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# A Welfare Comparison Between Export Subsidies and Exchange Rate Depreciation

Adelina Gschwandtner





INSTITUT FÜR HÖHERE STUDIEN  
INSTITUTE FOR ADVANCED STUDIES  
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**Author(s):**

Adelina Gschwandtner

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Institut für Höhere Studien  
Stumpergasse 56, A-1060 Wien  
Fax: +43/1/599 91-163

Adelina Gschwandtner  
BWZ – Department of Economics  
University of Vienna  
Bruennerstr. 72, A-1210 Wien  
Phone: +43/1/4277-37480  
Fax: +43/1/4277-37498  
E-mail: Adelina.Gschwandtner@univie.ac.at

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## **Abstract**

This paper develops a Bertrand Price Competition model with differentiated goods in which export subsidies are compared to exchange rate depreciation as different government policies for promoting exports. National governments may wish to help domestic firms to expand market shares in profitable areas and might do this through either one of these two tools. Their effects on equilibrium values are analyzed and compared. It is shown that while the two examined trade policies give rise to the same highest welfare, they could produce some significant differences according to circumstances. If the exchange rate is sufficiently high and the level of the nominal wage sufficiently low, the marginal effect of the subsidy will be higher. But if unions are strong (and demand a high nominal wage) and the exchange rate is sufficiently low, the governments could also consider a depreciation as an alternative policy to export subsidies.

## **Keywords**

Export subsidies, exchange rate depreciation, international trade, Bertrand competition

## **JEL Classifications**

F13, F31

**Comments**

I would like to thank Guiseppe Colangelo for supervising this paper and helping me with his very useful comments and ideas. I would also like to thank Leo Kaas for reviding this paper and finding a much easier way to prove one of the final results. His idea is presented in Appendix A.3.

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# 1 Introduction

Export subsidies and exchange rate depreciation are two most obvious tools for export promotion. Traditionally, these two tools are the mostly used trade policies for increasing exports and for helping the trade balance. Because improving exports is a vital problem for a small developing country, it is important to answer the question what is the best way to do it?

This question is very interesting also because there seems not to be much literature trying to compare the two policy tools. There is much literature about export subsidies but there seems not to be much literature about exchange rate depreciation and even less literature comparing it to export subsidies.

The classic international trade theory provides a strong and robust welfare argument against export subsidies. The main loss from export subsidies is the so-called "terms of trade loss", which appears due to the worsening in the terms of trade (the price of a country's exports divided by the price of its imports) of the domestic country. Adding up the effects, an export subsidy unambiguously leads to costs that exceeds its benefits.<sup>1</sup>

Yet, Brander and Spencer (1985) showed that export subsidies can be profitable from a domestic point of view. In a very simple Cournot-duopoly model they showed that export subsidies, which enable the domestic firm to capture a larger share of a profitable international market share, can be attractive for the domestic country. The main idea of the article is that the government subsidy alters the successive noncooperative equilibrium in the inter-firm rivalry in favor of the domestic firm by giving it a cost advantage over the competitor that deters him from entering the market. This argument for industrial policy based on imperfect competition is often referred to as the "strategic trade policy" argument.

This argument has attracted much interest but it also received much criticism. It is important to remind the main critical arguments against export subsidies in order to explain why the government might take into consideration also other policy tools for export promotion.

The first point of critique refers to the unavailability of information. The problem of insufficient information has two aspects. The first concerns information about the subsidized industry and the second concerns information about the other industries with whom the subsidized industry competes for resources. If one industry is subsidized, it will draw resources from other industries and lead to increases in their costs. To decide whether the policy is justified or not the government needs to weight all effects, and even if it had a precise understanding of one industry it needs equally precise understanding of those industries with which that industry competes for resources. This information is very unlikely to

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<sup>1</sup>See for instance Krugman and Obstfeld (1994).

be available.<sup>2</sup>

The second critique point refers to foreign retaliation. Strategic trade policies are so-called "beggar-thy-neighbor policies" that increase domestic welfare at other countries' expense and therefore risk a trade war that leaves everyone worse off.

The third critique point is concerned with political misuse. Once a procedure of market intervention is established, it might be susceptible to political pressure of special interest groups. Experience shows that trade policy fails to take into account the interest of consumers and usually serves only the interest of those fortunate enough to gain favor. "The invisible hand of the economic market could then be replaced by the even more invisible hand of the political market."<sup>3</sup>

These points of critique to export subsidies and many others (like targeting, unfavorable income distribution, etc.) are very well summarized in an article by Gene M. Grossman called "Strategic Export Promotion: A Critique" (1992). Grossman suggests that government could help exports by other measures like support of education and industrial R&D, redeployment of resources released from declining industries, favourable (software) taxes or macroeconomic policies like lower interest rates for investment or depreciation of overvalued exchange rates.

As some Eastern European Countries have an overvalued exchange rate and at the same time mean to subsidize exports (although they want to become members of the European Community, which prohibits this), I found it interesting to make a comparative welfare analysis between export subsidies and exchange rate depreciation. The main results are that the optimal export subsidy and the optimal exchange rate depreciation give the same level of welfare but their marginal effects might differ according to circumstances. The subsidy and the exchange rate depreciation are not always equivalent. Under certain particular conditions one tool might have a greater marginal effect than the other.

