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Real and Nominal UK Interest Rates, ERM Membership and Inflation Targeting

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Abstract

This paper models the time-varying mean of the UK real and nominal short-term interest rate. Both rates mean revert to a time-varying central tendency in continuous-time interest rate models. Before and during British membership in the ERM, the mean of the real and nominal short rate have a strong negative correlation. Afterwards, when the UK implemented an inflation targeting policy, the mean of the real and nominal short rate are no longer negatively correlated, but instead have a strong positive correlation. The paper also reports empirical evidence of a relationship between the mean of the real and nominal short rate and inflation in the period before the departure from the ERM.

Keywords

ERM, inflation targeting, nominal and real rates, term structure model, UK

JEL Classification

E52, F33, G12

Comments

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1. Introduction

Various authors report significant negative correlations between real rates and inflation in terms of both realized changes and expected values (e.g., Barr and Campbell 1997, Campbell and Shiller 1996, Campbell and Viceira 2001). This negative relation between real rates and inflation is known as Mundell-Tobin effect and constitutes evidence against the Fisher neutrality hypothesis. However, the relation between real interest rates and inflation appears to have changed over time. Goto and Torous (2002) use a regime-switching model and find that the negative relationship between expected inflation and real rates in the US has switched sign in 1981, which coincides with a major change in US monetary policy. The UK implemented a major change in its monetary policy after Sterling's departure from the Exchange Rate Mechanism (ERM) of the European Monetary System in September 1992. Monetary policy in the UK was oriented towards a stable exchange rate with other European currencies and membership in the ERM up to September 1992. Afterwards, the British government established the first inflation target in the history of the UK in October 1992. Since then the UK has had low and stable inflation rates (e.g., Benati and Mumtaz 2006). This suggests a change in the relationship between real and nominal UK interest rates in September 1992, because nominal rates comprise real and inflationary components.

The objective of this paper is to study the relationship between the UK real and nominal short rate during the run-up and membership in the ERM and the period of inflation targeting afterwards. The short-term interest rate is an important policy variable and several different single factor models of the short rate have been proposed in the literature (e.g., Chan, Karolyi, Longstaff, and Sanders, 1992). Two famous special cases are the Vasicek (1977) model and the Cox, Ingersoll and Ross (1985, henceforth CIR) model. These equilibrium models of the short rate include a mean reversion component that describes the tendency of the short rate to return to its average level over time. A characteristic of single factor models is that they assume a constant mean-reverting level or central tendency. The assumption of a constant central tendency of the short rate is empirically restrictive and multifactor models with a time-varying central tendency have been proposed in the literature. One of the first examples of a model with a time-varying mean is the double decay model of Beaglehole and Tenney (1991). It describes the central tendency of the short rate with another mean reverting process. Balduzzi, Das and Foresi (1998) use yields of different maturities to model the time-varying mean of the short rate. They show for US nominal interest rates that the time-varying mean is an important second risk factor besides the short rate in equilibrium models of the nominal short rate. However, it is not clear whether the time-varying central tendency factor is specific to the behavior of the nominal short rate or also applicable to the real short rate.

This paper first investigates whether the real short rate mean reverts to a constant mean or a time-varying central tendency similar to the behavior of the nominal short rate. Brown and Schaefer (1994) estimate short rate models for a cross-section of UK inflation-indexed bonds, but they do not consider models with a time-varying central tendency. Indexed bonds are designed to neutralize the impact of inflation and reveal the term structure of real yields (e.g., Barr and Campbell 1997, Evans 1998, Sack 2000). We use inflation-indexed government bonds to capture the time-varying real interest rate issue. The availability of real yields from British index-linked bonds allows us to estimate the time-varying mean model of Balduzzi et al. (1998) for the UK real short rate.[1] The empirical results suggest a time-varying mean for the UK real and nominal short rate.

The time-varying central tendencies of the real and nominal short rate should describe the behavior of the real and nominal short rate better than the level of the real and nominal rate, because the rates mean-revert to the time-varying central tendencies. Moreover, short rate deviations from the central tendency tend to be persistent and, therefore, the short rate returns only slowly to its time-varying mean over time. The correlation coefficient of the real and nominal short rate assumes a constant central tendency for the rates. With a time-varying mean for the real and nominal short rate, the correlation between the real and nominal short rate has two components. One is due to common movements in the central tendencies of the real and nominal short rate. The other one is common deviations of the real and nominal rate from the time-varying central tendencies. Therefore, we analyze common movements in the mean of the real and nominal short rate and assess the correlation of real and nominal short rate deviations from the time-varying means. This separates the impact of common movements in the central tendencies from common deviations of the two rates from the time-varying central tendencies.

The empirical results suggest a substantially different relationship between the time-varying mean of the real and nominal short rate before and after September 1992. For the period up to Sterling's departure from the ERM the mean of the real and nominal short rate have a strong negative correlation, but afterwards, when the UK set an inflation target, the two central tendencies have a strong positive correlation. The correlation coefficient between the level of the real and nominal short

[1] The UK issues inflation-indexed government bonds since the early 1980s and the British index-linked gilts market is one of the largest markets for inflation-indexed government debt. Other countries with inflation-indexed bonds include Israel, where most government debt is indexed to inflation and the US, with the introduction of Treasury Inflation Protected Securities in 1997. Since the introduction of inflation-indexed debt in the US, several other countries have started to issue indexed bonds. Among these countries are large economies such as France, Italy, Japan and since March 2006 also Germany.

rate does not reveal this fundamental change in the relationship between the real and nominal short rate.

A recent series of papers links the latent, or unobserved, factors in affine term structure models to macroeconomic fundamentals (e.g., Ang and Piazessi 2003, Dewachter and Lyrio 2006, Evans and Marshall 2002). A related issue is the macroeconomic source of the time-varying central tendency of the real and nominal short rate in the Balduzzi et al. (1998) model. Inflation is an intuitive candidate for the time-varying mean of the nominal short rate. Over the past twenty-five years nominal interest rates decreased considerably alongside similar decreases in inflation (e.g., Campbell, 1995). Changes in monetary policy with more emphasis on price stability are the likely source of this decrease in inflation. When inflation also affects the mean of the real rate, it may then explain the change in the relationship between the mean of the real and nominal short rate. We explore the relationship of the time-varying central tendency of the real and nominal short rate with current, past and market implied expected future inflation. We find some support for a relationship of the time-varying central tendencies with inflation for the period up to the departure of Sterling from the ERM in September 1992.

The remainder of the paper is organized as follows. Section 2 considers short rate models with a time-varying mean. Section 3 presents the empirical results and Section 4 concludes.

2. Short rate models with a time-varying mean

Balduzzi et al. (1998) propose an equilibrium model of the term structure of interest rates, in which longer term yields depend on the time-varying mean of the short-term rate. This allows them to extract information about the time-varying mean of the short rate from yields with different maturities.[2] As the short-term interest rate also affects longer term yields, Balduzzi et al. (1998) present a measure of the time-varying mean that is independent of the short rate. They derive the appropriate weights in a linear combination of two bond yields to reveal the time-varying mean of the short-term interest rate process. This results in a two-factor model that encompasses the single factor models of Vasicek and CIR with a constant central tendency as special cases. The following briefly outlines the time-varying mean model of Balduzzi et al. (1998).

[2] Babbs and Nowman (1999) point out that the double decay model of Beaglehole and Tenney (1991) does not uniquely identify the unobserved central tendency of the short rate. The time-varying mean model of Balduzzi et al. (1998) is not subject to this ambiguity.

