

ON COMPARING DISTRIBUTION
PROCESSES^x

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ABSTRACT

Distribution processes are usually only compared with respect to their result, i.e. the distribution of income among individuals (or households). This limitation is unsatisfactory when a broad welfare theoretic conception of economic justice is borne in mind: Not only the 'static' inequality as exhibited by a given distribution of income has to be taken into account. Full consideration has to be given as well to the underlying mechanism by which income is distributed, i.e. the 'imputation mechanism'.

Given a model of the imputation mechanism, it is possible to construct measures of 'process inequality' (as opposed to measures of 'static' inequality of a particular distribution of income). This is done for the 'chance' model which is related to the conception of 'unequal opportunities'. On the basis of a 'Dynamic Equity Principle' the measure δ of process inequality is derived. It is shown that the measure δ of process inequality contains normatively relevant information which could not be obtained by taking only 'static' inequality measures into account.

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I. PROCESS INEQUALITY: AN INTRODUCTION*

Economic justice refers to the process of distributing social welfare. This process consists of two components, each of which is important when income inequality is borne in mind:

- First, the outcome of the distribution process, i.e. a particular allotment of income among individuals.
- Secondly, the properties of the distribution process; i.e. the mechanism by which income is allotted among individuals.

This second component of a distribution process may conveniently be called the 'imputation mechanism'. The term refers to rules of economic interaction, determining what individuals actually get: Individuals get what the mechanism imputes to them. Thus an imputation mechanism models the 'distributional side' of economic activities (it is not just a device of calculating 'shadow prices')¹⁾.

The two components of a distribution process - viz, the imputation mechanism and the particular distribution of income - have to be discussed to some extent separately if one tries to give a full account of economic inequality. A restriction of equity considerations just to the outcome of a distribution process is bound to miss an important feature of economic justice. This may be illustrated by a simple example. Take the notion of 'discrimination' which certainly is related to economic justice: It is argued that women are discriminated against if, for example, women get lower pay than men though they have the same productivity

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and engage in identical occupations as men. In this case, it is not only inequality due to different earnings of men and women which matters. Additionally, the way by which this inequality is generated is thought to be unjust. Discrimination does not only refer to the result of the distribution process but to some features of the process itself, viz., the 'unequal treatment between male and female employees which does not directly result in cost minimisation in monetary terms in relation to labour utilisation'²⁾.

The example of discrimination illustrates an important aspect of economic justice, (being the background concept of inequality): People do not only care about how much they get, but also about the way they get their share³⁾. Or, to put it in more general terms: The working properties of an imputation mechanism are themselves of normative interest when inequality is borne in mind. This is due to the fact that there is generally no one-to-one correspondence between the structure of an imputation mechanism and the actual distribution of income. Consider once more the example of discrimination. Let us assume that the same distribution of income has emerged from two different processes. The first process exhibits discrimination against women, whereas in the second process women's productivity is lower, but they are not discriminated against by entrepreneurs. In the second case, it is the difference in acquisition of human capital rather than economic discrimination which causes income inequality⁴⁾. From a normative point of view, people may prefer the distribution process without discrimination, though the resulting distributions of income are identical in both cases. This implies, in terms of a social welfare function, that 'discrimination' is diminishing social welfare in addition to the losses caused by the unequal distribution of income. The existence of discrimination relates to a property of the

imputation mechanism itself and is not identifiable solely from a given distribution of income. Thus it is necessary to evaluate in addition to the static inequality of a given income distribution, the built-in inequality of the underlying imputation mechanism.

Any attempt to construct an operationally meaningful measure for process inequality has to cope with two difficulties:

- First, a decision has to be made about which models of the imputation mechanism are to be used in the analysis of process inequality.
- Secondly, it is necessary to state certain principles upon which the identification of process inequality can be based.

We can hardly expect to solve these two problems - of finding a descriptive framework and a normative standard - by discussing them in general terms. Rather it is preferable to consider the difficulties involved by these two decisions in relation to a particular model of the distribution process. Such a model is proposed in the following part II of the paper. It should, however, be made clear that the particular model proposed is quite a separate point from our general concern with the necessity of studying process inequality (as opposed to static inequality of a current distribution of income).

II. MEASURING PROCESS INEQUALITY: THE 'CHANCE' MODEL

It is widely held that economic inequality is unjust as far as unequal opportunities are concerned⁵⁾. However, it is by no means clear what an economic or social system offering equal opportunities should look like. As a starting point we may consider the income dimension of unequal opportunities. How are the opportunities to experience an increase (or a decrease) in income distributed? To answer this question, we need a model of the imputation mechanism relating a set of input variables to the distribution of income changes. The simplest models of this type available are stochastic models. Stochastic models relate the chances of income change to the actual level of income. The present level of income is used as an input variable to determine the chance of success and failure. Though the income level is a very rough approximation of an individual's position, it is by no means far-fetched to use the income level as a criterion to classify a population for the purpose of discussing economic inequality within the distribution process⁶⁾.

