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Time-Varying Uncertainty and the Credit Channel

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Founded in 1963 by two prominent Austrians living in exile – the sociologist Paul F. Lazarsfeld and the economist Oskar Morgenstern – with the financial support from the Ford Foundation, the Austrian Federal Ministry of Education and the City of Vienna, the Institute for Advanced Studies (IHS) is the first institution for postgraduate education and research in economics and the social sciences in Austria. The **Economics Series** presents research done at the Department of Economics and Finance and aims to share "work in progress" in a timely way before formal publication. As usual, authors bear full responsibility for the content of their contributions.

Das Institut für Höhere Studien (IHS) wurde im Jahr 1963 von zwei prominenten Exilösterreichern – dem Soziologen Paul F. Lazarsfeld und dem Ökonomen Oskar Morgenstern – mit Hilfe der Ford-Stiftung, des Österreichischen Bundesministeriums für Unterricht und der Stadt Wien gegründet und ist somit die erste nachuniversitäre Lehr- und Forschungsstätte für die Sozial- und Wirtschafts-wissenschaften in Österreich. Die **Reihe Ökonomie** bietet Einblick in die Forschungsarbeit der Abteilung für Ökonomie und Finanzwirtschaft und verfolgt das Ziel, abteilungsinterne Diskussionsbeiträge einer breiteren fachinternen Öffentlichkeit zugänglich zu machen. Die inhaltliche Verantwortung für die veröffentlichten Beiträge liegt bei den Autoren und Autorinnen.

Abstract

We extend the Carlstrom and Fuerst (1997) agency cost model of business cycles by including time varying uncertainty in the technology shocks that affect capital production. We first demonstrate that standard linearization methods can be used to solve the model yet second moment effects still influence equilibrium characteristics. The effects of the persistence of uncertainty are then analyzed. Our primary findings fall into three broad categories. First, it is demonstrated that uncertainty affects the level of the steady-state of the economy so that welfare analyses of uncertainty that focus entirely on the variability of output (consumption) will understate the true costs of uncertainty. A second key result is that time varying uncertainty results in countercyclical bankruptcy rates – a finding which is consistent with the data and opposite the result in Carlstrom and Fuerst. Third, we show that persistence of uncertainty affects both quantitatively and qualitatively the behavior of the economy.

Keywords

Agency costs, credit channel, time-varying uncertainty

JEL Classifications

E2, E4, E5

Comments
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1 Introduction

Business cycle research has long associated risk with cyclical movements in the economy. It is therefore somewhat surprising that modern macroeconomics has, until quite recently, ignored the interaction between uncertainty and cyclical dynamics. This lack of study did not, however, reflect an inherent lack of interest, but was instead driven by primarily technological constraints: the typical solution method used in solving calibrated business cycle models imposes certainty equivalence so that the influence of uncertainty (i.e. second moment effects) are ruled out by construction.

One of the first models in which the variance of shocks played a prominent role was that developed by Obstfeld and Rogoff (2000). While this analysis provided important insights into the effects of uncertainty (identified with the variance of money growth and productivity shocks), the setting abstracted from capital accumulation so that the impact of risk on investment could not be studied. Only very recently, have solution methods for quantitative macroeconomic models (i.e. models in which the parameters can be calibrated to key features of the economy) been developed that do not impose certainty equivalence. In particular, Collard and Juillard (forthcoming), Sims (2001) and Grohe-Schmitt and Uribe (2001) present different solution techniques in which the second moments influence equilibrium dynamics.

The analysis presented here continues the emphasis on second moments but departs from previous work by conducting the analysis within a setting in which information and uncertainty play prominent roles in equilibrium characteristics. Specifically, we use the business cycle model developed by Carlstrom and Fuerst (1997). This model is particularly

attractive for our purposes since it incorporates a lending channel (for investment) that is characterized by asymmetric information between lenders and borrowers. In addition, this model is a variant of a typical real business cycle model so that key parameters can be calibrated to the data. Within this setting, we model time varying uncertainty as a mean preserving spread in the distribution of the technology shocks affecting capital production and explore how changes in uncertainty affect equilibrium characteristics.

We first demonstrate that linearization solution methods can be employed yet this does not eliminate the effects of second moments. That is, in solving for the linear equilibrium policy functions, the vector of state variables includes the variance of technology shocks buffeting the capital production sector. We then examine how the persistence of a change in the variance influences equilibrium characteristics. One of the primary findings is that time varying uncertainty produces countercyclical bankruptcy rates. In contrast, Carlstrom and Fuerst's (1997) analysis of aggregate technology shocks produced the counterfactual prediction of procyclical bankruptcy rates. Hence, the analysis presented here demonstrates that second moment effects, not surprisingly, expand the set of equilibrium characteristics; moreover, in some instances, first and second moment effects move in opposite directions. This may have important consequences for understanding historical business cycles. That is, historical business cycles can be differentiated by whether the shocks are predominantly to aggregate supply or aggregate demand. Our analysis suggests that another useful distinction may be the role of information; namely, is it the first or second moments of the shocks hitting the economy that is dominant in influencing equilibrium characteristics.

The next section presents the model while the following section discusses equilibrium characteristics. The final section offers some concluding comments.

2 Model

We employ the agency cost business cycle model of Carlstrom and Fuerst (1997) to address the financial intermediaries' role in the propagation of productivity shocks and extend their analysis by introducing time-varying uncertainty. Since, for the most part, the model is identical to that in Carlstrom and Fuerst, the exposition of the model will be brief.

The model is a variant of a standard RBC model in which an additional production sector is added. This sector produces capital using a technology which transforms investment into capital. In a standard RBC framework, this conversion is always one-to-one; in the Carlstrom and Fuerst framework, the production technology is subject to technology shocks. (The aggregate production technology is also subject to technology shocks as is standard.) This capital production sector is owned by entrepreneurs who finance their production via loans from a risk neutral financial intermediation sector - this lending channel is characterized by a loan contract with a fixed interest rate. (Both capital production and the loans are intra-period.) If a capital producing firm realizes a low technology shock, it will declare bankruptcy and the financial intermediary will take over production; this activity is subject to monitoring costs. With this brief description, we now turn to an explicit characterization of the economy.

