

IHS Economics Series

Working Paper 113

April 2002

# Cross-Ownership Among Firms: Some Determinants of the Separation of Ownership from Control

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## Impressum

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### Title:

Cross-Ownership Among Firms: Some Determinants of the Separation of Ownership from Control

### ISSN: Unspecified

### 2002 Institut für Höhere Studien - Institute for Advanced Studies (IHS)

Josefstädter Straße 39, A-1080 Wien

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Reihe Ökonomie  
Economics Series

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Founded in 1963 by two prominent Austrians living in exile – the sociologist Paul F. Lazarsfeld and the economist Oskar Morgenstern – with the financial support from the Ford Foundation, the Austrian Federal Ministry of Education and the City of Vienna, the Institute for Advanced Studies (IHS) is the first institution for postgraduate education and research in economics and the social sciences in Austria. The **Economics Series** presents research done at the Department of Economics and Finance and aims to share “work in progress” in a timely way before formal publication. As usual, authors bear full responsibility for the content of their contributions.

Das Institut für Höhere Studien (IHS) wurde im Jahr 1963 von zwei prominenten Exilösterreichern – dem Soziologen Paul F. Lazarsfeld und dem Ökonomen Oskar Morgenstern – mit Hilfe der Ford-Stiftung, des Österreichischen Bundesministeriums für Unterricht und der Stadt Wien gegründet und ist somit die erste nachuniversitäre Lehr- und Forschungsstätte für die Sozial- und Wirtschaftswissenschaften in Österreich. Die **Reihe Ökonomie** bietet Einblick in die Forschungsarbeit der Abteilung für Ökonomie und Finanzwirtschaft und verfolgt das Ziel, abteilungsinterne Diskussionsbeiträge einer breiteren fachinternen Öffentlichkeit zugänglich zu machen. Die inhaltliche Verantwortung für die veröffentlichten Beiträge liegt bei den Autoren und Autorinnen.

## **Abstract**

This paper demonstrates that the current literature on cross-ownership among firms underestimates the true degree of separation between cash flow rights and voting rights. We use accounting identities to define coefficients of control, such that any (direct or indirect) control of a firm may be identified using these coefficients. This procedure is sufficient to show that under cross-ownership the voting rights associated with ownership are typically underestimated. We demonstrate by example that control and ownership of dividend rights may be entirely separated, and that multiple equilibria may exist in economies with cross ownership.

## **Keywords**

Cross-ownership, ownership and control, corporate governance

## **JEL Classifications**

G32, G34

**Comments**

The authors gratefully acknowledge support from the Austrian National Bank, Jubilaeumsfonds grant # 9048.

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# 1 Introduction

Workhorse Inc. is a stock market listed company. The two largest blocks of single held shares amount to 25% minus one share each. One of these blocks is held by Crooks Ltd., the other by Rogue & Co. None of the three companies has a majority shareholder. The CEO of the three firms is Mr. Dagobert Duck, the champion among CEO's when it comes to salaries. How come?

Crooks Ltd. holds 50% minus one share in Rogue & Co. Likewise, Rogue & Co. owns 50% minus one share of Crooks Ltd. Mr. Duck owns two shares in each of the latter two companies, and three shares in Workhorse Inc. When it comes to shareholder voting on the CEO (and his salary) for Crooks Ltd., Mr. Duck and Rogue & Co. vote for Mr. Duck and the maximum salary. Since this coalition amounts to 50% plus one share in Crooks Ltd., Mr. Duck gets established. For Rogue & Co. the shares held by Crooks Ltd. and Mr. Duck ensure that also here Mr. Duck gets paid a generous salary as the CEO. Finally, when it comes to Workhorse Inc., the votes cast by Crooks Ltd., Rogue & Co., and Mr. Duck amount to 50% plus one share and establish Mr. Duck as the CEO at the best salary ever. Any move to unseat Mr. Duck would be doomed to fail.

The only expenses that Mr. Duck faces in building his empire is the price for three shares in Workhorse Inc. and for two shares in each of Crooks Ltd. and Rogue & Co. Since the market value of the latter two amounts to approximately one quarter of the value of Workhorse Inc., his expenses are an equivalent of four shares in Workhorse Inc. Hence, four (indirectly owned) shares are enough for control.

The hypothetical CEO "Dagobert Duck" introduced above is a particular example of the separation of firm ownership from control of the firm, in the presence of cross-ownership between firms. The separation of ownership from control is a well-studied problem in economics, finance and law (see e.g. Shleifer and Vishny (1997) for a recent survey on corporate governance). It has long been recognized, for example, that pyramiding of firms, in which a chain of firms is constructed to control voting rights in a target firm, may allow an individual with only marginal cash flow rights in the target company to nonetheless control voting rights by controlling each firm in the chain. And in preferred voting stock arrangements, an investor with strict minority shareholding may control enough voting stock to dictate control of the company. [See Bebchuk, Kraakman and Triantis (1999), La Porta, Lopez-de-Silanes and Shleifer (1999), Faccio, Lang and Young (2001) and Faccio and Lang (2001) for examples of pyramiding and preferred voting stock in both theoretical and empirical analysis.]

Pyramids are little else than an example of cross-ownership relations among firms. The precise quantitative effect of cross-ownership between firms is, however, difficult to capture, both at the theoretical and the empirical level. Recently, authors such as Claessens, Djankov and Lang (2000) and Faccio and Lang (2001) have presented very detailed empirical results which investigate the ownership structures in East Asia and Western Europe, respectively. While they show ample evidence of pyramid structures (and somewhat less evidence of preferred voting stock) their treatment is hampered by the difficulty of capturing cross-ownership effects. Claessens et al. (2000), for example, state that

[t]he presence of cross-holdings creates some difficulties in measuring cash-flow and voting rights. Imagine that firm A owns 50% of firm B which, in turn, owns 25% of firm A. How should firm A be classified?...[W]e classify firm A as controlled by firm B at the 20% cutoff level. (Claessens et al. 2000, p. 93)

In Claessens et al. (2000) and Faccio and Lang (2001) cash flow rights for cross-ownership structures are left ambiguous, while voting control is taken as a floor (in the above example, 20%) of the respective cash-flow rights each firm holds in the other. The authors thus specify cross-ownership effects in much the same way as pyramiding, in which the final ownership of a company is the product of ownership shares along a chain of firms, while voting rights, on the other hand, are simply the *minimum* shareholding value along the entire chain.

On the theoretical side, several authors have attempted to define cross-ownership effects using the pyramid structure as a template. In Bebchuk et al. (1999), for example, cross-ownership effects are simply defined as the sum of an individual investor's *direct* ownership in a target firm, plus the shares of ownership of that investor in other firms, each of which own a part of the target firm (*indirect* ownership). This treatment is insufficient because cross-ownership so defined does not include the recursion between firms who own shares of each other ("A owns part of B, B owns part of A, so A owns part of B's ownership of A, which is also part of a part of A's ownership of B, which is..."). This recursion must be addressed, as it allows for a much greater dispersion of control mechanisms using incremental cash flow rights (as in Dagobert Duck, above) than a simple quasi-pyramid calculation would reveal. Bolle and Güth (1992) attempt to take this effect into consideration, but do not offer a full treatment of the possible ownership structures as there is assumed to exist a controlling shareholder *a priori* for each firm.

