RATIONAL EXPECTATIONS,
STOCHASTIC COEFFICIENTS
AND MONETARY STABILIZATION POLICY

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Zusammenfassung

Diese Arbeit überprüft die Aussage von Sargent & Wallace, die besagt, daß eine systematische Geldpolitik keine Auswirkungen auf reale ökonomische Variable hat, wenn (a) die ökonomischen Akteure rationale Erwartungen, (b) das Nationalprodukt von Fehlern in den Preiserwartungen abhängt, und (c) das Modell linear ist. Es wird gezeigt werden, daß diese Aussage nicht mehr unbedingt richtig ist, wenn das ökonomische Modell multiplikative Störterme besitzt. Unter diesen Umständen kann eine systematische Geldpolitik für eine wirksame Stabilisierungspolitik verwendet werden. In der Folge werden für verschiedene multiplikative Störterme optimale Geldangebotsregeln abgeleitet.

Summary

The paper examines the validity of the Sargent-Wallace proposition that systematic monetary policy is ineffective with regard to real economic variables whenever (a) economic agents have rational expectations, (b) national product depends on errors in price expectations, and (c) the model of the economy is linear. It is shown that this policy ineffectiveness proposition is not necessarily valid when the economic model contains multiplicative disturbances. Given such disturbances, systematic monetary policy may become a useful stabilization policy device. In this vein, optimal money supply rules associated with various multiplicative disturbances are derived.

In the recent literature on macroeconomic activity under rational expectations, it has been argued that systematic monetary policy is ineffective with regard to real economic variables. In other words, changes in money supply rules leave the distributions of real variables unaffected. This argument is purported to hold for any linear stochastic macroeconomic model which embodies the natural-rate hypothesis (whereby, in one of its variants, the level of national product depends on errors in product price expectations) and the rational-expectations hypothesis (whereby the product prices anticipated by the public are equal to the mathematical expectations of these prices). The purpose of this paper is to show that the policy ineffectiveness argument does not necessarily follow from these conditions; its validity depends on the stochastic structure of the model. Linear models can contain only additive and multiplicative random disturbance terms. Whereas the policy ineffectiveness argument holds when all the disturbances are additive, it is not necessarily valid when they are multiplicative.

If the disturbance terms enter the macroeconomic model multiplicatively (i.e. if coefficients of the model's endogenous variables are stochastic), systematic monetary policy may be able to influence the distributions of real variables such as production and employment. Thus, systematic monetary policy may become useful as a stabilization-policy device. In the context of a simple macroeconomic model, this paper examines what the existence of multiplicative disturbance terms implies for the formulation of optimal monetary stabilization policy.

The postwar literature on stabilization policies covers both "descriptive" and "optimizing" analyses of these policies. The former are concerned with the implications of exogenously given, ad hoc policy rules (relating policy control variables to state variables of the economic system) for the achievement of the policy-makers' objectives (specified in terms of desired levels of the state, and possibly control, variables). In their nondeterministic guises, these analyses assume that the parameters of the model are nonstochastic but unknown to the policy maker (e.g. Cooper (1969), Mundell (1968) and Phillips (1954)). The optimizing analyses are commonly concerned with the optimization of a quadratic objective function (specified in terms of deviations of state and control variables from their respective desired levels) subject to constraints relating the state variables to the control variables.

One of the major conclusions emerging from the optimizing analyses of stabilization policies is the "certainty equivalence theorem" (Theil (1958)). According to this theorem, the optimal values of the control variables (and, for that matter, the state variables) are the same for a stochastic linear system with only additive disturbances as for its deterministic counterpart. The argument for the ineffectiveness of systematic monetary policy under rational expectations and the natural rate is simply an application of this theorem. In linear macroeconomic models embodying the natural-rate hypothesis and the assumption of perfect foresight, changes in money supply rules have no effect on real variables.[2] What Sargent and Wallace (1975) and others have shown is that the inclusion of additive disturbance terms in such models leave this result intact.

However, Brainard (1967) has shown that the certainty equivalence theorem breaks down in the presence of multiplicative disturbances. The optimal policy under multiplicative risk (i.e. stochastic coefficients) is not necessarily identical to that under additive risk. This paper pursues an optimizing (not descriptive) analysis of stabilization policy. It extends Brainard's result to macroeconomic models containing the natural-rate and rational-expectations hypotheses and thereby provides an argument for the non-neutrality of systematic monetary policy.

In accordance with the conventional formulations of stabilization policy objectives, we assume that the monetary authority seeks to minimize the deviations of national product from its "natural rate" (at which no errors in product price expectations occur). To set the stage, we first show that systematic monetary policy is irrelevant to these objectives whenever all the random disturbance terms enter the model additively. Second, we examine how multiplicative disturbance terms provide scope for stabilization policy through systematic monetary rules. Lastly, we indicate that the existence of multiplicative disturbances does not guarantee the effectiveness of systematic monetary policy.