2.1 Households

The representative household is infinitely lived and has expected utility over consumption c_t and leisure $1 - l_t$ with functional form given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln (c_t) + \nu (1 - l_t) \right] \tag{1}$$

where E_0 denotes the conditional expectation operator on time zero information, $\beta \in (0, 1)$, $\nu > 0$, and l_t is time t labor. The household supplies labor, l_t, and rents its accumulated capital stock, k_t, to firms at the market clearing real wage, w_t, and rental rate r_t, respectively, thus earning a total income of $w_t l_t + r_t k_t$. The household then purchases consumption good from firms at price of one (i.e. consumption is the numeraire), and purchases new capital, i_t, at a price of q_t. Consequently, his budget constraint is

$$w_t l_t + r_t k_t \geq c_t + q_t i_t \tag{2}$$

The law of motion for households' capital stock is standard:

$$k_{t+1} = (1 - \delta) k_t + i_t \tag{3}$$

where $\delta \in (0, 1)$ is the depreciation rate on capital.

The necessary conditions associated with the maximization problem include the standard labor-leisure condition and the intertemporal efficiency condition associated with investment. Given the functional form for preferences, these are:

$$\nu c_t = w_t \tag{4}$$

$$\frac{q_t}{c_t} = \beta E_t \left[\frac{q_{t+1}(1-\delta) + r_{t+1}}{c_{t+1}} \right] \tag{5}$$

2.2 Firms

The economy's output of the consumption good is produced by firms using Cobb-Douglas technology[1]

$$Y_t = \theta_t K_t^{\alpha_K} H_t^{\alpha_H} (H_t^e)^{\alpha_{H^e}} \tag{6}$$

where Y_t represents the aggregate output, θ_t denotes the aggregate technology shock, K_t denotes the aggregate capital stock, H_t denotes the aggregate household labor supply, H_t^e denotes the aggregate supply of entrepreneurial labor, and $\alpha_K + \alpha_H + \alpha_{H^e} = 1$.[2]

The profit maximizing representative firm's first order conditions are given by the factor market's condition that wage and rental rates are equal to their respective marginal productivities:

$$w_t = \theta_t \alpha_H \frac{Y_t}{H_t} \tag{7}$$

$$r_t = \theta_t \alpha_K \frac{Y_t}{K_t} \tag{8}$$

$$w_t^e = \theta_t \alpha_{H^e} \frac{Y_t}{H_t^e} \tag{9}$$

where w_t^e denotes the wage rate for entrepreneurial labor.

[1] Note that we denote aggregate variables with upper case while lower case represents per-capita values. Prices are also lower case.

[2] As in Carlstrom and Fuerst, we assume that the entrepreneur's labor share is small, in particular, $\alpha_{H^e} = 0.0001$. The inclusion of entrepreneurs' labor into the aggregate production function serves as a technical device so that entrepreneurs' net worth is always positive, even when insolvent.

2.3 Entrepreneurs

A risk neutral representative entrepreneur's course of action is as follows. To finance his project at period t, he borrows resources from the Capital Mutual Fund according to an optimal financial contract. The entire borrowed resources, along with his total net worth at period t, are then invested into his capital creation project. If the representative entrepreneur is solvent after observing his own technology shock, he then makes his consumption decision; otherwise, he declares bankruptcy and production is monitored (at a cost) by the Capital Mutual Fund.

2.4 Optimal Financial Contract

The optimal financial contract between entrepreneur and the Capital Mutual Fund is described by Carlstrom and Fuerst (1997). But for expository purposes as well as to explain our approach in addressing the second moment effect on equilibrium conditions, we briefly outline the model.

The entrepreneur has access to a stochastic technology that transforms i_t units of consumption into $\omega_t i_t$ units of capital. Unlike Carlstrom and Fuerst (1997), who let the technology shock ω_t to be $i.i.d.$ with a $c.d.f.$ of $\Phi(\omega)$ and a $p.d.f.$ of $\phi(\omega)$ that has a mean of one with a constant standard deviation, we let the standard deviation vary over time. More specifically, we assume that the standard deviation of the capital production technology shock is governed by the following AR(1) process

$$\sigma_{\omega,t} = \bar{\sigma}_{\omega}^{1-\zeta} \sigma_{\omega,t-1}^{\zeta} \mu_t \tag{10}$$

where $\zeta \in (0,1)$ and $\mu_t \sim i.i.d$ with a mean of unity. The unconditional mean of the standard deviation is given by $\bar{\sigma}_\omega$. The realization of ω_t is privately observed by entrepreneur – banks can observe the realization at a cost of μi_t units of consumption.

The entrepreneur enters period t with one unit of labor endowment and z_t units of capital. Labor is supplied inelastically while capital is rented to firms, hence income in the period is $w_t + r_t z_t$. This income along with remaining capital determines net worth (denominated in units of consumption) at time t:

$$n_t = w_t + z_t \left(r_t + q_t \left(1 - \delta\right)\right) \tag{11}$$

With a positive net worth, the entrepreneur borrows $(i_t - n_t)$ consumption goods and agrees to pay back $\left(1 + r^k\right)(i_t - n_t)$ capital goods to the lender, where r^k is the interest rate on loans. Thus, the entrepreneur defaults on the loan if his realization of output is less then the re-payment, i.e.

$$\omega_t < \frac{\left(1 + r^k\right)(i_t - n_t)}{i_t} \equiv \bar{\omega}_t \tag{12}$$

The optimal borrowing contract is given by the pair $(i, \bar{\omega})$ that maximizes entrepreneur's return subject to the lender's willingness to participate (all rents go to the entrepreneur)[3] :

$$\max_{\{i,\bar{\omega}\}} qif\left(\bar{\omega}\right) \text{ subject to } qig\left(\bar{\omega}\right) \geq (i - n)$$

where

$$f\left(\bar{\omega}\right) = \left[\int_{\bar{\omega}}^{\infty} \omega \phi\left(\omega\right) d\omega - \left[1 - \Phi\left(\bar{\omega}\right)\right] \bar{\omega}\right]$$

[3] For convenience, all time subscripts are ignored.

which can be interpreted as the fraction of the expected net capital output received by the entrepreneur[4] ,

$$g(\bar{\omega}) = \left[\int_0^{\bar{\omega}} \omega\phi(\omega)\, d\omega + [1 - \Phi(\bar{\omega})]\,\bar{\omega} - \Phi(\bar{\omega})\,\mu\right]$$

which represents the lender's fraction of expected capital output, $\Phi(\bar{\omega})$ is the bankruptcy rate so that $\Phi(\bar{\omega})\,\mu$ denotes monitoring costs. Also note that $f(\bar{\omega}) + g(\bar{\omega}) = 1 - \Phi(\bar{\omega})\,\mu$: the RHS is the average amount of capital that is produced – this is split between entrepreneurs and lenders. Hence the presence of monitoring costs reduces net capital production.[5]

