formal publication. As usual, authors bear full responsibility for the content of their contributions.

Das Institut für Höhere Studien (IHS) wurde im Jahr 1963 von zwei prominenten Exilösterreichern – dem Soziologen Paul F. Lazarsfeld und dem Ökonomen Oskar Morgenstern – mit Hilfe der Ford-Stiftung, des Österreichischen Bundesministeriums für Unterricht und der Stadt Wien gegründet und ist somit die erste nachuniversitäre Lehr- und Forschungsstätte für die Sozial- und Wirtschafts-wissenschaften in Österreich. Die **Reihe Ökonomie** bietet Einblick in die Forschungsarbeit der Abteilung für Ökonomie und Finanzwirtschaft und verfolgt das Ziel, abteilungsinterne Diskussionsbeiträge einer breiteren fachinternen Öffentlichkeit zugänglich zu machen. Die inhaltliche Verantwortung für die veröffentlichten Beiträge liegt bei den Autoren und Autorinnen.

Abstract

This paper aims at analyzing the implications of individuals' consumption jealousy on the dynamic structure of a two-sector model economy. We find that status-seeking substantially influences both, the long-term properties and the adjustment behavior of the model. Depending on the status motive, productivity disturbances might induce countercyclical responses of work effort whereas preference shocks are expected to generate an overshooting relative capital intensity. Generally we find that, for empirically plausible values of the intertemporal elasticity of substitution, a higher degree of consumption jealousy induces agents to devote more time to education which stimulates human capital accumulation and hence promotes economic growth.

Keywords

Status-seeking, economic growth, transitional dynamics, human capital

JEL Classification

D91, E21, O41

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1 Introduction

Time-separable utility representations have a long pedigree in economics and can be found at the core of most macroeconomic settings in recent work. Typically, an agent's level of satisfaction is assumed to depend primarily on individual variables, like current consumption, leisure, or real money balances. There is, however, persuasive empirical evidence, some of which is provided by Oswald (1997), Frank (1997) and Fuhrer (2000) indicating that individual utility is to a large extent determined by comparing current variables like consumption or wealth to some specific standard, which is usually a given reference stock. This reference stock is typically assumed to comprise some measure of current and/or past economy-wide average consumption levels, which makes the utility representation time non-separable. The presence of this comparison element suggests that individuals derive utility from relative rather than absolute consumption. Hence, they are concerned with their relative position in society, which is also referred to as their status. Generally, this type of preferences has been labeled as "outward-looking" since agents base their decisions on an external comparison criterion[1]. The key issue of this paper addresses the implications of status seeking for the process of economic growth. The goal then, is to discuss whether - and if yes, to what extent - status, modeled as relative consumption exhibits a perceivable impact on the model's equilibrium and transitional dynamics.

The idea that individuals' choices are to some degree motivated by social rewards (including status) is in fact not a new one, but can be rather traced back to great thinkers such as Adam Smith (1776), Thorstein Veblen (1899) and David Hume (1978), among others. Nevertheless, a formal integration of this concept into economics was not until the work of Duesenberry (1949), who defined status as the ratio of an individual's own consumption to average consumption of the others. Even though, time separability still plays a dominant role in economic modeling, there have been recent efforts to incorporate comparison utility into dynamic macro-settings. For instance, in the field of asset pricing this concept has been invoked to explain certain asset pricing anomalies, such as the well-known equity premium puzzle raised by Mehra and Prescott (1985). Notable attempts in this regard have been made, for instance, by Constantinides (1990), Abel (1990) and Campbell and Cochrane (1999)[2]. Status orientation has also been successfully adopted to address issues of taxation. Ljungquist and Uhlig (2000) study the implications of a "catching up" preference structure on short-run macroeconomic stabilization policies[3]. Liu and Turnovsky (2005) study the effects of consumption and production externalities on capital

[1] "Outward-looking" preferences are frequently referred to as "Keeping/Catching up with the Joneses", "interdependent utility" or "external habit formation". There exists an alternative specification, which the literature has termed "habit formation", postulating an internal criterion as a comparison benchmark. In contrast to the "outward-looking" case, the reference stock here depends on the agent's own consumption history rather than the economy's average.

[2] Constantinides (1990) employs a model with habit forming agents to address and resolve the premium puzzle, while Campbell and Cochrane (1999) analyze the effects of consumption externalities, induced by outward-looking preferences, on asset pricing and equity premiums. Galí (1994) argues that "keeping up" has the same effect as increasing the degree of risk aversion. This is due to the fact that consumers with these preferences dislike large swings in consumption, hence the premium paid for holding risky assets must be high relative to time separable preferences.

[3] They find that the optimal tax policy affects the economy countercyclically via procyclical taxes and, hence, mitigates aggregate demand fluctuations.

accumulation and welfare[4]. Comparison utility need not necessarily be defined over relative consumption. Various studies in the recent literature argue that it can be rather relative wealth that determines individuals status seeking. In the spirit of this approach Van Long and Shimomura (2004) study the influence of wealth-inequality in a heterogeneous agents framework[5]. Recently relative wealth has also been studied by various authors in the context of open economy settings. Most notably, Fisher (2004) and Fisher and Hof (2005) discuss an open economy Ramsey model that has been augmented by relative holdings of net foreign assets. They find that making allowance for status seeking can eliminate some crucial counterfactual properties of the conventional Ramsey model under time-separable utility[6].

As mentioned at the outset, the time non-separable utility representation is strongly supported by empirical findings. Early studies by van de Stadt et al. (1985) confirm the hypotheses that an individual's well-being depends on her relative position in society. Using panel data for the Netherlands they test for in- and outward-looking elements in individual utility. However, van de Stadt et al. (1985) cannot reject the existence of absolute and relative components. Fuhrer (2000) clearly rejects the time separable utility specification. He finds that both current average consumption and an internal benchmark determine utility, where approximately 80% of the utility weight should be put on the latter. Moreover, Fuhrer and Klein (1998) argue that for G7 countries habit formation is a significant characteristic of people's consumption behavior. Interestingly, using data on British workers, Clark and Oswald (1996) provide empirical evidence that the level of individuals' satisfaction is inversely related to comparison wage rates. Taking into account that wages, or, more precisely, income, is directly related to the consumption level of an individual, these results - along with the work carried out by Oswald (1997), Frank (1997) and Neumark and Postlewaite (1998) - are strongly in line with the theoretical predictions of the comparison consumption framework. The existing empirical evidence is admittedly sparse, but it nevertheless provides convincing support for the relevance of comparison elements in determining individual utility.

So far, relatively few papers have focused on the role status might play for economic growth. Aside from early efforts by Ryder and Heal (1973) who incorporate habit formation into a neoclassical growth model it was not until the late 1990s that economists started to discuss status preference in the context of growth settings. Carroll et al. (1997) introduce two types of time non-separable elements - an internal and an external criterion - in a simple AK-framework and contrast the respective implications of each element on the model's transitional dynamics[7]. A similar attempt has been made by Alvarez-Cuadrado et al. (2004), who discuss how alterna-

[4]The consumption externalities in their paper are again due to a preference structure that includes comparison elements. They derive an optimal tax structure that corrects for the distortions created by these externalities.

[5]They find that the initially poor might catch up with the rich if the marginal utility of relative wealth exceeds the elasticity of marginal utility of consumption. However, when people don't care about their status, then the inequality will persist in the long run.

[6]Particularly the fact that an impatient economy - in the sense that the personal discount rate exceeds the world interest rate - mortgages all its wealth over time. They identified an endogenous effective rate of return that hinges on both own consumption and the net assets and ensures the existence of a long-run interior equilibrium, even if the discount rate exceeds the world interest rate.

[7]Observe that the conventional AK growth model exhibits no transitional dynamics. However, a modified preference structure that accounts for time non-separability alters this property.

tive assumptions about preferences affect the process of economic growth. They compare three different regimes, i.e. time separable, inward- and outward looking preferences and find that the departure from time-separability substantially alters the dynamic structure of the model. Incorporating the concept of relative wealth in an endogenous growth setting, Futagami and Shibata (1998) find that if individuals are identical, higher status aspiration leads to higher growth whereas individuals' heterogeneity might reduce growth through an induced misallocation of resources. Corneo and Jeanne (1997) assert that status orientation may lead to excessive growth resulting from an overaccumulation of physical capital, but it can also induce the social optimal rate of growth if the status aspiration is sufficiently strong. In a subsequent paper Corneo and Jeanne (2001) find that relative wealth incorporated into a neoclassical growth model - one that typically exhibits zero steady-state per capita growth - appears to be the engine of positive equilibrium growth. The vast majority of this work is primarily concerned with preference and demand-related issues hence the production side is rather oversimplified. But especially when addressing the dynamic process of economic growth one should be aware that - due to their one-sector design - AK or neoclassical modeling devices are rather restrictive.

The theoretical framework we employ accounts for the fundamental role human capital plays in explaining long-term growth of modern economies. Hence, in the spirit of Robert Lucas (1988) we choose a two sector production specification in which agents are assumed, first, to exhibit a certain degree of consumption jealousy. In other words, we assume that agents posses preferences over their relative position in society. Second, individuals allocate their disposable time across the two production sectors to maximize their utility. This setting explicitly allows us to address a variety of issues that most of the existing literature fails to explain. For example, we can investigate the implications of consumers' status seeking on the intersectoral allocation of productive resources, i.e. raw time, physical and human capital. Second to study how consumption jealousy affects the model's transitional and equilibrium dynamics. This includes a careful discussion about the extent to which varying degrees of envy affect the adjustment behavior and the equilibrium growth performance. For the purpose of analyzing the dynamics, we consider specific productivity and preference shocks and numerically simulate adjustment paths of some selected key variables. Finally, we establish necessary and sufficient conditions that ensure the existence and the uniqueness of an interior equilibrium and show that these depend on the status-indicating parameters of the model.

Before closing this section we wish to stress some key-results of this analysis. First and foremost, we find that consumption jealousy substantially alters the dynamic structure of the economy. Both in the balanced-growth equilibrium and during the transition the envy motive exhibits a non-negligible impact on the model's key variables. For the equilibrium growth rate we obtain ambiguous effects of status seeking that mainly depend on the intertemporal elasticity of substitution. For parameter values that are in close accord with empirical estimates we find that productivity shocks in the final goods sector might induce (counter-)cyclical behavior of labor allocation resulting from an interplay of intersectoral reallocation and catching up efforts. Preference shocks, in contrast, entail a monotonous response of labor but cause an overshooting long-run capital intensity. This can be traced back to interdependencies of temporary pro-

ductivity imbalances across sectors and the average product of physical capital, in response to that shock. Generally we can state that a higher degree of status seeking tends to increase the schooling efforts and hence stimulates economic growth. Moreover, there exists a unique interior balanced growth path equilibrium whose properties depend to a large extent on the parameters that determine the envy motive.