1. Stochastic processes as descriptive models. Stochastic models have been criticized for not providing an explanation for distribution processes in terms of economic variables⁷⁾. Though this criticism is correct, it is not an objection against the use of stochastic processes as descriptive models. We may interpret a stochastic model as a framework of organizing data about distribution processes. This particular way of organizing the data is of special interest since it reveals certain implications of a distribution process, viz., how chances of failure and success are related to income levels. It may be difficult to interpret this 'income chance' relation as the 'structural' form of a causal relationship. However, even if we consider the relation as a quasi 'reduced' form (of an unknown 'true'

model of the imputation mechanism) we gain important information⁸⁾. Since people care about chances and opportunities, we may use stochastic models as a framework to discuss desirable equity properties of an imputation mechanism. E.g., certain types of 'income chance' relationships may serve as points of reference in discussions concerning the design of economic institutions. An equity-minded economist may inquire about the implications of an observed set of property rights for the distribution of opportunities. One way to tackle this problem is offered by the use of stochastic models. Stochastic models organize the data of income mobility as if the imputation mechanism generates income distribution in patterns that observe the rules of stochastic processes. In this case stochastic processes serve as descriptive models of the imputation mechanism. Thus we may analyze built-in inequalities as exhibited in the framework of stochastic processes.

It should be stressed that such an analysis of built-in inequality is directly related to the general structure of the stochastic process and not to particular states (of income distribution). The stochastic process used in the following section will be first-order Markovian. Thus the general structure of the imputation mechanism is modelled by a transition matrix. To this transition matrix - rather than to a particular state, may it be an equilibrium or not - does the discussion of process inequality refer to.

2. Notation. It may be useful to give a list of the symbols used to describe the process before discussing some of its properties.

- i, j ... indices of income classes
- k ... number of income classes

- t ... index of time period
- η ... proportionality factor for grading income
- K_i ... income class i
- P_{ij} ... chance to ascend from income class i to income class j
- q ... chance to descend from top class
- r ... chance to ascend from bottom class
- c ... ratio between q and r; $c = r/q$
- P ... transition matrix $P = (p_{ij})$
- $h(t)$... distribution vector at time t
 $h(t) = (h_1(t), h_2(t), \dots, h_i(t), \dots, h_k(t))$
- g ... vector of equilibrium distribution
- λ ... index of 'static' inequality related either to $h(t)$, or g.
- δ ... index of process inequality related to P.

3. Basic properties of the CHANCE - model. The CHANCE - model is a special type of a stochastic process. The imputation mechanism exhibits the following properties:

- (i) The distribution of income is discrete. Income is divided into classes following a pattern of equiproportional distance.
- (ii) The chances to experience an increase or decrease of income or to stay at the same income level depend only on the current level of income. These chances are the same for all individuals within one income class.
- (iii) In each class there is a positive chance to stay as well as to move. Within one time period the change of income is limited to a step into one of the immediately adjacent classes.

The states are defined by income classes:

$$(1) \quad K_i = K_1 \cdot \eta^{i-1}, \quad 1 \leq i \leq k, \quad \eta > 1$$

The distribution process consists in the transformation of a given distribution at time t :

$$(2) \quad h(t) = (h_1(t), h_2(t), \dots, h_i(t), \dots, h_k(t))$$

into another distribution at time $t+1$:

$$(3) \quad h(t+1) = (h_1(t+1), h_2(t+1), \dots, h_i(t+1), \dots, h_k(t+1))$$

The imputation mechanism is modelled by the transition matrix:

$$(4) \quad P = (p_{ij}) \quad 1 \leq i, j \leq k$$

The transformation process is described by:

$$(5) \quad h(t+1) = P' h(t)$$

Assumption (iii) ensures that P is banddiagonal:

$$(6) \quad p_{ij} = 0 \quad |i-j| > 1$$

$$p_{ij} > 0 \quad |i-j| \leq 1$$

Thus P is indecomposable and the process is ergodic.

This implies that there exists a distribution g ,

$$(7) \quad g = (g_1, g_2, \dots, g_i, \dots, g_k)$$

such that

$$(8) \quad g = P' g$$

This equilibrium distribution g depends only on the off-diagonal elements of P :

$$(9) \quad g_i = \frac{P_{12} P_{23} \cdots P_{i-1,i}}{P_{21} P_{32} \cdots P_{i,i-1}} g_1 \quad 1 < i \leq k$$

where g_1 is to be chosen such that:

$$(10) \quad \sum_{i=1}^k g_i = 1$$

It may be of interest to note that the assumption about the banddiagonal structure of matrix P is not an analytically necessary restriction. It is rather a convenient technical simplification¹⁰⁾. The length of the transition period has to be chosen such that it is not too unrealistic to assume that an individual cannot transgress more than one income bracket within one period. Obviously the length of the time interval is related to the distances between the income classes i.e. to the choice of the grading factor η .