The necessary conditions for the optimal contract problem are

$$\begin{aligned}
\frac{\partial(.)}{\partial\bar{\omega}} \quad &: \quad qif'(\bar{\omega}) = -\lambda ig'(\bar{\omega}) \\
&\Rightarrow \quad \lambda = \frac{-f'(\bar{\omega})}{g'(\bar{\omega})} \\
\lambda \quad &= \quad \frac{f'(\bar{\omega})}{\phi(\bar{\omega})\,\mu + f'(\bar{\omega})} \\
\lambda \quad &= \quad \frac{1 - \Phi(\bar{\omega})}{1 - \Phi(\bar{\omega}) - \phi(\bar{\omega})\,\mu}
\end{aligned}$$

where λ is the shadow price of capital,[6] and

$$\frac{\partial(.)}{\partial i} : qf(\bar{\omega}) = -\lambda\,[1 - qg(\bar{\omega})]$$

[4] The deriviative of this function is $f'(\bar{\omega}) = \Phi(\bar{\omega}) - 1$. Thus, as $\Phi(\bar{\omega}) \in [0, 1]$, we have $f'(\bar{\omega}) \leq 0$. That is, as the lower bound for the realization of the technology shock (or the cutoff bankruptcy rate) increases, the entrepreneur's output share goes down.

[5] This suggests that monitoring costs are akin to investment adjustment costs - in fact, Carlstrom and Fuerst demonstrate that this is the case. The important difference between this model and a model with adjustment costs is that entrepreneurs' net worth is an endogenous state variable that affects the dynamics of the economy - this feature is not present in an adjustment cost model.

[6] Note that in the absence of monitoring costs, $\lambda = 1$ - the shadow price just covers the cost of capital production

Solving for q using the first order conditions, we have

$$\begin{aligned} q &= \left[\left(f\left(\bar{\omega}\right) + g\left(\bar{\omega}\right) \right) + \frac{\phi\left(\bar{\omega}\right)\mu f\left(\bar{\omega}\right)}{f'\left(\bar{\omega}\right)} \right]^{-1} \\ &= \left[1 - \Phi\left(\bar{\omega}\right)\mu + \frac{\phi\left(\bar{\omega}\right)\mu f\left(\bar{\omega}\right)}{f'\left(\bar{\omega}\right)} \right]^{-1} \\ &\equiv \left[1 - D\left(\bar{\omega}\right) \right]^{-1} \end{aligned} \tag{13}$$

where $D\left(\bar{\omega}\right)$ can be thought of as the total default costs.

Equation (13) defines an implicit function $\bar{\omega}\left(q\right)$ that is increasing in q, or the price of capital that incorporates the expected bankruptcy costs. The price of capital, q, differs from unity due to the presence of the credit market friction. That is, to compensate for the bankruptcy (monitoring) costs, there must be a premium on the price of capital. And this premium is set by the amount of monitoring costs and the probability of bankruptcy. (Note that $f'\left(\bar{\omega}\right) = \Phi\left(\bar{\omega}\right) - 1 < 0$.)

Finally, the incentive compatibility constraint implies

$$i = \frac{1}{\left(1 - qg\left(\bar{\omega}\right)\right)} n \tag{14}$$

Equation (14) implies that investment is linear in net worth and defines a function that represents the amount of consumption goods placed in to the capital technology: $i\left(q, n\right)$. The fact that the function is linear implies that the aggregate investment function is well defined.

2.5 Entrepreneur's Consumption Choice

To rule out self-financing by the entrepreneur (i.e. which would eliminate the presence of agency costs), it is assumed that the entrepreneur discounts the future at a faster rate than

the household. Thus, is represented by entrepreneur maximizes following expected utility function

$$E_0 \sum_{t=0}^{\infty} (\beta\gamma)^t c_t^e \tag{15}$$

where c_t^e denotes entrepreneur's consumption at date t, and $\gamma \in (0,1)$. This new parameter, γ, will be chosen so that it offsets the steady-state internal rate of return to entrepreneurs' investment.

At the end of the period, the entrepreneur finances consumption out of the returns from the investment project implying that the law of motion for the entrepreneur's capital stock is:

$$z_{t+1} = n_t \left\{ \frac{f(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)} \right\} - \frac{c_t^e}{q_t} \tag{16}$$

Note that the expected return to internal fund is $\frac{q_t f(\bar{\omega}_t) i_t}{n_t}$; that is, the net worth of size n_t is leveraged into a project of size i_t, entrepreneurs keep the share of the capital produced and capital is priced at q_t consumption goods. Since these are intra-period loans, the opportunity cost is 1.[7]

Consequently, the representative entrepreneur maximizes his expected utility function in equation (15) over consumption and capital subject to the law of motion for capital, equation (16), and the definition of net worth given in equation (11). The resulting Euler equation is as follows:

$$q_t = \beta\gamma E_t \left\{ [q_{t+1}(1-\delta) + r_{t+1}] \left[\frac{q_{t+1} f(\bar{\omega}_{t+1})}{(1 - q_{t+1} g(\bar{\omega}_{t+1}))} \right] \right\}$$

[7] As noted above, we require in steady-state $1 = \gamma \frac{q_t f(\bar{\omega}_t)}{(1 - q_t g(\bar{\omega}_t))}$.

2.6 Financial Intermediaries

The Capital Mutual Funds (CMFs) act as financial intermediaries who earn no profit and produce neither consumption nor capital goods. There is a clear role for the CMF in this economy since, through pooling, all aggregate uncertainty of capital production can be eliminated. The CMF receives capital from three sources: entrepreneurs sell undepreciated capital in advance of the loan, after the loan, the CMF receives the newly created capital through loan repayment and through monitoring of insolvent firms, and, finally, those entrepreneur's that are still solvent, sell some of their capital to the CMF to finance current period consumption. This capital is then sold at the price of q_t units of consumption to households for their investment plans.

