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Public Policy for Efficient Education

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Abstract

This paper studies the role of public policy to promote efficiency in human capital accumulation in the representative agent framework. Agents accumulate human capital by spending time in home study and in publicly provided schools. The individual faces an aggregate externality in the accumulation of skills. In addition, the return to time spent in school is subject to congestion. To correct these distortions, a tuition fee combined with personal stipends is required, which shifts education in schools and universities to noninstitutional forms of learning such as home study. The dynamic effects of shifts in education policy as well as their welfare implications are also calculated in the paper.

Keywords

Education, human capital accumulation, optimal policy

JEL Classifications

H21, H42, I21, I28

Comments
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Contents

1. Introduction

Manpower quality has long been recognized as one of the most important sources of economic growth.[1] Education and training are also believed to yield important social returns.[2] Promoting human capital formation has, thus, long been a prime public policy concern. In fact, most governments have devised a large array of programs to improve educational attainment. Most, if not all, countries have a system of public schools, with primary and secondary education virtually free. Not only are numerous grants, subsidies and loans available to encourage higher education, but students are also offered a wide variety of institutions. Access to public universities is free in continental Europe and in many other countries as well. What is the basis for the significant level of public funds that is channeled into the educational system? Should governments, apart from distributing personal subsidies, also provide free access to higher educational institutions, as in continental Europe? Or should they charge tuition fees, as is commonly the case in the United States? This paper discusses optimal policies to provide agents with the correct incentives for education in the presence of spillovers in learning and potential crowding of public educational infrastructure.

To address these questions, we present a simple model of education production that relies on private time inputs and public infrastructure, such as schools and universities, including their equipment. A novel feature of our approach is an additional decision margin that has been neglected by the existing literature. Apart from the overall time allocated to education, agents may also decide how to allocate this time budget to home study and school attendance. Home study activity, which includes learning within families and firms, may be viewed as a noninstitutional source of skill formation. Heckman (1999), for one, considers this type of learning to be as important a source of skill acquisition as formal

[1] See the evidence of the Bureau of Labor Statistics (1993), Jorgenson, Ho and Fraumeni (1994), and Griliches (1997).

[2] A strain in the endogenous growth literature assumes that educational spillovers are so strong that they result in constant returns to aggregate human capital accumulation. See Lucas (1988) and Chamley (1993).

schooling. This extension allows us to analyze how governments may prevent possible congestion of their educational infrastructure by creating incentives for noninstitutional learning outside of school facilities.

We believe that our model connects quite well to the current debate on higher education policies, especially in continental Europe. These countries have traditionally offered free access to universities and other institutions of higher education. The supply of public educational infrastructure has not, however, kept pace with the rise in the number of students in the last decades. Accordingly, universities are severely under-equipped to serve this increase.[3] This has lead to over-crowded classrooms, a rising number of students per professor, increasingly restricted access to libraries and computer equipment, and, finally, to an increase in the average length of time required to complete a degree.[4] Such indicators of congestion have prompted a number of governments to consider tuition fees with the explicit intention of shortening the average study length and discouraging attendance of marginal students. For example, the Austrian government will formally introduce tuition fees starting in 2001, which will be partly compensated by student stipends. A number of German federal states have proposed charging fees to students who exceed the average study length. We interpret such measures as an attempt to reduce congestion of educational infrastructure. If tuition fees are combined with personal stipends, students are encouraged to shift to other noninstitutional forms of learning such as home study, rather than to completely sacrifice the opportunity for advanced training. Our model implies that governments, for reasons of efficiency, should encourage education in general, but also charge tuition to avoid congestion of school infrastructure. We, thus, provide a

[3]Barro and Sala-i-Martin (1995) view congestion as a pervasive problem for publicly provided goods. Lazear (1999) argues that congestion is particularly important in classroom education and is, in addition, crucial in determining optimal student/teacher ratio.

[4]For the data on the increase in the duration of study in the Austrian case, see the *Report on Higher Education* by the Austrian Federal Ministry of Science (1999). This increase has taken place across almost all fields of study and been particularly severe in business administration, medicine, law, electrical engineering, and pharmacy. Using data from previous years of the *Report*, we also find evidence of rising student/faculty ratios.

rationale for granting educational subsidies independent of distributional considerations, which tend to dominate public discussions of this issue.

These arguments are formalized in a model that captures the key influences on human capital formation. These include individual and community effort, educational subsidies or taxes imposed on the time spent in private educational activity or in public schools, and the level of public educational infrastructure. Section 2 describes the model and develops the decentralized equilibrium. Section 3 then compares the market allocation with the social optimum. Here we show that the optimal policy can be to subsidize overall educational effort and, simultaneously, to tax time spent using the publicly provided educational infrastructure. Section 4 derives the log-linearized form of the model, with details to be found in an Appendix, and discusses the comparative dynamic effects of public infrastructure and subsidy/tuition policies. Section 5 briefly concludes.

2. Decentralized Equilibrium

2.1. The Model

In this section we will describe a representative agent model in which skills, or human capital, are accumulated through effort and educational infrastructure. Unlike Lucas (1988) and Chamely (1993), human capital formation is bounded, reflecting diminishing returns to education. Unbounded accumulation is prevented by the fact that old agents loose part of their skills in the process of aging and, eventually, all skills with death. In a state of demographic and economic equilibrium, education of young agents and death of old agents just balances to give a finite constant stock of aggregate skills. One way to incorporate bounded human capital accumulation in an aggregate model — short of explicitly modeling and aggregating the life-cycle education decisions of overlapping generations — is to assume depreciation of human capital and diminishing returns to education in an infinitely lived representative agent model.[5] We believe that the representative agent framework is useful for the purposes of this paper, since the emphasis is on efficiency aspects of government education policy rather than on questions of intergenerational distribution.

The process of individual skill accumulation in our model depends on own and aggregate effort, as well as on the services of schools. To acquire skills, individuals typically spend some time attending schools. Expenditures on education include salaries for teachers as well as outlays for maintaining and possibly expanding the number of schools and their facilities. These services encourage individual learning, i.e., the accumulation of human capital. We consider schools in this framework to be "impure" public goods.[6] Schools are also to a certain extent non-rival, since it is not automatically the case that an additional student "in class" reduces one-for-one the educational services received by another student. The government in our model provides an aggregate level of educational infrastructure, K. We will assume a complementarity between effort and the level of schools, so that individuals become more effective in improving their skills if the level of public expenditures on schools is high. To keep the dynamics simple, we will model K as a flow variable.[7] Like other forms of government expenditure, an increase in expenditures on schools will divert resources from private consumption. As indicated, we will study the case in which these educational services are subject to spillovers. We shall then specify individual human capital accumulation in the following way

$$\dot{h} = G(e, K^s)H^{\gamma} - \delta h, \qquad K^s = i\,(i/I)^{\sigma}\,K, \qquad 0 \le \sigma \le 1, \quad 0 < \gamma < 1, \tag{2.1}$$

where h is the individual, H the aggregate stock of human capital, and $G(e, K^s)$ the component of human capital accumulation that depends on individual effort and the supply

[5] Glomm and Ravikumar (1992) employ an OG model to analyze the effect on growth and income inequality of private versus public education regimes.

[6] We follow the definition of Oakland (1972), who cites congestion externalities as leading to "impurity" in public goods. Of course, the existence of tuition or fees implies that schools are also excludable goods whether or not they are publicly or privately provided.

