

ENVIRONMENTAL POLICY AND THE EFFECT
OF POLLUTION ON PRODUCTION

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CONTENTS

page

1. Introduction	1
2. Structure of the Model	3
3. Policy Responses to Changes in the Pollution Effect on Production	9
4. Policy Responses to Production-Capacity Changes and Preference Changes	16
FOOTNOTES	29
REFERENCES	31

Summary

This paper examines how the effect of pollution on production processes is relevant to the formulation of dynamic environmental policy. In the context of a macroeconomic model which includes consumption, production, pollution, and pollution-treatment activities, the reactions of optimal environmental targets to changes in the pollution effect on production are analyzed. Furthermore, it is shown how the pollution effect on production conditions the optimal environmental target responses to changes in social preferences and various technological improvements.

Zusammenfassung

Dieser Artikel untersucht wie die Formulierung optimaler dynamischer Umweltpolitik von dem Umweltverschmutzungseffekt auf die Produktion abhängt. An Hand eines makroökonomischen Modells das Konsum, Produktion, Umweltverschmutzung und Umweltreinigung beinhaltet, werden die optimalen Reaktionen umweltpolitischer Zielsetzungen auf Veränderungen des Umweltverschmutzungseffekts analysiert. Ausserdem wird untersucht wie der Umweltverschmutzungseffekt die optimalen Reaktionen umweltpolitischer Zielsetzungen auf technologischen Fortschritt und auf Veränderungen der wirtschaftspolitischen Präferenzen beeinflusst.

1. INTRODUCTION

Thus far, the economic literature on the optimal control to pollution has not dealt with the effect of pollution on production processes. The recent work in this field (e.g. D'Arge and Kogiku [2], Forster [3], Keeler, Spence, and Zeckhauser [4], Plourde [5], and Smith [6]) examines the optimal level of pollution, under the assumption that production and consumption are sources of pollution and that consumers are affected by it. In this paper we explore the environmental policy implications of assuming that producers are affected by pollution as well.

There are numerous classical examples of how pollution interferes with production processes. Water pollution (e.g. high phosphorous levels) makes the production of beer more difficult than it otherwise would be. Laundries suffer through air pollution. The cost of operating nuclear power plants depends on the degree to which water (used in condensers) is polluted. The provision of various outdoor recreational services is hampered by land, water, and air pollution. More generally, increased pollution may lower labor productivity in a wide variety of operations, on account of health and nuisance effects.

We show how the effect of pollution on production is relevant to the formulation of environmental policy in the context of a simple macroeconomic model. Consider an economy which uses its scarce resources to provide a consumption good and a pollution-treatment service. A pollutant flow is generated by the production, consumption, and treatment activities. This

flow may be cleansed by the treatment service or by nature. Producers are adversely affected by the pollutant stock. In particular, a rise in this stock implies that a given amount of labor can provide fewer consumption goods. In effect, the causality relation between production and pollution runs both ways: the act of production gives rise to a pollutant flow, which contributes to the pollutant stock; the latter, in turn, affects the capacity to produce.

In this context, we consider setting a socially optimal environmental target for the pollutant flow. The greater (smaller) the proportion of scarce resources used by the production sector, the smaller (greater) the proportion left over for the pollution-treatment sector, and the greater (smaller) the pollutant flow. Thus, the environmental target controls the allocation of resources between the production and pollution-treatment sectors. Since there is a one-to-one relation between the level of the environmental target and the level of the pollution-treatment service, we will use the latter as a proxy for the former. The socially optimal environmental target is set with reference to a social welfare function. Social welfare is assumed to be directly related to the consumption flow and inversely related to the pollutant stock. Since the current pollutant flow affects social welfare at present and in the future, the socially optimal environmental target must be dynamic (i.e. the levels of the target at different points in time are interrelated).

We examine how the environmental target reacts to technological improvements which alter the effect of pollution on production. We consider two such improvements: a rise

in the "marginal product of pollution" (i.e. the partial derivative of product supply with respect to the pollutant stock) and a rise in the "average product of pollution" (i.e. the ratio of product supply to the pollutant stock). It will be shown that although the short-run responses of the optimal environmental targets to these changes are qualitatively similar, the dynamic time profiles of these targets are quite different. A fall in the marginal product of pollution calls for a change in the direction of target movement through time, whereas a fall in the average product of pollution requires no such reversal.

Moreover, we evaluate the significance of the pollution effect on production with regard to the way in which the environmental target should respond to other exogenous changes. These changes constitute changes in social preferences and technological improvements (other than those described above). The optimal environmental target responds differently to these changes in our model than it would respond in a model which does not take the effect of pollution on production into account. Thus, even if the pollution effect on production remains invariant through time, our analysis suggests that this effect conditions the way in which environmental policy should respond to other economic phenomena.

2. STRUCTURE OF THE MODEL

We assume that labor (L) is the only factor of production. It is used to provide a consumption good (Q) and a pollution-treatment service (T). The supply of labor is exogenously given at L . L_Q is employed in the production of Q ; L_T is employed in the treatment sector.

The production and consumption of Q and the provision of T all generate a pollutant (or "residual") flow, $\dot{R} = dR/dt$ (where R is the pollutant stock). The pollutant stock adversely affects the production of Q, but does not influence the supply of T. The production functions of the two sectors may be expressed as follows:

$$(1) \quad Q = f(L_Q, R), \quad f_L > 0, f_{LL} < 0, f_R < 0, f_{RR} < 0, f_{LR} < 0;$$

$$(2) \quad T = g(L_T), \quad g_L > 0, g_{LL} < 0.$$

From these production functions, a production possibility frontier relating the supplies of Q and T may be derived:

$$(3) \quad Q = F(T, R), \quad F_T < 0, F_{TT} < 0, F_R < 0, F_{RR} < 0, F_{TR} > 0.^1$$

The pollutant flows generated by the production, consumption, and treatment activities may be described through the following pollution technologies:

$$(4) \quad \dot{R}_Q = h_1(Q), \quad h_1' > 0, h_1'' > 0;$$

$$(5) \quad \dot{R}_T = h_2(T), \quad h_2' > 0, h_2'' > 0;$$

$$(6) \quad \dot{R}_C = h_3(C), \quad h_3' > 0, h_3'' > 0;$$

where C is the consumption flow and \dot{R}_Q , \dot{R}_T , and \dot{R}_C are the pollutant flows from Q, T, and C, respectively.