The remainder of the paper is organized as follows. In Section 2 we lay out the basic structure of the theoretical model introducing a time non-separable preference representation. Section 3 derives the equilibrium conditions and discusses the necessary and sufficient conditions for existence and uniqueness of a balanced growth equilibrium. This section also contains a stability and sensitivity analysis that underlies the local saddle path stability of the model. Section 4 studies the off-equilibrium dynamics of the economy which is mainly done by numerical simulation. Section 5 concludes and discusses possible extensions.

2 The Model

2.1 Preferences

As a basic framework we consider a two-sector model economy populated by a continuum $[0,1]$ of identical atomistic individuals with unbounded horizon. Let agent i's utility at each point in time depend on the comparison of her own consumption, $c_i(t)$, to a certain reference stock which in this model is determined by the average level of consumption in the economy, $\tilde{c}(t) = \int_0^1 c_i(t)di$. In particular, preferences of agent i are characterized by a C^2 utility function $U\left(c_i(t), \tilde{c}(t)\right)$ satisfying the standard concavity and limiting-behavior properties in c, i.e. $U_c\left(\cdot\right) > 0$, $U_{cc}\left(\cdot\right) < 0$ and $\lim_{c\to 0} U_c\left(\cdot\right) = +\infty$, $\lim_{c\to\infty} U_c\left(\cdot\right) = 0$.

To be more concrete, the functional form of $U\left(c_i(t), \tilde{c}(t)\right)$ in this paper is postulated to be isoelastic which will allow us to derive explicit results.

$$U\left(c_i(t), \tilde{c}\right) = (1-\sigma)^{-1}\left(c_i(t)(\kappa\int_0^t e^{-\kappa(t-s)}\tilde{c}(s)ds)^{-(1-\beta)}\right)^{1-\sigma} \tag{1}$$

Each individual discounts the utility of her future consumption at a constant exogenously given rate $\rho \geq 0$ and let $\sigma > 0$ denote the inverse of the intertemporal elasticity of substitution[8]. Furthermore, in accordance with much of the existing literature on status preferences, it is assumed that individuals, in general, envy their neighbors consumption, hence $U_{\tilde{c}}\left(\cdot\right) < 0$ and $U_{c\tilde{c}}\left(\cdot\right) > 0$[9]. The latter condition implies that an agent values an additional amount of own consumption more, the higher the average level of consumption in the economy is. The reference stock, call it $x(t)$, is composed of the weighted sum of current and past average consumption levels, i.e. $x(t) = \kappa\int_0^t e^{-\kappa(t-s)}\tilde{c}(s)ds$ where $\kappa \in [0,\infty)$ indexes the relative importance of recent compared to past average consumption. A more intuitive interpretation of κ can be deduced

[8]Similar versions of (1) have been recently used, for instance by Alvarez-Cuadrado et al. (2004) and Carroll et al. (1997).

[9]The case of admiration that is characterized by $U_{\tilde{c}}\left(\cdot\right) > 0$ is considered by Dupor and Liu (2003) and Liu and Turnovsky (2005)

from the dynamic representation of $x(t)$. Using Leibnitz' rule we get $\dot{x}(t) = \kappa\left(\tilde{c}(t) - x(t)\right)$[10] from which we can infer that κ drives the speed of adjustment of the reference stock. For high values of κ adjustment is rather rapid implying that $x(t)$ is close to $\tilde{c}(t)$ with $\lim_{\kappa\to\infty} x(t) = \tilde{c}(t)$. In this case individuals preferences are more presence-orientated. On the other hand, extremely low values of κ lead to sluggish adjustment and hence $x(t)$ hardly varies. Moreover, it should be mentioned that the two polar cases, i.e. $\kappa \to 0, \infty$, nest important classes of economic models. First, for sufficiently high values of κ we have $x(t) \approx \tilde{c}(t)$ and, as a consequence, all that matters for an agent's utility is the current average consumption level in the economy. For this type of preference structure the recent literature has coined the expression "Keeping up with the Joneses" [11]. Second, $\kappa \to 0$, hence $x(t) \approx x(0),\ \ \forall t \in (0, \infty)$, implies that the reference stock is irrelevant to an agent's utility and as a result $U(\cdot)$ collapses to the standard time-separable utility representation. The cases implied by values of κ lying in between these extremal points, often called "Catching up with the Joneses", will be the main focus of this paper. The importance of comparison utility for an agent's well-being is governed by the parameter $\beta \in [0, 1]$. For $\beta = 0$ comparison utility is all that matters while if $\beta = 1$ individuals only care about their own absolute consumption. In case of $\beta \in (0, 1)$ both are attached with strictly positive utility.
Due to the fact that the reference stock is solely determined by $\tilde{c}(t)$, preferences are by definition outward-looking[12]. Moreover, individuals take the path of $x(t)$ as given when making their allocative decisions. This inevitably creates a negative externality since agents do not take into account the effects of their own choices on the current and future reference stock of the others.

2.2 Technology and Accumulation

The model economy considered in this paper comprises two sectors: a final goods sector producing a single, homogeneous, non-storable consumption good and an education sector creating additional human capital. Each agent is endowed with one unit of time. Let $\chi_{y,i}$ and $\chi_{h,i}$ denote the fraction of time individual i devotes to work in the goods sector and uses for educational purposes, respectively. For sake of simplicity leisure is not considered in this model, hence notation can be simplified by writing $\chi_{y,i} = \chi_i$ and $\chi_{h,i} = (1 - \chi_i)$. In each point in time an individual's final output $y_i(t)$ is determined by the stock of her accumulated physical, $k_i(t)$, and human capital, $h_i(t)$, and the level of raw labor according to a constant returns to scale technology[13]

$$y_i(t) = \phi k_i(t)^{\alpha} \left(\chi_i(t) h_i(t)\right)^{1-\alpha}, \tag{2}$$

[10] In this paper we denote $\dot{q}(t) \equiv \frac{\partial q(t)}{\partial t}$

[11] This specification of agents' preferences has found widespread use, for instance, in work concerned with asset pricing, (see Galí (1994)), taxation, (see Ljungquist and Uhlig (2000)), and open economy issues, (see Fisher (2004) and Fisher and Hof (2005)), to mention a few. Dupor and Liu (2003) go a step further and distinguish between "Jealousy" and Keeping up with the Joneses. For an increase in aggregate consumption the first is associated with a lower level of individuals utility whereas in the latter environment the marginal utility of individual consumption rises relative to that of leisure.

[12] This is in contrast to inward looking behavior that constitutes the basis for the concept of habit formation. It makes use of the assumption that agents' own past consumption builds a habit typed reference stock.

[13] For future reference note that "time" and "raw labor" are used interchangeably.

with $\alpha \in (0,1)$ and $\phi > 0$ denoting a technology shift parameter. Given that h_i reflects the efficiency per unit of raw labor supplied, $\chi_i h_i$ clearly signifies the amount of effective labor used in the goods production. Final output can be either consumed currently or saved and transformed into physical capital

$$\dot{k}_i(t) = \phi k_i(t)^{\alpha} \left(\chi_i(t) h_i(t)\right)^{1-\alpha} - c_i(t) - \delta k_i(t), \tag{3}$$

where $\delta \geq 0$ denotes the constant, exogenously given, capital depreciation rate. The education sector utilizes human capital together with time to produce new human capital. In light of recent empirical evidence I refrain from using the traditional assumption that the human capital growth rate is linear in time. As pointed out by Alonso-Carrera (2001), results from life-cycle earnings estimations strongly suggest strict concavity of the accumulation technology in schooling time, implying diminishing private returns to education. Accounting for this fact the human capital accumulation technology is postulated to be

$$\dot{h}_i(t) = \varphi \left(1 - \chi_i(t)\right)^{\zeta} h_i(t) - \eta h_i(t), \tag{4}$$

with constant and exogenous parameters $\varphi > 0$, $\eta \geq 0$ and $\zeta \in (0,1)$.

3 Equilibrium Analysis

In a decentralized economy each agent i faces the following problem

$$\max_{(c_i(t), \chi_i(t))} \int_0^{\infty} U\left(c_i(t), \tilde{c}(t)\right) e^{-\rho t} dt, \tag{5}$$

subject to

$$\begin{aligned} &\dot{k}_i(t) = \phi k_i(t)^{\alpha} \left(\chi_i(t) h_i(t)\right)^{1-\alpha} - c_i(t) - \delta k_i(t), \\ &\dot{h}_i(t) = \varphi \left(1 - \chi_i(t)\right)^{\zeta} h_i(t) - \eta h_i(t), \\ &c_i(t) \geq 0 \quad k_i(t) \geq 0 \quad h_i(t) \geq 0, \quad \chi_i(t) \in [0,1], \quad \forall t \in [0,\infty), \\ &k_i(0) = k_{0,i} \quad h_i(0) = h_{0,i}, \end{aligned} \tag{6}$$

taking the path of $x(t) \geq 0$ as given. Equations (5)-(6) denote a common dynamic optimization problem with control variables $c_i(t)$ and $\chi_i(t)$ and state variables $h_i(t)$ and $k_i(t)$. From the maximum principle we get a system of first-order necessary conditions for optimality

$$\frac{c(t)^{-\sigma}}{x(t)^{(1-\beta)(1-\sigma)}} - \pi(t) = 0, \tag{7}$$

$$(1-\alpha)\,\pi(t) \frac{\phi k(t)^{\alpha} \left(\chi(t) h(t)\right)^{1-\alpha}}{\chi(t)} - \lambda(t) \varphi \zeta h(t) \left(1 - \chi(t)\right)^{\zeta - 1} = 0, \tag{8}$$

$$-\pi(t)\left(\frac{\alpha\phi k(t)^{\alpha}\left(\chi(t)h(t)\right)^{1-\alpha}}{k(t)}-\delta-\rho\right)=\dot{\pi}(t), \tag{9}$$

$$-\pi(t)\frac{(1-\alpha)\,\phi k(t)^{\alpha}\left(\chi(t)h(t)\right)^{1-\alpha}}{h(t)}-\lambda(t)\left(\varphi\left(1-\chi(t)^{\zeta}\right)-\eta-\rho\right)=\dot{\lambda}(t). \tag{10}$$

The impact of consumption jealousy is clearly reflected by (7) that equates costs and benefits of an additional unit of own consumption. A higher reference stock or a higher degree of jealousy (reflected by a lower β) pushes-up the marginal utility of consumption, implying that an additional unit becomes more valuable. Given the fact that the Hamiltonian function associated with (5)-(6) is jointly concave in both choice variables c and χ the first order conditions are not only necessary but also sufficient for optimality if, in addition, the following transversality conditions are fulfilled

$$\lim_{t\to\infty} e^{-\rho t}\pi_i(t)k_i(t)=0, \qquad \lim_{t\to\infty} e^{-\rho t}\lambda_i(t)h_i(t)=0.$$

The terms π and λ denote the co-state variables associated with the physical and human capital stock, respectively. In order to get a clear picture of the equilibrium concepts used in the subsequent analysis, it is quite instructive to state the following definitions.