The standard models supporting the policy ineffectiveness argument are generally composed of four building blocks: (1) an aggregate product supply function, which portrays the natural rate hypothesis, (2) an aggregate product demand function, which may be reduced to a relation among the actual price level, the expected price level, and the money supply, (3) a money supply function, which portrays the systematic and unsystematic components of the money supply, and (4) a definition of rational price expectations.

To demonstrate the policy-ineffectiveness argument, we represent these four elements by four equations containing only additive disturbances. All variables are expressed in logarithms. Let Q be national product (viz. the logarithm of national product), P the actual product price level at time t, P_e the price level at time t anticipated by the public at time t, and u^S a product-supply disturbance term. Then the aggregate product supply function may be written as

$$(1) \qquad Q = a\left[P - P_e\right] + u^S$$

u^S is a random variable; its mean, QN (the natural rate of production), and its variance are both constant:

$$E(u^S) = QN$$

$$var(u^S) = \sigma_S^2$$

The coefficient "a" is a positive constant. The model is static, thus, the time subscripts are suppressed.

Let M be the money supply and u^D a product-demand disturbance term. Then a simple reduced form of the aggregate product demand function is

$$(2) \qquad P = M + u^D$$

u^D is a random variable with constant mean and variance:

$$E(u^D) = \bar{u}^D$$

$$var(u^D) = \sigma_D^2$$

This function differs from the one commonly found in the literature on policy ineffectiveness. The usual function is actually a product market clearing condition, whose demand side is obtained from an IS and LM curve and whose supply side is given by the natural rate hypothesis. Equation (2) is chosen for algebraic simplicity. The central conclusion of this note (viz. that systematic monetary policy may be effective in the presence of stochastic coefficients) is not affected by our choice.

For simplicity, but without loss of generality, let the systematic money supply be a constant, $\overline{M}$ (rather than a predictable function of the endogenous variables of the model). Let u^M be a random money-supply disturbance term. Then the money supply function may be expressed as

$$(3) \qquad M = \overline{M} + u^M$$

The mean of u^m is zero; its variance is constant:

$$E(u^M) = 0,$$
$$\text{var}(u^M) = \sigma_M^2$$

One version of the rational expectations hypothesis is

$$(4) \quad P_e = E(P|I)$$

where I is the public's information set at time t. In other words, the price level (at time t) anticipated by the public (at time t) is equal to the mathematical expectation (at time t) of the actual price level (at time t) conditional on I (at time t).

The policy-ineffectiveness argument emerges straightforwardly from Equations (1) - (4). By Equation (2) and (3),

$$(5) \qquad P = \overline{M} + u^m + u^D$$

and thus, by Equation (4),

$$(6) \qquad P_e = \overline{M}$$

Substituting Equations (5) and (6) into (1), we find

$$(7) \qquad Q = a \left[u^m + u^D \right] + u^s$$

From this equation it is evident that the systematic money supply has no effect on the distribution of Q. Hence, systematic monetary policy cannot be used as a stabilization policy device.

To show that systematic monetary policy may acquire a stabilization role in the presence of multiplicative disturbances, we reformulate the aggregate product demand function, Equation (2), in multiplicative form:

$$(8) \qquad P = M \cdot u^D$$

Substituting Equation (3) into (8) and taking the mathematical expectation, we obtain the anticipated price level:

$$(9) \qquad P_e = \overline{M} \cdot \overline{u}^D + \overline{u}^M . \overline{u}^D + \sigma_{MD}$$

where σ_{MD} is the covariance of u^M and u^D, which we assume to be constant. Substituting Equations (8) and (9) into (1), we obtain

$$(10) \qquad Q = a \cdot (u^D - \overline{u}^D) \cdot \overline{M} + a \cdot \left[u^M . u^D - \overline{u}^M . \overline{u}^D - \sigma_{MD}\right] + u^s$$

Here the systematic money supply has a role to play in stabilizing national product. To illustrate this role, suppose that the monetary authority aims to minimize the expectation of the squared deviations of actual production from its natural rate subject to the constraint (10):

$$(11) \qquad \text{Minimize } E\left[(Q - QN)^2\right]$$

subject to $Q = a.(u^D - \overline{u}^D) \cdot \overline{M} + a \cdot \left[u^M . u^D - \overline{u}^M . \overline{u}^D - \sigma_{MD}\right] + u^s$

Substituting the constraint into the objective function, taking the expectation, and minimizing with respect to $\overline{M}$, we obtain the optimal level of the money supply:

$$(12) \qquad \overline{M}^* = -\frac{\sigma_{DS} + a \cdot \sigma_{D\eta}}{a \cdot \sigma_D^2}$$

where σ_{DS} is the covariance of u^D and u^s (assumed constant), $\eta = u^M . u^D$, and $\sigma_{D\eta}$ is the covariance of u^D and η (assumed constant).

