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Adusei Jumah
Robert M. Kunst

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Author(s):
Adusei Jumah, Robert M. Kunst

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Institut für Höhere Studien (IHS), Wien
Institute for Advanced Studies, Vienna

Contact:

Adusei Jumah
Department of Economics at BWZ
Bruenner Strasse 72
A-1210 Wien / Vienna, Austria
email: adusei.jumah@univie.ac.at
and
European Central Bank

Robert M. Kunst
Institute for Advanced Studies
Department of Economics and Finance
Stumpergasse 56
A-1060 Wien / Vienna, Austria
☎: +43/1/599 91-255
email: kunst@ihs.ac.at
and
Department of Economics at BWZ
Bruenner Strasse 72
A-1210 Wien / Vienna, Austria
email: robert.kunst@univie.ac.at

Founded in 1963 by two prominent Austrians living in exile – the sociologist Paul F. Lazarsfeld and the economist Oskar Morgenstern – with the financial support from the Ford Foundation, the Austrian Federal Ministry of Education and the City of Vienna, the Institute for Advanced Studies (IHS) is the first institution for postgraduate education and research in economics and the social sciences in Austria. The **Economics Series** presents research done at the Department of Economics and Finance and aims to share "work in progress" in a timely way before formal publication. As usual, authors bear full responsibility for the content of their contributions.

Das Institut für Höhere Studien (IHS) wurde im Jahr 1963 von zwei prominenten Exilösterreichern – dem Soziologen Paul F. Lazarsfeld und dem Ökonomen Oskar Morgenstern – mit Hilfe der Ford-Stiftung, des Österreichischen Bundesministeriums für Unterricht und der Stadt Wien gegründet und ist somit die erste nachuniversitäre Lehr- und Forschungsstätte für die Sozial- und Wirtschaftswissenschaften in Österreich. Die **Reihe Ökonomie** bietet Einblick in die Forschungsarbeit der Abteilung für Ökonomie und Finanzwirtschaft und verfolgt das Ziel, abteilungsinterne Diskussionsbeiträge einer breiteren fachinternen Öffentlichkeit zugänglich zu machen. Die inhaltliche Verantwortung für die veröffentlichten Beiträge liegt bei den Autoren und Autorinnen.

Abstract

Using data from Germany, Japan, UK, and the U.S., we explore possible threshold cointegration in nominal short- and long-run interest rates with corresponding inflation rates. Traditional cointegration implies perfect mean reversion in real rates and hence confirms the Fisher hypothesis. Threshold cointegration accounts for the possibility that this mean reversion is active only conditional on certain threshold values in the observed variables. We investigate whether findings of such effects can be exploited for interest rate prediction.

Keywords

Nonlinear time series, Fisher equation, yield spread, forecasting

JEL Classifications

C32, C53, E43

Comments

Contents

1 Introduction

Empirical studies on the stationarity of real interest rates present mixed results (e.g., ROSE, 1988; PHILLIPS, 1998; CHOI AND AHN, 1999; RAPACH AND WEBER, 2001). Stationarity implies that each real interest rate variable, although fluctuating, tends to return to a constant mean (hence, the expression 'mean reverting'). Conversely, a non-stationary variable would display non-constant means and variances. Why should real interest rates be non-stationary? Several reasons have been advanced to explain why real interest rates, every so often, exhibit non-stationary behavior. ROSE (1988) attributed non-stationarity in real interest rates to the 'stationary behavior of the inflation variable', while BEVILACQUA AND ZON (2001) view the use of linear models to explain fluctuations in macroeconomic time series as flawed. Some have argued that stationarity in real interest rates can only be attained by accounting for the increasing integration of financial markets and allowing for global influences on national bond rates (e.g., ANDERSON, 1999; WU AND ZHANG, 1997). Others such as CAMPBELL AND SHILLER (1987) and HALL *et al.* (1992) suggest the joint analysis of interest rates at different maturities or on related markets, i.e., yield spread. BEVILACQUA AND ZON (2001) recommend a dynamic and non-linear explanation for the double aim of describing and forecasting more accurately the evolution of the macroeconomic system.

The incidence of non-stationary real interest rates is incompatible with many macroeconomic and finance theories. Non-stationarity in real interest rates implies that long-run variations in nominal interest rates are not simply reflections of variations in inflationary expectations but are also caused by changes in the real rate. For this reason, the effects of monetary policy on inflation will become less predictable. GALÍ (1992, p.717) adds that non-stationary behavior of real interest rates in empirical models seems rather implausible on *a priori* grounds, due to its inconsistency with standard equilibrium growth models. Also, following LUCAS (1978) and HANSEN AND SINGLETON (1982), among others, ROSE (1988) argues that non-stationarity in real interest rates is problematic for consumption-based asset pricing models.

Hence, the prevailing viewpoint is that nominal interest rates as well as the rate of inflation are to be modeled as first-order integrated processes. Differences among these variables then yield stationary 'cointegrating' relations, such as the stationary yield spread and the stationary real interest rate. Acceptance of this viewpoint is not unanimous. For example, WU AND ZHANG (1997) apply panel unit root tests to yields on treasury bills and provide evidence for stationarity, whereas BEYER AND FARMER (2001) find cointegrating relations among non-stationary interest rates and inflation that differ from simple differences and thus provide evidence for non-stationary real rates. In this paper, we show that such apparently divergent and incompatible pieces of evidence may be encompassed with the aid of non-linear time-series models.

Rather than directly focusing on real interest rates, we explore the joint behavior of nominal interest rates and inflation by means of bivariate time-series models. To achieve this aim, we modify the traditional linear framework of cointegrated vector autoregressions by introducing some non-linear characteristics. We adopt the threshold cointegration approach of Balke and Fomby (1997), henceforth BF, to establish a stable relationship between the respective nominal interest rates and inflation for Germany, Japan, the UK and the US. We hypothesize that the failure of former studies to find cointegration between the nominal interest rate and inflation is due to their inability to account for the threshold characteristics of nominal interest rates. Finally, we evaluate the forecasting performance of the threshold cointegration model against competing models.

The paper is structured as follows. Section 2 highlights some stylized facts on interest rates and threshold cointegration. Section 3 presents the results of the threshold cointegration analysis, while section 4 elaborates on the results of the forecasting experiments. Section 5 concludes.

2 Threshold cointegration

2.1 Stylized facts on interest rates

Although the challenge of applying time-series modeling to interest rates has been taken up repeatedly in the literature, most models have been found to be slightly unsatisfactory, either because they become invalid over longer time intervals or because they do not match important features of the data or because they are at odds with economic theory. Because time-series analysis aims at matching the empirical stylized facts as closely as possible, we first summarize the main characteristics.

The first stylized fact on interest rates is that deviations from the pure *random-walk model* are small but significant. The random-walk model has been found to be appropriate for stock-market prices. Contrary to stock-market returns, however, first differences of interest rates do contain some substantial autocorrelation at shorter and longer lags. It is not always possible to exploit this correlation patterns for reliable prediction. Many authors have found that the explained share of variance (R^2) decreases as the time to maturity increases, such that long-term bond rates come close to pure random walks.

The second stylized fact is that, in the longer run, interest rates *remain in an interval* that is approximately determined by the lower bound of zero and the upper bound of around 10%. This fact reflects the economic adjustment mechanism that is primarily enacted by the stabilizing influence of central banks. Although the paradox of negative nominal interest rates has been reported for specific episodes in specific countries, the lower bound of zero can be regarded as sharp, as economic agents will not lend money if they are rewarded by a loss. The upper bound is less sharp and may be pushed up during phases of high inflation. Because of widespread international agreement about the dangers of high inflation, even these phases will usually remain episodes of limited life span.