Equilibrium models of the short rate describe the behavior of the instantaneous interest rate, r, over time t.[3] Historically it has been observed that very high and very low interest rates tend to revert to normal level over time. The following process describes the behavior of the instantaneous short rate

$$dr_t = \phi(\mu_t - r_t)dt + \sqrt{\sigma_0^2 + \sigma_1^2 r_t}\, dB_t \tag{1}$$

where B is a standard Brownian motion and σ_0 and σ_1 are volatility measures of the diffusion term. The volatility of interest rate changes is constant in the Vasicek model and depends on the square root of the current level of the interest rate in CIR model. The diffusion process encompasses the volatility specifications of Vasicek when $\sigma_1 = 0$ and CIR when $\sigma_0 = 0$. Both volatility specifications have the same mean-reversion process. The short rate reverts to the time varying mean μ over time. With traditional models of the short rate this central tendency of the short rate is a constant (e.g., Chan et al. 1992). The mean reversion coefficient ϕ measures the speed of the mean reversion.

The second factor in the Balduzzi et al. (1998) model allows the mean-reverting level of the short rate to vary over time according to the following process

$$d\mu = (m_0 + m_1\mu)dt + \sqrt{s_0^2 + s_1^2 \mu}\, dW \tag{2}$$

where W is another standard Brownian motion and m_0, m_1, s_0 and s_1 are constant parameters. The covariance between the two factors, $drd\mu/dt$, is assumed constant. The required premium to compensate investor for the risk of fluctuations of r is assumed to be linear in the short rate, $\lambda_0+\lambda_1 r$, with λ_0 and λ_1 constant. The premium to compensate investors for the risk of fluctuations of μ, $\ell(\mu)$, is a smooth function of μ only. These assumptions result in an affine term structure model.[4] Therefore, the price of a risk-free discount bond with maturity τ, $P = P(r,\mu;\tau)$, has the form

$$P(r,\mu;\tau) = e^{-A(\mu;\tau)-B(\tau)r} \tag{3}$$

where $B(\tau)$ is a constant that depends only on the maturity τ of the bond, while $A(\mu,\tau)$ is a smooth function of μ for any given maturity τ. Yields are linear in r and μ when the drift and variance of the diffusion term in (2) as well as $\ell(\mu)$ are linear functions of μ and the bond price has the following form

$$P(r,\mu;\tau) = e^{-C(\tau)-B(\tau)r-D(\tau)\mu} \tag{4}$$

[3] We suppress the time subscript throughout the model section, because in continuous-time all variables that change over time have the same time subscript. A later subsection on the empirical estimation of the model shows time subscripts, because the estimation is based on discretely sampled data.

[4] With an affine term structure model the log price of a pure discount bond has a linear relation with the state variables that drive the process. Piazzesi (2005) provides a review of affine term structure models.

where $A(\mu,\tau) = C(\tau) + D(\tau)\mu$. Duffie and Kan (1996) provide sufficient conditions for yields to be linear in the underlying state variables. The solution for $B(\tau)$, subject to the initial condition that $B(0) = 0$, is known from Cox et al. (1985),

$$B(\tau) = \frac{2\left(e^{\delta\tau} - 1\right)}{\left(\lambda_1 + \delta + \phi\right)\left(e^{\delta\tau} - 1\right) + 2\delta} \tag{5}$$

and

$$\delta = \sqrt{\left(\lambda_1 + \phi\right)^2 + 2\sigma_1^2} \tag{6}$$

In equation (4) bond prices contain information about μ, but the short-term interest rate r also affects longer-term bond prices. Thus, we want to extract the information in the yield curve that the short rate does not capture.

Balduzzi et al. (1998) derive a measure for the time-varying mean from two bond yields which is independent of r and captures variations of μ only. Consider two bonds with maturities τ_1 and τ_2, respectively. The corresponding yields are

$$y(r,\mu;\tau_i) = \frac{P(r,\mu;\tau_i)}{\tau_i} = \frac{A(\mu;\tau_i) + B(\tau_i)r}{\tau_i} \quad \text{for } i = 1,2. \tag{7}$$

To obtain a measure that does not depend on r, we solve one of the above equations for r and substitute the resulting expression into the equation for the second yield. Rearranging terms yields the following outcome

$$\tau_1 B(\tau_2) y(r,\mu;\tau_1) - \tau_2 B(\tau_1) y(r,\mu;\tau_2) = A(\mu;\tau_1)B(\tau_2) - A(\mu;\tau_2)B(\tau_1) \tag{8}$$

This quantity is independent of r but captures movements in μ, as B depends only on τ and A depends only on τ and μ. When $A(\mu,\tau)$ is a linear function of μ as above in (4) then the second factor becomes

$$\mu = \frac{B(\tau_2)\left[\tau_1 y(r,\mu;\tau_1) - C(\tau_1)\right] - B(\tau_1)\left[\tau_2 y(r,\mu;\tau_2) - C(\tau_2)\right]}{B(\tau_2)D(\tau_1) - B(\tau_1)D(\tau_2)} \tag{9}$$

Defining the following constants

$$a_0 = -\frac{B(\tau_2)C(\tau_1) - B(\tau_1)C(\tau_2)}{B(\tau_2)D(\tau_1) - B(\tau_1)D(\tau_2)} \quad \text{and} \quad a_1 = \frac{1}{B(\tau_2)D(\tau_1) - B(\tau_1)D(\tau_2)} \tag{10}$$

simplifies the notation and yields for the time-varying mean factor of the short rate the relation

$$\mu = a_0 + a_1\left[B(\tau_2)\tau_1 y(r,\mu;\tau_1) - B(\tau_1)\tau_2 y(r,\mu;\tau_2)\right] \tag{11}$$

This shows that an appropriately weighted linear combination of two bond yields reveals the time-varying central tendency of the short rate. When the affine relation of equation (4) in terms of the time-varying mean, μ, holds exactly, then any maturity pair (τ_1, τ_2) results in the same estimate for the time-varying mean. Balduzzi et al. (1998) point out that the assumption of a linear relation between the drift and diffusion term with the time-varying mean μ in equation (2) may not hold. When this is the case, equation (11) for the time-varying mean should be still a reasonable linear approximation of the true functional form relating μ to any two bond yields.

2.1. Predictable changes of the short rate

The mean reversion of the short rate in equation (1) implies that the current short rate and the time-varying mean of the short rate should predict changes of the short rate. The model presented in the previous section suggests that the prices of longer maturity bonds incorporate information about the central tendency of the short rate process. Therefore, longer maturity yields should predict movements in the short-term rate. This suggests a preliminary analysis of whether longer maturity bond yields contain information about future short rate movements. Longer term bond yields should forecast movements in future short-term rates, even when we control for information contained in the current level of the short rate. Balduzzi et al. (1998) use the following regression to assess the predictive power of the bond yield with maturity τ in combination with the short rate

$$r_{t+1} - r_t = \alpha_0 + \alpha_1 r_t + \alpha_2 y_t(\tau) + e_t \tag{12}$$

The time-varying mean of the short rate in equation (11) is based on the difference between two bond yields with different maturities. In equation (12) the short rate is one of these two yields. When the short rate reverts to the time-varying mean factor of equation (11) the current short-term rate is not sufficient to predict movements in the short-term rate as the change in the short rate should be related to the difference between the yield on longer maturity bonds and the short rate. A positive coefficient for the yield with maturity τ and a negative coefficient for the short rate are consistent with movements in the short rate back to the mean. When longer maturity yields contain no information about the mean of the short rate then only the current short rate should predict changes of the short rate.

