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Equilibrium Involuntary Unemployment under Oligempory

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Keywords

Involuntary unemployment, multi-stage game, imperfect competition

JEL Classifications

D43, E24

Comments
We thank Woojin Lee and seminar participants in Manchester, Milan, Prague, Vienna and Warwick for helpful comments.

Equilibrium Involuntary Unemployment under Oligempory *

Leo Kaas[†] Paul Madden[‡]

May 1999

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We show that equilibrium involuntary unemployment emerges in a multi–stage game model where all market power resides with firms, on both the labour and the output market. Firms decide wages, employment, output and prices, and under constant returns there exists a continuum of subgame perfect equilibria involving unemployment. A firm does not undercut the equilibrium wage since then high wage firms would attract its workers, thus forcing the low wage firm out of both markets. Full employment equilibria may also exist, but only the involuntary unemployment equilibria are robust to decreasing returns.

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Contents

1 Introduction

The purpose of this paper is to show that involuntary unemployment can occur as an equilibrium of a model in which all market power resides with wage and price setting firms. More specifically, we consider an oligempory[1] market structure in which the same set of firms are oligopolists in their homogeneous output market and oligopsonists in their homogeneous input (labour) market. An example of such a market structure would be a sector or an industry with a finite number of firms in which the production of the sector output requires labour input of non–unionised workers with a sector–specific skill. We study the strategic interaction amongst firms in such a model and show that it provides a new explanation for the emergence of involuntary unemployment.

Our result stands in contrast to the oligempory results of Stahl (1988, Proposition 2) and Jones and Manuelli (1992, Proposition 3) in which there is no rationing of input supply (or output demand) in equilibrium. Stahl considers a 2-stage game in which firms announce input prices at the first stage when input quantities are also allocated following the usual Bertrand rule ("winner–takes–all" at asymmetric input prices, equal shares at symmetric prices), and in which output prices are set at the second stage, which therefore becomes a capacity–constrained Bertrand game. If output demand is elastic, the unique subgame perfect equilibrium (SPE) is in fact Walrasian. However, the assumption of the usual Bertrand input allocation rule ignores the strategic potential for input quantity decisions to influence the resulting second stage prices (e.g. firms may not want to "take all", so as perhaps to force up the ensuing output price). Instead, our model has 3 stages, but still considers elastic demand. Input prices (=wages) are set at stage I, input quantities (=employment levels) and hence output quantities (via production functions) are determined at stage II which precedes the stage III capacity–constrained Bertrand output price game. The explicit modelling of the input (and so output) decisions at stage II implies that the mechanism leading to Stahl's Walrasian conclusion is absent from our model.[2] Firms may indeed want to raise wages from the Walrasian level and ration labour supply at stage II so as to increase the ultimate output price. This influence of employment/output decisions on output price is also absent from Jones and Manuelli (1992) whose multi–stage game has firms committing to both wages and prices at the first stage, prior to subsequent employment/output

[1] The terms monempory (a single trader) and oligempory (a few traders) were coined by Nichol (1943). An extensive study of oligempory from the industrial organisation viewpoint can be found in Dobson (1990).

[2] Stahl's model (1988, Proposition 3) may also generate a non–Walrasian conclusion under inelastic demand. If at equal input prices it is assumed that a "winner" is randomly selected, the winner will take all the supply (so there is still no rationing) but may retain some of this as unsold stock to force up output price at the second stage.

stages. As a result, in our model, the no–rationing equilibria of Stahl and Jones and Manuelli typically disappear. On the other hand, they are replaced by a continuum of involuntary unemployment equilibria at non–Walrasian wages and prices. These equilibria are supported by essentially a classical Bertrand argument - as a firm lowers its wage slightly, other firms expand their employment/output so as to force the low wage firm out of both markets.

