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Lagged Network Externalities and Rationing in a Software Monopoly

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Abstract

The paper presents a model of a software monopolist who benefits from a lagged network externality arising from consumers' feedback through the so-called bug-fixing effect. That is, the software producer is able to correct errors in the software code detected by previous users, improving her products over time. Another feature of the model is that it responds to the short life cycle of software products, implying time-of-purchase depending utility functions, which are in contrast to the usual durable goods models. Both of these modifications are incorporated in a standard two-periods durable goods monopoly, analysing questions of introductory pricing and quantity rationing. The model suggests that neither of these two instruments is able to explain why we see so much free software in the markets.

Keywords

Software monopoly, lagged network externality, introductory pricing, rationing

JEL Classifications

L120, L860, D420, D450

Comments

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Contents

1 Introduction

Software as a good is exchanged in highly profitable and fast growing markets, whose most striking feature is the continuous innovation brought about by the fierce competition among producers. At the same time these markets are also extremely segmented, and every producer who successfully introduces a new product enjoys some monopoly power over a limited period of time, until her competitors eventually catch up in that particular market segment, and introduce their own, often improved, versions of the product. Innovation and technological progress are, thus, crucial features of the software market, and therefore the life cycle of a software product is rather short, although it is generally considered a durable good. As a consequence the utility to be derived from a given software product is not independent of the time of purchase, as is usually assumed for other durable goods.

The second feature of software products that we intend to focus on is that their quality generally improves over their life cycle, owing to the so-called bug-fixing. The large number of users of any single software product leads to the detection of a far larger number of errors in the code and in the workings of the software, than any team of software designers would be able to identify. Customers' feedbacks are thus fundamental to the quality improvements of the early releases through patches and other bug-fixing devices. The whole bug-fixing process generates a positive lagged network externality which depends on the number of previous users.

Having described these characteristics of the software industry, we feel that they are not captured well enough in the literature. Accordingly, our aim is to develop a two-period durable goods monopoly model to represent the software market, and to use it to investigate two issues which are well known in the Industrial Organisation literature: introductory pricing and quantity rationing.

One of the best known results in the field of Industrial Organisation is that a durable goods monopolist faces the competition of her own future output. If the monopolist cannot commit to a price sequence, she has an incentive to lower the price after serving the consumers with the greatest willingness to pay for the good; in so doing, though, the monopolist induces consumers to postpone purchases. As a consequence, the no-commitment equilibrium will exhibit declining prices. It follows that, although price discrimination can be used in equilibrium, the static monopoly price cannot be sustained, thus leading to a lower equilibrium profit than under precommitment[1]. The existence of a lagged network externality should increase the incentives of

[1] On this and other issues of Industrial Organisation see, for example, Tirole [11].

the monopolist to sell at a lower price in the first period and then reap the benefit of the lower competition in subsequent periods. We investigate this matter in the first part of this chapter.

An alternative approach to reduce the competition of future output could be to serve less consumers than would be otherwise willing to buy the good at the current price in the first period, to face higher demand in subsequent periods. This is the idea behind quantity rationing. Indeed a typical strategy of software companies is to give away a limited number of free beta-versions of their products prior to the official release to benefit form the externality arising from bug-fixing, as mentioned above. We want to investigate if this kind of rationing will be supported by the model. This is the object of the second part of the paper.

The literature dealing in one way or another with these issues is huge. We will relate to three lines of literature in the present paper. The first line of literature refers to the possibility that the monopolist's market power decreases as the time between sales becomes shorter in what is known as the Coase-conjecture. Coase [3] claimed that the price set by the monopolist will quickly converge to marginal cost as the time between sales tends to zero. Several contributions confirmed or refuted this conjecture in a variety of settings. As shown by Hart and Tirole [8], though, in all cases the equilibrium solutions obey the so-called Coasian dynamics: They satisfy the skimming property, according to which higher valuation buyers make their purchases no later than lower valuation ones; and the price monotonicity property, which states that equilibrium price is non-increasing over time. Cabral et al. [2] show that the latter does not necessarily hold in the presence of network externalities. In particular they show that if consumers are "large", equilibria can obtain in which discounted prices rise over time. If consumers are small, to the contrary, Coasian dynamics prevail. Our task is to analyse if in the presence of a lagged externality the software monopolist finds it profitable to use introductory pricing.