2.2. Estimation of the time-varying mean model

The estimation of the time-varying mean model uses discretely sampled data. Following Balduzzi et al. (1998) we use an Euler discretization of the stochastic differential equation (1) for the change in the short rate over the interval Δ for the estimation of the model[5]

$$r_{t+\Delta} - r_t = \phi(\mu_t - r_t)\Delta + \sqrt{\sigma_0^2 + \sigma_1^2 r_t}\,\varepsilon_t\sqrt{\Delta} \tag{13}$$

[5] The stochastic differential equation of the short rate could be estimated as an exact stochastic difference equation when the short rate mean reverts to a constant instead of a time varying mean.

where ε_t is drawn from the standard normal distribution $N(0,1)$. The estimate of the model parameters maximizes the following non-constant part of the log-likelihood function

$$-0.5\sum_{t=1}^{T}\left[\ln\left[(\sigma_0^2+\sigma_1^2 r_t)\Delta\right]+\frac{\left[r_{t+1}-r_t-\phi(\mu_t-r_t)\Delta\right]^2}{(\sigma_0^2+\sigma_1^2 r_t)\Delta}\right] \tag{14}$$

where

$$\mu_t = a_0 + a_1\left[B(\tau_2)\tau_1 y_t(\tau_1) - B(\tau_1)\tau_2 y_t(\tau_2)\right] \tag{15}$$

The time-varying mean uses a linear combination of two zero coupon bond yields at time t, $y_t(\tau_1)$ and $y_t(\tau_2)$, with maturities τ_1 and τ_2, respectively. Following Balduzzi et al. (1998), we set the risk premium parameter λ_1 equal to zero for the estimation. This makes the risk premium for variations in the short rate a constant and, thus, it depends no longer on the level of the short rate. The term structure model parameters become

$$B(\tau) = \frac{2\left(e^{\delta\tau}-1\right)}{(\delta+\phi)\left(e^{\delta\tau}-1\right)+2\delta} \tag{16}$$

and

$$\delta = \sqrt{\phi^2 + 2\sigma_1^2} \tag{17}$$

So we estimate a_0 and a_1 instead of the parameters m_0, m_1, s_0 and s_1 for the second factor. When a_1 is insignificantly different from zero, or restricted to zero, then the mean of the short rate is constant. On the other hand, when a_1 is significantly different from zero the mean of the short rate varies over time. We estimate the short rate model for the real and nominal short rate. The real short rate is the nominal short rate adjusted for inflation. The shortest measurement interval for inflation is monthly and Δ = 1/12 for monthly observations of annual interest rates. To model the time-varying mean of equation (15) we use yields on conventional bonds for the nominal short rate and yields on inflation-indexed bonds for the real short rate.

3. Empirical results

The empirical section looks at the predictability of real and nominal short rate changes with longer maturity yields, models the time-varying mean of the real and nominal short rate and analyzes the relationship between the two central tendencies. Finally, we assess the impact of inflation on the central tendencies.

3.1. Data

We proxy the nominal short rate with the three-month Treasury Bill rate and adjust it for inflation to obtain the real short rate.[6] To model the time-varying central tendency of the real and nominal short rate we use the Bank of England real and nominal yield curve data. The Bank of England yield curve data is derived from the prices of inflation-indexed and conventional bonds. For further details on this data set see Anderson and Sleath (1999). Balduzzi et al. (1998) model the time-varying central tendency from the yields of bonds with one and two years to maturity. The shortest maturity in our UK data set for the yield on inflation-indexed bonds is three years. Thus, we use for both the real and nominal short rate the yields with three and four years to maturity to model the time-varying mean. The monthly sample starts in January 1982 and ends in January 2005.

3.2. Predicting real and nominal short rate changes

This section presents a preliminary analysis of the information in longer maturity inflation-indexed and conventional bond yields about movements in future real and nominal short rates, respectively. Table 1 reports the results of the regression (12) of short rate changes on the current short rate and longer maturity yields. Panel A reports the results for the real short rate. The positive coefficients for the yields on inflation-indexed bonds are consistent with movements back to the mean. The corresponding t-ratios are all larger than one, but none of them is significantly different from zero. This offers only a weak indication that longer term real yields contain information about the mean of the real rate process. The coefficient estimates for the real short rate are highly significant and they have the correct negative sign to bring the short rate back to a constant mean. The coefficient for the real rate is very similar for different maturities of the inflation-indexed bond yield.

Panel B of Table 1 shows the results for the nominal short rate. The results suggest that nominal yields contain information about future short rate changes in addition to the nominal short rate. Nominal yields with three and four years to maturity are significant, but conventional bond yields with maturities of five years and longer are insignificantly related to changes in the short rate. This suggests that bonds with long maturities do not track the time-varying mean of the short rate as well as bonds with shorter maturities. The negative sign of the coefficient for the nominal short rate and the positive

[6] We do not observe the instantaneous interest rate and, therefore, we have to use a proxy for it. The shorter the maturity of the proxy the lower is the maturity-induced approximation error. However, Duffee (1996) shows that the one-month and two-month rate are strongly affected by idiosyncratic variations. Thus, we use the three-month Treasury Bill rate as proxy for the short rate.

Table 1
Predicting changes of the real and nominal short rate

	τ (years)								
variable	-	3	4	5	7	10	15	20	25
Panel A: Real short rate									
r_r	-0.067	-0.087	-0.094	-0.098	-0.100	-0.098	-0.094	-0.092	-0.088
(t-ratio)	(-2.98)	(-2.91)	(-2.97)	(-3.05)	(-3.16)	(-3.21)	(-3.18)	(-3.14)	(-3.03)
$y_r(\tau)$	-	0.069	0.093	0.106	0.113	0.107	0.095	0.085	0.072
(t-ratio)	-	(1.02)	(1.23)	(1.38)	(1.55)	(1.66)	(1.70)	(1.69)	(1.50)
R^2	0.029	0.032	0.035	0.037	0.038	0.038	0.036	0.035	0.033
Panel B: Nominal short rate									
r_n	-0.015	-0.091	-0.073	-0.062	-0.047	-0.035	-0.025	-0.019	-0.016
(t-ratio)	(-1.71)	(-2.60)	(-2.48)	(-2.35)	(-2.10)	(-2.81)	(-1.53)	(-1.38)	(-1.27)
$y_n(\tau)$	-	0.100	0.080	0.065	0.046	0.030	0.016	0.008	0.002
(t-ratio)	-	(2.20)	(2.00)	(1.81)	(1.48)	(1.10)	(0.67)	(0.38)	(0.11)
R^2	0.005	0.036	0.029	0.023	0.015	0.008	0.003	0.002	0.001

Notes: Results of the regression of changes in the short rate on a constant, the short rate and the zero coupon yield with τ years to maturity. Panel A reports the results for the real short rate, r_r, with zero-coupon yields from inflation-indexed bonds, $y_r(\tau)$. Panel B investigates the nominal short rate, r_n, and uses nominal yields from conventional bonds, $y_n(\tau)$. The t-ratios of the parameter estimates are based on heteroscedasticity consistent standard errors. The R^2 is adjusted for the number of explanatory variables.

coefficient for longer-maturity conventional bond yields are both consistent with movements back to the mean. When only the short rate is included in the regression the estimate suggests an insignificant relation of the short rate with the change in the short rate. Thus, for the nominal short rate a time-varying mean may be more important than for the real short rate. The real short rate also appears to revert much faster to a constant mean than the nominal short rate.