Pollution may be cleansed (i.e. transformed into socially harmless residuals) through the anthropogenic treatment activity and nature. Nature's cleansing depends on the pollutant stock:

$$(7) \quad T_N = \delta(R), \delta' > 0, \delta'' < 0.$$

Both T and T_N are measured in terms of the amount of residual cleansed.
The net pollutant flow is

$$(8) \quad \dot{R} = \dot{R}_Q + \dot{R}_T + \dot{R}_C - T - T_N.$$

Given equations (3) - (6) and given that $Q = C$, equation (8) may be rewritten as follows:

$$(9) \quad \dot{R} = h_1(F(T,R)) + h_2(T) + h_3(F(T,R)) - T - \delta(R) = k(T,R).$$

This emission function has the following properties:

$$(9a) \quad k_T = F_T \cdot (h_1' + h_3') - (1 - h_2') < 0;$$

$$(9b) \quad k_{TT} = F_T \cdot (h_1'' + h_3'') + F_{TT} \cdot (h_1' + h_3') + h_2''.$$

With regard to equation (9b), we assume that the third right-hand term (a rising marginal pollutant flow from T with respect to T) outweighs the first two right-hand terms (a falling marginal pollutant flow from Q with respect to T , due to a rising marginal pollutant flow with respect to Q , and a diminishing marginal rate of transformation from Q into T). Thus, $k_{TT} > 0$.

$$(9c) \quad k_R = F_R \cdot (h_1' + h_3') - \delta' < 0;$$

$$(9d) \quad k_{RR} = F_{RR} \cdot (h_1' + h_3') + (F_R)^2 \cdot (h_1'' + h_3'') - \delta''.$$

For equation (9d), we assume that the first right-hand term is outweighed by the last two right-hand terms; thus $k_{RR} > 0$.

$$(9e) \quad k_{TR} = F_{TR} \cdot (h_1' + h_3') > 0.$$

At any point in time, social welfare may be defined by the following utility function:

$$(10) \quad U = U(Q,R), \quad U_Q > 0, \quad U_{QQ} < 0, \quad U_R < 0, \quad U_{RR} < 0, \quad U_{RQ} = 0.$$

Given that the policy maker's time horizon extends from the present (time $t = 0$) into the infinite future, the optimal anthropogenic pollution-treatment target may be found by solving the following dynamic optimization problem:

$$(11) \quad \text{Maximize} \quad W = \int_0^{\infty} e^{-rt} \cdot U(F(T,R), R) dt$$

$$\text{subject to} \quad \dot{R} = k(T,R),$$

where r is the rate of time discount. T is the control variable and R is the state variable. The optimal time path of the environmental target (i.e. the target for the pollutant flow) is implied by the optimal trajectory of T .

The current-value Hamiltonian for problem (11) is

$$H = U(F(R,T), R) + \mu \cdot k(T,R)$$

where μ is the current-value shadow price of the pollutant flow. The necessary conditions for an interior optimal solution are

$$(12) \quad \frac{\partial H}{\partial T} = 0 \quad \rightarrow \quad \mu = - \frac{U_Q \cdot F_T}{k_T}$$

$$(13) \quad - \frac{\partial H}{\partial R} = \dot{\mu} - r \cdot \mu \quad \rightarrow \quad \dot{\mu} = \mu \cdot (r - k_R) - U_Q \cdot F_R - U_R$$

$$(9) \quad \dot{R} = k(T,R).$$

Substituting equation (12) into equation (13), we find

$$(14) \quad \dot{T} = - \frac{T}{\phi_1} \cdot \left[(r - k_R) + \frac{F_R}{F_T} \cdot k_T + \frac{U_R}{U_Q} \cdot \frac{k_T}{F_T} + \phi_2 \cdot \frac{\dot{R}}{R} \right] = j(T,R),$$

where $\phi_1 = -\sigma_{TT}^F + \sigma_{TT}^k - \sigma_{QQ}^U \cdot \sigma_T^F,$

$\phi_2 = -\sigma_{TR}^F + \sigma_{TR}^k - \sigma_{QQ}^U \cdot \sigma_R^F,$

and the following elasticities are assumed to be constants:

$\sigma_{TT}^F = \frac{F_{TT}}{F_T} \cdot T > 0$ (the elasticity of the marginal rate of transformation of T into Q with respect to T);

$\sigma_{TT}^k = \frac{k_{TT}}{k_T} \cdot T < 0$ (the elasticity of the marginal pollutant flow from T with respect to T);

$\sigma_{TR}^F = \frac{F_{TR}}{F_T} \cdot R < 0$ (the elasticity of the marginal rate of transformation of T into Q with respect to R);

$\sigma_{TR}^k = \frac{k_{TR}}{k_T} \cdot R < 0$ (the elasticity of the marginal pollutant flow from T with respect to R);

$\sigma_{QQ}^U = \frac{U_{QQ}}{U_Q} \cdot Q < 0$ (the elasticity of the marginal utility from Q with respect to Q);

$\sigma_T^F = \frac{F_T}{F} \cdot T < 0$ (the elasticity of Q with respect to T);

$\sigma_R^F = \frac{F_R}{F} \cdot R < 0$ (the elasticity of Q with respect to R).

Thus, $\phi_1 < 0$.

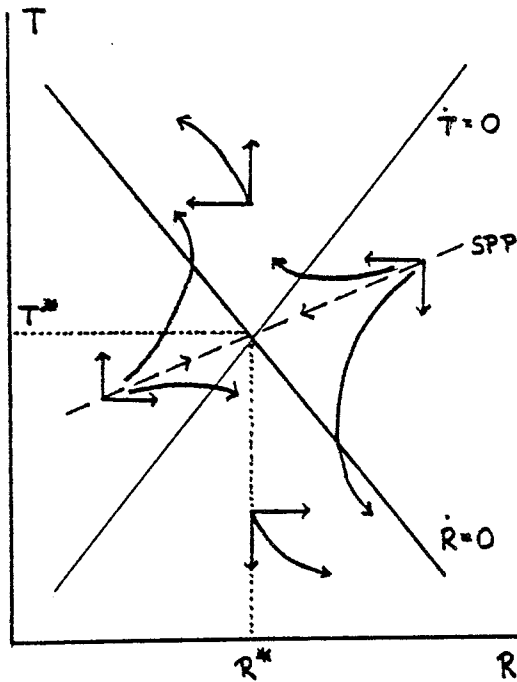


FIGURE 1

The differential Equations (9) and (14) yield the time paths of R and T which satisfy the first order conditions. These time paths are depicted in Figure 1. The $\dot{T} = 0$ function is upward-sloping in $R - T$ space since $(j_R/j_T) > 0$.² The $\dot{R} = 0$ function is downward-sloping since $-(k_R/k_T) < 0$. The stationary state, at which $\dot{T} = \dot{R} = 0$, is represented by the point (R^*, T^*) . The non-stationary states are illustrated by the arrows.