Using accounting identities we present measurements of both cash flow rights and voting rights which differ from previous research. First, they

result from a full treatment of cross-ownership relationships between firms, including the infinite recursion between many firms all of whom may have cross-holdings in each other. These measurements lead to the possibility of greater control of voting rights for a given degree of cross-ownership and cash flow rights than the minimum-along-a-chain or floor criterion described above. That is, current estimates of the separation of ownership and control are generally too low when the full identities are taken into consideration.

In addition, the identities also allow us to define *control coefficients* that indicate when an investor exercises full control over a particular firm. These control coefficients are used to calculate both the necessary conditions and the sufficient conditions for control of a firm, in which the required shareholding percentage for control need not be greater than the 50% benchmark commonly used in current research. Both the identities and the necessary and sufficient conditions are developed with several examples, showing along the way that it is possible for a given economy (i.e. a given set of investors and firms with a given level of cross-ownership) to possess multiple equilibria: in one equilibrium there is full voting rights control by the shareholder with majority cash flow rights in a particular firm, while in the other equilibrium cash flow rights and voting rights are completely separated. In fact, in the second equilibrium an investor may exercise full control of voting rights in a given firm while at the same time owning an arbitrarily small amount of the firm's cash flow rights.

Section 2 introduces the accounting identities and also demonstrates that under cross-ownership the book value of a firm will tend to be overestimated with respect to the underlying cash flows. Section 3 presents the mechanism for calculating the control coefficients. The existence of multiple equilibria is also demonstrated by example. Section 4 calculates the necessary conditions and the sufficient conditions for controlling a firm, and defines the voting equilibrium in which relative majority ownership is sufficient for control. An algorithm for passing from the sufficient to the necessary condition is also outlined. Section 5 concludes and provides a brief summary of current research into the specification of a full model, and the empirical testing of data using the algorithm defined in Section 4.

## 2 Accounting Identities

Consider an economy with consumers/investors  $i = 1, \dots, n$  and firms  $j = 1, \dots, m$ . Let  $\vartheta_{ij} \geq 0$  denote the share of firm  $j$  owned by consumer/investor  $i$ , and denote by  $\sigma_{ij} \geq 0$  the share in firm  $j$  owned by firm  $i$  (i.e., the first subscript denotes the owner and the second the firm which is owned). Denote

the  $n \times m$  matrix of firm shares held by consumers/investors by  $\Xi = [\vartheta_{ij}]$  (where  $i = 1, \dots, n$  and  $j = 1, \dots, m$ ) and the  $m \times m$  matrix of firm shares held by other firms by  $\Sigma = [\sigma_{ij}]$  (where  $i, j = 1, \dots, m$ ). By definition, for each firm  $j = 1, \dots, m$  it must be true that

$$\sum_{i=1}^m \sigma_{ij} + \sum_{i=1}^n \vartheta_{ij} = 1$$

for all  $j = 1, \dots, m$ . Writing  $e = (1, 1, \dots, 1)$  for the summation (row) vector, in matrix notation this boils down to

$$e\Sigma + e\Xi = e \quad (1)$$

Assume that there is an upper limit  $\delta$  on the share of a firm that can be held by other firms with  $0 < \delta < 1$ . Then  $e\Sigma \leq \delta e \ll e$  implies that the matrix  $(I - \Sigma)$  (where  $I$  denotes the identity) has a dominant diagonal, because  $1 - \sigma_{jj} > \sum_{i \neq j} |-\sigma_{ij}|$  for all  $j = 1, \dots, m$ . Hence,  $(I - \Sigma)^{-1}$  exists and (1) can be rewritten as

$$e\Xi (I - \Sigma)^{-1} = e \quad (2)$$

The  $n \times m$  matrix  $\Theta$  of *imputed ownership shares*  $\theta_{ij}$  in firm  $j = 1, \dots, m$  for consumers/investors  $i = 1, \dots, n$  is, therefore, given by

$$\Theta = [\theta_{ij}] = \Xi (I - \Sigma)^{-1} \quad (3)$$

(Note that (2) states that  $e$  is a left eigenvector of  $\Theta$  with associated real eigenvalue 1.) In other words, if consumer/investor  $i$  holds *direct* shares  $\vartheta_i = (\vartheta_{ij})_{j=1, \dots, m}$  in firms, her ultimate shares in distributed profits (imputed shares) are given by (the row vector)

$$\theta_i = (\theta_{ij})_{j=1, \dots, m} = \vartheta_i (I - \Sigma)^{-1} \quad (4)$$

for all  $i = 1, \dots, n$ .

Let  $\pi = (\pi_j)_{j=1, \dots, m}$  denote the (column) vectors of firms' *cash flows* (or liquidation values, or net present value of dividends) and  $\Pi = (\Pi_j)_{j=1, \dots, m}$  the (column) vector of effective profits of firms. The *effective profit* (or *book value*)  $\Pi_j$  of firm  $j$  consists of its cash flow plus the revenue from ownership in other firms,

$$\Pi_j = \pi_j + \sum_{i=1}^m \sigma_{ji} \Pi_i$$

for all  $j = 1, \dots, m$ . In matrix notation this boils down to

$$\Pi = \pi + \Sigma \Pi = (I - \Sigma)^{-1} \pi \quad (5)$$

In other words, the sum of consumer/investor  $i$ 's *direct* shares in *profits* equals the sum of her imputed shares in *cash flows*,

$$\vartheta_i \Pi = \vartheta_i (I - \Sigma)^{-1} \pi = \theta_i \pi \quad (6)$$

for all  $i = 1, \dots, n$ . And, therefore, in the aggregate the sum of all distributed profits equals the sum of all cash flows,

$$e \Xi \Pi = e \Xi (I - \Sigma)^{-1} \pi = e \Theta \pi = e \pi \quad (7)$$

using (2).

This also holds for firms' output vectors, rather than cash flow. Suppose firm  $j = 1, \dots, m$  produces an output (row) vector  $x_j$  net of what it receives from other firms. Then, its gross output (row) vector  $y_j$  is given by

$$y_j = x_j + \sum_{i=1}^m \sigma_{ji} y_i$$

or, with  $X = [x_{jl}]$  resp.  $Y = [y_{jl}]$  (where  $l$  denotes the commodity index) denoting the matrix of firms' net resp. gross outputs,

$$Y = X + \Sigma Y = (I - \Sigma)^{-1} X$$

Therefore, consumer  $i = 1, \dots, n$  receives

$$\vartheta_i Y = \vartheta_i (I - \Sigma)^{-1} X = \theta_i X$$

so that  $e \Xi Y = e \Xi (I - \Sigma)^{-1} X = e \Theta X = e X$  guarantees that the total outputs by all firms is entirely distributed to consumers.