3.3. The real short rate

This section investigates the models of Vasicek and CIR with a constant central tendency and the time-varying central tendency specification of Balduzzi et al. (1998) for the real short rate. Panel A of Table 2 reports the estimation results for the real short rate. The short rate mean reverts to the constant parameter a_0 in the models with a constant mean. The estimate of the constant mean parameter is highly significant and suggests that the real short rate mean reverts to a rate of around 3.7% per annum. The different volatility specifications of Vasicek and CIR affect this estimate of the central tendency only little. The mean reversion coefficient is also significant for both volatility specifications.

Table 2
Short rate models with constant and time-varying central tendencies

Model	ϕ	a_0	a_1	σ_0	σ_1	Log-likelihood
Panel A: Real short rate						
Vasicek	0.757	0.038		0.020		1273.989
(t-ratio)	(2.48)	(5.36)		(39.16)		
CIR	0.427	0.037			0.100	1289.377
(t-ratio)	(2.11)	(3.64)			(42.88)	
Vasicek*	1.104	0.005	-1.264	0.020		1275.695
(t-ratio)	(3.28)	(0.26)	(-1.82)	(38.44)		
CIR*	0.989	-0.011	-1.747		0.099	1292.247
(t-ratio)	(2.97)	(-0.88)	(-3.16)		(39.21)	
Panel B: Nominal short rate						
Vasicek	0.194	0.061		0.017		1309.398
(t-ratio)	(1.09)	(1.44)		(46.84)		
CIR	0.163	0.057			0.057	1340.717
(t-ratio)	(1.10)	(1.58)			(49.01)	
Vasicek*	0.452	0.004	-0.794	0.017		1310.423
(t-ratio)	(1.82)	(0.08)	(-1.91)	(39.82)		
CIR*	0.555	-0.006	-0.910		0.057	1343.250
(t-ratio)	(2.08)	(-0.20)	(-3.29)		(37.52)	

Notes: Maximum likelihood estimation results of short rate models with a constant and time-varying central tendency for the real short rate (Panel A) and the nominal short rate (Panel B). Models indicated with a '*' allow for a time-varying central tendency. With the volatility specification of Vasicek (1977) $\sigma_1 = 0$ and with the CIR model specification $\sigma_0 = 0$.

The coefficient a_1 distinguishes the time-varying and constant mean specification. The time-varying mean specification collapses to the constant mean model when this additional parameter is insignificant. In Table 2 the entries Vasicek* and CIR* indicate models with a time-varying central tendency and the volatility specifications of Vasicek and CIR, respectively. With the heteroscedastic volatility specification of CIR the coefficient for the time-varying mean is highly significant with a t-ratio larger than three. Moreover, the speed of the mean reversion more than doubles when the time-varying mean factor is added in the CIR model. With the volatility specification of Vasicek the coefficient for the time-varying mean is not significant with a t-ratio equal to -1.82. This suggests that the heteroscedastic volatility specification of CIR is an important feature of the model for the real short rate.[7]

[7] The results for the Vasicek model are consistent with the insignificant coefficients for the index-linked bond yields in Table 2. The Vasicek model and the regression for the change in the real short rate both assume constant volatilities and, therefore, do not account for the CIR volatility structure.

Likelihood-ratio tests also allow us to assess the constant mean model against the time-varying central tendency model. The test statistic is twice the difference in the log-likelihood of the time-varying and constant mean model with the same volatility specification. The test is chi-square distributed with one degree of freedom, since the constant mean model restricts one parameter to zero. For the CIR model the likelihood-ratio test strongly favors the time-varying mean specification. This suggests a time-varying central tendency for the real short rate and that the volatility of the real rate depends on the level of the real rate as suggested by the CIR model.

The upper graph in Fig. 1 shows the real short rate and the fitted time-varying mean for the real short rate with the CIR volatility specification. The lower graph in Fig. 1 plots deviations of the real short rate from the time-varying and constant mean in the CIR model specification. The fitted mean of the real short rate varies approximately between one and six percent over the sample period. During the first half of the sample the time-varying mean rather exceeds the constant mean estimate of 3.7% per annum. This results in predominantly positive deviations of the real short rate from the constant mean. This suggests that the prevailing mean of the real short rate is larger than the estimate of the constant mean during this early part of the sample. Moreover, for earlier parts of the sample the volatility of rates is much higher than during the second half of the sample. The CIR volatility specification may capture this, because it assigns higher volatilities to higher rates. For the second half of the sample the fitted time-varying central tendency of the real short rate is rather smaller than the estimate of the central tendency from the constant mean model. The constant mean estimate of the real short rate is too high for this period and results in mostly negative deviations of the real short rate from the constant mean. The deviations of the real short rate from the time-varying mean are evenly distributed around zero over the sample period. This suggests that the time-varying mean model describes the mean of the real short rate considerably better than the constant mean specification.

3.4. The nominal short rate

Panel B of Table 2 reports the estimation results of the constant and time-varying central tendency model for the nominal short rate. The constant central tendency specification suggests a mean of around 6% for the nominal short rate. For the two different volatility specifications this estimate of the constant mean differs only slightly. However, the estimate of the constant mean for the nominal short rate is insignificant for both volatility structures. Moreover, for the nominal short rate the mean reversion coefficient is not significantly different from zero in the models with a constant mean. This contrasts sharply with the result for the real rate. For the real rate the constant mean model performs much better than for the nominal short rate.

