STABILIZATION POLICIES VERSUS INTERTEMPORAL POLICY REVERSALS

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Summary

This paper deals with the question whether stabilization policies are socially desirable. It is assumed that these policies do not induce cyclical fluctuations in national or sectoral outputs. The paper argues that whenever (a) government policy impulses have welfare effects in the present and future and (b) there is an intertemporal tradeoff between these welfare effects, cyclical output fluctuations may be in the best public interest. In the context of two-sector models, cyclical production paths are shown to be the socially optimal responses to two macroeconomic problems: (1) an output mix which is not appropriate to society's preferences and (2) an inefficient use of productive factors. The optimal production paths reverse their directions of movement through time and thus suggest that "intertemporal policy reversals" may be preferable to stabilization policies.

Zusammenfassung

Dieser Artikel befaßt sich mit der Frage, ob wirtschaftliche Stabilisierungspolitik wünschenswert ist. Es wird angenommen, daß solche wirtschaftlichen Maßnahmen nicht mit zyklischen Produktionsschwankungen vereinbar sind. Anhand verschiedener Zwei-Sektoren Modelle wird demonstriert, daß zyklische Produktionsschwankungen die optimale Reaktion auf zwei makroökonomische Probleme darstellen: (1) eine Output-Allokation, die nicht den sozialen Präferenzen entspricht, und (2) ein ineffizienter Gebrauch der vorhandenen Produktionsfaktoren. Die Produktionsschwankungen sind wünschenswert, wenn wirtschaftspolitische Impulse intertemporale Nutzeneffekte hervorrufen, die nicht unabhängig voneinander sind. Unter solchen Umständen ist die wirtschaftliche Stabilisierungspolitik nicht optimal.

CONTENT

1. Introductory Remarks

In a broad sense, this article is concerned with an argument why stabilization policies may be socially suboptimal. For this purpose, stabilization policies will be specified in a special sense - one that is not commonly found in the theoretical literature, but appears to be frequently encountered in practical discussions of these policies by politicians and the news media. Quite simply, it will be assumed here that stabilization policies do not induce cyclical fluctuations in national or sectoral outputs.

Suppose, for example, that an economy is producing a socially suboptimal array of products, whereupon a government comes to power which wishes to rectify this deficiency. If the government does so by first inducing a reduction in the output of a particular sector and then stimulating the output of that sector, it will be maintained here that the government is not employing stabilization policies. In order for the government policies to satisfy our stabilization criteria, the government must approach its long-term production objectives monotonically through time.

The possible - indeed, generally common - undesirability of such stabilization policies will be demonstrated in the context of a dynamic, non-stochastic economic model. As noted, our concept of stabilization policies differs somewhat from those prevalent in the theoretical literature and thus it is important to specify where this difference lies. The literature may be divided into "descriptive" and "optimizing" analyses of stabilization policies. The descriptive analyses are largely confined to the early attempts (in the fifties and sixties) to examine the properties of various specified policy reaction functions, i.e. functional relations between policy target and policy instrument variables. These reaction functions are given by assumption, rather than derived through optimization criteria. Seminal work in the area was done by Phillips (1954, 1957), (who analysed the functioning of "proportional", "integral", and "derivative" policies in the context of the multiplier-accelerator model of Samuelson (1939) and Hicks (1950)). In this body of research, the concept of "stabilization" is inherent in the specification of the policy reaction functions. (For example, in Phillips' proportional policy, the desired level of government spending is inversely proportional to the difference between current and desired national income).

The optimizing analyses of stabilization policies may be categorized with regard to whether the underlying economic system is static or dynamic and deterministic or stochastic. Tinbergen (1952) provided the first rigorous treatment of linear, static, deterministic systems. His rule that a policy maker needs only as many instrument variables as target variables (in order to achieve any desired levels of the target variables) is well known. In his analysis, as in all the major contributions thereafter, the desired levels of the target variables are taken to be exogenously given. Theil (1958, 1964) and Brainard (1967) extended this analysis to stabilization policies of stochastic (but still linear and static) systems. Their concepts of "stabilization" are inherent in their policy objective functions, whereby the sum of the squared deviations of the actual target variables from their desired levels is minimized.[1] The literature on stabilization policies of dynamic systems, of both the deterministic and the stochastic variety, rest on the same conceptual foundations; here the same types of objective functions are minimized subject to differential or difference equations (which may or may not include random variables). This literature has grown rapidly over the past decade; its prominent contributors include Chow (1970, 1972, 1973a,1973b), Prescott (1971,1972), Simon (1956), Theil (1957), and Turnovsky (1974, 1976).

Thus, for the optimizing analyses, there is one common litmus test of whether a given set of policies satisfied stabilization criteria. It rests on whether these policies succeed in minimizing the deviations of the actual from the desired levels of the target variables. In this article a different litmus test is proposed; this test depends on whether a given set of policies induce cyclical fluctuations in real economic variables.

The motivation for our test arises straightforwardly from several shortcomings of the traditional test. One well-known shortcoming is that the quadratic objective functions, which predominate in the literature, imply that positive and negative deviations of equal magnitude are associated with identical social costs. For example, if the government's desired level of the unemployment (or inflation) target is 4%, then a 3% rate is assumed to be just as socially harmful as a 5% rate. In recognition of the obvious unrealism of such an assumption, several attempts have been made to develop asymmetric social loss functions (e.g. Friedman (1972, 1975).

However, a much more fundamental criticism remains to be made - one that is equally valid for symmetric and asymmetric loss functions. A benevolent government must be assumed to maximize a social welfare function (provided we make the strong assumption that one can be unambiguously derived), yet the traditional loss functions in the stabilization-policy literature bear no compelling (or even vague) relation to plausible characterizations of social welfare functions.

An example may help to clarify this difficulty. Suppose that the government wishes to maximize an intertemporal social welfare function, depending on national output and the aggregate rate of inflation, subject to an expectations-augmented Phillips curve. Under what conditions can this problem be convincingly approximated by the maximization of an intertemporal social loss function, depending on the deviation of output and inflation from their desired levels, subject to the same Phillips curve?

It must be emphasised that in the latter problem, the desired levels of output and inflation are exogenously given; they are not determined endogenously through the model's social optimality criteria. In order for the two policy problems to be viable approximations of each other, these desired levels must be set equal to the long-run optimal levels of output and inflation (which define the stationary equilibrium) in the former problem. (Otherwise the two problems must yield different rates of output and inflation in the long run).

Suppose, for the sake of argument, that this equivalence is assured (although the latter problem provides no means for doing so). Then the latter problem gives rise to the following implications: (a) if inflation falls short of its desired level, then a rise in inflation promotes social welfare, and (b) if output exceeds its desired level, then a rise in output diminishes social welfare. These implications are not reconcilable with the plausible and commonly adduced social welfare functions in which output is always a boon and inflation is always a bane.

An obvious response to this shortcoming of the traditional approach to stabilization policy is to abandon loss functions which depend on deviations between actual and desired target variables, and to adopt explicit social welfare functions instead. A socially optimal policy, quite simply, is one that maximizes a social welfare function subject to the given constraints. What can be said about the optimality of stabilization policies in this context? Stabilization policies in the traditional sense (i.e. ones that minimize deviations between actual and desired target variables) are obviously non-optimal. This conclusion emerges straightforwardly from the difference between the traditional loss functions and the social welfare functions. Thus, we consider stabilization policies in a different sense: policies which do not induce cyclical swings in real economic variables. It is not obvious whether or not these types of stabilization policies are socially optimal. This question is the object of our concern.

It will be argued here that, under a commonly encountered set of circumstances, these stabilization policies are suboptimal. In particular, cyclical production behavior may well be in the best public interest. Broadly speaking, this is the case whenever (a) the government is able to control two (or more) economic entities which are relevant to social welfare, (b) on account of technological of behavioral reasons there exists a tradeoff between these entities, such that the augmentation of social welfare through one entity implies the reduction of social welfare through the other, and (c) one of the entities is short-lived (and thus affects welfare only at present) whereas the other is long-lived(and thus affects welfare both at present and in the future).

It is not difficult to think of a wide variety of macroeconomic problems -- to which stabilization policies have been applied -- which satisfy these conditions. Consider the tradeoff between nondurable consumption and investment which is commonly encountered in macroeconomic growth theory. The nondurable consumption goods are short-lived, while the investment goods are long-lived. Given the factors of production at the economy's disposal, an increase in the production of consumption goods implies a decrease in the maximal amount of investment goods that can be produced. The increase in consumption implies a rise in present welfare; the decrease in investment implies that the stock of capital goods (whereby future consumption and welfare could be augmented) is lower than it otherwise would have been. The government may be able to influence consumption and investment through such instruments as income taxes, interest rates, or investment tax credits. In fact, if the market mechanism ensures full employment and efficient production (so that consumption and investment lie on the economy's production possibility frontier), control over consumption implies control over investment and vice versa.

Another prominent example is the tradeoff between inflation and unemployment as described by, say, a shifting Phillips curve. In the short run, inflationary expectations may be assumed constant and a rise in the actual rate of inflation implies a fall in unemployment. In the longer run, inflationary expectations adjust to the actual rate of inflation and thus the short-run Phillips curve shifts upwards. Unemployment may be considered to have a short-lived adverse effect on welfare because it implies, say, a deficient amount of nondurable consumption; but inflationary expectations may have a long-lived adverse effect on welfare since it may take time to bring these expectations into conformity with the actual rate of inflation. The government may be able to control unemployment and changes in inflationary expectations through monetary and fiscal policies.

Yet another example is to be found in the tradeoff betweennondurable consumption and the depletion of renewable natural resources. The higher the rate of consumption, the lower the growth rate of resource supplies. An increase in consumption implies a rise in present welfare and a fall in resource growth implies a diminished ability to satisfy consumption demand in the future. The government may affect consumption or resource depletion through taxes and subsidies.

In this article, the tradeoff between consumption and pollution will be used to illustrate why stabilization policies may be socially suboptimal. There is no overwhelming reason for the choice of this tradeoff rather than the ones described above (or many others which satisfy the requisite conditions). Each would serve our purposes adequately. However, the dynamic interface between the macroeconomy and the environment has received little attention in the formulation of macroeconomic policies. (Some prominent exceptions to this rule are D'Arge and Kogiku (1973), Forster(1973), Keeler, Spence, and Zeckhauser (1972), Mäler (1974), Plourde (1972), and Smith (1972).) Since the adverse effects of pollution arising from a multitude of essential production and consumption activities are certainly not negligible in most countries nowadays -- consider, for example, carbon monoxide from fuel combustion, halocarbons from refrigerators and airconditioners, lead from automobile exhausts, asbestos from building demolition, chlorinated hydrocarbons as pesticides, sewage in waterways, thermal pollution, radioactive wastes, etc. -- it appears that the macroeconomic-environmental interface may have received insufficient attention, and the present article may serve as a small step along this neglected path.

Besides, the tradeoff between consumption and pollution provides somewhat more latitude in describing dynamic macroeconomic policies than the tradeoffs above, since some serious macroeconomic-environmental problems concern short-lived consumption goods and long-lived pollutants, while others involve long-lived consumption goods and short-lived pollutants. This distinction will emerge to be of crucial importance for the following reason. As noted, stabilization policies will be considered undesirable whenever it can be shown that a cyclical swing in production is uniquely optimal. Yet there can be no justification for cyclical swings whenever both consumption goods and pollutants are short-lived (i.e. whenever present consumption and present pollution affect social welfare at present, but not in the future). Then, regardless of how long the government's time horizon is, the optimal levels of consumption and pollution are derivable from a <u>static</u> optimization problem; the adjustment to the optimal levels cannot involve cyclical swings.

Yet the situation may be different if the consumption goods or the pollutants are long-lived and if the government's time horizon is not infinitely short. Then the government must recognize that the decision to consume more now affects welfare not only at present (directly or via the associated increment in pollution), but also in the future (directly if the consumption good is long-lived or indirectly if the pollutant is long-lived). Thus, the optimal levels of consumption and pollution are derivable from a dynamic optimization program and the social optimality of stabilization policies becomes open to question. The analysis below will show not only that stabilization policies may not be optimal, but that (under conditions to be specified later) the properties of the optimal fluctuations in sectoral outputs depend on whether the consumption good or the pollutant is long-lived.

With this in mind, we will examine two model economies. The first, to be called the "pollutant-accumulation economy", contains a nondurable consumption good and a long-lived pollutant. This economy may be understood as an analytical abstraction covering a wide variety of economic-environmental problems in which the consumption good does not last as long as the pollutant which it generates: aerosol deodorants releasing halocarbons, foods in plastic or metal containers, sulfur oxides from coal and oil burned for space heating, nuclear wastes from electricity generation, bottles from beer consumption, and so on.

This economy also produces a treatment service, which "cleanses" the pollutant (i.e. transforms it into socially harmless substances). Nature -- through diffusion, dilution, chemical decomposition, and biodegradation -- is assumed to cleanse the pollutant as well. A flow of pollutant emissions is generated by the production and consumption of the nondurable good and by the anthropogenic treatment service.

There is a fixed amount of factors available for the production of the good and the provision of the treatment service. At the socially optimal state of this economy, the factors are fully employed and used efficiently. We assume that external production economies are absent. Hence, an increase in the production of the good means a fall in the provision of the treatment service. The government can influence the allocation of factors between the production and treatment sectors by means of, say, taxes and subsidies.