Denote by  $v = (I - \Sigma)^{-1} e$  the (column) vector of row sums of the matrix  $(I - \Sigma)^{-1}$  and by  $V = \text{diag } v$  the diagonal matrix with the  $v_j$ 's on its diagonal ( $j = 1, \dots, m$ ). Since

$$(I - \Sigma)^{-1} = \sum_{t=0}^{\infty} \Sigma^t \quad (8)$$

it follows from  $\Sigma \geq 0$  that  $v = \sum_{t=0}^{\infty} \Sigma^t e \geq e$  with at least one strict inequality if  $\Sigma$  is not identically zero. Therefore,  $V^{-1} (I - \Sigma)^{-1} e = e$  implies that  $(I - \Sigma)^{-1}$  is the product of a matrix  $V^{-1} (I - \Sigma)^{-1}$  the rows of which (are nonnegative and) sum to 1 and a diagonal matrix  $V$  the diagonal elements of which are at least 1, because  $(I - \Sigma)^{-1} = V V^{-1} (I - \Sigma)^{-1}$ .

## 2.1 Firm Book Values

Using this construction, the profit  $\Pi_j$  of firm  $j$  ( $= 1, \dots, m$ ) can be perceived as a weighted average  $e_j V^{-1} (I - \Sigma)^{-1} \pi$  (where  $e_j$  is a row vector of zeros except for a 1 at the  $j$ -th position) of all firms' cash flows times an *expansion factor*  $v_j \geq 1$ , i.e.,

$$\Pi_j = v_j e_j V^{-1} (I - \Sigma)^{-1} \pi \quad (9)$$

which increases the firm's book value beyond a weighted average of all firms' cash flows. Obviously, if  $\Sigma \equiv 0$  then  $\Pi_j = \pi_j$  for all  $j = 1, \dots, m$ . Hence, the vector  $v = (I - \Sigma)^{-1} e$ , in a sense, measures the bias that is introduced by cross-ownership among firms in the transition from cash flows to book values. By (8) this is always an upward bias. Hence, cross-ownership among firms leads to an *overvaluation* of book values.

If there is an isolated group of companies in the economy which own each other, but no company outside this conglomerate, and no company outside the conglomerate owns any company belonging to it, then the matrix  $\Sigma$  decomposes (possibly by suitable permutations) into zero matrices and a smaller matrix  $\tilde{\Sigma} \geq 0$ ,

$$\Sigma = \begin{pmatrix} \tilde{\Sigma} & 0 \\ 0 & 0 \end{pmatrix}$$

such that also the matrix  $(I - \Sigma)^{-1}$  decomposes by

$$(I - \Sigma)^{-1} = \begin{pmatrix} (I - \tilde{\Sigma})^{-1} & 0 \\ 0 & I \end{pmatrix}$$

and the effects of cross-ownership within the conglomerate can be analysed independently from firms outside of it. The term "conglomerate" will be reserved for such cases where there is a subset  $J \subset \{1, \dots, m\}$  such that  $\sigma_{ij} > 0$  implies  $i, j \in J$ . The following is an example of the book valuation bias which is introduced when cross-ownership in a conglomerate is present. In this case, two companies in a conglomerate owning each other is equivalent to both owning themselves, and drives their book values upwards.

**Example 1** Consider a conglomerate of two companies  $j = 1, 2$  where each either owns shares in itself or in the other company. For  $\alpha, \beta \in (0, 1)$  let

$$\Sigma_1 = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} \text{ and } \Sigma_2 = \begin{pmatrix} 0 & \beta \\ \alpha & 0 \end{pmatrix}$$

be the associated matrices of cross-ownerships. Then

$$(I - \Sigma_1)^{-1} = \begin{pmatrix} \frac{1}{1-\alpha} & 0 \\ 0 & \frac{1}{1-\beta} \end{pmatrix} \text{ and } (I - \Sigma_2)^{-1} = \frac{1}{1-\alpha\beta} \begin{pmatrix} 1 & \beta \\ \alpha & 1 \end{pmatrix}$$

and the two companies' book values are given

$$\Pi^1 = (I - \Sigma_1)^{-1} \pi = \begin{pmatrix} \frac{\pi_1}{1-\alpha} \\ \frac{\pi_2}{1-\beta} \end{pmatrix} \text{ and } \Pi^2 = (I - \Sigma_2)^{-1} \pi = \begin{pmatrix} \frac{\pi_1 + \beta\pi_2}{1-\alpha\beta} \\ \frac{\pi_2 + \alpha\pi_1}{1-\alpha\beta} \end{pmatrix}$$

where  $\pi \gg 0$  is assumed for convenience. Now let  $\beta$  be chosen such that

$$\beta = \beta(\alpha, \pi) = \frac{\alpha\pi_1}{(1-\alpha)\pi_2 + \alpha\pi_1} \in (0, 1)$$

Then  $1 - \alpha\beta = (1 - \alpha) [\pi_2 + \alpha\pi_1] / [(1 - \alpha)\pi_2 + \alpha\pi_1]$  implies that  $\Pi_1^2 = \frac{\pi_1}{1-\alpha}$  and  $\Pi_2^2 = \frac{\pi_2}{1-\beta}$ , precisely as in the case where each company owns shares in itself, i.e.,  $\beta = \beta(\alpha, \pi) \Rightarrow \Pi^2 = \Pi^1$ .

Now start from a situation without cross-ownership,  $\Sigma = 0$ . The managers of the two companies hold stock options (on stocks of their own companies) with exercise prices  $q_j$  above  $\pi_j$  for  $j = 1, 2$ . They decide to merge the two companies. The merger is financed by exchanging  $\alpha$  newly issued firm 1 shares against  $\beta$  newly issued firm 2 shares. Issuing shares will dilute seasoned shares by  $1 - \alpha$  for firm 1 and by  $1 - \beta$  for firm 2 shares. If, however,  $\beta$  is chosen such that  $\beta = \beta(\alpha, \pi)$  as above, then a pre-issue share  $\vartheta_{i1}$  (resp.  $\vartheta_{i2}$ ) held by shareholder  $i$  amounts to pre-issue claims  $\vartheta_{i1}\pi_1$  (resp.  $\vartheta_{i2}\pi_2$ ) and to (diluted) post-issue claims  $(1 - \alpha)\vartheta_{i1}\Pi_1^2 = \vartheta_{i1}\pi_1$  (resp.  $(1 - \beta)\vartheta_{i2}\Pi_2^2 = \vartheta_{i2}\pi_2$ ). Therefore, all shareholders in both companies are indifferent as to the financial consequences of the merger.