Roberts (1987, 1989) also generates involuntary unemployment equilibria in a model in which firms commit to wages and prices at the first stage, before subsequent employment/output determination. The strategic influence of employment/output on prices which is central to our model is therefore again absent from Roberts's model and the Walrasian outcome remains an equilibrium. But now there is also a continuum of involuntary unemployment equilibria supported by a Keynesian coordination failure mechanism emanating from the assumption that labour supplies and output demands are signalled simultaneously to the firms. Jones and Manuelli (1992) show how these involuntary unemployment equilibria disappear without this simultaneity.

The literature on efficiency wages provides another explanation of involuntary unemployment equilibrium where firms rationally fail to lower wages (e.g. Weiss (1980) and Shapiro and Stiglitz (1984)). Firms refuse to reduce wages even though there are workers willing to work for less because wage cuts not only reduce costs, but also cause a drop in the productivity of labour.

Also relevant are the examples of Heal (1981) and Böhm et al. (1983) where price–setting agents impose binding quantity constraints on other agents in equilibrium, because of certain nonconvexities (Heal) or agent heterogeneities (Böhm et al.).[3] In our model such nonconvexities and heterogeneities are absent. Moreover, there are neither efficiency wage effects nor Roberts's coordination failures – it is essentially the classic Bertrand mechanism which sustains the involuntary unemployment equilibria.

In what follows, we focus on an example (constant returns, unit elastic demand, 2 firms, perfectly inelastic labour supply), and then investigate the robustness of the results for this example in various directions. In particular, we show that the results are robust to decreasing returns, unlike the well–known Bertrand paradox for a single market. Moreover, we also prove the robustness to a uniformly elastic demand and more than 2 firms. The perfectly inelastic labour supply assumption is maintained for simplicity reasons, but this relaxation would make no difference.

The paper is organized as follows. In the next section we introduce the 2 firm model

[3] Of course there are many models where the equilibrium actions of agents with market power on one side of a market lead to no rationing of the other side of the market but to implicit rationing of their own trades (e.g. workers with wage setting power may well produce involuntary unemployment equilibria); see Madden and Silvestre (1991, 1992) for a discussion of the relation between some of these models and fixprice equilibria.

and show that the duopsony special case in which firms have no output market power (i.e. in which output demand is perfectly elastic) generates the Walrasian equilibrium and also a continuum of involuntary unemployment equilibria when firms produce under constant returns, but that all the involuntary unemployment disappears when firms produce at decreasing returns. In Section 3 we consider the duempory (2 firm) model with a unit elastic demand and constant returns and show that it possesses again a continuum of involuntary unemployment equilibria, but no full employment equilibrium. In Section 4 we investigate the robustness of these results to variations of the model in Section 3 to (a) decreasing returns, (b) uniformly elastic demand, and (c) more than 2 firms.

2 The model

There are 2 profit–maximizing firms, a continuum of workers of mass 2 who each wish to supply inelastically 1 indivisible unit of labour to one of the firms, and a continuum of consumers who demand output according to an elastic inverse demand function $p = Y^{-\eta}$, $0 \leq \eta \leq 1$, where $Y > 0$ is aggregate output (the case $Y = 0$ will be discussed later). The 2 firms produce the homogeneous output from inputs of homogeneous labour under, for now, unit constant returns (so output equals labour input). The process of wage/price setting and employment/output determination is modelled as a 3 stage game, as follows.

At stage I, firms simultaneously announce money wages $w_1, w_2 \geq 0$. At stage II, if $w_i > w_j$ (throughout $i, j = 1, 2$, $i \neq j$), then all workers would like to work for the high wage firm i. Firms do not necessarily want to hire all workers willing to work for them at their announced wage. Therefore, we assume that firms at stage II announce simultaneously labour demands $J_1, J_2 \geq 0$, and that the actual employment levels (ℓ_1, ℓ_2) are

$$\begin{aligned} \ell_i &= \min(2, J_i) \quad , & (1) \\ \ell_j &= \min(2 - \ell_i, J_j) \quad . & (2) \end{aligned}$$

Here (1) assumes the high wage firm gets as much of the labour force as it wants, and (2) says that the low wage firm hires as much of the residual as it wants.