At any instant of time, social welfare depends on the flow of consumption (since the consumption good is nondurable) and the stock of pollutants (since pollutants can be accumulated through time). The government seeks to maximize social welfare over time through the appropriate intersectoral factor allocation (or, equivalently, through the appropriate mix of production and treatment services). If factors are transferred from the treatment sector to the production sector, then there is an immediate welfare gain from consumption (for the entire supply of the nondurable good is consumed) and a future welfare loss from pollution (since more pollution is generated through production and consumption and less is cleansed through the treatment service).

In this analytical context, we focus on the following question. If the current use or allocation of factors is socially suboptimal, should production and treatment change monotonically through time -- as they would under stabilization policies -- or should there be "intertemporal reversals" of the production or treatment levels (e.g. a fall in production followed by a rise or vice versa)?

The second model economy is called the "product-accumulation economy". In contrast to the pollutant-accumulation economy, it contains a durable consumption good and a short-lived pollutant. This economy may serve as an analytical abstraction of economic-environmental problems in which the consumption good lasts longer than the pollutant which it generates: harmful concentrations of carbon monoxide and hydrocarbons from automobiles, noise from construction and demolition activities, harmful concentrations of oil from ships, and so on.

The stock of the durable good is increased through the production activity and diminished through depreciation. The pollutant may be cleansed by a treatment service (e.g. oil can be skimmed from the ocean surface, noise from construction can be reduced through sound-proofing and more careful handling of meterials). Nature's cleansing activity takes a particularly simple form: each pollutant is made harmless shortly after it is generated. (In other words, nature is responsible for the short life span of the pollutant.) The pollutant flow is generated by the production (and possibly also the consumption) of the durable good and by the anthropogenic treatment service.

At any instant of time, social welfare depends on the stock of consumption goods and the flow of pollutants. The government maximizes social welfare through time by inducing the appropriate allocation of factors between the production and treatment sectors. As in the pollutant-accumulation economy, we assume that an increase in production implies a drop in pollution treatment. If factors are transferred from the treatment sector to the production sector, there is an immediate welfare loss from pollution (since a greater pollutant flow is generated and a smaller pollutant flow is cleansed) and a future welfare gain from consumption (since the stock of consumer durables will be greater than it otherwise would have been). Once again, we inquire whether a suboptimal use or allocation of factors calls for monotonic changes in the levels of production and treatment -- as yielded by stabilization policies -- or for intertemporal reversals of these levels.

What deserves emphasis in our analysis of the pollutant-accumulation economy and the production-accumulation economy is that the social desirability of stabilization policies will be evaluated on the basis of a single criterion: the ability of these policies to rectify an inappropriate use or allocation of factors. Since factor services in both economies can only be devoted to socially beneficial uses (viz, the production of consumption goods or the treatment of pollutants), an "inappropriate use" of factors can occur only if they are left unemployed or used inefficiently (so that more of at least one sectoral output could be produced with the same factor input). On the other hand, an "inappropriate allocation" of factors between the two productive sectors may prevail even through all factors are used fully and efficiently. It exists whenever a transfer of factors from one sector to another could raise social welfare.

Inappropriate use and allocation of factors are common and serious macroeconomic problems and stabilization policies are frequently used to combat them. In fact, these problems may be the major raison d'être of stabilization policies. We will examine whether stabilization policies constitute the most socially desirable way of handling these problems in the pollutant-accumulation economy and the product-accumulation economy. To facilitate a comparison of the two economies, we subject them to the same economic problem and derive their optimal dynamic paths is response to this problem. We will then inquire whether these paths are compatible with stabilization policies or whether they involve intertemporal reversals of production or anthropogenic treatment. Furthermore, we will examine whether the optimal, dynamic responses of the two economies to the same problem are qualitatively identical. In this manner it is possible to take a first step toward understanding how the longevity of consumption goods or pollutants is relevant for the formulation of dynamic macroeconomic policy.

The next section contains a prescriptive model of the pollutant-accumulation economy and the product-accumulation economy and derives the optimal paths of production, consumption, pollution generation, and pollution treatment for these economies. Section 3 analyzes the optimal dynamic responses of the economies to inappropriate use or allocation of factors and evaluates stabilization policies from this perspective. Finally, Section 4 provides a general overview of the argument against stabilization policies and summarizes the major analytical results.

2. The Pollutant-Accumulation Economy and the Product-Accumulation Economy

In the pollutant-accumulation economy, production and consumption are always equal. Both may be denoted by QP. Since the consumption good is nondurable, it cannot be accumulated and thus consumption cannot exceed production; since the good promotes social welfare, it is always desirable to consume whatever is produced and thus consumption cannot fall short of production. However, the pollutant is durable and therefore a distinction must be made between the stock of pollutants (P) and the flow of pollutant emissions ($\dot{P} = dP/dt$).

A fixed supply of factors can be used to produce the consumption good and provide the treatment service (T). Since both of these activities promote social welfare, it is always desirable to employ these factors fully and efficiently. Hence, the optimal combination of production and treatment provided by the economy is to be found on a production possibility frontier, which may be depicted as follows:

$$(1)\quad T = F(QP), \qquad F', F'' < 0.$$

The flow of pollutant emissions (P_E) arises from the production, consumption, and treatment activities. Each of these activities are characterized by positive and increasing emissions per unit of output:

$$(2)\quad P_E = g_1(QP) + g_2(T), \qquad g_1', g_2' > 0;\ g_1'', g_2'' > 0.$$

The emission flow may be cleansed anthropogenically and naturally. Nature's treatment (T_N) depends directly on the stock of pollutants:

(3) $T_N = f(P)$, $f' > 0,\ f'' < 0.$

Both anthropogenic and natural treatment services are scaled in such a way that one unit of treatment service is equal to one unit of emission flow cleansed.

Thus, the net flow of pollutant emissions is

(4) $\dot{P} = P_E - T - T_N.$

Substituting Equations 1, 2, and 3 into Equation 4, we obtain

(5) $$\dot{P} = g_1(QP) + g_2(F(QP)) - F(QP) - f(P)$$

$$= k(QP) - f(P)$$

where $k_Q = g_1' - (1 - g_2') \cdot F' > 0$

and $k_{QQ} = g_1'' - (1-g_2') \cdot F'' + g_2'' \cdot (F')^2 > 0.$

Social welfare is augmented by the consumption flow and reduced by the pollutant stock. Whereas additional units of QP raise social welfare by smaller and smaller amounts (asymptotic satiation), additional units of P reduce social welfare by larger and larger amounts:

(6) $U = U(QP,P)$, $U_Q > 0,\ U_{QQ} > 0;$

$U_P < 0,\ U_{PP} < 0;$

where $U_Q = (\partial U/\partial QP)$ and $U_P = (\partial U/\partial P)$.

The government seeks to maximize social welfare from the present to the infinite future. (For a finite span of time, this goal is equivalent to maximizing social welfare over this span and valuing the terminal pollutant stock optimally.) We assume that the social rate of time preference (r) is constant and that utilities gleaned at different points in time are functionally independent of one another.

We assume that the government can control the level of QP through some fiscal instrument, e.g. a tax or subsidy on production or the treatment service. By setting a time path for QP, the government determines the time paths of all other endogenous variables in our model. At the initial point in time (t=0), the govenment inherits the initial pollutant stock (P(0)) and, with it, the initial amount of natural treatment ($T_N(0)$). By setting the initial level of QP, it determines the initial level of T. QP, T, and T_N together determine the net flow of pollutant emissions, $\dot{P}$. Thereby, the pollutant stock at the next instant of time may be computed (along with the corresponding T_N). Then, the government again sets the level of QP, and the sequence of events is repeated.

Thus, the government's policy problem may be stated as follows:

$$(7)\quad \text{Maximize } W = \int_0^\infty e^{-rt} \cdot U(QP,P)\, dt$$

$$\text{subject to } \dot{P} = k(QP) - f(P),$$

where QP is the control variable and P is the state variable. We restrict our attention to interior optima. (Optima at which nothing is consumed, no treatment service is provided, or no pollutant stock exists appear to have little practical relevance.) Hence, the nonnegativity constraints

$$QP \geq 0, \quad F(QP) \geq 0, \text{ and } P \geq 0$$

need not be included in the specification of the policy problem.

The current-value Hamiltonian is

$$H = U(QP,P) + \mu \cdot [k(QP) - f(P)] ,$$

where μ is the shadow price of a net of pollutant emissions (or, alternatively, $-\mu$ is the social value of a unit of treatment service). The necessary conditions for social optimality are

$$(8) \qquad \frac{\partial H}{\partial QP} = 0 \Rightarrow \qquad U_Q = -\mu \cdot k_Q$$

$$(9) \qquad -\frac{\partial H}{\partial P} = \dot{\mu} - r \cdot \mu \Rightarrow - U_P + \mu \cdot f' = \dot{\mu} - r \cdot \mu$$

$$(5) \qquad \frac{\partial H}{\partial \mu} = \dot{P} \Rightarrow \qquad \dot{P} = k(QP) - f(P)$$

where $k_Q = (dk/dQP)$.

Condition 8 concerns the tradeoff between the marginal welfare gains and losses from consumption. Each incremental unit of consumption is responsible for a welfare gain (U_Q) and also for an emission flow (k_Q) whose social cost is $-\mu \cdot k_Q$. Condition 8 requries that these costs and benefits be equal at each point in time.

Condition 9 concerns the tradeoff between the welfare loss from an incremental increase in the pollutant stock and the welfare costs associated with an incremental reduction in the pollutant stock through anthropogenic treatment. An incremental increase in the pollutant stock gives rise to a direct welfare loss (U_P), but it also leads to an increase in natural treatment (f'), and the value of this treatment ($-\mu \cdot f'$) must be subtracted from the direct welfare loss. An incremental reduction in the pollutant stock implies two costs: an opportunity cost of treatment, $-r \cdot \mu$ (which arises when the social damage of pollutant increments are discounted through time) and a capital loss from pollution treatment, $\dot{\mu}$ (which arises when the social value of reducing the pollutant stock, $-\mu$, falls through time). By Condition 9, the marginal welfare costs of pollution must always be equal to the marginal welfare costs of pollution treatment.

Condition 8 implies that there is a unique relation between consumption and the social value of treatment. Let $\phi(QP) = (U_Q/k_Q)$; then

$$(8a) \quad QP = \phi^{-1} (-\mu).$$

Since
$$\phi'(QP) = \frac{\mu_{QQ} \cdot k_Q - k_{QQ} \cdot U_Q}{(k_Q)^2} < 0.$$

$(\phi^{-1})'$ is also negative.

Substituting Equation 8a into Equation 5, we obtain

$$(10) \quad \dot{P} = k\,[\phi^{-1}(-\mu)] - f(P).$$

Equations 9 and 10 yield the time paths of the pollutant stock and the shadow price of emissions which satisfy the necessary conditions for social optimality. These time paths are depicted in Figure 1a.[1a] By Equation 8a (depicted in Figure 1b), the time path of QP may be derived from the time path of μ , as shown in Figure 1c.

The time paths above are socially optimal if the following second-order conditions are satisfied[2]:

(A) The Hamiltonian, maximized with respect to the control variable, is a concave function of the state variable (for a given time and costate variable), and

(B) $\lim_{t\to\infty} e^{-rt} \cdot \mu \geq 0$ and $\lim_{t\to\infty} e^{-rt} \cdot \mu \cdot P = 0.$

Condition A is always satisfied[3], but condition B is satisfied only if μ and P approach stationary values in the long run. This is the case only if μ and P lie along the "saddlepoint path", denoted by SPP in Figure 1a. The corresponding optimal path for consumption is given by SPP in Figure 1c.