Not all of the paths generated by the necessary conditions for a social optimum satisfy the sufficient conditions as well. It can be shown that all economically meaningful paths which satisfy both sets of conditions lie on a single monotonically increasing function in $R - T$ space, illustrated by the dashed line in Figure 1. We call this function the "saddle-point path".³

At time $t = 0$, the policy maker inherits a pollutant stock at an arbitrary initial level, R_0 . The initial environmental target is set so that just sufficient pollution treatment service is forthcoming to place (R_0, T_0) on the saddle-point path. The successive changes of the environmental target must be such as to maintain the level of the treatment service and the pollutant stock on the saddle-point path.

In the next two sections, we use the model above to assess the significance of the pollution effect on production in formulating dynamic environmental policy.

3. POLICY RESPONSES TO CHANGES IN THE POLLUTION EFFECT ON PRODUCTION

We now examine the dynamic policy implications of changes in the pollution effect on production. We assume that the economy is initially at its optimal stationary state (point (R^*, T^*) in Figure 1) and then an exogenous change occurs in the effect of the pollutant stock on the economy's capacity to produce the consumption good. How must the environmental target change through time in order to maintain the economy on the socially optimal "trajectory", i.e. the socially optimal transition path from the initial stationary state to the final stationary state?

A change in the effect of the pollutant stock on production capacity may be interpreted as a technological change. We confine our attention to the optimal policy responses to two types of technological progress:

- (a) a rise in the "marginal product of pollution", f_R (which is equal to F_R), and
- (b) a rise in the "average product of pollution", (Q/R) .

The responses of the environmental target to the changes above may be divided into short-run, medium-run, and long-run responses. In the short run, the environmental target can affect the level of the treatment service, but the pollutant stock remains at its initial level. In the medium run, both the level of the treatment service and the pollutant

stock can change, but the final stationary state is not achieved. In the long run, the economy moves from the initial to the final stationary state.

First, we consider a rise in the marginal product of pollution and assume that, at the initial stationary state, the average production of pollution remains unaffected thereby. Such a change causes the $\dot{T}=0$ function to shift downwards in R-T space, as may be shown by the following argument. Substituting Equation (9c) into Equation (14), the optimal time path of the treatment service may be written as

$$\dot{T} = -\frac{T}{\phi_1} \cdot [(r - F_R \cdot (h_1' + h_3') + \delta') + \frac{F_R}{F_T} \cdot k_T + \frac{U_R}{U_Q} \cdot \frac{k_T}{F_T} + \phi_2 \cdot \frac{\dot{R}}{R}].$$

The response of \dot{T} to a change in F_R is

$$\frac{d\dot{T}}{dF_R} = -\frac{T}{\phi_1} \cdot [-(h_1' + h_3') + \frac{k_T}{F_T}].$$

If $0 < h_2' < 1$ (i.e. the treatment activity cleanses more pollutants than it generates), Equation (9a) implies that $k_T < F_T \cdot (h_1' + h_3')$. Thus, $-(h_1' + h_3') + (k_T/F_T) > 0$. Consequently,

$(d\dot{T}/dF_R) > 0$, i.e. a rise in F_R increases \dot{T} . Since $(d\dot{T}/dT) > 0$ (from Equation (14)), a fall in T is required to offset the effect of a rise in F_R on \dot{T} . Thus, the $\dot{T}=0$ function shifts downwards, as shown in Figure 2. A rise in F_R implies a rise in k_R . Consequently, the slope of the $\dot{R}=0$ function, $-(k_R/k_T)$, rises. As result of these changes, the stationary state moves from point α to point γ . The saddle-point path shifts

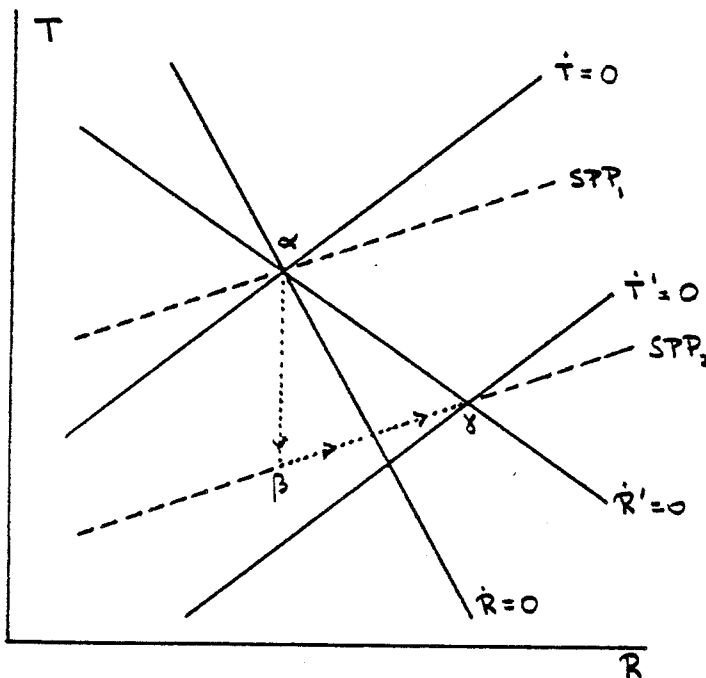


FIGURE 2

downward from SPP_1 to SPP_2 .

In the short run, the economy moves from point α to point β . To occasion this movement, the environmental target for the pollutant flow must rise, causing labor to be transferred from the treatment sector to the production sector. In the medium run, the economy moves from point β to point γ . The environmental target must fall steadily in order for the level of the treatment service to rise steadily. As result, labor returns to the treatment sector from the production sector. Since the contraction of the treatment sector in the short run outweighs the expansion of this sector in the medium run, not all of the labor which left the treatment sector in the short run returns to the treatment sector in the medium run. A rise in the treatment service level must be sufficiently small so that the pollutant flow generated continues to exceed the pollutant flow cleansed. (Otherwise the pollutant stock would fall.) In the long run, the economy moves from point α to point γ . Hence, the environmental target should be set so that the treatment service level falls and the pollutant stock rises. These conclusions may be summarized by the following proposition:

Proposition 1: A rise in the marginal product of pollution (f_R) should be met by

- (a) a short-run rise in the environmental target (causing the production activity to expand and the treatment activity to contract); and
- (b) a medium-run fall in the environmental target (causing the production activity to contract and the treatment activity to expand).

