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On Aghion's and Blanchard's "On the Speed of Transition in Central Europe"

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Abstract

In this paper we derive the correct solution of optimal closure of the state sector studied in Section 6.4 of Aghion and Blanchard (1994). Aghion and Blanchard only present an 'approximate' solution which entails a constant unemployment rate in what they call a turnpike approximation. We show that optimal unemployment paths have two features. First, unemployment is increasing up to a certain point in time, when, second, the remaining inefficient state sector is closed down. At that point in time, which we may define as the end of transition, unemployment is discontinuous. The approximate solution presented by Aghion and Blanchard is thus found to lead to welfare losses compared to the optimal policy. In particular, the unemployment rate corresponding to the solution presented in Aghion and Blanchard is too low. Our solution is formally based on transforming the dynamic optimization problem to a scrap value problem with free terminal time.

Keywords

Transition, optimal unemployment rate, dynamic optimization

JEL Classification

C61, E61, P20

Comments

Part of this work was done whilst Eric Nævdal was a post doctoral fellow at Princeton University and Martin Wagner was a visiting scholar at Princeton University and the European University Institute. The hospitality of these institutions is gratefully acknowledged.

Contents

1 Introduction

Aghion and Blanchard (1994) is an early, important contribution to the literature on economic transition from centrally planned to market economies. In their Section 6.4 they present a dynamic optimization model to determine the optimal speed of transition and the resultant optimal unemployment rate (see also the description in Roland, 2000). The solid microeconomic foundations of this dynamic model make it a valuable tool for analyzing optimal macro-behavior of a transition economy.

When solving the dynamic optimization problem, Aghion and Blanchard do not, in fact, derive the exact solution but only an 'approximate' solution, which neglects the behavior of the economy after the state sector has been closed down, see in particular their footnote 33 on p. 305. We derive in this note the correct solution to the dynamic optimization problem and show that the optimal path has several interesting features that have not been noticed by Aghion and Blanchard. It turns out that a proper analysis of the model gives rise to richer dynamics than might be expected when resorting to what Aghion and Blanchard label 'turnpike' approximation.

We show that (correct) optimal paths have the following properties: Up to a certain point in time, say τ, the government assumes an active role on the labor market by shrinking the inefficient state sector. This is done at an increasing rate, hence the optimal unemployment rate is not constant. At τ the government closes down the (remaining) inefficient state sector and does not intervene in the labor market any further. Hence, at τ the unemployment rate jumps and from there on gradually moves towards zero. Thus, in this model we may define the end of transition as τ, where the remaining state sector is closed down in a discontinuous fashion. It holds that τ is endogenous and has to be chosen optimally by the government. Further, it holds that the correct optimal unemployment rate is larger than the rate proposed by Aghion and Blanchard over the transition period. Thus, the path obtained by Aghion and Blanchard leads to welfare losses.

The result presented here may perhaps be best understood by noticing that the problem is formally similar to the problem of extracting an exhaustible resource. The stock of individuals employed in the state sector is the resource that can be 'mined'. The difference to the resource problem lies in that the process of mining a resource yields profits that represent instantaneous benefits, whereas in the present model, mining (i.e. unemployment) is costly. This explains why models of exhaustible resources predict that resources should be mined at a decreasing rate whereas the present model prescribes that unemployment should increase over the interval $[0, \tau)$.

The paper is organized as follows: In Section 2 we set up and analyze the Aghion and Blanchard (1994) model in detail and Section 3 briefly concludes.

2 The Model and the Solution of Aghion and Blanchard

We restrict the description of the model on the dynamic optimization problem presented in Section 6.4 of Aghion and Blanchard (1994) and discuss only those parts of the analysis presented in their paper in detail that are immediately relevant here.