The paper proceeds as follows: In section 2, I present the model. Next, I solve the game by backward induction and begin with the consumers (section 3). After solving the consumer problem and getting the direct demand functions I present the producers problem (section 4). Next, I look at the welfare maximization of the government (section 5). In the last section I present some conclusions.

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<sup>2</sup>For an analysis of the "informational" criticism of the strategic-trade-policy theory see also Maggi (1996).

<sup>3</sup>As G. M. Grossman (1992) noticed.

## 2 The Model

The analysis will be done with the use of a Bertrand model similar to the "third-market model" used by Brander and Spencer. The main differences to their model refer to price competition and product differentiation.<sup>4</sup>

Consider the following situation:

There are two countries A and B each with one government and one firm. The two firms, produce a substitutable but differentiated good from that produced by the rival.<sup>5</sup> Each firm produces both for the domestic and for the foreign market. The subsidy is paid only for the quantity exported (specific export subsidy). The two firms compete upon prices. The technology is such that labor is the only input. One unit of input gives one unit of output (constant returns to scale).

The workers are paid in the local currency. The governments have to simultaneously set the exchange rate against a third reference currency, say the USD. From this, their relative exchange rate will be set.<sup>6</sup>

In each country there is a representative consumer who works (and earns a nominal wage that is exogenously given) and buys the two goods produced by the firms.

So each country is represented by three agents:

- \* one government,
- \* one firm,
- \* one representative consumer.

The order of moves in this three-stage model is the following:

In a first stage, the governments simultaneously set their exchange rate against the third currency or alternatively a specific (per unit) export subsidy  $s$  (in USD) in order to maximize welfare. Welfare is defined as the utility of the consumer plus the profit of the firm (see equation (22)).

In the second stage, given the exchange rate (or alternatively the export subsidy) chosen by the government, the firms compete upon prices. From the first-order conditions of profit maximization, the reaction functions and then the equilibrium prices are derived.

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<sup>4</sup>For surprising results in regard of how Brander's and Spencer's results could change in an Bertrand environment without product differentiation see Eaton, Jonathan and Grossman (1986).

<sup>5</sup>For the relation between product differentiation, market structure and exchange rate pass through see Martin S. and Philips L. (1995), and Kirman A. and Philips L. (1996).

<sup>6</sup>Say for example that the relative exchange rate of country A is  $e_a$  =price of 1 USD in the currency of the country A (8400 Lei for 1 USD if A is Romania) and the exchange rate of country B is  $e_b$  =price of 1 USD in the currency of the country B (12 ATS for 1 USD if B is Austria).

In the third stage, the consumers, given their nominal wages and all the variables set in the previous stages, determine the optimal quantity that maximizes their utility.

Solving the game by backward induction, I start with the third stage, the utility maximization of consumers.

### 3 The Consumer Problem

#### 3.1 Utility Maximization in Country A

The consumers of country A choose quantities  $x$  and  $y^*$  in order to maximize the following quasilinear utility function:<sup>7</sup>

$$U_A(x, y^*, m) = U(x, y^*) + m$$

subject to the following budget constraint:

$$xp_x + y^*p_y + m = \frac{w_a}{e_a}(x + x^*) - sx^*$$

where:

$x$  = quantity of good 1 produced for the domestic market (in country A)

$x^*$  = quantity of good 1 produced in A for export

$y$  = quantity of good 2 produced for the domestic market (in country B)

$y^*$  = quantity of good 2 produced in B for export

$m$  = demand for the numeraire good in country A

$w_a$  = the nominal wage of workers in country A (fixed)

$e_a$  = the relative exchange rate in country A (in Lei/USD)

and everything is expressed in the reference currency (USD) except for the nominal wage  $w_a$ .

They consume quantity  $x$  of the goods produced domestically (and pay  $p_x$  per unit) and quantity  $y^*$  of the goods produced in country B, for which they pay  $p_y$  per unit. They also consume the numeraire good  $m$  (with unit price).

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<sup>7</sup>As the utility function is quasilinear, there will be no income effect.

Consumers earn a wage of  $\frac{w_a}{e_a}$  for the whole amount produced domestically ( $x+x^*$ ) and have to pay the subsidy  $s$  (for the exported goods  $x^*$ ).

I assume that consumers are taxed by a lump sum tax by the government. If the government has a balanced budget constraint, then the amount of the lump sum tax must be equal to the amount of the subsidy that the government wants to pay to the firm. Under such conditions, the government is only a veil and in practice it is as if the consumer pays the subsidy directly to the firm.

Furthermore the consumers are permanent residents of a country and cannot change residence. Because they have to pay the subsidy in the domestic country the workers/consumers will have an incentive to emigrate to the foreign country and work there. So I assume that the foreign country does not allow such an immigration (or that the domestic country does not allow an emigration).<sup>8</sup>

Preferences towards the choice among the two goods under consideration are represented by a quadratic utility function of the following form:<sup>9</sup>

$$U(x, y^*)_A = \alpha(x + y^*) - \frac{1}{2}(\beta x^2 + 2\gamma xy^* + \beta y^{*2}) \quad (1)$$

with  $\alpha > 0, \beta > |\gamma| \geq 0$ . The goods are substitutes, independent, or complements according to whether  $\gamma \leq 0$ . When  $\beta = \gamma$  the goods are perfect substitutes.  $\frac{\gamma}{\beta}$  goes from 1 to -1.

In order to have substitutes we need  $\gamma > 0 \Rightarrow |\gamma| = \gamma \Rightarrow \beta > \gamma$ .