2.7 Equilibrium

There are four markets: labor markets for households and entrepreneurs, goods markets for consumption and capital.

$$H_t = (1 - \eta)\, l_t \tag{17}$$

where η denotes the fraction of entrepreneurs in the economy.

$$H_t^e = \eta \tag{18}$$

$$C_t + I_t = Y_t \tag{19}$$

where $C_t = (1 - \eta)\, c_t + \eta c_t^e$ and $I_t = \eta i_t$.

$$K_{t+1} = (1 - \delta)\, K_t + I_t \left[1 - \Phi\left(\bar{\omega}\right) \mu\right] \tag{20}$$

A competitive equilibrium is defined by the decision rules for $\{K_{t+1}, Z_{t+1}, H_t, H_t^e, q_t, n_t, i_t, \bar{\omega}_t, c_t, c_t^e\}$ where these decision rules are stationary functions of $\{K_t, Z_t, \theta_t, \sigma_{\omega,t}\}$ and satisfy the following equations[8]

$$\nu c_t = \theta_t \alpha_H \frac{Y_t}{H_t} \tag{21}$$

$$\frac{q_t}{c_t} = \beta E_t \left[\frac{1}{c_{t+1}} \left(q_{t+1}(1-\delta) + \theta_{t+1}\alpha_K \frac{Y_{t+1}}{K_{t+1}} \right) \right] \tag{22}$$

$$q_t = \left[1 - \Phi(\bar{\omega}_t)\mu + \frac{\phi(\bar{\omega})\mu f(\bar{\omega}_t)}{f'(\bar{\omega}_t)} \right]^{-1} \tag{23}$$

$$i_t = \frac{1}{(1 - q_t g(\bar{\omega}_t))} n_t \tag{24}$$

$$q_t = \beta\gamma E_t \left\{ \left[q_{t+1}(1-\delta) + \theta_{t+1}\alpha_K \frac{Y_{t+1}}{K_{t+1}\,t+1} \right] \left[\frac{q_{t+1} f(\bar{\omega}_{t+1})}{(1 - q_{t+1} g(\bar{\omega}_{t+1}))} \right] \right\} \tag{25}$$

$$n_t = \theta_t \alpha_{H^e} \frac{Y_t}{H_t^e} + z_t \left[q_t(1-\delta) + \theta_t \alpha_K \frac{Y_t}{K_t} \right] \tag{26}$$

$$Z_{t+1} = \eta n_t \left\{ \frac{f(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)} \right\} - \eta \frac{c_t^e}{q_t} \tag{27}$$

$$\theta_{t+1} = \theta_t^{\rho} \xi_{t+1} \text{ where } \xi_t \sim i.i.d. \text{ with } E(\xi_t) = 1 \tag{28}$$

$$\sigma_{\omega,t+1} = \bar{\sigma}_{\omega}^{1-\zeta} \sigma_{\omega,t}^{\zeta} \mu_{t+1} \text{ where } \mu_t \sim i.i.d. \text{ with } E(\mu_t) = 1 \tag{29}$$

3 Equilibrium Characteristics

3.1 Steady-state analysis

While our focus is primarily on the cyclical behavior of the economy, an examination of the steady-state properties of the economy is useful for two reasons. First, by studying the interaction between uncertainty (i.e. the variance of the technology shock affecting the capital production sector) and the steady-state, the intuition for how time-varying uncertainty

[8] A more thorough presentation of the equilibrium conditions are presented in the Appendix.

affects the cyclical characteristics of the economy will be improved. Second, it is important to point out that changes in the second moment of technology shocks affect the level of the economy - most notably consumption and output. That is, since the cyclical analysis presented in the following section is characterized in terms of deviations from steady-state, the impact of changes in uncertainty on the *level* of economic activity is lost.[9]

For this analysis, we use, to a large extent, the parameters employed in Carlstrom and Fuerst's (1997) analysis. Specifically, the following parameter values are used:

Table 1: Parameter Values

β	α	δ	μ
0.99	0.36	0.02	0.25

Agents discount factor, the depreciation rate and capital's share are fairly standard in RBC analysis.[10] The remaining parameter, μ, represents the monitoring costs associated with bankruptcy. This value, as noted by Carlstrom and Fuerst (1997) is relatively prudent given estimates of bankruptcy costs (which range from 20% (Altman (1984) to 36% (Alderson and Betker (1995) of firm assets).

The remaining parameters, (σ, γ), determine the steady-state bankruptcy rate (which we denote as *br* and is expressed in percentage terms) and the risk premium (denoted *rp*)

[9] This statement is in reference to Lucas's analysis of the cost of business cycles (Lucas (1987) in which the trend and cycle are treated as distinct. In contrast, our analysis demonstrates that the cyclical behavior of the economy has implications for the level of the steady-state. If one were using an endogenous growth model, cyclical behavior may well have implications for the trend.

[10] The fraction of households in the economy, η, is purely a normalization and does not influence equilibrium steady-state. See Appendix 1 for details.

associated with bank loans.[11] (Also, recall that γ is calibrated so that the rate of return to internal funds is equal to $\frac{1}{\gamma}$.) While Carlstrom and Fuerst found it useful to use the observed bankruptcy rate to determine σ, for our analysis we treat σ and br as exogenous and examine the steady state behavior of the economy under different scenarios. In particular we consider the following four economies:[12]

Table 2: Four Economies

Economy	$\boldsymbol{\sigma}$	br (%)	$\boldsymbol{\gamma}$
Economy I ($C\&F$)	0.207	0.974	0.9474
Economy II	0.30	0.974	0.9538
Economy III	0.30	1.2	0.9458
Economy IV	0.35	1.8	0.9287

Hence Economy II departs from the Carlstrom and Fuerst economy by having greater uncertainty in the technology shock but holds the bankruptcy rate at the same level used by Carlstrom and Fuerst (note that this implies that the internal rate of return $\left(\frac{1}{\gamma}\right)$to entrepreneurs falls). Economy III then permits the bankruptcy rate to increase by roughly a third. The final economy increases both the degree of technological uncertainty and the steady-state bankruptcy rate. To examine the effects of these changes, Table 3 reports the behavior of several key variables; for all but two variables, these are presented as percentage

[11] The equations defining the steady-state are presented in the Appendix. This derivation also demonstrates that the parameter η (the fraction of entrepreneurs in the economy) is strictly a normalization and does not influence equilibrium characteristics.