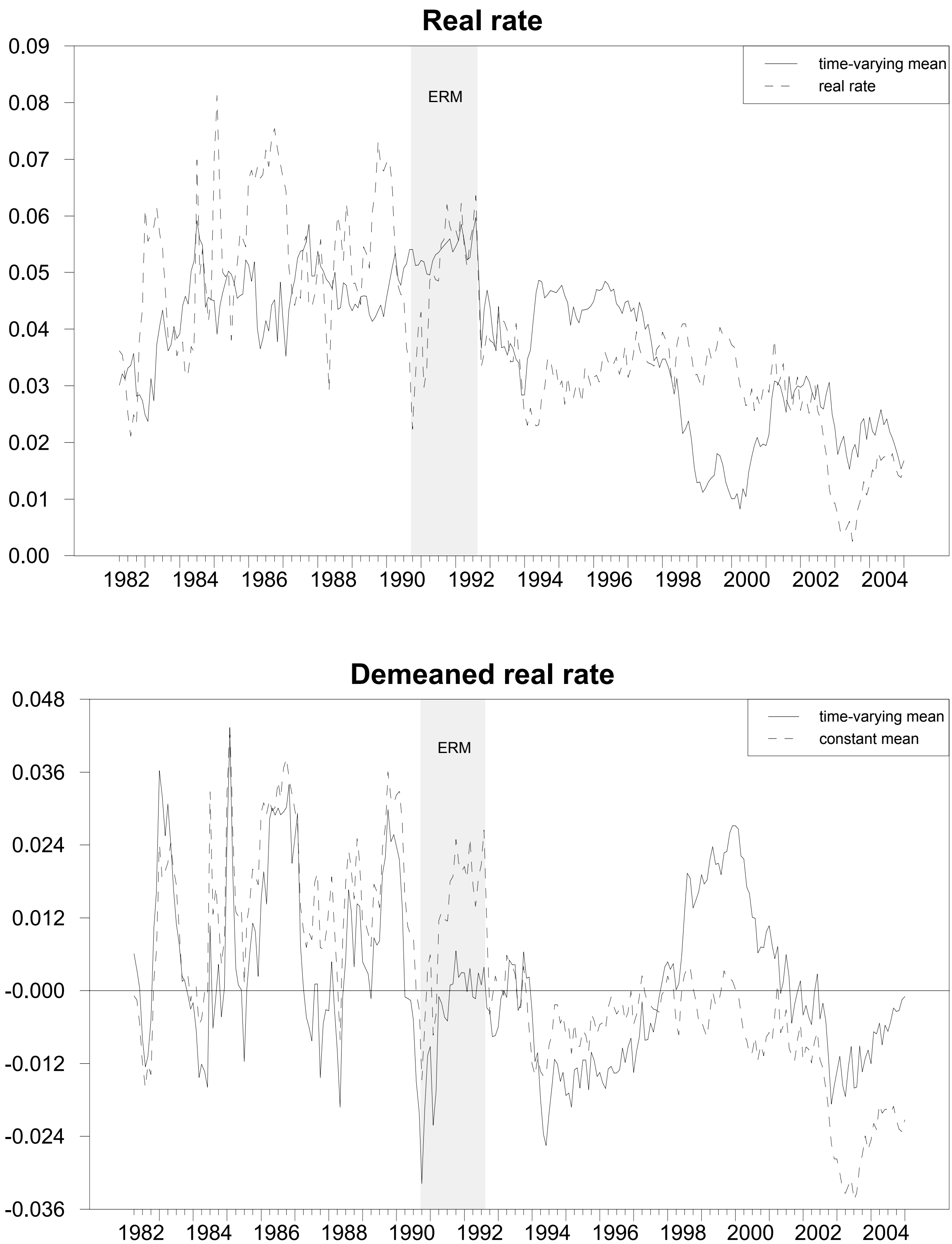


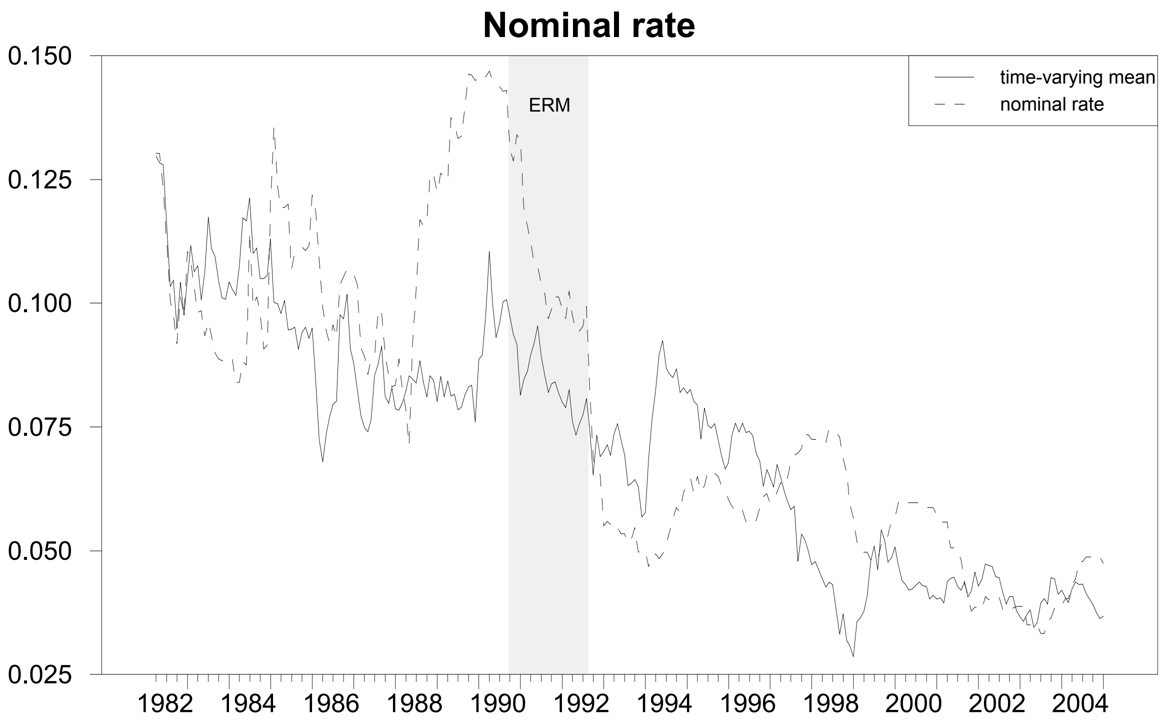
Fig. 1: The real short rate with the CIR model

The coefficient for the time-varying mean is highly significant in the model with the CIR volatility structure. For the volatility specification of Vasicek the time-varying mean coefficient is close to significant. The log-likelihood ratio test statistic also suggests a significant increase in the fit of the CIR model when the constant mean is replaced by the time-varying mean specification. The positive sign of the mean reversion coefficient leads to short rate movements back to equilibrium with both the constant and the time-varying mean specification. The mean reversion coefficient is considerably larger for the models with a time-varying mean. This indicates that the nominal short rate mean reverts much faster to the time-varying central tendency than to the constant central tendency. The mean reversion parameter is only for the CIR model with a time-varying mean specification significantly different from zero. This suggests a time-varying mean for the nominal short rate with a CIR volatility specification.

The upper graph in Fig. 2 shows the nominal short rate and its time-varying mean from the CIR model specification. Over the full sample period the estimate of the time-varying central tendency varies considerably in a range from approximately 3% to 13% per annum. The nominal short rate and the estimate of its time-varying mean both decrease over time, as large values are predominately found in the earlier part of the sample and low values characterize the later part of the sample. This suggests that a constant central tendency is too low for earlier parts of the sample and too high for later parts of the sample. The lower graph in Fig. 2 plots deviations of the nominal short rate from the time-varying and constant mean in the CIR model. For earlier parts of the sample a constant central tendency is too low and results in only positive deviations of the nominal short rate from the constant mean. For later parts of the sample the constant mean is too high and the deviations of the nominal short rate from the constant mean are mostly negative. On the other hand, deviations of the nominal short rate from the time-varying mean are positive and negative at various time periods. This suggests that a time-varying mean describes the behavior of the nominal short rate, but a constant mean does not. This demonstrates the need for a model with a time-varying mean of the nominal short rate.

3.5. The relationship between the real and nominal central tendency

The estimation results of the CIR model with a time-varying mean for the real and nominal short rate suggest that both have a time-varying central tendency. In the following we examine the relation between the central tendency of the real and nominal short rate for the full sample period and two subperiods. We investigate the period while the UK shadowed the deutsche mark and became a member of the ERM versus the period after ERM membership when the UK started to implement an inflation target. Thus, we split the sample at the date of the exit of the British pound from the ERM in September 1992. This should reveal whether the relation between the real and nominal short rate is different for these two periods.