In order to have a strictly positive marginal utility (a monotonic increasing utility function) we need to impose the following:

$U_x = \alpha - \beta x - \gamma y^* > 0, U_{y^*} = \alpha - \gamma x - \beta y^* > 0$ . From this we get the following conditions: if  $x < \frac{\alpha}{\beta + \gamma}$  then  $y^* \leq \frac{\alpha - \beta x}{\gamma}$  and if  $x > \frac{\alpha}{\beta + \gamma}$  then  $y^* \leq \frac{\alpha - \gamma x}{\beta}$ .<sup>10</sup>

The simple quadratic utility function expressed in equation (1) allows us to get the following linear demands:

$$p_x = \alpha - \beta x - \gamma y^*$$

$$p_y = \alpha - \gamma x - \beta y^*$$

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<sup>8</sup>If we think of the foreign country as an EU country and the domestic country as one of the former Communist Block (Eastern European Country), this assumption is not unrealistic.

<sup>9</sup>This form of quadratic utility function is frequently used. See for example: Xavier Vives (1984).

<sup>10</sup>Note that for  $\alpha = \beta = 1, \gamma = 0,5$  and  $x = y^* = 1$  we get  $U(1, 1) = 0,5$  and  $U(2, 2) = -2$  and the utility function would not be monotonically increasing in quantity.

And assuming now  $a = \frac{\alpha}{\beta+\gamma}$   $b = \frac{\beta}{\beta^2-\gamma^2}$  and  $c = \frac{\gamma}{\beta^2-\gamma^2}$  we get the following linear direct demand functions:

$$x = a - bp_x + cp_y \quad (2)$$

$$y^* = a - bp_y + cp_x \quad (3)$$

$$m = I - a(p_x + p_y) + b(p_x^2 + p_y^2) - 2cp_xp_y$$

where  $I = \frac{w_a}{e_a}(x + x^*) - sx^*$  is the total income of the representative consumer in country A.

From the condition of substitutability we get the result that  $a, b, c > 0$  and  $b > c$ .

This condition is also satisfied because of strategic complementarity. The strategic complementarity condition just says that the effect of a change in the own price ( $p_x$ ) must be higher than the effect of a change in the price of the other good ( $p_{y^*}$ ).

### 3.2 Utility Maximization in Country B

The representative consumer in country B choose  $y$  and  $x^*$  in order to maximize a similar quasilinear utility function:

$$U_B(x^*, y, m^*) = U(x^*, y) + m^*$$

subject to the following budget constraint:

$$x^*p_x + yp_y + m^* = \frac{w_b}{e_b}(y + y^*)$$

with

$w_b$  = nominal wage of workers in country B (fixed),

$e_b$  = relative exchange rate of country B (in ATS/USD),

$m^*$  = demand for the numeraire good in country B

and

$$U(x^*, y)_B = \alpha(x^* + y) - \frac{1}{2}(\beta x^{*2} + 2\gamma x^*y + \beta y^2) \quad (4)$$

The only difference is that workers/consumers in country B do not pay the subsidy.

If we now again solve the maximization problem (by setting  $MRS = \frac{p_i}{p_j}$ ) and substitute for  $\alpha$ ,  $\beta$ ,  $\gamma$ , we get the following linear demand functions for the representative consumer in country B:

$$x^* = a - bp_x + cp_y \quad (5)$$

$$y = a - bp_y + cp_x \quad (6)$$

which are just the same as equation (2) and equation (3).

By inspection of equations (2), (3), (5) and (6) it is easy to see that it must be that:

$$x = x^* \text{ and } y^* = y.$$

We are now ready to look at the firm's competition.

## 4 Price Competition between Firms

### 4.1 The Profit Functions

Firms compete upon prices (Bertrand Price Competition) given the demand of the consumers in order to maximize the following profit functions:

$$\pi_A = xp_x + x^*p_x - \frac{w_a}{e_a}(x + x^*) + sx^* \quad (7)$$

$$\pi_B = yp_y + y^*p_y - \frac{w_b}{e_b}(y + y^*) \quad (8)$$

I assume that everything is expressed in the reference currency (the USD). So consumers have to purchase all the goods (not only the imported ones) in USD but receive their wages in the local currency.

The two firms earn revenue from the goods they produce ( $(x + x^*)$  for firm A and  $(y + y^*)$  for firm B) and have to pay their workers the wages  $\frac{w_a}{e_a}$  and  $\frac{w_b}{e_b}$ . Firm A also receives a specific subsidy for each unit it is exporting to country B ( $sx^*$ ).

In order to get a higher export subsidy the firm would also have an incentive to declare a higher  $x^*$ . I have assumed that the government is able to distinguish between  $x$  and  $x^*$ .<sup>11</sup>

Anticipating that the demand is the same in both countries the two profit functions can be written simplified as follows:

$$\pi_A = x(p_x, p_y) \left( 2p_x - 2\frac{w_a}{e_a} + s \right)$$

$$\pi_B = 2y(p_x, p_y) \left( p_y - \frac{w_b}{e_b} \right)$$

## 4.2 First-Order Conditions

The First-Order Conditions are:

$$\frac{\partial \pi_A}{\partial p_x} = -b \left( 2p_x - \frac{2w_a}{e_a} + s \right) + 2(a - bp_x + cp_y) = 0$$

$$\frac{\partial \pi_B}{\partial p_y} = -b \left( p_y - \frac{w_b}{e_b} \right) + a - bp_y + cp_x = 0$$

yielding reaction functions:

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<sup>11</sup>There is evidence that for example during the seventies Italian exporters tried to declare more exports by filling up empty boxes and transporting them over the border in order to get a higher subsidy. This could also be made in agreement with the importing firm, and then share the profit of the subsidy. So the government must have the possibility to control at the border how much the domestic firm really exports (to know how much  $x^*$  really is).

$$p_x = \frac{c}{2b}p_y + \frac{w_a}{2e_a} + \frac{a}{2b} - \frac{s}{4} \quad (9)$$

$$p_y = \frac{c}{2b}p_x + \frac{w_a}{2e_b} + \frac{a}{2b} \quad (10)$$

So, the two reaction functions are symmetric with the only difference that the first one contains the subsidy.