Apparently, the second stylized fact contradicts the first one, as random walks display non-stationary behavior due to an ever-increasing variance and an unbounded support. In the literature, the typical way out has been to view the random-walk model or its weakened form, the first-order integrated model, as an approximation for a limited time span. Building on the integrated model, for example CAMPBELL AND SHILLER (1987) and HALL *et al.* (1992) have gained interesting insights into the joint movements of interest rates at different maturity, which leads to the third stylized fact.

The third stylized fact on interest rates is that rates at different maturities or on related markets, such as the bond market and the money market, strongly indicate parallel movements as they develop through time. Accepting the integrated model as a working hypothesis, CAMPBELL AND SHILLER (1987), HALL *et al.* (1992), among others, have found that interest rates tend to be *cointegrated* in the sense that a linear combination is stationary. Usually, this linear combination has been found to be the difference or *yield spread*, such that short and long rates are separated by a stationary term premium with a time-constant mean.

Other empirical features of interest rates have been identified, such as evidence on non-normal distributions, highly non-normal kurtosis, conditional heteroskedasticity, long memory etc. It appears impossible to take up all these issues simultaneously. Hence, within the limits of this paper we refrain from modeling higher-order moments and exclusively focus on modeling the conditional expectation. We also set aside fractional and long-memory models, which may be seen as an alternate approach to the one outlined here. This model class suffers from a severe increase in complexity when multivariate applications are studied. In contrast, the threshold approach rests on only slight adjustments to the standard linear models and is explicitly designed for multivariate time series.

As BF observed, the forces which have been identified by the cointegration modelers and which tend to tie together different interest rates may be absent for very small deviations and gain strength as the yield spread increases. This observation has led BF to consider the concept of *threshold cointegration* in the sense that two (or more) variables behave like mutually independent integrated processes if the yield spread is small and behave like cointegrated processes if the yield spread surpasses a certain threshold value, which triggers an error-correcting mechanism. We also consider another threshold, above which all interest rates are stabilized by political forces, such that the support remains bounded in the longer run. Although this second mechanism may be the more important one, it has been ignored in most of the extant literature.

2.2 Stylized facts on inflation

At a first glance, the longer-run characteristics of inflation time series resemble those of interest rates. However, unlike interest rates, neither price nor wage inflation face strict lower bounds. Prolonged periods of deflation, i.e., negative inflation, are known from historical data. While most observations on annual inflation in most countries fall into the range between zero and ten percent, sizeable violations of these limits are known to occur.

Another distinction to interest rates is that inflation is significantly *predictable*. There is a widespread belief that last year's inflation is a good forecast for this year's inflation. This view is also reflected in textbook descriptions of variants of the Phillips curve (see BLANCHARD, 1999). Assuming expectations to be rational would imply that inflation obeys a random walk. The fact that *changes in inflation* are also predictable invalidates this random-walk model. In analogy to interest rates, one also observes that a fall in inflation is more probable when inflation is high and a rise in inflation is more probable when inflation is low. One reason for this 'mean reversion at the extremes' is likely to be the policy reaction of monetary authorities.

In summary, inflation like interest rates behaves like a first-order integrated process for lengthy episodes, while its behavior will be governed by mean reversion at extreme and socially unacceptable values.

2.3 Threshold cointegration, the yield spread, and the Fisher hypothesis

Let us assume a long rate i_{Lt} and a short rate i_{St}. Then, we search for a time-series model that mirrors the aforementioned features: firstly, both series should individually resemble first-order integrated series over substantial time horizons; secondly, both series should be stochastically bounded over long time horizons; thirdly, their difference should be recognizable as being stationary even over relatively short horizons. These three features are mirrored in the following threshold cointegration model

$$\begin{aligned} \begin{bmatrix} \Delta i_{St} \\ \Delta i_{Lt} \end{bmatrix} &= \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} (i_{Lt} - i_{St} - \rho) \\ &+ \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} (i_{Lt} - \iota)\, I(\{i_{Lt} > \iota_o\} \cup \{i_{Lt} < \iota_u\}) \\ &+ \textstyle\sum_{j=1}^{p-1} \Gamma_j \begin{bmatrix} \Delta i_{St} \\ \Delta i_{Lt} \end{bmatrix} + \varepsilon_t \quad . \end{aligned} \tag{1}$$

In this model, one error-correction vector is *always* operating, i.e., the vector $(1, -1)$, whereas the second vector $(0, 1)$ is only 'switched on' when the long interest rate i_{Lt} falls beneath a certain lower bound ι_u, which may be interpreted as the situation known as 'liquidity trap' from the Keynesian literature and which is typically avoided by financial markets, or when an upper bound ι_o is surpassed, which may delineate a maximum of tolerable inflation. In these extreme cases, where the policy makers' attention is aroused, a target rate ι is focused. Once the system returns to its normal state, the target rate is forgotten and a term premium ρ separates the long and short rates, which are furthermore subjected to the short-run dynamics as dictated by $\Gamma_j, j = 1, \ldots, p-1$ which may reflect inertia or the business cycle. The stability and geometric ergodicity of the model (1) follows from general statistical results (see TONG, 1990), given that the parameters $\alpha_j, \gamma_j, \Gamma_j$ follow the stability conditions for the linear model and that the target rate ι is in the range (ι_u, ι_o). We have also experimented with the short rate and an average rate, i.e., with the vectors $(1,0)$ and $(1,1)$ as candidates for the second error-correcting influence, and we have found from empirical analysis that the long rate generally yields the best results.

The model permits an insightful geometric interpretation in the (i_S, i_L)–plane. In a linear world, the cointegrating rank dictates a rather limited choice set of forms of dynamic equilibrium. A rank of zero yields a system without any equilibrium or, equivalently, with the whole plane representing equilibria; a rank of one yields a system with a straight line as the locus of equilibria; a rank of two has a properly defined distributional mean as a unique equilibrium point. The threshold model restricts the set of equilibria to a line *segment*. The line segment evolves from intersecting the line $i_L = i_S + \rho$ with the area $\{\iota_u < i_L < \iota_o\}$. Within this strip, the line segment serves as an attractor, while from outside the intersection of the line with the horizontal line $i_L = \iota$ is targeted, i.e., the point $(\iota - \rho, \iota)$. The location of the line segment reflects the term premium, while the position of the strip reflects the concerns of controlling inflation.

In our second model, we model one of the two interest rates jointly with price inflation, as defined from a consumer price index p_t via $\pi_t = 100 \ln (p_t/p_{t-12})$. The threshold model is very similar, with the stationary term premium ρ being replaced by a stationary real rate as suggested by the Fisher hypothesis.