These results imply that the pollutant-accumulation economy has a unique optimal stationary state, characterized

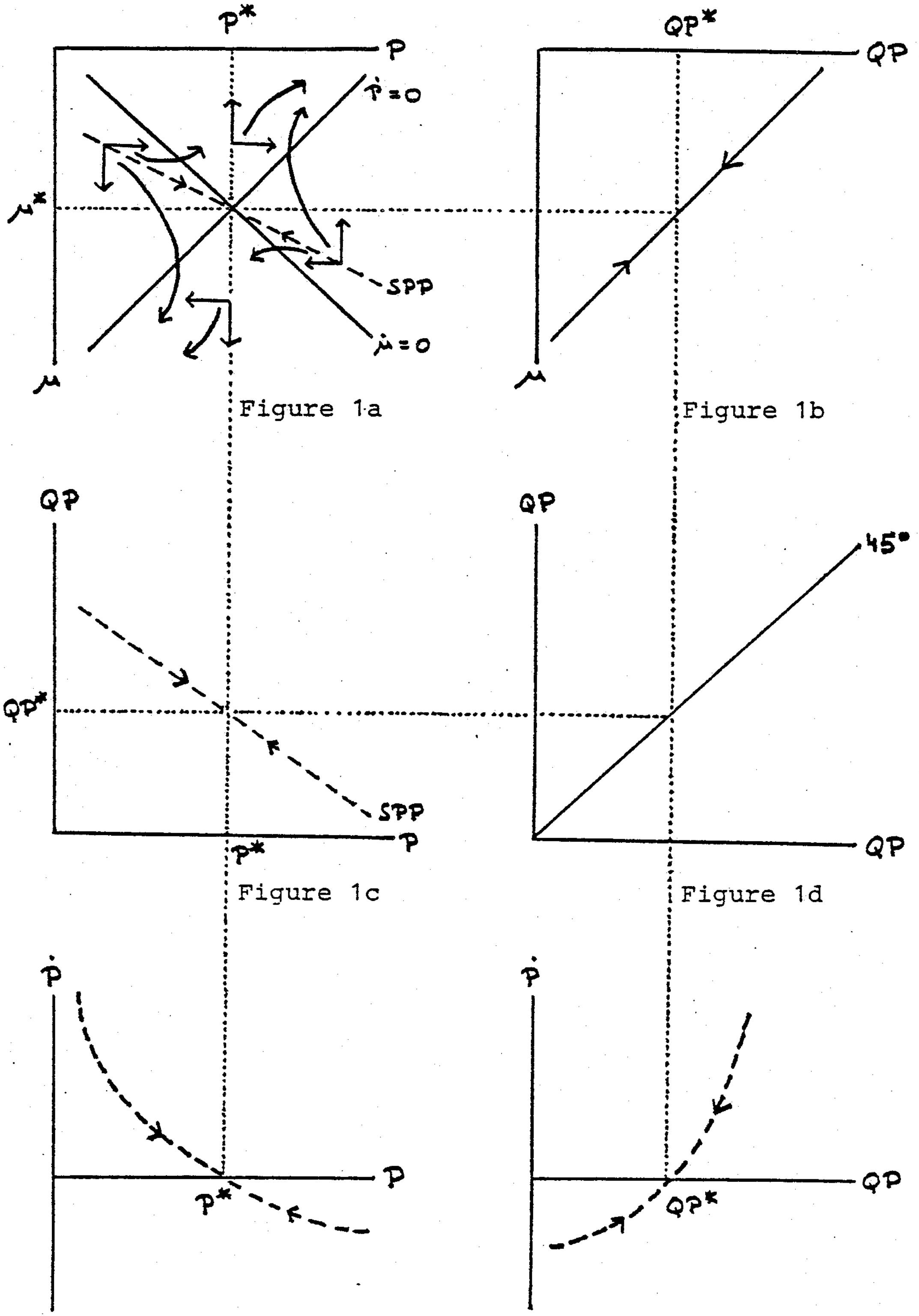

Figure 1a

Figure 1b

Figure 1c

Figure 1d

Figure 1e

Figure 1f

FIGURES 1

by constant levels of consumption and production (QP^*), anthropogenic treatment (T^*), natural treatment ($f(R^*)$), emission flow ($\dot{P}^*$), and pollutant stock (P^*). The optimal trajectories of these variables converge to this stationary state, regardless of what the current state of the economy is.

The optimal current state depends solely on the current stock of pollutants. The nature of this dependence is illustrated in Figure 2. If the government inherits a "dirty" environment -- i.e. if the current stock of pollutants exceeds the long-run optimal pollutant stock P^* (as shown in link (a) of Figure 2) -- then the marginal welfare cost of the pollutant stock is high relative to its long-run optimal level (link (b) of Figure 2). (The reason is that the marginal disutility of pollution rises with the stock of pollution and the marginal rate of natural treatment falls with the stock of pollution; in other words, $- U_P + \mu . f'$ is an increasing function of P.) By Condition 9, the marginal welfare cost of pollution must be equal to the marginal welfare cost of anthropogenic treatment; thus, if the former lies above its long-run optimal level, the latter must do so as well (link (c) of Figure 2).

In turn, the marginal welfare cost of treatment lies above its long-run optimal level whenever
(I.) the social value of a unit of treatment ($-\mu$) exceeds its long-run optimal level (link (d) of Figure 2), and
(II.) the capital loss from pollution treatment ($\dot{\mu}$) is positive[4] (link (i) of Figure 2).
Let us examine the economic implications of each of these conditions.

As for condition (I.) (link (d)), the social value of a unit of treatment exceeds its long-run optimal level only if production falls short of its long-run optimal level (link (e)). The reason is to be found in Condition 8, which may be rewritten in the following form:

$$(8b) \qquad - \mu = \frac{U_Q}{k_Q} .$$

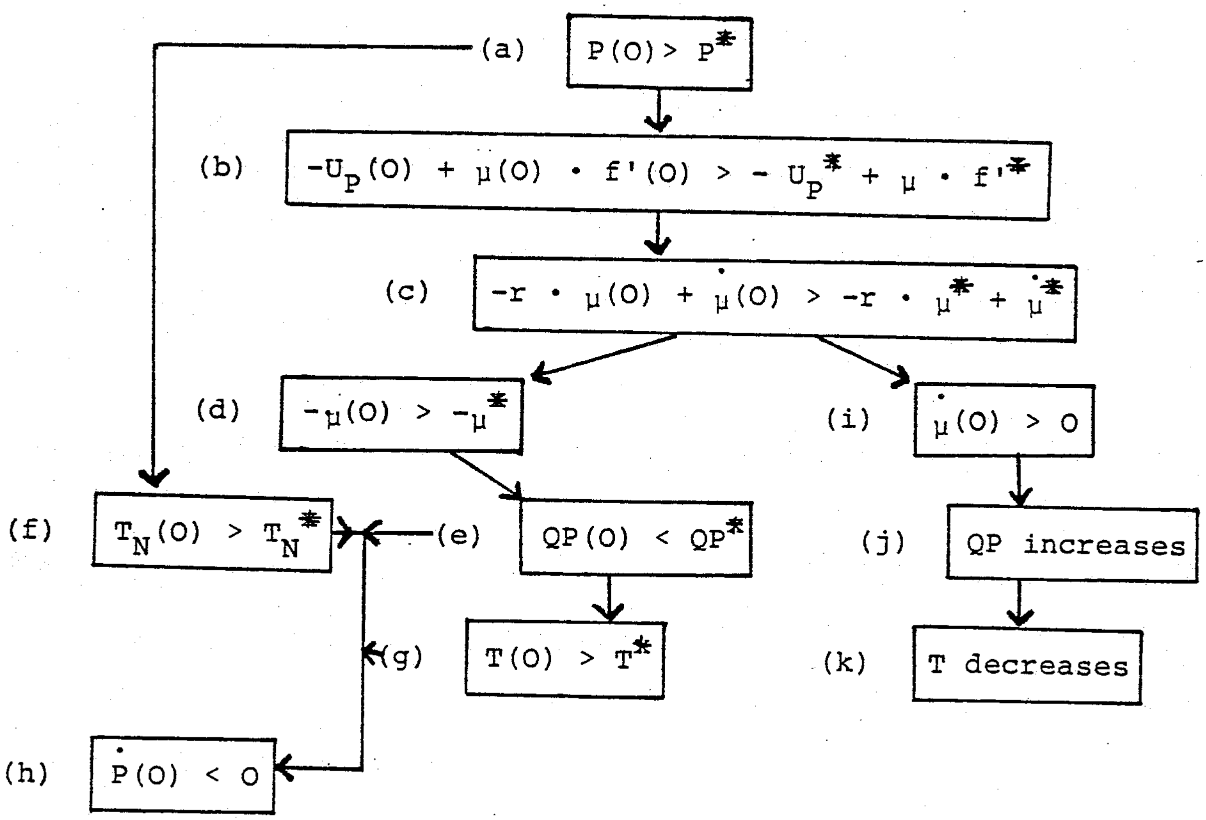
(a)
P(O) > P*
(b)
-U_P(O) + μ(O) • f'(O) > - U_P* + μ • f'*
(c)
-r • μ(O) + μ̇(O) > -r • μ* + μ̇*
(d)
-μ(O) > -μ*
(i)
μ̇(O) > O
(f)
T_N(O) > T_N*
(e)
QP(O) < QP*
(j)
QP increases
(g)
T(O) > T*
(k)
T decreases
(h)
Ṗ(O) < O

FIGURE 2

i.e. the social value of a unit of treatment must be set equal to the social welfare gain per unit of emission associated with the marginal production and consumption of QP. Since the marginal utility of consumption falls with QP and the marginal emission from consumption and production rises with QP, (U_Q/k_Q) is a decreasing function of QP. Thus, if $-\mu$ is to exceed its long-run optimal level, QP must fall short of its long-run optimal level as well.

Moreover, if QP is below its long-run optimum, then (given the slope of the production possibility frontier) T is above its long-run optimum (link (g)). Thus, the production, consumption, and treatment activities taken together generate a smaller pollutant flow than that in the optimal long run. In addition, if the initial pollutant stock exceeds P^*, then natural treatment services lie above their long-run optimal level (link (f)). Hence, the net pollutant flow (from production, consumption anthropogenic treatment, and natural treatment) lies below its long-run optimum; i.e. $\dot{P}$ is negative (link (h)). In other words, if the initial state of the economy is characterized by a "dirty" environment, then the initial levels of consumption and production should be sufficiently small so that the pollutant stock contracts.

According to Condition (II.), however, it is not desirable for the economy to remain in this initial state. For a positive capital loss from pollution treatment (link (i)) means that the value of a unit of treatment falls through time. By Condition 8b, the social welfare gain per unit of emission associated with an increment in QP must fall as well. For this purpose, QP must rise (link (j)) and, with it, T falls (link (k)). Thus, the emission flow generated increases relative to the emission flow cleansed. As result, the improvement in environmental quality -- arising from the initial levels of consumption and production -- proceeds at a slower and slower rate through time (link (l) of Figure 2 and Figure 1e).

Moreover, as the value of a unit of treatment falls ($\dot{\mu} > 0$) and as environmental quality improves ($\dot{P} < 0$), the capital loss from pollution treatment becomes smaller and smaller through time, since

$$(11) \qquad \frac{\partial \dot{\mu}}{\partial \mu} = r - f' > 0 \text{ and } \frac{\partial \dot{\mu}}{\partial P} = - U_{PP} - \mu \cdot F'' > 0.$$

Thus, the value of a unit of treatment falls by smaller and smaller amounts through time and (by Condition 8b) the production rises by smaller and smaller amounts through time. In this manner, the economy gradually approaches its optimal long-run stationary state.

So far, so good. Yet this story of optimal dynamics, as it stands, is missing a crucial building block. We noted that if the environment is "dirty" (link (a)), then the welfare cost of pollution is relatively high (link (b)), and thus the welfare cost of treatment must be relatively high as well (link (c)). Clearly, the conditions $-\mu(0) > -\mu^*$ (link (d)) and $\dot{\mu}(0) > 0$ (link (i)) are <u>sufficient</u> to ensure that the welfare cost of treatment is high relative to its long-run optimum. But do these two conditions <u>necessarily</u> hold along the optimal path of the pollutant-accumulation economy?

This question may be answered in the affirmative. For the sake of argument, suppose that the first condition is fulfilled ($-\mu(0) > -\mu^*$), but that the second condition is not (i.e. the capital loss from pollution treatment is nonpositive). If $\dot{\mu}$ is negative, then QP falls (link (j)) and T rises (link (k)) through time. Ignoring nature's treatment service for the moment, the flow of emissions cleansed falls relative to the flow of emissions treated. Hence, not only does the pollutant stock fall through time (link (h)), but it falls at a faster and faster rate. The falling pollutant stock and the falling μ both cause $\dot{\mu}$ to fall (by Equations 11). Thus, μ falls at a faster and faster rate, which implies that QP falls without limit while T rises without limit. In that case, however, no long-run stationary state is approached and the sufficient conditions for social optimality (given above) are not fulfilled.

(The concomitant dynamics of natural treatment do not vitiate this result. As QP falls and T rises, the pollutant stock falls; and while this is happening, nature's treatment service diminishes, which has a counterveiling effect on the pollutant stock. Yet clearly the natural treatment effect can never be dominant. Natural treatment falls only if the pollutant stock falls, which implies that the combined effect of QP and T is dominant.)

If $\dot{\mu}$ is zero, then -- in the first instant -- QP and T remain constant through time and thus the pollutant stock falls (link (h)). However, by Equations (11), a fall in the pollutant stock causes $\dot{\mu}$ to fall, i.e. to turn negative. But as we have seen, a negative $\dot{\mu}$ implies a violation of the sufficiency conditions.

On the other hand, suppose that the second condition is fulfilled ($\dot{\mu}(0) > 0$), but that the first condition is not (i.e. the social value of a nit of treatment, $-\mu$, falls short of its long-run optimal level. Then QP must exceed QP^* (link (e)) and T must fall short of T^* (link (g)). Thus, the stock of pollutants must rise through time (link (h)). (Once again, the resulting increase in natural treatment cannot dominate this chain of effects.) The rise in the pollutant stock and the rise in μ cause $\dot{\mu}$ to rise (by Equations 11). Thus, μ rises at a faster and faster rate and, with it, QP rises without limit and T falls without limit. But then no long-run stationary state can be approached and thus the sufficient conditions for optimality are not satisfied. (Besides, if μ rises at a faster and faster rate, it must turn positive after a finite span of time, which contradicts optimality condition 8.)

The argument above not only demonstrates that $-\mu > -\mu^*$ and $\dot{\mu} > 0$ at the initial point in time (t=0), but it also shows that these two conditions must hold throughout the entire period of adjustment preceeding the long-run stationary state. For as soon as one of these conditions is violated (at any point in time prior to the attainment of the long-run stationary state), μ either rises or falls at a faster and faster rate and consequently the sufficiency conditions are violated.

