

THE STRUCTURE OF AUSTRIAN INTEREST RATES

A BOX-JENKINS AND SPECTRAL
ANALYSIS APPROACH

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Abstract

New statistical methods gave rise to consider the structure of interest rates in Europe again (compare PORSIUS (1977)). This paper intends to apply some of these methods to Austrian monetary time series. Particularly, we compared different methods for the analysis of interest rates: Regression Analysis, Spectral Analysis, and the Box-Jenkins Approach.

Four interest rates, rates on new bonds, rates on bonds in circulation, the call money rate and the Euro-Dollar rate, and the Shares Index of Austria were analysed in the frequency and the time domain. Their dynamic properties together with the seasonal structure are compared. In difference to the first draft of the paper, we splitted up the time period in two halves and included spectral analysis.

Zusammenfassung

Die Entwicklung neuer statistischer Methoden gab Anlaß zu einer Reihe von Untersuchungen, die sich mit dem Zusammenhang zwischen lang- und kurzfristigen Zinssätzen, dem Problem der sogenannten Zinsstruktur, befassen. In der vorliegenden Arbeit wird versucht, einige dieser Methoden auch auf Zeitreihen für österreichische Zinssätze anzuwenden. Im besonderen wurden dabei die Ergebnisse von Regressions-, Spektral- und Box-Jenkins-Analyse verglichen.

An Datenmaterial herangezogen wurden der Geldmarktsatz, der Euro-Dollar-Satz für Dreimonatsgelder, die Kredite der Neuemissionen und die Sekundärmarktrendite sowie der Aktienindex. Diese wurden im Frequenz- und im Zeitbereich analysiert, ihre dynamischen Eigenschaften und saisonale Struktur verglichen. Die Ergebnisse deuten auf relativ schwache Zusammenhänge zwischen lang- und kurzfristigen Zinssätzen.

1. Introduction *)

Since the early 1960s there has been an impressive revival of interest in an old problem of monetary theory and policy, namely in the examination of the question for the determinants of the term structure of interest rates and for the possibilities to influence it. Main steps in taking up these problems were (among others) Meiselman's (1962) test of the expectations hypothesis, the approach by Culbertson (1957) according to which creditors and debtors behave according to certain preferences for different maturity structures of debts, leading to segmentation of capital markets and independent movements of interest rates on bonds with different maturities as well as the 'preferred habitat' - hypothesis by Modigliani and Sutch (1966).

For policy considerations there are mainly two points of concern in this context, namely

- a) the problem of efficiency of policies following the 'Operation Twist' pattern. A policy of this kind tries - by manipulating the maturity structure of debts - to raise short term interest rates in order to stimulate capital imports and to stabilize long term rates at a relatively low level in favour of domestic investment levels, and
- b) the question of adequacy of so-called 'Bills Only' policies as an instrument of countercyclical stabilization policy according to which capital market rates can be influenced sufficiently in an indirect way, namely by regulations of money markets rates.

Models of the term structure of interest rates dealt with in the literature referred especially to bond rates. Until

*) Especially we want to thank Professor H. ABELE, who gave the authors valuable advices in improving the analysis.

recently, there was less empirical work on interrelationships between a broader spectrum of interest rates. However, during the last years there was increased interest in this category of studies, see for example Cagan (1966), Fand (1966), Sargent (1968), Smith and Marcis (1972), Fase (1973), Porsius (1977a, 1977b), and others. Methods used in this context range from the method of principal components to time series analysis by Box-Jenkins-methods and spectral analysis. Ample room in these studies is dedicated to the examination of seasonalities in interest rates. The results reported are very inconclusive and even contradictory¹⁾.

There are no studies of the above-mentioned kind for Austria. This paper presents some preliminary results which have to be seen, however, in the light of a rather restricted data base and of some peculiar institutional facts.

A study of this kind may be useful for a number of reasons: Some general insight could be gained into the functioning of capital and money market; at the same time a study can be performed on the interaction between money and capital market and on the influence of short term monetary policy upon level and structure of interest rates.

In detail, the following points can be raised which this study (and further research) will try to answer:

- 1) It is to be examined whether Austrian interest rates showed systematic movements over the period under consideration or whether their development over time was of a random character as could, for instance, be observed for New York stock market prices (Granger and

1) For a short review see BRICK and THOMPSON (1978).

Morgenstern (1963)). In a number of studies on other countries seasonalities in interest rate movements could frequently be watched, see, for example, Barth and Bennet (1975), König and Wolters (1969), Porsius (1977a, 1977b), Smith and Marcis (1972), and others.

- 2) Possible systematic correlations between variations in interest rate policy and money market rates are to be investigated.
- 3) It should be checked whether there exists a systematic connection between long-term and short-term interest rates (for reference see the discussion on the Bills-Only-Policy followed by the U.S. between 1953 and 1961 according to which control of short-term rates by monetary authorities suffices to influence long-term rates via an assumed high interrelationship between them, normally presuming a lead reaction of short rates over long rates (Fand (1966)). The contradicting opinion is presented by the above mentioned theory of market sequencing.
- 4) The problem of seasonalities in interest rates is of importance for the specification of money demand and other behavioral functions of the monetary sector as was shown by Lombra and Kaufman (1975): If seasonalities exist use of unadjusted interest series in money demand equations together with adjusted series for M1 and GNP may cause biases in the implied elasticities.
- 5) A further question relates to the use of Euro-Dollar-Rates in monetary models which frequently have to be treated as exogenous; endogenization by structural models or reduced forms is difficult because of data problems (Freedman (1977)). However, Euro-Dollar Rates

play an important role in modelling international capital movements and in demonstrating interdependencies of national financial markets. This unsatisfactory state of treating Euro-Dollar Rates as purely exogenous variables can be remedied at least partially if it can be shown that this rate may be explained in a satisfactory way from its own past values by means of time series methods.

The plan of the study is as follows: Section 2 presents some institutional facts to show the environment in which interest rates in Austria are guided. In Section 3 the available data base is shown and some preliminary results about the structure of these rates are gained. Sections 4 - 6 discuss the detailed outcome of applying regression analysis, spectral analysis, and Box-Jenkins-analysis to the interest rate series. Finally, section 7 presents the general conclusions which have to be drawn for the structure of Austrian interest rates.

2. Institutional Background

An investigation into interest rates gains some peculiar aspects by the institutional situation into which interest rate policy is embedded in Austria. An official statement in this context sounds: "Significant for the Austrian interest rate policy is the aim for interest rates which do not allow each disturbance in the market to affect these rates. The reason for this interest in keeping rates as stable as possible is the desire for an assured and continuing raise of funds in the capital market; this is regarded as a contribution to a secure financing of the volume of investments in the economy".²⁾

This simply means that the interest rate - at least in the capital market - is not a market price. Instead, there is a key interest rate fixed by collective bargaining between central bank, ministry of finance, commercial banks, and 'Social Partners'. Only short term rates, however, are to a larger extent determined by the banking sector and scarcities, although also in this case the aim at stabilising can be observed. It will be the scope of this study to examine - given these conditions - to which extent correlations between long term and short term rates may exist. Usually the market for financial assets is divided into the money market and the capital market, maturities and also participants being used as criteria. In the money market assets of very short term up to a maximum of one year are traded thus being relatively close money substitutes. In most cases participation is restricted to banks exchanging surpluses and deficits of base money - be it by use of credits or of buying and selling of appropriate papers. To a smaller extent also non-banks, e.g. enterprises, may participate purchasing from and selling

²⁾ Mitteilungen des Direktoriums der Oesterreichischen Nationalbank, 10/1974, p.734, authors' translation.

short term papers to commercial banks. Also in Austria a certain exclusivity of market participants is to be observed: Almost exclusively banks appear in the money market. However, there exists a number of 'internal' money markets between groups of commercial banks such that only peak money demands are balanced in the 'open' money market. Main form of money market transactions is the granting of credits. The volume of transactions in money market papers is very small. This market also has to be denoted as imperfect because of the existence of internal money markets mentioned above and because of many traditional business relations (W. Weber und Mitarbeiter (1972)).

To an increasing extent the market for Euro-currencies offers possibilities of substitution for the participants in the money market leading to inclusion of the Euro-market into the analysis (Fase (1973)). Especially for an open economy like the Austrian this fact will be of interest.

The capital market is defined as the segment of the market for financial assets with maturities of more than one year. An essentially distinctive feature compared with the money market is that the capital market is to be seen as an instrument for investment financing, whereas the money market serves the function of transferring liquidity. The capital market can be further divided into the market for fixed-interest-bearing securities (bonds) and risk-bearing capital (shares). The relation of the volumina of these two markets was approximately 3:1 in 1975 (volume of fixed-interest-bearing-securities: face value of shares).

Interest rates in the capital market are kept as stable as possible as mentioned above. The federal government dominates emissions of new fixed-interest securities (1975: almost 60 %). Approximately 60 % of the total volume of bonds are held by the commercial banks.

In general, a marked segmentation of Austrian financial markets can be observed which in the following will have to be examined by means of the data. The intense influence taken by economic policy on interest rates and the aim to keep them stable seems to recommend the application of time series methods. Short term influences also are not likely to be given room to work themselves out though sometimes they lead to revisions of the interest level regarded as 'normal'.

3. Data Base

The following time series could be made available for this study:

- a) The call money rate (GMS) used in interbank lending and borrowing on a very short term basis (daily).
- b) The Eurodollar rate (ER) on 3-months deposits as observed in London.
- c) The long term bond yield on new issues (R) if held up to maturity without taking into account tax relief (approximately eight years, in general).
- d) The long term bond rate on bonds in circulation with an average remaining maturity (R2); included are government and municipal bonds, bonds issued by electric power plant companies and others.
- e) The discount rate set by the Austrian central bank (DR).
- f) An index of shares quotations (AKI): unweighted index of 36 industrial companies and six quotations of banking and insurance companies, together representing 75 % of total nominal capital of quoted companies; averages of weekly observations taken from last day's quotations of stock exchange week.