In the long run, the environmental target should promote an expansion of the production activity, a contraction of the treatment activity, and a rise in the level of the pollutant stock.

It is interesting to note that the environmental policy

response to a rise in the marginal effect of pollution on producers (f_R) is qualitatively the same as the policy response to a rise in the marginal effect of pollution on consumers (viz, U_R). A rise in U_R leaves the $\dot{R}=0$ function unchanged. Yet it causes a downward shift in the $\dot{T}=0$ function, since

$$\frac{dT}{dU_R} = -\frac{T}{\phi_1} \cdot \left[\frac{1}{U_Q} \cdot \frac{k_T}{F_T} \right] > 0.$$

Thus, the environmental target should rise in the short run and fall in the medium run, whereas in the long run it should cause the treatment service level to fall and the pollutant stock to rise.

Next consider a rise in the average product of pollution (Q/R), leaving the marginal product of pollution unaffected in the neighborhood of the initial stationary state. As consequence of this change, a given pollutant stock and a given amount of labor employed in the production sector (L_Q) is associated with a larger amount of Q than

previously and therefore also with a larger pollutant flow from the production and consumption of Q . For every given R , \dot{R} is larger than before. Since $(d\dot{R}/dT) = k_T < 0$, an increase in T is required to offset the effect of a drop in the average product of pollution on \dot{R} . Thus, the $\dot{R}=0$ function shifts upwards, as shown in Figure 3. We assume that the drop in the average product of pollution does not affect F_R or F_T in the neighborhood of the initial

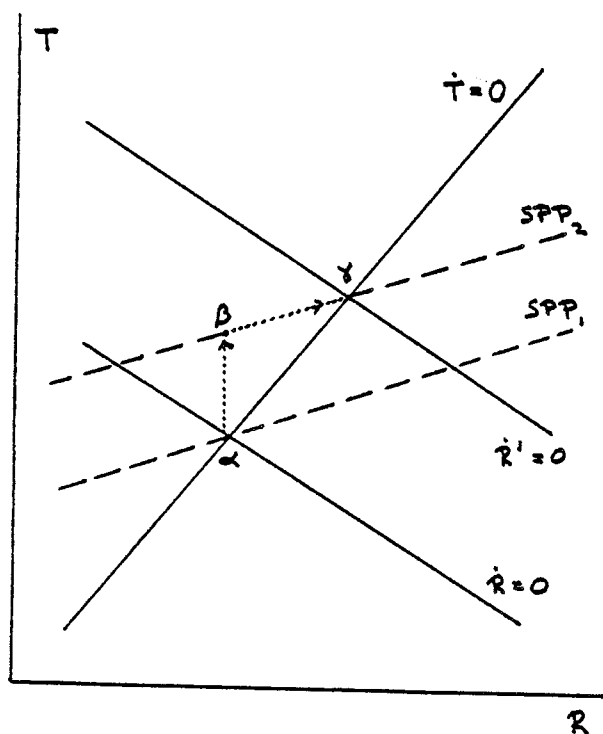


FIGURE 3

stationary state and consequently the $\dot{T}=0$ function remains unaffected in such a neighborhood.

The stationary state moves from point α to point γ . The saddle-point path shifts upwards from SPP_1 to SPP_2 . In the short run, the environmental target for the pollutant flow must be raised, since \dot{R} is zero at point α and positive at point β .⁴ In medium run, the environmental target must fall, since the level of the treatment service rises at the expense of the production activity. In the long run, the levels of the treatment service and the pollutant stock both rise.

A rise in the average product of pollution is a technological improvement which permits the given labor force to produce more than the original bundle of Q and T (for any given level of the pollutant stock). The labor which is no longer needed to produce this bundle may be viewed as labor "released" by the technological improvement. Since the level of the treatment service expands in the short run, it is apparent that some of the released labor must be allocated to the treatment activity. Since the pollutant flow is positive in the short run, the pollutant flow generated by the production and consumption activities must exceed the pollutant flow cleansed by the treatment activity and by nature. Given that nature's treatment activity remains unchanged in the short run, this development is possible only if the production of Q rises above its original level. Hence, some of the released labor must be allocated to the production activity in the short run.

In sum, the labor released through the technological improvement in the short run is allocated to both the treatment and production activities. Yet in order for the treatment activity to expand in the medium run, labor must be transferred from the production sector to the pollution-treatment sector. As shown in Figure 3, the pollutant stock rises throughout the medium run; hence, the transfer of labor must remain sufficiently small so that the pollutant flow generated

by Q never falls short of the pollutant flow cleansed naturally and anthropogenically. Clearly, not all of the released labor entering the production sector in the short run is transferred to the treatment sector in the medium run.

Proposition 2: A rise in the average product of pollution (Q/R) should be met by

- (a) a short-run rise in the environmental target (so that both the production and treatment activities expand); and
- (b) a medium-run fall in the environmental target (causing the treatment activity to expand and the production activity to contract).

In the long run, the target should promote an expansion of the production and treatment activities and an increase in the level of the pollutant stock.

Comparing Propositions (1) and (2), it is evident that the optimal dynamic responses of the environmental target to increases in the marginal and average products of pollution are qualitatively the same: the environmental target should rise in the short run and fall in the medium run. Yet the effect of these target movements on the production and treatment sectors is not identical for the two cases. In the short run, an increase in the marginal product of pollution calls for a contraction of the treatment sector and an expansion of the production sector, whereas an increase in the average product of pollution calls for an expansion of both sectors. (In the medium run, both technological improvements require an expansion of the treatment sector and a contraction of the production sector.)

