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Robustness of the CUSUM and CUSUM-of-Squares Tests to Serial Correlation, Endogeneity and Lack of Structural Invariance: Some Monte Carlo Evidence

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Abstract

This paper investigates by means of Monte Carlo techniques the robustness of the CUSUM and CUSUM-of-squares tests (Brown et al., 1975) to serial correlation, endogeneity and lack of structural invariance. Our findings suggest that these tests perform better in the context of a dynamic model of the ADL type, which is not affected by serial correlation or non-predetermined regressors even if over-specified. In this case, the empirical sizes of both tests are close to the nominal ones, whether a stationary or a cointegration environment is considered. The CUSUM-of-squares test is to be preferred, as it is very powerful to detect changes in the conditional model parameters, whether or not the variance of the regression error is included in the set of parameters shifting, especially towards the end of the sample.

Keywords

CUSUM and CUSUM-of-squares tests, parameter instability, structural invariance, marginal and conditional processes, ADL model

JEL Classification

C12, C15, C22

Contents

1 Introduction

Two of the most frequently employed tests for parameter constancy in the context of a linear regression are the CUSUM and CUSUM-of-squares tests proposed in the seminal paper of Brown et al. (1975). Their widespread use is due to a large extent to the fact that they are designed to test the null hypothesis of parameter stability against a variety of alternatives. By constrast, other tests require prior knowledge of the type of coefficient variation (or the timing of any structural shifts) exhibited by model parameters, whilst have no power against other alternatives. Typically, the alternative being considered is that the parameters follow a randow walk. Examples include the F-test of LaMotte and McWhorter (1978), the point optimal invariant (POI) test of King (1980, 1985, 1988), the locally best invariant (LBI) test of Nyblom and Makelainen (1983), King and Hillier (1985), Leybourne and McCabe (1989) and Nyblom (1989).

The wide applicability of the CUSUM and CUSUM-of-squares tests has to be set against several drawbacks from which they suffer, as has become increasingly clear.[1] The discussants of the original Brown et al. (1975) paper had already detected some potential factors that might affect the distribution of the test statistics under the null hypothesis of parameter stability or under the alternative of parameter variation, thereby affecting size and power respectively. Subsequent papers have investigated these issues further. Below we summarise in chronological order the main studies concerned with factors that are likely to affect the size of the test.

Smith (1975) and Quandt (1975) raised the question whether the null distributions might be affected by the presence of a serially correlated error. Brown et al. (1975) had speculated that these effects are likely to be substantial. Priestley (1975) pointed out that that these tests require the regressors to be non-stochastic. This rules out lagged dependent variables on the right-hand side of the regression, thus making the applicability of the tests in the context of general dynamic models questionable. In this respect, one should note that the effects of a non-exogenous regressor on their null distribution might be significant. If the regressor is non-exogenous, then the OLS estimator is inconsistent, which in turn implies that the residual is not a consistent estimator of the regression error. Kramer et al. (1988) examined whether the CUSUM and CUSUM-of-squares tests generalise to dynamic models. They crucially assumed that the regression

[1]Note that the other tests mentioned in the text also have several shortocomings, as shown in the literature (see, e.g., Moryson, 1998). However, most of the criticisms refer to their low power, whilst the focus in this paper is on the size of the test (see below).

error is a martingale difference sequence with respect to the $\sigma - fields$ generated by contemporaneous and past regressors and the past regression errors. Theorem 1 (pp. 1358) proves that the null distribution of the CUSUM test remains asymptotically the same regardless of whether or not a lagged endogenous variable is included in the set of regressors. However, their analysis is not informative about how the presence of a non-exogenous regressor might affect the properties of the CUSUM test, since it is based on the assumption that the regression error is orthogonal to the set of regressors.

More recently, Hansen (2000) raised a general issue concerning tests for parameter constancy (including the CUSUM and CUSUM-of-squares tests), i.e. whether they can distinguish between instability in the regression parameters and instability in the process driving the regressors. This question is related to the concept of superexogeneity, according to which a necessary condition for a regressor to be superexogenous with respect to a parameter of interest is that the parameters of the conditional moment be stable even if the parameters of the marginal model change. Such a property is known as *structural invariance* (see Engle et al., 1983). According to Hansen (2000), the tests suffer from size distortions when a change in the parameters of the marginal process occurs.

He also proposed a variation of the CUSUM test for parameter stability in the context of a cointegrating regression (see Hansen, 1992), and showed that it can be seen as testing the null of cointegration against the alternative of no-cointegration. Further, the modifications required for its validity involve removing nuisance parameter dependencies which characterise the simple OLS estimator under cointegration. These second-order effects are present if the regressor is not strictly exogenous, and if the regression error is serially correlated. It should be obvious, therefore, that the null distribution of the CUSUM test depends on the type of regression, i.e. stationary or cointegrated. The effects of a non-exogenous regressor are also different: first-order effects are present in the former case (the OLS estimator is inconsistent), but second-order ones in the latter (the OLS estimator is superconsistent, but its asymptotic distribution contains nuisance parameters). This means that, in the presence of a non-exogenous regressor, the residuals are not consistent estimates of the regression errors in the stationary case, but they are so in the cointegrating case. Consequently, it is worthwhile to examine any possible changes in the size of the CUSUM test as one moves from a stationary to a cointegrating environment.

This paper sheds further light on the size properties of the CUSUM and CUSUM-of-squares tests by conducting a number of Monte Carlo experiments. [2]

[2] The power properties of these tests have been investigated in a number of papers. For

In particular, we examine possible deviations of the empirical from the nominal size when:

1) The regressor in not exogenous, and exhibits various degrees of persistence, ranging from a white noise process to a random walk (in the latter case, the regression becomes a cointegrating one).

2) The regression error is serially correlated, both in a stationary and in a cointegrating environment. We isolate the effects of serial correlation by assuming that the regressor is exogenous.

3) The regressor is subject to structural change.

The layout of the paper is the following. Section 2 describes the Data Generation Process (DGP) considered in the analysis. Section 3 presents the Monte Carlo evidence. Section 4 summarises the main findings and briefly discusses their implications for the applied researcher.