The time paths of these series are shown in Figure 1 and 1a. The largest oscillations are exhibited by the Eurodollar Rate with a minimum of 2 percent in 1958 and almost 14 percent in 1974. The movements of the call money rate seem to follow the business cycle to some extent; this may be caused by the cyclical swings in liquidity related to the development of the balance of payments. The long term rates R and R2 which move very closely together are rather stable and do not seem

FIGURE 1 TIME PATHS OF AUSTRIAN INTEREST RATES

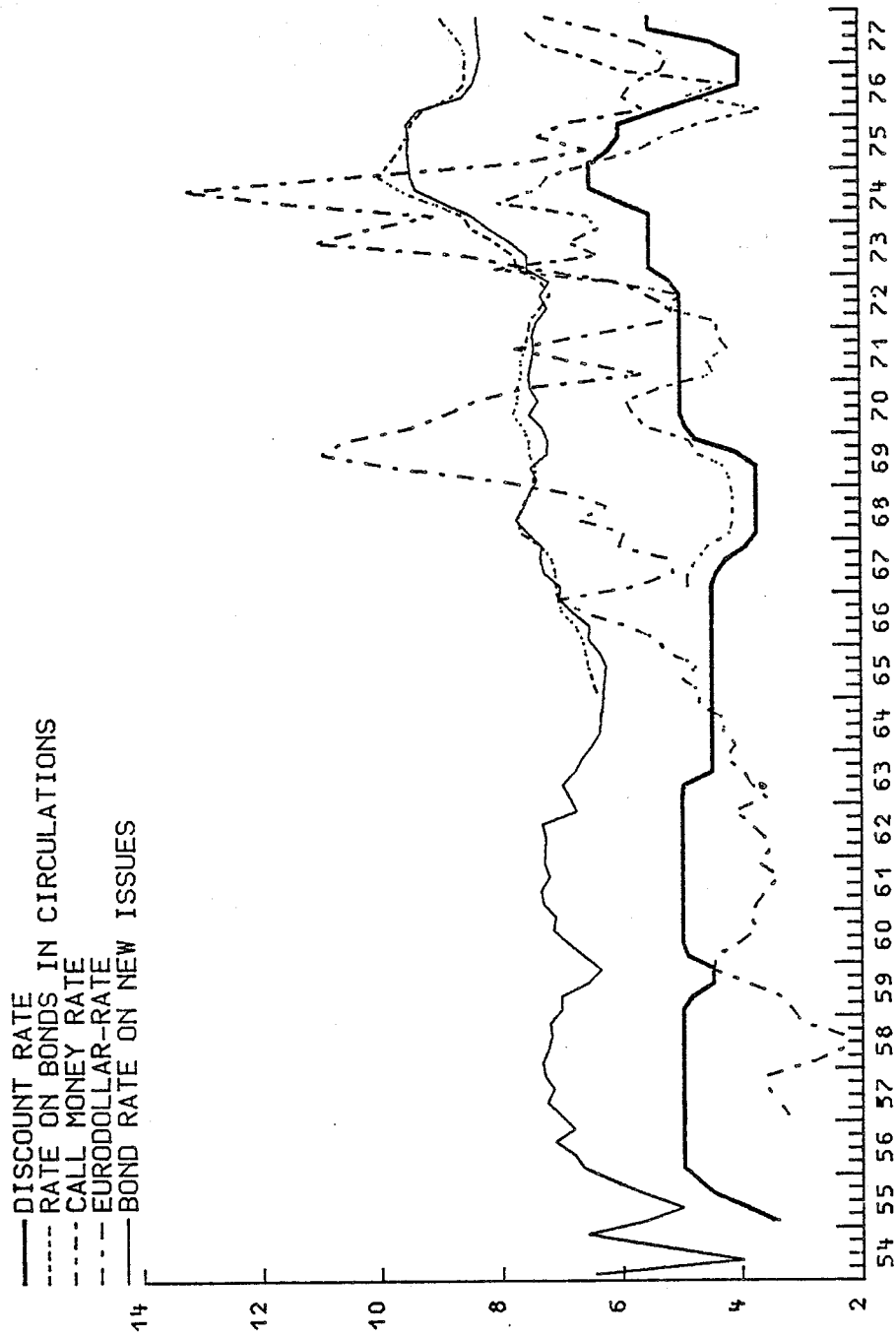
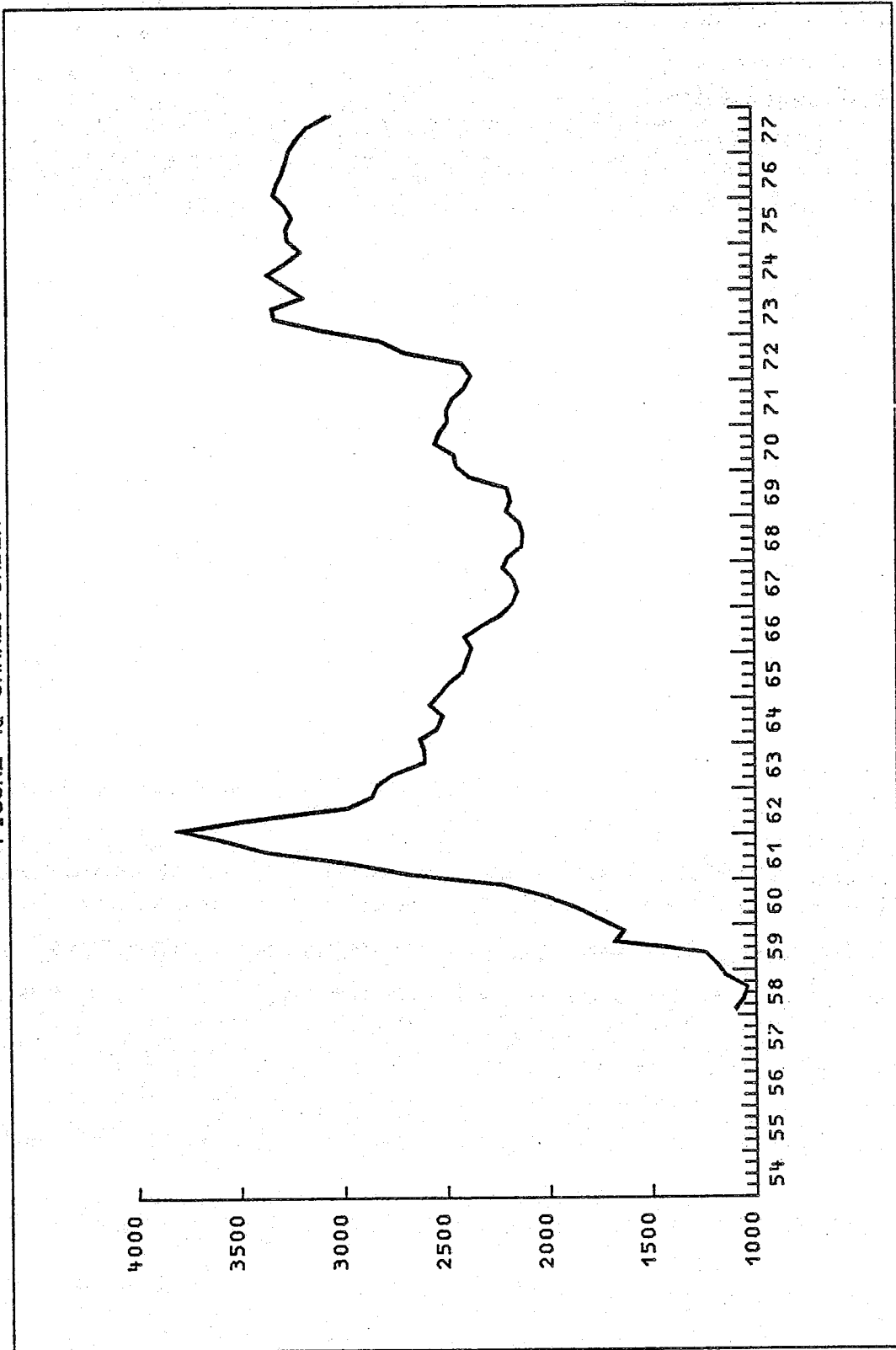


FIGURE 1a SHARES INDEX



to be influenced by the business cycle; their high values during 1974 and 1975 were caused by the high inflationary pressure of these years.

The shares index seems to be influenced by long swings with an impressive rise in the late fifties and early sixties and a new upswing in the early seventies. Some connection with the business cycle can be observed. Calculations of means and variances of the series yield the following results:

Table 1: Means and Variances of Time Series

	Mean	Variance	Coefficient of Variation
GMS (Nov.66-Sept.77)	5.40458	1.22296	0.22628
ER (Jan.57-Sept.77)	5.80133	2.37171	0.40882
R (first quarter 54- second quarter 77)	7.20947	0.96917	0.13443
R2 (Nov.64-Oct.77)	7.84304	0.94576	0.12059
DR (Jan.55-Dec.77)	4.84388	0.62801	0.12965
AKI (Jan.58-Oct.77)	2564.58658	632.93941	0.24680

One would expect that the more unbalanced and/or the more active the market for a certain asset is, the larger should be the variance of the corresponding interest rate. It can be observed in Table 1 that the variances of long term rates and, of course, also for the discount rate are relatively small vis-a-vis the means of these series compared with the respective relations for the Eurodollar rate and the call money rate.

For the policy considerations mentioned above it is, of course, of more interest to investigate common movements of the time series under consideration. Table 2 gives simple correlation coefficients between the time series.