[7] A more complete, although more complicated, model would specify that K is a stock that depreciates at a certain rate. Such an approach has been recently adopted by Fisher and Turnovsky (1998).

of public school infrastructure.[8] The agent allocates his time endowment, (normalized to one), to total educational effort or work, l. We further assume that the time devoted to education can be broken-down into a home study component, e, and a component representing the time spent in schools, i, so that $l = 1 - e - i$. We term the time spent in acquiring skills away from school as "home study", but this should be thought of as any type of educational activity independent of public educational facilities. Home study does not generate external effects in our framework.

In addition to devoting time to home study, the student gains skills by using the services, K^s, of schools. We assume in (2.1) that the services of schools depend on the individual, i, and the aggregate amount of time, I, spent in schools as well as on the supply of schools, K. In our formulation of K^s, the influence of the aggregate time spent in school depends on the function $(i/I)^\sigma$. The degree of congestion in school depends on the parameter σ. If $\sigma = 0$, then schools are strictly non-rival public goods that can be consumed simultaneously by all students. In this case each student appropriates iK in public school services if he spends i units of time attending classes. If, on the other hand, $0 < \sigma \leq 1$, then the services of public schools are subject to congestion externalities. Here, the same amount of time spent in school yields only $i[(i/I)^\sigma K]$ in services if classes are "crowded", because congestion deteriorates the quality of services, with a larger value of σ corresponding to a higher degree of congestion.[9]

Along with the spillovers arising from school attendance, we specify in (2.1) that individual human capital accumulation depends positively on the level of aggregate skills, H, where γ parametrizes the extent of this spillover. In other words, we consider, as do

[8] Lower case variables will refer to individual quantities, while upper case variables will denote aggregate levels. Unless indicated, we will suppress a variable's functional dependence on time. A dot over a variable indicates a time derivative.

[9] If $\sigma = 1$, then schools have an effective capacity equal to $(i/I)K$. This case represents proportional congestion, since school capacity remains constant only if the aggregate level of schools, K, increases in direct proportion to aggregate attendance, I. Our interpretation is similar to that used by Fisher and Turnovsky (1998). Observe, finally, that we exclude the case of positive aggregate attendance spillovers. This would correspond to $-1 < \sigma < 0$.

Cooper and John (1988), that it is easier to acquire skills if others also have them.[10] For simplicity, and with no loss in generality, we impose linear homogeneity on G. Since, as indicated, this is a model of bounded human capital accumulation, we specify in (2.1) that skills deteriorate at the constant rate δ.

A simple specification of production is to consider a dynamic Ricardian model in which human capital is the sole input. Given a human capital stock h inherited from the individual's previous educational decisions, effective labor supply is $lh = (1 - e - i)h$, which produces the following output level:

$$y = (1 - e - i)h. \tag{2.2}$$

Effective labor earns its marginal product $w = 1$, i.e. the real wage is fixed at unity. Our framework then implies that if the individual decides to spend more time in acquiring skills, either in home study or in school, he sacrifices current wage income in exchange for a higher level of human capital in the future.

We assume that a continuum of identical agents with unit mass maximizes the discounted time-separable utility of consumption over an infinite horizon. Intertemporal preferences for each agent are given by

$$U_0 = \int_0^\infty e^{-\rho t} u(c) dt, \tag{2.3}$$

where ρ is the exogenous rate of time preference. A further simplifying specification restricts instantaneous preferences to the logarithmic case, $u(c) = \ln c$. In addition to the time constraint $l = 1 - e - i$, the individual's actions are subject to the accumulation equations for assets and human capital

$$\begin{array}{llcl} (a) & \dot{a} & = & ra + w[1 - (1-\tau)e - (1 - \tau + z)i]h - \chi - c, \\ (b) & \dot{h} & = & G(e, K^s)H^\gamma - \delta h, \end{array} \tag{2.4}$$

[10]This formulation is akin to the models of Romer (1986, 1989) in which the aggregate capital stock embodies the stock of knowledge. Some suggestive empirical evidence for this has been provided by Glaeser (1994), who shows that the level of schooling has a powerful effect on the growth of schooling.

where a represents financial assets that yield a real return r, c is consumption, and χ are lump-sum taxes, (the government budget constraint will be introduced below). In equation (2.4a), educational subsidy policy is introduced. We specify that the total time devoted to education, $e+i$, receives a subsidy, or stipend, τ. This is a reasonable assumption, we believe, because educational stipends do not in general discriminate between the time spent in home study and that spent attending school. We assume, in addition, that the time spent using the educational infrastructure attracts a specific fee, denoted by z. For reference, equation (2.4b) repeats (2.1).[11]

2.2. Optimality Conditions and Market Equilibrium

In solving the utility-maximizing problem, we attach the multipliers λ and μ, respectively, to the dynamic constraints $\dot{a}$ and $\dot{h}$. The optimizing choices satisfy the following first-order conditions:

$$
\begin{array}{llll}
(a) & c: & 1/c & = \lambda, \\
(b) & e: & (1-\tau)\,wh & = (\mu/\lambda)G_e(e,K^s)H^\gamma, \\
(c) & i: & (1-\tau+z)\,wh & = (\mu/\lambda)G_k(e,K^s)H^\gamma\,(1+\sigma)\,(i/I)^\sigma\,K, \\
(d) & a: & \dot{\lambda}/\lambda & = \rho - r, \\
(e) & h: & \dot{\mu}/\mu & = \rho+\delta-(\lambda/\mu)w[1-(1-\tau)e-(1-\tau+z)i].
\end{array}
\tag{2.5}
$$

According to (2.5a), the agent chooses a level of consumption so that its marginal utility equals its shadow value, λ. In deciding how much time to spend on private study or in school, agents compare the marginal foregone wage income today, net of subsidies and fees, with the present value (μ/λ) of future wage incomes that accrues because education raises the future stock of human capital. The existence of aggregate human capital spillovers and the externalities arising from the decision to accumulate human capital in school

[11] We could also incorporate an explicit labor/leisure choice into the problem and specify that educational subsidies and infrastructure expenditure are financed using taxes on labor income. By making the admittedly strong assumption of lump-sum taxation, we focus on the allocating role of educational policies and keep the algebraic solution of the model as simple as possible.

modify the optimality conditions for e and i. The return on home study, $G_e H^\gamma$, depends on the aggregate stock of human capital as well as on individual effort.

The return on time spent in school depends on the term $H^\gamma (1+\sigma)(i/I)^\sigma K$, which, in addition to H^γ, is a function of the ratio (i/I), the supply of public schools K, and the congestion parameter σ. Under conditions of congestion, $(\sigma > 0)$, an individual student may not only exploit a given effective capacity per student, $(i/I)^\sigma K$, for a longer time-span i, but may also attain for his own use a larger *share* of the overall effective capacity $(i/I)^\sigma K$. In other words, congestion leads individuals to believe that they may capture a larger fraction of the overall capacity by attending classes more intensively. The existence of aggregate human capital spillovers and the externalities arising from public school attendance must be taken into account in evaluating the overall benefits of time spent at home and in school and will, as we will show below, determine socially optimal values for τ and z. The last two optimality conditions describe, respectively, the evolution of the shadow values of wealth and human capital if optimal paths of these two variables are chosen. In addition, the accumulation of financial assets and human capital must satisfy the usual transversality conditions.