Denote with $E(t)$ the number of people employed in the state sector (with constant marginal productivity x), with $N(t)$ the number of people employed in the emerging private sector (with constant marginal productivity y) and with $U(t)$ the number of unemployed people at time t. Population is normalized to one, i.e. $E(t)+N(t)+U(t) = 1$. At the outset of transition, employment in the state sector drops from 1 to $E(0) < 1$. Aghion and Blanchard (1994) develop an efficiency-wage based explanation for costly labor adjustment between the old state and the new private sector. In particular, they derive the following relationship for the speed of job creation in the new private sector (developed in equation (9) on page 298):[1]

$$\dot{N} = f(U) = a\left[\frac{U}{U+ca}\right]\left[y - rc - \left(\frac{b}{1-U}\right)\right] \tag{1}$$

with a, b, c and r constants. The cost of job creation in the private sector is given by $\frac{1}{2ar}(f(U))^2$. The state sector declines over time and the government

[1] To avoid overloaded notation we sometimes skip the time index t.

chooses the speed of closure and hence of unemployment.

The government is only concerned with efficiency and chooses employment in the state sector[2] to maximize the present discounted value of output. This optimization problem is given by:

$$\max_{E(t)} \int_0^\infty \left[E(t)x + N(t)y - \frac{1}{2ar} f\left(U(t)\right)^2 \right] e^{-rt} dt \tag{2}$$

subject to:

$$\dot{N}(t) = f(U(t)) \tag{3}$$

$$N(0) = 0 \tag{4}$$

$$E(t) + N(t) + U(t) = 1 \tag{5}$$

and non-negativity of $E(t)$, $N(t)$ and $U(t)$.

Based on the relation that $E(t)+N(t)+U(t) = 1$, one immediately observes that the problem can equivalently be formulated by using $U(t)$ as the control and by eliminating $E(t)$, which leaves us with only $U(t)$ and $N(t)$ in both the objective function and the constraints.[3] This formulation of the problem is given by:

$$\max_{U \in [0,1]} \int_0^\infty \left[(1 - N(t) - U(t))x + N(t)y - \frac{1}{2ar} f\left(U(t)\right)^2 \right] e^{-rt} dt \tag{6}$$

subject to:

$$\dot{N}(t) = f(U(t)) \tag{7}$$

$$N(t) \in [0, 1] \tag{8}$$

$$N(t) + U(t) \leq 1 \tag{9}$$

Note first that an optimal path may have one of the following properties: There exists a $\tau < \infty$ such that $\tau = \inf_{t \geq 0}(N(t) + U(t) = 1)$ or condition (9) is not binding for any finite t. These two cases will be discussed separately below. Before doing so, an important property of the model is derived in Proposition 1.

Proposition 1 *Along any path it holds that $N(t) < 1$ for all $t < \infty$.*

[2] See below that this is equivalent to choosing unemployment.

[3] We perform this substitution to have U, postulated to be constant along optimal paths by Aghion and Blanchard (1994), as the control variable.

Proof: For values of $N(t)$ sufficiently close to 1, the largest possible value of $\dot{N}(t)$ is given by setting $U(t) = 1 - N(t)$. The ordinary differential equation $\dot{N}(t) = f(1 - N(t))$ has a stable steady state at $N = 1$, hence $N(t)$ approaches 1 only asymptotically. $\square$

An additional problem with the model is that that the Hamiltonian may have two local maxima with respect to U. However, this problem can easily be dispensed with: If $\widehat{U}$ is the larger of these two maxima, then it is easy to show that there is some value $\widetilde{U} < \widehat{U}$ such that $f\left(\widetilde{U}\right) = f\left(\widehat{U}\right)$. If this is the case, then $\widetilde{U}$ leads to the same rate of job creation at a lesser cost, so $\widehat{U}$ cannot be optimal. Hence, we can disregard the possibility of two local maxima of the Hamiltonian in the sequel.