Table 2: Correlation Coefficients (full period)

	GMS	ER	R	R2	DR	AKI
GMS	1	.35954	.24401	.38811	.75255	.46432
ER		1	.42271	.35284	.26006	.34682
R			1	.91660	.50106	.43924
R2				1	.79310	.55293
DR					1	.38021
AKI						1

In general, the results of Table 2 show relatively weak connections between the six time series. Correlation is highest between the two long term rates R and R2, but surprisingly low between the call money rate and the Eurodollar-rate. The call money rate is much more connected to variations in the domestic interest level via the discount rate than it is to foreign influences. The discount rate is also relatively highly correlated with R2, to a smaller extent with R. Rather weak seems the correlation between the call money rate and the long term rates R and R2. Correlations of medium size are shown for the shares' index with the highest value for the correlation between the shares' index and R2.

Dividing the period into two parts by intersecting at the end of 1969 does not change the result of low correlations between long term and short term rates but gives some interesting details about changing climate and behavioral pattern over the two subperiods.

Table 2a: Correlation Coefficients (-Dec. 69)

	GMS	ER	R	R2	DR	AKI
GMS	1	.06558	-.29079	-.34755	.68050	.04166
ER		1	.09594	.60521	-.65425	.05211
R			1	.72445	.08764	-.03066
R2				1	.57503	-.18112
DR					1	.06965
AKI						1

Table 2b: Correlation Coefficients (Jan. 70-)

	GMS	ER	R	R2	DR	AKI
GMS	1	.50375	.17338	.27211	.35900	.44739
ER		1	.22679	.27891	.57393	.22055
R			1	.97552	.55726	.77258
R2				1	.56002	.74526
DR					1	.29363
AKI						1

The connections between the long-term rates, discount rate and shares' index seem to have improved in the second sub-period. The same is true for a rising influence of the Euro-Dollar-Rate on almost all domestic rates. Call money rate and Euro-Dollar-Rate were not correlated in the first subperiod but showed some correlation in the second period which may be explained by the rising importance of foreign transactions of commercial banks during the 70's; also some limitations on foreign transactions have been cancelled in this period.

There seems to have been no connection at all between the shares' index and any other rate in the first subperiod. On the other hand, the correlation between call money rate and discount rate was almost twice as high in the first than in the second period.

Discount rate policy seems to have been counteracting the Euro-Dollar-Rate in the first subperiod whereas it was much more accommodating during the second part of the period under consideration.

The reasons for the relatively low correlations between short term and long term rates as reported also in other studies (e.g. König & Wolters 1969) can be regarded as a confirmation of Culbertson's hypothesis, but might also be found in the existence of lead and lag relationships and different cyclical movements. In the next sections of this paper we will examine these possibilities by means of traditional regression analysis and spectral methods.

4. Regression Analysis

The following statistical analysis attempts to investigate interest rate movements by traditional regression analysis. The low correlations between long-term and short-term rates are to be examined in more detail by means of lag distributions.

If long-term rates are dominated by secular influences, short-term rates by cyclical and seasonal swings, then low correlation is to be expected. Can we - despite of this - find evidence that regulation of the short-term rate by monetary authorities will result in desired changes of the long term rate? This question, of course, is of no concern for the Austrian environment as a policy of this kind has not been pursued. The Austrian policy on the other hand relies more on soft loans and subsidies to make money cheap.

Another point may, however, be of interest in this respect, namely, whether movements in the short-term rate are able to influence long-term rates in spite of the regulation of the latter. Or, whether knowledge of current and past values of the short-term rate allows the prediction of the long-term rate to a satisfactory degree. Some experiments³⁾ to clarify these questions are presented in this section.

Model I: Autoregression Hypothesis

Here we test the hypothesis that the long-term rate is more or less determined by its own past values and that it is relatively independent from oscillations of the short-term rate. This hypothesis is supported by the correlation results. The outcome of this experiment serves for comparison with results from the other models where the short-term rate is used as a predictor for the long-term rate. We applied these models to monthly

3) We follow the approach taken by Fand (1966).

data using the call money rate (GMS) as the short-term and the bond rate (R2) as the long-term rate. Results are presented in Table 3. As criteria for the quality of the regressions we use their standard errors.

Model I has the form:
$$R2 = \sum_{i=1}^n w_i R2_{-i}$$

Model II: Regression of long-term rate on short-term rate.

The long-term rate is to be explained by current and past values of the short-term rate alone. Standard errors of the results are considerably higher than in Model I; these errors reduce when lags become longer, but cannot reach the low values of the autoregressive model with a reasonable length of the lag. Obviously, knowledge of past values of R2 seem to be of much more importance for the prediction of it than the knowledge of current and past values of the short-term rate is.

Model II is of the form:
$$R2 = \sum_{i=0}^n w_i \text{GMS}_{-i}$$

Model III: Combination of Model I and Model II

This model is to test the question whether knowledge of GMS gives at least some additional information when predicting R2. For this purpose we regress R2 on lagged R2 and current and lagged values of GMS. The results show some moderate improvements over the autoregressive model indicating that knowledge of the short-term rate does give additional information.

Model III has two variants:

$$a) R2 = a_0 R2_{-1} + a_i \text{GMS}_{-i}; i=1, \dots, n$$

$$b) R2 = b_0 R2_{-1} + \sum_{i=0}^n w_i \text{GMS}_{-i}$$

The evidence from regression analysis can be summed up in the statement that current and lagged values of short-term interest rates in Austria by far cannot predict the long-term rate as

well as lagged values of the long-term rate themselves can do. A typical behavioral pattern or a causal chain from short- to long-term rates seemingly does not exist. However, the improvements of the regression results when adding the short-term rate as explanatory variable to the autoregressive model let us suppose that the monthly movements of the long-term rate are not completely independent from seasonal and cyclical swings of the short-term rate. Some seasonal and/or cyclical periodicities therefore have to common both to the short-term and the long-term rate. The extent of these similarities, however, seems to be so small that changes in the short-term rate lead only to marginal changes in the long-term rate.

The question of common periodicities will be examined in the next section.

Table 3: Standard Errors of Models I-III

Periods (n)	Model I	Model II	Model III	
			a)	b)
0	-	.76899	.08881	.08881
1	.09417	.75071 (1S)	.08992	.08869 (1E)
2	.08908	a) .73433 (2S)	.09213	.08893 (1E)
3	.09131 (2E)	.71466 (2S)	.09333	.08928 (1E)
4	.09147 (2N)	.69386 (2S)		.08959 (1E)
5	.09251 (2N)	.67220 (2S)		
6	.09235 (2N)		.09497	.08985 (2N)
...				
12	.09835 (3N)	.51740 (2N)	.10552	.09198 (3N)
...				
18	.12018 (3N)	.36213 (3S)		.08680 (3N)

a) Symbols in parentheses indicate degree and restrictions of the polynomial used in the lag distribution with
 N: no restriction
 S: start restriction
 E: end restriction

5. Spectral Analysis

The hypothesis that two or more interest rates show common cyclical movements should be tested by means of their covariation. One way to do that is to determine the particular periodicities that are present in all the series concerned. To isolate those periodicities we calculate spectral densities of the interest rates concerned.

5.1. Spectral Methods

5.1.1. The spectrum of a time series

We assume that the observed time series x_t , $t=1, \dots, T$ is a realisation of a stationary (covariance-stationary) stochastic process with

$$Ex_t = \mu$$

$$\text{Var}(x_t) = \sigma_x^2$$

The autocovariance function (ACovF) is defined as

$$\gamma_k = E(X_{t-\mu})(x_{t+k-\mu}) \quad k \in \mathbb{Z}$$

and the autocorrelation function (ACF) is given by

$$\rho_k = \frac{\gamma_k}{\gamma_0} \quad k=0,1,2,\dots$$

The spectrum of a time series is the Fourier-transformation of the ACovF

$$\begin{aligned} \text{SP}(\omega) &= \mathcal{F}(\gamma_k) = \\ &= \int_{-\infty}^{\infty} \gamma(k) e^{-i\omega k} dk \\ &= \sum_{k=-\infty}^{\infty} \gamma_k e^{-i\omega k} \quad \omega \in \mathbb{R} \end{aligned}$$

Because γ_k is an even function the spectrum reduces to a real function

$$SP(\omega) = \sum_{k=-\infty}^{\infty} \gamma_k \cos \omega k \quad -\pi \leq \omega \leq \pi$$

and we restrict to $|\omega| < \pi$, because the spectrum is an even periodic function with period 2π

$$SP(\omega + 2\pi) = SP(\omega)$$

5.1.2. Spectral analysis

The estimation of the spectrum of a time series is called spectral analysis. Three problems arise in spectral analysis of univariate time series:

a) The continuous theoretical spectrum has to be approximated by a discrete one; b) The theoretical ACF is not known and has to be replaced by an estimate; c) Because of the fact that observed time series are finite, one can only use a finite number of autocorrelations and one has to weight the ACF by an appropriate weighting function ("spectral window").

The estimation formula used is

$$SP\left(\frac{2\pi t}{T_p}\right) = \gamma_0 + 2 \sum_{k=0}^K c_k g(k) \cos\left(2\pi k \frac{t}{T_p}\right)$$

where $g(k)$ is the Parzen weighting function

$$g(k) = \begin{cases} 1 - 6\left(\frac{k}{K}\right)^2 + 6\left(\frac{k}{K}\right)^3 & 0 \leq k \leq K/2 \\ 2\left(1 - \frac{k}{K}\right)^3 & K/2 \leq k \leq K \end{cases}$$

The Parzen weight has the advantage that it provides no negative estimates of the spectrum. K is the number of autocorrelations used for the estimation of the spectrum and T_p is the truncation point or length of the spectrum. With the appropriate choice of T_p one can influence the approximation of the spectrum and the interesting length of the periods, which should appear explicitly

in the spectrum. Usually $T_p = K$ but the calculation of the spectrum for different truncation points T_p has been proposed by Jenkins/Watts (1968) as so-called "window closing"-operation to test the stability of peaks in the spectrum.

In plotting the graph of the spectrum it is convenient to plot the logarithm of the spectrum against the frequencies (or periods) in order to damp the spectrum at the low frequencies.