While model (1) focuses on a solution for the problem of the long-run behavior of yield spreads and interest rates, the original contribution by BF concentrates on another aspect. These authors find evidence for threshold behavior in the primary error-correcting mechanism in the sense that this mechanism might only be activated in the presence of severe deviations from the equilibrium. While the BF model deserves attention, our own view is that the puzzle of the long-run behavior is conceptually more important. Both ideas can be integrated into the model structure of (1) in the following way:

$$\begin{aligned} \left[\begin{array}{c} \Delta i_{St} \\ \Delta i_{Lt} \end{array} \right] = \left[\begin{array}{c} \alpha_1 \\ \alpha_2 \end{array} \right] &(i_{Lt} - i_{St} - \rho)\, I(\{i_{Lt} - i_{St} - \rho > \kappa\} \cup \{i_{Lt} - i_{St} - \rho < \kappa\}) \\ &+ \left[\begin{array}{c} \gamma_1 \\ \gamma_2 \end{array} \right] (i_{Lt} - \iota)\, I(\{i_{Lt} > \iota_o\} \cup \{i_{Lt} < \iota_u\}) \\ &+ \textstyle\sum_{j=1}^{p-1} \Gamma_j \left[\begin{array}{c} \Delta i_{St} \\ \Delta i_{Lt} \end{array} \right] + \varepsilon_t \quad . \end{aligned} \tag{2}$$

In this model world, the attractor is not a line segment but a segment of a strip with the boundaries (in clockwise order) $\{i_L = \iota_o\}$, $\{i_L = i_S + \rho - \kappa\}$, $\{i_L = \iota_u\}$, $\{i_L = i_S + \rho + \kappa\}$. There is no predilection of the market for the center of the quadrangle. As long as the long rate i_L is inside the horizontal strip $\{\iota_u < i_L < \iota_o\}$, the entire quadrangle serves as an attractor, while once the long rate leaves the strip, only the horizontal line segment $(\iota - \rho - \kappa, \iota - \rho + \kappa) \times \{\iota\}$ is targeted. This paper considers both models (1) and (2), which will be referred to, respectively, as the "single-threshold" and the "double-threshold" models.

The bivariate system in (1) and (2) for the two interest rates i_L and i_S will be referred to as the yields-spread model. An analogous system for a specific interest rate and inflation π will be referred to as the Fisher-effect model. In the Fisher-effect model, the real rate $i_j - \pi$ for $j = S, L$ replaces the yield-spread as the first error-correction variable and inflation π replaces the second error-correction variable.

2.4 Estimation and inference

Due to their non-linear form and their discontinuities, the models (1) and (2) entail special statistical problems. Firstly, their validity may be of interest as compared with simplified structures, such as a vector autoregression. Unfortunately, most of the known statistical procedures aiming at testing for threshold cointegration, such as modifications of the Dickey-Fuller test, are asymptotic in nature. Their feature of interest, such as a break or threshold of a particular form, may be superseded by a variety of other sample-specific features, such as strong deviations from distributional assumptions, and may be extremely unreliable in comparatively small samples. It is certainly of more interest to *estimate* the indicated model and to subject it to potential simplification steps. We then use a comparative evaluation of out-of-sample prediction based on selected models. While this check may not provide reliable evidence on the 'validity' of the models, as misspecified models are often reported to be good forecasting 'workhorses', we view model selection as an intermediate step toward the final aim of prediction.

Estimation of threshold models brings in additional problems. Maximum-likelihood estimation is possible in principle, though it may suffer from poor convergence properties. Particularly, estimation of narrow-sense threshold parameters may hinge critically on few observations, which discourages any statistical refinement. Hence, we prefer to apply a sequence of specification steps that may yield a reasonable quality for the estimated structures:

1. A linear lag-order search is guided by the AIC criterion and a lag order p is determined.

2. A linear VAR cointegration analysis uses $p-1$ conditioning lags and yields a cointegrating rank (see JOHANSEN, 1995). In our bivariate models, a rank that is 'close' to the rank of one suggested by theory is of special interest. Our simulation experiments have confirmed that such an empirical rank of one is typical for structures such as (1), when the true rank is two, as the system is stationary, and a rank of one is in operation for the vast majority of the observations. The test decision will be reported but it is not seen as binding, due to the potential nonlinear effects. The first canonical vector from the JOHANSEN analysis is a candidate for an error-correction vector, though we generally focus on the theory-based vectors $(1, -1)$ and $(0, 1)$.

3. For model (1), the threshold values ι_u and ι_o are determined by varying the cut-off points of i_L over empirical fractiles such as 1% (99%), 5% (95%), 10% (90%). This step results in an optimum threshold model, while we impose a restriction of symmetry, which can be compared to the

linear model of the previous step. Because the properties of the likelihood-ratio test are somewhat uncertain, due to failure of regularity conditions and due to the optimization step, we will not focus on the decision by hypothesis tests.

For model (2), we add another step similar to step 3, in which we vary the empirical fractiles of the second error-correction variables. By construction, this estimation procedure is designed for sample sizes of 100 to 200. For very large samples, it may be convenient to improve the procedure by iterating the indicated sequence of steps, by bootstrapping the rank determination, and by refining the grid in step 3, in order to obtain consistent estimates. For smaller samples, particularly the fact that the parameter estimates from the 'outer' or tail regimes rest on a few observations only discourages such modifications.

3 Empirical results: model estimation

We apply the modeling ideas to monthly observations from January 1986 to December 2000 on interest rates for four main economies: Germany, Japan, the United Kingdom, and the United States. In the results tables 1–6, we report some statistics for each of the applied models. Firstly, we show the lag order p that is identified by AIC on the basis of an unrestricted VAR for the original data, i.e., for bivariate systems comprising two interest rates or one interest rate and the inflation rate π. Based on the order p for the 'level' model, $p-1$ lags of the differenced variables were included in the conditioning step of the JOHANSEN procedure and as 'short-run effects' in all linear and non-linear models. In some cases, the increased descriptive power of the more sophisticated threshold models may suggest reducing the lag order but we feel that this modification is of little importance.

Secondly, we evaluate the descriptive power of all models by the log-determinant criterion, which is proportional to the likelihood and which may be viewed as an information criterion without penalty term. These log-determinants could be used for formal likelihood-ratio tests. For reasons given above, we prefer to view them as descriptive guidelines for the specification procedure. The final aim of our models is prediction, hence a comparative evaluation will preferably rely on the actual predictive power of the model, not on in-sample hypothesis tests.

Thirdly, we give t–values for all untruncated and truncated error-correction variables, i.e., for the α coefficients in (1) and (2). We denote the α coefficients in the long-rate equation by α_L, those in the short-rate equation by α_S, and those in the inflation equations by α_π. In these equations, the respective 'regressands' are Δi_L, Δi_S, and $\Delta\pi$. Again, the null distributions of these coefficients may be different from the usual standard normal approximation, due to the pre-testing problem and other sources. We also give t–values for a linear cointegration model with a prespecified rank of one and a prespecified untruncated first error-correction variable, i.e., the yield spread or the real rate in the corresponding models.

3.1 The Fisher-effect experiments

For each country, we consider two versions of the Fisher-effect systems: one with the long rate and inflation, and one with the short rate and inflation. According to economic theory, each of the two interest rates individually cointegrates with π, via the real-interest cointegration vector $(1, -1)$. We already remarked, however, that the original variables i_L or i_S and π may not be first-order integrated when they are observed for long time spans. As potential second error-correcting vectors, we could consider the interest rates, the inflation rate π, or an average (or sum) of the two, as inflationary indicators. We have found that the inflationary indicator π yields the best results for most cases, hence we restrict attention to this specification. While *any* second cointegrating vector in a bivariate system makes the system and hence both variables stationary, this choice considerably affects the threshold variants.

For the bivariate models that consist of the *long* rate and inflation, the preliminary VAR analyses via AIC yields shorter optimum lag lengths of $p = 4$ and $p = 5$ for the United States and Japan and rather long lag orders for Germany and the United Kingdom. The null hypothesis of no cointegration is marginally rejected in general, while the second canonical roots remain small and insignificant. The freely identified error-correction vectors come close to $(1, -1)$, which is quite in line with the real interest rate and the Fisher hypothesis. In most cases, the real interest rate $i_L - \pi$ exerts its error-correcting effect mainly in the $\Delta\pi$ equation of the system. This would indicate that inflation tends to correct shocks to the real rate. The performance of the threshold models differs across countries. The strongest threshold effects are visible in the British data set. In Germany and in the United States, the double-threshold model does not attain the performance of the single-threshold model. Excepting the United Kingdom, also the (truncated) second error-correction variable exerts its main influence on inflation. The long interest rate does not react directly to inflationary tendencies that are represented by deviations from the mean in π. In Germany and Japan, i_L is found to be exogenous for the long-run parameters. In other words, equilibrium adjustment is performed by adjusting π in all specifications.