Definition 1 *A symmetric perfect-foresight equilibrium consists of paths* $\{c(t),\chi(t),k(t),h(t),x(t)\}_{t=0}^{\infty}$ *that solve the optimal control problem indicated in (5)-(6) for given initial conditions* $k(0)=k_0$, $h(0)=h_0$.

Note that this definition already accounts for the fact that due to agents' symmetry in preferences and endowments we have $\tilde{c}=c_i=c$ and $\chi_i=\chi$.

Definition 2 *A balanced growth path equilibrium (BGP) is a set of paths* $\{c(t),\chi(t),k(t),h(t),x(t)\}_{t=0}^{\infty}$ *satisfying Definition 1 such that* $c(t)$, $k(t)$, $h(t)$, $x(t)$ *grow at a constant rate and* $\chi(t)$ *is constant.*

The optimality conditions together with the laws of motion for k, h and x can now be combined yielding a five-dimensional system that fully describes the underlying dynamics of the model economy [14]

$$\gamma_c=\left(\alpha\phi\chi(t)^{1-\alpha}\xi(t)^{-(1-\alpha)}-\delta-\rho-(1-\beta)(1-\sigma)\kappa\left(\omega(t)-1\right)\right)\sigma^{-1}, \tag{11}$$

[14] In order to facilitate the analysis and to reduce notational clutter we define the following ratios that are constant along the BGP $\omega=c/x$, $\nu=c/k$ and $\xi=k/h$. Furthermore, we use $\gamma_q\equiv\frac{\dot{q}}{q}$.

$$\gamma_\chi = \left[-\alpha\nu + \varphi\left(1-\chi(t)\right)^\zeta\left(\frac{1-\chi(t)\left(1-\zeta\right)}{1-\chi(t)} - \alpha\right) + \left(1-\alpha\right)\left(\delta-\eta\right)\right] \left(\alpha + \frac{\left(1-\zeta\right)\chi(t)}{1-\chi(t)}\right)^{-1}, \tag{12}$$

$$\gamma_k = \phi\chi(t)^{1-\alpha}\xi(t)^{-(1-\alpha)} - \nu(t) - \delta, \tag{13}$$

$$\gamma_h = \varphi\left(1-\chi(t)\right)^\zeta - \eta \qquad \gamma_x = \kappa\left(\omega(t)-1\right). \tag{14}$$

Applying Definition 2 to the system above, we can immediately conclude that the key variables not only grow at a constant, but also common rate given by

$$\gamma_{c^*} = \gamma = \frac{\alpha\phi\left(\chi^*/\xi^*\right)^{1-\alpha} - \delta - \rho}{1-\beta\left(1-\sigma\right)}, \tag{15}$$

and implying, $\gamma_c = \gamma_k = \gamma_h = \gamma_x = \gamma_y \equiv \gamma$. Before we continue our steady state analysis let us stop at this point and take some time to reflect on the implications of (15). At a first glance the equilibrium growth rate looks similar to the well-known Keynes-Ramsey rule in the conventional Uzawa-Lucas two sector growth model. For comparison reasons let us denote it by γ^{KR}. The numerator reflects the difference between the net marginal product of capital and the discount rate. However, the denominator makes the crucial difference. For values of the intertemporal elasticity of substitution sufficiently large implying $\sigma < 1$ and for $\beta \in (0,1)$ it is always true that $\gamma^{KR} > \gamma$. However, for empirically plausible values of σ, i.e. $\sigma > 1$ we get the reverse case, $\gamma^{KR} < \gamma$. In addition, this relationship becomes more pronounced the higher agents value status, i.e. for lower values of β. To develop the economic intuition underlying this dependence, we need to go deeper into the steady state analysis. First, for convenience, we rewrite the dynamic system in (11)-(14) using the ratio-definitions

$$\gamma_\omega = \left[\alpha\phi\chi(t)^{1-\alpha}\xi(t)^{-(1-\alpha)} - \delta - \rho - \left[1-\beta\left(1-\sigma\right)\right]\kappa\left(\omega(t)-1\right)\right]\sigma^{-1}, \tag{16}$$

$$\gamma_\xi = \left[\phi\chi(t)^{1-\alpha}\xi(t)^{-(1-\alpha)} - \nu(t) - \delta - \varphi\left(1-\chi(t)\right)^\zeta + \eta\right], \tag{17}$$

$$\gamma_\nu = \left[\phi\left(\chi(t)/\xi(t)\right)^{1-\alpha}\left(\alpha-\sigma\right) - \left(1-\sigma\right)\left[\left(1-\beta\right)\kappa\left(\omega(t)-1\right)+\delta\right] + \nu(t)\sigma - \rho\right]\sigma^{-1}, \tag{18}$$

$$\gamma_\chi = \left[-\alpha\nu + \varphi\left(1-\chi(t)\right)^\zeta\left(\frac{1-\chi(t)\left(1-\zeta\right)}{1-\chi(t)} - \alpha\right) + \left(1-\alpha\right)\left(\delta-\eta\right)\right] \left(\alpha + \frac{\left(1-\zeta\right)\chi(t)}{1-\chi(t)}\right)^{-1}. \tag{19}$$

Next, we assume that an interior BGP exists[15].

Definition 3 *An interior balanced growth path equilibrium is a set of paths* $\{c(t), \chi(t), k(t), h(t), x(t)\}_{t=0}^{\infty}$ *satisfying Definition 2 and* $0 < \chi(t) < 1 - \left(\frac{\eta}{\varphi}\right)^{\frac{1}{\zeta}} \quad \forall t \in [0, \infty)$.

Note that Definition (3) implies two things; first an interior BGP can only exist if both sectors are active and second, the equilibrium growth rate is strictly positive, implying endogenous growth. Ad hoc activity could in principle be ensured by the weaker condition $\chi \in (0,1)$ but the case $\chi \in \left(1 - (\eta/\varphi)^{\frac{1}{\zeta}}, 1\right)$ would not be sustainable since it implies $\hat{h}(t) < 0$ resulting in $\lim_{t\to\infty} h(t) = 0$ and a shutdown of the goods sector.
Recall that by definition $\dot{\nu}(t) = \dot{\xi}(t) = \dot{\omega}(t) = \dot{\chi}(t) = 0$ must hold along an interior balanced growth path. Consequently, after setting (16)-(19) equal to zero we get, after a fair amount of algebra, a system $\{\nu^*(\chi^*), \xi^*(\chi^*), \omega^*(\chi^*), \chi^*\}$ that implicitly determines the equilibrium values of the corresponding variables[16].

To get a clear-cut picture of the steady state's quantitative properties, we calibrate the system and compare the results to features of actual economies. The parametrization we employ is compactly summarized in Table (1).

Parametrization of the Benchmark economy	
Preferences	$\sigma = 2.5$, $\rho = 0.04$, $\beta = 0.8$, $\kappa = 0.05$
Production	$\alpha = 0.4$, $\phi = 1$, $\varphi = 0.08$, $\zeta = 0.8$
Accumulation	$\delta = 0.05$, $\eta = 0.02$

Table 1: Benchmark Parameters

The conventional preference and technology parameters are set in accordance with the literature, see e.g. Mulligan and Sala-i-Martin (1993) and Ortigueira and Santos (2002), Lucas (1990), Gong et al. (2004). Moreover, κ is chosen to fit plausible rates of convergence while the value of β is set such that both, absolute and relative consumption are attached with positive utility. Table (2) provides some rough impression about the features of the resulting benchmark economy.

χ^*	$\frac{k^*}{h^*}$	$\frac{k^*}{y^*}$	$\frac{c^*}{y^*}$	ι_1	γ
0.680	5.288	3.425	0.787	0.056	0.0122

Table 2: Benchmark Economy

In fact, the results in Table 2 fit actual economic data remarkably well. Since the model abstracts from population growth, aggregate values are associated with per capita values. In light of this an equilibrium growth rate of 1.2% per annum seems to be quite plausible. The

[15]The proof of existence and uniqueness is provided below.
[16]The equilibrium expressions can be found in the appendix. Notice that a * attached to a variable denotes its steady-state value.

capital-output ratio taking a value of 3.4 is very close to results obtained by Alvarez-Cuadrado et al. (2004) and Eicher and Turnovsky (2001). Furthermore, the derived value of the consumption-output ratio indicates that almost 79% of final output are devoted to individuals consumption. In this case Alvarez-Cuadrado et al. (2004) obtain a similar result. Given that we neglect governmental activities, this value appears to be highly plausible. In addition, Table 2 attests that individuals engage for slightly less than one third of their disposable time in education. Accounting for the fact that the concept of education used in this paper includes all sorts of human capital accumulation - not only formal schooling but also activities such as acquiring job specific skills and knowledge, learning by doing or on the job training - the seemingly low value in Table 2 turns out to be quite realistic. The capital stocks ratio indicates that in equilibrium the physical capital stock of the economy is more than 5 times higher than its human capital stock. Finally, ι_1 which is the eigenvalue that satisfies $0 > \iota_1 > \iota_2$ implies that the asymptotic speed of convergence of the economy is around 5.6% per annum. This value exactly lies in the consensus range of $3\% - 11\%$ implying a high degree of empirical consistency. On the whole, the remarkable fit of the indicated equilibrium features to the characteristics of actual economies makes us highly confident of the model's ability to generate also realistic and insightful transitional dynamics. Using these results we can conduct some comparative statics analysis by computing the effects of parametric changes on the equilibrium values. These results are displayed in Table 3 below.

	χ^*	c/k^*	c/x^*	k/h^*	γ^*
$\Delta\kappa$	0	0	-	0	0
$\Delta\beta$	+	+	-	+	-
$\Delta\phi$	0	0	0	+	0
$\Delta\varphi$	-	+	+	-	+
$\Delta\sigma$	+	+	-	+	-
$\Delta\rho$	+	+	-	+	-

Table 3: Effects of parametric changes on equilibrium values

Notice that the table also includes the equilibrium growth rate. The impact of the conventional technology and preference parameters ϕ, φ, σ and ρ are fairly standard and well documented and interpreted by earlier studies, see e.g. Mulligan and Sala-i-Martin (1993). However, a thorough analysis on the influence of both new parameters β and κ, reflecting, respectively, individuals' desire for social status and the persistence of past average consumption, has been lacking, as of yet, in an endogenous growth environment.