[12] In Table 2, the values of γ are reported strictly for comparison. That is, once the values of σ and br are specified, the value of γ is determined endogenously.

deviations from the values in the Carlstrom and Fuerst economy. The risk premium differential is reported as an absolute change while the minimum technology shock defined in the lending contract (i.e. $\bar{\omega}$) in the three modified economies is reported

Table 3: Steady-state behavior

(comparison to Carlstrom & Fuerst Economy)

variable	Economy II	Economy III	Economy IV
c	-0.19	-0.44	-1.18
k	-0.51	-1.13	-3.05
rp	0.54	1.06	2.98
q	0.35	0.78	2.11
$\bar{\omega}$[13]	0.47	0.49	0.45
z	28.4	25.0	28.4
n	28.7	25.7	30.4

Note that increases in uncertainty reduce the steady-state level of consumption and the aggregate capital stock monotonically. For the high variance, high bankruptcy rate economy (Economy IV), the reduction in consumption is greater than 1% - a non-trivial amount and similar in magnitude to welfare losses reported by Lucas for moderate inflations (Lucas (2000)). Clearly, more research is needed to examine the welfare consequences of uncertainty - in particular, one of the lessons of the equity premium puzzle literature is that logarithmic preferences are not consistent with agents' treatment of aggregate risk;

[13] In the Carstrom and Fuerst economy, the minimum technology realization for solvency is: $\bar{\omega} = 0.60$.

introducing habit persistence or other preferences that more accurately represent agents' risk tolerance would be a first step. We leave this to future research.

The risk premium associated with the lending contract as well as the price of capital are also monotonically increasing in the variance and the bankruptcy rate. However, note that this is not the case for the last three variables. In particular, the comparison between Economies II and III shows that holding the variance of the technology shock constant but increasing the steady-state bankruptcy rate results in a fall (again relative to Economy II) in both the entrepreneurs' capital stock (z) and net worth (n). This occurs despite the fact that the price of capital is greater and reflects the impact that the greater bankruptcy rate has on the level of lending in the economy.

We now examine the cyclical behavior of the economy with time-varying uncertainty in the capital production sector.

3.2 Cyclical Behavior

As described in Section 2, eqs. (21)through (29) determine the equilibrium properties of the economy. To analyze the cyclical properties of the economy, we linearize (i.e. take a first-order Taylor series expansion) of these equations around the steady-state values. This numerical approximation method is standard in quantitative macroeconomics. What is not standard in this model is that the second moment of technology shocks hitting the capital production sector will influence equilibrium behavior and, therefore, the equilibrium policy rules. That is, linearizing the equilibrium conditions around the steady-state typically imposes certainty equivalence so that variances do not matter. In this model, however, the

variance of the technology shock can be treated as an additional state variable through its role in determining lending activities and, in particular, the nature of the lending contract. Linearizing the system of equilibrium conditions does not eliminate that role in this economy and, hence, we think that this is an attractive feature of the model.

While the previous section analyzed the steady-state behavior of four different economies, in this section we employ the same parameters as in the Carlstrom and Fuerst model (Economy I in the previous section). We depart from Carlstrom and Fuerst by relaxing the *i.i.d.* assumption for the capital sector technology shock. This is reflected in the law of motion for the standard deviation of the technology shock which is given in eq. (29); for convenience this is rewritten below:

$$\sigma_{\omega,t+1} = \bar{\sigma}_{\omega}^{1-\zeta} \sigma_{\omega,t}^{\zeta} \mu_{t+1}$$

As in Carlstrom and Fuerst, the standard deviation of the technology shock ω_t is, on average, equal to 0.207. That is, we set $\bar{\sigma}_{\omega} = 0.207$. We then examine two different economies characterized by the persistence in uncertainty, i.e. the parameter ζ. In the low persistence economy, we set $\zeta = 0.05$ while in the high persistence economy we set $\zeta = 0.95$. The behavior of these two economies is analyzed by examining the impulse response functions of several key variables to a 1% innovation in σ_{ω}. These are presented in Figures 1-3.

We first turn to aggregate output and household consumption and investment. With greater uncertainty, the bankruptcy rate increases in the economy (this is verified in Figure 2), which implies that agency costs increase. The rate of return on investment for the economy therefore falls. Households, in response, reduce investment and increase consumption

and leisure. The latter response causes output to fall. Note that the consumption and leisure response is increasing in the degree of persistence. This is not the case, however, for investment - this is due to the increase in the price of capital (see Figure 2) and reflects the behavior of entrepreneurs. This behavior is understood after first examining the lending channel.

The increase in uncertainty affects, predictably, all three key variables in the lending channel: the price of capital, the risk premium associated with loans and the bankruptcy rate. As already mentioned, the bankruptcy rate increases and, in the high persistence economy, this increased rate of bankruptcy lasts for several quarters. This result implies that the bankruptcy rate is countercyclical in this economy; in contrast, in the analysis by Carlstrom and Fuerst the bankruptcy rate was, counterfactually, procyclical. Their focus was on the effects of innovation to the aggregate technology shock and, because of the assumed persistence in this shock, is driven by the change in the first moment of the aggregate production shock. Our analysis demonstrates that second moment effects may play a significant role in these correlations over the business cycle. Further research, both empirical and theoretical, in this area would be fruitful . Returning to the model, the increased bankruptcy rate implies that price of capital is greater and this increase lasts longer in the high persistence economy. The same is true for the risk premium on loans.

Figure 3 reports the consumption and net worth of entrepreneurs in the economies. In contrast to all other variables, persistence has a dramatic qualitative effect on entrepreneurs' behavior. With low persistence, entrepreneur's exploit the high price of capital to increase

consumption: the lack persistence provides no incentive to increase investment. Since the price of capital quickly returns to its steady-state values, the increased consumption erodes entrepreneurs' net worth. To restore net worth to its steady-state value, consumption falls temporarily. The behavior in the high persistence economy is quite different: the price of capital is high and forecast to stay high so investment increases dramatically. Initially, the investment is financed by lower consumption, but as entrepreneurs net worth increases (due to greater capital and a higher price of capital) consumption also increases. This endogenous response by entrepreneurs is why, in the high persistence economy, the initial fall in aggregate investment is not as great in the high persistence economy.