For the bivariate models that consist of the *short* rate and inflation, the threshold models fail to offer much advantage over error-correction models without thresholds. The freely estimated cointegrating vectors differ from the theoretical real-rate vector $(1, -1)$. The closest vector is obtained for the United States. In contrast, evidence on cointegration on the basis of JOHANSEN's trace statistic increases. Hence, the general evidence on the Fisher effect is mixed. Compared with the results for the long rate, reaction to disequilibrium is more diversified, with the short rate bearing part of the burden of the adjustment. This conforms well to the impression that short rates usually react faster than long-term interest rates.

3.2 The yield spread experiments

For the *German data*, the preliminary VAR analysis via AIC yields an optimum lag length of $p = 2$. The JOHANSEN trace test accepts the null hypothesis of a zero cointegrating rank. Given the fact that the unit-root test as a consequence of the JOHANSEN procedure is more powerful than any univariate test of the DICKEY-FULLER type, one is tempted to conclude that both interest rates are first-order integrated, without cointegration.

As in the Fisher-effect experiments, we at first considered the freely estimated canonical vectors from the JOHANSEN algorithm. Due to the uncertainty of estimating these vectors in a possibly non-linear system, we generally preferred to replace these with the theory-based vectors $(1, -1)$ and $(0, 1)$ that occur in the representation (1). The absolute t–values of the tentatively included lagged yield spread remain small, 0.03 for the long-rate equation and 1.57 for the short-rate equation. If the latter value is seen as indicating an error-correcting influence, the former value supports weak exogeneity of the long rate 'for the long-run parameters', in accordance with the literature. The log-determinant criterion (AIC without penalty) is only slightly better than for a VAR in differences. Padding this system by additionally including the lagged long rate as another error-correcting influence again results in small t–values and an impalpable improvement of the log-determinant. Note that this system is equivalent to an unrestricted VAR in levels and only uses a different parameterization. In the models with threshold cointegration, the truncated variable i_L is used as a second error-correction vector. The log-determinant criterion is optimized for a 50% truncation, i.e., values outside the sample quartiles are included. The t–values for $i_{L,t-1}$ are -0.10 for the long-rate equation and 1.63 for the short-rate equation. The latter, only marginally significant, value may indicate that inflationary stabilization works through the short rate. These t–values, however, may be distorted by a search bias. The influence of the yield spread $i_{Lt} - i_{St}$ remains insignificant in the long-rate equation (t–value of -0.10), thus also indicating weak exogeneity of the long rate with respect to mean reversion in the yield spread.

For the *UK data*, AIC identifies a lag order of $p = 2$. The JOHANSEN trace statistic is insignificant and suggests the absence of error correction. If the first canonical vector $(1.13, -0.79)$ is applied as a cointegrating vector nevertheless, its influence remains insignificant, though the t–value is slightly higher in the long-rate equation. The second canonical vector shows its best fit for the 50% truncation, when both vectors, i.e., $(1.13, -0.79)$ with its influence unrestricted and $(0.98, 0.20)$ reduced to its tail areas, cause marginally significant error correction. We again prefer to replace these vectors with their theory-based counterparts $(1, -1)$ and $(0, 1)$. Also for the theoretical vectors, the minimum is obtained for 50% truncation. For this specification, *both* error correction vectors become significant in the long-rate equation, while the truncated and lagged long rate also reaches a t–value of -1.75 in the short-rate equation. The log determinant is markedly below all other versions. We conclude that the long rate is indeed stationary, that hence the entire UK system is stable, and that the UK short rate is simply not in the focus of the action, as both regularization of the yield curve and anti-inflationary policy are working primarily via the long rate. Hence, while the long rate appears weakly exogenous for Germany, it is rather the short rate that is almost, though not quite, weakly exogenous for the UK.

An even better result is obtained if also the *first* error correction vector, i.e., the yield spread, is used in a truncated version in the spirit of (2). Reducing the influence of this vector to the 20% tails region reduces the log determinant considerably and increases the significance of the error-correcting effects in the long-rate equation. This result is in line with the idea of BF that error correction operates at stronger deviations from the equilibrium only. Together with the idea that is in focus here, i.e., that long-run stationarity is attained by the self-stabilizing forces of the socioeconomic system, we obtain the most accurate description. We also attempted an exhaustive investigation over combinations with respect to the fractile areas with regard to the first and second vectors. The outlined structure was confirmed as the optimum.

For the *US data* set, AIC selects a lag order of $p = 4$. Based on this lag order, the JOHANSEN procedure again yields an insignificant trace statistic, which suggests a system of first-order integrated interest rates without error correction. If the yield spread is nevertheless viewed as cointegrating, its influence remains small. The estimated first canonical vector does not match the theoretical considerations. The positive linear combination of interest rates is far from the yield spread and rather points to a weighted average. This would literally mean that the interest rates are 'more stationary' than the spread, which is hard to believe. Hence, we discard the first estimated canonical vector from further experiments and replace it by the imposed theoretical yield spread. For the threshold variants, the error-correcting influence by the yield spread remains insignificant. The second potential error-correcting variable, the lagged truncated long rate, attains marginal significance in the short-rate equation. Again, the exogeneity of the long rate is confirmed.

The *Japanese* data are exceptional in some aspects. The AIC–selected lag order of $p = 7$ is larger than for the other countries. The same holds for the trace statistic, though it does not attain statistical significance. Similarly to the British data, the error correction mechanism appears to work via the long-rate equation. The yield spread, in contrast to the estimated canonical variable, remains insignificant in the linear cointegration model. However, it attains significance in the long-rate equation, once the truncated lagged long rate is inserted as a second error-correction variable. In the double-threshold model, the picture is reverted, with the short rate reacting to the truncated long rate. The observed effects can be interpreted in the following sense. An unusually high yield spread or an inverted yield curve cause a reaction of the long rate that re-establishes the typical term premium. Unusually high interest rates, which may indicate inflation, are stabilized via the short rate that reacts faster to monetary policy. There is no evidence on exogeneity of the long rate in the Japanese economy.

4 Empirical results: Forecasting

In this section, we focus on a comparative evaluation of forecasting performance of the identified model structures. We note that the relative performance of models in the identification and estimation stage need not coincide. Two different types of evaluation will be presented. Firstly, an extensive ex-post evaluation indicates the quality of in-sample tracking. Secondly, an ex-ante prediction exercise is executed over a limited time period at the end of the sample.

4.1 Ex-post tracking performance

For the ex-post evaluation, the parameters are taken from the full-sample estimates. Because some of the models are non-linear, stochastic forecasting is utilized. Innovations are drawn from a normal distribution, with the standard deviation corresponding to the estimated standard deviation from the estimation stage. Due to the comparative mildness of the non-linearity, we restrict ourselves to 100 replications. The forecast x_t^f, say, for x_t is defined as the average over these replications. For the forecast errors $x_t - x_t^f$, we give means, means of absolute values, and means of squares, in all cases over $t = t_0, \ldots, T$, where t_0 is set at 20, in order to exclude all potential problems with longer lags at the start of the sample. We note that $T = 180$.