2 The Model

Let $\mathbf{z}_t$ and $\mathbf{u}_t$ be two bivariate processes, with $\mathbf{z}_t = [y_t, x_t]^\top$ and $\mathbf{u}_t = [u_{1t}, u_{2t}]^\top$, and let the Data Generation Process (DGP) for y_t be the following:

$$y_t = \theta x_t + u_{1t} \tag{1}$$

$$x_t = \rho x_{t-1} + u_{2t} \tag{2}$$

Next, assume that $\mathbf{u}_t$ follows a stable VAR(1) process:

$$\begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} u_{1t-1} \\ u_{2t-1} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} \tag{3}$$

and

$$\begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} \tilde{}\, NIID \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix} \right] \tag{4}$$

Both eigenvalues of the matrix $A = [a_{ij}], i, j = 1, 2$ are assumed to be less than one in modulus. This means that $\mathbf{u}_t$ is a stationary process (I(0)). If $\rho < 1$, then (1) is a stationary regression, whereas if $\rho = 1$, (1) is a cointegrating regression.

instance, Kendall (1975) questioned their ability to distinguish betwen changes in regression coefficients and residual variance. Kramer et al.. (1988) showed that their local power depends on the angle between structural shift and mean regressor. Hansen (1991) presented evidence that neither test can detect changes in the slope coefficient in the case of a zero-mean regressor.

Therefore, the "cointegrability" of the system depends solely on the parameter ρ. The DGP (1) - (4) implies the following:

Stationary case: $\rho < 1$.

a) The condition for x_t to be predetermined in the context of (1) amounts to $a_{12} = \sigma_{12} = 0$. If either a_{12} or σ_{12} are different from zero, then application of OLS to (1) result in inconsistent estimates of θ. Concerning the degree of persistence of the regressor, as measured by ρ, Ploberger and Kramer (1992) have shown that the limit distributions of the CUSUM and CUSUM-of-squares test statistics are asymptotically invariant to the limit behaviour of the regressor.

b) The regression error u_{1t} admits an ARMA(2,1) representation. Specifically, after some tedious algebra, one can show that

$$u_{1t} - (a_{11} + a_{22})u_{1t-1} + (a_{11}a_{22} - a_{21}a_{12})u_{1t-2} = \epsilon_{1t} + \gamma_1\epsilon_{1t-1} \tag{5}$$

where ϵ_{1t} is i.i.d with

$$var(\epsilon_{1t}) = \frac{\sigma_{11}(1 + a_{22}^2) - 2\sigma_{12}a_{22}a_{12} + \sigma_{22}a_{12}^2}{1 + \gamma^2} \tag{6}$$

and γ solves

$$\sigma_{11}a_{22}\gamma^2 + \left(\sigma_{11}(1 + a_{22}^2) - 2\sigma_{12}a_{22}a_{12} + \sigma_{22}a_{12}^2\right)\gamma + a_{22}\sigma_{11} = 0 \tag{7}$$

This means that a sufficient condition for the regression error u_{1t} to be serially uncorrelated is $A = 0$. If $A \neq 0$, then, in general, the regression error u_{1t} will be serially correlated. The implications of serial correlation are different for the case $\rho < 1$ and $\rho = 1$ respectively (see below).

c) Let us turn now to the issue of structural invariance, which implies that the parameters of the conditional model remain unchanged even when those of the marginal model change. Assume that the regressor x_t in (1) is weakly exogenous for θ in the sense of Engle et al. (1983). It is easy to show that, with $\rho < 1$, the necessary and sufficient condition for weak exogeneity is given by $a_{12} = a_{21} = \sigma_{12} = 0$, i.e. weak exogeneity coincides with *strict exogeneity* for this particular model. Moreover, if the error is serially uncorrelated, i.e. $a_{11} = 0$, then $A = 0$, and the conditional expectation $E(y_t \mid x_t, F_{t-1})$ becomes equal to θx_t (where F_{t-1} is the $\sigma - field$ generated by the past values of y_t and x_t), namely (1) coincides with the conditional model. In this case the variance of the regressor error is equal to σ_{11}. This means that the parameters of the conditional model are θ and σ_{11}, whilst those of the marginal model are ρ and σ_{22}. Obviously, structural invariance holds for this parameter configuration, as the parameters of the conditional and of the marginal model are not related in any way. Consequently, x_t is superexogenous for θ in the context of (1). Under these circumstances, the CUSUM or CUSUM-of-squares tests should not be affected by structural breaks in the marginal model. If, in the presence of a structural shift in ρ, say, the percent rejections of the tests exceed the nominal size, then the tests reject the true null of parameter stability too frequently, thus being over-sized.

Let us now assume that structural invariance fails, with $A \neq 0$ and $\sigma_{12} \neq 0$. Then, the conditional model that corresponds to the DGP (1) - (4) is no longer (1) but rather the following dynamic regression:

$$y_t = \theta_1 x_t + c_1 y_{t-1} + c_2 x_{t-1} + c_3 x_{t-2} + \nu_t \tag{8}$$

where

$$\theta_1 = \theta + \frac{\sigma_{12}}{\sigma_{22}} \tag{9}$$

$$c_1 = a_{11} - a_{21}\frac{\sigma_{12}}{\sigma_{22}} \tag{10}$$

$$c_2 = a_{12} - \frac{\sigma_{12}}{\sigma_{22}}(a_{22} + \rho - a_{21}\theta) - a_{11}\theta \tag{11}$$

$$c_3 = (a_{22}\frac{\sigma_{12}}{\sigma_{22}} - a_{12})\rho \tag{12}$$

$$Var(\nu_t) \equiv \sigma_\nu^2 = \sigma_{11} - \frac{\sigma_{12}^2}{\sigma_{22}} \tag{13}$$

The ‘exogeneity status of x_t in (8) is different from that in (1). In the latter regression x_t is neither strictly nor weakly exogenous for θ. On the other hand, in the context of the regression model (8) x_t is predetermined, in the sense that $E(\nu_{t+i}x_t) = 0$, $\forall i \geq 0$, implying that the parameters of this model can be consistently estimated by OLS. It should be noted, however, that if the parameter of interest is still θ, then x_t is not weakly exogenous for θ even in the context of (8), since, as implied by equations (9)-(12), θ cannot be identified from the parameters of the conditional model alone.[3] Nevertheless, we focus on the conditional model (8) instead of the static model (1)[4] in order to isolate the effects

[3]The necessary condition for weak exogeneity is restored if either $\rho = 1$ or $a_{12} = \sigma_{12} = 0$.

[4]We could argue that the parameters of interest are now the parameters of the conditional model themselves.

of structural change in the parameters of the marginal model on the properties of the CUSUM test applied to the conditional model, when structural invariance fails. The reason is that the dynamic model, as opposed to the static (1), is not contaminated by other types of misspecification arising, say, from non-orthogonal regressors and serially correlated errors.