The second line of literature to which we relate is the vast and growing literature on network externalities. We develop our model following the ideas of Ruiz [10] on lagged externalities, although our focus is not on innovation and planned obsolescence but rather on pricing in this context. We think of a software monopolist introducing a new software and then releasing free upgrades or posting add-ons and patches on her website, to fix the bugs discovered by the first generation of users. Clearly, the larger the number of these first generation users, the faster and more effective the feedbacks and the bug-fixing process[2].

[2] Ruiz [10] presents a number of different examples also from more traditional types of

Finally, we also relate to the literature on rationing. In the rationing literature it is common to introduce some form of precommitment to increase the market power of the monopolist. Van Cayseele [12], for example, assumes that the seller can commit to a given second-period output, so that high-valuation consumers are not certain of obtaining the good if they postpone their purchases. Other models, for example Gilbert and Klemperer [6] and Allen and Faulhaber [1], posit some kind of consumption externality or incomplete information and, in general, assume fixed capacity. We avoid any form of precommitment in either output or prices, assume complete information and no capacity constraints, and follow Denicolò and Garella [4] in analysing the possibility that rationing may be profitable in a standard durable goods monopoly model. Denicolò and Garella [4] find circumstances where, under proportional rationing, it pays the monopolist to ration consumers in the first period. Yet, we feel that in their setup the decision on rationing, and more specifically on the scale of rationing, is not truly the monopolist's choice, as we will explain later on. We investigate the same issue of rationing in a modified setup, commenting on the original results by Denicolò and Garella.

To summarise, we will develop a simple two-periods monopoly which is similar to the model in Denicolò and Garella [4], but differs in three important aspects: First, our utility functions are designed to better match the special features of software products, as mentioned above. Second, we introduce a lagged externality. Third, we completely endogenise the monopolist's decision concerning rationing. In what follows we pursue two lines of analysis: In Section 2 we investigate whether, in this framework, there can obtain equilibria with increasing discounted prices or if Coasian dynamics prevail. As will become clear afterwards, our results confirm those in Cabral et al. [2]. In the second part of the paper, in Section 3, we ask ourselves if the results in Denicolò and Garella [4] remain valid also with the modified utility functions. We find this not to be the case, indeed we show that rationing can never be an equilibrium strategy in our modified setup.

2 The Non-Rationing Case

2.1 The Model

A monopolist sells her software product in two trading periods. The consumption of the good is subject to a lagged network externality which depends on the number of users in the previous period. The monopolist pro-

industry.

duces at constant marginal cost, which without loss of generality can be set to zero. There is no resale market. Price commitments as well as the possibility of renting the good are ruled out.

There is a continuum of consumers, all buying at most one unit of the indivisible good. Consumers are indexed by their per-period utility θ for the good. They are uniformly distributed over the interval $[0, 1]$, with distribution function $F(\theta) = \theta$ for all $\theta \in \Theta = [0, 1]$. This implies a linear (static) demand function.

The sequence of actions is as follows: At the beginning of period 1, the monopolist sets the price p_1 for the current period. Consumers then decide whether to buy the good or not. No information about the price of the good in period 2 becomes available. In period 2 the lagged network externality created by consumption in period 1 sets in. That is, in period 2 the monopolist offers an improved version of the product. The monopolist sets a price p_2, and all consumers who have not bought in period 1 decide whether to buy or not. Furthermore we assume that consumers who bought in period 1 are offered a free update of the product so that they also benefit from bug-fixing[3]. Letting x_1 be the fraction of consumers who bought the good at $t = 1$, the (positive) network externality they create is $f(x_1)$, with $f'(x_1) > 0$. For simplicity we assume it to be linear, i.e. $f(x_1) = ax_1$ with $a > 0$[4].

Let us now turn to payoff functions. A consumer with valuation θ who buys the good in period 1 earns an overall payoff of

$$\theta - p_1 + \delta(\theta + ax_1), \tag{1}$$

where $\delta \in [0, 1]$ is the discount factor. A consumer who buys at $t = 2$ receives

$$\delta(\theta + ax_1 - p_2). \tag{2}$$

In the standard durable goods monopoly, consumers' payoffs are based on the discounted present value of future utility. In these models, a consumer buying at $t = 1, 2$ receives a payoff of $\delta^{t-1}(\theta - p_t)$. Hence, a consumer's valuation for the good is independent of the time of purchase since the durability of the good is assumed to be constant. The only loss from waiting for the

[3] Alternatively, one could assume that the monopolist opens a new market in period 2 to sell patches to period 1 consumers. This would, of course, leave some low-valuation period 1 consumers without the patch. We find our assumption of a free upgrade more realistic, though.