When $w_1 = w_2$ workers are indifferent between working at either firm, and, as usual in Bertrand models, we assume that the workers would allocate themselves symmetrically across firms if firms want to hire them, i.e.

$$\ell_1 = \ell_2 = 1 \quad \text{if} \quad J_1 \geq 1, \; J_2 \geq 1 \quad . \tag{3}$$

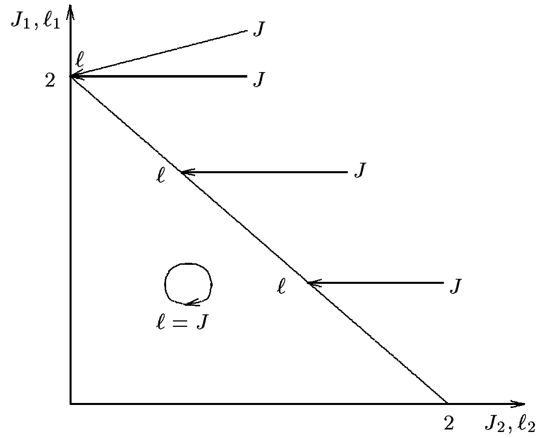
If $J_1 + J_2 \geq 2$ but $J_i < 1$, then we think of workers re–allocating from the now involuntary trade symmetric allocation to

$$\ell_i = J_i, \; \ell_j = 2 - \ell_i \quad \text{if} \quad J_i + J_j \geq 2, \; J_i < 1 \quad . \tag{4}$$

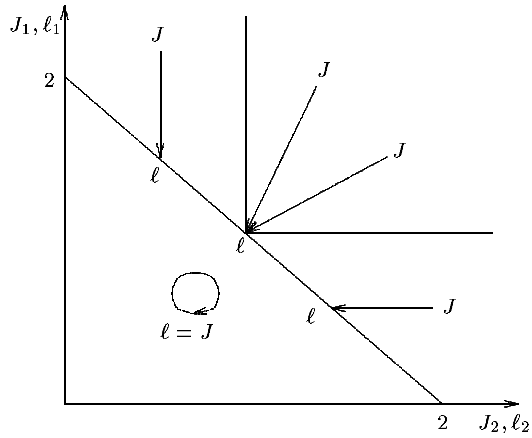
Finally, if $J_i + J_j < 2$, all full employment allocations are involuntary and we assume the re–allocation leads to

$$\ell_i = J_i,\ i = 1, 2 \quad \text{if} \quad J_1 + J_2 < 2 \quad . \tag{5}$$

Generally, (1) – (5) assume that workers re–allocate from their symmetric "Bertrand" labour supplies to the nearest efficient employment allocation consistent with the labour demands. Figure 1 illustrates the $J = (J_1, J_2) \mapsto \ell = (\ell_1, \ell_2)$ function defined by (1) – (5) which we denote in the sequel by $\ell_i = f(w_i, w_j, J_i, J_j)$.



(a) $w_1 > w_2$



(b) $w_1 = w_2$

Figure 1: The labour allocation ℓ as a function of labour demands J.

The employment levels (ℓ_1, ℓ_2) determined at stage II produce outputs (y_1, y_2) (= (ℓ_1, ℓ_2) under the constant returns assumption) which are then "taken to market" at stage III where firms simultaneously announce prices for the sale of (up to) these output levels. Stage III thus becomes a Bertrand–Edgeworth game with "capacities" (y_1, y_2). Because demand is uniformly elastic, there are unique Nash equilibrium payoffs to the stage III subgame which are those that emerge from the firms setting the market clearing price:

$$p_1 = p_2 = \frac{1}{Y^\eta} = \frac{1}{(y_1 + y_2)^\eta} \quad . \tag{6}$$

This result does not depend on the way consumers' demand is rationed at asymmetric prices (efficient, proportional etc.) and ensures that the Bertrand price setting produces an essentially Cournot outcome (see Madden (1998)).

Our 3 stage game is thus summarised as follows.

<u>Stage I</u> Firms announce (w_1, w_2) simultaneously.