It is not difficult to rationalize this difference in the short run sectoral reactions. The short run contraction of the treatment sector in response to an increase in the marginal product of pollution may be understood in terms of Equation (13). Substituting Equation (9c) into this equation,

we obtain the following optimality condition:

$$U_Q \cdot F_R - U_R = -\dot{\mu} + \mu \cdot r - \mu \cdot F_R \cdot (h_1' + h_3') + \mu \cdot \delta'$$

According to this condition, the size of the treatment sector should be such that the marginal social cost of extra pollution is equal to the marginal social cost of extra pollution treatment. The marginal social cost of extra pollution is the sum of (a) the utility loss from the production foregone due to extra pollution ($U_Q \cdot F_R$) and (b) the direct utility loss attributable to extra pollution (U_R). The marginal social cost of extra pollution treatment is the sum of (a) the capital loss from pollution treatment ($-\dot{\mu}$), (b) the opportunity cost of pollution treatment due to time discounting ($\mu \cdot r$), (c) the utility loss from the pollution flow which accompanies the increased production induced through the treatment of pollution ($-\mu \cdot F_R \cdot (h_1' + h_3')$), and (d) the utility loss from the reduction in nature's treatment activity through anthropogenic treatment ($\mu \cdot \delta'$).

An increase in the marginal product of pollution upsets the optimality condition above: the marginal social cost of extra pollution treatment falls relative to the marginal social cost of extra pollution. In order to bring these two social costs into equality, the treatment sector must contract.

On the other hand, an increase in the average product of pollution means (as shown above) that labor is released for further productive use. This released labor must be allocated between the two sectors in such a way that the marginal social welfare from production remains equal to the marginal social welfare from pollution treatment (as dictated by Equation (12)). In other words, the released labor should enter both sectors and consequently both sectors should expand in the short run.

In any event, an increase in the average product of pollution calls for a short-run rise in the environmental target, but this rise should be small enough to permit the

treatment sector to expand along with the production sector. A rise in the environmental target should induce a contraction of the treatment sector in response to an increase in the marginal product of pollution, but not in response to an increase in the average product of pollution.

4. POLICY RESPONSES TO PRODUCTION-CAPACITY CHANGES AND PREFERENCE CHANGES

Whereas Section 3 showed that the optimal environmental target should respond to changes in the pollution effect on production, this section describes how the pollution effect on production influences the manner in which the optimal environmental target responds to production-capacity changes and preference changes. The production-capacity changes described here are those arising from changes in the size of the labor force and from technological improvements (other than the changes in the pollution effects on production considered in Section 3). Preference changes include changes in the social rate of time preference and changes on the marginal social valuations of the consumption flow and the pollutant stock. The optimal environmental target may respond differently to these various exogenous changes if the pollution effect on production is present than if it is absent.

In the absence of the pollution effect on production, the production possibility frontier $Q = F(T, R)$, as represented in equation (3), becomes $Q = \hat{F}(T)$. Moreover, ϕ_1 and F_R both take on the value of zero. Given these modifications, equations (9) and (14) -- which describe the optimal time paths of R and T -- may be reduced to the following forms:

$$(15) \quad \dot{R} = h_1(\hat{F}(T)) + h_2(T) + h_3(\hat{F}(T)) - T - \delta(R) = \hat{k}(T, R)$$

$$(16) \quad \dot{T} = -\frac{T}{\phi_1} \cdot \left[(r - k_R) + \left(\frac{U_R}{U_Q} \right) \cdot \frac{k_T}{F_T} \right] = \hat{j}(T, R),$$

where " $\hat{\cdot}$ " identifies the functions which must be changed in order to eliminate the pollution effect on production. We denote equations (15) and (16) as Model I and equations (9) and (14) as Model II.

The behavior of Model I can be illustrated in R-T space, similarly to the way in which the behavior of Model II is captured in Figure 1. For both models, the $\dot{R}=0$ function is downward-sloping and the $\dot{T}=0$ function is upward-sloping. Yet Model II contains a steeper $\dot{R}=0$ function than Model I. The slope of the $\dot{R}=0$ function in Model II is

$$- \frac{F_R \cdot (h'_1 + h'_3) - \delta'}{F_T \cdot (h'_1 + h'_3) - (1 - h'_2)},$$

whereas the slope of its counterpart in Model I is

$$\frac{\delta'}{F_T \cdot (h'_1 + h'_3) - (1 - h'_2)}$$

The slope of the $\dot{T}=0$ function in Model II may be steeper or flatter than the slope of its counterpart in Model I.

We now compare Models I and II in terms of the optimal environmental target responses to exogenous economic changes. Our comparisons will not depend on an evaluation of the relative slopes of the $\dot{T}=0$ function in the two models. Consider first the target responses to a change in social preferences. Suppose that a society becomes more "pollution conscious", in the sense that its marginal disutility from pollution rises, its marginal utility from consumption falls, or its valuation of future utilities rises relative to its valuation of present utilities. Such a change in preferences may be represented by a fall in U_R , U_Q , or r . In both Models I and II, these changes affect the position of the $\dot{T}=0$ function in R-T space, but leave the $\dot{R}=0$ function unchanged. For both models, the deriva-

tives of \dot{T} with respect to U_R , U_Q , and r are

$$\frac{\partial \dot{T}}{\partial U_R} = - \frac{T}{\phi_1} \cdot \frac{1}{U_Q} \cdot \frac{k_T}{F_T} > 0,$$

$$\frac{\partial \dot{T}}{\partial U_Q} = \frac{T}{\phi_1} \cdot \frac{U_R}{(U_Q)^2} \cdot \frac{k_T}{F_T} > 0,$$

$$\frac{\partial \dot{T}}{\partial r} = - \frac{T}{\phi_1} > 0.$$

For Model I,

$$\left. \frac{\partial \dot{T}}{\partial T} \right|_I = - \frac{T}{\phi_1} \cdot \left[\left(\frac{U_R}{U_Q} \right) \cdot \left(\frac{k_{TT} \cdot F_T - F_{TT} \cdot k_T}{(F_T)^2} \right) \right] > 0,$$

in the neighborhood of $\dot{T}=0$. Thus, for any given level of R , a fall in U_Q , U_R , or r elicits a fall in \dot{T} , which (in turn) may be offset by rise in T . Consequently, an increase in pollution consciousness causes the $\dot{T}=0$ function to shift upwards to $T_I=0$, as shown in Figure 4.