To calculate the decentralized macroeconomic equilibrium, we use the assumption of a continuum of identical agents to derive that the aggregate times devoted to work, home study, and time in school are given, respectively, by $L = l$, $E = e$, and $I = i$. Applying these relations to (2.1), we rewrite the aggregate human capital accumulation function as

$$G = G(e, iK) = eg(Ki/e), \tag{2.6}$$

where $K^s = iK$. Linear homogeneity allows us to write G in the intensive form, $eg(Ki/e)$, where g is increasing and concave. Since there is no net trading of financial assets in equilibrium and, by assumption, no government debt, we can set $\dot{a} = a = 0$ and $\dot{A} = A = 0$. Using this fact and the aggregate relationships, the economy-wide private sector budget constraint is given by

$$C + T + ziH = [L + \tau(e+i)] H, \tag{2.7}$$

where C, T, L and H correspond to aggregate consumption, lump-sum taxes, labor supply and human capital, respectively. The government pays for its expenditures on schools and finances its subsidy policies by levying lump-sum taxes and tuition fees. This implies that the aggregate government budget constraint equals

$$K + \tau(e+i)H = T + ziH. \tag{2.8}$$

Substituting the government into the private sector budget constraint and using the aggregate version of (2.2), we obtain the following market-clearing condition

$$C + K = LH \equiv Y, \tag{2.9}$$

where $Y = (1-e-i)H$ is aggregate output.[12]

To derive the economy's dynamics, we will first obtain an equilibrium restriction on the ratio of time spent in school to time spent in home study. Without loss of generality, we can simplify our analysis by assuming that the human capital accumulation function G has a Cobb-Douglas specification, so that $\alpha = eG_e/G$ and $1-\alpha = iKG_K/G$.[13] Using the equilibrium versions of (2.5b-c), the ratio i/e is given by:

$$\frac{i}{e} = \frac{(1+\sigma)(1-\alpha)}{\alpha} \cdot \frac{1-\tau}{1-\tau+z}. \tag{2.10}$$

Observe that the ratio i/e depends, in addition to the technology and congestion parameters, exclusively on τ and z. Rewriting (2.5b) and using our technological assumptions and the intensive form of G, we obtain

$$\frac{\lambda}{\mu} = \frac{\alpha g(Ki/e)H^{\gamma-1}}{1-\tau}, \tag{2.11}$$

which we substitute into (2.5e) to obtain the dynamic equation for the shadow price. This, together with (2.4b), forms the dynamic system in (μ, H) that determines the evolution of intertemporal macroeconomic equilibrium

$$\begin{array}{llll} (a) & \dot{\mu} & = & (\rho+\delta-r^H)\mu, \quad r^H \equiv \frac{\alpha g(Ki/e)H^{\gamma-1}}{1-\tau}\left[1-(1-\tau)e-(1-\tau+z)i\right], \\ (b) & \dot{H} & = & eg\left(Ki/e\right)H^{\gamma} - \delta H, \end{array} \tag{2.12}$$

[12]Hereafter, we shall designate the equilibrium time spent in home study and in school, e and i, with lower case letters, while all other variables will be denoted with upper case.

[13]The Cobb-Douglas form implies $G = eg(Ki/e)$ and $g = (Ki/e)^{1-\alpha}$.

where r^H denotes the rate of return on human capital (so that the implicit return on financial assets is $r = r^H - \delta$). Consider the dynamics of the shadow value, μ. If the return to human capital r^H is high, the shadow value μ of new skills is also high. Agents increase their efforts at education. The shadow value declines as the skill levels improve and the return on human capital is driven down to the long-run rate equal to $\rho + \delta$.

To complete the description of intertemporal equilibrium, we next show how e and i depend on the dynamic variables and exogenous policy parameters. The time e devoted to home study and the time i spent in school are interdependent according to (2.10). Note further that the ratio i/e does not depend on μ and H, but only on the subsidy and fee rates τ and z, the share parameter α, and the congestion parameter σ. Nevertheless, both e and i can be expressed as a function of the dynamic variables μ and H. Substituting aggregate consumption, $C = 1/\lambda$, into (2.5b), using the Cobb-Douglas specification of g, and replacing C from the product market clearing condition (2.9) yields:

$$(1-\tau) = \mu\alpha g(Ki/e)H^{\gamma}\left[1 - [1 + (i/e)]e - K/H\right]. \tag{2.13}$$

This equation solves for $e(\mu, H; \tau, z, K)$ and, since (2.10) fixes the ratio i/e, also the value of $g(Ki/e)$.[14] In the Appendix, we derive a log-linearized version of the dynamic equations (2.12). Using this system, discussed below in Section 4, we can obtain analytical solutions for μ and H and can calculate the long-run comparative dynamics for these variables with respect to government infrastructure expenditure and stipend/tuition policy.

The steady state equilibrium occurs when $\dot{H} = \dot{\mu} = 0$. It consists of the following relationships

$$(a)\ H_{\infty}^{1-\gamma} = \frac{e_{\infty}g(\cdot)}{\delta}, \qquad (b)\ H_{\infty}^{1-\gamma} = \frac{\alpha g(\cdot)}{\rho+\delta}\left[\frac{1}{1-\tau} - \left(1 + \frac{(1+\sigma)(1-\alpha)}{\alpha}\right)e_{\infty}\right], \tag{2.14}$$

where (2.10) has been used to obtain (2.14b) and the subscript ∞ denotes a steady state value. We can now state the first proposition of the paper.

Proposition 1. (Steady State Time Allocations in Decentralized Equilibrium) *The steady state allocations of time spent in home study and time spent in school are*

[14] Once $e(\mu, H; \tau, z, K)$ is obtained, $i(\mu, H; \tau, z, K)$ is calculated using (2.10).

given by:

$$(a) \quad e_\infty = \tfrac{1}{1-\tau} \cdot \tfrac{\alpha\delta}{\rho+\delta+\delta[\alpha+(1+\sigma)(1-\alpha)]}, \qquad (b) \quad i_\infty = \tfrac{1}{1-\tau+z} \cdot \tfrac{\delta(1+\sigma)(1-\alpha)}{[\rho+\delta+\delta\{\alpha+(1+\sigma)(1-\alpha)\}]}. \tag{2.15}$$

The long-run expression for e_∞ was found by equating the two conditions (2.14), while the long-run solution for i_∞ was determined by substituting (2.15a) into the ratio (2.10). Observe that e_∞ and i_∞ depend on the parameters of the model, such as σ, and on the overall subsidy rate τ, but not on the aggregate human capital spillover parameter γ or on the level of educational infrastructure K.[15] The latter is a consequence of employing the Cobb-Douglas specification in the human capital accumulation function.[16] Nevertheless, even if the time spent in skill accumulation does not depend on the level of infrastructure, the stock of human capital, H_∞, always does. With the i/e-ratio, and, consequently $g(Ki/e)$, determined by (2.10), the long-run skill level H_∞ is then inferred from (2.14a) and its shadow value μ_∞ from (2.13). In examining the solutions for e_∞ and i_∞, we can also investigate the influence of the congestion parameter σ. It is straightforward to show that e_∞ is smaller and i_∞ is larger if there are congestion externalities in school, $0 < \sigma \leq 1$. The individual then strives to acquire human capital by attending school more intensively while reducing the time spent in home study.

To consider the steady state influence of subsidy, tuition, and infrastructure policy on the time spent at home and at school as well as on the stock of human capital and its shadow value, we next state the following proposition.[17]

Proposition 2. (Impact of Education Subsidy and Infrastructure Policy) *The impact of education subsidies on the long-run time allocations to home and school effort is equal to:*

$$(a) \qquad \hat{e}_\infty = \frac{1-\tau+z}{1-\tau}\hat{\tau}, \qquad (b) \qquad \hat{i}_\infty = \hat{\tau} - \hat{z}. \tag{2.16}$$

[15] While the aggregate human capital spillover affects neither the level nor the division of time spent in home study or in school, it does affect their productivity.