The effects of κ on the model's steady-state values are not dramatic. Only the status-indicating variable ω responds to changes in κ which is, after recalling the definition of ω, self-explanatory. An increase in κ leads to faster adjustment of the reference stock, which in the limit approaches $\tilde{c}$. Since $\tilde{c} = c$ due to symmetry, we have $\lim_{\kappa\to+\infty}\omega(\kappa) = 1$. Despite modest steady-state effects, κ nevertheless plays a crucial role in determining the model's off-equilibrium behavior, as we will see later on. In contrast, the impact of β on the model's equilibrium is far more complex. As indicated in the second row of Table 3, a lower degree of consumption enviousness leads to a higher equilibrium final good employment, consumption-capital ratio, and physical to

human capital ratio, whereas the steady state growth rate and relative consumption experience a decline. The economic intuition behind these phenomena can be best elucidated using the agent's optimality conditions with respect to consumption, (7), and the state variable k, (9). Given the fact that $(\partial U_c/\partial\beta) < 0$, we can conclude that an incremental increase in β lowers the marginal utility of own consumption, which, as implied by the optimality condition, leads to a drop in the shadow value of capital. As a consequence, its (negative) growth rate is stimulated. The reestablishment of equality in equation $\hat{\pi}(t) = -MPK(t) + \delta + \rho$, that results from the agent's first order conditions, requires an increase in the marginal product of capital (MPK) which is achieved by a sufficiently large increase in activity in the final goods sector, χ. Since we have $(\partial\xi^*(\chi^*)/\partial\chi^*) > 0, \quad \forall\chi \in \left(0, 1-(\eta/\varphi)^{1/\zeta}\right)$, the physical to human capital ratio, ξ^* increases but by less than χ is rises, i.e. $\Delta\chi > \Delta\xi$ [17]. This yields the required condition $\partial\left(\frac{\chi^*}{\xi^*}\right)/\partial\chi^* > 0$ which causes the final rise in the marginal product. As shown in Table 3, the long run growth impact of a higher β is clearly negative. Note that a rise in β in fact entails two opposing effects on the equilibrium growth rate, which becomes clear by inspecting equation (15). An indirect positive effect is triggered by the higher steady state marginal product of physical capital which enhances the numerator of (15) and, as a result, also growth. Weaker status motives, on the other hand, also create a direct growth depressing effect. As can be inferred from the human capital accumulation technology (4), the induced intersectoral reallocation of time toward final goods production impacts growth in a negative way. For future reference, let's denote these effects by "marginal product effect" and "reallocation effect", respectively [18]. For the parametrization used above we get that the negative reallocation effect outweighs the positive marginal product effect which finally results in the indicated negative overall effect. However, the picture changes substantially when we vary the value of σ around one. In particular, if σ is small enough, i.e. $\sigma < 1$, the signs in the second row of Table 3 are reversed, indicating the opposite effects as described above. This leads us directly to our first result.

Result 1 *For $\sigma \in (0,1)$ (respectively $\sigma \in (1,+\infty)$) a lower degree of consumption jealousy, i.e. a higher value of β, implies that the reallocation effect is positive (negative) and stronger (weaker) than the negative (positive) marginal product effect, resulting in an overall positive (negative) impact on the equilibrium growth rate. For the knife edge case, $\sigma = 1$, changes in the status motive have no impact on equilibrium allocations and growth.*

Furthermore, the positive response of the consumption-capital ratio, ν^* depicted in Table 3 is intimately connected to the increase in labor since $(\partial\nu^*(\chi^*)/\partial\chi^*) > 0$. Finally, it remains to explain the negative impact of weaker status motives on relative consumption. In fact, this turns out to be pretty straightforward. Rearranging the instantaneous utility function given in (1) to explicitly model the status dependence yields $U\left(c(t),\omega(t)\right) = (1-\sigma)^{-1}\left(c(t)^{\beta}\omega(t)^{1-\beta}\right)^{1-\sigma}$,

[17] This result can be established using the steady state value of ξ^*, see equation (27) in the appendix. Values of χ outside this range would not fulfill this condition but since we are considering only interior equilibria we can omit them.

[18] To be precise, the *marginal product effect* in this paper generally denotes the positive stimulus for consumption growth caused by a higher marginal product of capital. It is, therefore, equivalent to what Alvarez-Cuadrado et al. (2004) call the *rate of return effect*.

from which we can infer that for an increasing β the benefit of an additional unit of relative consumption declines steadily, i.e. $\partial U_\omega/\partial\beta < 0$. A low marginal utility of ω, caused by a high β, induces agents to accumulate less of it which finally results in the negative response displayed in Table 3. This section can be concluded by stating another important result.

Result 2 *An economy that awards an agent's level of consumption with social status devotes more time to education and as a consequence exhibits a relatively higher human than physical capital stock than an economy with weaker or no status valuation.*

3.1 Existence and Uniqueness of a Steady State Equilibrium

For notational convenience, let us first define $m \equiv 1-(\eta/\varphi)^{\frac{1}{\zeta}}$ and $\Xi = \{\chi \in \Re : 0 < \chi < m\}$. To establish existence of an interior equilibrium, we need to find a $\chi \in \Xi$ that solves the system $\{\nu^*(\chi^*), \xi^*(\chi^*), \omega^*(\chi^*), \chi^*\}$. For this purpose we define the function $F : [0,1) \to \Re$ using the implicit equilibrium expression for χ^*.

$$F(\chi) = \varphi(1-\chi)^{\zeta-1}[\zeta\chi + \beta(1-\sigma)(1-\chi)] - \rho - \beta\eta(1-\sigma) \tag{20}$$

Any χ that solves $F(\chi) = 0$ and satisfies $\chi \in \Xi$ is a candidate for an interior equilibrium. This leads us to the following proposition.

Proposition 1 *If $\beta(1-\sigma)(\varphi-\eta) < \rho < \zeta\eta\left(\frac{m}{1-m}\right)$, then there exists a unique $\chi^* \in \Xi$ such that the quadruple $(\nu^*(\chi^*), \xi^*(\chi^*), \omega^*(\chi^*), \chi^*)$ constitutes an interior balanced growth path equilibrium.*

Proof. The proof itself is straightforward. Notice that $F(\chi)$ is at least twice continuously differentiable for all χ in the domain. As a first step we have to check the properties of the function $F(\chi)$ at the boundaries, i.e. $\lim_{\chi\to 0} F(\chi)$ and $\lim_{\chi\to m} F(\chi)$. For these we get

$$\lim_{\chi\to 0} F(\chi) = \beta(1-\sigma)(\varphi-\eta) - \rho,$$

and

$$\lim_{\chi\to m} F(\chi) = \zeta\eta\left(\left(\frac{\varphi}{\eta}\right)^{\frac{1}{\zeta}} - 1\right) - \rho.$$

Another important ingredient is the slope of $F(\chi)$.

$$F_\chi = \frac{\varphi\zeta}{(1-\chi)^{1-\zeta}}\left[\frac{1-\chi\zeta}{1-\chi} - \beta(1-\sigma)\right]$$

Since $\frac{\varphi\zeta}{(1-\chi)^{1-\zeta}} > 0$ and $\frac{1-\chi\zeta}{1-\chi} \geq 1$, $\forall\chi \in [0,1)$, we see that $F_\chi > 0$, $\forall\chi \in \Xi$. The picture that emerges from this exposition suggests that $F(\chi)$ is a strictly increasing function $\forall\chi \in \Xi$. Accounting for this fact, the proof essentially reduces to verifying that $\exists\chi \in \Xi$ such that $F(\cdot) = 0$. Since $F(\cdot)$ is strictly increasing, it is clearly the case that an interior equilibrium can exist iff $F(0) < 0$ and $F(m) > 0$. From the limiting behavior properties of $F(\chi)$, we can infer that the

necessary and sufficient conditions for these cases are $\sigma > 1 - \frac{\rho}{\beta(\varphi-\eta)}$ and $\zeta\eta\left(\left(\frac{\varphi}{\eta}\right)^{\frac{1}{\zeta}} - 1\right) - \rho > 0$. After some simple rearrangement, we obtain the condition used in Proposition 1. To finalize the proof, note that the solution candidate needs to fulfill both transversality conditions. We know that in the limit the transversality conditions can be expressed as $-\rho + \gamma_\lambda + \gamma_h < 0$ and $-\rho + \gamma_\pi + \gamma_k < 0$. Using (9) and (10) yields

$$-\frac{\chi(t)}{(1-\chi(t))^{1-\zeta}} < 0,$$

and

$$-\frac{\alpha\varphi\zeta\chi(t)}{(1-\chi(t))^{1-\zeta}} < 0,$$

which both hold for all $\chi \in \Xi$. As a result, we can conclude that an interior balanced growth equilibrium exists if both conditions $\sigma > 1 - \frac{\rho}{\beta(\varphi-\eta)}$ and $\zeta\eta\left(\left(\frac{\varphi}{\eta}\right)^{\frac{1}{\zeta}} - 1\right) - \rho > 0$ are satisfied. Furthermore, uniqueness is given by the fact that $F(\chi)$ is strictly increasing and, hence, the $F(\chi) = 0$ line can be crossed only once.

3.2 Local Stability Analysis

The local stability properties of an equilibrium can, in general, be studied by analyzing the structure of the eigenvalues of the dynamic system that has been linearized around its steady state. For this purpose we take the fourth-order dynamic system in (16)-(19) and apply first order Taylor expansions to approximate the models dynamics

$$\begin{pmatrix} \dot{\omega} \\ \dot{\xi} \\ \dot{\nu} \\ \dot{\chi} \end{pmatrix} = \begin{pmatrix} \frac{-\omega^*\kappa[1-\beta(1-\sigma)]}{\sigma} & j_{12} & 0 & -\frac{\xi^*}{\chi^*}j_{12} \\ 0 & j_{22} & -\xi^* & \xi^*\left(\frac{\sigma}{\omega^*\alpha}j_{14} + \frac{1}{\chi^*}j_{44}\right) \\ \frac{-\nu^*(1-\sigma)(1-\beta)\kappa}{\sigma} & \frac{\nu^*(\alpha-\sigma)}{\omega^*\alpha}j_{12} & \nu^* & -\frac{\xi^*}{\chi^*}j_{32} \\ 0 & 0 & j_{43} & \zeta\varphi\left(1-\chi^*\right)^{\zeta-1}\chi^* \end{pmatrix} \begin{pmatrix} \Delta\omega \\ \Delta\xi \\ \Delta\nu \\ \Delta\chi \end{pmatrix}. \tag{21}$$

For tractability, we define, $j_{22} = -(1-\alpha)\left(\nu^* + \delta + \varphi\left(1-\chi^*\right)^{\zeta} - \eta\right)$, $j_{12} = -\omega^*\alpha\phi(1-\alpha)\left(\chi^*/\xi^*\right)^{1-\alpha}\left(\xi^*\sigma\right)^{-1}$, $j_{43} = \frac{-\alpha\chi^*(1-\chi^*)}{\alpha(1-\chi^*)+\chi^*(1-\zeta)}$ and $\Delta\omega = \omega - \omega^*$. The terms j_{mn} denote the matrix element in the mth row and nth column. To check the local stability properties of the system it would, in principle, be sufficient to determine the signs of the eigenvalues of the Jacobian. Doing this in an analytical way can be quite challenging, however. A proper way to circumvent difficulties that are due to a high degree of complexity is to solve for sign-indicating conditions, i.e. conditions from which one can draw conclusions about the properties of the eigenvalues and hence the stability of the system[19]. Nevertheless, solving for these conditions in

[19] In the case of a two dimensional system these conditions are simply the trace and the determinant of the corresponding Jacobian matrix, while in a three-dimensional system the term $j_{12}j_{22} + j_{13}j_{31} + j_{23}j_{32} - j_{11}j_{22} - j_{11}j_{33} - j_{22}j_{33} < 0$ is an additional condition.

our fourth-order system turns out to be intractable as well. An elegant way out is to evaluate the roots by calibrating the model's structural parameters with precise values. The strategy that is adopted in this procedure, however, accounts for the fact that the literature is sparse about precise or even the range of values of non-conventional preference parameters such as β and κ. This problem is tackled by simply computing the models eigenvalues for a broad range of possible values of β and κ, while the remaining parameters are set in accordance with the empirical literature. Results are reported in Table 4 below. Notice that the table contains only the negative eigenvalues.