In sum, if the government inherits a "dirty" environment ($P(O) > P^*$), it should set the level of QP sufficiently low to permit the pollutant stock to contract (as implied by the condition $-\mu > \mu^*$) and it should thereupon allow QP to rise so that the pollutant stock contracts at a slower and slower rate through time (as implied by the condition $\dot{\mu} > O$).

Analogously, if the government inherits a "clean" environment -- i.e. if the current pollutant stock falls short of P^* -- then it should set the level of QP sufficiently high to cause a deterioration of environmental quality ($\dot{P} > O$) and it should thereupon allow QP to fall so that the deterioration of environmental quality proceeds at a slower and slower rate.

In sum, the optimal pollutant stock and the optimal pollutant emission flow should always move in opposite directions through time (except when the optimal stationary state is attained). The same holds for the optimal value of the treatment service and the capital loss from the treatment service. Due to these counterveiling movements, the economy approaches its optimal stationary state gradually. Furthermore, the optimal level of production moves in the same direction as the optimal emission flow (and in the opposite direction to the pollutant stock).

The optimal dynamic relation between production and the emission flow is qualitatively the same in the product-accumulation economy, although for quite different reasons. In this economy, the consumption good is long-lived; thus, consumption (Q) is measured by the stock of consumer durables, whereas production (QP) is a flow. The pollutant is short-lived; thus, there is no difference between the net amount of pollutants generated by the economy and the amount of pollutants which affect social welfare.

As in the pollutant-accumulation economy, there is a production possibility frontier which specifies the tradeoff between production and treatment services:

$$(1)\quad T = F(QP), \qquad F', F'' < 0.$$

The stock of consumer durables is augmented by production and diminished by depreciation, $\delta(Q)$:

$$(12)\quad \dot{Q} = QP - \delta(Q),$$

where $\dot{Q} = (dQ/dt)$ and $\delta' > 0$, $\delta'' \geq 0$.

Pollutants are emitted by the production and treatment activities[5] and cleansed by the latter:

$$(13)\quad P = g_1(QP) + g_2(T) - T, \quad g_1', g_2' > 0;\ g_1'', g_2'' > 0.$$

Substituting Equation 1 into Equation 13,

$$(13a)\quad P = g_1(QP) + g_2(F(QP)) - F(QP) = h\ (QP)$$

where $h' = g_1' - F' \cdot (1 - g_2') > 0$ and

$$h'' = g_1'' + (F')^2 \cdot g_2'' - (1 - g_2) \cdot F'' > 0.$$

Hence,

$$(13b)\quad QP = h^{-1}(P),\ (h^{-1})' > 0,\ (h^{-1})'' < 0.$$

Substituting Equation 13b into Equation 12,

$$(14)\quad \dot{Q} = h^{-1}(T) - \delta(Q)$$

Social welfare depends on the <u>stock</u> of consumer durables and the flow of pollutant emissions:

(15) $U = U(Q,P)$, $U_Q > 0$, $U_{QQ} < 0$;
where U_Q is now $(\partial U/\partial Q)$. $U_P < 0$, $U_{PP} < 0$.

The government seeks to maximize social welfare from the present to the infinite future. It can control QP. If it raises QP, it stimulates present pollution as well as present and future consumption. Since QP is uniquely related to P (in accordance with Equation 13b), P may be used as control variable.

The government's policy problem is

(16) Maximize $W = \int_0^{\infty} e^{-rt} \cdot U(Q,P)\, dt$

subject to $\dot{Q} = h^{-1}(P) - \delta(Q)$,

where P is the control variable and Q is the state variable. Once again, we are interested only in interior optima; thus, the nonnegativity constraints $P \geq 0$, $Q \geq 0$, $QP = h^{-1}(P) \geq 0$, and $T = F(h^{-1}(P)) \geq 0$ may be ignored.

The current-value Hamiltonian is

$$\hat{H} = U(Q,P) + \lambda \cdot \left[h^{-1}(P) - \delta(Q)\right],$$

where λ is the social value of a unit of consumer durable accumulation. The first-order conditions are

(17) $\frac{\partial \hat{H}}{\partial P} = 0 \Rightarrow -U_P = \lambda \cdot (h^{-1})'$

(18) $-\frac{\partial \hat{H}}{\partial Q} = \lambda - r \cdot \lambda \Rightarrow U_Q - \lambda \cdot \delta' = r \cdot \lambda - \dot{\lambda}$

(14) $\frac{\partial \hat{H}}{\partial \lambda} = \dot{Q} \Rightarrow \dot{Q} = h^{-1}(P) - \delta(Q)$.

Condition 17 concerns the tradeoff between the marginal welfare gains and losses from the pollutant emissions. An incremental increase in emissions is associated with a welfare loss ($-U_P$) and with an increase in the production of consumer durables $(h^{-1})'$, whose social value is $\lambda \cdot (h^{-1})'$. The optimality condition 17 requires that the marginal welfare gains and lossess from emissions be equal at each point in time.

Condition 18 concerns the tradeoff between the welfare gain from an incremental increase in the consumer durable stock and the welfare cost associated with this incremental increase. An incremental increase in Q gives rise to a direct welfare gain (U_Q) and to an increase in depreciation (δ'), which reduces welfare by $\lambda \cdot \delta'$. The welfare cost of consumer durable accumulation is determined by the difference between the opportunity cost, $r \cdot \lambda$, and the capital gain, $\dot{\lambda}$. By Condition 18, the welfare gains and losses from an incremental increase in Q must always be equal.

By Condition 17, there is a unique relation between pollution and the social value of consumer durable accumulation. Let $\psi(P) = U_P/(h^{-1})'$;
then

(17a) $P = \psi^{-1}(-\lambda)$.

Since

$$\psi'(P) = \frac{U_{PP} \cdot (h^{-1})' - (h^{-1})'' \cdot U_P}{[(h^{-1})']^2} < 0,$$

$(\psi^{-1})' < 0$ as well.

Substituting Equation 17a into Equation 14,

(19) $\dot{Q} = h^{-1}[\psi^{-1}(-\lambda)] - \delta(Q)$.

The time paths for the consumer durable stock and the social value of consumer durable accumulation may be found through the simultaneous solution of Equations 18 and 19. These paths, satisfying the first-order conditions and associated with arbitrary initial values of Q and λ , are illustrated in Figure 3a. Since there is a monotonic relation between λ and R (as shown in Figure 3b), the time path of pollutant emissions can be derived (as shown in Figure 3c).

Not all of these time paths are socially optimal. The only ones that satisfy the sufficient conditions enumerated above are those which lie on the "saddle-point path",[6] denoted by SPP in Figures 3a and 3c.

A comparison of Figures 1 and Figures 3 reveals that the optimal dynamics of the pollutant-accumulation economy and the product-accumulation economy are similar in some respects and quite dissimilar in others. Both economies have an optimal stationary state which they approach regardless of what their optimal current state is. In both economies the optimal production and optimal emission flow move in the same direction through time. (The latter result, however, is determined entirely by technological considerations in the product-accumulation economy -- as indicated by Equation 12a -- but partly by optimality considerations in the pollutant-accumulation economy.) Yet whereas consumption and pollutant emissions optimally move in the same direction in the pollutant-accumulation economy (as shown in Figure 1f), they move in opposite directions in the product-accumulation economy (as shown in Figure 2c).

The optimal path of the product-accumulation economy obviously does not depend on whether the government inherits a "dirty" or "clean" environment; here pollutants cannot be inheritted at all. Instead, the optimal path depends on whether the government inherits a "large" or "small" stock of consumer durables. If it inherits a "large" stock -- i.e. if the current stock exceeds the long-run optimal stock, Q^* --then the marginal welfare gain is low relative to its long-run optimal level (since $U_Q - \lambda \cdot \delta$ is a decreasing function of Q). Social optimality requires (by Condition 17) that the marginal welfare cost of consumer durable

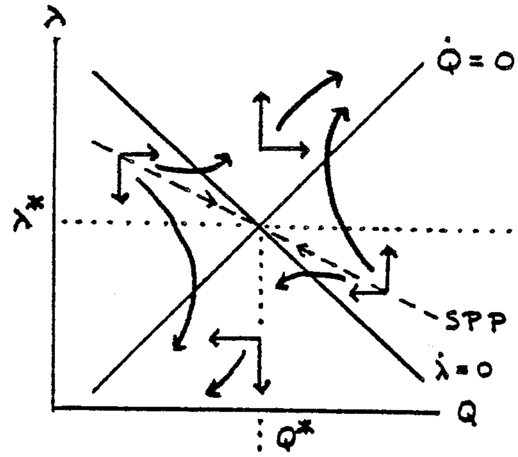

Figure 3a

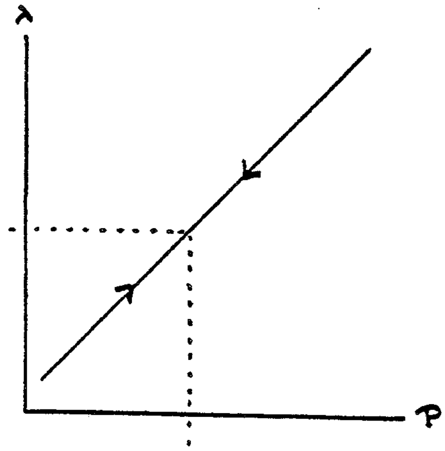

Figure 3b

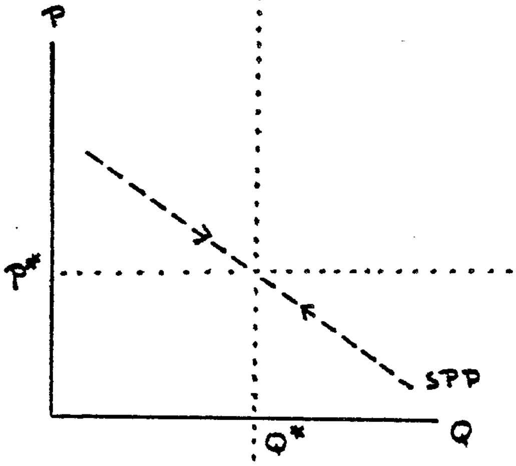

Figure 3c

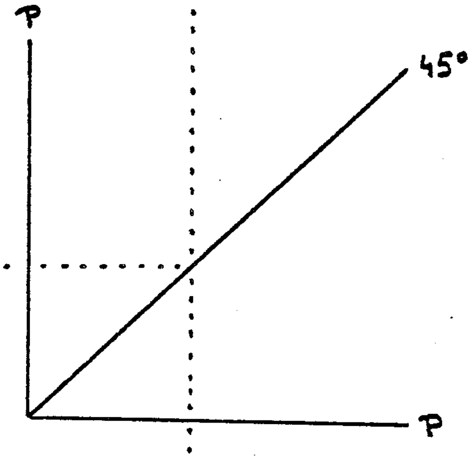

Figure 3d

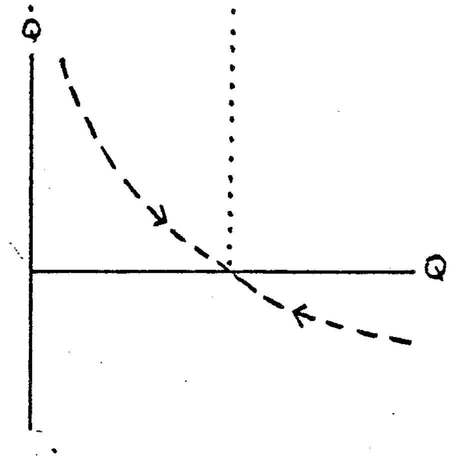

Figure 3e

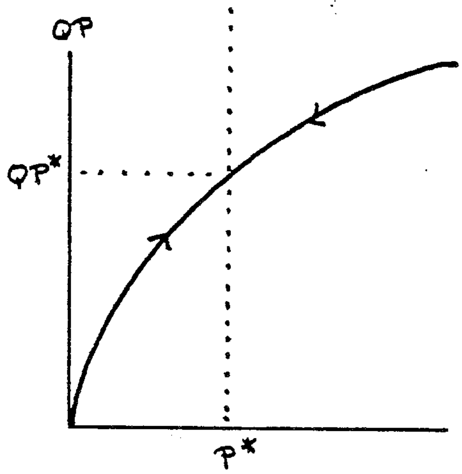

Figure 3f

FIGURES 3

accumulation lies beneath its long-run optimal level as well. This occurs whenever two conditions are fulfilled:
(I.') the social value of a unit of consumer durable accumulation falls short of its long-run optimal level ($\lambda < \lambda^*$) and
(II.') the capital gain from accumulation is positive ($\dot{\lambda} > 0$).

Condition (I.') implies that the level of pollution falls short of its long-run optimal level ($P < P^*$). This implication is contained in Equation 17, which may be rewritten as follows:

$$(17b) \qquad \lambda = - \frac{U_P}{(h^{-1})'} ,$$

i.e. the social value of accumulation (λ) must be equal to the social welfare per unit of accumulation associated with a rise in pollution ($- U_P/(h^{-1})'$). The latter is an increasing function of P, since there is declining marginal utilitiy from P and decling production associated with P. Thus, if $\lambda \nless \lambda^*$, the $P < P^*$, which implies (in turn) that $QP \quad QP^*$ (by Equation 13b). Condition (II.') implies that the social value of a unit of product accumulation rises through time. By Condition 17, the level of pollution should rise and, with it, the level of production should rise as well.