[4] Note that the externality is independent of the consumer's type. In principle one could argue that higher valuation consumers could benefit more from any given improvement. Accordingly, one could devise a form for the externality such as, for example, $g(x_1, \theta) = a\theta x_1$. Again, for the sake of tractability, we will not consider externalities of this kind in this paper.

consumer arises from the discount factor, i.e. from her impatience. As already mentioned, we do not consider this assumption appropriate in the case of software. We think that software does indeed "expire" at a certain point in time in the future due to technological progress. Therefore consumers experience an additional loss in payoffs by waiting: According to equations (1) and (2), the software product expires after $t = 2$. A consumer buying in period 2 not only loses payoffs from impatience, but also forgoes the per-period valuation θ related to the first period. In other words, a consumer has to be aware that by buying software tomorrow rather than today, she will face a shorter lifetime of the product.

Finally, the payoff of the monopolist is

$$p_1 x_1 + \delta p_2 x_2, \tag{3}$$

where x_2 is the demand at $t = 2$.

2.2 The Subgame-perfect Equilibria

The monopolist cannot commit to a price sequence, thus the relevant solution concept is that of subgame-perfect equilibrium: We start looking for the Nash equilibrium at the last stage of the game and proceed backwards. At the second period, given first period output x_1 and the price set by the monopolist p_2, each consumer who did not buy in the previous period will buy if $p_2 \leq \theta + ax_1$. The marginal consumer, that is the consumer who is indifferent between buying and not buying, has a valuation $\theta_2 = \max\{0, p_2 - ax_1\}$. As it is standard in the Industrial Organisation literature, we will only consider those subgame perfect equilibria that have exactly one indifferent consumer. This assumption implies the skimming property, i.e. higher valuation consumers make their purchases no later than lower valuation consumers. Generally, there can very well be more than one indifferent consumer, giving rise to subgame perfect equilibria not featuring the skimming property[5].

Letting θ_1 denote the valuation of the marginal consumer in period one, it follows that $\theta_2 \in [0, \theta_1]$[6]. The market size in period 2 will then be

[5] For a discussion on this topic and on how to avoid this problem see Güth and Ritzberger [7].

[6] The level of the externality will determine how much of the market the monopolist finds optimal to cover, and thus what price to choose. In particular, the larger the externality the higher the share of consumers that the monopolist will decide to supply with the good. Thus the requirement that θ_2 is (weakly) positive, as well as the analogous requirement for θ_1, will determine the existence of different solutions, depending on the level of the externality. We will present the different cases as we proceed with the analysis.

$$x_2 = F(\theta_1) - F(\theta_2) = \theta_1 - \theta_2. \quad (4)$$

The monopolist at $t = 2$ sets the price

$$p_2 = \theta_2 + ax_1, \quad (5)$$

such that she maximises

$$\pi_2 = \delta p_2 x_2 = \delta p_2[\theta_1 - \theta_2]. \quad (6)$$

Depending on the size of the externality, $\theta_2 = p_2 - ax_1$ can be larger or equal to zero, implying the existence of two different cases.

We start by considering the case when the externality is not large enough for the monopolist to cover the whole market in the second period, so that $\theta_2 > 0$. In this case we can rewrite (6) as

$$\pi_2 = \delta p_2[\theta_1 - p_2 + ax_1]. \quad (7)$$

The price set is $p_2^{NR1} = \arg\max_{\{p_2\}} \delta p_2[\theta_1 - p_2 + ax_1]$, or

$$p_2 = \frac{\theta_1 + ax_1}{2}, \quad (8)$$

which corresponds to an output level of

$$x_2 = \frac{\theta_1 + ax_1}{2}, \quad (9)$$

and to a second period's profit of

$$\pi_2^{NR} = \delta \frac{(\theta_1 + ax_1)^2}{4}. \quad (10)$$

Consumers at $t = 1$ decide whether to buy or to wait. They will buy if their valuation is such that

$$\theta - p_1 + \delta(\theta + ax_1) \geq \max\{0, \delta[\theta + ax_1 - p_2]\}. \quad (11)$$

That is, they will buy now if the payoff of doing so is positive and exceeds the payoff of waiting, assuming that they perfectly foresee p_2.