			κ			
β	0.05	0.1	0.3	0.7	2	10
0	-0.0334	-0.0574	r = -0.1359	-0.1591	-0.1501	-0.1484
	-0.1357	-0.1324	+/- 0.0354i	-0.2746	-0.8054	-4.0085
0.3	-0.0428	-0.0774	r = -0.1596	-0.1414	-0.1391	-0.1384
	-0.1294	-0.1244	+/- 0.0197i	-0.4110	-1.1683	-5.8096
0.6	-0.0511	-0.0960	-0.1354	-0.1324	-0.1317	-0.1314
	-0.1262	-0.1197	-0.2331	-0.5407	-1.5298	-7.6103
1	-0.0609	-0.1109	-0.1248	-0.1248	-0.1248	-0.1248
	-0.1248	-0.1248	-0.3109	-0.7109	-2.0109	-10.0109

Note: the remaining parameters are the same as in Table 3

Table 4: Asymptotic Speed of Convergence

This simple calibration exercise indeed sheds some light on the structure of the eigenvalues. As clearly indicated in Table 4, the system possesses two stable and two unstable eigenvalues, from which we can deduce the existence of local saddle path stability[20]. Recalling, however, that for parameter values with less empirical support the model might display contrary stability features, we deal the subsequent analysis with great care. Notice that apart from stability considerations, the eigenvalues in Table 4 can also be consulted to draw conclusions on the asymptotic speed of convergence of the model's variables. In principle, we know that the asymptotic behavior of a m-dimensional dynamic system exhibiting saddle path stability is essentially governed by the eigenvalue ι_i that satisfies $0 > \iota_i > \iota_j > ... > \iota_{m/2}$. From Table 4 we can infer that the displayed eigenvalues ι_1 with $\iota_1 > \iota_2$ span a range of -0.033 to -0.15, indicating that the economy asymptotically converges at rates of 3% to 15% per annum toward its steady state. The empirical evidence on the adjustment speed is, in fact, far from being in unison. Early studies in this field (see e.g. Barro and Sala-i-Martin (1992)) found values around 2% per annum whereas more recent results suggest annual rates up to 11% (e.g. Islam (1995) and Caselli et al. (1996)). Our model perfectly matches these results for the whole range of β and sufficiently low values of κ, i.e. $\kappa < 0.3$. High values of κ appear to be implausible, since they would imply rates of convergence up to 16%.

[20]Observe that for selected combinations of parameters we get complex roots indicating cyclical behavior of the system.

4 Transitional and global dynamics

4.1 The Benchmark Economy

Having conducted a fruitful equilibrium and stability analysis, we now concentrate our attention to the characterization of the model's behavior outside the steady state. The transitional dynamics in one-sector neoclassical and two-sector endogenous growth models is now, in general, well understood thanks to insightful studies by Mulligan and Sala-i-Martin (1993), Bond et al. (1996) and Eicher and Turnovsky (2001), to mention just a few. Nevertheless the importance of transition in the recent literature is generally underestimated and, hence, carelessly handled. As a result, the focus of numerous studies is on balanced growth properties only, without paying sufficient attention to the off-equilibrium behavior. This is problematic insofar as, first, the authors implicitly or even explicitly assume existence of a steady state without ensuring that the economy actually converges to an equilibrium at all. And second, as shown by Mulligan and Sala-i-Martin (1992), periods of transition can be rather long implying that off-equilibrium conditions might have a substantial quantitative impact on the overall economic performance. Noteworthy exceptions to this practice are Alvarez-Cuadrado et al. (2004), Alonso-Carrera (2001) and Caballe and Santos (1993). The stability analysis above indeed gives rise to the conjecture that transitional dynamics might play a substantial role in our model. Given that the conditions for saddle path stability are satisfied the off-equilibrium dynamics is governed by two negative eigenvalues so that the stable manifold is two-dimensional[21]. As pointed out by Eicher and Turnovsky (2001), a two-dimensional transition path substantially enriches the model's dynamics by introducing additional flexibility that allows its behavior to mimic important features of economic data. They mention that as a direct consequence of a higher dimensional adjustment locus the speed of convergence might differ across variables, which permits a more flexible time path[22]. Moreover, adjustment paths need not be monotonic. Since the dynamics is determined by two negative eigenvalues, their respective influence on the adjustment is subject to variations as time proceeds implying also the possibility of overshooting. Finally, systems with two or higher dimensional stable manifolds might respond asymmetrically to positive or negative shocks of equal magnitude. This is in clear contrast to conventional one-sector growth models in which the response to a positive shock is just the mirror image of the same negative shock (Eicher and Turnovsky, 2001). To study the transitional dynamics in our model, we assume that the system summarized in (16)-(19) is initially in equilibrium. Thereafter, we confront the model by specific shocks and trace out the subsequent time path of some selected key variables. The strategy we adopt therein implicitly makes use of the fact that due to the

[21]The stable manifold is the locus of points from which the respective variables asymptotically converge to their equilibrium values if they are allowed to evolve according to their laws of motion. In the well-known Ramsey optimal growth model, for instance, the stable manifold is one-dimensional implying that for each value of k (physical capital) there exists a unique c (consumption) that places the economy on the stable arm of the saddle path. The same applies to most classes of one-sector growth models including the AK model.

[22]More precisely, this allows variables to evolve virtually independently from the time path of other variables. In their paper, Eicher and Turnovsky (2001) consider a two-sector non-scale growth model that permits technology and physical capital to evolve independently and at different adjustment speeds that is consistent with empirical evidence.

local stability characteristics, the economy returns to the BGP equilibrium after it was hit by a shock. Consequently, after implementing a specific disturbance, we can numerically simulate the time path of selected variables to reproduce the transitional dynamics until the economy reaches the equilibrium path again[23].

4.2 The Set Up

Given that the transitional adjustment locus is a two-dimensional stable saddle path, we can express the stable solution of (16)-(19) in terms its eigenvalues and -vectors. This enables us to separate the interdependent system and get self-contained equations, which greatly facilitates the subsequent analysis. In particular, making use of the two stable roots denoted by ι_i, with $\iota_2 < \iota_1 < 0$, and the associated normalized eigenvectors, $\upsilon_i = (\upsilon_{1i}, 1, \upsilon_{3i}, \upsilon_{4i})'$, we can reformulate the generic form of any two variables in (16)-(19) to explicitly express their time path during transition.

$$\omega(t) = \omega^* + \left[(\omega_0 - \omega^*)\left(\upsilon_{11}e^{\iota_1 t} - \upsilon_{12}e^{\iota_2 t}\right) - (\xi_0 - \xi^*)\,\upsilon_{11}\upsilon_{12}\left(e^{\iota_1 t} - e^{\iota_2 t}\right)\right](\upsilon_{11} - \upsilon_{12})^{-1} \tag{22}$$

$$\xi(t) = \xi^* + \left[(\omega_0 - \omega^*)\left(e^{\iota_1 t} - e^{\iota_2 t}\right) - (\xi_0 - \xi^*)\left(\upsilon_{12}e^{\iota_1 t} - \upsilon_{11}e^{\iota_2 t}\right)\right](\upsilon_{11} - \upsilon_{12})^{-1} \tag{23}$$

For expositional convenience, we depict here only the path of the capital stocks ratio, ξ and comparison consumption ω [24]. However, the remaining ones, i.e. for χ and ν, can be established analogously. From (22) we can immediately infer that if $t \to +\infty$, $\omega(t) = \omega^*$. The same, of course, applies to ξ, χ and ν implying that the economy asymptotically returns to its equilibrium path. However, this adjustment behavior need not be monotonous. The off-equilibrium dynamics of the variables under scrutiny can now analytically be approximated by

$$\begin{pmatrix} \dot{\omega}(t) \\ \dot{\xi}(t) \end{pmatrix} = \frac{1}{\upsilon_{12} - \upsilon_{11}} \begin{pmatrix} \upsilon_{12}\iota_2 - \upsilon_{11}\iota_1 & \upsilon_{12}\upsilon_{11}(\iota_1 - \iota_2) \\ \iota_2 - \iota_1 & \upsilon_{12}\iota_1 - \upsilon_{11}\iota_2 \end{pmatrix} \begin{pmatrix} \omega(t) - \omega^* \\ \xi(t) - \xi^* \end{pmatrix}. \tag{24}$$

It might come as surprise that the original interrelated fourth-order system expressed in (16)-(19) can be reduced to two-dimensional systems without loosing any feedback information coming from the respective isolated variables. Or to state it more clearly, why should the original interdependence of ξ and χ in (16)-(19) be decoupled and simply disappear? Concerning this issue, Eicher and Turnovsky (2001, p.97, footnote 13) point out that despite there is no apparent link between the imbedded and the detached variables the system takes fully account of the feedback namely through the two eigenvalues. As a result, the dynamics expressed in

[23]Notice that the equilibrium is in fact reached when $t \to +\infty$, hence the simulation is just an approximation, even though a very precise one.

[24]Notice that equations (22)-(23) can be derived by using the generic form $\omega(t) - \omega^* = K_1\upsilon_{11}e^{\iota_1 t} + K_2\upsilon_{12}e^{\iota_2 t}$ and $\xi(t) - \xi^* = K_1e^{\iota_1 t} + K_2e^{\iota_2 t}$. The constant coefficients $K_i, i = 1, 2$ can be computed from the initial conditions. In particular, set $t = 0$ and express K_i by successive substitution. Re-substituting finally yields (22)-(23).

phase-space (24) can be illustrated graphically by means of a two-dimensional phase diagram. By construction, this system possesses two eigenvalues with $0 > \iota_1 > \iota_2$, implying that (24) represents a stable sink. To develop a proper intuition about the underlying structure of the adjustment dynamics, we use Figure 1 to illustrate the corresponding ω, ξ phase-space with the $\dot{\omega} = 0$ and the $\dot{\xi} = 0$ isoclines. Observe that both the $\dot{\omega} = 0$ and the $\dot{\xi} = 0$ isoclines are downward-sloping. Lower comparison consumption, $\omega \equiv c/x$, decelerates reference stock growth which necessitates a lower γ_c to remove the induced imbalance, i.e. to reinstall $\dot{\omega} = 0$. This can be achieved by reducing the marginal product of physical capital, which requires a higher k or equivalently a lower h both resulting in a higher ξ. On the other hand, an increased ω implies a higher c which disequilibrates $\dot{\xi} = 0$ through depressing $\dot{k}$. Since $\dot{h}$ is unaffected, balancing $\dot{\xi} = 0$ again requires a higher average product of physical capital, procurable by a lower ξ.