Let us now turn to study the possible optimal paths in detail. We start with the case that the constraint (9) becomes binding for the first time at some point $\tau < \infty$. Given that state sector employment is monotonically non-increasing, it follows that for $t \geq \tau$ the control problem has a trivial optimal solution. Denote with $N(t, N_\tau)$ the solution to the differential equation $\dot{N}(t) = f(1-N(t))$ solved over (τ, ∞) with initial condition $N(\tau) = N_\tau$. Note that it trivially holds that $\frac{\partial N(\tau, N_\tau)}{\partial N_\tau} = 1$. Also note that up to now both τ and N_τ are unspecified.

The objective function of the optimization problem from τ onwards is given by:

$$V(\tau, N_\tau) = \int_\tau^\infty \left[N(t, N_\tau)\, y - \frac{1}{2ar} f(1 - N(t, N_\tau))^2 \right] e^{-rt} dt \qquad (10)$$

Note the following relationships for the partial derivatives of the objective function (10):

$$\frac{\partial V(\tau, N_\tau)}{\partial \tau} = -\left[N(t, N_\tau)\, y - \frac{1}{2ar} f(1 - N(t, N_\tau))^2 \right] e^{-r\tau} \qquad (11)$$

$$\begin{aligned} \frac{\partial V(\tau, N_\tau)}{\partial N_\tau} &= \int_\tau^\infty \left[y + \frac{1}{ar} f(1 - N(t, N_\tau))\, f'(1 - N(t, N_\tau)) \right] e^{-rt} dt \\ &= \frac{y}{r} e^{-r\tau} + \int_\tau^\infty \left[\frac{1}{ar} f(1 - N(t, N_\tau))\, f'(1 - N(t, N_\tau)) \right] e^{-rt} dt \qquad (12) \end{aligned}$$

Now the optimization problem corresponding to the case considered can be rewritten as a scrap value problem with free terminal time, i.e. τ is to be chosen optimally as well:

$$\max_{U\in[0,1],\tau\in[0,\infty)}\left[\int_0^\tau\left[(1-N-U)x+Ny-\frac{1}{2ar}f\left(U\right)^2\right]e^{-rt}dt+V\left(\tau,N\left(\tau\right)\right)\right] \tag{13}$$

subject to (7), (8) and (9).

Problems of this type are studied in Seierstad and Sydsæter (1987, Theorem 3 and Note 2, p. 182–184) where necessary conditions for optimality are presented. The Hamiltonian corresponding to this problem is given by $H=(1-N-U)x+Ny-\frac{1}{2ar}f(U)^2+\mu f(U)$, where we ignore, for brevity, the other constraints (8) and (9) and the associated multipliers. It is straightforward but cumbersome to present the solution including these additional terms in the Hamiltonian. It can be shown that these constraints will not be binding, except possibly at $t=0$ and $t=\infty$.[4]

Necessary conditions for optimality are given by:

$$-x-\frac{1}{ar}f\left(U\right)f'\left(U\right)+\mu f'\left(U\right)=0 \tag{14}$$

$$\dot{\mu}=r\mu+x-y \tag{15}$$

Furthermore, the following transversality condition has to hold:

$$\mu\left(\tau\right)e^{-r\tau}=\frac{\partial V\left(\tau,N_\tau\right)}{\partial N_\tau} \tag{16}$$

The optimal terminal time τ is found from:

$$He^{-r\tau}+\frac{\partial V\left(\tau,N_\tau\right)}{\partial\tau}=0 \tag{17}$$

Equation (15) gives the following solution for $\mu(t)$:

$$\mu(t)=\frac{y-x}{r}+Ke^{rt} \tag{18}$$

Here K is a constant whose value has to be determined from the transversality condition (16).

[4]In fact, it can be shown that the only possible case where any other constraint than $U(t)+N(t)\leq 1$ is binding for $t<\infty$ is the case where $U\left(0\right)=1$, in which case $\tau=0$.