For Model II,

$$\left. \frac{\partial \dot{T}}{\partial T} \right|_{II} = - \frac{T}{\phi_1} \cdot \left[\left(F_R + \frac{U_R}{U_Q} \right) \cdot \left(\frac{k_{TT} \cdot F_T - F_{TT} \cdot k_T}{(F_T)^2} \right) - \frac{F_{RT}}{F_T} \cdot (1-h'_2) \right] > 0$$

and since $F_R < 0$ and $F_{RT} > 0$,

$$\left. \frac{\partial \dot{T}}{\partial T} \right|_{II} > \left. \frac{\partial \dot{T}}{\partial T} \right|_I.$$

Thus, the upward shift of the $\dot{T}=0$ function (to $\dot{T}_{II}=0$ in Figure 4) in Model II is not as great as the upward shift of this function in Model I. By implication, the saddle-point path of Model II (SPP_{II}) lies above the saddle-point path

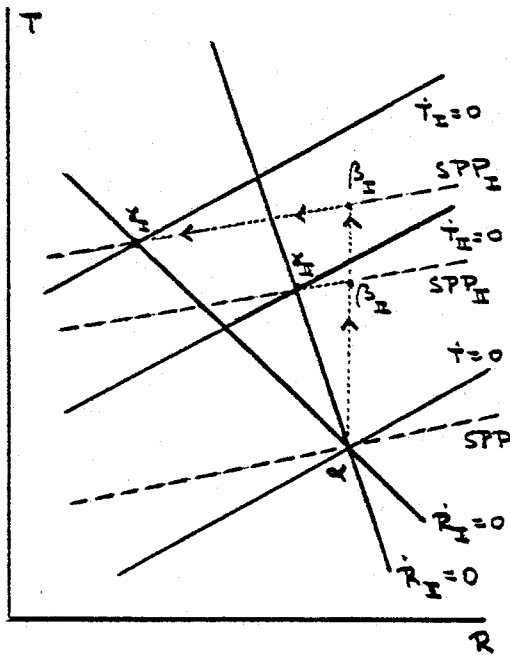


FIGURE 4

of Model I (SPP_I).

The optimal short-run movement of the economy extends from point α to point β_I in Model I, but extends only from point α to point β_{II} in Model II. Since the short-run rise of the treatment service level is smaller in Model II than in Model I, the drop of the environmental target must be smaller in the presence of the pollution effect on production than in the absence of this effect. In the medium run, the economy of Model I moves from point β_I to point γ_I ,

whereas the economy of Model II moves from point β_{II} to point γ_{II} . It is apparent that the long run fall in the pollutant stock is smaller for Model II than for Model I.

In sum, the pollution effect on production exerts an influence on the optimal response of the environmental target to a rise in pollution consciousness. This influence may be summarized by the following proposition:

Proposition 3: In response to a rise in pollution consciousness (as represented by a fall in U_R , U_Q , or r),

- (a) the optimal short-run rise of the treatment service level,
- (b) the optimal short-run fall of the environmental target, and
- (c) the optimal long-run fall of the pollutant stock

is smaller in the presence of the pollution effect on production than in the absence of this effect.

Next, consider the optimal environmental target response to the following technological improvement: a drop in the marginal pollutant flow from the treatment service (h'_2), leaving the average pollutant flow from T , R_T/T , unchanged in the

neighborhood of the initial stationary state. The pollution effect on production conditions this response in qualitatively the same way as it conditions the response to the rise in pollution consciousness described above. In the absence of the pollution effect on production (Model I),

$$\frac{\partial \dot{T}}{\partial h_2'} = - \frac{T}{\phi_1} \cdot \frac{1}{F_T} \cdot \left(\frac{U_R}{U_Q} \right) > 0$$

whereas in the presence of the pollution effect on production (Model II),

$$\frac{\partial \dot{T}}{\partial h_2'} = - \frac{T}{\phi_1} \cdot \frac{1}{F_T} \cdot \left(F_R + \frac{U_R}{U_Q} \right) > 0$$

Thus, in both models a drop in h_2' induces a fall in \dot{T} , which may be offset by rise in R (since $(\partial \dot{T} / \partial T) > 0$ in Models I and II). As result, the $\dot{T}=0$ function shifts upwards in R - T space in both cases.

It is evident that a given drop in h_2' has a stronger contractionary influence on \dot{T} in Model II than in Model I. Moreover, recall that a given rise in T elicits a stronger expansionary influence on \dot{T} in Model II than in Model I. It can be shown that, if $k_{TT} < 1$, the effect of T on \dot{T} exceeds the effect of h_2' on \dot{T} . Thus, the upward shift of the $\dot{T}=0$ function in R - T space is not as great in Model II than in Model I. (A drop in h_2' also causes the slope of the $\dot{R}=0$ function to rise; yet this effect has no bearings on the conclusions of our analysis.) The optimal dynamic evolution of T and R in response to a drop in h_2' may be illustrated in Figure 4. Clearly, the way in which the pollution effect on production influences the response of T and R to a drop in h_2' is analogous to the way in which it influences the response of T and R to a rise in pollution consciousness:

Proposition 4: In response to a drop in the marginal pollutant flow from the treatment service (h'_2),

- (a) the optimal short-run rise of the treatment service level,
- (b) the optimal short-run fall of the environmental target, and
- (c) the optimal long-run fall of the pollutant stock

is smaller in the presence of the pollution effect on production than in the absence of this effect.

In accordance with the analysis above, it is clear that a rise in pollution consciousness and a drop in h'_2 induce a short-run rise in the environmental target, a medium-run fall in this target, and a long-run fall in the pollutant stock. It is also clear that the pollution effect on production dampens the magnitude of these responses. What may not be so obvious at first glance is that a drop in the marginal pollutant flow from production or consumption (h'_1 or h'_3) induces qualitatively the opposite responses and that the pollution effect on production also mitigates these responses.

A drop in h'_1 or h'_3 (leaving the average pollutant flows from production and consumption, R_Q/Q and R_C/C , unchanged in the neighborhood of the initial stationary state) is a technological improvement which calls for the opposite dynamic response of the environmental target from that required by a drop in h'_2 . For both Models I and II,

$$\frac{\partial \dot{T}}{\partial h'_1} = \frac{\partial \dot{T}}{\partial h'_3} = - \frac{T}{\phi_1} \cdot \frac{U_R}{U_Q} < 0$$

Thus, a drop in h'_1 or h'_3 elicits a rise in \dot{T} , which may be offset by a fall in T (since $(\partial \dot{T} / \partial T) \gg 0$). Hence, the $\dot{T}=0$ function shifts downwards in R - T space for both models. Since the positive effect of T on \dot{T} is greater in Model II than in Model I, the downward shift of the $\dot{T}=0$ function is smaller

in Model II than in Model I. (A drop in h_1' or h_3' also induces a change in the slope of the $\dot{R}=0$ function; however, this influence is not relevant for the conclusions of our analysis.)