But the emerging cross-ownership increases the firms' profits from  $\pi_j$  to  $\Pi_j^2 = \Pi_j^1 > \pi_j$  for  $j = 1, 2$ . Hence, since profits and, thus, market values of shares now rise, the managers of the two companies can exercise their stock options (at  $q_j$  for  $j = 1, 2$ ) and sell the so acquired shares at stock prices  $\Pi_j^2 = \Pi_j^1$  at the spot market. This is a profitable operation for both managers, provided  $\alpha$  is chosen sufficiently large such that

$$\pi_1 < q_1 < \frac{\pi_1}{1-\alpha} \text{ and } \pi_2 < q_2 < \frac{\pi_2}{1-\beta(\alpha, \pi)}$$

which is always possible by making  $\alpha$  sufficiently large (because  $\partial\beta(\alpha, \pi)/\partial\alpha > 0$ ,  $\beta(0, \pi) = 0$ , and  $\beta(1, \pi) = 1$ ).

The managers' gains  $\Pi_j^2 - q_j > 0$  come at no cost and are undetectable as insider trading, because the option contracts were written before the merger. Moreover, the transaction of newly issued shares between the two firms satisfies a quid-pro-quo in post-issue stock values, because at  $\beta = \beta(\alpha, \pi)$  we have  $\beta\Pi_2^2 = \frac{\alpha}{1-\alpha}\pi_1 = \alpha\Pi_1^2$ , so that managers cannot be blamed for negotiating distortive pricing either.

In general, a conglomerate can *always* increase (the sum of) its book value(s) beyond (the sum of) its cash flow(s). For, if  $\tilde{\Sigma}e \gg 0$  then

$$\left(I - \tilde{\Sigma}\right)^{-1} e = \sum_{t=0}^{\infty} \tilde{\Sigma}^t e \gg e$$

so that  $v_j > 1$  for all  $j$  which belong to the conglomerate. Note that in the previous example the two companies could achieve  $\Sigma_2$  from  $\Sigma_1$  by simply swapping the shares they hold in themselves.

Hence, if firms have an incentive to increase their profits (book values) beyond their cash flows, then they can achieve this by a suitable choice of cross-ownership structure. As a consequence, if there is nonzero cross-ownership among firms in an economy the aggregate book value of the firm sector overestimates the aggregate value of cash flows produced by firms. This is because consumers/investors hold less than the total stock of firms, yet ultimately receive the total liquidation value of the firm sector.

### 3 Separating Ownership and Control

Complicated ownership structures may divorce stock ownership from control over a company at almost no cost. The most obvious example of this is a pyramid:

**Example 2** Let  $\Sigma$  be given by  $\sigma_{ij} \in (0, 1)$  for  $j = i - 1 \geq 1$  and  $\sigma_{ij} = 0$  otherwise, i.e., for  $i \geq 2$  company  $i$  owns  $\sigma_{i,i-1} > 0$  shares in company  $i - 1$ , but in no other company, and company  $i = 1$  owns no firm shares. If

$$a_{ij} = \begin{cases} 1 & \text{if } i = j \\ \prod_{k=j}^{i-1} \sigma_{h+1,h} & \text{if } 1 \leq j \leq i - 1 \\ 0 & \text{if } i < j \end{cases}$$

then  $-\sigma_{i,i-1}a_{i-1,j} + a_{ij} = 0$  if  $i < j$ ,  $-\sigma_{i,i-1}a_{i-1,j} + a_{ij} = 1$  if  $i = j$ , and  $-\sigma_{i,i-1}a_{i-1,j} + a_{ij} = \prod_{k=j}^{i-1} \sigma_{h+1,h} - \sigma_{i,i-1} \prod_{k=j}^{i-2} \sigma_{h+1,h} = 0$  if  $i > j$ . Therefore,  $A = [a_{ij}] = (I - \Sigma)^{-1}$ .

Assume now that  $\sigma_{j,j-1} = \sigma \in (0, 1)$  and that  $\pi_1 > 0 = \pi_j$  for all  $j \geq 2$ . Then  $a_{ij} = \sigma^{i-j}$  if  $1 \leq j \leq i$  and  $a_{ij} = 0$  if  $i < j$ . Hence,  $\Pi_j = \sigma^{j-1}\pi_1$  and  $\theta_{kj} = \sum_i a_{ij}\vartheta_{ki} = \sum_{i \geq j} \sigma^{i-j}\vartheta_{ki}$  for any shareholder  $k$  and all firms  $j$ . Now consider a shareholder  $k$  who owns  $\vartheta_{km} > \frac{1}{2}$  shares of company  $j = m$  and suppose that  $\sigma > \frac{1}{2}$ . Then each company  $j \geq 2$  holds a majority in company  $j-1$  and, therefore, shareholder  $k$  has full control over all companies  $j = 1, \dots, m$ . Yet, the cost of buying majority control in company  $j = 1$  by



buying  $\vartheta_{km}$  amounts only to  $\vartheta_{km}\sigma^{m-1}\pi_1 \approx 2^{-m}\pi_1 \rightarrow_{m \rightarrow \infty} 0$  if  $\vartheta_{km}$  and  $\sigma$  are close enough to  $\frac{1}{2}$ .

Buying control in a company without buying a significant claim to its returns may be motivated, for example, by risk preferences or by the mere attempt of management to retain control against the will of shareholders.<sup>1</sup> Cross-ownership in particular may also be motivated by a desire for collusion between firms under imperfect competition (Macho-Stadler and Verdier 1991, Spagnolo 1998). It is, however, more difficult to capture the necessary and sufficient conditions for control under cross-ownership than it is for pyramids, as the entire matrix of firm cross-ownerships must be considered. Developing a complete model to assess the necessary conditions for control would require specifying a game where investors strategically attempt to gain control by buying into or founding holding companies, or cartels which own each other (this is one direction of our current research—see section 5 for some brief remarks).

But as a first step in this direction it is useful to develop a tool which allows one to check if, for a *given* structure of cross-ownership, a particular control structure is an equilibrium. For example, it is clear that the “Dagobert Duck” CEO from the Introduction can exercise full control while holding only a fraction of the cash flow rights of the target firm. But is this the only outcome? Is it possible that in this same economy there might also exist an equilibrium in which a majority holder of cash flow rights exercises full control? We seek in this case a sufficient condition for control, so that it might be possible to check if a given control structure is in fact an equilibrium.

### 3.1 Control Coefficients

To formalize such a sufficient condition, introduce a “control parameter”  $c_{ij}$  for  $i = 1, \dots, n$  and  $j = 1, \dots, m$  with  $c_{ij} \in \{0, 1\}$ . The interpretation is that  $c_{ij} = 1$  if consumer/investor  $i$  *controls* company  $j$ , and  $c_{ij} = 0$  if not. For each  $j = 1, \dots, m$  let  $h_j : [0, 1] \rightarrow \{0, 1\}$  be the heavyside function with respect to a parameter  $\eta_j \in (0, 1)$ , i.e.,

$$h_j(x) = \begin{cases} 1 & \text{if } x > \eta_j \\ 0 & \text{if } x \leq \eta_j \end{cases} \quad (10)$$

and for an  $m$ -vector  $x = (x_1, \dots, x_m) \in [0, 1]^m$  define

$$h(x) \equiv (h_1(x_1), \dots, h_m(x_m)) \quad (11)$$

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<sup>1</sup>See e.g. Hansen and Lott, Jr. (1996) for a treatment of cross-ownership as a means for portfolio diversification and risk-smoothing.

componentwise.<sup>2</sup> Then, require that the vectors  $c_i = (c_{i1}, \dots, c_{im}) \in \{0, 1\}^m$  satisfy the  $(n + 1)m$  conditions

$$h(\vartheta_i + c_i \Sigma) = c_i \text{ and } \sum_{k=1}^n c_k \leq e \quad (12)$$

for all  $i = 1, \dots, n$  simultaneously. In other words, if investor  $i$  controls companies, whose joint shareholdings in company  $j$  together with her own share in company  $j$  exceed the fraction  $\eta_j$  of the shares in  $j$ , then she controls company  $j$ . In the previous example both  $(c_1, c_2) = ((1, 1, 1), (0, 0, 0))$  and  $(c_1, c_2) = ((0, 0, 0), (1, 1, 1))$  are solutions to the system (12) for  $\eta_j = 1/2$  for  $j = 1, 2, 3$ .