We report the results from the Fisher-effect models, with the long rate and inflation (Table 7) as well as with the short rate and inflation (Table 8). In these and all following tables, 'winners' with the lowest forecasting statistic are indicated by underlined numbers. In most cases, the threshold models turn out to track better than the unrestricted VAR, excepting some inflation forecasts. The pure model in differences comes in last, in spite of the statistical support for no cointegration. For the short rate, with the results given in Table 8, the performance is more amenable to the unrestricted VAR. Inflation forecasts are better in the short-rate version for Japan and the United Kingdom, whereas German inflation is described better using the long rate in Table 7.

For the interest-rate or yield-spread systems, results are given in Table 9. The predictive accuracy for interest rates deteriorates relative to the systems with inflation, excepting Japan. Hence, inflation is more informative for interest-rate forecasts than just another interest rate, as the MAE and MSE values are lower than in Table 9.

Generally, the differences among the models are small, excepting the single-threshold model for the US data set, which is distorted by a local outlier. Note that the double-threshold model does not show this problem. A cursory comparative evaluation of a horse race among the four specifications—single threshold, double threshold, unrestricted VAR, VAR in differences—sees each of the threshold specifications and the unrestricted VAR in front in a third of all cases, with the differenced VAR coming in way behind the other models. Only two cases are dominated by the VAR in differences, which is often reported in the literature as a good forecasting workhorse even for data-generating processes where it is misspecified, such as cointegrated VARs. It appears that the single-threshold model wins the horse race with respect to in-sample tracking performance at hair's width.

4.2 Ex-ante forecasting performance

Of special interest for prediction is the out-of-sample forecasting performance. In these experiments, parameter estimates are taken from a sample that is truncated at the end so that it covers $t = t_0, ..., t_1$, where t_0 is chosen large enough that all required lags exist. These parameter estimates and the sample are used to predict the observation at $t = t_1 + \tau$ by means of stochastic prediction. This experiment can be repeated by increasing t_1 to $t_1 + 1, t_1 + 2, \ldots$ until $T - \tau$ is reached. Finally, measures of predictive accuracy are averaged. In the interest of brevity, we focus on the cases $\tau = 1$, i.e., the one-step forecast that is comparable with the ex-post tracking experiments, and $\tau = 12$ which corresponds to a forecast for the next year.

It is debatable how much of the model structure is to be kept constant during these experiments. In most reported evaluations, lag orders and cointegrating ranks are identified from the full sample and are then imposed on all reduced samples. We follow this convention, including the percentage of truncation but excepting the thresholds, which are updated continually. For the Fisher-effects systems that consist of the long rate and inflation, results are given in Table 10, whereas, for the short rate and inflation, results are summarized in Table 12. For twelve-step forecasts, an analogous evaluation is reported in Tables 11 and 13. For the long rates, we observe that all four models are of comparable predictive value if only one step is done into the future. If the forecast horizon is extended to twelve, the double-threshold model fails to keep pace with the simple linear models. For half of the variables, the VAR in differences dominates, which indicates that mean-reverting behavior cannot improve prediction even over a time span of twelve months. The poor estimates of the threshold parameters are likely to be responsible for this feature. For the short rates, the general impression changes. At the one-step horizon, the unrestricted VAR and the double-threshold models show the best performance, with the threshold model dominating for interest rates and the unrestricted VAR dominating for inflation. At the longer horizon, the double-threshold model takes the lead in even more cases. We note that none of these features corresponds to the rankings in the tracking performance.

Results for the systems that consist of the two interest rates are given in Table 14 and 15. While for the one-step prediction, as in in-sample tracking, the performance is generally worse than for the Fisher-effects systems that include inflation, the impression is reversed for twelve-step prediction. In other words, a dynamic model that consists of interest rates gives more reliable longer-run predictions than a dynamic model that consists of the interest rate to be predicted and inflation. Generally, the simple VAR in differences dominates at both horizons. Mean reversion and threshold effects can be exploited for some series only, such as for the U.S. rates and the short rate in the United Kingdom. This comparative impression hardly changes between the one-step experiment in Table 14 and the twelve-step experiment in Table 15.

These results corroborate the traditional rule that simplicity is an asset for prediction. Even when the threshold forms are correct, there may be too few observations in the distributional tails to estimate crucial parameters with the necessary precision. Although first-order integrated models cannot be valid representations of the data-generating mechanisms of interest rates and inflation, the VAR model in differences, which is implied by first-order integrated processes without cointegration, is the most robust workhorse tool for prediction. The noteworthy exception is the system that comprises short rates and inflation, where the double-threshold model dominates. We again note that only the double-threshold model and the unrestricted VAR 'in levels' imply the stationary behavior of all variables that is visible from historical data.

At even longer horizons, mean reversion and error correction should become more important. This is obvious from stochastic simulations of the data-generating processes that are implied by the full-sample estimates. Only the double-threshold models yield a longer-run performance that visually matches the features known from the sample. A reliable evaluation of the forecasting performance over even longer horizons, such as several years, would need much longer observation periods and is therefore not in the focus of this paper.

5 Summary and conclusion

The incidence of non-stationary real interest rates is incompatible with many macroeconomic and finance theories, yet empirical studies present mixed results on the issue. Why should real interest rates be non-stationary? Several reasons have been advanced to explain this inconsistency. These include—among other things—the 'stationary behavior of the inflation variable' (Rose, 1988), the flaw in the use of linear models to explain fluctuations in macroeconomic time series (Bevilacqua and Zon, 2001), and the exclusion of integration and global influences on national bond rates (e.g., Andersen, 1999; Wu and Zhang, 1997). By adopting the threshold cointegration approach of Balke and Fomby (1997), we hypothesize that the failure of former studies to find stationarity in real interest rates is due to the studies' inability to account for the threshold characteristics in nominal interest rates. Finally, we evaluate the forecasting performance of the threshold cointegration model against competing models.

As in other studies of predictive accuracy, we found that the statistical evaluation of rival model descriptions is not reflected one-for-one in the ranking of these models in a comparative prediction evaluation. However, the derived conclusions may be viewed differently depending on whether the evaluating agent is a statistician, an economist, or a forecaster. While an economist may doubt whether prediction evaluations are appropriate tools of model validation, a forecaster may take the recommendations from the prediction evaluation more seriously than the issue of model validity.

Notwithstanding the fact that the statistical procedures are adversely affected by the small number of observations in the distributional tails that constitute the leverage for the hypothesized threshold effects, the picture emanating from the statistical evaluation conforms to our theory. In the linear framework, error correction to the average yield spread or to a time-constant real interest rate is often found to be weak. The long rate is often found to be exogenous and adjustment is performed by the short rate or by inflation. While neither interest rates nor inflation can be integrated variables, due to the implied infinite variance and undefined expectations, these variables behave *like* integrated variables for considerable time spans. Most bivariate data sets indicate that mean reversion, indeed, takes place whenever unusually low or unusually high observations occur. This mechanism can only be monitored by using the suggested threshold models.

The explanatory power of the threshold models is confirmed in our ex-post tracking experiments with stochastic simulation. Mean reversion from the distributional tails, i.e., threshold cointegration using the variables themselves, appears to be more important than threshold effects for the yield spread and the Fisher equation. In the ex-ante prediction experiments, the comparatively complex threshold model lose ground, as some of their parameters estimated from the tails contain high uncertainty. The simple model in differences, that ignores all error correction and cointegration, performs remarkably well, although a threshold specification is still found to predict optimally in almost one third of the reported cases.

The basically linear vector autoregression with threshold cointegration offers a convenient and plausible tool that overcomes the inherent problems in using linear cointegration models for economic variables that are apparently bounded in the long run, such as inflation, interest rates, or unemployment rates. With regard to economic forecasting, the real power of the approach may be felt more strongly in long-run scenarios or in policy evaluations—a topic for future research. The self-stabilizing forces of economic systems should not be ignored in time-series modeling, whenever the models are designed to capture the main features of economic reality.