Since a spectrum can be considered an analysis of variance of a time series over the range of frequencies one can also calculate how much percentages can be accounted to a certain frequency band. Due to the leakage effect harmonic components of a time series are distributed over a certain range in the spectrum.

In the interpretation of the spectrum one has also to observe the harmonic components of a dominant period movement. They appear on length of periods T/n with $n=1, \dots, [T/2]$ where T is the length of the dominant periodic movement. These harmonic components are the more observable, the more the dominant movement is not a pure sine wave.

5.2. Spectral Analysis of Interest Rates

In the appendix we include the print-outs of the graphs of the available interest rates (except for the discount rate).

The power spectra of all series in level form were similar in shape to the so-called "typical spectral shape of economic variables" (Granger (1966)). All time series were non-stationary. By taking first differences stationarity in general could be achieved. In the following we discuss the spectra of first differences.

The spectrum of the first differences of the call money rate (GMS) shows a very pronounced peak at three months as well as some less important cycles at 21, 8 and 5 months. .

A different picture is shown by the spectrum of the rate on circulating bonds (R2): In this case a marked medium range cycle of approximately five years can be observed, but also a three-months-cycle - like for the call money rate - can be determined. A two-months-cycle is shown, too.

Also for the rate on new bonds a medium-term cycle of 24 quarters is seen as well as a very pronounced three-quarter-swing.

The most significant cycle for the Euro-Dollar-Rate lies at 12 months. A pronounced four-months-swing can be observed, too.

The shares' index exhibited a rather unstable behavior. Whereas for the whole period a long-term cycle seemed to be most important, a much more diversified spectrum is to be observed for a sub-period starting with 1963. In this case the long-term swing is still to be seen, but now also cycles at 10 and six months are very pronounced as well as some less marked cycles at approximately 4 and 2 months can be seen.

Summing up it may be stated that

- 1) Long-term rates show a marked long-term cycle of 5-6 years which cannot be observed for the short-term rates.
- 2) GMS and R2 show pronounced common swings at three months. The movements in the call money rate which can be explained by tax laws, window-dressing by commercial banks, and by seasonal oscillations of economic activity in general do have some effect on the rate on "old" bonds, but do not influence the rate on new bonds which is fixed by economic policy.
- 3) Long-term rates seem to be dominated rather by cyclical, call money rate and Euro-Dollar-rate rather by seasonal movements.

6. Box-Jenkins-Analysis of Interest Rates

6.1. The Box-Jenkins-Method

The Box-Jenkins-method is an abbreviation for a system of methods for identification, estimation and forecasting time series. The time series are estimated in the time domain belonging to the class of ARIMA (autoregressive integrated moving average)- or SARIMA (seasonal autoregressive integrated moving average)- processes. These methods are fully described in G.E.P. Box and G. Jenkins (1976), further references are O.D. Anderson (1976), C. Chatfield (1975), for forecasting economic time series see C.W.J. Granger & P. Newbold (1977).

6.1.1. Interactive Model Building

The application of these methods involves an interactive model building philosophy:

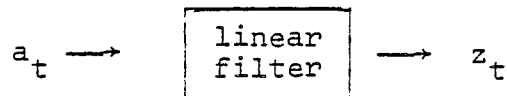
Identification - Estimation - Diagnostic Checking.

- a) Identification: Plot the time series and the ACF (auto-correlation function) and PACF (partial ACF) for various differences. From the ACF-structure one can choose a tentative SARIMA-model.
- b) Estimation: With the help of preliminary parameter values one starts the tentative SARIMA-model (using a nonlinear estimation program).
- c) Diagnostic Checking: Now one plots the ACF and PACF of the residuals of the estimated model. Is the model adequate? This question is difficult to answer if there are either still "significant" contributions in the residual ACF, or the residual ACF is similar to an ACF of a white-noise-process. So the residual ACF gives rise for a new model and one can repeat the estimation procedure.

If the diagnostic checking of the residuals is satisfactory one can use the model for forecasting and control. Since the prognostic performance is also important for adequate representation, we decided to include the mean square error of forecast in the diagnostic checking section.

6.1.2. The Linear Filter Model

For the description of SARIMA-processes we use the linear filter model as a part of the following black-box-scheme:



The observed time series (stochastic process) is explained as an output of a linear filter, where the input is a white-noise-process. A white-noise-process has the following properties:

$$E(a_t) = 0$$

$$\text{Var}(a_t) = \sigma_a^2$$

$$\text{Cov}(a_t, a_{t+s}) = 0 \quad \forall t, s; t \neq s$$

Now we can write a linear filter process

$$\begin{aligned} z_t &= \sum_{j=1}^{\infty} \psi_j a_{t-j} \\ &= \psi(B) a_t \end{aligned}$$

where $\psi(B)$ is the so-called "transfer function" of the linear filter. The transfer function is an operator polynomial in B , where B is the backshift operator.

$$Bz_t = z_{t-1}$$

$$B^m z_t = z_{t-m}$$

The transfer function states how the past values of the white-noise-process (the past disturbances) influence the current value of the time series. Since the transfer function could include an infinite number of values one has to assume the existence of the sum. This fact also ensures stationarity of the time series. Stationary processes are characterized by means and variances which do not depend on time (they are constant), the covariance is only a function of the time difference. This generally means that observations with larger distance in time are less correlated.

In the linear filter model the generation of a time series is described from left to right. In fitting SARIMA-processes one has to go the other way round: From the observed time series (and the ACF/PACF-structure) one has to make the inference to the transfer function to obtain a white-noise-process of residuals.

6.1.3. ARIMA-processes

As the name indicates, ARIMA-processes consist of three processes:

a) A Stationary Autoregressive Process (AR(p)-process)

$$\phi(B)z_t = a_t$$

To ensure stationarity the AR-polynomial of degree p

$$\phi(B) = 1 - \phi_1 B - \dots - \phi_p B^p = 0$$

has roots which lie outside the unit circle.

b) An Invertible Moving Average Process (MA(q)-process)

$$z_t = \theta(B)a_t$$

To ensure invertibility the MA-polynomial of degree q

$$\theta(B) = 1 - \theta_1 B - \dots - \theta_q B^q = 0$$

has roots which lie outside the unit circle.

c) A Nonstationary Component

$$\nabla^d z_t = a_t$$

where

$$\nabla^d = (1-B)^d$$

means the d-th difference of the time series. Nonstationary time series can be made stationary by successive differencing. One determines d as the order of the differenced process which comes first closest to the ACF of a stationary process.

The result of combining these three components is the ARIMA(p,d,q)-process

$$\phi(B)\nabla^d z_t = \theta(B)a_t$$

or

$$\psi(B)z_t = \theta(B)a_t$$

where $\psi(B) = \phi(B)\nabla^d$ is the so-called ARI-operator or generalized autoregressive operator.

6.1.4. SARIMA-processes

SARIMA(p,d,q)x(P,D,Q)_s-processes are seasonal models for the time series. One usually combines nonseasonal ARIMA(p,d,q)-processes with seasonal ARIMA(P,D,Q)_s-processes where s is the length of the season. The operatorpolynomials of "multiplicative" models can be separated in products of nonseasonal and such of seasonal components, while in additive or mixed models this separation is not possible. The multiplicative version has the form

$$\phi(B)\phi(B^s)\nabla_s^D\nabla^d z_t = \theta(B)\theta(B^s)a_t$$

$\phi(B)$, ∇^d and $\theta(B)$ denote the polynomials of the ARIMA-components. Now we have three additional seasonal operator polynomials

a) Seasonal Autoregressive Process

The AR_s-polynomial

$$\phi(B^s) = (1-\phi_1 B^s - \phi_2 B^{2s} - \dots - \phi_P B^{Ps})$$

has P seasonal autoregressive coefficients and is of order Ps.

b) Seasonal Moving Average Process

The MA_s -polynomial

$$\Theta(B^s) = (1 - \theta_1 B^s - \theta_2 B^{2s} - \dots - \theta_Q B^{Qs})$$

has Q seasonal moving average coefficients and is of order Qs .

c) Seasonal Differences

Besides ordinary differences a SARIMA-process could contain seasonal differences

$$\nabla_s^D = (1 - B^s)^D$$

The order of the seasonal and regular differences can be determined by "variate-difference-matrices" (see O.D.Anderson (1976), p. 128).

For economic time series a very often fitted model is the $SARIMA(0,1,1) \times (0,1,1)_s$ -process. It can be interpreted as a combined exponential smoothing process in the seasonal and the non-seasonal component.

$$\nabla_s \nabla z_t = (1 - \theta B)(1 - \theta B^s) a_t$$

In the extended form the process reads

$$z_t = z_{t-1} + z_{t-s} + z_{t-s-1} + a_t - \theta a_{t-q} - \theta a_{t-s} + \theta \theta_{t-s-1}$$

6.1.5. Fitting and Diagnostic Checking

After identifying the tentative model we fitted the model with the help of the Marquardt-algorithm of nonlinear least squares. The preliminary parameter values are chosen in the identification stage, proportional to certain values of the ACF(PACF). The convergence of the algorithm was in all cases achieved without difficulties.

In the stage of diagnostic checking one has to examine the tentative estimated model carefully. The main question is as follows: Is the model adequately estimated and what do we understand by an "adequate estimation"?

6.1.6. Forecasting ARIMA-Processes

The theory of forecasting economic time series (compare C.W.J. Granger and P. Newbold (1977)) demands the specification of the forecasting problem with respect to forecast time, forecast criterion and properties of the forecast-function.

An autoprojective time invariant linear forecast function has the form

$$\hat{z}_{t+1} = \sum_{j=1}^T \alpha_j(1) z_{t-j}$$

where $\alpha_j(1)$ are the forecast coefficients depending on the leadtime l and the distance to the time origin t .