Stage II Firms announce (J_1, J_2) simultaneously leading to employment/output levels $\ell_i = y_i = f(w_i, w_j, J_i, J_j)$, $i = 1, 2$.

Stage III Firms announce (p_1, p_2) simultaneously.

We are interested in subgame perfect equilibria (SPE) of this game. Given the stage III Nash equilibrium noted in (6), payoffs become, for $i = 1, 2$,

$$\pi_i(w_i, w_j, J_i, J_j) = \frac{y_i}{(y_1 + y_2)^\eta} - w_i \ell_i \quad \text{where} \quad y_i = \ell_i = f(w_i, w_j, J_i, J_j) \ . \tag{7}$$

Notice that payoff functions are only defined as long as $J_1 + J_2 > 0$ (implying $y_1 + y_2 > 0$). However, when $\eta < 1$, the payoff functions can be continuously extended to $\pi_i(w_i, w_j, 0, 0) = 0$, $i = 1, 2$. When $\eta = 1$, consumers' expenditures are constant and equal to one for all $y_1 + y_2 > 0$, and we assume that at $y_1 + y_2 = 0$ consumers share these expenditures equally between firms, so that $\pi_i(w_i, w_j, 0, 0) = 1/2$, $i = 1, 2$.

Before we analyze SPE of this game in the next sections, we will consider first the case of a Bertrand duopsony, i.e. we assume a perfectly competitive output market where $\eta = 0$ and so firms can supply as much as they wish with no effect on the market price $p = 1$. As in a Bertrand duopoly, it is immediate that in all pure strategy SPE of this (now 2 stage) game both firms announce the Walrasian wage $w_1 = w_2 = 1$ and make zero profit.[4] However, firms do not necessarily employ the whole labour force in these equilibria. In fact, all labour demands (J_1, J_2) signalled at stage II after Walrasian wages announced at stage I are SPE, supported by the following argument. If firm 1 undercuts the Walrasian wage set by firm 2, firm 2 can credibly employ all workers at stage II ($J_2 = 2$) and leave no workers for the undercutting firm 1. Thus, there exists a continuum of SPE involving involuntary unemployment, but these equilibria are not robust to decreasing returns. To see this, suppose that firms produce the output good with a Cobb-Douglas production function $y = \ell^\alpha$, $0 < \alpha < 1$. Again, as in a Bertrand duopoly, it will be shown below that this game possesses no pure strategy equilibrium, and, even more, that all mixed strategy equilibria involve full employment.

Proposition 1: The duopsony model ($\eta = 0$) with firms producing under constant returns ($\alpha = 1$) has a continuum of SPE with involuntary unemployment in which firms announce the Walrasian wage at stage I and signal arbitrary labour demands at stage II. The duopsony model with firms producing under decreasing returns ($\alpha < 1$) has no pure strategy SPE. Moreover, in all mixed strategy SPE there is full employment with probability 1.

[4]There may be also mixed strategy equilibria in which firms make positive expected profits, see Baye and Morgan (1997).

Proof: It remains to consider the case of decreasing returns. Suppose that wages w_1, w_2 have been announced at stage I and suppose that firms signal their Walrasian labour demands

$$J_1 = L(w_1) := \left(\frac{\alpha}{w_1}\right)^{1/(1-\alpha)} \quad , \; J_2 = L(w_2)$$

at stage II. Either the resulting labour allocation is feasible, i.e. $J_1 + J_2 \leq 2$, in which case no firm could be better off by signalling another labour demand, or $J_1 + J_2 > 2$ in which case at least one firm is rationed. Since the assumed rationing mechanism is nonmanipulable (see e.g. Benassy (1982) for a definition), a rationed firm can equally signal any other labour demand higher than its maximally achievable trade, but this has no effect on the labour allocation to both firms. Thus, payoffs induced in any stage II Nash equilibrium equal the payoffs resulting from both firms signalling their Walrasian labour demands.