The marginal model for x_t can be written as:

$$x_t = \lambda_1 x_{t-1} + \lambda_2 x_{t-2} + \lambda_3 y_{t-1} + e_{2t} \tag{14}$$

where

$$\lambda_1 = \rho - a_{21}\theta + a_{22} \tag{15}$$

$$\lambda_2 = -a_{22}\rho \tag{16}$$

$$\lambda_3 = a_{21} \tag{17}$$

Assume now that the parameter ρ undergoes structural change as a result of a change in the persistence of the regressor. The parameters of the marginal model λ_1 and λ_2 will also change according to equations (9) and (10) respectively. Furthermore, the parameters of the conditional model c_2 and c_3 will be affected according to (11) and (12) respectively. In other words, the conditional model is not structurally invariant with respect to these specific changes in the parameters of the marginal model. Similarly, if the parameter of the marginal model $\lambda_3 = a_{21}$ changes (with all the other parameters in the DGP constant), this will induce a change in the parameters of the conditional model c_1 and c_2. Moreover, if the variance σ_{22} of the marginal model changes, then all the parameters of the conditional model, including the variance of the conditional model σ_ν^2, will also change according to (9)- (13). This means that, if a researcher applies the CUSUM or CUSUM-of-squares test to the correct model (8), then he should expect a rejection of the null hypothesis of parameter stability if one of the parameters of the marginal process has shifted. In such a case, if the percent rejections of the CUSUM or CUSUM-of-squares tests exceed their nominal sizes, this evidence should be interpreted not as an indication that the tests are over-sized, but rather in terms of their power, since the null hypothesis is obviously false.

Cointegration case: $\rho = 1$.

If $\rho = 1$ the DGP (1)-(4) becomes the triangular cointegration system analysed by Phillips and his co-authors in a series of papers (see, for example, Phillips

1988, Phillips and Hansen 1990). In this context, the effects of a non-exogenous regressor are different from those in the stationary case. Applying OLS to (1) yields superconsistent estimates of θ, regardless of the values of A and σ_{12}. This means that the OLS residuals $\widehat{u}_{1t}$ are consistent estimates of the regression error u_{1t}. Nevertheless, 'second-order' asymptotic bias effects are present in the asymptotic distribution of the OLS estimator. To be more precise, let us define the long-run covariance matrix Ω and the one-sided covariance matrix Δ given by

$$\Omega = (I - A)^{-1}\Sigma(I - A^{\top})^{-1} \tag{18}$$

$$\Delta = G(I - A^{\top})^{-1} \tag{19}$$

where Σ denotes the innovations covariance matrix of the VAR, and G is the unconditional covariance matrix of $\mathbf{u}_t$ given by

$$vecG = (I - A \otimes A)^{-1} vec\Sigma \tag{20}$$

One can now distinguish two nuisance parameters in the asymptotic distribution of the OLS estimator. The first is the parameter, ω_{12}/ω_{22}, that describes the "long-run correlation" effect, due to the non-diagonality of the long-run covariance matrix $\Omega = [\omega_{ij}]\,, i, j = 1, 2$. The second is the parameter $\delta_{21} = \sum_{k=0}^{\infty} E(u_{20}u_{1k})$, that corresponds to the "endogeneity" effect.[5] The presence of these second-order effects requires a modification of the standard CUSUM and CUSUM-of-squares tests in order to correct for endogeneity and serial correlation, along the lines suggested by Xiao and Phillips (2002). Nevertheless, it is easy to show that, if $a_{12} = a_{21} = \sigma_{12} = 0$, the asymptotic nuisance parameters are zero and standard OLS is optimal. In other words, the presence of a serially correlated error, that is $a_{11} \neq 0$, has no effects asymptotically on OLS, as shown by Kramer (1986) and Park and Phillips (1988). Unlike in the stationary case, under the assumption that the regressor is strictly exogenous, serial correlation has no asymptotic effects on the null distributions of the CUSUM and CUSUM-of-squares test statistics.

As for structural invariance, similar considerations to those for the stationary

[5]Using some tedious algebra, it can be shown that ω_{12}/ω_{22} and δ_{21} are related to the parameters of the VAR as follows:

$$\omega_{12} = k_1^2 \left\{\sigma_{12}[(1-a_{11})(1-a_{22}) + a_{12}a_{21}] + a_{12}\sigma_{22}(1-a_{11}) + a_{21}\sigma_{11}(1-a_{22})\right\} \tag{21}$$

$$\omega_{22} = k_1^2 \left\{a_{21}^2\sigma_{11} + 2a_{21}(1-a_{11})\sigma_{12} + (1-a_{11})^2\sigma_{22}\right\} \tag{22}$$

and

$$\delta_{21} = k_2[\sigma_{11}a_{21}\zeta_1 + \sigma_{12}\zeta_2 + \sigma_{22}a_{12}\zeta_3] \tag{23}$$

where

$$\begin{aligned}\zeta_1 \;=\;& -a_{11} - a_{12}a_{21}a_{22} + a_{11}a_{22}^2 + a_{11}a_{22} + a_{12}a_{21}a_{22}^2 - a_{11}a_{22}^3 - a_{12}a_{21} + \\ & + a_{12}^2a_{21}^2 - a_{11}a_{21}a_{22}a_{12}\end{aligned} \tag{24}$$

$$\begin{aligned}\zeta_2 \;=\;& a_{12}^2a_{21}^2 - 1 + a_{11}^2 + a_{22}^2 - a_{11}^2a_{22}^2 - a_{12}^2a_{21}^2a_{22} + a_{22} - a_{11}^2a_{22} - a_{22}^3 + \\ & a_{11}^2a_{22}^3 - 2a_{11}a_{21}^2a_{12}^2 - 2a_{21}a_{22}a_{12} + 2a_{11}^2a_{21}a_{22}a_{12}\end{aligned} \tag{25}$$

$$\begin{aligned}\zeta_3 \;=\;& -a_{11}a_{21}a_{12} - a_{22} + a_{11}^2a_{22} + a_{11}a_{21}a_{22}a_{12} + a_{22}^2 - a_{11}^2a_{22}^2 - 1 + a_{11}^2 + \\ & a_{21}a_{12} + a_{11}^2a_{21}a_{12} + a_{11}a_{22} - a_{11}^3a_{22}\end{aligned} \tag{26}$$

and

$$k_1 = [\det(I-A)]^{-1} \tag{27}$$

$$k_2 = [(-1+\det A)(\det(I-A))^2(1+trA+\det A)]^{-1} \tag{28}$$

case apply. The only difference is that structural change in the marginal process cannot arise from a change in ρ, since now this parameter is fixed to unity. Of course, one can consider a change in the parameter $\lambda_2 = -a_{22}$ affecting c_2 and c_3, or in $\lambda_3 = a_{21}$ affecting c_1 and c_2, or in σ_{22} affecting all the parameters of the conditional model.