4.3 Numerical Analysis

4.3.1 Case 1: Productivity Shocks

In the first case we consider a one per cent positive, permanent productivity shock occurring in the final goods sector. In the language of the model this can simulated by an increase in the productivity parameter ϕ. From Table 3 we can infer that in the long run this affects only the capital stocks ratio ξ. Since $\partial\xi^*/\partial\phi > 0$, i.e. the actual capital stocks ratio at impact is too low, combined with the fact that capital adjustment can not be instantaneous suggests that the transition of ξ toward its new equilibrium value entails a variety of adjustment effects. This is further supported by the close interdependence of the involved variables, implying a multiplicity of feedback effects. As noted at the outset, higher dimensional adjustment loci, basically, allow for non-monotonic adjustment behavior. In the case of a productivity shock, however, the capital stocks ratio, ξ, evolves monotonically, which is implied by (22). Due to $\omega_0 = \omega^*$, the time path essentially reduces to $\xi(t) = \xi^* - (\xi^* - \xi_0)\left(\upsilon_{12}e^{\iota_1 t} - \upsilon_{11}e^{\iota_2 t}\right)(\upsilon_{12} - \upsilon_{11})^{-1}$. $\iota_1 > \iota_2$ and $\xi^* > \xi_0$, which further implies that we must have $\left(\upsilon_{12}e^{\iota_1 t} - \upsilon_{11}e^{\iota_2 t}\right)(\upsilon_{12} - \upsilon_{11})^{-1} > 0$, resulting in a monotonic adjustment path. This need not be the case for the remaining variables. Figure 1 now sketches the off-equilibrium dynamics of some selected variables. In particular, Panel a.) depicts the phase space describing the transition process of comparison consumption, ω and the capital stocks ratio, ξ. The dynamics of working time, χ, and the consumption-capital ratio, ν, are, respectively, displayed in Panel b.). There, the solid (dotted) line representing χ (ν) is associated with the left (right) vertical axes. From a general perspective, one might also be interested in the growth path of some conventional variables such as output, consumption and human capital. Panel c.) therefore summarizes the dynamics of these variables. Panels d.) - e.) will be referred to later on. The model's parametrization is chosen to fit yearly observations, hence, the simulated time horizon spans about 100 years. However, most of the adjustment variation takes place in the first 25 years.

Panels a.) - c.), indeed, display a remarkable, mostly non-monotonous, adjustment behavior of the indicated variables. Most of them undergo periods of rising and falling growth rates respectively levels, though in the end only ξ exhibits a permanent change. Furthermore, panels

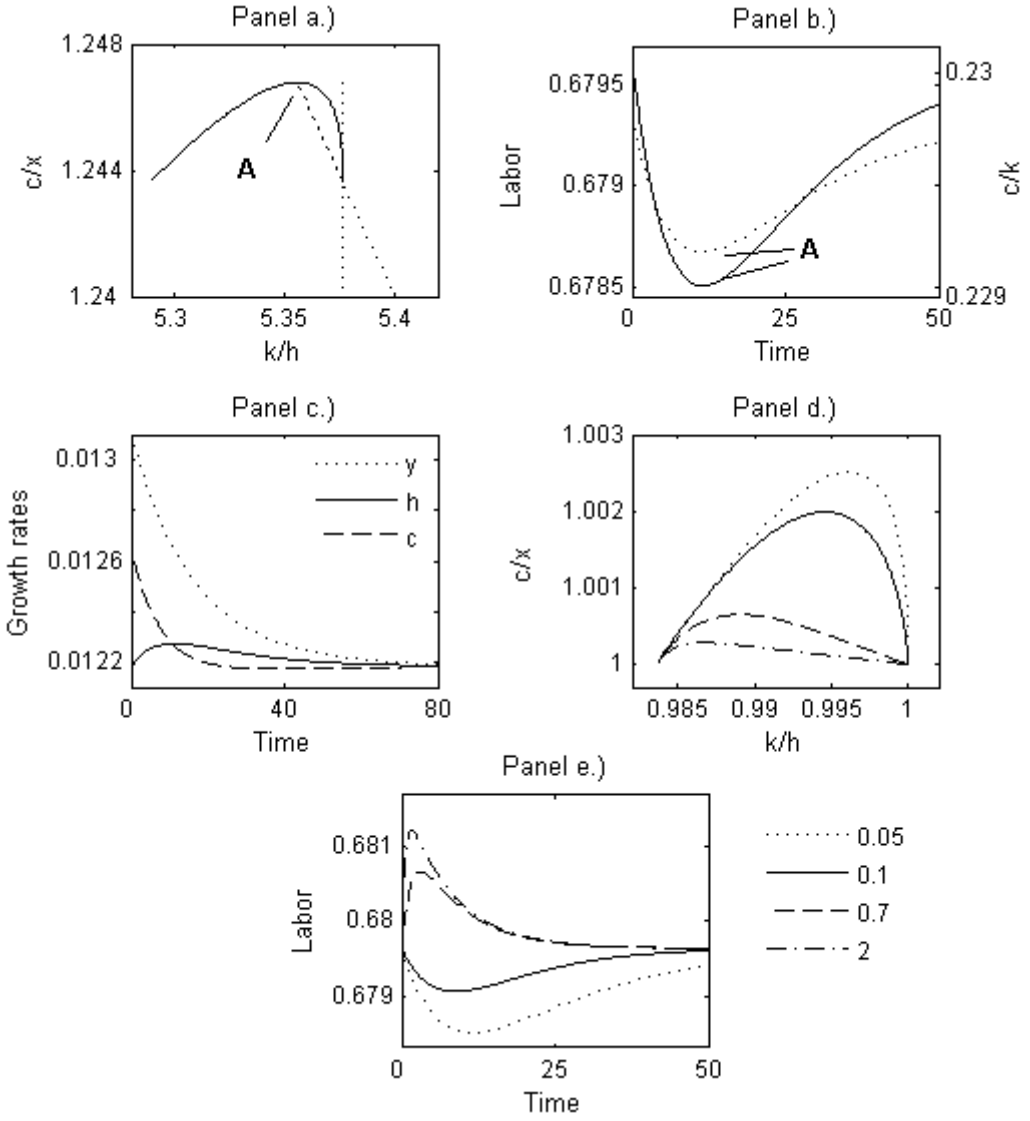

Figure 1: Adjustment Paths in Response to a Productivity Shock

a.)-c.) suggest a natural division of the whole time horizon in two consecutive intervals, i.e. $[0, A)$ and (A, ∞).

Considering the transitional dynamics in detail, we observe that a positive productivity shock pushes-up the steady-state ratio of the two capital stocks, implying a too low ξ at impact. Since agents intersectorally allocate their disposable time such that marginal benefits equalize across sectors, a lower-than equilibrium ξ depresses the marginal product of effective labor in the final goods sector. This induces agents to shift time toward education in order to recover the balance called by optimality condition (8). But since $\partial\chi^*/\partial\phi = 0$, this shift poses a departure from the equilibrium path entailing a variety of repercussions. The following section is devoted to analyzing the first-period dynamics. Most importantly, a low ξ is intimately associated with a high average and marginal product of physical capital[25]. Combined with the increased

[25]For notational convenience let APK and MPK denote the average and marginal product of physical capital respectively.

productivity ϕ this encourages agents to save and hence stimulates investment. The case $\alpha < \sigma$ implies that capital growth is initially stimulated by more than consumption growth, hence $\Delta\gamma_k > \Delta\gamma_c$ causes a decline in ν. Moreover, the high MPK accelerates consumption growth such that $\gamma_c > \gamma_{c^*}$. This leads to an upward movement of ω which, in turn, pushes the reference stock growth rate, γ_x. From $U_{cx} > 0$ we know that an increased reference stock growth additionally stimulates consumption growth since agents appreciate higher own consumption more for greater values of x. To sum it up we can conclude that there are two reinforcing effects driving the growth rate of consumption. The well known marginal product effect and an effect which the literature calls the status effect[26]. In equation (7) these are represented by the first and the last term respectively[27]. For the baseline parameterization, however, the latter is negligible due to a low value of κ which implies only sluggish adjustment of the reference stock. This result can be used to finalize the line of argument about the behavior of comparison consumption, ω. Consumption growth is stimulated mainly due to a high MPK, the reference stock x on the other hand increases only moderately implying that γ_ω is initially positive as displayed in Panel a.).

We have briefly mentioned that due to $\Delta\gamma_c < \Delta\gamma_k$, ν experiences a decline. In fact, this is a final outcome resulting from the interplay of two opposing effects that deserve explicit mention since they will play a key role in determining the location of the turning point A. First, as mentioned above, $\alpha < \sigma$ implies a greater stimulus for capital than for consumption growth caused by the low ξ. In terms of equation (14), this impacts γ_ν in a negative way. Second, because the status effect exhibits a tendency to promote consumption growth it implicitly hampers the decline in ν. Initially the first dominates the latter effect implying a negative overall impact which is also displayed by the dotted line in panel b.). As afore mentioned, the adjustment in ξ can not be immediate, but it is monotonous. The latter point should not conceal, however, that also here there are opposing forces at work. The reallocation of raw labor χ toward education stimulates human capital accumulation, as indicated by the solid lines in Panels b.)-c.). This would, in principle, lower ξ, but the effect is negligible compared to the increase in γ_k, however.

The local stability and monotonicity properties imply that $\xi < \xi^*$ is associated with $\gamma_\xi > 0$ and $\lim_{t\to\infty} \xi(t) = \xi^*$. As a result, if ξ is lower than its long run value, due, for instance, to a shock, it returns to the - probably new - equilibrium path again. In the case at hand, the reestablishment of ξ can be accomplished in two different ways either through increased savings, which is reflected by a lower ν, or through higher work effort. Given the initial decrease in χ, the investment effect needs to dominate the outflow of raw labor to stay in accordance with the monotonicity property.

Clearly, the underlying dynamics in period $[0, A)$ is not sustainable, since it implies $\chi < \chi^*$, $\nu < \nu^*$, $\omega > \omega^*$. At point A the ongoing outflow of raw labor, respectively, the increase in ξ, have reduced the MPK and APK considerably. Simultaneously, the positive γ_ω has boosted ω sufficiently so that the induced higher reference stock growth rate exactly balances consumption growth, leading to a subsequent decline in comparison consumption. At the same time the

[26]The creation of this name can be awarded to Alvarez-Cuadrado et al. (2004).