[16] In the case of non-unitary elasticity of substitution, the results would be more complicated. In general, the steady state levels of e and i would then depend on K.

[17] A logarithmic derivative of a variable x is denoted $\hat{x} = dx/x$. The exceptions are $\hat{\tau} \equiv d\tau/(1-\tau+z)$ and $\hat{z} \equiv dz/(1-\tau+z)$, respectively. We also make the innocuous assumption that $(1-\tau+z) > 0$.

The effect of subsidy and infrastructure policy on steady state human capital is:

$$\hat{H}_\infty = \frac{1}{1-\gamma}\left[(1-\alpha)(\hat{K}-\hat{z}) + \left(1+\frac{\alpha z}{1-\tau}\right)\hat{\tau}\right]. \tag{2.17}$$

The effect on the long-run shadow value of human capital equals

$$\hat{\mu}_\infty \quad = \quad -\tfrac{1-\alpha-(1-\gamma)k}{(1-\gamma)(1-k)}\hat{K} + \tfrac{1-\alpha-(1-\gamma)s_I}{(1-\gamma)(1-k)}\hat{z} - \tfrac{1-(1-\gamma)\bar{s}-[(1-\gamma)(\bar{s}-s_I)-\alpha]z/(1-\tau)}{(1-\gamma)(1-k)}\hat{\tau}, \tag{2.18}$$

where $s_I \equiv i/l$*,* $\bar{s} \equiv (1-l)/l$*, and* $k \equiv K/Y$*.*

Logarithmic derivatives of (2.15) were taken to obtain (2.16). Observe that the long-run response of home study, $\hat{e}_\infty$, depends solely, though more than proportionately, on the rate of change of the overall subsidy rate $\hat{\tau}$. In contrast, the adjustment of school attendance, $\hat{\imath}_\infty$, depends on the *difference* between the rates of change of the overall subsidy and the tuition fee, $\hat{\tau}-\hat{z}$. Both $\hat{e}_\infty$ and $\hat{\imath}_\infty$ are, however, independent of $\hat{K}$.[18]

To calculate the long-run impact of subsidy policy on $\hat{H}_\infty$, we substituted $\hat{e}_\infty$ and $\hat{\imath}_\infty$ into the percentage change of g, $\hat{g} = (1-\alpha)[\hat{K}+\hat{\imath}_\infty-\hat{e}_\infty] = (1-\alpha)\left[K-\hat{z}-\frac{z}{1-\tau}\hat{\tau}\right]$, and the resulting expression into the log-linearized version of (2.14a), which is equal to $\hat{H}_\infty = (1-\gamma)^{-1}(\hat{e}_\infty+\hat{g})$. This yields (2.17). According to (2.17), improved educational infrastructure and more generous subsidies serve to expand human capital. Higher tuition fees would tend to lower $\hat{H}_\infty$. The extent of the changes depends on the factor shares, the spillover parameter γ, and the initial subsidy and tuition rates τ and z.

Next, substituting $\hat{e}_\infty$, $\hat{g}$, and $\hat{H}_\infty$ into equation (A.6) (derived in the appendix), we solve in (2.18) for the long-run comparative statics for the shadow value of human capital. Whether $\hat{\mu}_\infty$ rises or falls depends in general on the resource cost of the policy change relative to its effect on the returns to educational activities. Consider the following illustrative cases. In the absence of human capital spillovers, $\gamma = 0$, an increase in $\hat{K}$ will lower (raise) $\hat{\mu}_\infty$ if the output share of schooling infrastructure k is less (greater) than the infrastructure share in skill production, $1-\alpha$. With positive spillovers, the same effect

[18] The congestion parameter σ drops out of the steady state expressions for $\hat{e}_\infty$ and $\hat{\imath}_\infty$. This is a consequence of having imposed a unitary elasticity of substitution in skill production.

continues to hold for even higher values of k. Similarly, if we set the initial subsidy and tax rates at zero, $\tau = z = 0$, assume there is no congestion, $\sigma = 0$, and substitute for $\bar{s}$ and s_I, we can then show that increases in $\hat{\tau}$ and decreases in $\hat{z}$ reduce $\hat{\mu}_\infty$.[19]

3. Social Optimum

3.1. Optimality Conditions and Equilibrium

Private agents take as given the number of schools and the educational efforts of other agents. Society at large, however, faces a trade-off between the resource cost of public infrastructure and the returns that result from changing τ and z, which include any externalities involved in the process of acquiring skills. Weighing the per-capita benefits against the resource costs of marginally expanding the size of educational infrastructure determines a welfare maximizing number of schools. This trade-off and its welfare implications are addressed by a social planning approach in which per-capita intertemporal utility is maximized subject to the aggregate resource constraints, i.e., the market-clearing condition for the output and the law of motion for human capital. In deciding on behalf of the entire community, the planner chooses the effort of all agents simultaneously. This takes into account the aggregate level of externalities in school attendance. The planner internalizes these spillovers by setting $i = I$, which results in $K^s = iK$. By equating $h = H$, the planner also takes into account how the aggregate level of human capital contributes to skill accumulation. This planning problem can, therefore, be stated as:

$$\max \int_0^\infty e^{-\rho t} \ln(C) dt \quad s.t. \quad C + K = (1 - i - e)H, \quad \dot{H} = G(e, iK)H^\gamma - \delta H. \qquad (3.1)$$

[19] Evaluating (2.15) in this case, we have $e = \alpha\delta/(\rho + 2\delta)$ and $i = e(1-\alpha)/\alpha$. Equation (A.1) then gives $\bar{s} = \delta/(\rho + \delta)$ and $s_I = (1-\alpha)\bar{s}$.

The social optimum is characterized by the following maximum conditions:

$$\begin{array}{llcl}(a) & e: \frac{H^{1-\gamma}}{C} & = & \mu G_e(e, iK), \\ (b) & i: \frac{H^{1-\gamma}}{C} & = & \mu G_K(e, iK)K, \\ (c) & K: \frac{H^{-\gamma}}{C} & = & \mu G_K(e, iK)i, \\ (d) & H: \dot{\mu}/\mu & = & \rho + \delta - \gamma G(e, iK)H^{\gamma-1} - \frac{\theta}{\mu C}(1-e-i). \end{array} \tag{3.2}$$

Condition (3.2a) weighs the utility cost of forgone consumption against the utility gain from increased skills due to a socially optimal time spent in home study. Conditions (3.2b,c) apply the same criterion to determine the optimal intensity of school attendance and educational infrastructure. Condition (3.2d) describes the socially optimal evolution of the shadow value of skills. Using the intensive form of G described in footnote 13 and assuming Cobb-Douglas technology, we obtain from (3.2a),

$$\frac{1}{\mu C} = \alpha g(Ki/e)H^{\gamma-1}, \tag{3.3}$$

which is the socially optimal version of (2.11). Substituting (3.3) into (3.2d) yields

$$\dot{\mu}/\mu = \rho + \delta - r^{H^*}, \qquad r^{H^*} = [\gamma e + \alpha(1-i-e)]\, g(Ki/e)H^{\gamma-1}, \tag{3.4}$$

where r^{H^*} is the social return of human capital (henceforth, we will denote a socially optimal value by the superscript $*$). Next, combining (3.2a,b) yields the socially optimal ratio of time spent in school to time in home study:

$$\left(\frac{i}{e}\right)^* = \frac{1-\alpha}{\alpha}. \tag{3.5}$$

Equation (3.5) implies that $(i/e)^*$ should be set equal to the ratio of the factor shares of school services and home effort, $(1-\alpha)/\alpha$. It is instructive to compare (3.5) to (2.10), the ratio in the decentralized equilibrium in which individual decisions are distorted by externalities, subsidies, and fees. If there were no congestion, $\sigma = 0$, and the subsidy and tuition rates were set to zero, $\tau = z = 0$, then (2.10) would coincide with (3.5). Observe also that the optimal ratio (3.5), like its decentralized counterpart (2.10), is independent of γ, the spillover parameter for human capital accumulation. As we will show below, it is the "wedge" between private and social returns that implies a role for optimal government policy. Next, using (3.2b,c) or, alternatively, (3.2a,c) we obtain

$$K^* = Hi^* = \frac{1-\alpha}{\alpha} He^*, \tag{3.6}$$

which says that the fiscal policy authorities, given the historically accumulated stock of human capital, should supply the level of infrastructure given in (3.6) in order to accommodate optimal school attendance. We now state