If some parameter  $\eta_j$  is strictly smaller than  $1/2$  a possible complication arises. There could be  $i, h \in \{1, \dots, n\}$  such that  $i \neq h$  and  $\vartheta_{i\ell} + \sum_{k=1}^m c_{ik} \sigma_{kj} > \eta_j$  for both  $\ell = i, h$ . Then  $c_i = c_h = 1$  would hold. Yet,  $1 \geq \vartheta_{ij} + \vartheta_{hj} + \sum_{k=1}^m (c_{ik} + c_{hk}) \sigma_{kj} > 2\eta_j$  implies  $\eta_j < 1/2$ . Therefore,  $\eta_j = 1/2$  for all  $j = 1, \dots, m$  is sufficient for (12) to have a solution. This follows from the fact that the left hand side of the first equation is nondecreasing in  $c_i$  and has a finite range, for all  $i = 1, \dots, n$ .

If  $\eta_j$  is smaller than  $1/2$  for some  $j = 1, \dots, m$  we need a different technique to assign control coefficients. Let  $C = [c_{ij}]$  with  $i = 1, \dots, n$  and  $j = 1, \dots, m$  be the matrix of control coefficients. They need to be (real) solutions to the following optimization problem:

$$\begin{aligned} & \max_C eCe \\ & \text{s.t. } \left[ \vartheta_i + c_i \Sigma - \sum_{j \neq i} (\vartheta_j + c_j \Sigma) \right] \text{diag}(c_i) \geq 0 \text{ and} \\ & c_i [I - \text{diag}(c_i)] = 0 \text{ for all } i = 1, \dots, n, \\ & \text{and } eC \leq e \end{aligned} \quad (13)$$

Note that the second of the three constraints is only there to ensure that  $c_{ij} \in \{0, 1\}$ .

A sufficient condition for  $c_{ij} = 0$  is that  $\vartheta_{ij} + \sum_{k=1}^m \sigma_{kj} \leq \eta_j$ , because

$$c_{ij} = 1 \Rightarrow \eta_j < \vartheta_{ij} + \sum_{k=1}^m c_{ik} \sigma_{kj} \leq \vartheta_{ij} + \sum_{k=1}^m \sigma_{kj}$$

Hence, if cross ownership among firms is legally restricted such that  $\sum_{k=1}^m \sigma_{kj} \leq \eta_j - \alpha$ , for  $\alpha \in (0, 1/2)$ , then it takes a minimum of  $\alpha$  privately held shares

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<sup>2</sup>The parameter  $\eta_j$  will usually be  $\eta_j = 1/2$  if all shares are voting stock. If there is preferred stocks outstanding,  $\eta_j$  may be less than  $1/2$ .

to control a company. In this sense the sum of outstanding (voting) stock in the hands of other firms,  $\sum_{k=1}^m \sigma_{kj}$ , measures how vulnerable company  $j$  is to control divorced from ownership (if, say,  $\eta_j = 1/2$  for all  $j$ ).

Let the  $n \times m$  matrix  $C = [c_{ij}]$  be a solution to the system (12) and partition firms into a set  $J_1(C) = \{1, \dots, k\}$  (without loss of generality) of firms for which there is  $i = 1, \dots, n$  such that  $c_{ij} = 1$ , and a set of firms  $J_0(C) = \{k+1, \dots, m\}$  (w.l.o.g.) which do *not* have a majority shareholder, i.e.,  $\sum_{i=1}^n c_{ij} = 0$  for all  $j \in J_0(C)$ . Denote by  $\Lambda^* = \Lambda^*(C)$  the diagonal  $m \times m$  matrix which has  $\lambda_{jj}^* = 1$  as its diagonal element if and only if  $j \in J_0(C)$  and zero entries otherwise, i.e.,  $\Lambda^*(C) = \text{diag}(e - eC)$ , and define

$$\Sigma^* = \Sigma^*(C) \equiv \Sigma \Lambda^* \text{ and } \Xi^* = \Xi^*(C) \equiv \Xi \Lambda^* + C \quad (14)$$

Since  $e\Sigma^* = e\Sigma \Lambda^* \leq e\Sigma$ , the matrix  $I - \Sigma^*$  has an inverse, whenever  $I - \Sigma$  has, i.e., whenever  $e\Sigma \ll e$ . Using (1) one obtains from (14) that

$$e\Sigma^* + e\Xi^* = e(\Sigma + \Xi) \Lambda^* + eC = e\Lambda^* + eC = e - eC + eC = e$$

i.e., the modified matrices  $\Sigma^*(C)$  and  $\Xi^*(C)$  form a consistent share distribution. Since  $e\Xi^*(I - \Sigma^*)^{-1} = e$ , the voting/controlled effective shares of investor  $i$  can now be defined by

$$\theta_i^* = \theta_i^*(C) \equiv (\vartheta_i \Lambda^* + c_i) (I - \Sigma^*)^{-1} \quad (15)$$

The voting/controlled effective shares from (15) will be the ones which determine firm decisions. Hence, if it comes to voting in a shareholder assembly at all, then only investors vote in it, and not other firms. So, there is no issue of cross-voting.