The marginal consumer's valuation is derived by equalising (11) and substituting for p_2, to obtain[7]:

$$\theta_1 = \frac{2p_1 - \delta ax_1}{2 + \delta}. \quad (12)$$

[7] It is clear that in the subgame-perfect equilibrium, $\delta[\theta_1 + ax_1 - p_2]$ is always positive. Suppose to the contrary that $\theta_1 + ax_1 < p_2$. Substituting $p_2 = \frac{\theta_1 + ax_1}{2}$ yields $\theta_1 + ax_1 < \frac{\theta_1 + ax_1}{2}$, which contradicts. To put it in other words, the monopolist cannot commit to a price sequence, therefore she will always maximise profits in period 2 given the residual demand, which forces her to set $p_2 \leq \theta_1 + ax_1$.

Substituting $x_1 = 1 - \theta_1$ and solving for θ_1 yields

$$\theta_1 = \frac{\delta a - 2p_1}{\delta(a-1) - 2}. \tag{13}$$

Finally, the monopolist sets p_1 to maximise the overall non-rationing profit

$$\pi^{NR1}(p_1) = p_1 \left(1 - \frac{\delta a - 2p_1}{\delta(a-1) - 2}\right) + \frac{\delta}{2}\left((1-a)\frac{\delta a - 2p_1}{\delta(a-1) - 2} + a\right)^2. \tag{14}$$

Maximising $\pi^{NR1}(p_1)$ gives the non-rationing equilibrium in which

$$p_1^{NR1} = \frac{1}{2}\frac{\delta^2(a-1) + \delta(2a^2 - 4) - 4}{\delta(a^2 - 1) - 4}. \tag{15}$$

Substituting p_1^{NR1} in the respective equations yields

$$p_2^{NR1} = \frac{1}{2}\frac{\delta(a-1) - 2a - 2}{\delta(a^2 - 1) - 4}, \tag{16}$$

$$\theta_1^{NR1} = \frac{\delta a^2 + \delta a - \delta - 2}{\delta a^2 - \delta - 4}, \tag{17}$$

$$\theta_2^{NR1} = \frac{1}{2}\frac{\delta a - \delta - 2 + 2a + 2\delta a^2}{\delta a^2 - \delta - 4}, \tag{18}$$

$$\pi^{NR1} = \frac{[4 + \delta(4 + 4a + \delta)]}{4\,(4 + \delta - \delta a^2)}. \tag{19}$$

From equation (18) it is now possible to derive the bounds for a such that θ_2 is strictly positive: $a \in [0, \frac{1}{4\delta}\left(\sqrt{(9\delta^2 + 20\delta + 4)} - \delta - 2\right))$. For introductory prices, i.e. $p_1^{NR1} \leq \delta p_2^{NR1}$, the externality should be larger than $\frac{1}{2\delta}\left(\sqrt{(5\delta^2 + 8\delta)} - \delta\right)$. As it can be shown the critical value of the externality is always outside of the bounds which are relevant to this case. Hence, introductory pricing is never an optimal strategy in this case.

The second case we discuss here is the case in which $\theta_2 = 0$. The monopolist finds it optimal to cover the whole market in the second period.

Following the backwards induction reasoning again, we know that in the second period the market size is $x_2 = \theta_1$. Accordingly, the monopolist sets the second period price $p_2 = ax_1$.

Consumers in period 1 decide whether to buy or to wait. They buy if

$$\theta - p_1 + \delta(\theta + ax_1) \geq \delta\theta, \tag{20}$$

and for the marginal consumer in period 1 it has to be true that (substituting $x_1 = 1 - \theta_1$)

$$\theta_1 - p_1 + \delta(\theta_1 + a(1-\theta_1)) = \delta\theta_1. \tag{21}$$

Solving for θ_1 yields

$$\theta_1 = \frac{p_1 - \delta a}{1 - \delta a}. \tag{22}$$

The monopolist maximises overall profits

$$\pi^{NR2}(p_1) = p_1(1 - (\frac{p_1 - \delta a}{1 - \delta a})) + \delta(a(1 - (\frac{p_1 - \delta a}{1 - \delta a})))(\frac{p_1 - \delta a}{1 - \delta a}), \tag{23}$$