Proposition 3. (Time Allocation and Infrastructure in the Social Optimum) *The socially optimal steady state allocations of time spent in home study and in public schools are given by:*

$$(a) \quad e^*_\infty = \frac{\alpha\delta}{\rho + \delta + \delta(1-\gamma)}, \qquad (b) \quad i^*_\infty = \frac{(1-\alpha)\delta}{\rho + \delta + (1-\gamma)\delta}. \tag{3.7}$$

The socially optimal share of infrastructure in output is equal to:

$$k^* \equiv \frac{K^*}{Y^*} = \frac{(1-\alpha)e^*_\infty}{\alpha - e^*_\infty} = \frac{(1-\alpha)\delta}{\rho + \delta(1-\gamma)} < 1. \tag{3.8}$$

To derive equations (3.7) of proposition 3, we note that the law of motion for skills, as in the decentralized case, equals $H^{1-\gamma}_\infty = e_\infty g(\cdot)/\delta$ in the steady state equilibrium. On the other hand, $\dot{\mu} = 0$ in the social optimum requires $H^{1-\gamma}_\infty = \frac{g(\cdot)}{\rho+\delta}[\gamma e^*_\infty + \alpha - e^*_\infty]$, where we used (3.5) to eliminate i^*_∞. Equating the two relationships for $H^{1-\gamma}_\infty$, we solve for e^*_∞ and then use (3.5) again to obtain i^*_∞.

Comparing (3.7) to (2.15) reveals how externalities cause the decentralized level of home effort and the time spent in school to depart from their socially optimal levels. For example, the decentralized length of time devoted to education (setting $\tau = z = \sigma = 0$), whether at home or in school, will fall short of its socially optimal counterpart if the aggregate level of skills have a positive impact on human capital accumulation, i.e., if $\gamma > 0$. On the other hand, assuming $\tau = z = \gamma = 0$, but allowing for congestion, $0 < \sigma \leq 1$, agents spend too much time in school relative to the social optimum. The optimal policies we will calculate below will eliminate these discrepancies.[20]

To compute (3.8), the socially optimal output share of school infrastructure, we divide (3.6) by output $Y = (1 - i - e)H$, and use (3.5). We find, after substituting for (3.7a), the expression for k^*. As long as $\gamma < \rho/\delta$, the optimal share of infrastructure is less than proportional to the share of school services in the human capital accumulation function, $(1 - \alpha)$. Clearly, the socially optimal share of total resources devoted to school infrastructure is greater the larger is γ, the aggregate human capital spillover parameter.[21]

3.2. Optimal Tax/Subsidy Policy

We will show how the private equilibrium stated in section 2.2 can replicate first-best equilibrium through the use of optimal government policy. We shall limit our attention to the replication of the steady state equilibrium, though a time-varying government policy can be used to reproduce the first-best equilibrium along a dynamic path.

Proposition 4. (Optimal Subsidy/Tuition Policy) *For the decentralized equilibrium to become socially optimal, the two economies must attain the same time allocation of educational activities for any given level of public infrastructure. The first step is to set the subsidy rates so that the private ratio i/e replicates the socially optimal one. Equating (2.10) with (3.5), we obtain the restriction:*

$$(1 - \tau^* + z^*) = (1 + \sigma)(1 - \tau^*). \tag{3.9}$$

where $0 < \sigma \leq 1$. *To generate the same incentives for skill accumulation, we equate the private and social rates of return to skills given in (2.12a) and (3.4), i.e.,* $r^H = r^{H^*}$.

[20] As in the decentralized case, H^*_∞ can be determined: substitution of (3.5) and (3.7) into the stationary condition $\dot{H} = 0$ implicitly solves for H^*_∞. In turn, the shadow value of human capital μ^*_∞ can be found by combining (3.3) with (3.1) and substituting for the socially optimal values we have calculated.

[21] Note that the long-run shadow value in (2.18) will always decline for an increase in $\hat{K}$ as long as the output share k is less than k^*, or does not exceed k^* by too much. Substituting (3.8) into $[1 - \alpha - (1 - \gamma)\, k]$ yields $(1 - \alpha)\, \rho / \left[\rho + (1 - \gamma)\, \delta\right]$, so that the coefficient on $\hat{K}$ in (2.18) is negative.

Using (2.10) and the restriction on z^* in (3.9), we obtain the optimal overall subsidy:

$$\tau^* = \frac{e\left[(1-\alpha)\sigma + \gamma\right]}{\alpha + e\left[(1-\alpha)\sigma + \gamma\right]}. \tag{3.10}$$

The relationship (3.9) implies $z^* = \sigma(1-\tau^*)$, which is combined with (3.10) to derive the optimal tuition fee:

$$z^* = \frac{\alpha\sigma}{\alpha + e\left[(1-\alpha)\sigma + \gamma\right]}. \tag{3.11}$$

Proposition 4 states that while a *tuition fee* should be charged for school attendance, $z^* > 0$, overall effort should receive a *subsidy*, $\tau^* > 0$. Observe, however, that if schools are strictly non-rival public goods, $\sigma = 0$, then no tuition fee should be charged, $z^* = 0$, while an overall subsidy should still be offered, $\tau^* > 0$, as long as there are aggregate human capital spillovers, $\gamma > 0$.

Consider, on the other hand, the case in which there is no aggregate human capital externality, $\gamma = 0$, but public education is subject to congestion, $\sigma > 0$. The optimal values of (3.10, 11) then become:

$$\tau^* = \frac{(1-\alpha)\sigma e}{\alpha + (1-\alpha)\sigma e}, \qquad z^* = \frac{\alpha\sigma}{\alpha + (1-\alpha)\sigma e}. \tag{3.12}$$

This implies, as in the general case, that an overall subsidy to skill accumulation should be offered, while, at the same time, a tuition fee should be imposed. In the absence of spillovers from aggregate human capital, the trade-off between education and work is not distorted.[22] Nevertheless, if agents fail to correctly evaluate the benefits of attending public schools due congestion externalities, private decisions regarding the correct time allocation between the two educational activities will be. As before, congestion ($0 < \sigma \leq 1$) calls for charging tuition fees and subsidizing overall effort. Charging a tuition fee to prevent congestion must, therefore, be accompanied by an overall subsidy in order to preserve the overall incentives for educational effort.

[22] According to Acemoglu and Angrist (1999), it is difficult to establish empirically the existence of aggregate human capital spillovers.