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Tables

Table 1: Threshold models with a single threshold for Fisher effects: long rates

	Germany	Japan	United Kingdom	USA
p	10	5	9	4
Joh trace	11.64	7.01	13.97	12.82
β_π/β_L	-1.33	-1.66	-1.12	-1.11
logdet (UR–VAR)	3.571	5.566	4.967	4.201
logdet (D–VAR)	3.632	5.609	5.003	4.264
$t(\alpha_{1.L})$ linear	-1.15	-0.03	-0.95	-1.36
$t(\alpha_{1.\pi})$ linear	1.95	2.09	1.61	2.33
logdet (T–VAR 1)	3.541	5.565	4.922	4.199
optimal truncation	0.9	0.98	0.9	0.8
$t(\alpha_{1.L})$ T–VAR 1	-1.15	0.09	-1.62	-1.47
$t(\alpha_{1.\pi})$ T–VAR 1	1.91	2.13	1.00	1.94
$t(\alpha_{2.L})$ T–VAR 1	-0.20	-1.47	-2.40	-1.45
$t(\alpha_{2.\pi})$ T–VAR 1	-2.61	-0.63	-1.87	-1.09

Notes. p is the VAR lag order determined by AIC. 'Joh trace' is Johansen's trace statistic formed from both canonical correlations. β_π/β_L is the coefficient ratio for the first canonical vector as identified by the Johansen algorithm. 'logdet' denotes the logarithm of $\det(\hat{\Sigma})$, where $\hat{\Sigma}$ is the estimated variance matrix of errors calculated from the respective model residuals. 'UR–VAR' denotes the unrestricted VAR in levels, 'D–VAR' the VAR in first differences, 'T–VAR 1' the VAR with one unrestricted and one truncated error-correction vector. $t(.)$ are t–statistics for coefficients in a linear VAR with one cointegrating vector and in the T–VAR 1 model, where $\alpha_{j.x}$ denotes the coefficient of the j–th error-correction variable in the equation for variable x.

Table 2: Threshold models with two thresholds for Fisher effect: long interest rates

	Germany	Japan	United Kingdom	USA
p	10	5	9	4
logdet (T–VAR 1)	3.541	5.565	4.922	4.199
optimal truncation				
2nd vector	0.9	0.98	0.9	0.8
1st vector	0.5	0.98	0.5	0.9
$t(\alpha_{1.L})$ T–VAR 2	-1.36	0.12	-3.36	-2.37
$t(\alpha_{1.\pi})$ T–VAR 2	-1.16	3.31	-0.28	-0.06
$t(\alpha_{2.L})$ T–VAR 2	-0.24	-1.47	-2.24	-1.08
$t(\alpha_{2.\pi})$ T–VAR 2	-2.70	-0.56	-2.27	-1.67
logdet (T–VAR 2)	3.565	5.525	4.879	4.210

Notes. See Table 1. 'T–VAR 2' denotes the threshold cointegration model with two truncated error-correction vectors.

Table 3: Threshold models with a single threshold for Fisher effects: short rates

	Germany	Japan	United Kingdom	USA
p	11	8	12	4
Joh trace	22.11	9.03	57.72	15.79
Joh 2	8.64	2.68	0.62	1.49
β_π/β_S	-2.50	-2.57	-1.55	-1.29
logdet (UR–VAR)	3.884	3.661	5.031	3.596
logdet (D–VAR)	3.967	3.717	5.147	3.654
$t(\alpha_{1.S})$ linear	-2.47	-1.27	-1.81	-1.15
$t(\alpha_{1.\pi})$ linear	0.59	1.11	0.72	2.16
logdet (T–VAR 1)	3.902	3.641	5.111	3.569
optimal truncation	0.9	0.5	0.98	0.8
$t(\alpha_{1.S})$ T–VAR 1	-2.20	-0.75	-1.68	-0.67
$t(\alpha_{1.\pi})$ T–VAR 1	1.66	-0.82	0.84	2.44
$t(\alpha_{2.S})$ T–VAR 1	-0.28	2.29	-0.91	-2.37
$t(\alpha_{2.\pi})$ T–VAR 1	-3.06	-2.78	-1.06	-1.58

Notes. See Table 1.

Table 4: Threshold models with two thresholds for Fisher effect: short interest rates

	Germany	Japan	United Kingdom	USA
p	11	8	12	4
logdet (T–VAR 1)	3.902	3.641	5.111	3.569
optimal truncation				
2nd vector	0.9	0.5	0.98	0.8
1st vector	0.9	0.9	0.8	0.5
$t(\alpha_{1.S})$ T–VAR 2	-1.76	0.70	-1.72	-1.27
$t(\alpha_{1.\pi})$ T–VAR 2	-1.21	1.46	-1.03	1.71
$t(\alpha_{2.S})$ T–VAR 2	0.58	-1.42	1.01	-2.10
$t(\alpha_{2.\pi})$ T–VAR 2	-2.59	-2.16	-1.03	-1.54
logdet (T–VAR 2)	3.934	3.664	5.109	3.579

Notes. See Tables 1 and 2.

Table 5: Threshold models with a single threshold for interest rates

	Germany	Japan	United Kingdom	USA
p	2	7	2	4
Joh trace	6.59	10.39	5.63	5.87
β_S/β_L	-0.526	-0.817	-0.722	1.919
logdet (UR–VAR)	3.863	3.196	5.342	3.589
logdet (D–VAR)	3.897	3.245	5.378	3.616
$t(\alpha_{1.S})$ linear	1.57	1.19	1.62	1.31
$t(\alpha_{1.L})$ linear	-0.03	-1.61	-0.19	-0.08
logdet (T–VAR 1)	3.861	3.161	5.299	3.579
optimal truncation	0.5	0.5	0.5	0.9
$t(\alpha_{1.S})$ T–VAR 1	2.10	-1.26	-0.06	1.04
$t(\alpha_{1.L})$ T–VAR 1	-0.10	-2.45	-2.30	-0.24
$t(\alpha_{2.S})$ T–VAR 1	1.63	-1.84	-1.76	-1.97
$t(\alpha_{2.L})$ T–VAR 1	-0.19	-2.92	-3.09	-1.22

Notes. See Table 1.

Table 6: Threshold models with two thresholds for interest rates

	Germany	Japan	United Kingdom	USA
p	2	7	2	4
logdet (T–VAR 1)	3.861	3.161	5.299	3.579
optimal truncation				
2nd vector	0.5	0.5	0.5	0.9
1st vector	0.8	0.5	0.8	0.8
$t(\alpha_{1.S})$ T–VAR 2	2.98	-0.67	-0.50	0.05
$t(\alpha_{1.L})$ T–VAR 2	0.37	-1.04	-3.32	-1.43
$t(\alpha_{2.S})$ T–VAR 2	1.93	-2.84	-2.31	-2.12
$t(\alpha_{2.L})$ T–VAR 2	-0.01	-0.62	-3.53	-1.13
logdet (T–VAR 2)	3.838	3.189	5.264	3.573

Notes. See Tables 1 and 2.

Table 7: Ex-post tracking performance of stochastic prediction: long rate and inflation.