4. Dynamic Equity Principle. Using the transition matrix as a model of the imputation mechanism, we may proceed to the next question: What principles should guide the analysis of process inequality? Such principles should reflect an egalitarian point of view (otherwise the whole attempt to evaluate process inequality will be of little normative interest). Furthermore, it must be possible to classify and order at least partially the alternative transition matrices according to the equity principle¹¹⁾.

Instead of attempting at a very general equity principle, we will restrict ourselves to a rather particular aspect

of the equity considerations related to the imputation mechanism. Furthermore, these equity considerations will be put into a particular formulation which is related to our previous choice of the CHANCE-model as model of the imputation mechanism.

Dynamic Equity Principle:

- (1) The chance of an increase in income should be negatively correlated to the current level of income.
- (2) The chance of a decrease in income should be positively correlated to the current level of income.

Though it may be hard to find a general ethical postulate from which this particular principle could be derived there are several intuitive conceptions of economic justice which are expressed in this principle. First of all, the Dynamic Equity Principle asks for a kind of 'regression towards the mean' built-in to the imputation mechanism. This reflects an egalitarian point of view. Another equity argument raised in favour of this principle may run as follows: Those who are already well off are less deserving of an increase in income than those who are in a worse income position¹²⁾. This point of view brings out the 'dynamic' aspect of this principle, viz., that the change of income should be related to the current level of income¹³⁾.

The difficulty to find a coherent normative background for the Dynamic Equity Principle is partly due to the 'reduced form' in which it is expressed. This notion of a (normative) reduced form parallels the econometric concept of a (descriptive) reduced form in which income data are organized by stochastic models. Though a reduced form expresses a relation which is thought to be valid it is difficult to

justify the relation directly; may that be either with reference to causality (descriptive model) or regarding economic justice (normative standard)¹⁴⁾.

5. Reflecting Barriers. To illustrate some applications of the Dynamic Equity Principle consider the so called 'vicious circle of poverty'. The existence of such a vicious circle indicates that the underlying imputation mechanism does make it very difficult for the poor to ascend above the poverty line. Under such circumstances there exists a kind of reflecting barrier which inhibits upwards income mobility for the lower income classes. This obviously violates part 1 of the dynamic equity principle. Another example is provided by bureaucratic systems where it is very unlikely to fail once you are already at the top. In this case there are quasi-reflecting barriers with respect to downwards mobility.

The concept of quasi-reflecting barriers¹⁵⁾ (i.e. very small non-zero off-diagonal elements in the transition matrix P) fits well into our intuitive notion of process inequality within the dynamics of income distribution. The existence of reflecting barriers is a special case of the violation of the monotonicity property of the Dynamic Equity Principle. We may classify transition matrices into a simple dichotomy according to whether the off-diagonal non-zero elements are monotonic functions of the income level. If the imputation mechanism does not exhibit the monotonicity property there is a violation of the dynamic equity principle. Though it is of interest for empirical research in income inequality¹⁶⁾ to study the existence of reflecting barriers, we have to restrict our theoretical analysis to imputation mechanisms observing the monotonicity property. By this restriction we will gain further analytical results without

excluding imputation mechanisms which are underlying the standard stochastic models, i.e. the 'proportionate effect models'.

6. MONOTONE-model. Consider the class of CHANCE - models exhibiting the monotonicity property: The non-zero off-diagonal δ elements of the transition matrix P are a monotonic function of the income level. Let us call imputation mechanisms of this type MONOTONE. A simple example of a MONOTONE model is provided by the following transition matrix P.

$$P = \begin{pmatrix} 1-r & r & 0 & \dots & 0 \\ q\delta^{k-2} & 1-(q\delta^{k-2}) - (r\delta) & r\delta & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & q\delta^{k-i-1} & 1-(q\delta^{k-i-1}) - (r\delta^{i-1}) & r\delta^{i-1} & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & r\delta^{k-2} & \dots \\ 0 & \dots & 0 & q & 1-q \end{pmatrix}$$

P is characterized by four parameters:

- q ... downwards mobility in the highest income class
- r ... upwards mobility in the lowest income class
- δ ... the factor by which upwards mobility is increased and downwards mobility decreased.
- k ... number of income classes

These parameters have to observe the following restrictions

$$(11) \quad \begin{aligned} 0 < r\delta^{i-1} < 1, \quad 0 < q\delta^{k-i-1} < 1 \quad \text{where } 1 \leq i \leq k-1 < \infty \\ 0 < 1 - (r\delta^{i-1}) - (q\delta^{k-i-1}) < 1 \quad \text{where } 2 \leq i \leq k-1 < \infty \end{aligned}$$

The three parameters characterizing P can be employed to compare different imputation mechanisms of MONOTONE type. First we may classify MONOTONE models with respect to the dynamic equity postulate. This can be done by referring to parameter δ . There are three cases:

- $\delta > 1$ 'amplifier of dynamic inequality' (AMPL)
- $\delta = 1$ 'proportionate effect' (PROPOR)
- $\delta < 1$ 'regression towards the mean' (REGRESS)

The AMPL case obviously exhibits a tendency to increase income inequality: The richer you are the less are the chances to fail and the greater are the chances to succeed. The poorer you are, the higher the chances to fail and the less are the chances to experience an increase of income.