### 4.3 The Equilibrium Prices

Calculating the intersection point of the reaction functions (equation (9) and equation (10)), we get the following equilibrium prices:

$$p_x = \frac{1}{4b^2 - c^2} \left( -b^2s + bc\frac{w_b}{e_b} + 2b^2\frac{w_a}{e_a} \right) + \frac{a}{2b - c} \quad (11)$$

$$p_y = \frac{1}{4b^2 - c^2} \left( -\frac{bc}{2}s + 2b^2\frac{w_b}{e_b} + bc\frac{w_a}{e_a} \right) + \frac{a}{2b - c} \quad (12)$$

Now, we can calculate the effects of the subsidy and the exchange rate depreciation.

Differentiating equation (11) first with respect to the subsidy ( $s$ ) and then with respect to the exchange rate ( $e_a$ ) we get:

$$\frac{\partial p_x}{\partial s} = -\frac{b^2}{4b^2 - c^2}$$

$$\frac{\partial p_x}{\partial e_a} = -\frac{2b^2w_a}{(4b^2 - c^2)e_a^2}$$

The first observation is that both effects have the same sign. Both the subsidy and the exchange rate depreciation lead to a reduction of the price of the good produced in the domestic country. They act for the domestic firm like a reduction in the marginal cost.

The second observation is that the subsidy is not fully passed over to the consumers because:

$$\frac{\partial p_x}{\partial s} = \frac{-b^2}{4b^2 - c^2} = \frac{-1}{4 - \frac{c^2}{b^2}} > -1 \Rightarrow \left| \frac{\partial p_x}{\partial s} \right| < 1$$

$b > c$  and  $a, b, c > 0$ .

This means that an increase of the subsidy of one dollar reduces the price of the good produced by the firm that receives the subsidy by less than one dollar, so a part of the subsidy increases the profit margin of the firm, due to its oligopolistic power.

The third observation is that:

$$\frac{\frac{\partial p_x}{\partial e_a}}{\frac{\partial p_x}{\partial s}} = \frac{2w_a}{e_a^2} \quad (13)$$

This says that the effect of the depreciation compared with the effect of the subsidy on the price increases (in absolute value) with  $\frac{w_a}{e_a^2}$ . The higher the wage, the higher the effect on price of the depreciation compared to the effect of the subsidy and the higher the exchange rate the lower the effect of the depreciation compared to the effect of the subsidy.

The effects on the price of the foreign firm are somehow similar and can be calculated from equation (12):

$$\frac{\partial p_y}{\partial s} = -\frac{bc}{2(4b^2 - c^2)} \quad (14)$$

$$\frac{\partial p_y}{\partial e_a} = -\frac{bcw_a}{(4b^2 - c^2)e^2} \quad (15)$$

We can observe that both effects are negative and smaller than the effects on the own price.<sup>12</sup> So the subsidy and the depreciation reduce also the price in the foreign country but to a smaller extent.<sup>13</sup>

From equation (14) and equation (15) we can see that if the goods are independent ( $c = 0$ ), the subsidy and the depreciation have no effect on the price of the

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<sup>12</sup>From the equations above we get that:  $\frac{\frac{\partial p_y}{\partial s}}{\frac{\partial p_x}{\partial s}} = \frac{\frac{\partial p_y}{\partial e_a}}{\frac{\partial p_x}{\partial e_a}} = \frac{c}{2b} < 1$  since  $b, c > 0; b > c$ .

<sup>13</sup>This is what we would have expected. That the effect on the own price is greater than the effect on the price of the competitor.

competitor ( $p_y$ ). In this case, even if there is a subsidy, the competitor has no incentive to reduce the price.<sup>14</sup>

The magnitude of the effect of the depreciation compared to the effect of the subsidy depends again on the "wage" in the reference currency.

$$\frac{\frac{\partial p_y}{\partial e_a}}{\frac{\partial p_y}{\partial s}} = \frac{2w_a}{e_a^2}$$

The higher the nominal wage and the lower the exchange rate, the stronger the effect of the depreciation compared to that of the subsidy.

Next, I am going to look at the effect on the exchanged quantity.