We highlight the control coefficient technique by reconsidering the Dagobert Duck example from the Introduction, showing that in addition to the equilibrium originally presented, in which the CEO controls all voting rights with no cash flow rights, there also exists an equilibrium in which full cash flow rights also imply full voting rights control:

**Example 3** (*Dagobert Duck*) *Let there be two consumers/investors  $i = 1, 2$  and three firms  $j = 1, 2, 3$  with*

$$\Sigma = \begin{pmatrix} 0 & 0 & 0 \\ \frac{1}{4} - \varepsilon & 0 & \frac{1}{2} - \varepsilon \\ \frac{1}{4} - \varepsilon & \frac{1}{2} - \varepsilon & 0 \end{pmatrix} \text{ and } \Xi = \begin{pmatrix} 3\varepsilon & 2\varepsilon & 2\varepsilon \\ \frac{1}{2} - \varepsilon & \frac{1}{2} - \varepsilon & \frac{1}{2} - \varepsilon \end{pmatrix}$$

for some small  $\varepsilon > 0$ . Using (3) the true ownership distribution is given by

$$\Theta = \frac{1}{1 + 2\varepsilon} \begin{pmatrix} 5\varepsilon - 2\varepsilon^2 & 4\varepsilon & 4\varepsilon \\ 1 - 3\varepsilon + 2\varepsilon^2 & 1 - 2\varepsilon & 1 - 2\varepsilon \end{pmatrix} \xrightarrow{\varepsilon \downarrow 0} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

i.e., consumer/investor  $i = 2$  (approximately) owns all three firms. When it comes to control, however, the system (12) has multiple solutions. In one equilibrium, investor  $i = 2$  does not control any of the firms, despite owning all of them. For the control coefficient  $c_2 = (0, 0, 0)$  is a solution to system (12): for each firm  $j = 1, 2, 3$  we have

$$h\left(\frac{1}{2} - \varepsilon\right) \rightarrow_{\varepsilon \downarrow 0} 0 = c_{2j}.$$

In this case investor  $i = 1$  (“Dagobert Duck”) controls all three firms while owning an arbitrarily small amount of each firm.

In the other equilibrium investor  $i = 2$  controls all three firms, i.e.  $c_2 = (1, 1, 1)$ . System (12) also has the solution

$$h\left(\frac{1}{2} - \varepsilon + \sum_{j=1}^3 \sigma_{j1}\right) = h(1 - 3\varepsilon) \rightarrow_{\varepsilon \downarrow 0} 1 = c_{21},$$

$$h\left(\frac{1}{2} - \varepsilon + \sum_{j=1}^3 \sigma_{j2}\right) = h(1 - 2\varepsilon) \rightarrow_{\varepsilon \downarrow 0} 1 = c_{22},$$

$$h\left(\frac{1}{2} - \varepsilon + \sum_{j=1}^3 \sigma_{j3}\right) = h(1 - 2\varepsilon) \rightarrow_{\varepsilon \downarrow 0} 1 = c_{23}.$$

Here full control of cash flow rights and voting rights coincide for investor  $i = 2$ .

In Example 3 we have for the two solutions to (12),

$$C_1 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } C_2 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

and the two associated matrices  $\Lambda_l^* = \text{diag}(e - eC_l) = 0$  for  $l = 1, 2$  that  $\Theta^*(C_l) = (\Xi\Lambda_l^* + C_l)(I - \Sigma\Lambda_l^*)^{-1} = C_l$ , i.e.,  $J_0(C_l) = \emptyset$  for  $l = 1, 2$  and all companies are controlled by one of the two investors.

However, it is *not* true that a shareholder who owns more than half the dividend rights according to (4) will always be able to control the company. The following modification of Example 3 shows this.

**Example 4 (resumed)** Replace in Example 3 the second shareholder, who originally owned  $1/2 - \varepsilon$  shares in all companies, by two shareholders, letting

$$\Xi = \begin{pmatrix} 3\varepsilon & 2\varepsilon & 2\varepsilon \\ \frac{1}{2} - \varepsilon & \frac{1}{2} - \varepsilon & 0 \\ 0 & 0 & \frac{1}{2} - \varepsilon \end{pmatrix}$$

yielding imputed shares

$$\Theta = \begin{pmatrix} \frac{\varepsilon(5-2\varepsilon)}{1+2\varepsilon} & \frac{4\varepsilon}{1+2\varepsilon} & \frac{4\varepsilon}{1+2\varepsilon} \\ \frac{3(1-2\varepsilon)}{4(1+2\varepsilon)} & \frac{2(1-2\varepsilon)}{3+4\varepsilon-4\varepsilon^2} & \frac{(2\varepsilon-1)^2}{3+4\varepsilon-4\varepsilon^2} \\ \frac{(1-2\varepsilon)(1-4\varepsilon)}{4(1+2\varepsilon)} & \frac{(2\varepsilon-1)^2}{3+4\varepsilon-4\varepsilon^2} & \frac{2(1-2\varepsilon)}{3+4\varepsilon-4\varepsilon^2} \end{pmatrix} \xrightarrow{\varepsilon \downarrow 0} \begin{pmatrix} 0 & 0 & 0 \\ \frac{3}{4} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

by (3). For small enough  $\varepsilon > 0$  investor  $i = 2$  owns the majority of dividend rights in companies  $j = 1$  and 2, and investor  $i = 3$  owns the majority of dividend rights (imputed shares) in company  $j = 3$ . But  $c_{2j} = 0$  for all  $j = 1, 2, 3$ . For, since  $\vartheta_{23} = 0$ , we have from (12) that

$$\vartheta_{23} + \sum_{j=1}^3 c_{2j}\sigma_{j3} = c_{22} \left( \frac{1}{2} - \varepsilon \right) \leq \frac{1}{2} - \varepsilon < \frac{1}{2}$$

implies  $c_{23} = 0$  which, in turn, implies

$$\vartheta_{22} + \sum_{j=1}^3 c_{2j}\sigma_{j2} = \frac{1}{2} - \varepsilon + c_{23} \left( \frac{1}{2} - \varepsilon \right) = \frac{1}{2} - \varepsilon < \frac{1}{2}$$

and, therefore,  $c_{22} = 0$ . It follows that

$$\vartheta_{21} + \sum_{j=1}^3 c_{2j}\sigma_{j1} = \frac{1}{2} - \varepsilon < \frac{1}{2}$$

implies  $c_{21} = 0$ , i.e.,  $c_2 = 0$ . That is, while investor  $i = 2$  owns approximately three quarters of all dividend rights in company  $j = 1$  (and two thirds of dividend rights in company  $j = 2$ ), she cannot control any of the companies.

Similarly, because for  $j = 1, 2$

$$\vartheta_{3j} + \sum_{k=1}^3 c_{3k}\sigma_{kj} = \sum_{k=1}^3 c_{3k}\sigma_{kj} \leq \frac{1}{2} - \varepsilon < \frac{1}{2}$$

it follows from (12) that  $c_{31} = c_{32} = 0$ . Therefore,  $\vartheta_{33} + \sum_{j=1}^3 c_{3j}\sigma_{j3} = \frac{1}{2} - \varepsilon < \frac{1}{2}$  implies also  $c_{31} = 0$ , i.e.,  $c_3 = 0$ . That is, investor  $i = 3$  cannot control any of the companies either - not even company  $j = 3$ , where she owns two thirds of the dividend rights (imputed shares) according to (4).

On the other hand,

$$\vartheta_1 + e\Sigma = \left( \frac{1}{2} + \varepsilon \right) e \gg \frac{1}{2}e$$

still implies that investor  $i = 1$ , who owns negligible dividend rights in all three companies, can control all companies, i.e., for  $i = 1$  (12) has the solution  $c_i = c_1 = e$ .

This calculus of control also reveals that in Example 2 majority control of company  $m$  is necessary and sufficient for controlling *all* companies.