The PROPOR case is frequently discussed in the literature on stochastic models of income distribution. Those models are built on the assumption that the 'law of proportionate effect' holds¹⁷⁾. This implies that the transition probabilities depend only on the relative size of the change of income. This amounts within the framework of the CHANCE-model to identical non-zero off-diagonal elements of the transition matrix P. Looking at the 'proportionate effect' assumption not as a 'law', but rather from a normative point of view, we observe that it violates the Dynamic Equity Postulate. Thus we may conclude that the 'proportionate effect' is an undesirable property of an imputation mechanism.

Only the 'regression towards the mean' (REGRESS) case is compatible with the Dynamic Equity Postulate. Within this class of REGRESS imputation mechanisms the analysis can be carried one step further.

7. δ -inequality in REGRESS-models. The class of REGRESS-models is characterized by the range of the three parameters p , q , and d :

$$\begin{aligned}
 & 0 < r, q < 1 \\
 (12) \quad & 0 < \delta < 1 \\
 & 0 < 1 - r\delta^{i-1} - q\delta^{k-i-1} < 1 \qquad 2 < i < k-1 < \infty
 \end{aligned}$$

For any given p and q the parameter δ represents the strength of the equalizing forces built into the imputation mechanism. The larger δ is, the weaker are the regression towards the mean tendencies. Thus we may use the parameter δ as a measure of process inequality.

The parameter δ has some advantages and some drawbacks. The major advantage of δ is to be found in the possibility of ordering REGRESS-models. The use of a scalar as dynamic inequality index offers a simple way of ordering. Such an ordering is independent of the levels of q and r , provided the parameter δ is interpreted as 'relative strength' of the regression towards the mean tendency. The value of δ measures the REGRESS property relative to the level of q and r . A slight disadvantage in using δ as inequality measure arises from the restrictions imposed on it. These restrictions are necessary to preserve the 'stochastic' properties of the matrix P . This implies that δ has to be chosen from a set which observes for given q , r and k the restrictions of (12). However, these restrictions are no substantial limitation in the applicability of measure δ . This is for two reasons: First, whenever the CHANCE - model is estimated from empirical data then restriction (12) is observed automatically. Secondly, in constructing examples there is always the possibility to observe formula (12) a priori. There is only one case in which the restriction (12) is a substantial limitation to the analysis. These

are the AMPL-models with $\delta > 1$. However, this class of models is not of special interest in the context of income distribution since such a system is only stable when income is concentrated mainly in the bottom and the top class.

From these comments on various 'chance'-model of the imputation mechanism, on the dynamic equity principle, and on the measure derived from it, we may proceed to the third problem to be discussed: Does 'process inequality' contain normatively relevant information about income inequality? Or is process inequality anyway fully reflected in the outcome of the distribution process, i.e., in the static inequality exhibited by the distribution of income? These two questions amount to the problem whether there exists a monotonic function between the measures of process inequality and the static inequality measures related to the distribution of income. The next section will show that there does not exist generally such function within the framework of REGRESS-models. The two types of inequality measures are in some sense independent from each other.

8. Independence properties of the δ -measure. It is the advantage of using stochastic models of the imputation mechanism that we can distinguish 'short-run' effects from 'long-run' effects of process inequality. Within the framework of the 'chance'-model we may call the change of a given distribution of income from one period to the next, the short-run effect of the imputation mechanism P. This change of static inequality may be indicated by some inequality index λ_t .

$$(13) \quad \lambda_t = \lambda(f(t))$$

where $f(t)$ is the discrete distribution of income at time t . Thus the short-run effect of P on λ is:

$$\begin{aligned}(14) \quad \Delta\lambda_t &\equiv \lambda_t - \lambda_{t+1} \\ &= \lambda(f(t)) - \lambda(f(t+1)) \\ &= \lambda(f(t)) - \lambda(P'f(t))\end{aligned}$$

The short run effect of process inequality is the change of a static inequality index defined on the distribution of income.