Following Box-Jenkins (1976) we denote the forecast function for ARIMA-processes with

$$\hat{z}_t(1) = \hat{z}_{t+1}$$

A time series z_t modelled as ARIMA-process relates an observation z_t by the means of the transfer function $\psi(B)$

$$\psi(B) = 1 - \psi_1 B - \psi_2 B^2 - \dots$$

to the disturbances a_t . Therefore we can express the forecast function of an ARIMA-process also as a function of current and past a_t

$$\hat{z}_t(1) = \sum_{j=1}^T \beta_j(1) a_{t-j}$$

If we assume an autoprojective, linear and time invariant forecast function and use the mean square error(MSE)-criterion we can show that the coefficients of the forecast function of an ARIMA-process are

$$\beta_j(1) = \psi_{j+1}$$

where the ψ_j are the coefficients of the transfer function. Therefore we have for the forecast function

$$\hat{z}_t(1) = \sum_{j=0}^{\infty} \psi_{j+1} a_{t-j}$$

6.1.7. Forecast Checking

To observe the forecasting performance of an estimated model or between two estimated models, we apply some methods of "forecast-checking". In comparing forecasts, we use the actual values and the percentage change to the previous year:

$$\gamma_t = \frac{z_t - z_{t-s}}{z_{t-s}}$$

where s is the length of the season. For nonseasonal (yearly) time series we have $s=1$.

To measure the average forecast error during a given forecast period we calculate the mean square error (MSE) and the root mean square error (RMSE)

$$MSE_L = \frac{1}{L} \sum_{l=1}^L (\hat{z}_t(1) - z_{t+l})^2$$

$$RMSE_L = \sqrt{MSE_L}$$

The index L indicates that there is one forecast origin t and L forecasts of the forecast function. For seasonal time series one can choose $L=s$ or $L=2s$, and then the $RMSE_L$ -coefficients between different models can help to decide which model has the better seasonal forecasting performance over one or two seasonal periods of a specific time series.

Besides this procedure we compared the one step ahead forecast error over the last K periods. For the sequence of the last K time origins

$$t-k, \quad k=1, \dots, K$$

we estimated the sequence of the one step ahead forecasts

$$\hat{z}_{t-k}(1) \quad k=1, \dots, K$$

and calculated the MSE of this sequence

$$MSE_1 = \sum_{k=1}^K (\hat{z}_k(1) - z_{t-k+1})^2$$

$$RMSE_1 = \sqrt{MSE_1}$$

If the forecast period was L then we take for $K=L$, too. This procedure gives information about the forecasting performance of the model within the last period and with respect to one step ahead forecasts.

6.2. The Seasonal Structure of Interest Rates

In this section we briefly discuss the phases of model identification and model checking of the Box-Jenkins models applied to the individual time series.

GMS (Call money rate) 66M11-77M9

(131 monthly observations)

The identification gives one regular difference, but instead of a rich seasonal component around lag 12 no seasonal difference is necessary. The ACF shows a rich seasonal activity from lag 11-15.

The first tentative model is $\nabla z_t = (1 - .27B_{11} - .26B_{13})a_t$ with $\sigma_a = 0,491$ and $RMSE_1 = 5,71\% (0,327)$. Both estimated coefficients are significant. θ_{12} could not be estimated significantly.

The next model $\nabla z_t = (1 - .29B11 - .22B13 - .34B14 + .13B15)a_t$ shows the lowest $\sigma_a = 0,467$ with $RMSE_1 = 5,98\% (0,34)$. This model explains almost all contributions of the ACF but θ_{15} is not estimated significantly. Autoregressive components of the form ϕ_1, ϕ_2, ϕ_3 or ϕ_4 were tried but without significant estimation or increasing forecasting properties. This is also the case for the θ_1 component. So the best compromise is the model

$$\nabla z_t = (1 - .30B11 - .24B13 - .33B14)a_t \quad \sigma_a = 0,470$$

with

$$RMSE_1 = 6.23\% (.356)$$

ER (Euro-Dollar-Rate) 57M1-77M9

(249 monthly observations)

Identification: No seasonal differences, one regular difference. The ACF shows non-diminishing values at lags 12, 24 etc. Together with the PACF which has a cut off at lag 12, these are indicators for a seasonal AR-process. The first tentative model leads to

$$\nabla(1 - .23B12)z_t = (1 + .38B1)a_t \quad \sigma_a = 0,481$$

with

$$RMSE_{12} = 8,85\% (0,5)$$

and

$$RMSE_1 = 6,13\% (0,37)$$

In contradiction to our identified model, P. Porsius estimated a SARIMA(0,1,1x(0,1,1))₁₂ process with $\theta_1 = -0,14$ and $\theta_{12} = -0,18$ with standard error $\sigma_a = 0,699$.

Since our identification procedure did not lead to this model, we made a comparison with the Porsius-model. The estimated parameters of the Porsius-model were used as preliminary parameter values. This leads to

$$\nabla z_t = (1 + .35B1 + .17B12)a_t \quad \sigma_a = 0,472$$

All parameters could be estimated significantly. The $RMSE_{12}=8,98\%$ (0,53) and $RMSE_1=5,56\%$ (0,32). So we got the interesting case that this model shows a lower standard error and a lower one step ahead forecast error, but the forecast properties for a whole seasonal period worsens. Since the residual ACF shows a contribution at lag 3 we tried a final version of this model

$$\nabla z_t = (1-.33B1+.18B3+.19B12)a_t \quad \sigma_a=0,469$$

with

$$RMSE_{12} = 9,94\% (0,60)$$

and

$$RMSE_1 = 5,39\% (0,31)$$

This model shows an additional decrease in the standard error and the root mean square error of the one step ahead forecast error, but a worse seasonal forecast behaviour.

Finally, we can say that the seasonal structure of the model is at lags 3 and 12 present.

R (Rate on new bonds) 53M4-77M2

(95 quarterly observations)

The identification stage gives no rise for seasonal differences, only for one regular difference. For the first 3 values the ACF shows significant correlations. Surprisingly, the PACF shows the same correlation pattern as the ACF. This leads to the first tentative model $(1+.37B1+.31B2)\nabla z_t=(1+.53B1)a_t$. The standard error is 0,263 and the root mean square error is $RMSE_4^*)=4,41\%$ and $RMSE_1=2,98\%$. The diagnostic checks give a significant residual correlation for lag 2.

One interesting fact is that an AR(3)-process, written as parsimonious multiplicative AR(2)-model gives better forecasting properties: The $(1-.30B1)(1+.33B2)\nabla z_t=a_t$ model has a standard error of 0,236 and a $RMSE_4=3,1\%$ and a $RMSE_1=2,78\%$. The seasonal θ_4 and θ_8 -components are not significant.

*) $RMSE_4$ for the period 76.3-77.2, the $RMSE_1$ for the period 75.3-77.2 (one step ahead forecasts)

We tried some moving average components for this model.
The best parsimonious model turned out to be

$$(1-.17B1)(1+.44B2)Vz_t = (1+.53B2)a_t$$

The standard error is 0,214 and the $RMSE_4 = 0,545\%$ (0,045) and the $RMSE_1 = 2,74\%$ (0,26). All parameters are significantly estimated.

R2 (Rate on bonds in circulation) 66M11-77M10

(132 monthly observations)

The identification gives one regular difference, but no seasonal difference. The ACF shows a rich seasonal structure.

The first tentative model $(1-.34B1)Vz_t = (1+.17B3+.25B4)a_t$ with $\sigma_a = .0902$ has $RMSE_{12} = 1,66\%$ and $RMSE_1 = .726\%$. The residual ACF shows high correlations at the lags 5, 11 and 16. All parameters are estimated significantly. An enlarged model shows no significant θ_1 but a significant θ_{11} with worse residual ACF. Greater models show a divergent behaviour. So there can θ_5 , θ_6 and θ_{11} be vice-versa significantly estimated by a good residual ACF behaviour, but the forecasting properties are decreasing.

E.g.: The model $(1-.38B1)=Vz_t(1+.21B3+.27B4+.23B5+.19B6+.16B11)a_t$ has the lowest estimated standard error ($\sigma_a = .850$) but the short term forecasting properties are increasing. $RMSE_{12} = 2,1\%$ (0,18) and $RMSE_1 = .74\%$ (0,067). In conclusion one can say that R2 has a rich but alternating seasonal structure, when lags of the length 3, 4, 5, 6 and 11 seem to be important.

AKI (Shares index) 58M1-77M10

(238 monthly observations)

Identification: No seasonal differences. One (or two) regular differences. The estimated model is an AR(1)-process in the first differences with $\phi = .48$

$$Vz_t = 0,48Vz_{t-1} + a_t \quad \sigma_a = 57,9$$

$$V(1-.48B)z_t = a_t$$

The RMSE's are

$$\text{RMSE}_{12} = 2,86\% \quad 76\text{M}11-77\text{M}10$$

$$\text{RMSE}_1 = 0,64\% \quad 75\text{M}11-77\text{M}10$$

Seasonal autoregressive and moving average parameters (of the order 12 and 6) could not be significantly estimated. An alternative MA(1)-model which has been estimated, yields worse forecasting properties.

TABLE 4:

	MODELS FITTED	STANDARD ERROR
GMS	month. $Vz_t = (1-.30B11-.24B13-.33B14)a_t$	0.470
ER	month. $Vz_t = (1+.33B1+.18B3+.19B12)a_t$	0.469
R	quart. $(1-.17B1)(1+.44B2)Vz_t = (1+.53B2)a_t$	0.214
R2	month. $(1-.34B1)Vz_t = (1+.17B3+.25B4)a_t$	0.902
AKI	month. $Vz_t = .48Vz_{t-1}+a_t$	57.9

7. Conclusions

This paper presents an attempt to find empirical evidence on the interrelationships between various interest rates in Austria. Methods used were regression analysis, spectral analysis, and Box-Jenkins-analysis.

First inspection of the data by simple correlation analysis in general showed low correlations between the available interest rates, especially between short term and long term rates.