#### 4.4 The Effect on Equilibrium Quantities

Substituting the price equations in the direct demand functions (equation (2) and equation (3)), we get the exchanged quantities of the two goods depending only on the desired parameters ( $a, b, c, e_a, e_b, w_a, w_b, s$ ):

$$x = \frac{1}{4b^2 - c^2} \left[ \frac{b(2b^2 - c^2)}{2}s + b^2c \frac{w_b}{e_b} - b(2b^2 - c^2) \frac{w_a}{e_a} \right] + \frac{ab}{2b - c} \quad (16)$$

$$y = \frac{1}{4b^2 - c^2} \left[ -\frac{b^2c}{2}s - b(2b^2 - c^2) \frac{w_b}{e_b} + b^2c \frac{w_a}{e_a} \right] + \frac{ab}{2b - c} \quad (17)$$

Now we can look at the effects of the subsidy and of the depreciation. Differentiating equation (16) with respect to  $s$  and  $e_a$  we get the effects on  $x$ :

$$\frac{\partial x}{\partial s} = \frac{b(2b^2 - c^2)}{2(4b^2 - c^2)}$$

$$\frac{\partial x}{\partial e_a} = \frac{b(2b^2 - c^2)w_a}{(4b^2 - c^2)e_a^2}$$

So, both the subsidy and the exchange rate depreciation increase  $x$ . This is not that obvious. If we look at the demand equation (2), we see that we have two opposing effects. The reduction in the own price ( $p_x$ ) increases  $x$  but the

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<sup>14</sup>You can see this also from the demand equation (3). If you set  $c = 0$ , then the demand of the second good does not depend on the price of the first good anymore.

reduction in the price of the competitor ( $p_y$ ) decreases  $x$ . Since the price of  $x$  falls (due to the subsidy or the depreciation)  $x$  rises but because of competitive substitutability also the price of the other good falls and also  $y$  rises. This reduces the demand for  $x$ . But we have seen that  $p_x$  is falling than  $p_y$  and we also know that  $b > c$ , so the own effect is stronger and  $x$  rises.

The effects on  $y$  are:

$$\frac{\partial y}{\partial s} = -\frac{b^2 c}{2(4b^2 - c^2)}$$

$$\frac{\partial y}{\partial e_a} = -\frac{b^2 c w_a}{(4b^2 - c^2) e_a^2}$$

We can observe that both effects are negative. If we look again at the demand equation (6), we can see that the own price reduction would make  $y$  increase, but the reduction of the price of the competitive good makes  $y$  fall. The effect on  $p_x$  is so much greater than the effect on  $p_y$  that even the fact that  $b > c$  cannot compensate for this and  $y$  falls. Because of the asymmetry in the prices  $p_y$  is so much less affected that the effect of  $p_x$  prevails and  $y$  falls.

Again, the effect on  $y$  is smaller (in absolute value) than that on  $x$ .<sup>15</sup> This is what we would have expected. Since the effect on the own price dominates, also the effect on the own quantity ( $x$ ) must be greater than the effect on the foreign quantity ( $y$ ).

Comparing the effects of the subsidy and the depreciation we obtain for both quantities the same result as before:

$$\frac{\frac{\partial x}{\partial e_a}}{\frac{\partial x}{\partial s}} = \frac{\frac{\partial y}{\partial e_a}}{\frac{\partial y}{\partial s}} = \frac{2w_a}{e_a^2}$$

Again, the magnitude of the effect of the depreciation compared to the effect of the subsidy seems to increase with the level of the nominal wage and decrease with the effect of the exchange rate.

A very interesting result is that concerning both the prices and the demand the signs of the two effects are the same. So the subsidy and the depreciation have the same effect. Only their magnitude differs.

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<sup>15</sup>  $\frac{\frac{\partial y}{\partial s}}{\frac{\partial x}{\partial s}} = \frac{\frac{\partial y}{\partial e_a}}{\frac{\partial x}{\partial e_a}} = \frac{-bc}{2b^2 - c^2} = \frac{-\frac{c}{b}}{2 - \frac{c^2}{b^2}} < |1|$  since  $b > c \Rightarrow \frac{c}{b} < 1$

$$\text{sign} \left( \frac{\partial \theta}{\partial s} \right) = \text{sign} \left( \frac{\partial \theta}{\partial e_a} \right) \quad (18)$$

$\theta = p_x, p_y, x, y$

Comparing the magnitude of the effect of the two tools (subsidy versus exchange rate depreciation) for the domestic and the foreign profit we obtain the same result as before. The exchange rate depreciation increases profit more than the subsidy only for a high nominal wage and a low exchange rate.

$$\frac{\frac{\partial \pi_A}{\partial e_a}}{\frac{\partial \pi_A}{\partial s}} = \frac{\frac{\partial \pi_B}{\partial e_a}}{\frac{\partial \pi_B}{\partial s}} = \frac{2w_a}{e_a^2}$$

This result is valid also for the effects on the utility of the consumers (see Appendix A.3).

In the next stage we are going to look at the welfare maximization of the government.

## 5 The Government Problem

### 5.1 Comparing the Two Tools

Solving the problem by backward induction we have come to the last stage, the welfare maximization of the government.

As discussed at the beginning, the government is maximizing a "welfare function" of the form:

$$W = \underbrace{CS}_{\text{Consumer Surplus}} + \underbrace{PS}_{\text{Producer Surplus}}$$

Here I measure the Consumer Surplus as the net utility of the representative consumer (equation (24)) and the Producer Surplus as the profit of the firms (equation (7)).

The government has two tools. It either can choose the subsidy level  $s$  or the exchange rate  $e_a$ . The government is able to act first and set subsidy/exchange rate levels, using its understanding of how this will influence the prices in equilibrium. A very important assumption is that the government understands everything that will occur afterwards and takes these anticipations into consideration at the moment of setting the trade policy (export subsidy or exchange rate depreciation).