The long run effect of process inequality is related to the question what will the distribution of income look like when the imputation mechanism is applied repeatedly on to the same population. To this we can give a straightforward answer, since all 'chance' processes are ergodic having invariant distributions. Thus the long run effect of process inequality can be represented by using an inequality index for the equilibrium distribution g :

$$(15) \quad \lambda(g) = \lambda(g(P))$$

With help of the two concepts - short and long run effect - the independence problem can be divided into three questions:

- (i) Does the existence of regression towards the mean, i.e. $\delta < 1$, imply a short run reduction of static inequality?
- (ii) Does short run reduction of static inequality imply that the underlying PMIL-model does not violate the dynamic equity axiom?
- (iii) Does a lower process inequality, as measured by δ , imply a lower static inequality of the corresponding equilibrium distribution?

If the process inequality index is expected to exhibit information additional to the static inequality indices

(related to the distribution of income) then the questions i - iii have to be answered in the negative. This is actually the case.

8.1.'Short run independence I'. Consider a distribution $f(t)$, where

$$(16) \quad f(t) = (0, 0, \dots, f_i(t), \dots, 0)$$

All persons have the same income in time t . The banddiagonal P operates such that:

$$(17) \quad f(t+1) = (0, \dots, 0, f_{i-1}(t+1), f_i(t+1), f_{i+1}(t+1), 0, \dots, 0)$$

where

$$f_{i-1}(t+1) > 0, f_i(t+1) > 0, f_{i+1}(t+1) > 0$$

This implies

$$(18) \quad \lambda(f(t)) < \lambda(f(t+1))$$

Thus there is a short run increase of static inequality, though P exhibits the REGRESS property.

This example does not only prove an independence property of measure δ . It indicates another important feature of process inequality:

Whenever $\delta > 0$, $1 > p$, $q > 0$ then there exists process inequality.

The inequality parameter δ reflects only the relative degree of inequality. Thus it is not counter intuitive

that a process exhibiting even comparatively low built-in inequality may transform a state of equal personal income into a state to which an increased index of inequality corresponds.

8.2. 'Short run independence II' Consider the following REGRESS-model, in which P is characterized by the properties of (19):

- (i) k is an odd number
- (ii) $r = q = 0.5$
- (19) (iii) $\delta = \delta > 0$
- (iv) For $m = (k+1)/2$
 let $p_{m,m-1} < q\delta^{k-m-1}$
 and $p_{m,m+1} < r\delta^{m-1}$

Condition (iii) implies the existence of quasi-reflecting barriers in income class K_m . However, this does not imply necessarily a short run increase in static inequality. Consider the following example. Let a P as characterized by (19) operate on an distribution $f(t)$:

(20) $f(t) = (f_1(t), f_2(t), \dots, f_i(t), \dots, f_k(t))$

where

(21) $f_1(t) = f_2(t) = \dots = f_i(t) = \dots = f_k(t)$

When P operates on $f(t)$, then

(22) $\lambda(f(t)) > \lambda(f(t+1))$

Thus there is a reduction in short run inequality though the matrix P does not observe the Dynamic Equity Principle.

8.3. 'Long run independence'. Consider the following class of PMIL-models of the imputation mechanism. Income is graded into five classes, starting with $K_1 = 2000$ and a grading factor $\eta = 0.58$. Furthermore $c = r/q = 3.0$. This implies that the chance to ascend from the lowest income class K_1 is three times larger than the chance to descend from the highest income class K_5 . What is going to happen to the values of the inequality index of the equilibrium distributions, when the process inequality index varies monotonically?

Index of process inequality δ	Gini-coefficient of the equilibrium distribution corresponding to δ
.1	.098
.3	.149
.5	.189
.7	.189
.9	.150
1.0	.128

The discussion of the three questions (i), (ii), and (iii) has proved that the measure δ exhibits an independence property with respect to the traditional measures like the Gini coefficient. This holds true for the short and the long run¹⁹⁾. The measure δ of process inequality contains information that is not reflected in the inequality indices for the current (or equilibrium) distribution of income. However, two questions may arise:

- Do MONOTONE-models resemble any process compatible with empirically observed data?

- Granted that the δ -measure contains information not contained in static measures. Is this additional information relevant to a normative evaluation of economic inequality?

The following two sections will be devoted to these problems.

9. England vs Shorrocks' Land. In a recent paper A.F. Shorrocks²⁰⁾ presents data on income mobility of English male employees during the period 1963-1969. Income is graded into five classes, but no assumption is made about banddiagonality. The matrix M is based on direct observations of individual income mobility.

$$(23) \quad M = \begin{bmatrix} 0.64 & 0.29 & 0.04 & 0.03 & 0.00 \\ 0.14 & 0.56 & 0.26 & 0.03 & 0.01 \\ 0.02 & 0.22 & 0.54 & 0.21 & 0.01 \\ 0.01 & 0.04 & 0.27 & 0.54 & 0.14 \\ 0.00 & 0.01 & 0.05 & 0.27 & 0.67 \end{bmatrix}$$

The entries of this matrix M reflect the chances to move from one income class into another income class within a time period of three years. Assuming that the matrix M represents the results of a Markov process which is based on year-to-year changes, we may ask the following two questions: What does the equilibrium distribution corresponding to a process characterized by M look like? Secondly, does a process of MONOTONE type exist which leads to approximately the same equilibrium distribution as the process M?