Figure 1: Impulse Response Functions
Output, Consumption, Investment

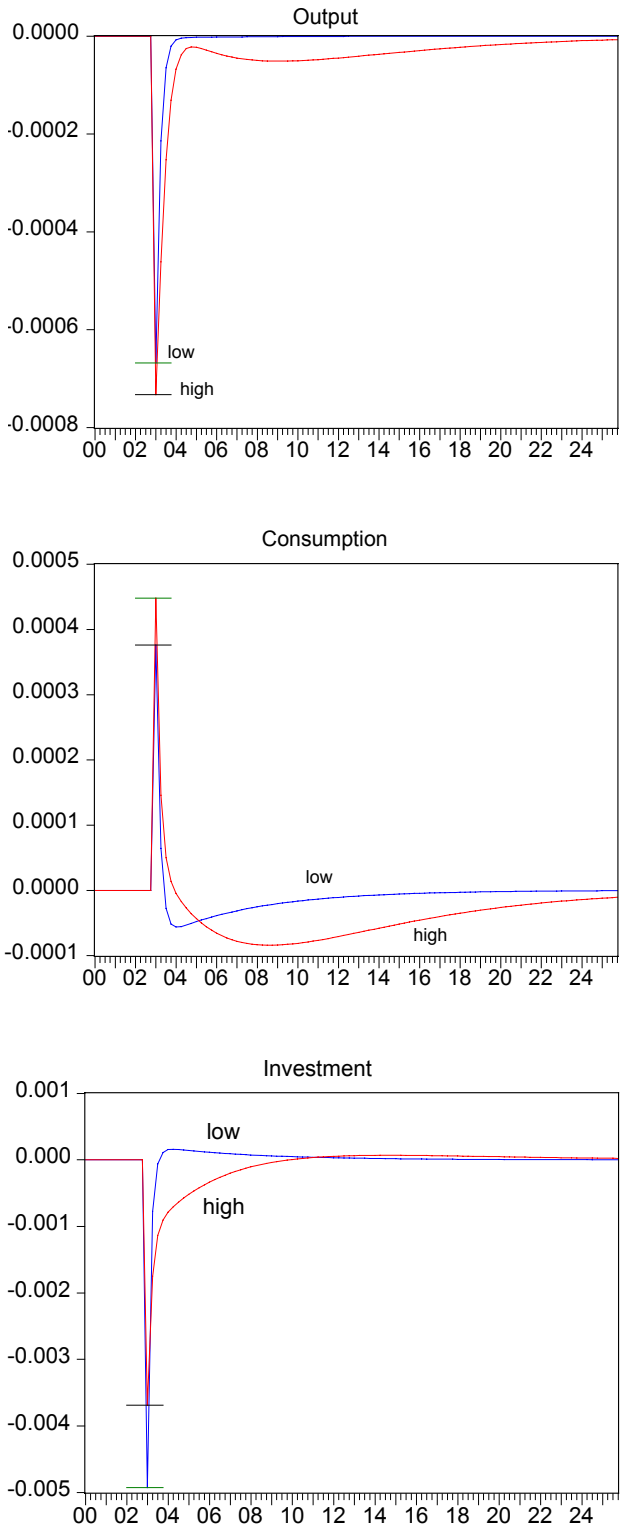


Figure 2: Impulse Response Functions
Price of Capital, Risk Premium, Bankruptcy Rate

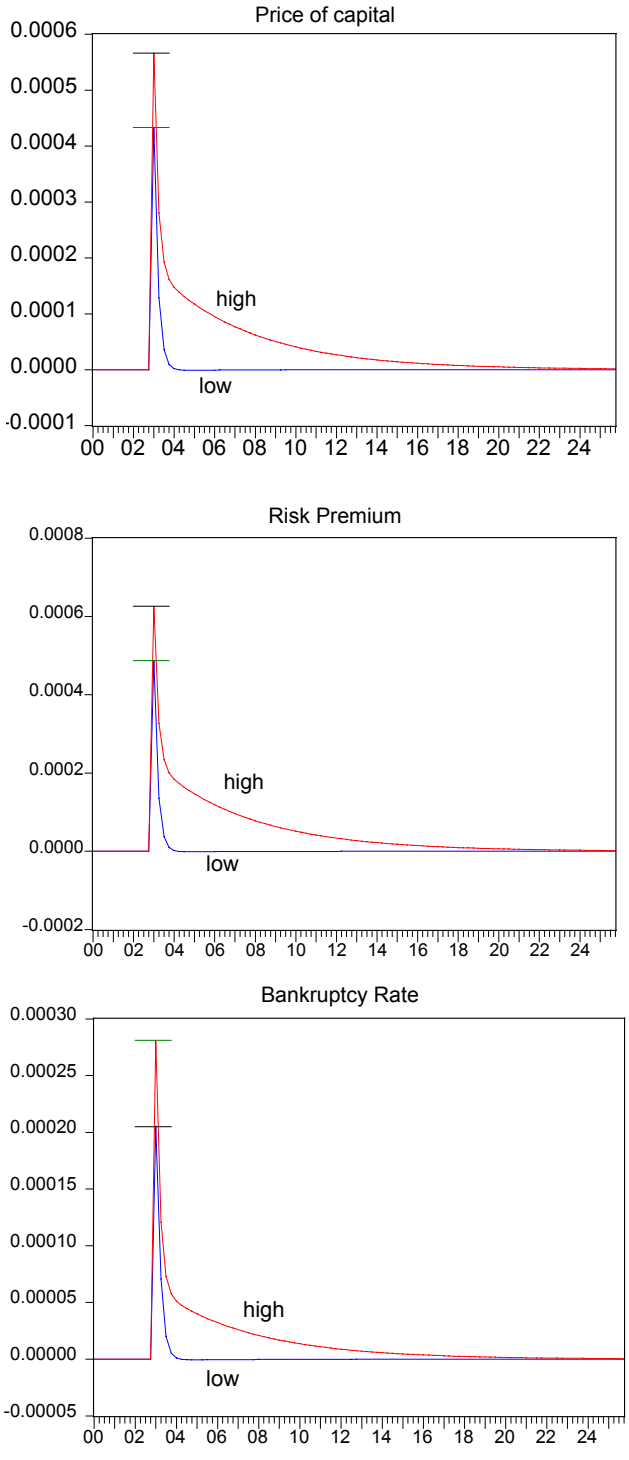
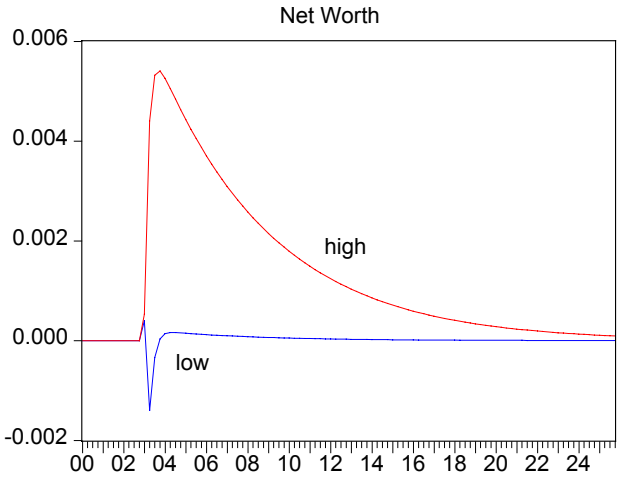
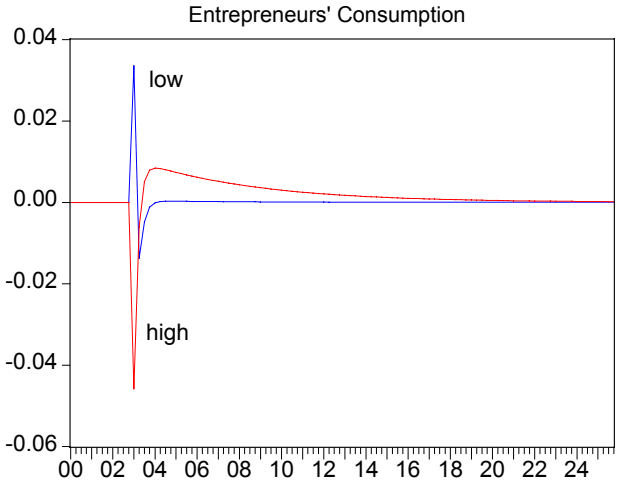