4. Comparative Dynamics of Education Reform

In this section we shall discuss the impact of educational policies on the dynamics of human capital investment. The basis of our analysis will be the log-linearized version of the system derived in (2.12). Since the derivation of this system involves some lengthy algebra, we relegate it to the Appendix and simply state it here in matrix form:

$$\begin{bmatrix} \dot{\hat{\mu}}_t \\ \dot{\hat{H}}_t \end{bmatrix} = \begin{bmatrix} \varepsilon_\mu & \varepsilon_H \\ \eta_\mu & \eta_H \end{bmatrix} \begin{bmatrix} \hat{\mu}_t \\ \hat{H}_t \end{bmatrix} + \begin{bmatrix} \varepsilon_K & \varepsilon_\tau & \varepsilon_z \\ \eta_K & \eta_\tau & \eta_z \end{bmatrix} \begin{bmatrix} \hat{K} \\ \hat{\tau} \\ \hat{z} \end{bmatrix}. \tag{4.1}$$

The ε and η elements of the coefficient matrices are defined in the Appendix and the subscript t indicates the variables whose growth rates vary with time. For convenience, we use the following short-hand matrix notation to represent the dynamic system, $\dot{X} = AX + B$. The roots of the characteristic polynomial, $\Psi(\omega) = |\omega I - A| = 0$, of (4.1) determine a pair of eigenvalues $\zeta, \bar{\zeta}$. Since

$$\Psi(0) = \det A = \zeta\bar{\zeta} = \varepsilon_\mu \eta_H - \varepsilon_H \eta_\mu = -\eta_\mu (1-\gamma)(\rho + \delta + \phi) < 0, \tag{4.2}$$

where $\phi \equiv \delta[\alpha + (1+\sigma)(1-\alpha)] > 0$, there exists a positive and a negative root, $\zeta < 0 < \bar{\zeta}$, which implies that the equilibrium $X_\infty = -A^{-1}B$ of (4.1) is a saddle point. A particular solution of X is the steady state equilibrium

$$\begin{bmatrix} \hat{\mu}_\infty \\ \hat{H}_\infty \end{bmatrix} = \frac{-1}{\det A} \begin{bmatrix} \eta_H & -\varepsilon_H \\ -\eta_\mu & \varepsilon_\mu \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \qquad B \equiv \begin{bmatrix} \varepsilon_K \hat{K} + \varepsilon_\tau \hat{\tau} + \varepsilon_z \hat{z} \\ \eta_K \hat{K} + \eta_\tau \hat{\tau} + \eta_z \hat{z} \end{bmatrix}. \tag{4.3}$$

Equation (4.3) corresponds to the steady state solutions (2.17,18) below.

The solution to (4.1) is $X_t - X_\infty = De^{\zeta t}$ where $D = [d_1, d_2]'$ is the eigenvector corresponding to the stable root $\zeta < 0$. Solving $[A - \zeta I]D = 0$ yields the eigenvector up to a scalar multiple. From the second row, we obtain $d_1 = \frac{\zeta - \eta_H}{\eta_\mu} d_2$, where d_2 is determined by the initial condition human capital. In this case, $\hat{H}_0 = 0$, which implies that $d_2 = -\hat{H}_\infty$. The complete stable solution of (4.1) for time invariant policy shocks then equals

$$\hat{H}_t = (1 - e^{\zeta t})\hat{H}_\infty, \qquad \hat{\mu}_t = \hat{\mu}_\infty + \frac{\eta_H - \zeta}{\eta_\mu} \hat{H}_\infty e^{\zeta t}. \tag{4.4}$$

The stable saddlepath of the system is, in turn, given by

$$\hat{\mu}_t - \hat{\mu}_\infty = \frac{\zeta - \eta_H}{\eta_\mu}(\hat{H}_t - \hat{H}_\infty). \tag{4.5}$$

which implies that the forward looking variable $\hat{\mu}_t$ can jump at $t = 0$.

We can illustrate the dynamics of this system with two phase diagrams, Figures 1a and 1b. These depict the negatively sloped stable arm, denoted by XX, and the $\dot{\hat{H}}_t = 0$ and $\dot{\hat{\mu}}_t = 0$ loci, whose slopes equal $-\eta_H/\eta_\mu$ and $-\varepsilon_H/\varepsilon_\mu < 0$, respectively. The intersection of $\dot{\hat{H}}_t = 0$ and $\dot{\hat{\mu}}_t = 0$ at point A determines the steady state values of $\hat{H}_\infty$ and $\hat{\mu}_\infty$.[23] The distinction between the two diagrams is that Figure 1a depicts the case in which the $\dot{\hat{H}}_t = 0$ is positively sloped, while Figure 1b illustrates the case in which the slope of $\dot{\hat{H}}_t = 0$ is negative.[24] Since $\eta_\mu > 0$, Figure 1a corresponds to $\eta_H < 0$, while Figure 1b corresponds to $\eta_H > 0$. Rewriting the expression for η_H in (A.9) as $[k + \gamma/(1-\gamma) - \bar{s}]\,\delta(1-\gamma)/\bar{s}$, we can show that Figure 1a corresponds to the case in which $\bar{s} > k + \gamma/(1-\gamma)$, while Figure 1b illustrates the case $\bar{s} < k + \gamma/(1-\gamma)$. That is, the slope of $\dot{\hat{H}} = 0$ is positive (negative) as the ratio of the time spent in educational activities to the time spent working, $\bar{s}$, is greater (less) than the share of educational infrastructure, k, plus the term $\gamma/(1-\gamma)$. The latter term is greater, the larger is the human capital spillover.

We can use the phase diagrams to illustrate the paths taken by human capital and its shadow value in response to a time-invariant shift in infrastructure, subsidy, or tuition policy. To take one example, consider a permanent increase in $\hat{K}$, holding $\hat{\tau} = \hat{z} = 0$ and letting $\gamma = \alpha$. Using (2.17,18), the long-run comparative statics for H_∞ and μ_∞ are $d\hat{H}_\infty/d\hat{K} = -d\hat{\mu}_\infty/d\hat{K} = 1$. If we also assume that there is no congestion, $\sigma = 0$, and that $k = \frac{\delta(1-\alpha)}{\rho+\delta}$, which is *less,* according to (3.8), than the corresponding social optimum k^*, then the $\dot{\hat{H}}_t = 0$ locus is negatively sloped, since $\eta_H > 0$. Turning to Figure 2 and equations (4.4,5), we can describe how the dynamic system for these parameter values

[23]The stable arm XX is negatively sloped, since it is also the case that $d_1 = \frac{\varepsilon_H}{\zeta - \varepsilon_\mu} d_2$ where $\frac{\varepsilon_H}{\zeta - \varepsilon_\mu} < 0$.

[24]Note in Figure 1b that the slope of $\dot{\hat{\mu}} = 0$ is greater in absolute value than that of $\dot{\hat{H}} = 0$. The opposite case, i.e., the slope of $\dot{\hat{\mu}} = 0$ being *less* in absolute value than that of $\dot{\hat{H}} = 0$, can be ruled out by the saddlepoint property.

adjusts to this shift in education policy. Because $\varepsilon_k < 0$, $\eta_k > 0$ for these parameter values, an increase in $\hat{K}$ causes the $\dot{\hat{\mu}}_t = 0$ locus to shift to the right while the $\dot{\hat{H}}_t = 0$ locus shifts to the left. Their new intersection at point C illustrates that while human capital $\hat{H}_\infty$ increases its shadow value $\hat{\mu}_\infty$ falls. In order to reach their new steady state values at C, both $\hat{H}_t$ and $\hat{\mu}_t$ must start at point B and adjust down the new stable locus towards C. The shadow value, $\hat{\mu}_0$, must then *jump* instantaneously to point B when the policy change is implemented at $t = 0$. The rise in $\hat{\mu}_0$ reflects the initial drain on resources that a permanent increase in $\hat{K}$ imposes on the economy. Its subsequent fall reflects the productive impact of the rise in $\hat{K}$ on human capital accumulation.