To answer these questions consider first the equilibrium distribution corresponding to M:

$$(24) \quad g = (0.12, 0.25, 0.29, 0.30, 0.11)$$

There is an easy way to check whether the equilibrium distribution of (24) can be generated by a MONOTONE process⁽²¹⁾. Consider a MONOTONE-model with

$$(25) \quad c = r/q = 1.$$

For such processes the following holds true:

$$(26) \quad \frac{g_{i+1}}{g_i} = c \delta^{2i-k}$$

From this we obtain

$$(27) \quad \delta^2 = \frac{g_{i-1} \cdot g_i}{g_{i+1}}$$

Making use of (27) we may calculate from the equilibrium distribution (24) three values of δ . Each of these three values is close enough to 0.8 such that we may draw the following conclusion: A MONOTONE-model characterized by

$$(28) \quad \begin{aligned} c &= r/q = 1 \\ \delta &= 0.8 \end{aligned}$$

provides a sufficient explanation for the equilibrium distribution (24) of the process characterized by M which is based on empirical observations. This is confirmed by the equilibrium distribution(29) corresponding to a MONOTONE-model as defined by (28):

$$(29) \quad g = (0.12, 0.23, 0.29, 0.23, 0.12)$$

Though Shorrocks infers from his data that the assumption of proportionate effect cannot be rejected, it is fair to

argue that quite a different state of affairs is conceivable. Let us turn from England (through Shorrocks' glasses) to Shorrocks' Land (through MONOTONE-glasses): In Shorrocks' Land the poor had enough political power to bargain for a set of property rights which ensure that the imputation mechanism observes the Dynamic Equity Principle²²⁾. Agreement was reached that the poor in the lowest class should stand the same chance to ascend as the rich in the top class have to descend; thus $c = r/q = 1$. Then the δ -Inequality factor was fixed with $\delta = 0.8$. This was thought to be a victory of the Dynamic Equity Faction in the constitutional committee of Shorrocks' Land. When some critics confronted this faction with the fact that the equilibrium distribution for Shorrocks' Land was nearly the same as in England, the Dynamic Equity Faction took the following view: Though in the long run little is to be gained in static inequality over England, the resulting equilibrium distribution is achieved in Shorrocks' Land by a process to be preferred, viz., by a process observing the Dynamic Equity Principle, (whereas in England this principle is violated).

10. Normative interrelations. Does the argument of the Dynamic Equity Faction stand to reason? Does it not draw too strict a line of demarcation between the 'rules of the game' and the resulting outcomes? Though such a distinction is useful for analytical purposes it can be rather misleading when used in a normative evaluation of alternative distribution processes. One should not infer from the necessity to distinguish in descriptive models between the current distribution of income and the underlying imputation mechanism, that from a normative point of view these two components of the distribution process are to be evaluated separately. Such a separate evaluation could not be done unambiguously, since people frequently let their normative evaluation of a given distribution of income depend on the way it was achieved.

People are more willing to accept a given level of static inequality once they have been convinced that it is the outcome of a 'fair game' (whatever their criteria of 'justice' of 'fairness' may be)²³⁾. On the other hand, an imputation mechanism that is thought to be 'fair' in principle may lose substantially in normative evaluation when - from a normative point of view - disastrous distributions of income emerge. Even among those who believe that a competitive market imputes 'just' rewards to those engaged in economic activities, some may find a pure market imputation system unbearable if it would lead to large scale poverty. Thus the losses of welfare implied by static and by process inequality are interdependent.

To this interdependence Professor Sen has referred to indirectly when writing: 'The relation between inequality and rebellion is indeed a close one, and it runs both ways. That a perceived sense of inequity is a common ingredient of rebellion in societies is clear enough, but it is also important to recognize that the perception of inequity, and indeed the content of that elusive concept, depend substantially on possibilities of actual rebellion.'²⁴⁾ Obviously rebellions and even more revolutions try to change the imputation mechanism. Partly because they try to change the distribution of income, partly because they detest the working properties of the old imputation mechanism itself; as was the case in the Russian and Chinese revolutions. This illustrates once more the conclusion to be drawn from the above considerations: Process and static inequality are normatively interrelated but there is no one-to-one correspondence. Thus welfare economics should consider process inequality in an adequate analytical framework to gain a fuller concept of income inequality. A restriction to just the distribution of income will not do.