Figure 3: Impulse Response Functions
Entrepreneurs' Consumption, Net Worth





4 Conclusion

The effect of uncertainty as characterized by second moment effects has been largely ignored in quantitative macroeconomics due to the numerical approximation methods typically employed during the computational exercise. The analysis presented here uses standard solution methods (i.e. linearizing around the steady-state) but exploits features of the Carlstrom and Fuerst (1997) agency cost model of business cycles so that time varying uncertainty can be analyzed. While development of more general solution methods that capture second moments effects is encouraged, we think that the intuitive nature of this model and its standard solution method make it an attractive environment to study the effects of time-varying uncertainty.

Our primary findings fall into three broad categories. First, we demonstrate that uncertainty affects the level of the steady-state of the economy so that welfare analyses of uncertainty that focus entirely on the variability of output (or consumption) will understate the true costs of uncertainty. Second, we demonstrate that time varying uncertainty results in countercyclical bankruptcy rates - a finding which is consistent with the data and opposite the result in Carlstrom and Fuerst. Third, we show that persistence of uncertainty affects both quantitatively and qualitatively the behavior of the economy. Together, these results make a strong case for more research into the effects that uncertainty have on economic behavior.

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5 Appendix: Steady-state conditions in the Carlstrom and Fuerst Agency Cost Model

We first present the equilibrium conditions and express these in scaled (by the fraction of entrepreneurs in the economy) terms. Then the equations are analyzed for steady-state implications. As in the text, upper case variables denote aggregate wide while lower case represent household variables. Preferences and technology are:

$$\begin{aligned} U\left(\tilde{c}, 1-l\right) &= \ln \tilde{c} + \nu\left(1-l\right) \\ Y &= \theta K^{\alpha}\left[\left(1-\eta\right) l\right]^{1-\alpha-\phi} \eta^{\phi} \end{aligned}$$

Where η denotes the fraction of entrepreneurs in the economy and θ is the technology shock. Note that aggregate household labor is $L = (1-\eta)\, l$ while entrepreneurs inelastically supply one unit of labor. We assume that the share of entrepreneur's labor is approximately zero so that the production function is simply

$$Y = \theta K^{\alpha}\left[\left(1-\eta\right) l\right]^{1-\alpha}$$

This assumption implies that entrepreneurs receive no wage income (see eq. (9) in C&F.

There are nine equilibrium conditions:

The resource constraint

$$(1-\eta)\,\tilde{c}_t + \eta c_t^e + \eta i_t = Y_t = \theta_t K_t^{\alpha}\left[(1-\eta)\,l_t\right]^{1-\alpha} \tag{30}$$

Let $c = \frac{(1-\eta)\tilde{c}}{\eta}$ and $h = \frac{(1-\eta)}{\eta} l$, then eq(30) can be written as:

$$c_t + c_t^e + i_t = \theta_t k_t^{\alpha} h_t^{1-\alpha} \tag{31}$$

Household's intratemporal efficiency condition

$$\tilde{c}_t = \frac{(1-\alpha)}{\nu} K_t^{\alpha}\left[(1-\eta)\,l_t\right]^{-\alpha}$$

Defining $\nu_0 = \frac{\eta}{1-\eta}\nu$, this can be expressed as:

$$\nu_0 c_t = (1-\alpha)\,k_t^{\alpha} h_t^{-\alpha} \tag{32}$$

Law of motion of aggregate capital stock

$$K_{t+1} = (1-\delta)\,K_t + \eta i_t\left[1 - \Phi\left(\bar{\omega}_t\right)\mu\right]$$

Dividing by η yields the scaled version:

$$k_{t+1} = (1-\delta)\,k_t + i_t\left[1 - \Phi\left(\bar{\omega}_t\right)\mu\right] \tag{33}$$

Household's intertemporal efficiency condition

$$q_t \frac{1}{\tilde{c}_t} = \beta E_t \left\{ \frac{1}{\tilde{c}_{t+1}} \left[q_{t+1} (1-\delta) + \theta_{t+1} \alpha K_{t+1}^{\alpha-1} \left[(1-\eta) l_{t+1} \right]^{1-\alpha} \right] \right\}$$

Dividing both sides by $\frac{1-\eta}{\eta}$ and scaling the inputs by η yields:

$$q_t \frac{1}{c_t} = \beta E_t \left\{ \frac{1}{c_{t+1}} \left[q_{t+1} (1-\delta) + \theta_{t+1} \alpha k_{t+1}^{\alpha-1} h_{t+1}^{1-\alpha} \right] \right\} \tag{34}$$

The conditions from the financial contract are already in scaled form:

Contract efficiency condition

$$q_t = \frac{1}{1 - \Phi(\bar{\omega}_t)\mu + \phi(\bar{\omega}_t)\mu \frac{f(\bar{\omega}_t)}{f'(\bar{\omega}_t)}} \tag{35}$$

Contract incentive compatibility constraint

$$\frac{i_t}{n_t} = \frac{1}{1 - q_t g(\bar{\omega}_t)} \tag{36}$$

Where n_t is entrepreneur's net worth.

Determination of net worth

$$\eta n_t = Z_t \left[q_t (1-\delta) + \theta_t K_t^{\alpha-1} \left[(1-\eta) l_t \right]^{1-\alpha} \right]$$

or, in scaled terms:

$$n_t = z_t \left[q_t (1-\delta) + \theta_t k_t^{\alpha-1} h_t^{1-\alpha} \right] \tag{37}$$

Note that z_t denotes (scaled) entrepreneur's capital.