3 Monte Carlo Results

This section reports on Monte Carlo simulations aimed at investigating the issues discussed above and their implications for the size properties of the CUSUM and CUSUM-of-squares tests. The only parameter kept fixed in all the experiments that follow is $\theta = 1$. We generate 2000 series of length 150, starting with $u_{10} = u_{20} = 0$, and then discard the initial 50 observations, thus obtaining a sample size of 100. We consider four different cases, and for each of them report the percent rejections produced by the CUSUM and CUSUM-of-squares tests for the hypothesis of parameter stability in the context of both the static (and often heavily misspecified in a number of ways) regression (1) and the dynamic regression (8).

Case I: Benchmark Case

Here we assume that $A = 0$, and $\sigma_{12} = 0$. We also set $\sigma_{11} = \sigma_{22} = 1$. This is a case where the conditional expectation is equal to θx_t. This means that the static regression (1) is the correct model, whereas the dynamic (8) is over-specified. No serial correlation or endogeneity effects are present. We set $\rho = 0.5$ (the stationary case) and $\rho = 1$ (the cointegration case). The results are reported in Table 1. It can be seen that the empirical sizes of both tests are close to their nominal 5% value in both the stationary and the cointegration case. This is hardly surprising, since, even with $\rho = 1$, the OLS estimator in the case of the correct model (1) is the optimal estimator, and the modifications in the spirit of Xiao and Phillips (2002) are redundant.

Case II: Non-exogenous Regressor

In this case, it is still assumed that $A = 0$, but σ_{12} now differs from zero. This means that, in the context of the static regression (1), the regressor x_t is not predetermined, even though the error u_{1t} is serially uncorrelated. On the other hand, it is predetermined (and weakly exogenous for the parameters of the conditional model, though not for θ) in the context of the ADL model (8), and the regression error ν_t is, of course, serially uncorrelated. Nevertheless the

ADL model (8) is still over-specified, since the correct conditional expectation is given by $y_t = \theta_1 x_t + c_2 x$, where $c_2 = \frac{\sigma_{12}}{\sigma_{22}} \rho$. In order to examine the effects of different "degrees" of endogeneity on the size properties of the tests under study, we consider the following values of $\sigma_{12} : \sigma_{12} = 0.3, 0.5, 0.7, 0.9$. The results are reported in Table 2, again for $\rho = 0.5$ and $\rho = 1$. They can be summarised as follows:

1) The CUSUM-of-squares test is robust to the presence of $\sigma_{12} \neq 0$ for both $\rho = 0.5$ (the stationary case) and $\rho = 1$ (the cointegration case), since, although applied to the misspecified equation (1), it does not exhibit size distortions. It appears, therefore, that this source of misspecification does not affect the test in either a stationary or a cointegrating environment.

2) A different picture emerges for the CUSUM test. Here there are important differences between the two cases of a stationary or a cointegrating *static* regression. In the stationary case the size of the test applied to (1) remains close to the nominal one for all values of σ_{12}. If, however, $\rho = 1$, then the test is over-sized, with the size distortion proportional to the endogeneity effect as measured by σ_{12}. For example, for $\sigma_{12} = 0.7$ the nominal size is 28.27 and 4.90 percent for $\rho = 1$ and $\rho = 0.5$ respectively.

3) Nevertheless, the CUSUM test performs satisfactorily, even if $\rho = 1$, when the test is applied to the dynamic model (8). The explanation is that this model, unlike the static model (1), deals with the second-order endogeneity effects arising from $\sigma_{12} \neq 0$ in a parametric way, and hence they are not present in the residuals. We also find that the non-parametric corrections for the CUSUM test, suggested by Xiao and Phillips (2002), reduce the size distortions to a satisfactory level (these results, not reported, are available on request).[6] The implication is that, when testing the stability of a cointegration model by means of the CUSUM test, it is preferable to use the dynamic model (8) instead of the static model (1), unless one is willing to undertake the additional computational burden entailed by the Xiao and Phillips (2002) procedure. This is because the OLS-based CUSUM test misinterprets the second-order effects as parameter instability, thus leading to erroneous rejections of the null.

4) Finally, rather surprisingly, there appears to be no evidence that the CUSUM-of-squares test is affected by second-order effects when applied to (1) for $\rho = 1$ - a finding for which there is no obvious explanation.

Case III: Serially Correlated Regression Error

In this case it is again assumed that x_t is predetermined in the context

[6] Note that these modifications work well even if the choice of the bandwidth parameter, needed to implement them, is not optimal.

of the static regression (1), which implies that $a_{12} = \sigma_{12} = 0$. We also set $a_{21} = 0$. For this parameter settings, the conditional expectation is equal to $\theta x_t + c_1 y_{t-1} + c_2 x_{t-1}$, where $c_1 = a_{11}$ and $c_2 = -a_{11}\theta$. Of course, in the context of the conditional model, x_t is weakly exogenous for θ, and the regression error ν_t is serially uncorrelated.[7] In order to examine the effects of different degrees of serial correlation on the size properties of the tests we consider the following values of a_{11} : $a_{11} = 0.3$, 0.5, 0.7, 0.9. The results are reported in Table 3, again for $\rho = 0.5$ and $\rho = 1$. The main findings are the following:

1) In the stationary case $\rho = 0.5$, the effects of serial correlation on the size (especially in the case of the CUSUM test) are significant. As the degree of serial correlation increases, so does the size in the context of the static regression (1), reaching the value of 86.77 and 68.37 percent for the CUSUM and CUSUM-of-squares tests respectively. The implication is that, when testing for parameter stability in the context of a regression model with a serially correlated error, the researcher will erroneously infer that the model parameters are unstable.

2) In the cointegration case $\rho = 1$, a similar picture emerges. Both tests are considerably over-sized, although slightly less so than in the stationary case.[8] This is a rather surprising result in view of the fact that the effects of a serially correlated error on the estimation of a cointegration parameter are asymptotically negligible if the regressor is strictly exogenous (as in the present case - see Kramer 1986, Park and Phillips 1988). It should be noted that the Xiao and Phillips (2002) modifications do not improve substantially the size behaviour of the tests (these results are not reported, for reasons of space, but are available on request). For example, for $a_{11} = 0.7$ the size of the OLS-based CUSUM test is 50.20 percent, whereas the size of the ADL-based is 44.67 percent. This is hardly surprising, since in this case $a_{12} = \sigma_{12} = a_{21} = 0$, and therefore both the nuisance parameters ω_{12}/ω_{22} and δ_{21} are already equal to zero, implying that there is no possible gain from using the modifications.

3) In the context of the dynamic model (8) both tests (particularly the CUSUM-of-squares) perform well in both environments, i.e. for $\rho = 0.5$ and $\rho = 1$. Again, if one utilises the correct regression model, their performance is satisfactory.

Case IV: (Lack of) Structural Invariance

The final set of experiments focuses on the role of structural invariance. One would expect that, if this property holds, i.e. if changes in the parameters of

[7]Note that since $a_{21} = 0$, x_t is also strongly exogenous for θ.