which yields

$$p_1^{NR2} = \frac{1}{2}\delta^2 a^2 + \frac{1}{2}. \tag{24}$$

The rest of the solution is:

$$p_2^{NR2} = \frac{1}{2}a\left(1 + \delta a\right), \tag{25}$$

$$\theta_1^{NR2} = \frac{1}{2} - \frac{1}{2}\delta a, \tag{26}$$

$$\pi^{NR2} = \frac{1}{4}\left(\delta a + 1\right)^2. \tag{27}$$

Note that for (26) to be strictly positive the externality must satisfy $a \in [\frac{1}{4\delta}\left(-\delta - 2 + \sqrt{\left(9\delta^2 + 20\delta + 4\right)}\right), \frac{1}{\delta})$. Introductory pricing would be optimal in this case for a value of a larger than $\frac{1}{\delta}$. Once more this value lies outside the relevant range for a and, as in the previous case, introductory pricing cannot be an equilibrium strategy.

Finally, another possibility arises, that is the case when the externality is so large that the monopolist finds it optimal to saturate the market in the first period. When $a \in \left[\frac{1}{\delta}, +\infty\right)$, the monopolist sets the price in such a way as to make the zero valuation consumer indifferent to buying or waiting, hence $p_1^{NR3} = \delta a$ and $\theta_1^{NR3} = 0$. Since then the residual demand in period 2 is zero, any $p_2 \geq 0$ can be part of the subgame perfect equilibrium. Intuitively, though, in order to prevent the marginal consumer in period 1, i.e. the zero-valuation consumer, from postponing consumption to period 2, the monopolist has to set the second period price p_2 such that

$$\theta_1 - p_1 + \delta(\theta_1 + ax_1) \geq \delta(\theta_1 + ax_1 - p_2). \tag{28}$$

For $p_1 = \delta a, x_1 = 1$, and $\theta_1 = 0$ it has to be true that $p_2 \geq a$. Hence, if the monopolist wants to "punish" delayed consumption, she should set

$p_2^{NR3} \geq a$, which of course necessarily leads to introductory pricing. Yet, as already stated, $p_2^{NR3} \geq a$ is not a necessary condition for subgame-perfect equilibrium, any p_2 will do.

In this section we have shown that, apart from the last (degenerate) case in which no sales take place in the second period, under no circumstances would the monopolist find it optimal to charge increasing (discounted) prices over time. This confirms the result obtained by Cabral et al. [2] in their proposition 1, which states that if buyers are small and there is no uncertainty, then, in no subgame-perfect Nash equilibrium can discounted prices be rising between periods in which sales occur.

3 The Rationing Case

As already pointed out in the introduction, in a durable goods monopoly the monopolist faces the competition of her own future output. As a consequence, in the absence of precommitment prices will be lower than the static monopoly ones and the static monopoly profit cannot be sustained. Denicolò and Garella [4] contend that this problem could be alleviated by rationing demand. In a two period durable goods model, consumers who have been rationed in the first period - i.e. they were willing to buy at the given price but they were not served - will carry their demand over to the next period. Thus the demand functions in the second period will be increased[8], reducing the monopolist's incentive to cut second period prices. Rational costumers will realise this change in the incentives of the monopolist and respond by postponing less purchases, thus improving the first period demand. The monopolist would be in a position to better discriminate between high-valuation and low-valuation consumers, and total profit may rise. It is clear that in the second and final period the monopolist has nothing to gain from rationing: Rationing can only be profitable in the first period, when it can affect the future price. Denicolò and Garella in their 1999 paper [4] show that under appropriate conditions it pays the monopolist to ration demand in the first period, making use of a proportional rationing scheme.

Coming back to our model, we feel that given the result of Denicolò and Garella the monopolist might have high incentives to ration period 1 consumers. Indeed, in actual markets it is common practice for software companies to release a limited amount of free, but not fully tested versions of their new products (the so-called beta-versions), in order to benefit from the externality generated by user-based bug-fixing. We want to investigate

[8]Indeed the demand function will be kinked as shown by Denicolò and Garella [4]. In our framework, the lagged network externality will also shift the demand function upwards.

whether or not our model supports rationing, i.e. a strategy in which the monopolist sets a low period 1 price p_1, but does not serve all the consumers willing to buy at that price. This strategy would allow the monopolist to create the desired externality, but would leave a larger number of high valuation consumers for period 2 consumption, therefore reducing intertemporal competition.