**Example 5** (resumed) Assume that in Example 2 (the pyramid)  $\sigma_{ij} > 0 \Rightarrow \sigma_{ij} > 1/2$ . (Recall that  $\sigma_{ij} > 0$  implies  $i = j+1$ .) Now suppose that  $\vartheta_{im} > 1/2$  for some investor  $i$ . Then  $\vartheta_{ij} + \sum_{k=1}^m \sigma_{kj} \geq \sum_{k=1}^m \sigma_{kj} = \sigma_{j+1,j} > 1/2$  for all  $j = 1, \dots, m$  implies that  $c_i = e$  is a solution to (12).

Conversely, if  $c_{ij} = 1$  for some  $j = 1, \dots, m$ , then (12) implies  $\vartheta_{ij} + \sum_{k=1}^m c_{ik} \sigma_{kj} = \vartheta_{ij} + c_{i,j+1} \sigma_{j+1,j} > 1/2$  and, therefore,  $c_{i,j+1} = 1$ , because if  $c_{i,j+1} = 0$  then from  $\sigma_{j+1,j} > 1/2$  it follows that  $\vartheta_{ij} < 1/2$ , yielding  $\vartheta_{ij} + \sum_{k=1}^m c_{ik} \sigma_{kj} = \vartheta_{ij} < 1/2$  in contradiction to the hypothesis. Hence,  $c_{ij} = 1$  implies  $c_{ik} = 1$  for all  $k = j, \dots, m$ . But then  $\vartheta_{i,j-1} + \sum_{k=1}^m c_{ik} \sigma_{k,j-1} = \vartheta_{i,j-1} + c_{ij} \sigma_{j,j-1} \geq \sigma_{j,j-1} > 1/2$  implies  $c_{i,j-1} = 1$ , too. Therefore, that there is some  $j$  such that  $c_{ij} = 1$  implies  $c_i = e$ . In particular,  $c_{im} = 1$ . The latter implies  $\vartheta_{im} > 1/2$ , because  $\sigma_{jm} = 0$  for all  $j = 1, \dots, m$ .

We see that the control coefficients allow for a concise identification of the conditions of ownership—once the coefficients are determined, the economy may be recast into one in which the controlling investors own the entire firm that they (directly or indirectly) control, and the firm shareholdings are adjusted accordingly. The problem of firm cross-voting is thereby avoided. In addition, we see how regulations which limit cross-ownership help to determine the susceptibility of a particular firm to having its dividend rights separated from its voting rights.

In the presence of cross-ownership the standard approach of measuring cash flow right and voting rights (see Introduction) will generally underestimate the degree of separation of ownership from control. Current research generally selects a minimum level, or floor, of cash flow rights in a cross-holding of firms as the level of voting rights enjoyed by one firm over another. We may adopt with some qualification the term ‘weakest link’<sup>3</sup> for this voting rights floor, as the minimum of cash flow rights translates directly into the voting rights.

By contrast, the approach presented here relies solely upon the effective shareholdings to determine whether or not investor  $i$  controls a (set of) firm(s), using (14). These shareholdings are determined by the *entire matrix* of firm ownership, whereas the conventional approach avoids the recursion problem entirely (thus implicitly setting the other matrix entries to zero). The voting rights of the ‘weakest link’ approach thus offer a lower bound of the degree of voting rights for a given set of cash flow rights, under a given

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<sup>3</sup>Faccio et al. (2001) p. 56; the term as originally used was for the minimum cash flow rights of a chain of firms in a pyramid.

ownership structure. In general the control of voting rights will be greater under cross-ownership than this lower bound, which means that the degree of separation of ownership from control in the current literature is generally *underestimated* in the presence of cross-ownership.

The control coefficients also allow one to determine under what voting conditions other than absolute majority voting it may be possible to divorce cash flow rights from control. In this case we may define both necessary conditions and sufficient conditions for control, and show how one may recursively pass from one set of conditions to the other.

## 4 Conditions for Control

When a shareholder can control a company corresponds to a selection of an equilibrium in the subgame where shareholders would like to vote management out of office. This subgame - which, of course, will not be reached - may have several equilibria. One of those is always such that a *relative* majority is enough for control.

To see this, consider a fixed firm and assume that investor  $i = 1$  is its CEO. Suppose all other shareholders are dissatisfied with 1's performance. To unseat him, they have to call a shareholder assembly and vote him out of office. Therefore, each shareholder  $i = 2, \dots, n$  has two possibilities: either attend the assembly and vote against management (denoted  $a_i = 1$ ) or stay home (denoted  $a_i = 0$ ). For the moment, denote by  $\theta_i$  the shares owned by investor  $i = 1, \dots, n$ .

Let ordinal preferences of investors  $i = 2, \dots, n$  over outcomes be as follows. Each shareholder  $i$  strictly prefers to attend and vote against management over staying home if and only if her vote is pivotal. That is,<sup>4</sup>

$$a_i = 1 \succ_i a_i = 0 \text{ if and only if } \sum_{j \geq 2} a_j \theta_j > \theta_1 \geq \sum_{i \neq j \geq 2} a_j \theta_j$$

This is because, if the other shareholders (except  $i$  and 1) decide to vote against management and together already command more shares than the CEO  $i = 1$ , then an arbitrarily small cost for attending the shareholder assembly makes it optimal for  $i$  to stay home. Likewise, if too few shareholders decide to attend, so that adding  $i$ 's share is insufficient to vote management out of office, then it is again optimal to stay home, if attendance carries a small cost.

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<sup>4</sup>Assume that at a tie the CEO's vote is pivotal. Then the challengers need to combine a strictly larger share than the CEO.

On the other hand, if, given the other shareholders' (except  $i$  and 1) decisions, adding  $i$ 's share turns the vote from failing to unseating the CEO, then  $i$  strictly prefers  $a_i = 1$  over  $a_i = 0$ . With these (purely ordinal) preferences the following is straightforward.

**Proposition 6** *In the voting game there exists a pure Nash equilibrium with  $\sum_{i=2}^n a_i = 0$  if and only if  $\theta_1 \geq \theta_i$  for all  $i = 2, \dots, n$ .*

**Proof.** “if” Suppose  $\theta_1 \geq \theta_i$  for all  $i = 2, \dots, n$ . We claim that  $a_i = 0$  for all  $i = 2, \dots, n$  is a Nash equilibrium. Consider any shareholder  $i = 2, \dots, n$ . Given that all other shareholders decide to stay home,  $a_j = 0$  for all  $j \neq i$  with  $j \geq 2$ , the move to unseat the CEO can only succeed if  $\theta_i > \theta_1$ . Since this is ruled out by hypothesis,  $a_i = 0$  is optimal. Since  $i$  was arbitrary, this verifies that  $a_i = 0$  for all  $i = 2, \dots, n$  is an equilibrium.