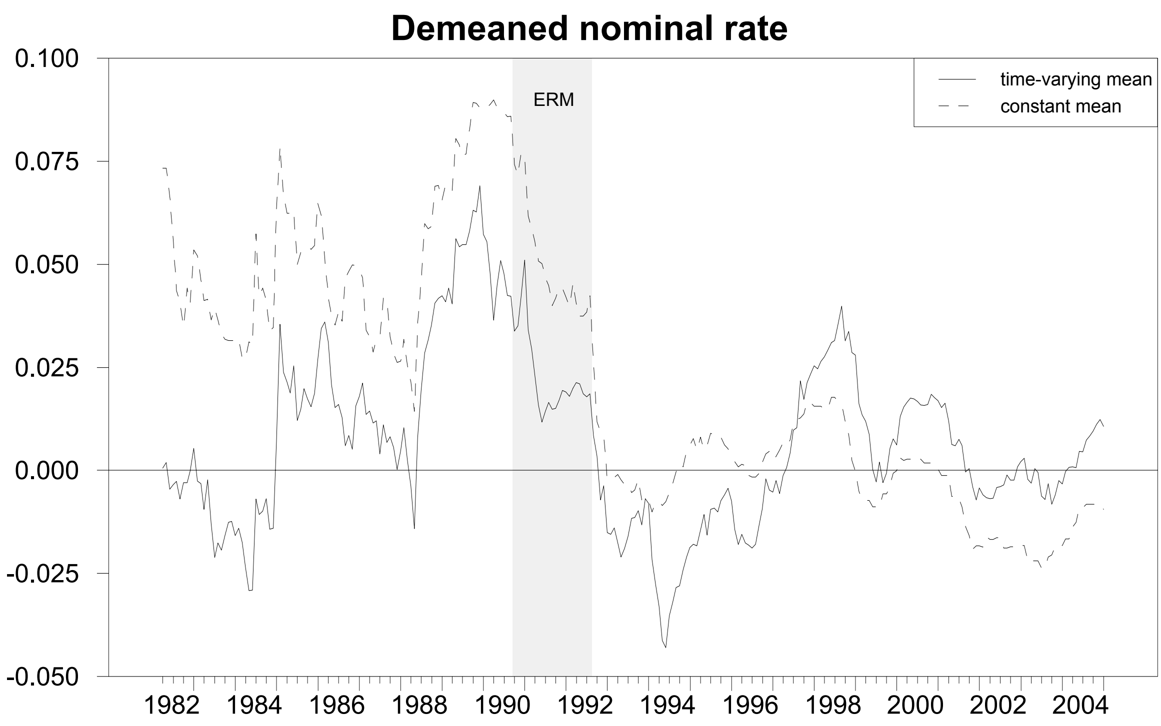


Fig. 2: The nominal short rate with the CIR model

Table 3
Analysis of the real and nominal central tendency

Statistic	Full sample period	Up to ERM exit	After ERM exit
	1982:1 to 2005:1	1982:1 to 1992:9	1992:10 to 2005:1
Panel A: Averages (% per annum)			
μ_r	3.76	4.60	3.04
μ_n	7.17	9.23	5.42
r_r	3.91	5.13	2.86
r_n	7.91	10.62	5.40
Panel B: Standard deviations (% per annum)			
μ_r	1.27	0.78	1.17
μ_n	2.41	1.32	1.61
r_r	1.59	1.28	0.95
r_n	3.11	1.89	1.08
Panel C: Correlation coefficients			
μ_n and μ_r	0.70	-0.38	0.85
$(r_n-\mu_n)$ and $(r_r-\mu_r)$	0.45	0.20	0.65
r_n and r_r	0.75	0.21	0.70
$\Delta\mu_n$ and $\Delta\mu_r$	0.30	0.18	0.51

Notes: Analysis of the real short rate, r_r, nominal short rate, r_n, the time-varying mean of the real short rate, μ_r, and the time-varying mean of the nominal short rate, μ_n, in the CIR model. Panel A reports averages, Panel B standard deviations and Panel C correlation coefficients for the full sample period from January 1982 to January 2005 and a sample split in September 1992.

Table 3 reports averages, variances and correlations for the real and nominal short rate and the time-varying central tendencies in the CIR model. The average of the real short rate is approximately 50% larger before and during ERM membership than afterwards. The fitted central tendency of the real short rate also tracks this change in the mean of the real rate. The average of the nominal short rate is twice as high during the first period than during the second period. The fitted time-varying mean for the nominal short rate mirrors this change in the average of the nominal short rate.

The variability of the rates and fitted central tendencies is larger for the full sample period than during the two subperiods. This is consistent with a change in the mean of the real and nominal short rate over time. For the full sample and the ERM period the variability of the real and nominal short rate is higher than the variability of the corresponding time-varying central tendency from the short rate model. However, for the period after the ERM the variability of the real and nominal short rate is lower than the variability of the corresponding time-varying central tendency. In each of the three sample periods the variability of the nominal short rate and its central tendency exceed the variability of the real short rate and its central tendency, respectively. The variability of the mean of the real rate is larger during the inflation targeting period than during the ERM period, but for the variability of the real rate the opposite is the case.

The two time-varying central tendencies characterize the equilibrium level of the real and nominal short rate. Table 3 reports the correlation between the time-varying mean of the real and nominal short rate. For the period up to the departure of Sterling from the ERM the mean of the real and nominal short rate have a strong negative correlation coefficient of -0.38. However, for the period after the ERM the mean of the real and nominal rate have a strong positive correlation coefficient of 0.85. The upper graph in Fig. 3 jointly plots the time-varying mean of the real and nominal short rate with the CIR volatility specification. During the 1980s the mean of the real short rate steadily increases and has a maximum at the date of the ERM departure.[8] On the other hand, the mean of the nominal short rate decreases. This explains the strong negative correlation between the mean of the real and nominal rate for the period before and during ERM membership. After September 1992 the UK implemented an inflation target and the two central tendencies move up and down together over time. This explains the strong positive correlation between the two central tendencies during the inflation targeting period.

The lower graph in Fig. 3 shows a scatter plot of the time-varying mean of the real and nominal short rate for the period before and during ERM membership and the inflation targeting period. For the full sample period the scatter plot indicates a positive relation between the two central tendencies. Table 3 reports a large positive correlation coefficient of 0.70 for the full sample. However, the relation between the two central tendencies is clearly different for the ERM and inflation targeting period. The graph suggests that the mean of the real and nominal rate scatter around a 45-degree line during the inflation targeting period. An approximate line for this relation crosses the axis of the mean of the nominal rate at around 2.5% per annum, which is the mid point of the early target band of 1% to 4% inflation for the UK.[9] Moreover, two 45-degree lines that represent the UK inflation target band

[8] The high real interest rates in the UK before the departure from the ERM put a substantial strain on the British economy and lead to poor economic conditions. Some market participants interpreted these conditions as not sustainable and the subsequent speculative attack against the pound forced Britain out of the ERM. Fig. 3 shows a sharp drop in the mean of the real short rate in September 1992.

[9] In June 1995 the initial target band of a 1% to 4% range (with a midpoint of 2.5% per annum) was reformulated to an explicit medium-term point target of 2.5% per annum. Subsequently, in the process of granting the Bank of England operational independence from the Treasury in May 1997, the Chancellor announced a symmetric inflation point target of 2.5% at all times with an equal weight to above and below target inflation. In December 2003 the target rate was reduced to 2% per annum.