[8]In the interpretation of Xiao and Phillips (2002), they reject too frequently the null hypothesis of cointegration.

the marginal model do not cause corresponding changes in the parameters of the conditional model, the percent rejections of the null hypothesis of parameter constancy (of the conditional model) should not be higher than the nominal ones for either the CUSUM or the CUSUM-of-squares tests. We consider the following cases:

(a) *Structural Invariance*: $A = 0$, $\sigma_{12} = 0$. For this parameter setting the static model (1) is the conditional model, whose parameters are θ and σ_{11}. The marginal model is $x_t = \rho x_{t-1} + u_{2t}$, with parameters ρ and σ_{22}. In this case structural invariance holds. We introduce a shift in the parameter ρ of the marginal model, which is set equal to 0.3 in the first half of the sample, and to 0.7 in the second half. This shift in ρ does not affect the conditional parameters θ and σ_{11}, and hence should not affect the size properties of the tests under study. The results are reported in Table 4A. It can be seen that both tests, whether they are applied to the correct static (1) model or to the over-specified dynamic one (8), have empirical sizes close to the corresponding nominal ones, confirming our prior that the size properties should be robust to changes in the marginal parameters in the presence of structural invariance.

We also adopt the following experimental design: we keep ρ constant for the whole sample, and change the other parameter of the marginal model, i.e. σ_{22}. This allows us to examine the effects of a structural change in the marginal model in both a stationary and a cointegrating framework (unlike in earlier cases where ρ was allowed to change - then we could not analyse the latter case, since cointegration requires ρ to be equal to one in the whole sample). Specifically, we set $\sigma_{22} = 1$ in the first half of the sample, and $\sigma_{22} = 2$ in the second half. The results are reported in Table 4B. The empirical sizes are close to the nominal ones in both the stationary case $\rho = 0.5$ and the cointegration one $\rho = 1$, and for both the static model (1) and the over-specified dynamic one (8). Again, despite a change in the marginal process, structural invariance ensures that the tests retain good size properties. Note that in the cases under study x_t is superexogenous for θ.

We also examine the robustness of the CUSUM and CUSUM-of-squares tests to a very large shift in $\sigma_{22} = 1$, i.e. from 1 in the first half to 10 in the second half of the sample. The results, reported in Table 4C, show that even such a sizeable change in the parameters of the marginal model does not affect the properties of the tests in the presence of structural invariance.

Further, we analyse the general case of structural invariance with $A \neq 0$ and $\sigma_{12} \neq 0$, the parameter settings being such that the conditional model is not affected by the shift in the underlying DGP parameters, whilst the marginal model is. Specifically, consider the following parameter settings for the first half of the sample: $a_{11} = 0.4$, $a_{12} = 0.7$, $a_{22} = 0$, $a_{21} = 0.2$, $\sigma_{12} = 0.7$, $\sigma_{11} = \sigma_{22} = 1$, $\rho = 0.5$. Then the parameters of the marginal model will be equal to $\lambda_1 = 0.36$, $\lambda_2 = 0$, $\lambda_3 = 0.2$, $\sigma_{22} = 1$, whilst those of the conditional model will be equal to $\theta_1 = 1.4, c_1 = 0.26$, $c_2 = 0.168, c_3 = -0.35 = \sigma_\nu^2 = 0.51$. Assume that there is a shift in the middle of the sample in the following parameters: $a_{11} = 0.61$, $a_{12} = 0.98$, $a_{22} = 0.4$, $a_{21} = 0.5$, $\sigma_{12} = 1.4$, $\sigma_{11} = 1.49, \sigma_{22} = 2$, $\rho = 0.5$. This will result in $\lambda_1 = 0.55$, $\lambda_2 = -0.2$, $\lambda_3 = 0.5$, $\sigma_{22} = 2$. As one can see, the parameters of the marginal model have changed, whereas those of the conditional model have not - in other words, structural invariance holds.

We have also investigated the effects of the same parameter shifts when $\rho = 1$ instead of $\rho = 0.5$, the values taken in the first subsample by the parameters of the conditional model now being $\theta_1 = 1.4, c_1 = 0.26$, $c_2 = -0.182, c_3 = -0.70 = \sigma_\nu^2 = 0.51$, and by those of the marginal model $\lambda_1 = 0.86$, $\lambda_2 = 0.0$, $\lambda_3 = 0.2$, $\sigma_{22} = 1$. The shifts do not affect the conditional model parameters, but the marginal ones change to $\lambda_1 = 1.05$, $\lambda_2 = -0.4$, $\lambda_3 = 0.5$, $\sigma_{22} = 2$ in the second subsample. The results for both the stationary and the cointegration case are presented in Table 4D. It appears that structural invariance holds again, as the percent rejections for both tests in the context of the correctly specified dynamic model (8) are always close to the nominal ones. Let us analyse the consequences of using the misspecified static model (1). It is apparent that its parameters, σ_u^2 and θ, change as a result of the shift. This is clearly the case for the former, as suggested by equations (5)-(7). As for the latter, although the shift does not affect it directly, it does affect the limiting parameter to which the OLS estimator applied to (1) converges. This point can be illustrated by noting that, if one applies OLS to (1), then this estimator, say $\widehat{\theta}_{LS}$, will converge in probability to $\theta + \widetilde{\theta}$, where $\widetilde{\theta}$ denotes the asymptotic bias due to the presence of a non-predetermined regressor. It is easy to show that $\widetilde{\theta}$ is a function of the parameters of the DGP changing between regimes. Therefore, although θ itself does not change, $\widehat{\theta}_{LS}$ does as a result of the shift in the DGP parameters, and so do the residuals on which the tests are based. The static regression (1) is also misspecified in other ways. First, the regression error u_{1t} is serially correlated in both subsamples, with different degrees of autocorrelation. For instance, the first and second autoregressive coefficients, $(a_{11} + a_{22})$ and $(a_{11}a_{22} - a_{21}a_{12})$ respectively, are equal to 0.4 and -0.14 in the first half of the sample, and to 1.01 and -0.146 in the second. Second, the regressor x_t is not predetermined in either case (a_{12}, $\sigma_{12} \neq 0$ in both subsamples). Therefore, one would expect both the CUSUM and CUSUM-of-squares tests to reject the null when carried out in the context of the static regression (1). However, it cannot be established whether such rejections are due to parameter instability (the correct reason for rejecting), or to other types of misspecification (the wrong reason). Indeed, the evidence presented in Table 4D confirms our conjecture that the tests will reject the null hypothesis quite often, especially if $\rho = 1$. This is in sharp contrast to their behaviour when applied to the correctly specified and structurally invariant ADL model (8).