We start by introducing some terminology. A rationing scheme is a function $\gamma(\theta) : \Theta \rightarrow [0, 1]$ (where Θ is the interval of definition of the willingness to pay θ) that determines the proportion of buyers of type θ not served in period 1. Proportional rationing implies that $\gamma(\theta)$ is a constant function in the interval $\theta \geq \theta_1$. A (time-consistent) rationing strategy is a price sequence (p_1, p_2) and a rationing function $\gamma(\theta)$ with $\int_{\theta_1}^{1} \gamma(\theta) dF(\theta) > 0$, where p_2 is determined in the second period to maximise second-period profit. A rationing equilibrium is an optimal rationing strategy that profit-dominates the equilibrium with no rationing.

Denicolò and Garella [4] show that under proportional rationing, for γ not too small, rationing can be an equilibrium strategy when the discount factor is large enough. This result is the object of their proposition 2. In what follows, our aim is to extend the framework of Denicolò and Garella to include lagged network externalities and investigate whether their result goes through in the modified setup. The model of this section differs from the one in the previous section in that the monopolist now sets the rationing parameter γ together with price p_1 at the beginning of period 1. That is, she decides what fraction γ of all the consumers willing to buy the good at price p_1 she will not serve. Note that, in contrast to Denicolò and Garella, we do not consider an exogenously given γ, but we make γ - i.e. the scale of rationing - part of the monopolist's decision.

3.1 Analysis

We now want to proceed with the analysis of the rationing case. We assume that rationing is proportional, that is the monopolist only serves a fraction $1 - \gamma$ of all the consumers who are willing to buy in period 1. Hence, sales in the first period are

$$x_1^R = (1 - \theta_1)(1 - \gamma). \tag{29}$$

We start by showing that, when the monopolist chooses prices, and possibly also γ, such that $\theta_1 \geq \theta_2$, rationing cannot pay. The argument follows proposition 1 of [4]: The optimal sales in the rationing case can also be achieved through non-rationing, but at a strictly higher profit. Consider the optimal sales x_1^R and x_2^R. Since $\gamma > 0$, the monopolist has to charge a price

p_1^R that is strictly lower than the price she would charge without rationing, in order to achieve the same amount of sales, so $p_1^{NR} > p_1^R$ and therefore $\theta_1^{NR} > \theta_1^R$. To achieve the same volume of sales x_2^R also in the second period, we simply set $p_2^{NR} = p_2^R$, or equivalently $\theta_2^{NR} = \theta_2^R$, which we can do since we hold x_1 constant for both rationing and non-rationing. But then of course, overall profits are higher when not rationing.

Therefore we only consider the complementary case when the monopolist sets a price sequence such that $\theta_1 < \theta_2$. Accordingly, sales in the second period are

$$x_2^R = (1-\theta_2)\gamma. \tag{30}$$

With $\gamma \geq 0$, the overall profit is

$$\pi^R = (1-\theta_1)(1-\gamma)p_1^R + \delta\gamma p_2^R(1-\theta_2). \tag{31}$$

In order to make the marginal consumers of period 1 and 2 indifferent we set

$$p_1^R = \theta_1 + \delta(\theta_1 + ax_1^R), \tag{32}$$

and

$$p_2^R = \theta_2 + ax_1^R. \tag{33}$$

Combining yields

$$\begin{aligned} \pi^R \quad = \quad & (1-\theta_1)(1-\gamma)(\theta_1 + \delta(\theta_1 + a(1-\theta_1)(1-\gamma))) + \\ & +\delta\gamma(\theta_2 + a(1-\theta_1)(1-\gamma))(1-\theta_2). \end{aligned} \tag{34}$$

As in the previous section we look for subgame-perfect equilibria. The monopolist maximises second period profit,

$$\delta\gamma(\theta_2 + a(1-\theta_1)(1-\gamma))(1-\theta_2), \tag{35}$$

over θ_2, taking γ and θ_1 as given, which yields

$$\theta_2^R = \begin{cases} \frac{1-a(\gamma-1)(\theta_1-1)}{2} & \text{if } a < \frac{1-2\theta_1}{(\gamma-1)(\theta_1-1)} \\ \theta_1 & \text{if } a \geq \frac{1-2\theta_1}{(\gamma-1)(\theta_1-1)} \end{cases}. \tag{36}$$

That is, we find an upper bound for a such that when a is within this bound, $\theta_2 > \theta_1$. Note also that in this case θ_2^R is bounded by $\frac{1}{2} \geq \theta_2^R$.