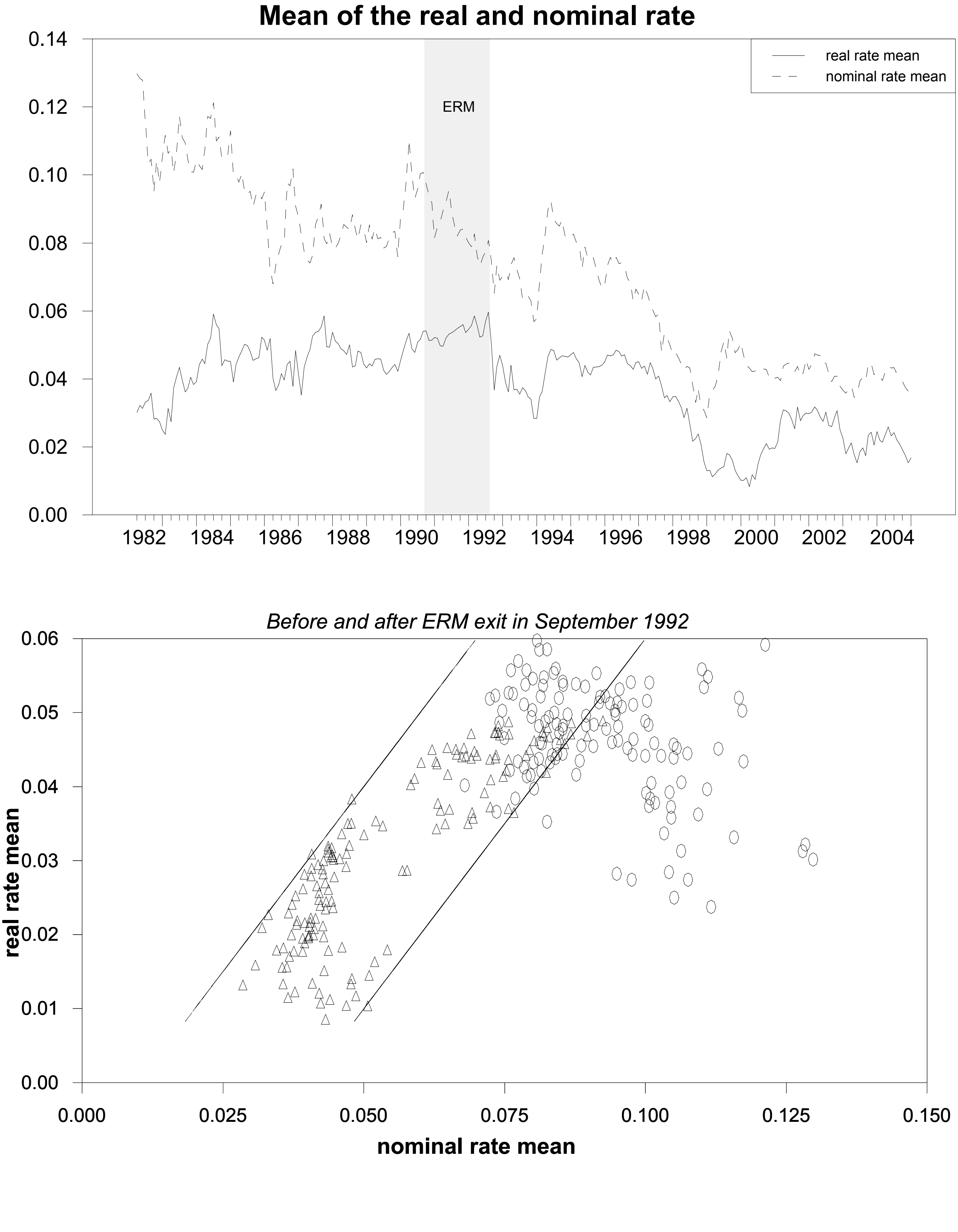


Fig. 3: The time-varying mean of the real and nominal short rate with the CIR model.

capture most of the observed combinations for the time-varying mean of the real and nominal short rate during the inflation targeting period.

The correlation of the real and nominal short rate is based on a constant mean of the real and nominal short rate. Modeling the time-varying mean of the real and nominal short rate allows us to analyze the correlation of real and nominal short rate deviations from the time-varying central tendencies. This correlation of real and nominal short rate deviations from the time-varying central tendencies is positive for all three sample periods and similar to the correlation of the real and nominal short rate. Fig. 1 and Fig. 2 reveal very similar tendencies for the real and nominal short rate in the deviations from the constant and time-varying mean, respectively. This suggests that deviations from the mean are the dominant force behind the correlation of the real and nominal rate. Movements in the time-varying central tendencies of the real and nominal rate appear to have only a small impact on the correlation of the two rates. This explains why the correlation coefficient between the real and nominal short rate does not pick up the change in the relation between the real and nominal rate. Only the time-varying central tendencies from the short rate model capture the change in the relation between the real and nominal short rate.

The correlation of changes in the time-varying mean of the real and nominal short rate is another measure of a link between the time-varying mean of the real and nominal short rate. The linkage between the two central tendencies is much stronger than the relation between changes in the two central tendencies. Furthermore, the correlation of changes in the mean of the nominal and real short rate is positive for the full sample period and the two sub-periods.

3.6. The real and nominal central tendency and inflation

The following assesses the relation of the central tendency of the real and nominal short rate with inflation. The analysis of the mean of the real and nominal short rate suggests that the relationship between the two central tendencies has changed over time. Inflation is one possible source for this variation. Thus, an interesting issue is the relation of the time-varying mean of the real and nominal short rate with current and past inflation. Moreover, as financial market participants are forward looking, the central tendencies may be related to market expectations about future inflation. The Bank of England yield curve data contains a measure of market-implied expected future inflation rates. This allows us to assess the relation of the central tendencies with expected future inflation.

Table 4

Relation of the mean-reverting level of the real and nominal rate with inflation

	Full sample period				Up to ERM exit				After ERM exit			
	1982:1 to 2005:1				1982:1 to 1992:9				1992:10 to 2005:1			
Panel A: Time-varying mean of the real short rate												
constant	2.73	3.03	1.94	1.94	4.72	5.40	6.70	7.20	2.73	2.54	-0.65	-0.75
(t-ratio)	(20.0)	(19.2)	(11.8)	(12.3)	(22.6)	(31.8)	(14.3)	(17.9)	(10.3)	(8.1)	(-2.5)	(-1.9)
π_t	0.26			-0.06	-0.02			0.12	0.13			-0.01
(t-ratio)	(10.1)			(-1.3)	(-0.7)			(4.1)	(1.3)			(-0.2)
π_{t-12}		0.17		-0.08		-0.12		-0.11		0.19		0.06
(t-ratio)		(5.2)		(-2.0)		(-4.1)		(-3.9)		(1.7)		(0.84)
$E_t\pi_{t+36}$			0.38	0.50			-0.31	-0.37			1.20	1.20
(t-ratio)			(11.5)	(11.1)			(-4.4)	(-5.4)			(16.3)	(17.0)
R^2	0.19	0.12	0.38	0.40	-0.00	0.16	0.16	0.33	0.00	0.01	0.54	0.53
[Significance]	[0.00]	[0.00]	[0.00]	[0.00]	[0.51]	[0.00]	[0.00]	[0.00]	[0.21]	[0.09]	[0.00]	[0.00]
Panel B: Time-varying mean of the nominal short rate												
constant	4.37	4.54	2.18	2.18	8.24	8.04	4.40	3.93	5.47	4.95	-0.74	0.31
(t-ratio)	(20.6)	(22.0)	(14.6)	(15.7)	(24.5)	(26.7)	(7.4)	(7.0)	(15.2)	(10.4)	(-3.2)	(1.0)
π_t	0.70			-0.21	0.17			-0.07	-0.02			-0.35
(t-ratio)	(15.6)			(-5.2)	(3.0)			(-1.8)	(-0.1)			(-5.2)
π_{t-12}		0.60		0.07		0.18		0.13		0.18		-0.15
(t-ratio)		(16.6)		(2.1)		(4.0)		(4.0)		(1.1)		(-2.3)
$E_t\pi_{t+36}$			1.04	1.15			0.71	0.71			2.01	2.1
(t-ratio)			(32.1)	(20.0)			(7.7)	(7.91)			(27.4)	(30.1)
R^2	0.40	0.44	0.80	0.82	0.06	0.12	0.31	0.36	-0.01	0.00	0.80	0.82
[Significance]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.90]	[0.28]	[0.00]	[0.00]

Notes: The table reports the results of univariate and multivariate regressions of the fitted central tendency of the real rate (Panel A) and the nominal rate (Panel B) on current inflation, past inflation and expected future inflation. The t-ratios of the parameters are based on heteroscedasticity consistent standard errors and the R^2 is adjusted for the number of explanatory variables. The [Significance] entry is the joint significance level of the explanatory variables.