Law of motion of entrepreneur's capital

$$Z_{t+1} = \eta n_t \left\{ \frac{f(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)} \right\} - \eta \frac{c_t^e}{q_t}$$

Or, dividing by η

$$z_{t+1} = n_t \left\{ \frac{f(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)} \right\} - \frac{c_t^e}{q_t} \tag{38}$$

Entrepreneur's intertemporal efficiency condition

$$q_t = \gamma \beta E_t \left\{ \left[q_{t+1}(1-\delta) + \theta_{t+1} \alpha K_{t+1}^{\alpha-1} \left[(1-\eta) l_{t+1} \right]^{1-\alpha} \right] \left(\frac{q_{t+1} f(\bar{\omega}_{t+1})}{1 - q_{t+1} g(\bar{\omega}_{t+1})} \right) \right\}$$

Or, in scaled terms:

$$q_t = \gamma \beta E_t \left\{ \left[q_{t+1}(1-\delta) + \theta_{t+1} \alpha k_{t+1}^{\alpha-1} h_{t+1}^{1-\alpha} \right] \left(\frac{q_{t+1} f(\bar{\omega}_{t+1})}{1 - q_{t+1} g(\bar{\omega}_{t+1})} \right) \right\} \tag{39}$$

5.1 Definition of Steady-state

Steady-state is defined by time-invariant quantities:

$$c_t = \hat{c}, c_t^e = \hat{c}^e, k_t = \hat{k}, \bar{\omega}_t = \hat{\omega}, h_t = \hat{h}, q_t = \hat{q}, z_t = \hat{z}, n_t = \hat{n}, i_t = \hat{\imath}$$

So there are nine unknowns. While we have nine equilibrium conditions, the two intertemporal efficiency conditions become identical in steady-state since C&F impose the condition that the internal rate of return to entrepreneur is offset by their additional discount factor:

$$\gamma \left(\frac{\hat{q} f(\hat{\omega})}{1 - \hat{q} g(\hat{\omega})} \right) = 1 \tag{40}$$

This results in an indeterminacy - but there is a block recursiveness of the model due to the calibration exercise. In particular, we demonstrate that the risk premium and bankruptcy rate determine $(\hat{\omega}, \sigma)$ - these in turn determine the steady-state price of capital.

From eq.(34) we have:

$$\hat{q} = \frac{\alpha\beta}{1-\beta(1-\delta)}\hat{k}^{\alpha-1}\hat{h}^{1-\alpha} = \frac{\alpha\beta}{1-\beta(1-\delta)}\frac{\hat{y}}{\hat{k}} \tag{41}$$

From eq.(32) we have:

$$\hat{h} = \frac{1-\alpha}{\nu_0}\frac{\hat{k}^{\alpha}\hat{h}^{1-\alpha}}{\hat{c}} = \frac{1-\alpha}{\nu_0}\frac{\hat{y}}{\hat{c}} \tag{42}$$

From eq.(33) we have:

$$\hat{k} = \frac{1-\Phi(\hat{\omega})\mu}{\delta}\hat{\imath} \tag{43}$$

Note that these three equations are normally (i.e. in a typical RBC framework) used to find steady-state $\left(\hat{k}, \hat{h}, \hat{c}\right)$ - because $\hat{q} = 1$. Here since the price of capital is endogenous, we have four unknowns.

From eq. (37) and eq. (34) we have

$$\hat{n} = \hat{z}\left(\hat{q}(1-\delta) + \alpha\frac{\hat{y}}{\hat{k}}\right) = \hat{z}\frac{\hat{q}}{\beta} \tag{44}$$

From eq. (38) and the restriction on the entrepreneur's additional discount factor (eq. (40)), we have

$$\hat{z} = \hat{n}\frac{1}{\hat{q}\gamma} - \frac{\hat{c}^e}{\hat{q}} \tag{45}$$

Combining eqs. (44) and (45) yields:

$$\frac{\hat{c}^e}{\hat{n}} = \frac{1}{\gamma} - \beta \tag{46}$$

We have the two conditions from the financial contract

$$\hat{q} = \frac{1}{1 - \Phi(\hat{\omega})\mu + \phi(\hat{\omega})\mu\frac{f(\hat{\omega})}{f'(\hat{\omega})}} \tag{47}$$

And

$$\hat{\imath} = \frac{1}{1 - \hat{q}(1 - \Phi(\hat{\omega})\mu - f(\hat{\omega}))}\hat{n} \tag{48}$$

Finally, we have the resource constraint:

$$\hat{c} + \hat{c}^e + \hat{\imath} = \hat{k}^{\alpha}\hat{h}^{1-\alpha} \tag{49}$$

The eight equations $(41), (42), (43), (44), (45), (47), (48), (49)$ are insufficient to find the nine unknowns. However, the risk premium, denoted as ζ, is defined by the following

$$\hat{q}\hat{\omega}\frac{\hat{\imath}}{\hat{\imath} - \hat{n}} = \zeta \tag{50}$$

But we also know (from eq.(48) that

$$\frac{\hat{n}}{\hat{\imath}} = 1 - \hat{q}(1 - \Phi(\hat{\omega})\mu - f(\hat{\omega})) = 1 - \hat{q}g(\hat{\omega})$$

Rearranging eq.(50) yields:

$$\frac{\hat{q}\hat{\omega}}{\zeta} = 1 - \frac{\hat{n}}{\hat{\imath}}$$

substituting from the previous expression yields

$$\hat{\omega} = \zeta g(\hat{\omega}) \tag{51}$$

Let κ = bankruptcy rate – this observable also provides another condition on the distribution. That is, we require:

$$\Phi(\hat{\omega}) = \kappa \tag{52}$$

The two equations eq.(51) and eq. (52) can be solved for the two unknowns - $(\hat{\omega}, \sigma)$. By varying the bankruptcy rate and the risk premium, we can determine different levels of uncertainty (σ) and the cutoff point $(\hat{\omega})$.

Note that the price of capital in steady-state, is a function of $(\hat{\omega}, \sigma)$ as determined by eq. (47). The other preference parameter, γ is then determined by eq. (40). Once this is determined, the remaining unknowns: $\left(\hat{c}, \hat{c}^e, \hat{h}, \hat{\imath}, \hat{k}, \hat{z}, \hat{n}\right)$ are determined by eqs. $(41), (42), (43), (44), (46), (48), (49)$.

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