Through an argument analogous to that presented for the pollutant-accumulation economy, it can be shown that Conditions (I.') and (II.') are not only sufficient to keep the welfare cost of accumulation below its long-run optimal value, but also necessary to maintain the economy on its optimal path. Since the argument for this purpose is strikingly similar to that presented with reference to the pollutant-accumulation economy, it need not be spelled out here.

In sum, a "large" initial stock of consumer durables requires that the initial social value of a unit of consumer durable accumulation be low and rising. To maintain social optimality, the inital level of production must also be low and rising. In particular, the initial level of production is sufficiently low to permit a reduction of the consumer durable stock. Yet as the level of production rises, the stock is reduced by smaller and smaller amounts through time.

As the consumer durable stock falls, the marginal welfare from the stock ($U_Q - \lambda \cdot \delta'$) rises, and, for social optimality, the marginal welfare cost of consumer durable accumulation must rise as well. A part of this rise occurs through the opportunity cost of accumulation ($r \cdot \lambda$) and a part through the capital gain from accumulation ($\dot{\lambda}$). As the capital gain falls, the social value of a unit of consumer durable accumulation rises by smaller and smaller amounts through time and, by Condition 17, pollution and production also rise more and more slowly. Thus, the economy appraches its stationary state gradually.

On the other hand, if the government inherits a "small" stock of consumer durables -- i.e. if the current stock falls short of the long-run optimal stock, Q^* -- then the initial social value of a unit of consumer durable accumulation should be high and falling. To maintin social optimality, the level of production must be high and falling as well. The level of production is high enough to induce a rise in the consumer durable stock, yet as production falls the stock rises at a slower and slower rate through time.

Thus far, we have been concerned only with the <u>optimal</u> dynamics of the pollutant-accumulation economy and the product-accumulation economy. Yet suppose that these economies provide a <u>suboptimal</u> mix of production and treatment services. What is the optimal policy response to this problem? Is the optimal policy response compatible with stabilization policy? This is the subject of the following section.

3. Intertemporal Reversals and Stabilization Policies

Let us examine the social optimality of stabilization policies as corrective devices for two types of economic problems:

(a) the economy uses its factors of production fully and efficiently, but its current mix of consumption goods and and treatment services is not appropriate to society's preferences and

(b) the current mix of consumption goods and treatment services is appropriate to social preferences, but the economy does not use its factors of production fully of efficiently. For simplicity, we assume that the initial state of the economy, which manifests either of these economic problems, is a stationary one. In other words, production, consumption, pollution, and pollution treatment are constant at the initial state.[7] We shall subject both the pollutant-accumulation economy and the product-accumulation economy to precisely the same economic problem and examine the optimal policy response in both economies. This procedure will indicate how the durability of the consumption good or the pollutant affects the optimal policies for a given problem and whether stabilization policies are optimal in either case.

To generate the first problem, suppose that both economies are initially at their respective optimal stationary states and then a change in social preferences takes place. Thus, although the initial state of each economy is one in which its factors are used fully and efficiently (otherwise it could not have been optimal), it is not optimal with regard to the new set of preferences. Two types of preference change will be considered: a drop in the social rate of time preference (i.e. society becomes more "future oriented") and a drop in the marginal utility of the pollutant stock (i.e. society becomes more "pollution conscious"). All other preference changes representable by our two macroeconomic models have policy implications which are qualitatively the same as those associated with one of the two preference changes above.

The second problem may be generated by supposing that both economies are initially at their respective optimal stationary states and then an increase in factor supplies occurs. For simplicity, this change is assumed to leave the marginal rate of transformation from QP into T (i.e. dT/dQP = F' of Equation 1) unaffected. With regard to the new set of factor supplies, the initial state of each economy may be regarded as one in which factors are not used fully of efficiently; yet given the existence of unemployment of inefficiency, the mix of consumption goods and treatment services is appropriate to the state of social preferences.

At the outset in our first comparative dynamic exercise, each economy uses its factor supplies fully and efficiently, yet its mix of consumption goods and treatment services is appropriate to a higher rate of social time preference than the one that actually obtains. Equivalently, let each economy be at its optimal stationary state, whereupon a drop in the social rate of time preference takes place. To derive the optimal dynamic response of the pollutant-accumulation economy, consider the effect of this exogenous shock on the Equation System 9 and 10. Clearly, r does not occur in Equation 10; thus, the $\dot{P}=0$ function remains unchanged in P-μ space. However, r does occur in Equation 9; a fall in r causes the $\dot{\mu}=0$ function to shift downwards in P-μ space,[8] as shown in Figure 4.

It is convenient to describe the movement of the economy from its initial state to its new optimal stationary state in terms of short-run, medium-run, and long-run components. In the short run, the government is free to set the level of production -- and thereby determine the level of anthropogenic treatment service, the social value of this service, and the net emission flow -- but the stock of pollutants does not change. In the medium run, the stock of pollutants is free to change as well, but the new optimal stationary state is not attained. Finally, in the long run, the entire transition from the initial state to the new optimal stationary state takes place.

The optimal movement of the pollutant-accumulation economy is described in Figures 4. The solid arrows of Figure 4a represent those time paths for P and μ generated by Equation System 9 and 10 which satisfy the sufficient conditions for optimality. The optimal time paths of all other endogenous variables may be derived from those of P and μ . Figure 4b represents Optimality Condition 8. Figure 4d describes the emission flow from the production and anthropogenic treatment activities and Figure 4c describes the entire net emission flow (given by Equation 5). Lastly, Figure 4e illustrates the production possibility frontier (Equation 1).

The optimal short-run movement of the pollutant-accumulation economy is from point α to point β in Figures 4. The optimal medium-run movement extends from point β to point γ . In the long run, the economy optimally moves from point α to point γ .

It is evident that the optimal adjustment of the pollutant-accumulation economy to a lowe rate of social time preference involves an intertemporal reversal of production (in particular, a short-run fall and a medium-run rise of production) and an intertemporal reversal of anthropogenic treatment (in particular, a short-run rise and a medium-run fall of the treatment service). Such behavior is not compatible with stabilization policy.

We have required that stabilization policies induce monotonic adjustments of sectoral outputs through time. A drop in the rate of social time preference means that society has become more "future oriented". In other words, the long-term welfare losses from pollution have become more important relative to the short-term welfare gains from consumption, both associated with an incremental rise in production. Naturally, the economy must reduce its production

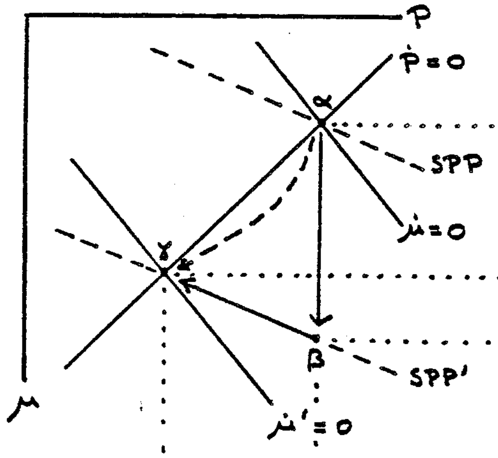

Figure 4a

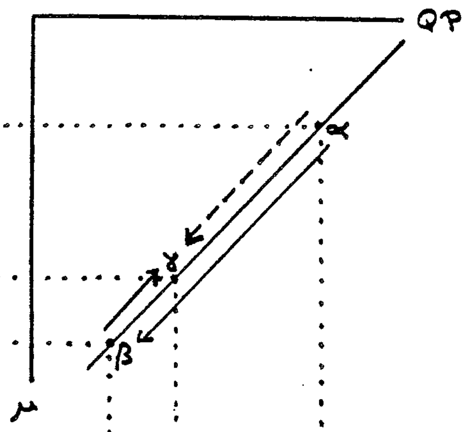

Figure 4b

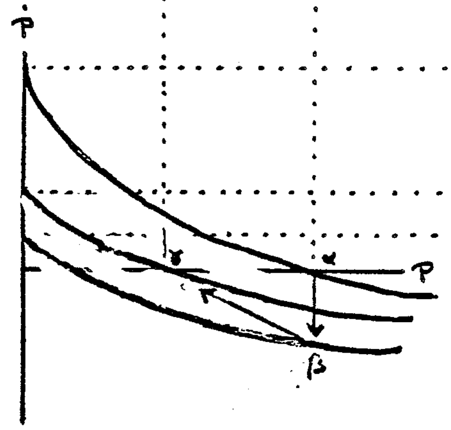

Figure 4c

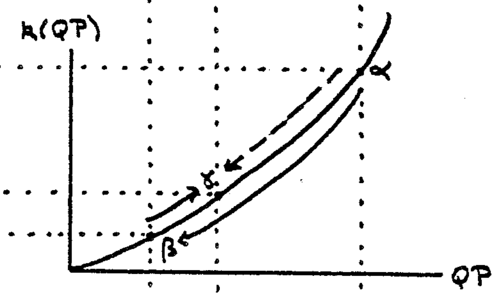

Figure 4d

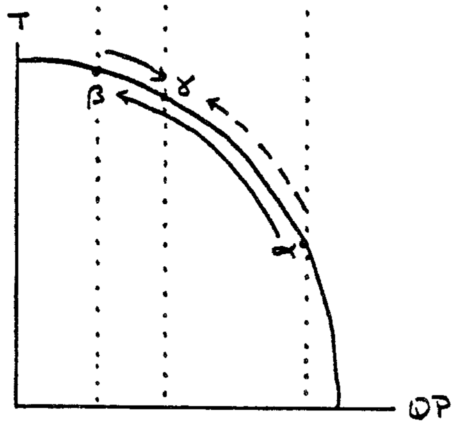

Figure 4e

FIGURES 4

and increase its treatment in the long run. If such adjustments are to proceed via stabilization policies, then production must fall and treatment must rise in the short and medium run as well. The resulting time path of the pollutant-accumulation economy is pictured by the dashed arrows leading directly from point α to points γ in Figures 4.

Figures 4 also show that the socially optimal time path is quite different. The reason why intertemporal reversals are socially desirable is not difficult to find. Initially, the economy's current mix of consumption goods and treatment services is inappropriate with regard to current social preferences. In particular, the current mix gives rise to too much consumption and too much pollution. In other words, with regard to current social preferences, the government inherits a "dirty" environment (i.e. $P > P^*$). Section 2 explained that the optimal policy response to this situation is twofold: (i) the level of production should be set low enough for the pollutant stock to contract and (ii) thereupon the level of production should be raised so that the pollutant stock contracts at a slower and slower rate. The first policy response requires a fall in production and rise in treatment in the short run: for at the initial state α, the pollutant stock is constant, and only a fall in QP and a rise in T can elicit a contraction of this stock. On the other hand, the second policy response requires that production and treatment both switch their directions of movement through time.

This optimal sequence of policy impulses is elucidated in Figure 5. A drop in the social rate of time preference calls for an immediate drop in QP. The magnitude of the drop in QP is such as to have two intertemporal effects on the pollutant-accumulation economy:

(1) the drop in QP is large enough to cause a reduction in the emission flow (and thereby a reduction in the future pollutant stock) and

(2) the drop in QP is small enough (and thus the associated

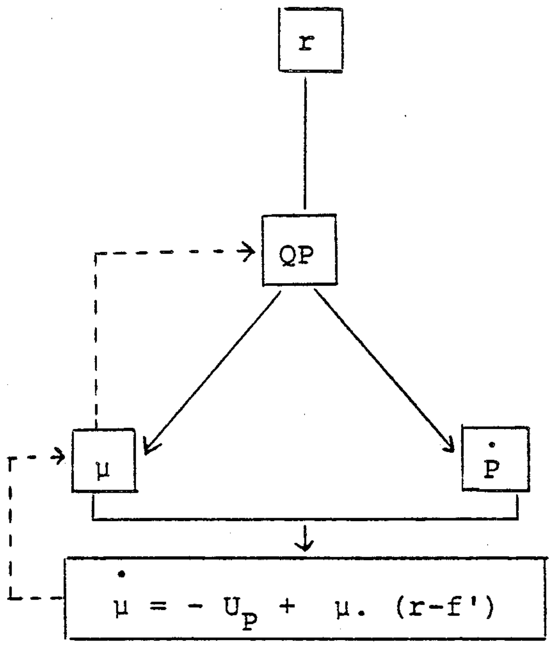

FIGURE 5

drop in μ is small enough, by Optimality Condition 8) to make a capital gain from treatment ($\dot{\mu} > 0$) necessary for the satisfaction of Optimality Condition 9.

The latter intertemporal effect means that the social value of treatment rises through time. Therefore, by Optimality Condition 8, QP must rise as well. In other words, QP switches its direction of movement through time.

Throughout the passage of the medium run, the fall in P reduces $\dot{\mu}$, whereas the rise in the social value of treatment stimulates $\dot{\mu}$. Yet the former intertemporal effect is large enough to cause $\dot{\mu}$ to fall through time. Thus, μ rises at a smaller and smaller rate and the economy approaches its long-run stationary state asymptotically.