All rates are marked by positive trends what has to be seen in connection with the inflationary movements in the period under consideration. This trend had to be removed by transformation of the data into first differences to result in stationary processes.

Results from the regression analysis gave first hints that the connection between short term and long term rates may not be very strong. Long term rates could be explained very well by their own lagged values; knowledge of current and past values of short term rates only gave marginal improvements.

These findings were backed by the results from spectral analysis: Call money rate and rate on "old" bonds showed common swings at three months. Long term rates displayed a long-range cycle of 5-6 years which cannot be observed for the short term rates.

Box-Jenkins-analysis gave more detail on the seasonal structure of Austrian interest rates. Seasonal components in general are present.

With exception of the shares' index which followed a random-walk-model in the first differences, systematic movements in Austrian interest rates are present. The often assumed high interrelationship between movements of short term and long term interest rates could not be observed. A typical seasonal figure and also a seasonal trend which was found in the model reported by Porsius (1977a) for the Euro-Dollar-Rate could not be confirmed though

we were successful in identifying an additional quarterly component which improved forecasting properties.

To draw some conclusion we may state that there seems to be considerable independence in the movements of short term and long term interest rates in Austria. Following Porsius (1977a) and others who found similar results, it can be maintained that at least in the short run strategies of influencing bond rates by means of money market operations will be of doubtful success.

Glossary of abbreviations

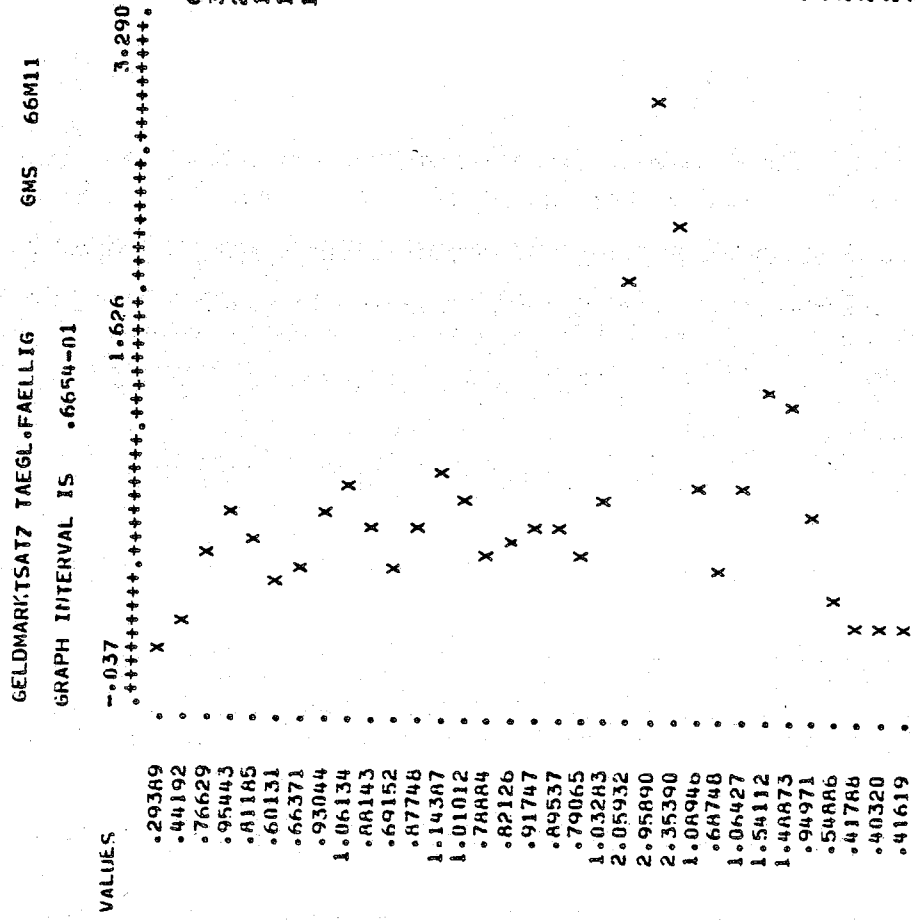
$\nabla^d = (1-B)^d$	regular differences of the order d
$\nabla_s^D = (1-B^s)^D$	seasonal differences of the order D where s is the seasonal length
ACF	auto-correlation function
PACF	partial auto-correlation function
RMSE	root mean square error
$RMSE_h$	root mean square error of the h-step forecast
$\hat{Z}_t(1)$	forecast function for the time origin t (fixed) and the lead time 1
L	length of the forecast horizon $l = 1, \dots, L$
$\hat{Z}_t(1)$	one step ahead forecast
θ_i	moving average parameter $i = 1, \dots, g$
θ_i	seasonal moving average parameter $i = 1, \dots, a$
ϕ_i	autoregressive parameter $i = 1, \dots, \gamma$
ϕ_i	seasonal autoregressive parameter $i = 1, \dots, P$
AR	autoregressive
MA	moving average
SAR	seasonal autoregressive
SMA	seasonal moving average
Z_t	time series $t = 1, \dots, n$
σ_a	standard error of the white noise process
a_t	white noise process
B	backshiftoperator $B^m Z_t = B Z_{t-m}$