5. Conclusion

Acquiring human capital is naturally considered a form of investment. While public policies, such as tax credits and depreciation allowances, to promote physical investment are very widespread, government intervention in the education sector, e.g., in terms of subsidies, grants, loans, tuition or the direct provision of educational personnel and infrastructure, has probably been even more pervasive in most countries, and surely so in terms of a share of GDP. In this paper we have developed a simple representative agent model to explore some of the rationales for the large scale public sector intervention in education. In our framework individuals accumulate human capital by private self-study and by attending publicly provided schools. The economic returns for the individual are, however, affected by spillovers. We focused on two distortions in this paper. One was the aggregate human capital spillover that encouraged the individual to acquire skills. The other was congestion externalities from school attendance.

Since the individual decides how much time to spend on acquiring human capital without regard to the aggregate externalities, a corrective subsidy/tuition policy can, in principle, elicit the socially optimal amount of time spent in home study and in school. We found that to eliminate an aggregate human capital externality, the total time spent in accumulating skills should receive a subsidy. In the case of congestion in schools, however, a specific tuition fee should be imposed. In making school attendance more expensive, the fee shifts education towards noninstitutional forms of learning such as home study. Furthermore, even in the case in which there are no aggregate human capital spillovers, we determined on efficiency grounds that both a subsidy to overall activity and a fees on school attendance must be imposed to prevent school congestion from impairing overall skill accumulation. We believe that this policy prescription rationalize some of the recent continental European reforms such as the introduction of tuition fees combined with individual student stipends.

6. Appendix

In this section we derive the log-linearized system (4.1). We start with the dynamic equation for human capital. Consider first the dynamics of working hours, $l = 1 - i - e$, and output, $Y = lH$. Working hours evolve according to:

$$\hat{l}_t = -s_I \hat{\imath}_t - s_E \hat{e}_t, \qquad s_I \equiv i/l, \qquad s_E \equiv e/l, \qquad \bar{s} \equiv s_E + s_I = (1-l)/l. \tag{A.1}$$

where the “hat”-notation indicates logarithmic (proportional) rates of change relative to the initial steady state. Note also that we have used the subscript t to distinguish those variables whose proportional rate of change is time variant from the policy variables that are not time variant. Taking the logarithmic derivative of (2.10) yields

$$\hat{\imath}_t = \hat{e}_t - \hat{z} - \frac{z}{1-\tau}\hat{\tau}, \tag{A.2}$$

where $\hat{\tau}$ and $\hat{z}$ are equal to $d\tau/(1-\tau+z)$ and $dz/(1-\tau+z)$. Substituting this result and (A.1) into the rate of growth of output gives us:

$$\hat{Y}_t = \hat{l}_t + \hat{H}_t = \hat{H}_t - \bar{s}\hat{e}_t + s_I \hat{z} + \frac{s_I z}{1-\tau}\hat{\tau}. \tag{A.3}$$

Turning to the demand side of the economy, income is devoted to consumption and public infrastructure, $Y_t = C_t + K$, implying:

$$\hat{Y}_t = (1-k)\hat{C}_t + k\hat{K}, \qquad k \equiv K/Y. \tag{A.4}$$

We next compute the percentage change of the time spent in home study, e_t. Substituting for $C_t = 1/\lambda_t$, we calculate the rate of change of (2.11), $(1-\tau)\theta H_t^{1-\gamma} = \mu_t \alpha g C_t$. This yields:

$$(1-\gamma)\hat{H}_t - \frac{1-\tau+z}{1-\tau}\hat{\tau} = \hat{\mu}_t + \hat{g} + \hat{C}_t. \tag{A.5}$$

Substituting (A.3, 4) into (A.5), we then obtain the following expression for $\hat{e}_t$:

$$\begin{aligned} \hat{e}_t \;=\; & \frac{1-k}{\bar{s}}\left[\frac{1}{1-k} - (1-\gamma)\right]\hat{H}_t + \frac{1-k}{\bar{s}}(\hat{\mu}_t + \hat{g}) \\ & -\frac{k}{\bar{s}}\hat{K} + \frac{1-k}{\bar{s}(1-\tau)}\left[(1-\tau+z) + \frac{s_I z}{1-k}\right]\hat{\tau} + \frac{s_I}{\bar{s}}\hat{z}. \end{aligned} \tag{A.6}$$

The log-linearized form of (2.12b) and the percentage change of $g(Ki/e)$ are given by

$$(a) \quad \dot{\hat{H}}_t = \delta\left[\hat{e}_t + \hat{g} - (1-\gamma)\hat{H}_t\right], \qquad (b) \quad \hat{g} = (1-\alpha)\left[\hat{K} - \frac{z}{1-\tau}\hat{\tau} - \hat{z}\right], \tag{A.7}$$

where we have used the steady state restriction $egH^{\gamma} = \delta H$ in (A.7a) and (A.2) in the expression for $\hat{g}$. Substitution of (A.6) and (A.7b) into (A.7a) then yields the dynamic equation in (4.1) for human capital in log-linearized form

$$\dot{\hat{H}}_t = \eta_H \hat{H}_t + \eta_\mu \hat{\mu}_t + \eta_k \hat{K} + \eta_\tau \hat{\tau} + \eta_z \hat{z}, \tag{A.8}$$

where the η coefficients are defined as:

$$\begin{aligned} \eta_H &= \delta\left[\tfrac{1}{\bar{s}} - (1-\gamma)\left(1 + \tfrac{1-k}{\bar{s}}\right)\right], \qquad \eta_\mu = \delta\tfrac{1-k}{\bar{s}} > 0, \\ \eta_K &= \delta\left[(1-\alpha)\left(1 + \tfrac{1-k}{\bar{s}}\right) - \tfrac{k}{\bar{s}}\right], \qquad \eta_z = \delta\left[\tfrac{s_I}{\bar{s}} - (1-\alpha)\left(1 + \tfrac{1-k}{\bar{s}}\right)\right], \\ \eta_\tau &= \tfrac{\delta}{1-\tau}\left[\tfrac{1-k}{\bar{s}}\left((1-\tau+z) + \tfrac{z s_I}{1-k}\right) - (1-\alpha)z\left(1 + \tfrac{1-k}{\bar{s}}\right)\right]. \end{aligned} \tag{A.9}$$

Turning to log-linearization of equation (2.12a), we obtain $\dot{\hat{\mu}}_t = -(\rho+\delta)\,\hat{r}_t^H$, where $r^H = \rho + \delta$ is the equilibrium interest rate. Using the definition of r^H in (2.12a), we denote the term in square brackets by $x = [1 - (1-\tau)e - (1-\tau+z)i]$. Using the steady state conditions $H_\infty^{1-\gamma} = e_\infty g(\cdot)/\delta$ and $x_\infty = \frac{(\rho+\delta)(1-\tau)e_\infty}{\alpha\delta}$, and inserting (2.10), we obtain:

$$(\rho+\delta)\,\hat{x}_t = -\delta\alpha\hat{e}_t + \delta\alpha\frac{1-\tau+z}{1-\tau}\hat{\tau} + \delta(1+\sigma)(1-\alpha)(\hat{\tau} - \hat{z} - \hat{\imath}_t). \tag{A.10}$$