It will be shown first that there exists no pure strategy equilibrium. Notice that $w = \alpha$ is the Walrasian wage at which each firm's labour demand equals one and at which firms make profit $1 - \alpha$. First, suppose firms announce symmetric wages $w_1 = w_2 = w < \alpha$, so both firms are rationed and make profit $1 - w$. But then each firm could increase its wage slightly and employ its Walrasian demand (which is strictly greater than 1), and make a strictly higher profit. Second, suppose firms announce symmetric wages $w_1 = w_2 = w \geq \alpha$ in which case both firms are not rationed. But if one firm undercuts this wage, it gets at least the same number of workers since the high wage firm will not attract more workers than before because of decreasing returns. Thus, undercutting is profitable. Third, suppose firms announce unequal wages. But then the high wage firm makes a strictly higher profit if it announces a slightly lower wage and employs the same number of people.

It remains to show that there is (almost surely) full employment in all mixed strategy equilibria. For achieving this, it suffices to show that the Walrasian wage dominates any higher wage, since then both firms announce wages higher than the Walrasian wage with probability zero in any best response, and so their labour demands exceed 1 with probability 1 in any SPE. If firm 1 announces a wage $w_1 \geq \alpha$, it gets its Walrasian labour demand $L(w_1)$ and makes a profit of $L(w_1)^\alpha - w_1 L(w_1)$, irrespective of the wage announced by firm 2 (if $w_2 < w_1$, firm 1 is the high wage firm getting as much as it wants; if $w_2 \geq w_1$, firm 2's labour demand is $L(w_2) < 1$, and so firm 1 gets its Walrasian labour demand $L(w_1) \leq 1$ at stage II). Since $L(w_1)^\alpha - w_1 L(w_1) < L(\alpha)^\alpha - \alpha L(\alpha)$ when $w_1 > \alpha$, the Walrasian wage dominates any higher wage. □

We will consider in the next section the duempory model with a unit elastic demand ($\eta = 1$) and show that, again under constant returns, there is a continuum of SPE with involuntary unemployment. However, there do not exist SPE with full employment.

3 The equilibria

Throughout this section we assume a unit elastic demand and constant returns.

To proceed with backward induction, consider the stage II game in J_1 and J_2 after wage announcements w_1 and w_2 at stage I. Notice first that this stage II game is continuous in J_1 and J_2 except at $J_1 = J_2 = 0$. But since $\pi_1 + \pi_2$ is continuous, and since each π_i, $i = 1, 2$, is weakly lower semi–continuous, Theorem 5 of Dasgupta and Maskin (1986) applies and guarantees the existence of a Nash equilibrium of the stage II game, at least in mixed strategies. In the following, we will focus on symmetric wage equilibria where we even find pure strategy Nash equilibrium continuations of the stage II game. We will also use only pure strategy Nash equilibrium continuations after all relevant unilateral wage deviations from symmetric wage situations.

Denote the best response correspondence of firm i in the stage II game by BR_i, i.e. $\mathrm{BR}_i(J_j) = \mathrm{argmax}_{J_i \geq 0}\, \pi_i(w_i, w_j, J_i, J_j)$, but consider for the moment the best response problem of firm i under the artificial assumption that there is no full employment constraint. Hence, consider the problem

$$\max_{\ell_i \geq 0} \hat{\pi}_i = \frac{\ell_i}{\ell_i + \ell_j} - w_i \ell_i \ . \tag{8}$$

$\hat{\pi}_i$ is strictly concave in ℓ_i, so the solution of (8) is characterized by the first order condition, and we denote this "unconstrained reaction function" by $\ell_i = \mathrm{URF}_i(\ell_j)$ which is

$$\mathrm{URF}_i(\ell_j) = \max\left(0, \sqrt{\frac{\ell_j}{w_i}} - \ell_j\right) \quad .$$

Suppose now that firms announce equal wages at stage I, $w_1 = w_2 = w$. Then the unique positive[5] intersection of URF_1 and URF_2 is

$$\ell_1 = \ell_2 = \frac{1}{4w} \quad \text{with} \quad \hat{\pi}_1 = \hat{\pi}_2 = \frac{1}{4} \ ,$$

which is feasible, i.e. satisfies the full employment constraint if $w \geq 1/4$ and exhibits involuntary unemployment if $w > 1/4$.