Remark 1 *The solution proposed by Aghion and Blanchard (1994) is derived from the above differential equation* (18) *by setting* $K = 0$. *This implies a constant value of the costate variable* $\mu(t) \equiv \frac{y-x}{r}$ *and thus a constant unemployment rate. Inserting* $\mu = \frac{y-x}{r}$ *in equation* (14) *leads to the solution proposed by Aghion and Blanchard (1994), see their equation (26) on p. 309.*

As noted in Aghion and Blanchard (1994) and also mentioned in the introduction, this cannot be the correct solution for all values of t*, since due to private sector job creation (which happens at a constant rate for constant unemployment) at some point the unemployment rate has to decline. We show below, however, that even before the end of transition, the optimal unemployment rate is not constant.*

Let us next determine K, or to be more precise, let us determine whether it is equal to 0 or not for optimal paths. This can be achieved by inserting (18) and (12) into the transversality condition (16). After some rearrangements this yields:

$$Ke^{r\tau} = \frac{x}{r} + \int_{\tau}^{\infty} \left[\frac{1}{ar} f\left(1 - N\left(t, N_\tau\right)\right) f'\left(1 - N\left(t, N_\tau\right)\right) \right] e^{-r(t-\tau)} dt \qquad (19)$$

In order to sign K, we need to sign the last term in the square brackets in this equation. The following proposition is helpful.

Proposition 2 *Along an optimal path,* $f'\left(U\left(t\right)\right) > 0$ *for all* t.

Proof: First, note that for any choice of τ and N_τ there is a segment $[\tau + d, \infty)$ such that $f'\left(1 - N\left(t, N_\tau\right)\right) = f'\left(U\left(t\right)\right) > 0$ for all $t \in [\tau + d, \infty)$. This is a straightforward implication of $U\left(t\right)$ becoming small as $N\left(t\right)$ goes to 1. In particular, this implies that paths where $f'\left(U\left(t\right)\right) > 0$ for all t are always feasible if $U\left(t\right)$ is chosen to be small enough. Second, note that for every $\widehat{U}$ such that $f'\left(\widehat{U}\right) < 0$, there is a value $\widetilde{U} < \widehat{U}$ such that $f\left(\widetilde{U}\right) = f\left(\widehat{U}\right)$ and $f'\left(\widetilde{U}\right) > 0$. Since $\widetilde{U}$ and $\widehat{U}$ give the same rate of job creation, but higher values of U are more costly, it follows that for the optimal choice of U it will always hold that $f'\left(U\left(t\right)\right) > 0$. Taken together these two facts imply that it is always possible to choose paths such that $f'\left(U\left(t\right)\right) > 0$ for all t and it is never optimal to choose

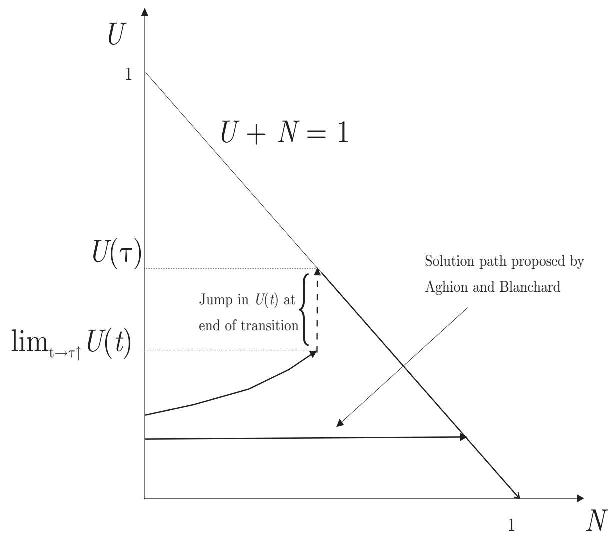

Figure 1: Illustration of optimal path of the unemployment rate, which is monotonically increasing until τ, where it jumps to $1 - N(\tau)$ to gradually decline to 0 afterwards. The figure also displays the unemployment rate corresponding to the solution proposed by Aghion and Blanchard, which is constant at a lower unemployment rate.