This rule has some interesting properties. The optimal money supply is inversely related to

- the covariance of u^D and u^s
- the covariance of u^D and η

and is positively related to

- the variance of u^D.

It does not depend on

- the variance of the unsystematic money supply or the correlation of u^M with either of the other random variables or
- the monetary authority's target level of production, QN.

The reason why the optimal money supply is unresponsive to the authority's choice of production target is that systematic monetary policy cannot affect the mean of the distribution of Q. To derive this result substitute Equation (12) into (10) and take the expectation of Q:

$$(13) \qquad E(Q) = QN$$

A different money supply rule emerges if the multiplicative disturbance occurs in the money supply function, rather than in product demand function:

$$(14) \qquad M = \bar{M} \cdot u^M$$

where it is assumed that $E(u^M) = \bar{u}^M$, a constant which may differ from zero.

Thus, the macroeconomic model under consideration comprises Equations (1), (2), (14), and (4). Substituting (14) into (2), we get

$$(15) \qquad P = \bar{M} \cdot u^M + u^D,$$

and taking the expectation, we find the anticipated price level:

$$(16) \qquad P_e = \bar{M} . \bar{u}^M + \bar{u}^D$$

Inserting (15) and (16) into (1), we obtain

$$(17) \qquad Q = a . (u^M - \bar{u}^M) \cdot \bar{M} + a \cdot (u^D - \bar{u}^D) + u^s$$

Maximizing the monetary authority's objective function subject to constraint (17), we obtain the following money supply rule:

$$(18) \qquad \overline{M}^{*} = -\left[\frac{\sigma_{MS} + a \cdot \sigma_{MD}}{a \cdot \sigma_M^2}\right],$$

where σ_{MD} is the covariance of u^M and u^D and σ_{MS} is the covariance of u^M and u^s (both covariances assumed constant).

Needless to say, the optimal money supply rules (12) and (18) are closely related. The role of the random variable u^D in the former rule is adopted by u^M in the latter. Thus, whereas in rule (12) the optimal money supply depends inversely on the variance of u^D and positively on the covariance of u^D and u^s, in rule (18) it depends inversely on the variance of u^M and positively on the covariance of u^M and u^s. Furthermore, the role of u^M $u^D = \eta$ in the former rule is adopted by u^D in the latter. Thus, in the former rule the optimal money supply is inversely related to the covariance of u^D and η, while in the latter rule it is inversely related to the covariance of u^M and u^D.[4]

Thus far, the inclusion of a multiplicative disturbance term in our macroeconomic model turned systematic monetary policy into an effective stabilization device. However, the existence of multiplicative disturbances is <u>not</u> a sufficient condition for the effectiveness of monetary rules. To demonstrate this negative result, suppose that the multiplicative disturbance occurs in the product supply function (rather than in the product demand of money supply functions). In particular, assume that the coefficient "a" (in Equation (1)) is a random variable with constant mean and variance:

$$E(a) = \overline{a} > 0$$

$$\text{var}(a) = \sigma_a^2 .$$

With this modification, our new macroeconomic model comprises Equations (1)- (4). The actual and anticipated price levels are given by Equations (5) and (6). National product is given by Equation (7). Here it is apparent that systematic monetary policy has no influence on production.

The primary message of this paper is that, even if the rational expectations hypothesis and the natural rate hypothesis both hold, systematic monetary rules may nevertheless have a role to play in stabilization policy, provided that the linear macroeconomic model contains stochastic coefficients. Whereas the absence of stochastic coefficients guarantees the ineffectiveness of systematic monetary policy, the presence of these coefficients does not ensure that such policy is effective.

Footnotes

1. I wish to express my gratitude to Ron Smith for his methodological coaching and Hugh Davies for his invaluable analytical insights.

2. Here the natural-rate hypothesis is interpreted narrowly: production is assumed to depend solely on the difference between the actual and the expected price level. If the natural rate hypothesis is amended to include the effects of capital accumulation on production (as in Fischer (1979)) or the effects of the real interest rate on production (as in Fair (1978)) and if there is a real money balance effect on consumption, then changes in money supply rules can influence real variables. In accordance with the certainty equivalence theorem, this result holds regardless of whether the model is deterministic or contains additive disturbances.

3. Recall that $\overline{M}$ is the logarithm of the optimal systematic money supply. Thus, Equation (12) does not violate a nonnegativity condition on the absolute level of the money supply.

4. The origin of these regularities may be clarified by re-writing Equation (10) as

$$Q = a\cdot(u^D - \overline{u}^D)\cdot\overline{M} + a\cdot\left[u^M\cdot u^D - E(u^M\cdot u^D)\right] + u^s$$

and comparing it with Equation (17).

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