In comparing the two tools, we obtained the following two main results:

$$\text{sign} \left( \frac{\partial \theta}{\partial s} \right) = \text{sign} \left( \frac{\partial \theta}{\partial e_a} \right)$$

and

$$\frac{\frac{\partial \theta}{\partial e_a}}{\frac{\partial \theta}{\partial s}} = \frac{2w_a}{e_a^2} \quad (19)$$

where  $\theta = p_x, p_y, x, y, \pi_A, \pi_B, U_A, U_B$ .

So, the effect of the two tools goes in the same direction but their magnitude differs depending on the nominal wage  $w_a$  and on the exchange rate  $e_a$ .

From equation (19) we can see that:

$$\begin{aligned} \text{if } w_a \rightarrow \infty \quad & \text{then } \frac{\frac{\partial \theta}{\partial e_a}}{\frac{\partial \theta}{\partial s}} \rightarrow \infty \\ \text{if } e_a \rightarrow 0 \quad & \text{then } \frac{\frac{\partial \theta}{\partial e_a}}{\frac{\partial \theta}{\partial s}} \rightarrow \infty \\ \text{if } w_a \rightarrow 0 \quad & \text{then } \frac{\frac{\partial \theta}{\partial e_a}}{\frac{\partial \theta}{\partial s}} \rightarrow 0 \\ \text{if } e_a \rightarrow \infty \quad & \text{then } \frac{\frac{\partial \theta}{\partial e_a}}{\frac{\partial \theta}{\partial s}} \rightarrow 0 \end{aligned}$$

So, we can see that if the exchange rate is not very high, the depreciation would do better than the subsidy. The higher the level of the exchange rate  $e_a$ , the lower the advantage of the depreciation. This means that a depreciation makes sense only at the beginning (like a demand shock) but its usefulness decreases as its level increases.

The magnitude of the effect of the depreciation compared to the effect of the subsidy also depends on the level of the nominal wage  $w_a$ . If there is a strong union with strong monopoly power, the exchange rate depreciation is better than the subsidy because it reduces the power of the union by reducing the workers' wage in terms of the international currency. This means that if the firm is too weak to bargain with the union, it would be better for the government to depreciate

in order to take the advantage from the union away and redistribute it to the consumer (through lower prices).

In many Eastern European Countries one can observe strong trade unions. In order to improve the competitiveness of their exports (lower prices), the governments should therefore use the exchange rate depreciation as an economic tool.

## 5.2 Optimal Values

Solving the utility function with respect to the subsidy and the exchange rate we obtain the following optimal values:

$$s^* = \frac{2(2ac^5e_ae_b + 12b^6w_ae_b + 3b^4c(4ae_a - 3cw_a)e_b + b^2c^3(-12ae_a + cw_a)e_b)}{b^2(12b^4 - 9b^2c^2 + c^4)e_ae_b} + \frac{2(b^3c^2e_a(5cw_b - 7ae_b) + bc^4e_a(-cw_b + ae_b) + 4b^5(-2ce_aw_b + ae_ae_b))}{b^2(12b^4 - 9b^2c^2 + c^4)e_ae_b} \quad (20)$$

$$e_a^* = \frac{2b^2(12b^4 - 9b^2c^2 + c^4)w_ae_b}{B} \quad (21)$$

$$B = -4ac^5e_b + 12b^6se_b + b^2c^3(24a + cs)e_b - 3b^4c(8a + 3cs)e_b - 2b^3c^2(5cw_b - 7ae_b) + 2bc^4(cw_b - ae_b) + 8b^5(2cw_b - ae_b)$$

If we substitute these values in the profit function (equation (23)) and in the utility function (equation (24)), we obtain the following results:

$$\pi_A(e_a^*) = \pi_A(s^*)$$

$$U_A(e_a^*) = U_A(s^*)$$

And since the welfare is just the sum of the two, it means that:

$$W_A(e_a^*) = U_A(e_a^*) + \pi_A(e_a^*) = W_A(s^*) = U_A(s^*) + \pi_A(s^*) \quad (22)$$

For the derivation of these results see Appendix A.4 .

This just means that in this model at optimum the exchange rate depreciation is exactly analogous to giving a subsidy. The export subsidy and the depreciation are equivalent. This is because the depreciation just means less wage for the worker ( $\frac{w_a}{e_a}$  falls as  $e_a$  rises) and is just the same as if the worker pays the tax for the subsidy. The welfare under the optimal subsidy and under the optimal depreciation rate would be the same, so actually the government will be indifferent between the two tools.

## 6 Conclusions

Under certain conditions (low exchange rate, high nominal wages) the government could consider, like Grossman suggests, the depreciation as an alternative tool to export subsidies.

If the level of the exchange rate depreciation is sufficiently high and the level of the nominal wage is sufficiently low, a subsidy will do better. But if the level of the nominal wage is high and the exchange rate is sufficiently low, the governments could also consider a depreciation as a tool for export promotion.

In the optimal case the two tools are identical for the government. Choosing an optimal subsidy or choosing an optimal depreciation rate gives the same level of welfare, so the government is indifferent between the two tools.

The main differences are that the depreciation is not targeted to only one industry, and its macroeconomic effect is not only the decrease of the real wage in the home country. A depreciation would affect all exporting industries and not only the particular one which the government might want to subsidize, while the subsidy goes directly to the targeted industry. At the same time through a depreciation other macroeconomic sectors might be affected, while through a subsidy not. If more differences between the two tools are to be found, these aspects should be analyzed.