[27]It should be noted that for values of σ low enough, i.e. $\sigma < 1$, the *status effect* appears to be negative.

increased reference stock growth has accelerated γ_c sufficiently such that $\gamma_c > \gamma_k$, implying that ν starts to rise [28]. To be precise, at A we have $\dot{\omega} = \dot{\nu} = \dot{\chi} = 0$, but since $\xi < \xi^*$, it is unstable and makes further adjustment necessary. In fact, in the subsequent interval (A, ∞), the effects which we have identified in $[0, A)$ are exactly reversed. Hence, ω starts to decline, the backflow of χ depresses human capital growth and $\gamma_c > \gamma_k$ helps to "recover" ν. Furthermore, ξ continues its monotonic increase. But in contrast to the previous time interval, its adjustment is now propelled by the increasing work effort, since the rising ν now puts downward pressure on ξ.

As previously mentioned, the parameter κ that governs the speed of reference stock adjustment and, hence, the size of comparison consumption ω has some significant impact on the transitional dynamics generated by the model. To get a clear picture of the actual influence, consider Panels d.) and e.) of Figure 1 illustrate the adjustment path of (ω, ξ) and χ for $\kappa = 0.05, 0.1, 0.7, 2$ respectively [29]. Panel d.) indicates that the deviation of ω from its equilibrium path gets smaller for higher κ. The initial impact, however, remains the same for all values. In contrast, a higher adjustment speed, entirely reverses the impact on working time χ, which can be observed in Panel e.). The underlying mechanics can be described as follows: to begin, note that a higher κ raises the reference stock x and, hence, reduces comparison consumption ω, although it boosts γ_x. The latter implies that the status effect now plays an important role through stimulating consumption growth, indeed the higher κ is, the more pronounced the status effect becomes. As a consequence, for κ sufficiently high we get $\gamma_c > \gamma_k$, resulting in $\hat{\nu} > 0$. Given that $U_{cx} > 0$, a higher reference stock pushes the marginal utility of own consumption and, therefore, encourages higher work effort in the final goods sector. This pattern is reflected in Panel e.), where paths of χ are traced out for different values of κ. For the purpose of separating the effects on initial labor supply caused by the respective status parameters β and κ, we carry out the same exercise as above for different values of β. Interestingly, we find that the value of β is completely irrelevant for the initial response of χ. This implies that for all possible degrees of consumption jealousy, i.e. $\beta \in [0, 1]$, the movement of χ at impact is purely governed by the adjustment speed κ. In the standard time-separable utility representation, a positive productivity shock is usually followed by an immediate expansion of individuals' labor supply. In contrast, our model suggests that for a fairly plausible parametrization and a utility representation that accounts for out- and backward-looking behavior, this might change for the reasons mentioned above. The intuition underlying Panel d.) can be explained rather briefly. As mentioned above, higher κ increases γ_x one for one, but consumption growth only by a factor $-(1-\beta)(1-\sigma)\sigma^{-1}$ which is strictly less than one. As a result the growth in comparison consumption is slowed down. In the limit ω takes the value of one implying a monotonous response during transition, purely governed by the marginal product effect[30]. In order to bring the reasoning above into a proper order let us state the following results.

Result 3 *The transition of ξ with $\xi < \xi^*$ entails two reinforcing effects on consumption growth.*

[28] This tendency is also supported by the reduced capital stock growth rate that is due to the decrease in APK.
[29] Note that the paths of (ω, ξ) were normalized to make them comparable.
[30] Limit in the sense that $\lim_{\kappa \to +\infty} \omega(\kappa)$.

First, the higher-than equilibrium marginal product of capital stimulates γ_c through the familiar marginal product effect already mentioned in Result 1. Second, the thereby induced higher reference stock growth enhances the envy motive, which pushes the marginal product of own consumption and, hence, additionally accelerates consumption growth. The magnitude of the latter so-called status effect is primarily determined by and positively related to the speed of reference stock adjustment κ.

Result 4 *The adjustment process following a positive productivity shock includes periods of increasing and decreasing work effort. For adjustment speeds sufficiently high (low), the economy responds to the productivity disturbance by initially increasing (decreasing) the work effort and correspondingly decreasing (increasing) educational activities. Having reached the turning point A, these effects are exactly reversed*[31].

4.3.2 Case 2: Preference Shocks

In the second case we address the adjustment effects in response to a preference shock. This shock which is induced by a joint change in the parameters β and κ is designed to mimic a certain phenomenon that was clearly observable in the recent past and substantially affected the market of a particular status good which is the mobile phone. This phenomenon can be characterized by the contraction of the product's life cycle that was accompanied by an increase in the degree of individuals' "consumption enviousness". When the mobile phone market started to take off in the mid-nineties, firms generally launched their products in half year or year intervals. Interestingly, during the boom period these periods have shortened significantly and so that every few months a "brand new" Nokia, Ericson, etc. enters the market. Surely, some fraction of these product-rollouts might be motivated by improved, or even innovative, technical features that potentially enrich and facilitate the wireless communication process. We claim, however, that a substantial fraction of launches is not necessarily driven by product advancements, but is rather targeted at the satisfaction of people's demand for "pseudo-innovative" products, which are "Pseudo-innovative" in the sense that it is primarily the product's design that changed rather than the underlying technology. Thus, it was simply "old wine in new bottles". At the same time, people became more and more keen to acquire new phones, no matter for how long they had their "old" ones. This specific habit was additionally stimulated by certain contract conditions set by the service providers that offer the customer to get a new mobile phone after a pre-specified period of time at particularly favorable terms. Since it became increasingly important to possess the newest type, mobile phones clearly deserve being labeled as a status good. In the language of our model the observed behavior can be simulated by a permanent increase in κ, i.e. recent consumption matters more, and a simultaneous decrease in β, reflecting a strengthened envy motive. As can be inferred from Table 3, variations in κ entail only moderate

[31] At a first glance the countercyclical movement in hours worked in the goods sector, as illustrated in Figure 1, can not be reconciled with the traditional *RBC* literature following Kydland and Prescott's (1982) seminal work. There a favorable productivity shock is usually followed by a boost in work effort. However, a number of recent studies, most notable Galí (1999), Francis and Ramey (2002) and Basu et al. (1998) have argued that positive technology shocks may reduce work effort in the short run which is actually consistent with our findings.

steady state effects. In particular, a boost in κ lowers comparison consumption ω, since it drives up the reference stock x, whereas all other key steady state variables remain unchanged. In contrast, a drop in β causes a multiplicity of equilibrium effects. For instance, work effort in goods production, χ, the consumption capital ratio, ν, and the capital stock ratio, ξ, are all subject to long-run reductions, while comparison consumption ω and the equilibrium growth rate γ experience a permanent boost. These features are all very well displayed in Figure 2 that summarizes the adjustment dynamics in response to the outlined preference shock. Similar to Figure 1, Panel a.) sketches the paths of comparison consumption ω and ξ in the respective phase space. Panel b.) again describes the transition of work effort and the consumption-capital ratio, whereas Panel c.) illustrates the behavior of some key aggregate variables. Panel d.) will be referred to later on. In the numerical analysis the value of $\kappa\,(=0.2)$ is chosen to generate reasonable asymptotic convergence behavior while $\beta\,(=0.7)$ is set to reflect only a modest increase in status aspiration. To trace out the transitional dynamics of the model, we observe that the joint variation in κ and β reduces the equilibrium work effort in the goods production, hence, the value of χ at impact is clearly too high relative to its new equilibrium value. Furthermore, recall that $\partial\left(\chi^{*}/\xi^{*}\right)/\partial\chi^{*}>0$. Combining both observations we can state that the ratio χ/ξ at impact is also higher than it is along the new BGP. This implies that the marginal product of effective labor in the goods producing sector, given by $(1-\alpha)\,\phi\,(\xi/\chi)^{\alpha}$, initially falls behind its new long-run value, creating an intersectoral imbalance. Reestablishing the marginal product equality across sectors necessitates, therefore, a reallocation of time toward education. The resulting reduction in working time is reflected by the solid line in Panel b.).

The adjustment dynamics of comparison consumption ω can be studied fairly briefly since we have discussed some fundamental (inter)dependencies of c and x previously. First, notice that due to $\chi/\xi>\chi^{*}/\xi^{*}$, the marginal product of physical capital at impact exceeds its new equilibrium value. This entails the familiar rate of return effect which pushes-up consumption growth. The final movement of ω is, however, composed of two opposing effects. The rate of return effect generally puts upward pressure on ω, while the accelerated reference stock growth - that is due to the jump in κ - potentially reduces it. Interestingly, even for small positive deviations of κ, the latter negative effect dominates the positive, leading to a downward-shift in comparison consumption ω. Intuitively, a reduction in β puts more weight on the comparison element in the utility representation, suggesting a higher equilibrium value of ω, while a higher reference stock, induced by an increased κ, ceteris paribus, reduces ω. For the parametrization chosen in this experiment the rise in κ suffices to generate a lower BGP value of comparison consumption. The corresponding transition process from an initially high ω to its new equilibrium value is illustrated in Panel a.). The adjustment locus of the capital stocks ratio ξ, implicitly depicted in the same phase space, features - in contrast to all other variables - a highly non-monotonic behavior. As the comparative statics exercise in Table 3 suggest, the capital ratio at impact is too high relative to its long run value. The initially high labor-capital ratio, χ/ξ, unambiguously drives up the average product of physical capital (APK) and hence stimulates investment. Consequently, the growth rate of physical capital γ_{k} experiences a boost. Even though the flow of raw time χ into the education sector pushes-up as well the human capital