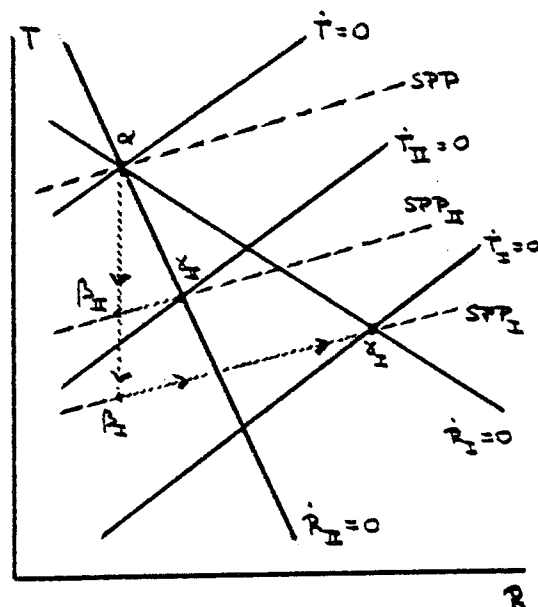


FIGURE 5

With regard to Figure 5, the Model I economy moves from point α to point β_I in the short run, but the Model II economy moves only from point α to point β_{II} in the short run. In the medium run, the former economy moves from point β_I to point γ_I , whereas the latter economy moves from point β_{II} to point γ_{II} . The policy implications of these results may be summarized as follows:

Proposition 5: In response to a fall in pollution consciousness (as represented by a rise in U_R , U_Q , or r) and to a drop in the marginal pollutant flow from production or consumption (h_1' or h_3' , respectively),

- (a) the optimal short-run fall of the treatment service level,
- (b) the optimal short-run rise of the environmental target, and
- (c) the optimal long-run rise of the pollutant stock

is smaller in the presence of the pollution effect on production than in the absence of this effect.

It is not difficult to see why the optimal short-run response of the environmental target to a drop in the marginal pollutant flow from T runs in the opposite direction from the optimal short-run response of the environmental target to a drop in the marginal pollutant flow from Q or C. At the initial stationary state, the marginal social welfare from pollution treatment is exactly equal to the marginal social welfare from production. When h_2' falls, the marginal

welfare contribution of the treatment service rises relative to the marginal welfare contribution of production. In order to restore the equality, the treatment sector must expand. However, when h_1' or h_3' falls, the marginal welfare contribution of the treatment service falls relative to the marginal welfare contribution of production. In this case, the treatment sector must contract.

Putting the analytical results above into a broad policy perspective, it is apparent that changes in social preferences (represented by rises and falls in pollution consciousness) and changes in the marginal pollutant flows from production, consumption and treatment call for dynamic environmental target responses which belong to a single behavioral paradigm. As shown above, a rise in pollution consciousness and a drop in h_2' require a short-run fall of the environmental target, a medium-rise of this target, a long-run rise in T , and a long-run fall in R . A drop in pollution consciousness and a drop in h_1' and h_3' require the dynamic obverse of these responses. For all of these preference changes and technological improvements, the short-run target responses and the long-run changes in the pollutant stock are of smaller magnitude in the presence of the pollution effect on production than in the absence of this effect.

We now consider a set of production capacity changes which give rise to dynamic environmental target responses belonging to a different behavioral paradigm. We examine the pollution effect on production in conditioning the target responses of this second paradigm. Suppose that the production possibility frontier (equation (3)) shifts outwards, leaving the marginal rate of transformation between Q and T unchanged. This change may be occasioned by a technological improvement or by an increase in the labor supply. In both Models I and II, this change affects the position of the $\dot{R}=0$ function in R - T space, but leaves the $\dot{T}=0$ function unchanged. For a given level of T and R , it is possible to produce more Q

than previously . Consequently, the locus of points in R-T space which was originally associated with a pollutant flow of zero is now associated with a positive pollutant flow. In other words, the $\dot{R}=0$ function shifts upwards (by the same amount in both models) as result of the technological improvement or the increase in the labor supply.

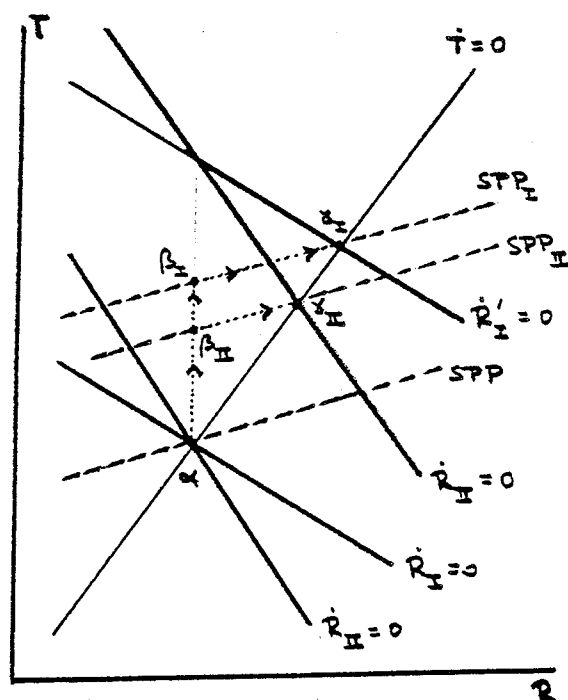


FIGURE 6

The optimal policy responses to the outward shift in the production possibility frontier are illustrated in Figure 6. Since the $\dot{R}=0$ function of Model II is steeper than that of Model I, the final stationary state of Model II is associated with smaller levels of T and R than the final stationary state of Model I. For levels of the pollutant stock between R_1 and R_2 , the final saddle-point path of Model II (SPP_{II}) lies beneath the final saddle-point path of Model I (SPP_I). In the short run, the economy of

Model I moves from point α to point β_I , but the economy of Model II moves only from point α to point β_{II} . In the medium run, the former economy moves from point β_I to point δ_I , whereas the latter economy moves from point β_{II} to point δ_{II} . The following proposition contains the major policy implications of these results:

Proposition 6: In response to an outward shift in the production possibility frontier (leaving the marginal rate of transformation between Q and T unchanged),

(a) the optimal level of the treatment service is lower in the medium run,

- (b) the optimal level of the environmental target is higher in the medium run,
- (c) the rise of the treatment service is smaller in the long run, and
- (d) the rise of the pollutant stock is smaller in the long run in the presence of the pollution effect on production than in the absence of this effect.