To summarise, these tests appear to reject the null when it is false (in the case of the static model), and not to reject it when it is true (in the case of the dynamic model). As already mentioned, the rejections in the former case might reflect several types of misspecification, such as serial correlation (primarily) or endogeneity, as well as parameter instability. We have reported earlier that autocorrelated errors mainly affect the empirical size of both tests, whereas the strongest effects of endogeneity occur in the case of the CUSUM test when cointegration holds. These findings might be interpreted as an indication that the tests under examination tend not to reject the null of parameter stability unless some other form of misspecification is also present. Consequently, although it is a desirable property that they should reject in the context of the static model (which is indeed characterised by parameter instability), they are not informative about the specific reason behind such rejections, which could be one of many forms of misspecification. Similarly, it is desirable that they should not reject when applied to a correctly specified structurally invariant model (i.e. the ADL), since the null in this case is true. But the question remains whether such behaviour is a consequence of a general tendency not to reject in the presence of any type of misspecification (which might not be linked at all to parameter variation). To address these issues, we investigate next the performance of these tests when the dynamic ADL model is not structurally invariant and the only source of misspecification is instability in the parameters of the conditional model.

b) *No Structural Invariance.* Consider the following parameter settings for the first half of the sample: $a_{11} = 0.7$, $a_{12} = 0.7$, $a_{22} = 0$, $a_{21} = 0.3$, $\sigma_{12} = 0.7$, $\sigma_{11} = \sigma_{22} = 1$, $\rho = 0.5$. The corresponding conditional model parameters are equal to $\theta_1 = 1.4, c_1 = 0.49$, $c_2 = 0.007, c_3 = -0.35, \sigma_\nu^2 = 0.51$. Assume a permanent shift in the middle of the sample in the following DGP parameters: $a_{11} = 0.9$, $a_{12} = 0.0$, $a_{22} = 0.0$, $a_{21} = 0.7$, $\sigma_{12} = 0.0$, $\sigma_{11} = 4, \sigma_{22} = 2$, $\rho = 0.5$. As a result, the conditional model parameters will become equal to: $\theta_1 = 0.7, c_1 = 0.90$, $c_2 =$ -0.63, $c_3 = 0.00$, $\sigma_\nu^2 = 4$, i.e. structural invariance does not hold. Consequently, the CUSUM and CUSUM-of-squares tests should reject the null hypothesis of parameter stability if applied to the dynamic regression (8). To put it differently, the desirable behaviour of these tests in this particular case is that they should "reject" as often as possible - and the percent rejections of the null should be interpreted in terms of "power" instead of size". Note, once again, that here the only form of misspecification is parameter instability: the ADL model does not have either a serially correlated error or a non-predetermined regressor.

Similar considerations apply when the tests are applied to the static model (1). This model is misspecified in several ways other than parameter instability: serial correlation of the error u_t (in both subsamples) and endogeneity of the regressor (in the first subsample) are both present. Therefore, rejections of the null do not lend themselves to a unique interpretation. The results for both the stationary and the cointegration case are shown in Table 4E, and can be summarised as follows:

(i) The CUSUM test lacks any power to detect the break that has occured in the parameters of the correct (ADL) conditional model, this being true in both the stationary and the cointegration case. In fact, its "power" appears to be more or less equal to its nominal size. On the contrary, the CUSUM-of-squares test is extremely powerful and rejects the false null in 100% of the cases, whether a stationary or a cointegration environment is considered. This is not a surprising finding, given the very large step change in the variance of the regression error σ_v^2, from 0.5 in the first to 4 in the second subsample. These results are in agreement with those of Ploberger and Kramer (1990) and Hansen (1991), who argue that neither test has asymptotic power to detect shifts in the slope parameters. Specifically, the CUSUM test has local asymptotic power to detect shifts only in the intercept, and the CUSUM-of-squares test only in the variance of the regression error.

(ii) When the tests are applied to the misspecified static model (1), the CUSUM test rejects more frequently (especially in the case of cointegration) than it did in the context of the correctly specified ADL model. This can be seen as further evidence that forms of misspecification other than parameter instability cause the test to reject: here the null of parameter instability is correctly rejected, but for the wrong reasons (serial correlation, endogeneity, etc.) The rejection frequencies for the CUSUM-of-squares test are also high, but can more plausibly be attributed to the correct reasons, as (a) it is the variance of u_{1t} which has changed, and therefore the test has a natural advantage over its CUSUM version to reject for the right reasons, and (b) according to the results presented in Tables 2 and 3, it is less affected than the CUSUM test by misspecification in the form of serial correlation or endogeneity.

Finally, we assume that the parameters of the DGP change in such a way as to affect the slope parameters of the conditional model, but at the same time leave the variance of the error σ_ν^2 unaffected. The motivation is the following. In the previous experiment, allowing σ_ν^2 to change whilst keeping the intercept constant meant that the CUSUM-of-squares test had an obvious advantage, as it is well known that this test has more power to detect shifts in the former relative to the CUSUM test, which performs better in detecting changes in the latter (see Hansen, 1991). Therefore, a fairer comparison between the performance of the two tests in the presence of a shift in the slope parameters of the conditional model can be made by keeping the variance of the error constant. Specifically, we assume that the parameters of the DGP in the first subsample are the same as in the previous case, whilst in the second one they are set equal to: $a_{11} = 0.9$, $a_{12} = 0$, $a_{22} = 0$, $a_{21} = 0.7$, $\sigma_{12} = 1.732$, $\sigma_{11} = \sigma_{22} = 2$. For the stationary case $\rho = 0.5$. The parameters of the conditional model are equal to $\theta_1 = 1.4, c_1 = 0.49$, $c_2 = 0.007, c_3 = -0.35$, $\sigma_\nu^2 = 0.51$ in the first subsample, and change to $\theta_1 = 1.566, c_1 = 0.294$, $c_2 = -0.638, c_3 = 0$, $\sigma_\nu^2 = 0.51$ in the second one. Similar parameter shifts are introduced for the cointegration case $\rho = 1$. The results are reported in Table 4F. It is apparent that the CUSUM-of-squares test has a superior performance: it has satisfactory power to detect parameter instability even when the conditional variance is kept constant. For instance, for $\rho = 0.5$, and in the context of the correctly specified ADL model, the percent rejections of the CUSUM and CUSUM-of-squares tests are 5.70 and 65.44 respectively. In the cointegration case, with $\rho = 1$, the corresponding figures are 6.30 and 68.17.