In future work the reaction possibilities of the foreign competitor should also be considered.

# A Appendix

## A.1 The Profit Function

Substituting the price  $p_x$  (equation 11) and the demand  $x$  (equation 16) in the profit function of the country A (equation 9) we get:

$$\pi_A = \frac{b(2b^2(e_a s - 2w_a)e_b + c(2ae_a - ce_a s + 2cw_a)e_b + 2be_a(cw_b + 2ae_b))^2}{2(c^2 - 4b^2)^2 e_a^2 e_b^2} \quad (23)$$

And if we derive this profit function with respect to  $s$  and  $e_a$ , we get the following results:

$$\frac{\partial \pi_A}{\partial s} = \frac{b(2b^2 - c^2)(2b^2(e_a s - 2w_a)e_b + c(2ae_a - ce_a s + 2cw_a)e_b + 2be_a(cw_b + 2ae_b))}{(c^2 - 4b^2)^2 e_a e_b}$$

$$\begin{aligned} \frac{\partial \pi_A}{\partial e_a} &= \frac{2b(2b^2 - c^2)w_a(wb^2(e_a s - 2w_a)e_b + c(2ae_a - ce_a s + 2cw_a)e_b + 2cw_a e_b)}{(c^2 - 4b^2)^2 e_a^3 e_b} + \\ &+ \frac{2b(2b^2 - c^2)w_a 2be_a(cw_b + 2ae_b)}{(c^2 - 4b^2)^2 e_a^3 e_b} \end{aligned}$$

and one can observe that if  $s \geq \frac{2w_a}{e_a}$  then both  $\frac{\partial \pi_A}{\partial s}$  and  $\frac{\partial \pi_A}{\partial e_a}$  are positive and that:

$$\frac{\frac{\partial \pi_A}{\partial e_a}}{\frac{\partial \pi_A}{\partial s}} = \frac{2w_a}{e_a^2}$$

## A.2 The Utility Function

From the budget constraint equation (1) we get:

$$m = 2x \frac{w_a}{e_a} - sx - p_x x - p_y y^*$$

Substituting this into the utility function (equation (1) together with the obtained values for  $x$  (equation (16)),  $y$  (equation (17)),  $p_x$  (equation (11)) and  $p_y$  (equation (12)) we obtain the following result for the utility function:

$$U_A(x, y^*, m) = U(x, y^*) + m = U(x, y^*) + 2x \frac{w_a}{e_a} - sx - p_x x - p_y y^* = \quad (24)$$

$$= \frac{A}{8(b^2 - c^2)(c^2 - 4b^2)^2 e_a^2 e_b^2}$$

$$A = b(4c^5(e_a s - 2w_a)(-2ae_a + ce_a s - 2cw_a)e_b^2 +$$

$$\begin{aligned}
& 4b^3ce_ae_b(17c^2w_b(e_as - 2w_a) + 32a^2e_ae_b + \\
& + ac(6e_aw_b + 17e_as e_b - 34w_ae_b)) - 16b^5e_ae_b(4cw_b(e_as - 2w_a) \\
& + a(2e_aw_b + 3e_as e_b - 6w_ae_b) + 4bc^3e_ae_b(-5c^2w_b(e_as - 2w_a) + \\
& + 6a^2e_ae_b + ac(6e_aw_b - 7e_as e_b + 14w_ae_b)) + 4b^6(28e_asw_ae_b^2 - \\
& - 28w_a^2e_b^2 + e_a^2(4w_b^2 - 7s^2e_b^2)) + b^2c^2(80ace_a^2w_ae_b + 104ae_a^2e_b^2 \\
& + 7c^2(4e_a^2w_a^2 - 3e_a^2s^2e_b^2 + 12e_asw_ae_b - 12w_a^2e_b^2)) \\
& + b^4(32a^2e_a^2e_b^2 + 16ace_ae_b(-6e_aw_b + e_as e_b - 2w_ae_b) \\
& + c^2(-60e_a^2w_a^2 + 41e_a^2s^2e_b^2 - 164e_asw_ae_b^2 + 164w_a^2e_b^2))
\end{aligned}$$

and differentiating this with respect to the subsidy (s) and to the exchange rate depreciation ( $e_a$ ), we get the following results:

$$\begin{aligned}
\frac{\partial U_A}{\partial s} = & -b \left[ \frac{b^4c(-8ae_a - 41c(e_as - 2w_a))e_b + 28b^6(e_as - 2w_a)e_b}{4(b^2 - c^2)(-4b^2 + c^2)^2e_ae_b} + \right. \\
& \frac{+21b^2c^4(e_as - 2w_a)e_b + 4c^5(ae_a - ce_as + 2cw_a)e_b}{4(b^2 - c^2)(-4b^2 + c^2)^2e_ae_b} - \\
& \left. \frac{-34b^3c^2e_a(cw_b + ae_b) + 8b^5e_a(4cw_b + 3ae_b) + 2bc^4e_a(5cw_b + 7ae_b)}{4(b^2 - c^2)(-4b^2 + c^2)^2e_ae_b} \right]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial U_A}{\partial e_a} = & bw_a \left[ b^4c \left[ \frac{8ae_a + 41c(e_as - 2w_a)e_b - 28b^6(e_as - 2w_a)e_b}{2(b^2 - c^2)(-4b^2 + c^2)^2e_a^3e_b} - \right. \right. \\
& \frac{-21b^2c^4(e_as - 2w_a)e_b + 4c^5(-ae_a + ce_as - 2cw_a)e_b}{2(b^2 - c^2)(-4b^2 + c^2)^2e_a^3e_b} + \\
& \left. \left. \frac{+34b^3c^2e_a(cw_b + ae_b) - 8b^5e_a(4cw_b + 3ae_b) - 2bc^4e_b(5cw_b + 7ae_b)}{2(b^2 - c^2)(-4b^2 + c^2)^2e_a^3e_b} \right] \right]
\end{aligned}$$

and one can observe that:

$$\frac{\frac{\partial U_A}{\partial e_a}}{\frac{\partial U_A}{\partial s}} = \frac{2w_a}{e_a^2}$$