Taking the differential of $r^H = \frac{\alpha g}{1-\tau}H^{\gamma-1}\cdot x$ yields:

$$\hat{r}_t^H = \hat{g} - (1-\gamma)\hat{H}_t + \frac{1-\tau+z}{1-\tau}\hat{\tau} + \hat{x}_t. \tag{A.11}$$

Substituting for $\hat{\imath}_t$ from (A.2) into (A.10) and then the resulting expression into (A.11), we find

$$\hat{r}_t^H = \hat{g} - (1-\gamma)\hat{H}_t + \frac{1-\tau+z}{1-\tau}\hat{\tau} + \frac{\phi}{\rho+\delta}\left[\frac{1-\tau+z}{1-\tau}\hat{\tau} - \hat{e}_t\right], \tag{A.12}$$

where $\phi \equiv \delta\left[\alpha + (1+\sigma)(1-\alpha)\right]$. Finally, we substitute for $\hat{g}$ and for $\hat{e}_t$ from (A.7b) and (A.6) into (A.12) and then substitute the resulting expression into $\dot{\hat{\mu}}_t = -(\rho+\delta)\hat{r}_t^H$ to obtain the log-linearized equation in (4.1) for the shadow value of human capital

$$\dot{\hat{\mu}}_t = -(\rho+\delta)\hat{r}_t^H = \varepsilon_H\hat{H}_t + \varepsilon_\mu\hat{\mu}_t + \varepsilon_K\hat{K} + \varepsilon_\tau\hat{\tau} + \varepsilon_z\hat{z}, \tag{A.13}$$

where the ε coefficients are given by:

$$\begin{aligned}
\varepsilon_H &= (\rho+\delta)(1-\gamma) + \tfrac{\phi(1-k)}{\bar{s}}\left[\tfrac{1}{1-k} - (1-\gamma)\right] > 0, \qquad \varepsilon_\mu = \tfrac{\phi(1-k)}{\bar{s}} > 0, \\
\varepsilon_K &= (1-\alpha)\left[\tfrac{\phi(1-k)}{\bar{s}} - (\rho+\delta)\right] - \tfrac{\phi k}{\bar{s}}, \\
\varepsilon_\tau &= -(\rho+\delta)(1+\tfrac{\alpha z}{1-\tau}) - \tfrac{\phi(1-\tau+z)}{1-\tau}(1 - \tfrac{1-k}{\bar{s}}) - \tfrac{\phi(1-k)z}{(1-\tau)\bar{s}}(1-\alpha-\tfrac{s_I}{1-k}), \\
\varepsilon_z &= \tfrac{\phi s_I}{\bar{s}} - (1-\alpha)\left[\tfrac{\phi(1-k)}{\bar{s}} - (\rho+\delta)\right].
\end{aligned} \tag{A.14}$$

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7. Figures

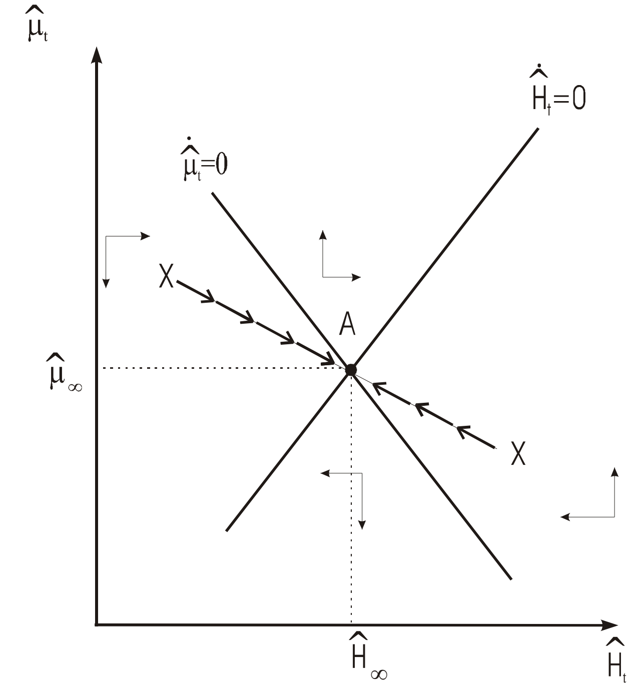


Figure 1a

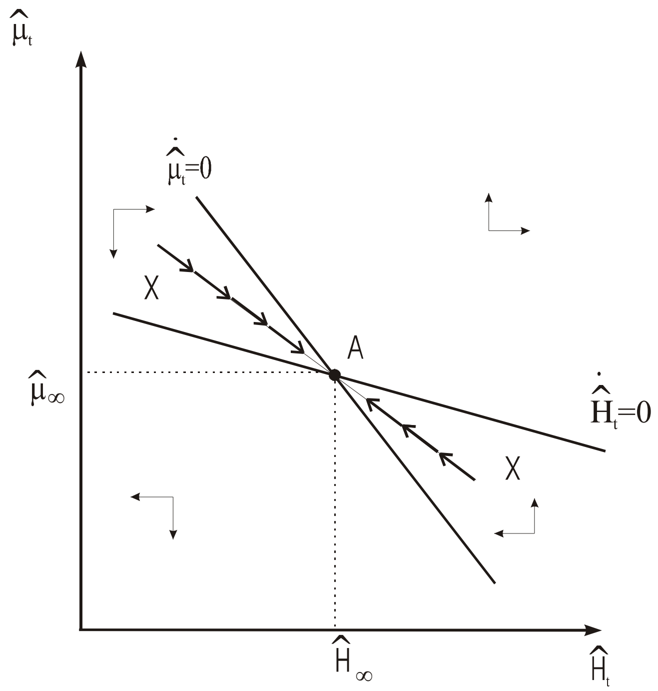
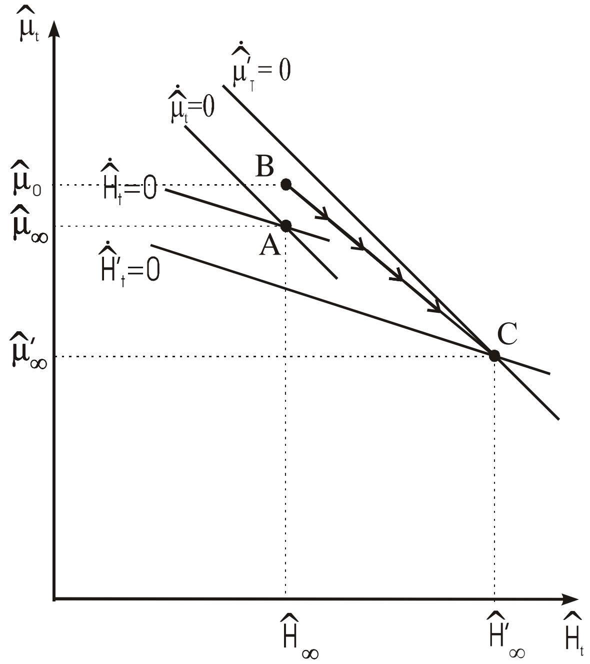


Figure 1b



$\hat{\mu}_t$
$\dot{\hat{\mu}}'_T = 0$
$\dot{\hat{\mu}}_t = 0$
$\hat{\mu}_0$
$\dot{\hat{H}}_t = 0$
B
$\hat{\mu}_\infty$
A
$\dot{\hat{H}}'_t = 0$
C
$\hat{\mu}'_\infty$
$\hat{H}_\infty$
$\hat{H}'_\infty$
$\hat{H}_t$

Figure 2

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