In case that a exceeds the bound given in (36), $\theta_2^R = \theta_1$ and rationing cannot be optimal as we have already shown above. Hence, we only consider the case when $a < \frac{1-2\theta_1}{(\gamma-1)(\theta_1-1)}$ and $\theta_2^R = \frac{1-a(\gamma-1)(\theta_1-1)}{2}$. The corresponding profit-maximising θ_1 is

$$\theta_1^R = \begin{cases} \frac{\delta a\gamma(a\gamma-a+3)+\delta(2-4a)+2}{\delta a\gamma(a\gamma+4-a)+\delta(4-4a)+4} & \text{if } a < \frac{\delta(4-3\gamma)-\sqrt{\delta(\gamma^2\delta-16\gamma\delta+16\delta+8\gamma-8\gamma^2)}}{2\gamma\delta(\gamma-1)} \\ 0 & \text{if } a \geq \frac{\delta(4-3\gamma)-\sqrt{\delta(\gamma^2\delta-16\gamma\delta+16\delta+8\gamma-8\gamma^2)}}{2\gamma\delta(\gamma-1)} \end{cases}. \quad (37)$$

Let us start by considering the latter case first. When $\theta_1 = 0$, the corresponding θ_2^R is

$$\theta_2^R = \frac{1}{2} + \frac{1}{2}a\gamma - \frac{1}{2}a. \quad (38)$$

Since $\theta_2 > \theta_1 = 0$ we immediately obtain an upper bound $a < \frac{1}{1-\gamma}$ for this case. Substituting (38) in the profit function (34) and maximising over γ yields

$$\gamma = \begin{cases} \frac{2a-2-\sqrt{(1+10a+a^2)}}{3a} & \text{if } a > 3+2\sqrt{2} \\ 0 & \text{if } a \leq 3+2\sqrt{2} \end{cases}. \quad (39)$$

Replacing $\gamma = \frac{2a-2-\sqrt{(1+10a+a^2)}}{3a}$ in the bound $a < \frac{1}{1-\gamma}$ yields, after some rearranging,

$$a \leq 1 - \sqrt{(1+10a+a^2)}. \quad (40)$$

Obviously, a cannot exceed 1 according to (40). But (39) requires $a > 3+2\sqrt{2}$ for $\gamma > 0$, which contradicts. Therefore, there exists no subgame-perfect equilibrium with $\gamma > 0$ in the case when $\theta_2 > \theta_1 = 0$.

It remains to analyse the case of $\theta_2 > \theta_1 > 0$. Then the profit-maximising γ, if $a > \frac{2(\delta-\sqrt{\delta^2+\delta})(\delta+1)}{\delta(\delta-1)}$, is

$$\gamma = \frac{\delta(a(4\delta-2-2\delta^2)-8(1+\delta))+2\sqrt{\delta(\delta^5a^2-\delta^4(4a+a^2)+\delta^3(4-4a-a^2)+\delta^2(12+a^2+4a)+12\delta+4\delta a+4)}}{2a\delta(3\delta-1)},$$

and 0 otherwise.

Consider now the upper bound $a < \frac{1-2\theta_1}{\gamma\theta_1+1-\gamma-\theta_1}$ for $\theta_2 > \theta_1$. Substituting θ_1^R and rearranging yields

$$1 < \delta + \gamma. \quad (41)$$

Substituting γ into the right hand side of this inequality yields an expression that turns out to be always less than 1, which contradicts again. Hence, also in this case there is no subgame-perfect equilibrium involving a strictly positive γ. This completes the analysis with the result that - given our utility functions and given an endogenous γ - there cannot exist any rationing equilibrium.

4 Conclusions

In the present paper we tackled two issues in the framework of a software monopoly with lagged network externalities: We first confronted the possibility of the existence of an equilibrium featuring introductory prices and subsequently turned our attention to the issue of proportional rationing as an equilibrium strategy.