Note that the short-run fall in production exceeds its medium-run rise, for in the long run production falls. The reason why the short-run movement must dominate the medium-run movement is also to be found in the social optimality conditions. For the sake of argument, suppose that the short- and medium-run movements were of equal magnitude, so that production returns to its original level. Hence, the treatment service must return to its original level as well. Thus, the long-run size of the pollutant stock must be equal to its original size and U_P returns to its original level. But then how can Optimality Condition 9 be satisfied? If U_P is identical in the initial and final state and $\dot{\mu}$ is iden- as well (viz, equal to zero), then a drop in r must be met by a rise in μ. Yet a rise in μ implies a rise in QP (by Optimality Condition 8), which contradicts our assumption that QP returns to its original level.

On the other hand, suppose that the medium-run movement of production dominates the short-run movement, so that production rises in the long run. Hence, the treatment service must fall in the long run and consequently the size of the pollutant stock must rise in the long run. Yet the pollutant stock can do so only if the flow of emission is positive during some interval of time (from the present to the infinite future). By Equation 11, the movement of P will stimulate $\dot{\mu}$ during this interval, and this effect is reinforced through the medium-run rise in μ. Thus, $\dot{\mu}$ is positive and rising during this interval. But then μ rises without limit and thereby the sufficient conditions for optimality would be violated.

For these reasons, the medium-run movement of production cannot exceed or equal the short-run movement of production. Consequently, the short-run movement of production must dominate and therefore production must fall in the long run.

In sum, a drop in the rate of social time preference calls for a short-run fall and a medium-run rise of production and for the obverse movements by the treatment service. Needless to say, a rise in the rate of social time preference would elicit the opposite results. Therefore we may conclude that, for a stationary pollutant-accumulation economy whose factors of production are used fully and efficiently but whose current mix of consumption goods and treatment services is inappropriate to society's "future orientation", stabilization policies are socially suboptimal.

For a product-accumulation economy facing the same economic problem, stabilization policies are not optimal either. Here, too, intertemporal reversals of production and treatment are socially desriable, but these reversals are the dynamic obverses of those derived for the pollutant-accumulation economy.

Suppose that the product-accumulation economy is initially at its optimal stationary state and then a drop in the social rate of time preference occurs. The optimal adjustment of the economy to society's new "future oriented" preferences is pictured in Figures 6. Figure 6a represents those time paths for Q and λ given by Equations 18 and 19 and satisfying the sufficient conditions for optimality. This figure provides a condensed overview of the optimal

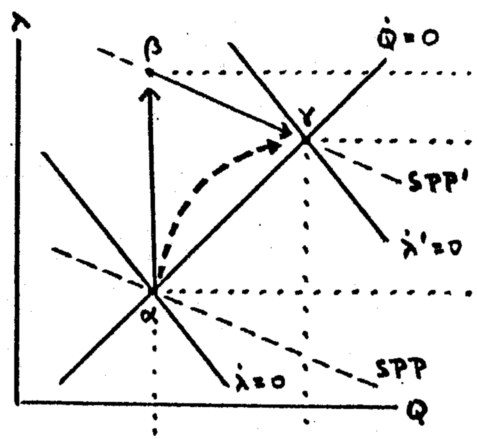

Figure 6a

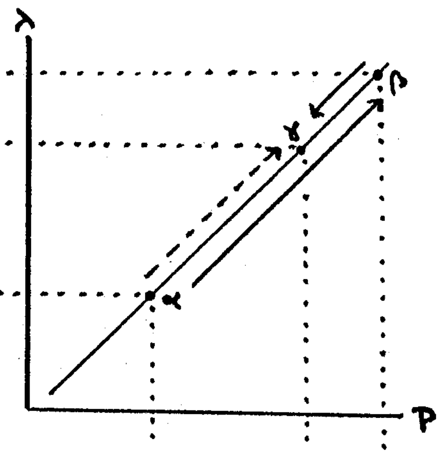

Figure 6b

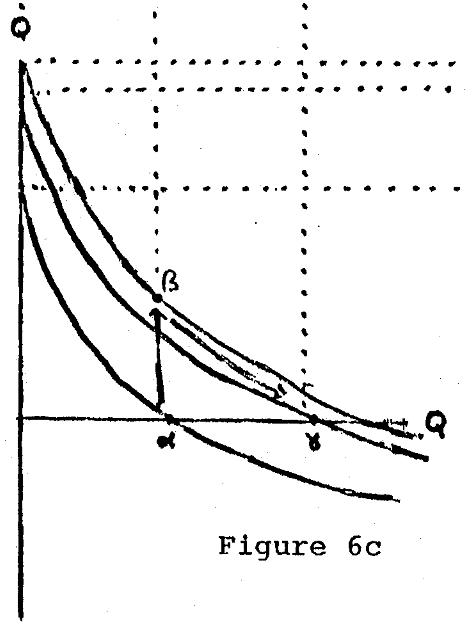

Figure 6c

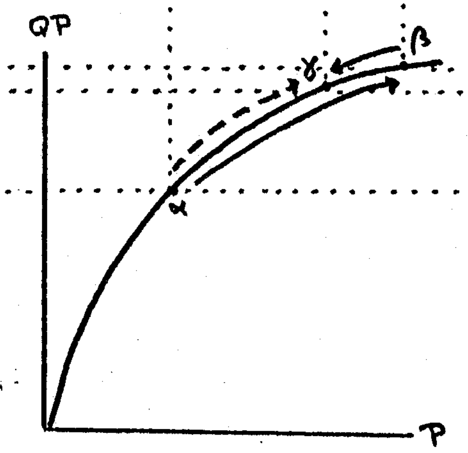

Figure 6d

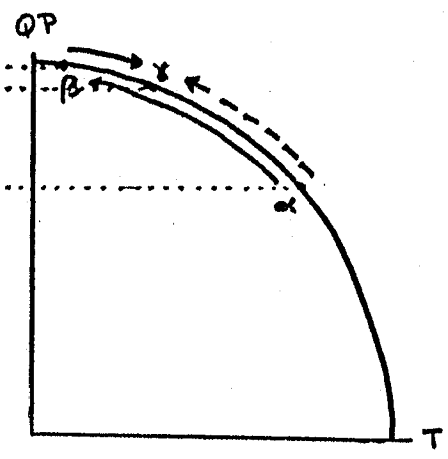

Figure 6e

FIGURES 6

adjustment path. A drop in r leaves Equation 19 unaffected; thus, the $\dot{Q}=0$ function of Figure 6a remains unchanged. However, a drop in r does affect Equation 18; it causes the $\dot{\lambda}=0$ function of Figure 6a to shirt upwards.[9]

Figure 6b represents Optimality Condition 17. Figure 6c describes the accumulation of consumer durables (by Equation 12). Figure 6d pictures the technological relation between production and pollution (from Equation 13b). Figure 6e represents the production possibility frontier.

In the short run the economy should move from point α to point β, in the medium run from point β to point γ, and in the long run from point α to point γ. This adjustment path involves intertemporal reversals of production and treatment; hence, stabilization policy cannot be optimal. The suboptimal path that would be induced by stabilization policy is given by the dashed arrows from point α to point γ in Figures 6.

The optimal intertemporal reversals run in the opposite directions from those of the pollutant-accumulation economy: in the short run production expands and anthropogenic treatment contracts, whereas in the medium run production contracts and treatment expands. The reason for this difference between the product-accumulation economy and the pollutant-accumulation economy is that a drop in the rate of social time preference does not have the same welfare implications in both economies. If the pollutant-accumulation economy becomes more "future oriented", then the long-lived effects of production on pollution become more important relative to the short-lived effect of production on consumption. Thus, production becomes <u>less</u> attractive than it was previously. By contrast, if the product-accumulation economy becomes more "future oriented", then the long-lived effects of production on consumption become more important relative to the short-lived effects of production on pollution. In this case, production becomes <u>more</u> attractive than it was previously. Thus, it is not surprising that the optimal paths of the two economies should be characterized by different intertemporal reversals.

For the product-accumulation economy, the current mix of consumption goods and treatment services is inappropriate for current social preferences in that there is too little consumption and too little pollution. Thus, with reference to these social preferences, the government inherits a "small" stock of consumer durables (i.e. $Q < Q^*$). According to Section 2, the optimal policy response to this initial state is (i) to set the level of production high enough so that the stock of consumer durables expands and (ii) thereupon to lower the level of production so that the durable stock expands at a slower and slower rate. Accordingly, production must rise in the short run, for at the initial state α, the durable stock is constant, and a rise in production is required to induce an expansion of the stock. Yet in the medium run production must fall.

As in the case of the pollutant-accumulation economy, this intertemporal reversal may be explained in terms of the intertemporal effects produced by the adjustment of the level of production. A drop in the rate of social time preference calls for a rise in the level of production. The rise in production has two intertemporal effects:
(1) it must be large enough to cause an expansion of the consumer durable stock in the future and
(2) it must be small enough (and thus the associated rise in λ, by Equations 13 and 17, must be small enough) to make a capital loss from accumulation ($\dot{\lambda} < 0$) necessary for the satisfaction of Optimality Condition 18.
Yet this latter intertemporal effect implies that the social value of accumulation falls and (by Equations 13 and 17) the level of production falls as well. Hence, the level of production switches its direction of movement through time.

Through this analysis it becomes clear that, for a stationary product-accumulation economy whose factors of production are used fully and efficiently, but whose current mix of consumption goods and treatment services is inappropriate to society's "future orientation", stabilization policies are not socially optimal. Instead, policies which induce intertemporal reversals of production and treatment are required, but these intertemporal reversals are the dynamic opposites of those requried in the pollutant-accumulation economy.

As the next economic problem, consider a mix of consumption goods and treatment services which is inappropriate to society's "pollution consciousness". In particular, consider a stationary economy which useds its factor supplies fully and efficiently, yet whose consumption good-treatment service mix is appropriate to a higher marginal utility of pollution (U_P) than the one that actually obtains. To generate this economic problem, we let the pollutant accumulation economy and the product-accumulation economy be at their respective optimal stationary states, whereupon a fall in the marginal utility of pollution takes place.

Stabilization policies are not the socially optimal way of overcoming this problem. Once again, intertemporal reversals of production and treatment are called for. Yet unlike the adjustment to a new "future orientation", the intertemporal reversals required of the pollutant-accumulation economy are qualitatively the same as those required of the product-accumulation economy.

This result may be demonstrated succinctly as follows. First, consider the optimal adjustment of the pollutant-accumulation economy as described by Equations 9 and 10. The $\dot{P}=0$ function (Equation 10) remains unaffected by a fall in U_P, but the $\dot{\mu} = 0$ function (Equation 9) shifts downwards in P- μ space.[10] Thus, the optimal path of the economy is qualitatively the same as that in response to a fall in the rate of social time preference. This path is illustrated by Figures 4.

Next, consider the optimal adjustment of the product-accumulation economy as described by Equations 18 and 19. A fall in U_P leaves the $\dot{\lambda} = 0$ function (Equation 18) unaffected, but causes the $\dot{Q}=0$ function (Equation 19) to shift downwards in $Q-\lambda$ space (as shown in Figure 7).[11] The economy should move from point α to point β in the short run, from point β to point γ in the medium run, and from point α to point γ in the long run.

Since the stock of consumer durables is constant at point α and falling at point β , the level of production must fall in the short run. Consequently, the level of anthropogenic treatment must rise in the short run. The social value of accumulation rises in the medium run. By the Optimality Condition 17, the flow of pollution must rise as well. Yet, by the technological relation 13b, this can happen only if the level of production falls. Consequently, the level of anthropogenic treatment rises in the medium run.

In sum, if society becomes more "pollution conscious" (i.e. if U_P falls), then the optimal adjustment of the product-accumulation economy to the new preferences involves intertemporal reversals of production and treatment. Similarly to the adjustment of the pollutant-accumulation economy to these new preferences, production falls in the short run and rise in the medium run while anthropogenic treatment moves in the opposite directions.

Analogously, if society becomes less "pollution conscious" (i.e. if U_P rises), then the optimal adjustment paths of both economies involve a short-run rise and a medium-run fall of production as well as a short-run fall and a medium-run rise of anthropogenic treatment. It can easily be shown -- it is redundant to do so here -- that (a) a rise in "consumption consciousness" (i.e. a rise in U_Q) calls for qualitatively the same adjustment paths as a fall in "pollution consciousness" for both economies and (b) a fall in "consumption consciousness" calls for qualitatively the same paths as a rise in "pollution consciousness" for both economies.

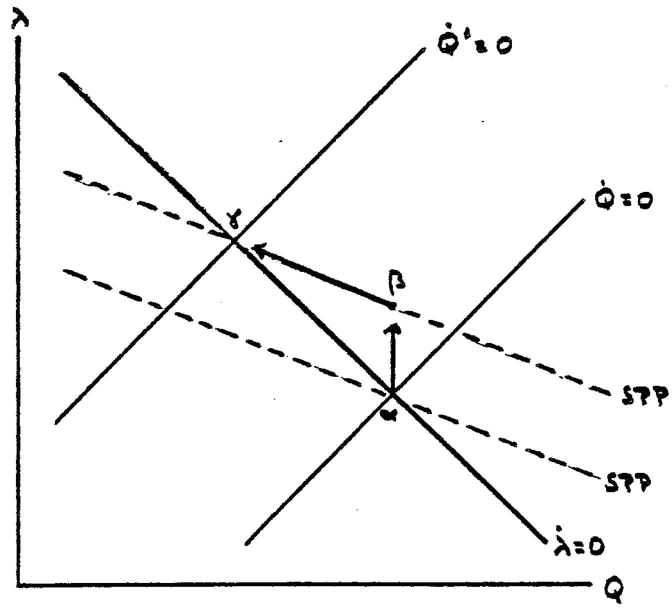

FIGURE 7

The following overall conclusion emerges from these considerations. If factors of production are used fully and efficiently in a stationary pollutant-accumulation economy and a stationary product-accumulation economy, but the current consumption good-treatment service mix is inappropriate to society's marginal valuation of consumption or pollution, then stabilization policies are socially suboptimal. A given inappropriate consumption good-treatment service mix calls for qualitatively the same intertemporal reversals of production and treatment in both economies.