11. The descriptive framework reconsidered. The construction of an analytical framework for studying process inequality is not only impeded by the difficulties of finding an adequate normative standard. Already the choice of the descriptive framework is a more complicated task in the case of process inequality than in the field of static inequality. (There is general agreement to use distribution functions as descriptive framework for measuring static inequality.) Already the choice of the descriptive framework, i.e. a model of the imputation mechanism, is crucial since not all models reveal built-in inequality to the same extent. Some models of the imputation mechanism start from assumptions which exclude a priori process inequality. Consider as an extreme example a model of a competitive tatonnement process of a pure exchange economy. Such a tatonnement process is always 'distributionally neutral'. As commodities are exchanged at equilibrium prices, no trader can lose or gain in terms of money income. Even less extreme models which are applied in empirical research in the field of income distribution make assumptions which partly beg the question. E.g. in the unrefined basic human capital model²⁵⁾ it is assumed that life income is equalized for the entire population. One has not to be a radical economist to question this assumption which makes it rather difficult to find any process inequality in the earnings function. Thus the choice of the descriptive framework already depends on the economist's sensitivity towards process inequality.

This sensitivity towards process inequality is clearly reflected in our choice of the CHANCE - model as descriptive framework for discussing built-in inequality. However, the CHANCE - model is subject to two substantial limitations. First, it is a reduced form model, and, secondly, it does only consider the income dimension of.

social welfare. These shortcomings are partly balanced by the favorable features of the CHANCE - model. Its analytical framework offers the opportunity:

- to organize systematically empirical data on distribution processes;
- to make a clear descriptive distinction between the imputation mechanism and a particular distribution of income;
- to consider changes of the imputation mechanism over time;
- to identify process inequality;
- to analyze the relation between static and process inequality.

With respect to this last item, viz., the relation between process and static inequality, the paper has by no means made use of all analytical possibilities offered by the CHANCE - model²⁵⁾. Especially one aspect of the CHANCE - model needs further consideration: Our analysis has proceeded on the assumption of an "eternal population". This is a useful device to bring out some characteristics of the imputation mechanism. In reality, however, we observe an influx and outflow of income units to and from the imputation mechanism. The structure of such a 'birth-and-death' processes is another important dimension of income inequality²⁶⁾. It indicates how unequally distributed starting positions are. It is an advantage of the CHANCE - model that it provides an explicit link between the structure of the influx of income units into the economic system, the dynamics of the imputation mechanism, and the observed distribution of income.

III. CONCLUSIONS

1. There are two components to be distinguished in the analysis of distribution processes. First, the current distribution of income and, secondly, the imputation mechanism. A comparison of distributional states is usually restricted to the output of a distribution process, i.e. to the distribution of income. This limitation seems to be unsatisfactory once it is recognized that people do not only care about how much they get, but as well about the way they get their share. Thus the imputation mechanism has to be included in the comparison of distribution processes, regardless whether the comparison is carried out for descriptive or normative purposes.
2. The construction of an analytical framework for comparisons of imputation mechanisms with respect to 'process inequality' has to face two difficulties: First, the choice of a descriptive framework, i.e. a model of the imputation mechanism. Secondly, the formulation of a normative standard upon which an operational measure of process inequality may be justified. Different models of the imputation mechanism imply different measures of process inequality. Thus such measures can only be used when comparisons are restricted to the same model of the imputation mechanism.
3. As a descriptive framework we have chosen a model of the imputation mechanism which refers to the conception of 'unequal opportunities'. The CHANCE - model relates the opportunities to experience an increase (or decrease) of income to the present level of income. The CHANCE - model organizes data on income mobility as if the imputation mechanism generates income distributions

observing the rules of some particular stochastic processes.

4. As a normative standard for our analysis we have formulated a Dynamic Equity Principle: The chance of experiencing an increase of income should be negatively correlated to the level of income; whereas the chance of experiencing a decrease of income should be positively correlated to the level of income.
5. Within the framework of CHANCE - models the Dynamic Equity Principle is applicable to two types of analysis. First, we may check whether an imputation mechanism observes the Dynamic Equity Principle. Secondly, we may try to evaluate the degree of built-in inequality. The 'reflecting barriers' are an example for the first type of analysis, whereas the measure δ can be used for a comparative evaluation of imputation mechanisms of MONOTONE type.
6. Measure ' δ ' of process inequality contains normatively relevant information sui generis which cannot be obtained by considering only the static inequality exhibited by the current (or equilibrium) distribution of income. There does not exist generally a monotonic function relating the static inequality indices to the measure ' δ ' of process inequality. This holds true for the short and the long run. Though there does not exist a one-to-one correspondence of process and static inequality, it is necessary from a normative point of view to evaluate both types of inequality simultaneously.