“only if” Suppose there is some  $i = 2, \dots, n$  such that  $\theta_i > \theta_1$ . Assume that there is an equilibrium with  $\sum_{j=2}^n a_j = 0$ . Then  $a_j = 0$  for all  $j = 2, \dots, n$ . But shareholder  $i$  can profitably deviate to  $a_i = 1$ , because her share is sufficient to vote management out of office by hypothesis - a contradiction. ■

This Proposition effectively states that a *relative* majority is enough for control, if in the subgame, where all shareholders (other than the CEO) would want to vote management out of office, a particular equilibrium is being played - the one where no shareholder challenges.

Determining control under such an equilibrium is somewhat more involved. Again, denote by  $c_i = (c_{i1}, \dots, c_{im}) \in \{0, 1\}^m$  the (row) vector of control coefficients for investor  $i = 1, \dots, n$  and by  $C = [c_{ij}]$  the  $n \times m$  matrix of control coefficients. The latter (viewed as reals) must solve the following system of inequalities and equalities:

$$\begin{aligned} [\vartheta_i - \vartheta_j + (c_i - c_j) \Sigma] \text{diag}(c_i) &\geq 0 \text{ for all } j = 1, \dots, n \text{ and} \\ c_i [I - \text{diag}(c_i)] &= 0 \text{ for all } i = 1, \dots, n, \\ \text{and } eC &= e \end{aligned} \tag{16}$$

By the second condition  $c_{ij} \in \{0, 1\}$  and, by the first,  $c_{ij} = 1$  implies  $\vartheta_{ij} + \sum_{k=1}^m c_{ik} \sigma_{kj} \geq \vartheta_{hj} + \sum_{k=1}^m c_{hk} \sigma_{kj}$  for all  $h = 1, \dots, n$ . The last condition ensures that for each company  $j = 1, \dots, m$  there is some investor  $i = 1, \dots, n$  such that  $c_{ij} = 1$ .

Note the formal similarity of these conditions to problem (13). In general, let  $N_{it} \subset \{1, \dots, n\}$  be a subset of the set of investors with cardinality  $n - t$  which does not contain  $i$ , i.e.  $|N_{it}| = n - t$  and  $i \notin N_{it}$ , and define control



coefficients  $C_t = [c_{ij}^t]$  of order  $t = 1, \dots, n - 1$  as (real) solutions to

$$\begin{aligned} & \max_{C_t} eC_t e & (17) \\ \text{s.t. } & \left[ \vartheta_i + c_i^t \Sigma - \sum_{j \in N_{it}} (\vartheta_j + c_j^t \Sigma) \right] \text{diag}(c_i^t) \geq 0 \text{ for all } N_{it} \subset \{1, \dots, n\} \\ & \text{and } c_i^t [I - \text{diag}(c_i^t)] = 0 \text{ for all } i = 1, \dots, n, \\ & \text{and } eC \leq e \end{aligned}$$

Hence, for  $t = 1$  this boils down to the *sufficient* condition (13) (resp. (12)) for control. And for  $t = n - 1$  it boils down to the *necessary* condition (16) for control. Note again that the second of the three constraints is only there to ensure that  $c_{ij}^t \in \{0, 1\}$  for all  $i = 1, \dots, n, j = 1, \dots, m$ , and  $t = 1, \dots, n - 1$ .

Passing from the sufficient condition to the necessary condition requires an algorithm for computing the control coefficients under investor ‘coalitions’ of varying sizes (the  $N_{it}$ ). First the largest coalition is tested for control. The control coefficients for that coalition are then calculated, and the economy then redefined (as in Section 3.1) so that those controlling investors are given full ownership of the firm. The algorithm then passes to the next level of coalitions, the control coefficients are recalculated, etc. and the algorithm proceeds until the necessary condition is reached. Structured in this way, each control coefficient calculation captures a new level of ownership not found in earlier stages—and by definition, the control coefficients of the previous stages must satisfy the inequalities of the following stages. This may be seen simply by comparing the sufficient and the necessary conditions—those investors who exercise full control at the sufficiency level, by having imputed shares greater than the entire population of other investors, must also have imputed shares greater than each investor—and thus full control at the necessary level. The development of this algorithm to calculate control coefficients from empirical data is a subject of current research (see Concluding Remarks).

## 5 Concluding Remarks and Current Research

In estimating the degree of ownership concentration in East Asia and Western Europe Claessens et al. (2000) and Faccio and Lang (2001) have found that the primary mechanisms for separating ownership from control appear to be dual-class shares and the pyramid structure. Pyramids, for example, are particularly prevalent in East Asia, where they comprise nearly 40% of the firms (nearly 67% in Indonesia alone).<sup>5</sup> Cross-ownership structures,

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<sup>5</sup>Claessens et al. (2000) p. 93.

although not insignificant, do not appear to play as important a role (the largest examples being, for example, only 2.69% of firms controlled at the 20% level in Germany, and 2.04% in Norway<sup>6</sup>). The foregoing analysis sheds light upon one possible reason for the low presence of cross-ownership, in that the methods of calculating voting rights from cash-flow rights lead to an underestimate of the separation of ownership from control in the presence of cross-ownership. Using accounting identities we demonstrate that it is possible to recover the control of a firm from the underlying cross-ownership structure in a way that preserves all possible connections between firms in an economy. It is also shown that a given economy may have multiple compatible ownership structures, in which control over both cash-flow rights and voting rights may either be jointly held or completely separated.

Of greater interest perhaps is how the identities and control coefficients fit into a full theoretical model of cross-ownership, in which firm investment levels are optimally selected by investors, and industry ownership structures are then determined. This would allow one to assess the general existence and likelihood of various cross-ownership structures (including pyramid structures) arising as the result of optimizing behavior on the part of investors and firms. As a full theoretical treatment appears to be lacking in the current literature, current research is focusing upon such a model of cross-ownership.

In addition, as defined in Section 4 the control coefficients also allow for empirical estimation of the degree of cross-ownership from data, using a recursive algorithm which traverses from the sufficient to the necessary conditions for control. Using this algorithm it should be possible to extract the controlling interest structure from data which uses the entire matrix of cross-ownership linkages. Naturally the application of such an algorithm is hampered not only by the size of such a matrix (which may be on the order of thousands of firms) but also by the fact that all possible coalitions must be tested in order to calculate the control coefficients of a particular coalition size. This is a combinatorial problem which, as the number of firms rises, becomes computationally prohibitive. For example, with only 1,000 firms one must naively check  $\binom{999}{1,000-t}$ ,  $t = 1 \dots 999$  different firm combinations for each  $t$ , for each firm, in order to calculate the control coefficients. Methods of optimizing the computation of these and similar conditions and applying the control coefficient technique to data are also currently underway.

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<sup>6</sup>Faccio and Lang (2001) p. 17.

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Authors: Klaus Ritzberger, Jamsheed Shorish

Title: Cross-Ownership Among Firms: Some Determinants of the Separation of Ownership from Control

Reihe Ökonomie / Economics Series 113

Editor: Robert M. Kunst (Econometrics)

Associate Editors: Walter Fisher (Macroeconomics), Klaus Ritzberger (Microeconomics)

ISSN: 1605-7996

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