As a final economic problem, consider a stationary economy whose current mix of consumption goods and treatment services is appropriate to society's preferneces, but whose factors are not used fully or efficiently. To generate this problem, we first set the pollutant-accumulation economy and the product-accumulation economy at their respective optimal stationary states, and then let an expansion of production capacity take place. This expansion may be the result of an increase in factor supplies or a technological improvement. For the sake of simplicity, we assume that the expansion does not affect the marginal rate of transformation from QP into T (i.e. dT/dQP = F' of Equation 1).

Stabilization policies are not the optimal corrective devices in this case either. Once again, the optimal adjustment paths of both economies involve intertemporal reversals, yet these reversals are different from the ones we have encountered thus far. In the previous comparative dynamic exercises, the optimal adjustment paths all involved intertemporal reversals of <u>both</u> production and anthropogenic treatment. In this case, an intertemporal reversal of only <u>one</u> sectoral output is required. In the pollutant-accumulation economy the treatment sector undergoes the intertemproal reversal, whereas in the product-accumulation economy the production sector undergoes the reversal.

With regard to the pollutant-accumulation economy, consider the system described by Equations 9 and 10. The expansion of production capacity makes it possible to increase both production and anthropogenic treatment in such a way that QP increases while the new emission flow ($\dot{P}$) remains unchanged. By Optimality Condition 8, this means that the social value of treatment (μ) may rise while $\dot{P}$ remains unchanged. Hence, the $\dot{P}=0$ function (Equation 10) shifts upwards in $P-\mu$ space. On the other hand, the $\dot{\mu}=0$ function (Equation 9) remains unaffected by the expansion of production capacity. Thus, the saddle-point path shifts upwards in $P-\mu$ space, as shown in Figure 8a. The expansion of production capacity is illustrated in Figures 8d and 8e.

From these figures it is clear that in the short run both production and anthropogenic treatment should increase, whereas in the medium run production should increase at the expense of anthropogenic treatment.[12] Thus, the optimal time path of the pollutant-accumulation economy involves an intertemporal reversal of anthropogenic treatment, but not of production.

The product-accumulation economy can be considered concisely in terms of Equations 18 and 19. The expansion of production capacity makes it possible to increase the level of anthropogenic treatment while the level of production remains unchanged. As result, the flow of pollutants (P) falls while the rate of consumer durable accumulation ($\dot{Q}$) remains the same. By Optimality Condition 17, the fall in P is accompanied by a fall in λ. Thus, the expansion of production capacity permits a fall in λ while $\dot{Q}$ is unchanged. In other words, the $\dot{Q}=0$ function (Equation 19) falls in $Q-\lambda$ space, as shown in Figure 9a. However, the $\dot{\lambda}=0$ function (Equation 18) remains unaffected in $Q-\lambda$ space. Figures 9d and 9e illustrate the expansion of production capacity.

As for the pollutant-accumulation economy, both the production and treatment sectors should expand in the short run. Yet in contrast to the pollutant-accumulation economy, the production sector should contract and the treatment sector should expand in the medium run.[13] In other words, the

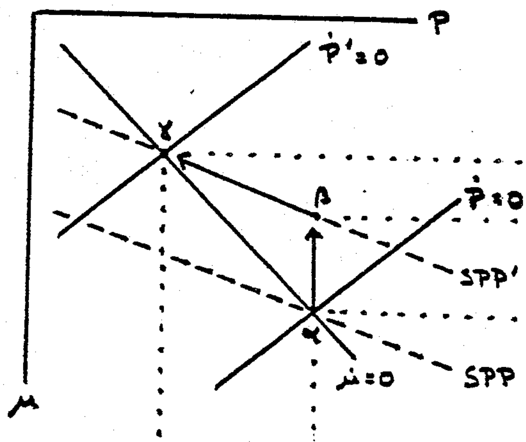

Figure 8a

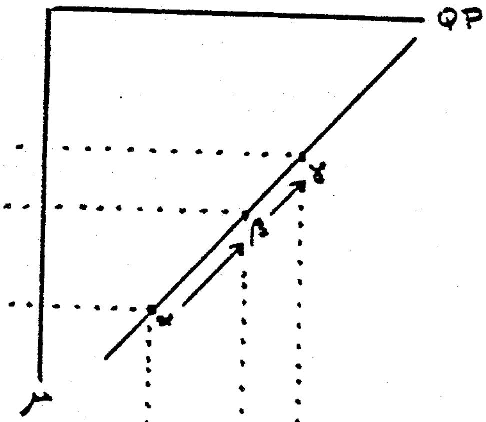

Figure 8b

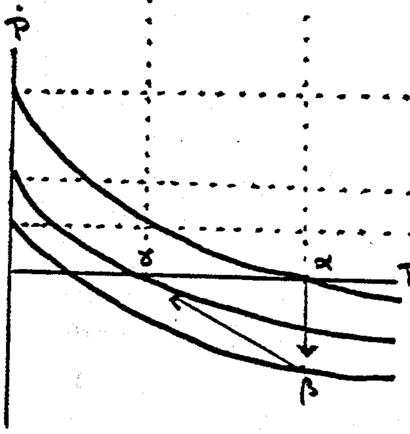

Figure 8c

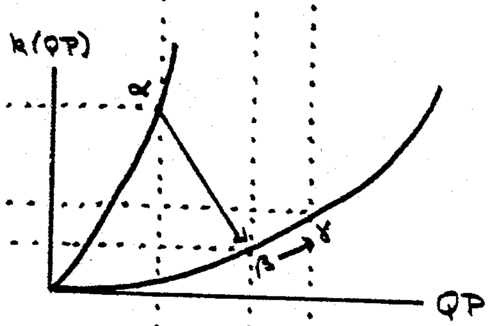

Figure 8d

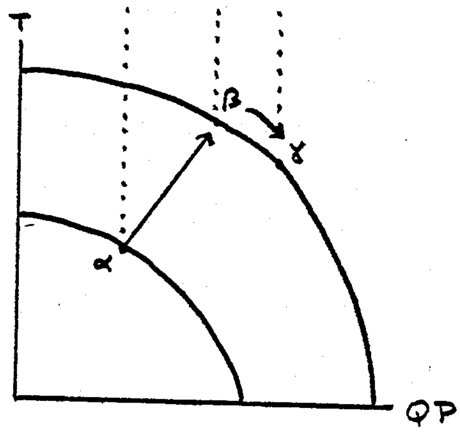

Figure 8e

FIGURES 8

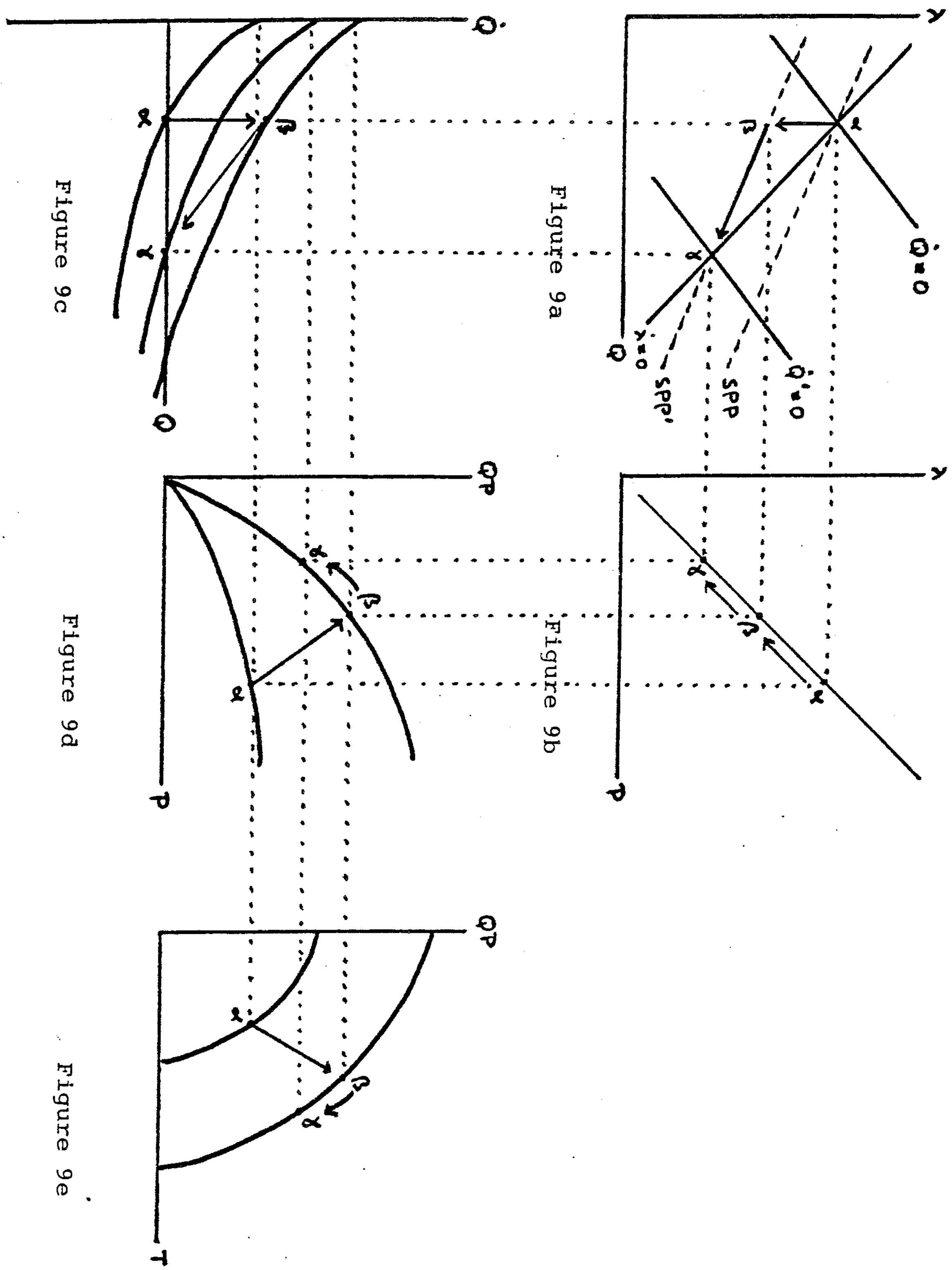

Figure 9a

Figure 9b

Figure 9c

Figure 9d

Figure 9e

FIGURES 9

optimal path involves an intertemporal reversal of production but not of anthropogenic treatment.

In sum, for a stationary economy whose consumption good-treatment service mix is appropriate to society's preferences but whose factor supplies are not used fully or efficiently, stabilization policies are not the most desirable corrective device. The intertemporal reversal of a single sectoral output is required. Whether this sectoral otuput comprises the treatment services or consumer durables depends on whether the pollutant or the consumption good is long-lived.

4. Conclusion and Overview

Although the analysis above proceeds in terms of two very simple aggregative models, it might conceivably lead to some scepticism concerning the inevitable desirability of real-world stabilization policies. Stabilization policies have commonly been applied to overcome the problems of unemployment or an undesirable sectoral output mix, and these are precisely the problems around which our analysis centers. We required that stabilization policies do not induce cyclical fluctuations in sectoral outputs and we then proceeded to show that such fluctuations may be the optimal policy responses to the problems above.

Naturally, a number of special assumptions are necessary to reach this conclusion, but these assumptions are not merely abstract curiosities. A crucial assumption is that government policy impulses have welfare effects in the present and in the future. Moreover, there is an intertemporal tradeoff between these welfare effects, so that a present welfare gain is associated with a future welfare loss, and a present welfare loss is associated with a future welfare gain. These are not far-fetched suppositions; many policy problems center around such welfare effects. The joint control of long-lived pollutants and short-lived consumption goods or of short-lived pollutants and long-lived consumption goods are not the only problems of this sort. As noted in Section 1, the joint control of nondurable consumption and investment, of inflation and unemployment (in a shifting-Phillips-curve world), and of nondurable consumption and renewable natural resources are further examples.

To investigate the social optimality of stabilization policies in this context, we constructed the pollutant-accumulation economy and the product-accumualtion economy, which both satisfy the assumptions above. We assumed that both economies are initially in a stationary state which manifests one of the following problems: (a) factor supplies are not used fully or efficiently or (b) the mix of consumption goods and treatment services is inappropriate to social preferences. We then derived the optimal dynamic paths which the two economies must follow in response to the problem at hand. Each of these paths involved an intertemporal reversal in the level of production or in the level of anthropogenic treatment or both.