A P P E N D I X :
=====

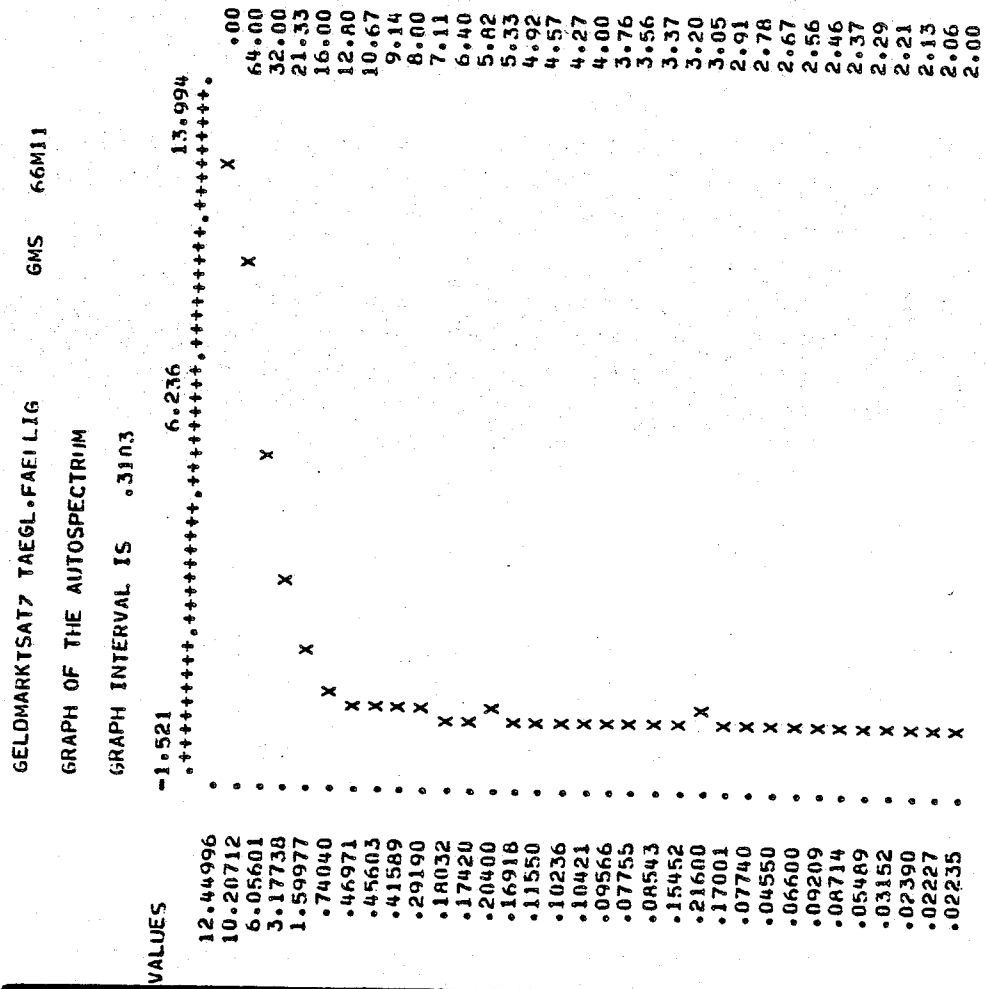
S P E C T R A

CALL MONEY RATE: SPECTRA

First Differences

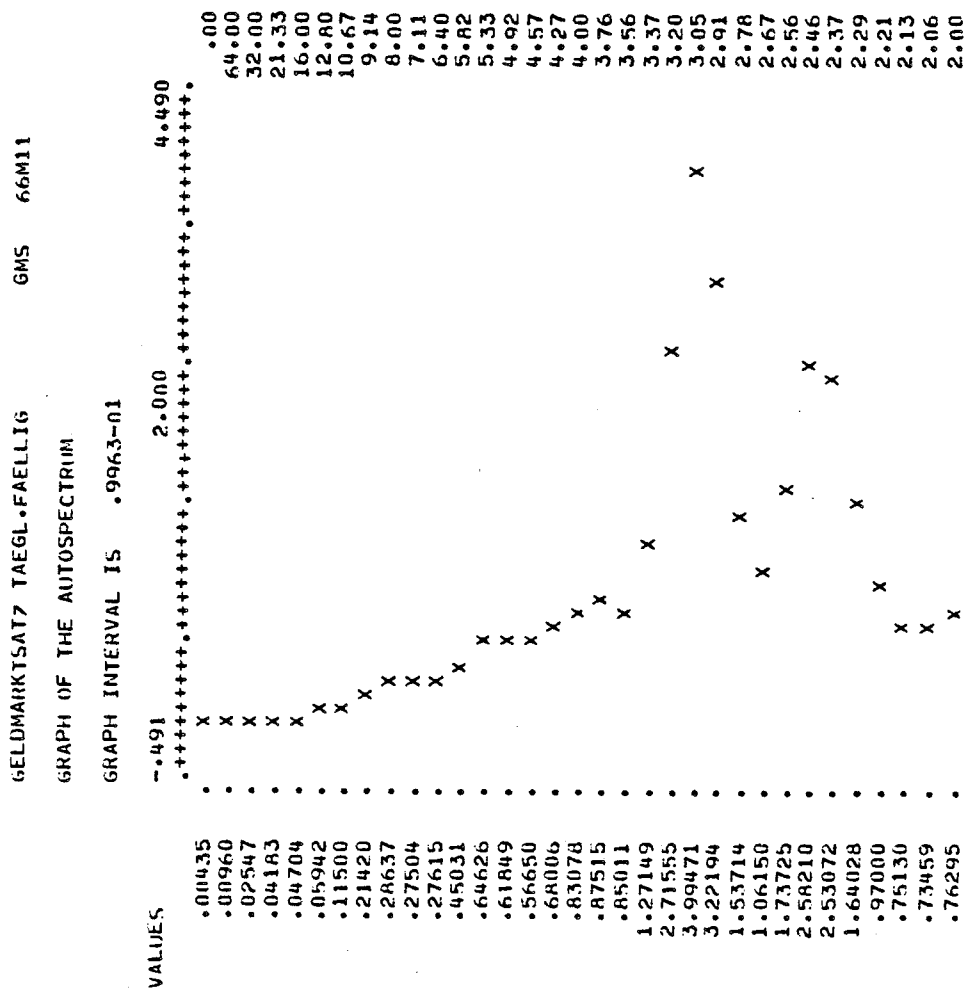


Level



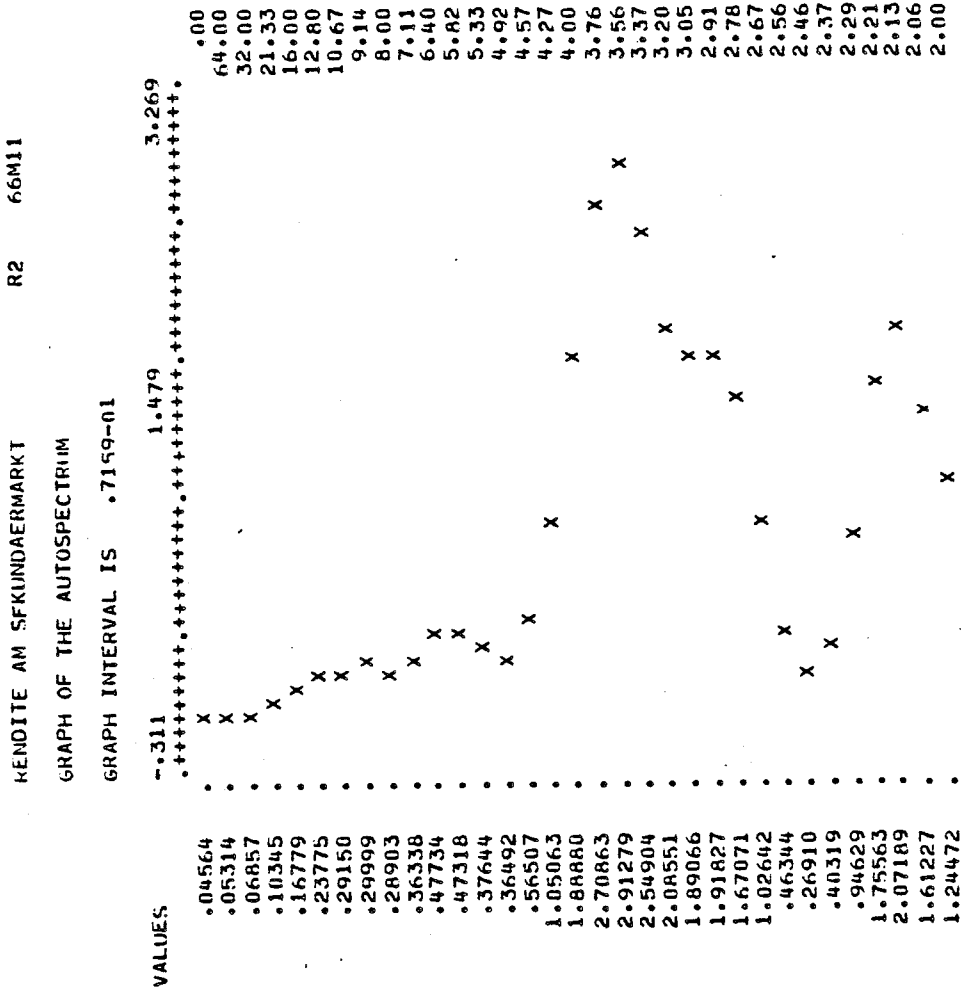
CALL MONEY RATE: SPECTRA

Second Differences



LONG TERM BOND YIELD (Old Issues): SPECTRA

Second Differences



LONG TERM BOND YIELD (New Issues): SPECTRA

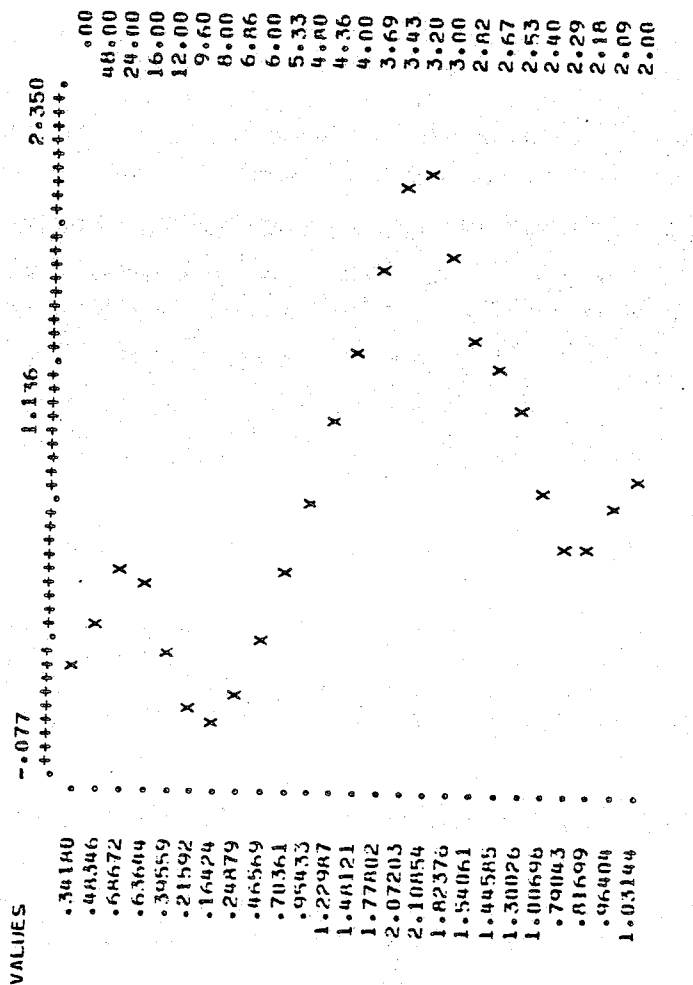
First Differences

Level

KENDITE DER NEUEMISSIONEN WENN RSECL 53.4 7

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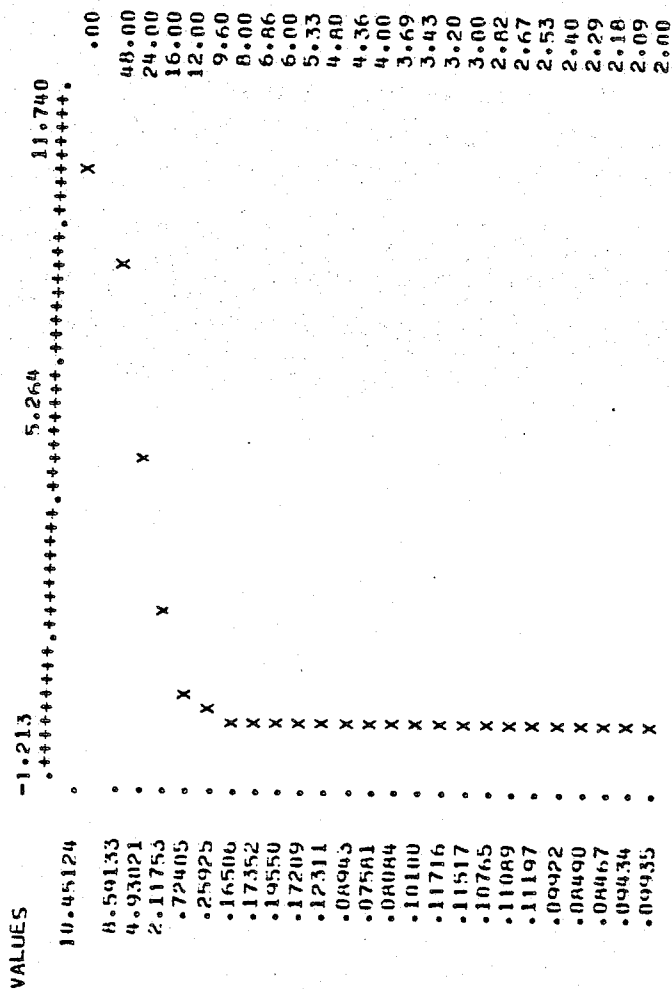
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KENDITE DER NEUEMISSIONEN WENN RSECL 53.4 7