NOTES

- 1 Our use of the term 'imputation mechanism' is closer to the meaning originally attached by the Viennese School rather than to the current usage in linear programming. Within game theory the original meaning has partly been preserved. Cf Neumann - Morgenstern 1953, esp. § 4.
- 2 Chiplin - Sloane 1976, 729
- 3 Consider a libertarian having the chance to get the same amount of income accruing from the market place or from a social security system. Would he be indifferent?
- 4 This systematic difference in acquiring human capital indicates, however, the existence of social discrimination against women.
- 5 Cf the introductory chapter in Jencks et al 1972.
- 6 At least not in societies influenced by the 'spirit of capitalism'.
- 7 Lydall 1968, 19 is quoted frequently on this subject.
- 8 Shorrocks 1975, takes a different line of defense of stochastic models. Cf also his comments in Atkinson, ed., 1976, 92.
- 9 Cf Feller 1968, 396 pass.
- 10 This simplification is necessary to obtain the simple algebraic expression (9) for the equilibrium distribution associated with the transition matrix. The recursive formula (9) for the equilibrium distribution is easily obtained from (30)

$$\begin{aligned} g_1 &= P_{11} g_1 + P_{21} g_2 \\ (30) \quad g_2 &= P_{12} g_1 + P_{22} g_2 + P_{32} g_3 \\ &\vdots \\ g &= P_{k-1,k} g_{k-1} + P_{kk} g_k \end{aligned}$$

The recursive formula is obvious for the first equation. In the second equation g_2 is substituted by the expression obtained from the first equation, and so on. The same procedure can be started from the last equation. The assumption of banddiagonality, however, is essential for obtaining such a simple formula.

- 11 The problem of gaining sufficient discriminatory power by using relatively weak normative assumptions has attracted interest in the discussions on the welfare theoretic background of inequality indices. Cf Atkinson, 1970; Dasgupta - Sen - Starrett 1973.
- 12 It is interesting to note, that the Dynamic Equity Principle may be related to an implicit postulate of efficiency. Such postulate would be built upon the following concept: Economic activities are carried out in a hierarchical social framework. This hierarchy determines two factors simultaneously: First, the extent to which a decision taken by a person affects the total population; (the influence is increasing from bottom to the top). Secondly, the income that is received by a person at a specified level of the hierarchy. The 'efficiency' version of the Dynamic Equity Principle asks the following. Those who are at the top have to make stronger efforts not to decline from their position, than those who are at a lower level of the hierarchy. This is justifiable on efficiency grounds because the relative social impact of decisions taken by a person increases-monotonically towards the top.
- 13 The Dynamic Equity Principle asks for a kind of intertemporal compensation by relating income mobility to the current level of income. A notion of compensation is also to be found in Sen's Weak Equity Axiom. Cf. Sen 1973, 18.
- 14 The justification of normative standards is a difficult task not only with respect to process inequality. In the field of measuring static inequality similar questions have been raised, e.g. whether from a normative point of view 'mean independence' is a desirable property of an inequality measure. Cf Sen 1973, 69 pass., Kolm 1976.
- 15 The term 'reflecting barrier' is borrowed from the theory of stochastic processes. Cf Feller 1968, 342 pass.
- 16 Empirical research is not bound to make rather general analyses of imputation mechanisms. Special cases attract as 'facts' interest even though no systematic comparisons with other imputation mechanisms can be carried out.
- 17 Cf Steindl 1965, 95 pass. this principle has played an important part in the derivation of Pareto's Law. Cf also Champernowne 1973.
- 18 This restriction is due to the fact that the second largest eigenvalue of the transition matrix is sensitive with respect to the number of income classes. For a detailed analysis. Cf Mitter 1977.

- 19 The 'independence property' refers to a negative result: There does generally not exist a monotonic function relating static inequality indices to the measure of process inequality. However, such functions do exist in special cases. A notable example are MONOTONE-models with $c = p/q = 1$ and $0 < \delta < 1$. The equilibrium distributions of such models are symmetrical with respect to the income classes. As δ decreases the equilibrium distribution loses symmetrically weight on its tails. Thus the new equilibrium distribution Lorenz dominates the old one. Cf Rothschild - Stiglitz 1973, Mitter - Wagner 1977.
- 20 Shorrocks 1976, 576.
- 21 Mitter 1977
- 22 An imputation mechanism is always related to a set of property rights which provides the framework of a penalty-reward structure. (Furubotn - Pejovich 1972, 1137 pass.) This penalty-reward structure of economic interaction may exhibit considerable asymmetry favouring some social groups and discriminating against others. Such an asymmetry corresponds to the built-in inequality of imputation mechanisms. Thus from the point of view of the 'contractual theories of distribution' we may consider the degree of process inequality as being part of the contractual agreement reached in a pre-constitutional stage. (Cf Bush - Buchanan 1974).
- 23 For a sample of alternative principles cf Sen 1973, 77 pass.
- 24 Sen 1973, 1.
- 25 Cf Mincer 1976.
- 26 Stochastic models, eg.g. mover - stayer models, can be used to identify directly discrimination. Cf McCall 1971.
- 27 The structure of such a 'birth and death' process can be used to control the distribution of income. Cf Bartholomew 1973, 95 pass.

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