### A.3 Marginal Effects

The end results in the previous appendices (A.1 and A.2) can also be derived in a more simple way:<sup>16</sup>

After substituting the demand functions (equations (2),(6),(5),(3)) in the utility functions (equation (1) and (4)) and in the profit functions (equation (7) and (8)), these become a function of the prices  $p_x$  and  $p_y$ . If we derive these function  $f$  with respect to the subsidy and use the result obtained in equation (13), we get:

$$\begin{aligned}
 \frac{df}{ds} &= \frac{df}{dp_x} \frac{dp_x}{ds} + \frac{df}{dp_y} \frac{dp_y}{ds} = \\
 &= \frac{e_a^2}{2w_a} \left( \frac{df}{dp_x} \frac{dp_x}{de_a} + \frac{df}{dp_y} \frac{dp_y}{de_a} \right) = \\
 &= \frac{e_a^2}{2w_a} \frac{df}{de_a} \Rightarrow \\
 \frac{\frac{df}{de_a}}{\frac{df}{ds}} &= \frac{2w_a}{e_a^2}
 \end{aligned}$$

where  $f(p_x, p_y) = \pi_A, \pi_B, U_A, U_B$

### A.4 Optimal Welfare

If we substitute the optimal subsidy (equation 20) and the optimal exchange rate (equation 21) in the profit function (equation 23), we obtain the following values for the optimal profit.

$$\begin{aligned}
 \pi_A(s^*) &= 2 \left[ \frac{-9ab^4ce_b + 9ab^2c^3e_b - 2ac^5e_b + b^5(cw_b - 8ae_b)}{b^3(12b^4 - 9b^2c^2 + c^4)^2e_b^2} + \right. \\
 &\quad \left. \frac{+bc^4(cw_b - ae_b) + b^3c^2(-2cw_b + 7ae_b)}{b^3(12b^4 - 9b^2c^2 + c^4)^2e_b^2} \right]_2
 \end{aligned}$$

$$\pi_A(e_a^*) = \pi_A(s^*)$$

If we substitute the same optimal values in the utility function (equation 24), we obtain the following results for the optimal utility.

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<sup>16</sup>For this hint I want to thank Leo Kaas from the Institute for Advanced Studies (IHS), Vienna.

$$\begin{aligned}
U_A(s^*) = & (b+c) \left[ \frac{(36b^{12}w_b^2 + 16a^2c^{10}e_b^2 - 72b^{11}w_b(cw_b + ae_b))}{2b^3(b-c)(12b^4 - 9b^2c^2 + c^4)^2e_b^2} - \right. \\
& \frac{-16abc^9e_b(cw_b + ae_b) + b^6c^4(-5c^2w_b^2 + 180acw_b e_b - 682a^2e_b^2)}{2b^3(b-c)(12b^4 - 9b^2c^2 + c^4)^2e_b^2} + \\
& \frac{+2b^5c^5(11c^2w_b^2 - 73acw_b e_b - 84a^2e_b^2) + 4b^2c^8(c^2w_b^2 + 6acw_b e_b - 34a^2e_b^2)}{2b^3(b-c)(12b^4 - 9b^2c^2 + c^4)^2e_b^2} + \\
& \frac{+b^{10}(17c^2w_b^2 + 88acw_b e_b - 28a^2e_b^2) - 4b^3c^7(2c^2w_b^2 - 21acw_b e_b - 23a^2e_b^2)}{2b^3(b-c)(12b^4 - 9b^2c^2 + c^4)^2e_b^2} - \\
& \frac{-12b^7c^3(c^2w_b^2 - 8acw_b e_b - 9a^2e_b^2) + 2b^9c(19c^2w_b^2 + 11acw_b e_b - 8a^2e_b^2)}{2b^3(b-c)(12b^4 - 9b^2c^2 + c^4)^2e_b^2} + \\
& \frac{+b^8c^2(-13c^2w_b^2 - 146acw_b e_b + 413a^2e_b^2)}{2b^3(b-c)(12b^4 - 9b^2c^2 + c^4)^2e_b^2} + \\
& \left. \frac{+b^4c^6(-7c^2w_b^2 - 114acw_b e_b + 449a^2e_b^2)}{2b^3(b-c)(12b^4 - 9b^2c^2 + c^4)^2e_b^2} \right]
\end{aligned}$$

$$U_A(e_a^*) = U_A(s^*)$$

Therefore:

$$W_A(s^*) = W_A(e_a^*)$$

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**Institut für Höhere Studien**  
**Institute for Advanced Studies**

Stumpergasse 56

A-1060 Vienna

Austria

Phone: +43-1-599 91-145

Fax: +43-1-599 91-163