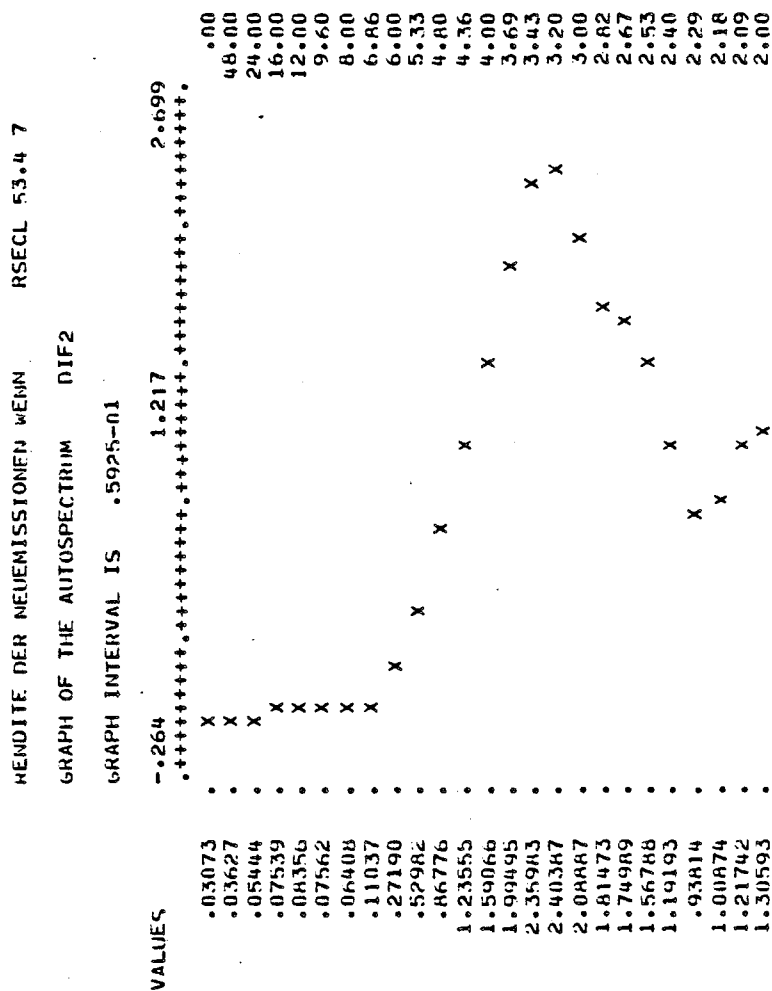
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LONG TERM BOND YIELD (New Issues): SPECTRA

Second Differences



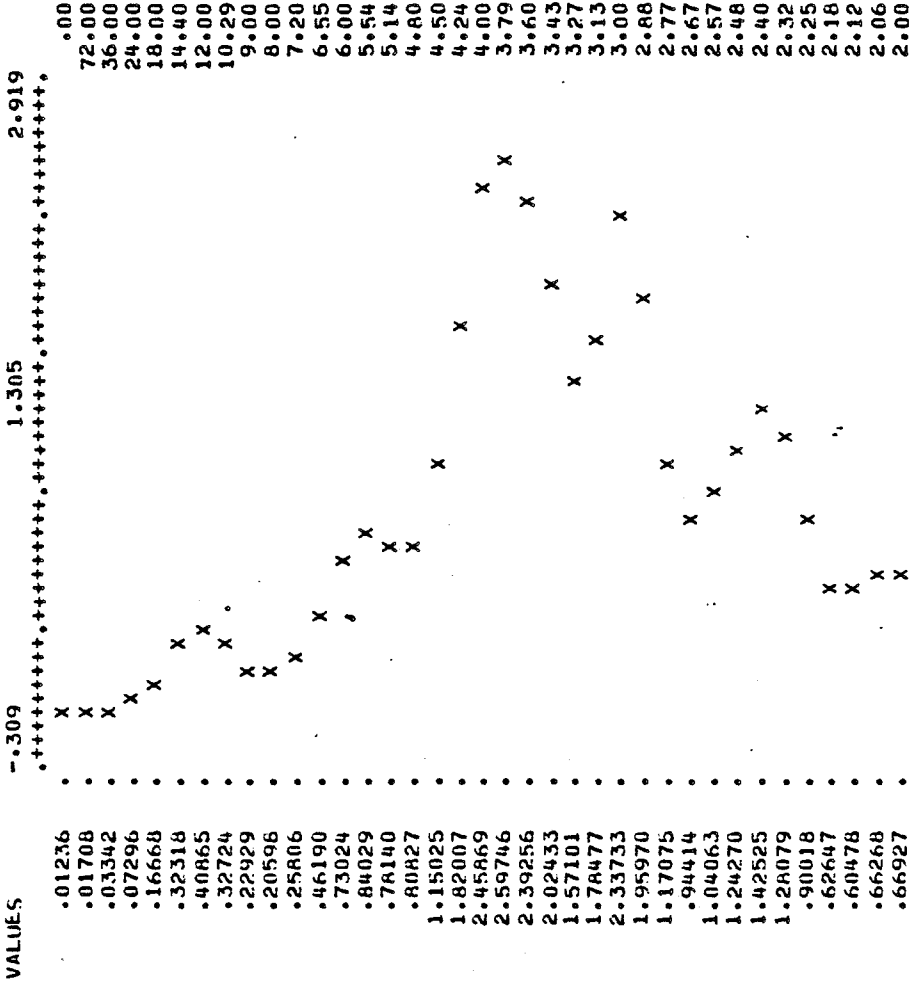
EURO-DOLLAR-RATE: SPECTRA

Second Differences

EURO-DOLLAR-5ATZ 3 M0 ER 57M1 7

GRAPH OF THE AUTOSPECTRUM

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Level

- 53 -

SHARES' INDEX: FULL PERIOD: SPECTRA

Second Differences

AKTIEKURSORINDEX	AKI	58M1 7
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.24957	+	+
.43608	+	+
.55246	+	+
.58003	+	+
.62437	+	+
.70361	+	+
.90399	+	+
1.04921	+	+
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.66063	+	+
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1.07733	+	+
1.44975	+	+
1.28845	+	+
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1.07726	+	+
1.10401	+	+
1.08529	+	+
1.08596	+	+
1.13029	+	+
1.33549	+	+
1.63085	+	+
1.82822	+	+
2.04161	+	+
2.18930	+	+
2.18737	+	+
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2.65090	+	+

Level

55

Second Differences

AKTIEKURSIINDEX AKI 63M1 7

GRAPH OF THE AUTOSPECTRUM

GRAPH INTERVAL IS .5775-01

[illegible]

SHARES' INDEX: LOGARITHMS FULL PERIOD

First Differences

Level

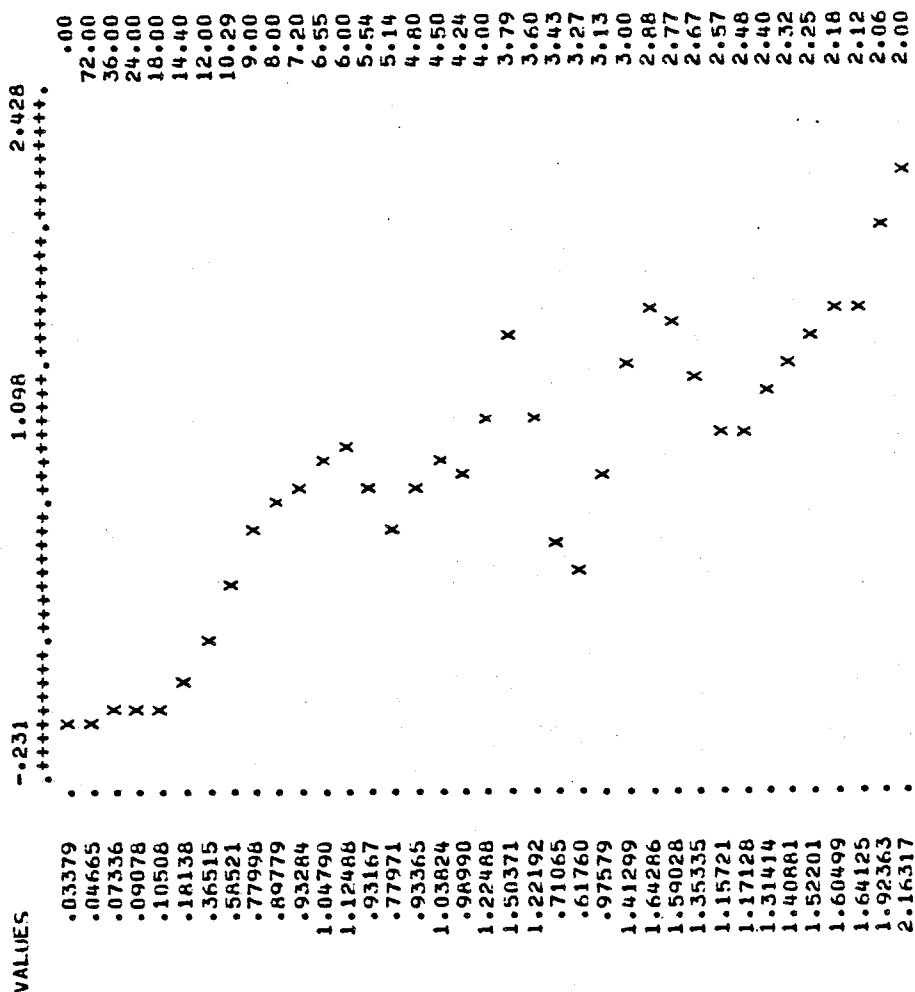
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1.90008		2.09586		2.09586		2.09586	
1.15316		.60502		.60502		.60502	
.94448		.29733		.29733		.29733	
1.31034		.22740		.22740		.22740	
1.73311		.15873		.15873		.15873	
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1.67743		.07335		.07335		.07335	
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.6986		.02855		.02855		.02855	
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.35594		.00916		.00916		.00916	
.38300		.00976		.00976		.00976	
.40235		.00971		.00971		.00971	
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AKI 58M1 7

AKTIENKURSIINDEX

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