Given the other types of robustness of the CUSUM-of-squares test already documented, one can conclude that this test is more reliable to detect parameter instability. It is important to note that its power to detect shifts in the slope parameters of the ADL model (even when the conditional variance is constant) increases further if the breaks occur later in the sample. For instance, if the shift takes place after 75 (rather than 50, as previously) observations, its power is 94.56 and 94.98 percent in the stationary and cointegration case respectively. This is in constrast to the findings of Kramer et al. (1988), who argued that "*power decreases as the shift moves toward the end of the sample*". Instead, we find a significantly lower power when the shift occurs after 25 observations (34.56 and 35.89 percent only in the two cases), i.e. as one moves towards the beginning of the sample (the full set of results is not reported for reasons of space). Also note that the power of this test is approximately equal to its nominal size regardless of the point where the shift occurs.

4 Conclusions

This paper has investigated by means of Monte Carlo techniques the size properties of the CUSUM and CUSUM-of-squares tests (see Brown et al., 1975), which are widely used in empirical applications because they are appropriate to test parameter instability against a variety of alternatives, but have been subjected to various criticisms concerning their performance both in terms of size and power (see, e.g., Kendall, 1975, Priestley, 1975, Kramer et al, 1988, Hansen, 1991). Our findings, which have important implications for the applied researcher, suggest the following.

The CUSUM-of-squares test is very robust to the presence of non-predetermined (endogenous) regressors in both a stationary and a cointegration environment. Instead, the CUSUM test is robust only in the former case, whilst it is characterised by large size distortions in the latter. These are proportional to the degree of correlation between the regression error and the regressor. Further, serial correlation has serious consequences in all cases, the CUSUM-of-squares test also being more robust to the presence of weakly serially correlated errors.

The implication is that it is preferable to use these tests in the context of a dynamic model of the ADL type, which is not affected by serial correlation or non-predetermined regressors even if over-specified. In this case, the empirical sizes of both tests are close to the nominal ones, whether a stationary or a cointegration environment is considered. This means that another advantage of performing cointegration analysis using the ADL framework is the availability of robust stability tests not requiring the modifications suggested by Xiao and Phillips (2002). Although such tests are not directly interpretable as tests for the null of cointegration, stability of the ADL parameter is a pre-requisite for cointegration.

The two tests being considered are also robust to changes in the parameters of the marginal process provided that structural invariance holds. In other words, they do not misinterpret shifts in the marginal process as changes in the parameters of the conditional model, i.e. the frequency of a type-I error is not higher than that implied by the nominal size. Structurally invariant parameters will be obtained by employing the appropriate conditional model, i.e.the dynamic ADL model, but not using a static model. For this reason, and also because such a model is misspecified in ways that favour rejections, the percent rejections will be higher.

When structural invariance fails, the CUSUM test is unable to detect parameter instability even if this is substantial and present in both the slope parameters and the variance of the regressor error. It tends to reject the null only if other forms of misspecification are also present, and hence is more useful as a test for serial correlation, say, than for parameter instability. Therefore, a rejection can only be seen as a general indication of misspecification rather than a specific sign of parameter variation. However, as is well known, this test does have power to detect intercept shifts (see Hansen, 1991). By constrast, the CUSUM-of-squares test is very powerful to detect changes in the conditional model parameters if the variance of the regression error is included in the set of shifting parameters. Its power is considerable even if this parameter is constant, especially towards the end of the sample.

In brief, our findings imply that the CUSUM-of-squares test should always be used in the context of a generalised dynamic ADL model, and never applied to a static model. Its performance is satisfactory in both stationary and cointegration environments. Importantly, it does not misinterpret shifts in the marginal process: if it is carried out within a dynamic ADL model, rejections of the null are highly likely to reflect actual parameter instability.

References

[1] Brown, R.L., Durbin, J. and J.M. Evans (1975), "Techniques for testing the constancy of regression relationships over time," *Journal of the Royal Statistical Society*, Series B, 149-162.

[2] Engle, R.F., Hendry, D.F. and J.-F. Richard (1983), "Exogeneity," *Econometrica*, 51: 2, 277-304.

[3] Hansen , B.E. (1991) A Comparison of tests for parameter instability: An examination of asymptotic local power, University of Rochester, working paper.

[4] Hansen, B.E. (1992), "Tests for parameter instability in regressions with I(1) processes," *Journal of Business & Economic Statistics*, 10: 3, 321-335.

[5] Hansen, B. E. (2000), "Testing for structural change in conditional models," *Journal of Econometrics*, 97, 93-115.

[6] King, M.L. (1980), "Robust tests for spherical symmetry and their application to least squares regression," *Annals of Statistics*, 8, 1265-1271.

[7] King, M.L. (1985), "A point optimal test for autoregressive disturbances," *Journal of Econometrics*, 27, 21-37.

[8] King, M.L. (1988), “Towards a theory of point optimal testing,” *Econometric Reviews*, 6, 169-218.

[9] King, M.L. and G.H. Hillier (1985), “Locally best invariant tests of the error covariance matrix of the linear regression model,” *Journal of the Royal Statistical Society*, B 47, 98-102.

[10] Kramer, W. (1986), “Least-squares regression when the independent variable follows an ARIMA process,” *Journal of the American Statistical Association*, 81, 150-154.

[11] Kramer, W., Ploberger, W., and R. Alt (1988), “Testing for structural change in dynamic models,” *Econometrica*, 56, 1355-1369.

[12] LaMotte, L.R. and A. McWhorter (1978), “An exact test for the presence of random walk coefficients ina linear regression model,” *Journal of the American Statistical Association*, 73, 816-820.

[13] Leybourne, S.J. and B.P.M. McCabe (1989), “On the distribution of some test statistics for coefficient constancy,” *Biometrika*, 76, 169-177.

[14] Moryson, M. (1998), Testing for Random Walk Coefficients in Regression and State Space Models, Physica-Verlag, Berlin.

[15] Nyblom, J. (1989), “Testing the constancy of parameters over time,” *Journal of the American Statistical Association*, 84, 223-230.

[16] Nyblom, J. and T. Makelainen (1983), “Comparison of tests for the presence of random walk coefficients in a simple linear model,” *Journal of the American Statistical Association*, 78, 856-864.

[17] Park, J.Y. and P.C.B. Phillips (1988), “Statistical inference in regressions with integrated processes,” *Econometric Theory*, 4, 468-497.

[18] Phillips, P.C.B. (1988), “Reflections on Econometric Methodology,” *Economic Record*, 64, 334-359.

[19] Phillips, P.C.B. and B.E. Hansen (1990), ”Statistical inference in instrumental regressions with I(1) processes,” *Review of Economic Studies*, 57, 99-125.

[20] Ploberger, W. and W. Kramer (1990), “The local power of the CUSUM and CUSUM of squares tests,” *Econometric Theory*, 6, 335-347.

[21] Ploberger, W. and W. Kramer (1992), “The CUSUM test with OLS residuals,” *Econometrica*, 60, 271-285.