Table 4 reports the results of univariate and multivariate regressions of the time-varying mean of the real and nominal short rate on current inflation, past inflation and expected inflation in three years. On average higher inflation should lead to higher nominal interest rates. Panel B of Table 4 reports a significant positive coefficient of the mean of the nominal short rate with current inflation during the ERM period. Moreover, the mean of the nominal short rate has also a positive relation with lagged inflation and expected future inflation. The negative correlation reported earlier between the central tendency of the real rate and nominal rate for the period up to the ERM departure may be due to a negative relation between real rates and inflation. The results in Panel A of Table 4 for the central tendency of the real short rate offer some support for this proposition. During the ERM period the mean of the real rate has an insignificant negative coefficient with current inflation. When we consider lagged inflation or expected future inflation the time-varying mean of the real short rate has a significant negative relation with inflation for the period up to the ERM departure. This is consistent with the negative correlation between of the two central tendencies during this period.

For the period after the ERM neither the mean of the real rate nor the mean of the nominal rate have a significant coefficient with current inflation. Similarly, lagged inflation is also insignificantly related with the central tendencies. However, both central tendencies have a strong positive relation with expected future inflation. These positive relations are consistent with a positive correlation between the two central tendencies since the government sets an inflation target. However, the results for expected inflation have to be interpreted with some caution, because the expected inflation figures and the time-varying central tendencies are both derived from the yields of nominal and indexed bonds. For the full sample period all three inflation measures have highly significant positive coefficients with the two central tendencies. This is also consistent with the reported positive correlation between the mean of the real and nominal rate for the full sample period.

To investigate the relative importance of the three inflation measures we estimate multivariate regressions of the central tendencies on current inflation, lagged inflation and expected future inflation. For the mean of the real rate the coefficient on lagged inflation is negative and significant for the full sample period and the ERM period. Expected inflation has a significant negative coefficient in the ERM period and a significant positive coefficient for the inflation targeting and full sample period. In the multivariate regression of the mean of the nominal short rate only the coefficient for expected inflation is significant and positive in all three sample periods. Current inflation has a negative coefficient and lagged inflation a positive coefficient for the full and ERM sample period, but they are both negative during the inflation targeting period.

4. Conclusions

This paper models the real and nominal short rate with term structure models that allow for a time-varying mean. We derive the time-varying central tendency of the real and nominal short rate from the yields of inflation-indexed and nominal government bonds, respectively. For both, the real and nominal UK short rate, the time-varying central tendency factor is significant in the CIR model. Modeling the time-varying mean of the real and nominal short rate provides interesting insights. During the run-up and British membership in the ERM a strong negative correlation between the mean of the real and nominal short rate emerges. After the departure from the ERM the UK implemented and maintained an inflation targeting policy. Since this switch in UK monetary policy towards inflation targeting the mean of the real and nominal short rate have a strong positive correlation.

Unlike the relation between the central tendencies, the correlation between the real and nominal short rate is positive throughout. Thus, only modeling the central tendency of the real and nominal short rate reveals this change in the relation between real and nominal interest rates. The paper also investigates the relation of the real and nominal central tendency with current, past and expected future inflation. For the period up to the departure from the ERM the mean of the nominal short rate has a positive relation with inflation, but the mean of the real short rate has a negative relation with inflation. This suggests that inflation may explain the negative relationship between the mean of the real and nominal short rate during the ERM period.

References

Anderson, N. and J. Sleath, 1999. New Estimates of the UK Real and Nominal Yield Curves. Bank of England Quarterly Bulletin, 384–92.

Ang, A. and M. Piazzesi, 2003. A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables. Journal of Monetary Economics 50, 745–787.

Babbs, S. H. and K. B. Nowman, 1999. Kalman Filtering of Generalized Vasicek Term Structure Models. Journal of Quantitative and Financial Analysis 34, 115–130.

Balduzzi, P., Das, S. R. and S. Foresi, 1998. The Central Tendency: A Second Factor in Bond Yields. Review of Economics and Statistics 80, 62–72.

Barr, D. G. and J. Y. Campbell, 1997. Inflation, Real Interest Rates and the Bond Market: A Study of UK Nominal and Index-Linked Government Bond Prices. Journal of Monetary Economics 39, 361–383.

Beaglehole, D. R. and M. Tenney, 1991. General Solution of Some Interest Rate-Contingent Claim Pricing Equations. Journal of Fixed Income 1, 69–83.

Benati, L. and H. M. Mumtaz, 2006. The 'Great Stability' in the U.K.: Good Policy or Good Luck?. forthcoming Journal of Money, Credit and Banking.

Brown, R. H. and S. M. Schaefer, 1994. The Term Structure of Real Interest Rates and the Cox, Ingersoll, and Ross Model. Journal of Financial Economics 18, 3–42.

Campbell, J. Y., 1995. Some Lessons from the Yield Curve. Journal of Economic Perspectives 9, 129–152.

Campbell, J. Y. and R. Shiller, 1996. A Scorecard for Indexed Government Debt. In NBER Macroeconomics Annual, edited by Bernanke, B. S. and J. J. Rotemberg, MIT Press, London.

Campbell, J. Y. and L. M. Viceira, 2001. Who Should Buy Long-Term Bonds?. American Economic Review 91, 99–127.

Chan, K., Karolyi, G., Longstaff, F. and A. Sanders, 1992. An Empirical Comparison of Alternative Models of the Short-Term Interest Rate. Journal of Finance 47, 1209–27.

Cox, J. C., Ingersoll, J. and S. A. Ross, 1985. A Theory of the Term Structure of Interest Rates. Econometrica 53, 385–407.

Dewachter, H. and M. Lyrio, 2006. Macro Factors and the Term Structure of Interest Rates. Journal of Money, Credit, and Banking 38, 119–140.

Duffee, G. R., 1996. Idiosyncratic Variation of Treasury Bill Yields. Journal of Finance 51, 527–552.

Duffie, D. and R. Kan, 1996. A Yield Factor Model of Interest Rates. Mathematical Finance 6, 379–406.

Evans, C. and D. Marshall, 2002. Economic Determinants of the Nominal Treasury Yield Curve. Federal Reserve Bank of Chicago, Working Paper 2001-16 Revised, September 3 2002.

Evans, M. D. D., 1998. Real Rates, Expected Inflation, and Inflation Risk Premia. Journal of Finance 53, 187–218.

Goto, S. and W. Torous, 2002. Evolving Inflation Dynamics, Monetary Policy and the Fisher Hypothesis. UCLA Working Paper.

Piazzesi, M., 2005. Affine Term Structure Models. Handbook of Financial Econometrics, Edited by Yacine Ait-Sahalia and Lars Peter Hansen, Elsevier Science.

Sack, B., 2000. Deriving Inflation Expectations from Nominal and Inflation-Indexed Treasury Yields. Journal of Fixed Income 10, 6–17.

Vasicek, O., 1977. An Equilibrium Characterization of the Term Structure. Journal of Financial Economics 5, 77–188.

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