The optimal response of the two economies is qualitatively the same for certain problems, but not for others. In particular, if the mix of consumption goods and treatment services is appropriate to a higher marginal valuation of pollution (U_P) than the one that actually obtains, then both economies required a short-run fall and a medium-run rise of production accompanied by a short-run rise and a medium-run fall of anthropogenic treatment. The same intertemporal reversals are necessary if the consumption good-treatment service mix is appropriate to a higher marginal valuation of consumption (U_Q) than the one that actually obtains. On the other hand, if the mix is appropriate to a lower marginal valuation of polluation or consumption than the actual one, then both economies require the dynamic obverses of these intertemporal reversals.

If the consumption good-treatment service mix is appropriate to a higher rate of social time preference than the one that actually obtains, then the two economies require different intertemporal reversals. For the pollutant-accumulation economy, production should fall in the short run and rise in the medium run, while anthropogenic treatment should rise in the short run and fall in the medium run; but for the product-accumulation economy, the intertemporal reversals run in the opposite directions.

The two economies also respond differently to unemployment or inefficient factor use. The pollutant-accumulation economy requires a short-run rise and a medium-run fall of anthropogenic treatment and a rise in production over the short and medium run; yet the product-accumulation economy requires a short-run rise and a medium-run fall of production and a rise in anthropogenic treatment over the short and medium run.

The reason why intertemporal reversals are the optimal responses to all of these economic problems is to be found in the intertemporal welfare effects of government policy. The government is able to influence the dynamic development of both economies by controling the level of production (QP). In the pollutant-accumulation economy, an increase in production implies an immediate social welfare gain from consumption and a future social welfare loss from pollution; whereas in the product-accumulation economy, an increase in production implies an immediate social welfare loss from pollution and a future social welfare gain from consumption.

In response to each of the economic problems above, the level of production must be adjusted in the short run. However, in the pollutant-accumulation economy, this adjustment affects the capital loss from anthropogenic treatment, and the change in the value of treatment in the medium run (in turn) affects the optimal elvel of production in the medium run. This feedback effect is responsible for the intertemporal reversal of production or treatment. The short-run adjustment of production also affects the pollutant stock in the medium run. This change in the pollutant stock is responsible for dampening the medium run movement of production and anthropogenic treatment, so that these sectoral outputs change at a slower and slower rate through time and the economy gradually approaches its optimal long-run stationary state.

Similarly, in the product-accumulation economy, the short-run adjustment of production affects the capital gain from consumer durable accumulation, and the change in the value of accumulation in the medium run (in turn) affects the optimal level of production in the medium. Here it is this feedback effect which is responsible for the intertemporal reversal of production or treatment. The short-run production adjustment also affects the consumer durable stock in the medium run, and the change in this stock dampens the medium-run movement of production and anthropogenic treatment. Thus, these sectoral outputs asymptotically approach their respective optimal long-run levels.

Our analysis of intertemporal reversals of production and treatment is not meant to imply that stabilization policies are never the optimal response to unemployment or an undesirable sectoral output mix. Obviously, the relaxation of some underlying assumptions of our analysis -- such as the existence of only two productive sectors or the stationarity of the initial state of the economy -- may give rise to different optimality properties of stabilization policies than the ones we have obtained. Nevertheless, there appears to be no a priori reason why complicating our models for the sake of greater realism should reveal stabilization policies to be invariably optimal. As long as government policy has social welfare effects at present and in the future and as long as there is an intertemporal tradeoff between these welfare effects, it may be unwise to give stabilization policies our unquestioned acceptance. Wether stabilization policies are optimal under these circumstances depends on the details of the problem at hand. Our analysis simply shows that they <u>may</u> be suboptimal and that therefore this question is worth exploring.

APPENDIX

The effect of consumption on pollution may be incorporated into our model of the product-accumulation economy as follows. The flow of pollutants is given by

$$\text{(A1)}\quad P = g_1(QP) + g_2(T) + g_3(Q) - T.$$

Substituting the production possibility frontier (Equation 1) into this equation,

$$\text{(A2)}\quad P = g_1(QP) + g_2(F(QP)) + g_3(Q) - F(QP)$$
$$= h(QP) + g_3(Q).$$

Thus,

$$\text{(A3)}\quad QP = h^{-1}\left[P - g_3(Q)\right].$$

Substituting Equation A3 into Equation 12,

$$\text{(A4)}\quad \dot{Q} = h^{-1}\left[P - g_3(Q)\right] - \delta(Q).$$

Consequently, the government's policy problem may be expressed as

$$\text{(A5)}\quad \text{Maximize}\quad W = \int_0^\infty e^{-rt} \cdot U(Q,P)\, dt$$

$$\text{subject to}\quad \dot{Q} = h^{-1}\left[P - g_3(Q)\right] - \delta(Q).$$

The first-order conditions are

$$\text{(17)}\quad -U_P = \lambda \cdot (h^{-1})'$$

$$\text{(A6)}\quad U_Q - \lambda \cdot \delta' - \lambda \cdot (h^{-1})' \cdot g_2' = r \cdot \lambda - \dot{\lambda}$$

as well as Equation A4.

Let $\psi(P,Q) = -(U_P/(h^{-1})')$.
(Note that this expression is not identical to the function ψ in the text.) By Equation 17,

(A7) $\lambda = \psi(P,Q)$.

Assuming that the following function exists

(A8) $P = \hat{\psi}(\lambda,Q)$,

we obtain

(A9) $\dot{Q} = h^{-1}\left[\hat{\psi}(\lambda,Q) - g_3(Q)\right] - \delta(Q)$

Soving Equations A6 and A9 simultaneously, the time paths for the consumer durable stock and the social value of consumer durable accumulation may be derived. These paths may be depicted by Figure 3a. The $\dot{Q}$=0 function is upward-sloping in Q- λ space:

$$\left.\frac{d\lambda}{dQ}\right|_{\dot{Q}=0} = -\frac{(h^{-1})' \cdot \left[\hat{\psi}_Q - g_3'\right] - \delta'}{(h^{-1})' \cdot \hat{\psi}_\lambda}$$

is positive since

$$\hat{\psi}_Q = -\frac{U_P \cdot g_3'}{(h^{-1})'^{2}} > 0 \quad \text{and}$$

$$\psi_P = \frac{(h^{-1})'' \cdot U_P - U_{PP} \cdot (h^{-1})'}{\left[(h^{-1})\right]^2} > 0$$

which implies that $\hat{\psi}_\lambda > 0$.
The $\dot{\lambda}$ =0 function is downward-sloping in Q- λ space:

$$\left.\frac{d\lambda}{dQ}\right|_{\dot{\lambda}=0} = -\frac{\lambda \cdot \left[(h^{-1})' \cdot g_3'' - (h^{-1})'' \cdot (g_2')^2 + \delta''\right] - U_{QQ}}{(h^{-1})' \cdot g_3' + \delta' + r} < 0.$$

Hence, it is clear that this model yields the same qualitative comparative dynamic results as those obtained from the model of the product-accumulation economy in the text.

FOOTNOTES

1. As a way of portraying the costs of instrument variable adjustment, Theil also included in his policy objective function the sum of the squared deviations of the actual instrument variables from their desired levels. Instrument adjustment costs have been given further attention by Henderson and Turnovsky (1972), Turnovsky (1977), and Ali and Greebaum (1976).

1a. The $\dot{P}=0$ function is upward-sloping since

$$\left.\frac{d\mu}{dP}\right|_{\dot{P}=0} = -\frac{f'}{k_Q \cdot (\phi^{-1})'} > 0.$$

the $\dot{\mu}=0$ function is downward-sloping since

$$\left.\frac{d\mu}{dP}\right|_{\dot{\mu}=0} = \frac{U_{PP} + \mu \cdot f''}{r + f'} < 0.$$

2. See, for example, Arrow and Kurz (1970).

3.

$$\frac{d^2H}{dP^2} = U_{PP} - \mu \cdot f'' - H_{QQ} \cdot \left(\frac{f'}{k_Q}\right)^2 + H_Q \cdot \left(\frac{f''}{k_Q}\right).$$

If the Hamiltonian is maximized with respect to the control variable, $H_Q = 0$. Moreover,

$$H_{QQ} = U_{QQ} + \mu \cdot k_{QQ} < 0$$

Thus, $(d^2H/dP^2) < 0$.

4. As shown by the saddle-point path of Figure 1a, the long-run optimal level of the capital loss from pollution treatment is zero.

5. Of course, pollutants may be emitted by the consumption activity as well. Exhausts from automobiles are a classic example. However, the inclusion of this possibility in our model complicates the mathematics considerably

without yielding analytical results which are qualitatively different from the ones that may be obtained in the absence of this possibility. Thus, we ignor the effect of consumption on pollution here, but outline the augmented model in the appendix.

6. The Hamiltonian, maximized with respect to R, must be a concave function of Q.

$$\frac{d^2\hat{H}}{dQ^2} = \hat{H}_{QQ} + \hat{H}_{PP} \cdot \left(\frac{\delta'}{(h^{-1})'}\right)$$
$$+ \hat{H}_P \cdot \left[\left(\frac{\delta''}{(h^{-1})'}\right) - \left(\frac{\delta'}{(h^{-1})'}\right)^2 \cdot \left(\frac{(h^{-1})''}{(h^{-1})'}\right) \right]$$

and if the Hamiltonian is maximized with respect to R, $H_R = 0$. Furthermore,

$$\hat{H}_{QQ} = U_{QQ} - \lambda \cdot \delta'' \quad \text{and}$$
$$\hat{H}_{PP} = U_{PP} + \lambda \cdot (h^{-1})''.$$

Thus, $(d^2\hat{H}/dQ^2) < 0$.

7. This is an important assumption. Through it, stabilization policies can be shown to be suboptimal corrective devices for both types of economic problems. If the assumption is violated, the suboptimality of stabilization policies may no longer emerge. Clearly, there are infinitely many ways in which the assumption could be violated -- infinitely many ways in which production, consumption, pollution, and pollution treatment could be changing at the initial state -- and the optimal corrective policy device is sensitive to thsi consideration. For particular non-stationary initial states, stabilization policies may prove to be optimal. Yet this possibility does not contradict the theme of this article. All that is demonstrated here is the <u>possibility</u> that stabilization policies may be suboptimal, not the necessity of their being so.

8. By Equation 9, it is evident that

$$\frac{\partial \dot{\mu}}{\partial r} = \mu < 0 \quad \text{and} \quad \frac{\partial \dot{\mu}}{\partial \mu} = r + f' > 0.$$

A fall in r causes $\dot{\mu}$ to rise and thus, for a given level of P, a fall in μ is required to restore $\dot{\mu}$ to zero. In other words, the $\dot{\mu}=0$ function must shift downwards in Figure 3a.

9. From Equation 18 it is evident that

$$\frac{\partial \dot{\lambda}}{\partial r} = \lambda > 0 \quad \text{and} \quad \frac{\partial \dot{\lambda}}{\partial \lambda} = r + \delta' > 0.$$

A fall in r causes $\dot{\lambda}$ to fall and therefore, for a given level of Q, a rise in λ is required to restore $\dot{\lambda}$ to zero. In other words, the $\dot{\lambda}=0$ function must shift upwards in $Q-\lambda$ space.

10. From Equation 9 it can be shown that

$$\frac{\partial \dot{\mu}}{\partial U_p} < 0 \quad \text{and} \quad \frac{\partial \dot{\mu}}{\partial \mu} > 0.$$

A fall in U_p causes $\dot{\mu}$ to rise. Thus, for a given level of P, a fall in μ is required to restore $\dot{\mu}$ to zero.

11. Since $\psi = (U_p/(h^{-1})')$ and $(h^{-1})' > 0$, $(\partial\psi/\partial U_p) > 0$. Thus, $(\partial\psi^{-1})/\partial U_p) > 0$ and also $\partial h^{-1}(\psi^{-1})/\partial U_p > 0$. Consequently, in Equation 19, $(\partial\dot{Q}/\partial U_p) > 0$. A fall in U_p causes $\dot{Q}$ to fall as well. Moreover, $(\partial\dot{Q}/\partial Q) < 0$ in Equation 19. Thus, for a given λ, a fall in Q is required to restore $\dot{Q}$ to zero.

12. These results may be deduced from Figure 8a alone. In this figure, the short-run movement from point α to point β implies an increase in μ, which (by Optimality Condition 8) is accompanied by an increase in QP. Thus, production must rise in the short run. Moreover, the pollutant stock is constant at point α and contracting at point β. Thus, the level of anthropogenic treatment must also rise in the short run. (The level of natural

treatment remains constant in the short run). The medium-run movement from point β to point γ involves a continued rise in μ and therefore also in QP. Yet in the medium run the production possibility frontier is constant and therefore the rise in production must go at the expense of anthropogenic treatment. Consequently, the pollutant stock is reduced at a slower and slower rate through time.

13. These results may be deduced from Figure 9a alone. Here the short-run movement from point α to point β implies and increase in production, since the stock of durables is constant at point α and increasing at point β. Furthermore, this short-run movement involves a fall in λ, which (by Optimality Condition 17) is accompanied by a fall in P. Yet given that production rises in the short run, pollution can fall only if anthropogenic treatment rises as well. The medium-run movement from point β to point γ involves a continued fall of λ and therefore also of P. Since the production possibility frontier is constant in the medium run, pollution can decline only if anthropogenic treatment expands at the expense of production.

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