	Germany		Japan		United Kingdom		USA	
	i_L	π	i_L	π	i_L	π	i_L	π
single threshold								
mean	-0.00059	0.00270	-0.00363	-0.00029	0.00102	-0.00628	-0.00601	0.00446
MAE	0.132	0.169	0.169	0.294	0.193	0.215	0.177	0.162
MSE	0.0286	0.0480	0.0502	0.1627	0.0578	0.0740	0.0451	0.0464
double threshold								
mean	-0.00097	-0.00088	-0.00376	-0.00802	0.00023	-0.00809	-0.00359	0.00118
MAE	0.131	0.168	0.170	0.290	0.185	0.216	0.180	0.165
MSE	0.0285	0.0491	0.0502	0.1655	0.0556	0.0744	0.0459	0.0483
unrestricted VAR								
mean	-0.00016	0.00351	-0.00406	0.00170	0.00083	-0.00705	-0.00736	0.00264
MAE	0.132	0.169	0.171	0.293	0.194	0.215	0.178	0.162
MSE	0.0286	0.0498	0.0514	0.1614	0.0601	0.0749	0.0453	0.0463
VAR in differences								
mean	0.00034	-0.00067	-0.00434	-0.00853	0.00234	-0.00894	-0.00366	-0.00028
MAE	0.132	0.171	0.171	0.294	0.196	0.219	0.180	0.168
MSE	0.0290	0.0524	0.0514	0.1670	0.0605	0.0770	0.0466	0.0492

Table 8: Ex-post tracking performance of stochastic prediction: short rate and inflation.

	Germany		Japan		United Kingdom		USA	
	i_S	π	i_S	π	i_S	π	i_S	π
single threshold								
mean	0.00170	0.00049	0.00004	0.00601	-0.00514	-0.01211	-0.00176	-0.00111
MAE	0.134	0.174	0.076	0.258	0.183	0.208	0.124	0.163
MSE	0.0316	0.0511	0.0136	0.1211	0.0618	0.0680	0.0268	0.0475
double threshold								
mean	0.00139	0.00105	0.00099	0.00002	-0.00376	-0.01269	-0.00124	-0.00173
MAE	0.134	0.176	0.076	0.260	0.181	0.207	0.124	0.164
MSE	0.0322	0.0521	0.0136	0.1224	0.0612	0.0676	0.0266	0.0481
unrestricted VAR								
mean	0.00278	0.00542	0.00002	0.00399	-0.00594	-0.00898	-0.00448	-0.00360
MAE	0.134	0.161	0.076	0.258	0.184	0.198	0.127	0.165
MSE	0.0314	0.0496	0.0136	0.1248	0.0620	0.0622	0.0273	0.0478
VAR in differences								
mean	0.00038	-0.00066	0.00018	-0.00392	-0.00446	-0.01248	-0.00380	-0.00167
MAE	0.136	0.178	0.077	0.260	0.183	0.207	0.127	0.167
MSE	0.0334	0.0555	0.0139	0.1297	0.0637	0.0682	0.0281	0.0497

Table 9: Ex-post tracking performance of stochastic prediction: two interest rates.

	Germany		Japan		United Kingdom		USA	
	i_L	i_S	i_L	i_S	i_L	i_S	i_L	i_S
single threshold								
mean	-0.00007	-0.00238	0.00127	-0.00212	0.00065	-0.00102	0.21787	-0.00243
MAE	0.148	0.157	0.171	0.070	0.217	0.209	0.278	0.129
MSE	0.0351	0.0533	0.0518	0.0121	0.0723	0.1005	0.1221	0.0270
double threshold								
mean	-0.00117	0.00796	-0.00099	0.00951	0.00013	0.00073	-0.00586	-0.00391
MAE	0.147	0.153	0.173	0.071	0.215	0.209	0.187	0.129
MSE	0.0352	0.0520	0.0535	0.0122	0.0708	0.1008	0.0520	0.0272
unrestricted VAR								
mean	-0.00092	0.00354	-0.00207	-0.00067	-0.00287	0.00217	-0.00918	-0.00329
MAE	0.146	0.157	0.172	0.068	0.215	0.208	0.188	0.131
MSE	0.0345	0.0519	0.0532	0.0125	0.0718	0.1018	0.0515	0.0280
VAR in differences								
mean	-0.00111	0.00221	-0.00401	0.00012	-0.00062	0.00366	-0.00813	-0.00275
MAE	0.146	0.154	0.173	0.069	0.217	0.208	0.189	0.131
MSE	0.0347	0.0529	0.0546	0.0127	0.0723	0.1029	0.0523	0.0286

Table 10: Ex-ante single-step forecasting performance of stochastic prediction: long rate and inflation.

	Germany		Japan		United Kingdom		USA	
	i_L	π	i_L	π	i_L	π	i_L	π
single threshold								
mean	-0.0175	-0.0097	<u>-0.0018</u>	0.0275	-0.0294	0.0098	-0.0063	0.0292
MAE	0.1718	0.1969	<u>0.1728</u>	0.1837	0.2178	0.2098	<u>0.2138</u>	0.1619
MSE	0.0442	<u>0.0663</u>	0.0561	0.0579	0.0758	<u>0.0700</u>	0.0684	0.0543
double threshold								
mean	-0.0220	-0.0328	-0.0323	-0.0263	-0.0214	<u>0.0014</u>	0.0079	0.0176
MAE	0.1774	0.1995	0.1758	0.1879	<u>0.2113</u>	0.2115	0.2152	0.1611
MSE	0.0474	0.0677	<u>0.0552</u>	0.0605	<u>0.0703</u>	0.0701	0.0699	0.0544
unrestricted VAR								
mean	-0.0332	<u>-0.0046</u>	-0.0504	<u>0.0165</u>	-0.0856	-0.0029	-0.0244	0.0112
MAE	0.1716	<u>0.1961</u>	0.1832	<u>0.1818</u>	0.2288	<u>0.2093</u>	0.2143	0.1614
MSE	0.0446	0.0665	0.0587	<u>0.0561</u>	0.0820	0.0701	<u>0.0680</u>	0.0535
VAR in differences								
mean	<u>-0.0068</u>	-0.0152	-0.0286	-0.0283	<u>-0.0103</u>	0.0166	<u>0.0053</u>	<u>0.0109</u>
MAE	<u>0.1704</u>	0.1969	0.1750	0.1845	0.2143	0.2099	0.2147	<u>0.1602</u>
MSE	<u>0.0440</u>	0.0670	0.0557	0.0588	0.0718	0.0704	0.0694	<u>0.0532</u>

Table 11: Ex-ante twelve-step forecasting performance of stochastic prediction: long rate and inflation.

	Germany		Japan		United Kingdom		USA	
	i_L	π	i_L	π	i_L	π	i_L	π
single threshold								
mean	-0.2366	-0.1207	0.0479	0.2502	-0.3463	-0.0176	0.2026	0.3859
MAE	0.9943	0.8716	0.6771	0.7424	1.1913	1.0231	0.9108	0.8738
MSE	1.5094	1.1594	0.7379	0.8771	2.2539	1.6719	1.4150	1.0110
double threshold								
mean	-0.0740	-0.4771	-0.3557	-0.3524	-0.2399	-0.1496	0.4187	0.2574
MAE	1.1528	0.9947	0.6721	0.8865	1.1299	1.0513	0.9529	0.6909
MSE	2.0906	1.3289	0.6616	1.0336	1.8962	1.6451	1.5306	0.7167
unrestricted VAR								
mean	-0.5493	-0.0342	-0.6034	-0.0225	-1.2934	-1.2349	-0.2104	0.0222
MAE	1.1490	0.8212	0.8488	0.7169	1.4548	1.4730	0.9631	0.7855
MSE	1.7265	1.0759	1.0289	0.7369	3.1564	3.9970	1.4577	0.8229
VAR in differences								
mean	0.0310	-0.2235	-0.2671	-0.4208	0.0506	0.3646	0.2069	0.2408
MAE	1.0368	0.9523	0.6294	0.9618	0.9149	0.9207	0.9755	0.6404
MSE	1.7067	1.2446	0.5915	1.2287	1.4462	1.2598	1.5361	0.6434

Table 12: Ex-ante single-step forecasting performance of stochastic prediction: short rate and inflation.