The dynamic opposite of the policy responses above are elicited by a technological improvement involving a downward shift of the pollution functions (equations (4), (5), and (6)), which leave the marginal pollutant flows from production, consumption, and treatment unchanged. Such a technological improvement induces a downward shift

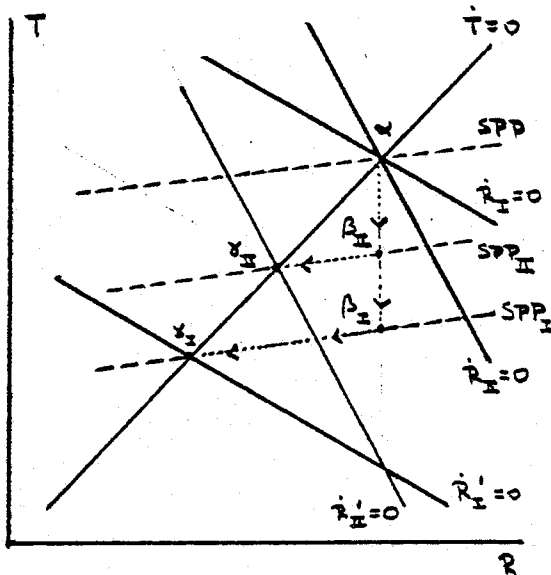


FIGURE 7

of the $\dot{R}=0$ functions of Models I and II, as shown in Figure 7. The manner in which the pollution effect on production influences the optimal policy responses to the technological improvement may be summarized as follows:

Proposition 7: In response to a downward shift of the pollution functions (leaving the marginal pollutant flow from consumption, production, and treatment unchanged),

- (a) the optimal level of the treatment service is higher in the medium run,
- (b) the optimal level of the environmental target is lower in the medium run,
- (c) the fall of the treatment service is smaller in the long run, and

(d) the fall of the pollutant stock is smaller in the long run

in the presence of the pollution effect on production than in the absence of this effect.

It is clear that outward shifts of the production possibility frontier and downward shifts of the pollution functions elicit dynamic policy responses which belong to a single behavioral paradigm. This second behavioral paradigm differs in an important way from the first behavioral paradigm (induced by changes in social preferences and changes in marginal pollutant flows from production, consumption, and treatment). In the first behavioral paradigm, the short-run movement of the environmental target is in the opposite direction from the medium-run movement of the target. (For example, a rise in pollution consciousness causes the environmental target to fall in the short run and to rise in the medium run.) Yet in the second behavioral paradigm, the short- and medium-run movements of the environmental target are in the same direction. In both paradigms, the pollution effect on production exerts a dampening influence on the magnitude of optimal policy responses to exogenous changes. In particular, the magnitude of the environmental target responses to changes in social preferences and to changes in production capacity are generally smaller in the presence of the pollution effect on production than in the absence of this effect.

In the analysis above, we have examined the role which the pollution effect on production should play in the formulation of environmental policy. Exogenous changes of this effect call for dynamic adjustments of the environmental target. Both a rise in the marginal product of pollution and a rise in the average product of pollution call for a short-run increase and a medium-run decrease in the environmental target. Moreover, the pollution effect on

production influences the manner in which the environmental target responds to changes in production capacity and to changes in social preferences. In sum, the pollution effect on production is significant for environmental policy not only in its own right but also on account of its influence on environmental policy responses to other economic phenomena.

FOOTNOTES

$$1. \quad L_T = g^{-1}(T), \quad Q = f(L - g^{-1}(T), R) = F(T, R).$$

$$F_R = f_R < 0 \text{ and } F_{RR} = f_{RR} < 0.$$

$$F_T = -f_L \cdot (g^{-1})_T < 0;$$

$$F_{TT} = -f_L \cdot (g^{-1})_{TT} + ((g^{-1})_T)^2 \cdot f_{LL} < 0$$

$$F_{TR} = - (g^{-1})_T \cdot f_{LR} > 0.$$

2. For the purposes of our analysis below, it is only necessary to examine the slope of the $\dot{T} = 0$ function in the neighborhood of the $\dot{R} = 0$ function.

In this neighborhood,

$$j_R = -\frac{T}{\phi_1} \cdot \left[\frac{k_T}{F_T} \cdot \left(\frac{U_{RR}}{U_Q} + F_{RR} \right) - k_{RR} + \frac{k_T}{F_T} \cdot \left(\frac{U_R}{U_Q} + k_R \right) \cdot \left(\frac{\sigma_{TR}^k - \sigma_{TR}^F}{R} \right) \right],$$

which is negative if we assume that $\sigma_{TR}^k - \sigma_{TR}^F > 0$;

$$j_T = \frac{T}{\phi_1} \cdot \left[k_T \cdot \left(\frac{\sigma_{TR}^k - \sigma_{TR}^F}{R} \right) + k_T \cdot \frac{F_{TT} \cdot F_R}{(F_T)^2} - k_{TT} \cdot \frac{F_R}{F_T} \right. \\ \left. - \left(\frac{U_R}{U_Q} \right) \cdot \frac{k_{TT} \cdot F_T - F_{TT} \cdot k_T}{(F_T)^2} + \left(\frac{k_T}{F_T} \right) \cdot \frac{U_{QQ} \cdot F_T \cdot U_R}{(U_Q)^2} \right].$$

which is positive.

3. The saddle-point path satisfies the following sufficient conditons adduced by Arrow and Kurz [1]:

FOOTNOTES (Continued)

- (a) for a given value of μ , the Hamiltonian is a concave function of R since $(\partial^2 H / \partial R^2) = U_Q \cdot F_{RR} + U_{RR} + \mu \cdot k_{RR} < 0$;
- (b) the present value of R (evaluated through λ) approaches zero as time approaches infinity (since $\lim_{t \rightarrow \infty} e^{-rt} \cdot \mu \cdot R = 0$).
4. Note that the environmental target rises in the short run even though the level of the treatment service rise in the short run as well. As shown below, both T and Q increase in the short run and the generation of pollutants through Q outweighs the cleansing of pollutants through T .

R E F E R E N C E S

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