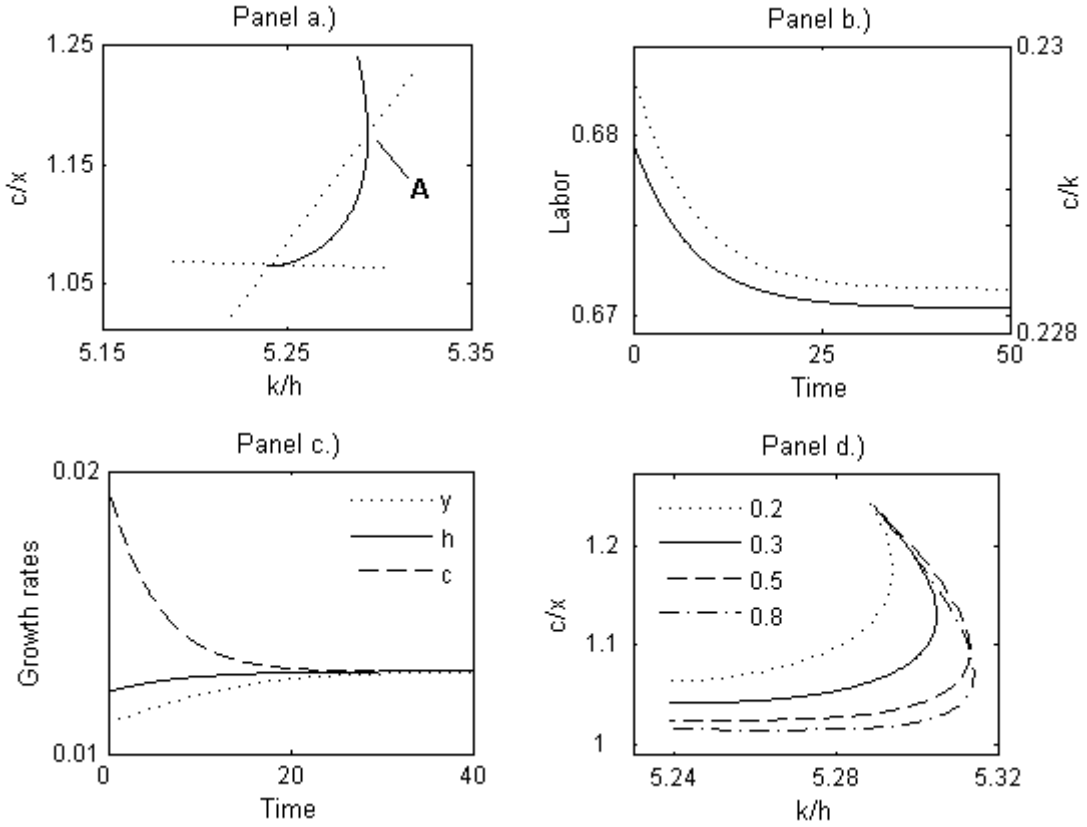

Figure 2: Adjustment Paths in Response to a Preference Shock

growth rate γ_h, the increase in γ_k initially outweighs the rise in γ_h. Hence, $\Delta\gamma_k > \Delta\gamma_h$ leads to a shift in the relative capital intensity toward physical capital, i.e. ξ rises. This situation, clearly, is not sustainable. Both the continuous outflow of χ and the rise in ξ reduce the APK of physical capital and hence γ_k, while increasing schooling efforts further push γ_h. At the turning point A, γ_k (γ_h) is reduced (increased) sufficiently so that subsequently $\gamma_k < \gamma_h$ and, thus, $\Delta\xi < 0$. The observed overshooting behavior of the relative capital intensity becomes more and more pronounced for rising values of κ as Panel d.) illustrates. This can be best explained using the agent's optimality condition for consumption. Given own consumption c, a higher κ pushes the reference stock x to a higher path and, hence, raises the marginal utility of consumption at each point in time. To re-establish optimality, a higher shadow value of physical capital is then required, which automatically poses an additional incentive to invest. The increased capital stock growth rate further promotes the overshooting. However, for very high adjustment speeds this pattern changes again, since a sufficiently high κ implies $x \approx c$ and, hence, $\omega \approx 1$. This causes an almost immediate jump to the new equilibrium value of ω, while ξ adjusts sluggishly. As a final remark concerning the equilibrium growth rate, we can state that due to the shift in raw time toward education, the human capital growth rate and, hence, the economy's aggregate growth rate rises over time. To conclude this section, we can summarize the observed (ir)regularities in the following Result.

Result 5 *The adjustment process following a preference shock - that is induced by an upward-jump, respectively reduction, in the degree of consumption jealousy and backward-looking behavior - implies an overshooting relative capital intensity, which becomes more pronounced for a more presence orientated comparison behavior.*

The quantitative implications for the economy's steady state of both, the productivity and the preference shock, are compactly summarized in Table 5. For comparison reasons it reports also pre-shock benchmark values of the respective variables.

	Labor	k/h	k/y	c/y	c/x	Growth rate
Benchmark	0.680	5.288	3.425	0.787	1.244	0.0122
Productivity Shock	0.680	5.377	3.425	0.787	1.244	0.0122
Preference Shock	0.670	5.239	3.434	0.784	1.065	0.0129

Table 5: Productivity and Preference Shock

The status augmented model proposed in this paper basically nests important classes of macro-models as polar cases. First, the conventional two sector growth model based on work by Lucas (1988) and extensively analyzed by Eicher and Turnovsky (2001), Ortigueira and Santos (2002) and Bond et al. (1996) can be mimicked by setting $\beta = 1$. In this case the interdependent fourth-order system in (16)-(19) is partly decoupled and comparison consumption ω evolves without generating any feedback effects. The underlying equilibrium and transitional dynamics of the reduced third-order system has been analyzed by Mulligan and Sala-i-Martin (1993) at great length. It crucially differs from the model at hand mainly with respect to the (non)monotonicity in the adjustment behavior. The introduced flexibility that is due to the two dimensional adjustment locus allows for explaining movements in and features of the data that the conventional two-sector growth model is not able to reproduce. Particularly, the fluctuation pattern of labor supply in response to a productivity shock appears to be more realistic than the monotonous adjustment paths implied by traditional two-sector modeling devices. Second, as mentioned at the outset for $\kappa \to \infty$, the set-up comprises a certain type of preference structure for which the literature coined the name "Keeping up with the Joneses". It makes use of the assumption that agents compare their own consumption to the current rather than past average consumption[32]. Although this type is used in a variety of applications, it features an unrealistic high degree of shortsightedness. It is, in fact, implausible that envious agents do not remember the recent consumption activities of their contemporaries. And if they do, why should they not take them into account?

5 Conclusion and Extensions

In a novel synthesis this paper incorporates status preferences - modeled as relative consumption - into a two-sector growth model in order to investigate the implications of an individual's

[32]In contrast, the non-degenerate model in this paper is of the so-called "Catching up with the Joneses" type since it assumes that agents are backward looking.

consumption jealousy on the economy's equilibrium and transitional dynamics. First and foremost we find that the presented model possesses a unique interior equilibrium that features a two-dimensional stable manifold. The properties of the associated balanced growth path depend crucially on the structural parameters that govern the status motive. For empirically plausible values of the intertemporal elasticity of substitution, we find that higher degrees of consumption jealousy tend to "push-up" human capital accumulation, hence, the economy's aggregate growth rate. This result is due to a phenomenon that we call the *reallocation effect*. More intensive status seeking drives up the marginal utility of own consumption, which induces agents to shift raw time to education in order to capitalize on the higher marginal utility in the future. We also identify a second effect in this context - labeled as the *marginal product effect* - that tends to reduce economic growth through a diminished steady-state marginal product of physical capital. However, we show that for an intertemporal elasticity of substitution consistent with empirical estimates the first effect always dominates the latter. The observed intersectoral allocation of disposable time suggests an inverse relationship of the steady-state relative capital intensity to the status motive. Hence, our model predicts that a society that values relative consumption possesses a relatively higher human capital stock than a comparable economy with a weaker status valuation. Moreover, we wish to emphasize that the calibrated benchmark economy features steady state properties that are highly consistent with actual economies. This makes us highly confident about its potential usefulness for further (probably empirical) research. One of the primary goals of the paper is to study the transitional dynamics of the two-sector model augmented with a time-dependent preferences structure. For this purpose we introduce two different shocks - a productivity shock in the final goods sector and a preference shock - into the model and analyze the resulting transitional behavior. A striking result we derive from this exercise concerns the response of work effort to a positive productivity disturbance. If agents' consumption jealousy is more history-orientated in the sense that she cares about her neighbors past (rather than recent) consumption, then labor supply in the final goods sector initially declines, generating a seemingly counterfactual behavior. This result is, however, consistent with the recent literature claiming that favorable productivity disturbances are likely to cause short-run contractions in work effort. In our case this behavior is due to temporary productivity imbalances across sectors, which motivates an initial shift of raw time toward education. If the jealousy motive is, however, more present-orientated, then we get the reverse case, which can be traced back to a phenomenon which we call the status effect. More specifically, present-orientation causes rapid reference stock adjustment and, hence, temporarily pushes-up consumption growth, which can be sustained only if labor moves to the consumption goods sector. The case of a preference shock that is designed to mimic a shift to stronger and more presence orientated consumption envy teaches us different lessons. An induced permanent shift to education pushes the economy's long-term growth rate and alters the steady-state relative capital intensity in favor of human capital. The short-run response of the latter is, however, characterized by an overshooting behavior. This results from an initial stimulus for physical capital investment that is due to a higher-than equilibrium average product of physical capital. As the latter declines over time and approaches its long-run value, the physical capital growth rate

falls short the human capital growth rate, resulting in a falling physical to human capital ratio. Additionally, the overshooting gets more pronounced for more present-orientated jealousy. As mentioned previously, rapid reference stock adjustment (caused by present-orientation) pushes the marginal utility of own consumption which poses an additional incentive (given by a higher shadow value) to invest in physical capital.

We believe this paper provides in general terms useful guidelines for studying the dynamics of a status augmented two-sector model. Moreover, due to its relatively concise structure it allows for several extensions. Introducing individuals' heterogeneity (in, e.g. initial endowments or status valuation), for instance would allow for discussing possible catching behavior that might be relevant especially in an outward-looking environment. Empirical work by Featherman and Stevens (1982) suggests a fundamental role of human capital in people's status considerations. In other words, it is relative human capital, instead of consumption (or wealth), determines individuals' position in society. In a two-sector model this would imply different incentives for agents to engage in education, which might alter the dynamic structure of the model. The presence of externalities implies that there is clearly room for welfare improving policy interventions. The discussion of various taxes and subsidies is, thus, also a worthwhile extension. In the context of relative consumption, welfare is per se an interesting issue, since consumption jealousy implies distorted individuals' decisions with respect to factor accumulation and supply. Hence, growth and welfare effects are expected to differ.

6 Appendix

6.1 Steady State Values

Setting the system in (16)-(19) equal to zero yields

$$\nu^* = \frac{\varphi}{\alpha}\left(1-\chi^*\right)^{\zeta}\left[1-\alpha+\frac{\zeta\chi^*}{1-\chi^*}\right]+\frac{(1-\alpha)}{\alpha}\left(\delta-\eta\right) \tag{25}$$

$$\omega^* = 1+\frac{\varphi\left(1-\chi^*\right)^{\zeta}-\eta}{\kappa} \tag{26}$$

$$\xi^* = \left[\frac{1}{\alpha\phi}\left[\varphi\left(1-\chi^*\right)^{\zeta}\left[1+\frac{\zeta\chi^*}{1-\chi^*}\right]+\delta-\eta\right]\right]^{-\frac{1}{1-\alpha}}\chi^* \tag{27}$$

$$\varphi\left(1-\chi^*\right)^{\zeta}\left[\left(\frac{\zeta\chi^*}{1-\chi^*}\right)+\beta\left(1-\sigma\right)\right] = \rho+\beta\eta\left(1-\sigma\right) \tag{28}$$

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