In the first part of the paper we show that introductory pricing does not, in general, obtain. Our conclusions are in line with the results in the first part of Cabral et al. [2], who show that, under perfect information, unless consumers are "large", no subgame-perfect equilibrium can obtain in which prices rise between periods in which sales occur. In our framework the monopolist faces a continuum of consumers and information is perfect, so this result appears natural. Yet we also show that for an externality large enough the subgame-perfect price sequence leads to increasing discounted prices over time. Such introductory pricing can only occur, though, when all the consumers already buy in period 1, so the residual period 2 demand is zero. In this case the externality created by consumers is so extensive that even the consumer who would have no valuation for the good would still like to buy immediately, foreseeing the immense improvement of the product in period 2. This result is, anyway, an artifact of our model and is driven by our assumption concerning the finite domain in the distribution of θ. This corner solution would disappear if we were to assume a distribution of θ defined over the $(-\infty, +\infty)$ interval.

In the second part of the paper we show that in our model there is no subgame-perfect equilibrium that involves rationing. The reason behind this result is that the monopolist can always increase her profits by rationing less; in fact the monopolist's profit is always maximised at $\gamma = 0$, i.e. when not rationing. Since in our model the scale of rationing is an explicit choice variable for the monopolist, she cannot credibly commit to rationing. In a certain sense, introducing rationing into the model moves the monopolist's problem of credible threats from period 2 prices to period 1 rationing. In the standard durable goods problem, the monopolist always faces a residual demand which she seeks to satisfy, selling to low valuation consumers at a low price in period 2. As a consequence she cannot credibly commit to a high period 2 price. When rationing, though, she could credibly commit to a high period 2 price, if only she were able to commit to rationing in period 1! Yet, as she can always increase her profit by rationing less, she is always tempted to sell to all the people willing to buy in period 1, up to the point when she is not rationing any more. But then, of course, we are back to the non-rationing case.

This result hinges crucially on the incorporation of the rationing parameter into the monopolist's decision, but neither on the lagged externality nor on the specific form of the payoff functions. It is straightforward to see that also in a standard durable goods monopoly, as in Denicolò and Garella [4], once γ becomes part of the monopolist's choice variables, the same result obtains: The monopolist's profit under rationing is maximised when $\gamma = 0$. In contrast to our model, Denicolò and Garella derive a necessary condition for the rationing parameter γ such that i) rationing profit-dominates non-rationing; and ii) the monopolist has no incentives to set p_2 such that $\theta_2 < \theta_1$. The monopolist's decision is then to decide between not rationing and rationing at a given scale. They show that for a discount factor large enough the monopolist will choose to ration in equilibrium. An exogenously given γ, though, completely ignores the monopolist's incentives to reduce rationing in the first period to increase her profits.

Naturally the question arises if such a γ under which rationing profit-dominates non-rationing exists in our model. We were not able to find such a γ, but we could not derive this result analytically due to the complexity of the model. So this result constitutes a conjecture, and it stems from the time-of-purchase depending payoff functions[9]. Hence, in our model not only is rationing not subgame-perfect, in fact rationing never pays the monopolist in the sense of Denicolò and Garella. The reason for this becomes clear when reconsidering payoff functions (1) and (2). By postponing consumers' purchases from period 1 to period 2 through rationing, consumers lose their first-period valuation for the good. The monopolist, then, cannot charge for this lost valuation, hence she loses some of the consumer's willingness-to-pay for the good. Consequently, she has to considerably cut down the price in period 2. The monopolist can obviously not recoup this loss by reducing intertemporal competition.

The model we presented is in many aspects a very simple one, as we were driven in our modelling choices by the willingness to obtain tractable closed form solutions. Several extensions are, of course, possible. In the paper we already suggested that the first and most straightforward extensions should certainly concern the domain of the distribution of consumers and a more general, possibly consumer-depending, functional form for the lagged externality. These two changes would increase the degree of generality of the model while further specialising it to better fit the software market's features.

At the end of this paper, we can say that the question of why we see so

[9] Calculating the model with the standard durable goods payoff functions plus a lagged externality, we were able to find such a γ, so the result is due to the alternative payoff functions and not to the externality.

much free software in the markets remains open. A large body of literature is forming on this issue and more generally on the behaviour of monopolists operating in markets characterised by fast obsolescence of "durable" goods, and many interesting, alternative answers are currently under investigation. Yet, we feel that we have indeed made a contribution, as we believe that the results discussed in this paper imply that, whatever the answer to that question, neither introductory pricing nor quantity rationing are the right candidates.

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