	Germany		Japan		United Kingdom		USA	
	i_S	π	i_S	π	i_S	π	i_S	π
single threshold								
mean	-0.0356	0.0167	-0.0255	0.0429	-0.0670	$\underline{-0.0009}$	0.0147	-0.0116
MAE	0.1372	0.1943	0.0739	0.2078	0.1787	0.2123	0.1376	0.1671
MSE	0.0329	0.0629	0.0145	0.0706	0.0518	0.0701	0.0341	0.0542
double threshold								
mean	$\underline{0.0173}$	-0.0174	-0.0152	-0.0304	-0.0068	-0.0035	$\underline{0.0060}$	0.0078
MAE	0.1395	0.1939	$\underline{0.0690}$	0.2142	$\underline{0.1679}$	0.2129	$\underline{0.1358}$	0.1671
MSE	0.0337	0.0631	$\underline{0.0133}$	0.0709	$\underline{0.0478}$	0.0691	$\underline{0.0340}$	0.0547
unrestricted VAR								
mean	-0.0339	0.0044	-0.0302	$\underline{0.0201}$	-0.0669	-0.0121	-0.0136	-0.0524
MAE	0.1369	$\underline{0.1786}$	0.0764	$\underline{0.1928}$	0.1792	$\underline{0.2051}$	0.1426	0.1765
MSE	$\underline{0.0328}$	$\underline{0.0564}$	0.0151	$\underline{0.0648}$	0.0524	$\underline{0.0652}$	0.0342	0.0576
VAR in differences								
mean	0.0226	$\underline{-0.0036}$	$\underline{0.0025}$	-0.0289	$\underline{-0.0024}$	-0.0010	0.0064	$\underline{-0.0052}$
MAE	$\underline{0.1367}$	0.1902	0.0705	0.2038	0.1689	0.2113	0.1403	$\underline{0.1651}$
MSE	0.0330	0.0612	0.0139	0.0664	0.0487	0.0688	0.0342	$\underline{0.0536}$

Table 13: Ex-ante twelve-step forecasting performance of stochastic prediction: short rate and inflation.

	Germany		Japan		United Kingdom		USA	
	i_S	π	i_S	π	i_S	π	i_S	π
single threshold								
mean	-0.4765	0.2242	-0.6397	0.4412	-1.9013	-1.0362	0.5414	0.0366
MAE	0.9025	0.8507	0.7570	0.7643	2.2487	1.6752	0.8049	0.7436
MSE	1.3918	1.0757	1.4538	0.9496	8.0885	4.4436	1.3659	0.7575
double threshold								
mean	0.3933	-0.1913	-0.5153	-0.2895	-0.0038	-0.1526	0.5100	0.1951
MAE	0.8530	0.8782	0.6755	0.8925	1.2212	1.1986	0.8096	0.7194
MSE	1.1226	1.1019	0.9112	1.0847	2.0040	1.9276	1.3490	0.7352
unrestricted VAR								
mean	-0.6645	-0.0722	-0.9936	0.1481	-2.1733	-1.3794	-0.1263	-0.5775
MAE	0.9538	0.5807	1.0830	0.6308	2.3817	1.6537	0.8222	0.8892
MSE	1.3575	0.4786	2.1556	0.6135	9.3136	4.1514	1.0341	1.0758
VAR in differences								
mean	0.5461	0.0233	0.1299	-0.3883	0.1584	-0.0138	0.3635	0.1830
MAE	0.8876	0.8610	0.6781	0.9662	1.2285	1.2773	0.8174	0.6703
MSE	1.2268	1.0354	0.9671	1.2666	2.1057	2.0908	1.2898	0.6734

Table 14: Ex-ante single-step forecasting performance of stochastic prediction: two interest rates.

	Germany		Japan		United Kingdom		USA	
	i_L	i_S	i_L	i_S	i_L	i_S	i_L	i_S
single threshold								
mean	-0.0222	-0.0628	0.0144	-0.0142	0.0241	0.0189	0.0056	0.0221
MAE	0.1696	0.1364	<u>0.1763</u>	0.0733	0.2075	<u>0.1236</u>	0.2136	0.1417
MSE	0.0450	0.0356	0.0557	0.0131	0.0698	0.0289	0.0726	0.0345
double threshold								
mean	-0.0192	-0.0454	<u>-0.0015</u>	-0.0109	0.0388	0.0391	<u>0.0036</u>	0.0131
MAE	0.1681	0.1345	0.1802	0.0727	0.2082	0.1302	0.2126	0.1414
MSE	0.0449	0.0353	0.0574	0.0128	0.0707	0.0313	0.0717	0.0343
unrestricted VAR								
mean	-0.0389	<u>-0.0318</u>	-0.0738	-0.0389	-0.0529	<u>-0.0086</u>	-0.0379	<u>0.0011</u>
MAE	0.1695	0.1416	0.1991	0.0742	0.2086	0.1253	0.2127	0.1381
MSE	0.0452	0.0352	0.0596	0.0132	0.0735	0.0287	<u>0.0669</u>	<u>0.0329</u>
VAR in differences								
mean	<u>-0.0175</u>	-0.0360	-0.0195	<u>-0.0006</u>	<u>-0.0044</u>	0.0177	0.0054	0.0086
MAE	<u>0.1667</u>	<u>0.1310</u>	0.1788	<u>0.0688</u>	<u>0.2007</u>	0.1242	<u>0.2107</u>	<u>0.1373</u>
MSE	<u>0.0444</u>	<u>0.0338</u>	<u>0.0553</u>	<u>0.0115</u>	<u>0.0670</u>	<u>0.0284</u>	0.0670	0.0332

Table 15: Ex-ante twelve-step forecasting performance of stochastic prediction: two interest rates.

	Germany		Japan		United Kingdom		USA	
	i_L	i_S	i_L	i_S	i_L	i_S	i_L	i_S
single threshold								
mean	-0.1720	-0.7061	0.0698	-0.2275	0.4278	0.4208	0.3655	0.9475
MAE	1.0114	1.1395	0.8222	0.8975	0.9401	1.0930	1.4567	1.5169
MSE	1.4405	2.0086	1.3337	2.0551	2.1383	1.6023	3.6310	4.0680
double threshold								
mean	-0.1555	-0.6515	0.0031	-0.1311	0.7002	0.7456	0.2977	0.6515
MAE	0.9903	1.1678	0.8788	0.8858	1.0523	1.3172	1.4390	1.4146
MSE	1.3744	1.9654	1.4698	1.9458	2.6599	2.3034	3.5953	3.8097
unrestricted VAR								
mean	-0.4675	-0.4436	-1.0263	-1.2254	-0.7564	-0.1831	-0.5043	0.0920
MAE	0.9906	1.2163	1.1052	1.2690	1.1584	1.1149	1.0261	0.7261
MSE	1.2257	2.2940	1.7350	2.3925	2.0906	1.7228	1.4650	0.8645
VAR in differences								
mean	-0.1225	-0.3528	-0.1101	0.1148	0.0128	0.4714	0.1964	0.3563
MAE	0.9210	1.0081	0.7147	0.6591	0.9035	1.0985	0.9665	0.8237
MSE	1.2016	1.4943	0.7923	0.8993	1.4008	1.6805	1.5310	1.1689

Authors: Adusei Jumah, Robert M. Kunst

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