[22] Priestley, M.B. (1975), discussion of Brown, R.L., Durbin, J. and J.M. Evans (1975), "Techniques for testing the constancy of regression relationships over time," *Journal of the Royal Statistical Society*, Series B, 166-168.

[23] Quandt, R.E. (1975), discussion of Brown, R.L., Durbin, J. and J.M. Evans (1975), "Techniques for testing the constancy of regression relationships over time," *Journal of the Royal Statistical Society*, Series B, 183-184.

[24] Smith, A.F.M. (1975), discussion of Brown, R.L., Durbin, J. and J.M. Evans (1975), "Techniques for testing the constancy of regression relationships over time," *Journal of the Royal Statistical Society*, Series B, 175-177.

[25] Xiao, Z. and P.C.B. Phillips (2002), "A CUSUM Test for Cointegration Using Regression Residuals," No 1329, Cowles Foundation Discussion Papers, Cowles Foundation, Yale University.

TABLE 1
Benchmark Case, A=0, Σ=I.
Percent Rejections of the Null Hypothesis of Parameter Stability

	CUSUM	CUSUM-SQ
	ρ=0.5	
OLS	4.20	5.43
ADL	4.37	5.83
	ρ=1	
OLS	3.77	5.43
ADL	4.50	5.83

TABLE 2
Endogeneity: A=0, $\sigma_{12}\neq 0$
Percent Rejections of the Null Hypothesis of Parameter Stability

	CUSUM	CUSUM-SQ
	ρ=0.5, σ_{12}=0.3	
OLS	4.03	5.43
ADL	4.07	5.77
	ρ=1, σ_{12}=0.3	
OLS	8.17	4.77
ADL	5.63	5.87
	ρ=0.5, σ_{12}=0.5	
OLS	4.63	5.67
ADL	4.20	5.90
	ρ=1, σ_{12}=0.5	
OLS	15.00	5.63
ADL	4.97	5.50
	ρ=0.5, σ_{12}=0.7	
OLS	4.90	5.87
ADL	4.73	6.17
	ρ=1, σ_{12}=0.7	
OLS	28.27	5.13
ADL	4.70	5.50
	ρ=0.5, σ_{12}=0.9	
OLS	7.40	5.70
ADL	4.20	5.50
	ρ=1, σ_{12}=0.9	
OLS	46.53	5.50
ADL	4.57	5.27

TABLE 3
Serial Correlation: $a_{11} \neq 0$
Percent Rejections of the Null Hypothesis of Parameter Stability

	CUSUM	CUSUM-SQ
	ρ=0.5, a_{11}=0.3	
OLS	16.40	8.67
ADL	5.07	5.90
	ρ=1, a_{11}=0.3	
OLS	14.07	9.03
ADL	5.07	6.33
	ρ=0.5, a_{11}=0.5	
OLS	31.73	15.80
ADL	5.53	6.17
	ρ=1, a_{11}=0.5	
OLS	27.97	16.50
ADL	6.13	6.37
	ρ=0.5, a_{11}=0.7	
OLS	56.11	33.83
ADL	6.80	6.30
	ρ=1, a_{11}=0.7	
OLS	50.20	34.20
ADL	8.00	6.37
	ρ=0.5, a_{11}=0.9	
OLS	86.77	68.37
ADL	10.10	6.40
	ρ=1, a_{11}=0.9	
OLS	79.43	68.93
ADL	11.73	6.57

TABLE 4
Structural Breaks
Percent Rejections of the Null Hypothesis of Parameter Stability

A. Structural Invariance Holds, A=0 σ_{12}=0
ρ: 0.3 $\rightarrow$0.7 at 50th observation

	CUSUM	CUSUM-SQ
OLS	3.80	5.37
ADL	4.93	6.00

B. Structural Invariance Holds, A=0 σ_{12}=0
σ_{22}: 1 $\rightarrow$2 at 50th observation

	CUSUM	CUSUM-SQ
	ρ=0.5	
OLS	4.20	5.43
ADL	4.37	5.83
	ρ=1	
OLS	3.77	5.40
ADL	4.50	5.86

C. Structural Invariance Holds, A=0 σ_{12}=0
σ_{22}: 1 $\rightarrow$10 at 50th observation

	Cusum	Cusum-SQ
	ρ=0.5	
OLS	5.10	5.99
ADL	5.65	6.24
	ρ=1	
OLS	4.87	5.87
ADL	5.76	6.87

D. Structural Invariance Holds, A≠0, σ_{12}≠0

a_{11}: 0.4 →0.61	**σ_{11}: 1 .0→1.49**
a_{12}: 0.7 →0.98	**σ_{12}: 0.7 →1.40**
a_{21}: 0.2 →0.50	**σ_{22}: 1.0 →2.00**
a_{22}: 0.0 →0.40	

ALL at 50th observation

	CUSUM	CUSUM-SQ
	ρ=0.5	
OLS	55.41	41.34
ADL	5.80	5.86
	ρ=1	
OLS	81.20	94.39
ADL	5.15	5.02

E. Structural Invariance Fails

a_{11}: 0.7 →0.90	**σ_{11}: 1.0 →4.00**
a_{12}: 0.7 →0.00	**σ_{12}: 0.7 →0.00**
a_{21}: 0.3 →0.70	**σ_{22}: 1.0 →2.00**
a_{22}: 0.0 →0.00	

ALL at 50th observation

	Cusum	Cusum-SQ
	ρ=0.5	
OLS	59.86	100
ADL	5.80	100
	ρ=1	
OLS	90.11	95.30
ADL	7.03	100

F. Structural Invariance Fails
(Conditional variance remains constant)

a_{11}: 0.7 →0.90 **σ_{11}: 1.0 →2.00**
a_{12}: 0.7 →0.00 **σ_{12}: 0.7 →1.732**
a_{21}: 0.3 →0.70 **σ_{22}: 1.0 →2.00**
a_{22}: 0.0 →0.00
ALL at 50th observation

	CUSUM	CUSUM-SQ
	ρ=0.5	
OLS	79.66	69.39
ADL	5.70	65.44
	ρ=1	
OLS	93.33	78.55
ADL	6.30	68.17

G. Structural Invariance Fails
(Conditional variance remains constant)

a_{11}: 0.7 →0.90 **σ_{11}: 1.0 →2.00**
a_{12}: 0.7 →0.00 **σ_{12}: 0.7 →1.732**
a_{21}: 0.3 →0.70 **σ_{22}: 1.0 →2.00**
a_{22}: 0.0 →0.00
ALL at 75th observation

	CUSUM	CUSUM-SQ
	ρ=0.5	
OLS	70.19	81.89
ADL	5.40	94.56
	ρ=1	
OLS	93.82	78.95
ADL	6.50	94.98

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