any other paths. Thus, the proposition follows. $\square$

Proposition 2 implies that the second term on the right hand side of (19), $f'(1 - N(t, N_\tau))$, is positive and hence, the right hand side is positive. Consequently, it follows that K is positive. This implies that $\mu(t)$ is not constant over time and thus the optimal unemployment rate is also not constant over time. In fact it follows that the optimal unemployment rate is increasing over time until τ, which is to be determined from equation (17). The fact that $K > 0$ implies that $\mu(t)$ is larger than $\frac{y-x}{r}$ for all $t < \tau$. This implies that $U(t)$ corresponding to the optimal solution is larger than derived in Aghion and Blanchard. Consequently it follows that the transition period is shorter than suggested by Aghion and Blanchard.

There is another interesting feature: The optimal unemployment rate is discontinuous at time τ and hence the optimal path for the unemployment rate is as illustrated in Figure 1. Let us now derive the result illustrated in Figure 1 analytically.

Proposition 3 *For an optimal path of the unemployment rate it holds that* $\lim_{t \to \tau^-} U(t) < 1 - N(t)$*. This implies that* $U(t)$ *is discontinuous at* τ*.*

Proof: The proof is by contradiction, therefore assume that $\lim_{t \to \tau^-} U(t) = 1 - N(t)$. Then equation (17) implies that $\mu(\tau) f(1 - N(\tau)) = 0$. This in turn implies, since $N(\tau) < 1$ (which follows from Proposition 1), that $\mu(\tau) = 0$. Then equation (18) implies that $K < 0$, since $y > x$ by assumption. However, $K < 0$ is in contradiction with (19). This shows the proposition. $\square$

To complete our analysis it remains to be shown that the second case, where condition (9) does not become binding for any finite t, cannot lead to optimal paths. Note first that, also in this case, $K \neq 0$, because $K = 0$ implies a constant unemployment rate (compare Remark 1). This follows from inserting (18) in (14), which now have to hold for all $t \geq 0$ for optimal paths. Since a constant unemployment rate implies a constant job creation rate, eventually the unemployment rate has to decrease because of constant population size. Thus, $K \neq 0$. This implies that $\mu(t)$ diverges to either plus or minus infinity, depending upon the sign of K. However, such a path of $\mu(t)$ cannot fulfill the necessary condition (14) for all $t \geq 0$, since both $f(U)$ and $f'(U)$ are bounded. This shows that indeed such paths cannot be optimal.

3 Conclusions

In this note we have studied the optimal solution for the dynamic optimization problem concerning the optimal speed of transition introduced in Aghion and Blanchard (1994, Section 6.4).

Aghion and Blanchard (1994) mention in footnote 33 of their paper that their solution is a 'turnpike' approximation to the solution and they obtain a constant optimal unemployment rate over time, implicitly assumed to hold until the state sector is shut down entirely. Neither how nor when that happens exactly is discussed in Aghion and Blanchard (1994). These questions are addressed here by transforming the dynamic optimization problem in a scrap value problem with free terminal time.

In this note we have discussed correct optimal unemployment paths, which have been found to differ in two respects from the partial solution presented in Aghion and Blanchard (1994). First, the optimal unemployment rate is increasing over time until, second, the state sector is shut down entirely at a certain point in time. This leads to a discontinuity in the unemployment rate at this point in time. The point in time where the government closes the inefficient remaining state sector entirely can be defined as the end of transition. Afterwards the government does not assume an active role in the labor market.

Finally also note that the path with constant unemployment rate as found in Aghion and Blanchard (1994) leads to welfare losses since a constant unemployment rate (until the end of transition) is not optimal. The non-constancy of the optimal unemployment rate is a similarity to the solutions typically found for exhaustible resource extraction problems. As discussed in the introduction such problems are equivalent to the problem of closing an inefficient state sector. We speculate, based on this observation, that the transition literature may borrow further insights from resource economics. Since transition is still ongoing or about to start in countries around the world, this may be a relevant line of research.

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