It is now straightforward to see that, when $w \geq 1/4$, $J_1 = J_2 = 1/(4w)$ is also a Nash equilibrium of the original game. Given $J_2 = 1/(4w)$, the full employment constraint is not binding for all $J_1 \leq 2 - 1/(4w)$, and therefore $J_1 = 1/(4w)$ is optimal amongst all $J_1 \in [0, 2 - 1/(4w)]$. By definition of the labour allocation map f, firm 1 cannot expand employment beyond $2 - 1/(4w)$ by signalling a higher labour demand, and therefore $J_1 = 1/(4w) \in \mathrm{BR}_1(J_2 = 1/(4w))$. Hence we have

[5] Notice that the other intersection $\ell_1 = \ell_2 = 0$ is not a Nash equilibrium of the stage II game.

Lemma 1: If $w_1 = w_2 = w > 1/4$, there is a Nash equilibrium of the stage II game with involuntary unemployment, where

$$J_1 = J_2 = \ell_1 = \ell_2 = \frac{1}{4w}$$

and with profits $\pi_1 = \pi_2 = 1/4$.

We now look at full employment equilibria of symmetric wage stage II subgames. Notice first that, when $w_1 = w_2 > 1/2$, the unconstrained best responses always fulfil the full employment constraint, i.e. $\mathrm{URF}_i(\ell_j) + \ell_j \leq 2$ for all $\ell_j \in [0, 2]$. Therefore, in this case $\mathrm{BR}_i = \mathrm{URF}_i$, and the only stage II Nash equilibrium is the unemployment equilibrium of Lemma 1. However, the following Lemma shows that in all other cases there exist stage II Nash equilibria with full employment.

Lemma 2: If $w_1 = w_2 = w \leq 1/2$, there is a Nash equilibrium of the stage II game with full employment where $J_1 = J_2 = 2$, $\ell_1 = \ell_2 = 1$, and with profits $\pi_1 = \pi_2 = 1/2 - w$.

Proof: When $J_2 = 2$, firm 1's best response problem becomes

$$\max_{J_1 \geq 0} \frac{\ell_1}{2} - w\ell_1 \quad \text{where} \quad \ell_1 = \min(1, J_1) \ ,$$

for which $J_1 = 2$ is a solution because of $w \leq 1/2$. Thus, $2 \in \mathrm{BR}_1(2) = \mathrm{BR}_2(2)$. □

The relation between the (symmetric) wage w and aggregate employment in the stage II Nash equilibria of Lemma 1 and 2 is illustrated in Figure 2.

Notice that $w = 1/2$, $\ell_1 = \ell_2 = 1$ is the unique Walrasian equilibrium of this economy, and notice also that for $1/4 < w \leq 1/2$ the full employment equilibria are Pareto inferior (for the firms) to the unemployment equilibria.

Now turn to the analysis of SPE of the full game. In particular, we address the question; which symmetric wages are announced in SPE of the above game and on which of the above stage II continuations firms coordinate?

To answer this question, consider first an unemployment equilibrium of Lemma 1 at wages $w_1 = w_2 = w > 1/4$, and suppose that firm 1 undercuts a little. Then firm 2 is the high wage firm, and consider the best response problem of firm 2, given a labour demand J_1 of firm 1. For $J_2 \in [0, 2 - J_1]$ the full employment constraint is not binding, and therefore aggregate output increases and the equilibrium price $p = 1/(y_1 + y_2)$ falls as J_2 is raised. However, for $J_2 \in [2 - J_1, 2]$, firm 2 attracts workers from firm 1 (so $\ell_2 = J_2$ and $\ell_1 = 2 - \ell_2$), but the aggregate output and the output price remain constant at $y_1 + y_2 = 2$ and $p = 1/2$. This "kinked demand"

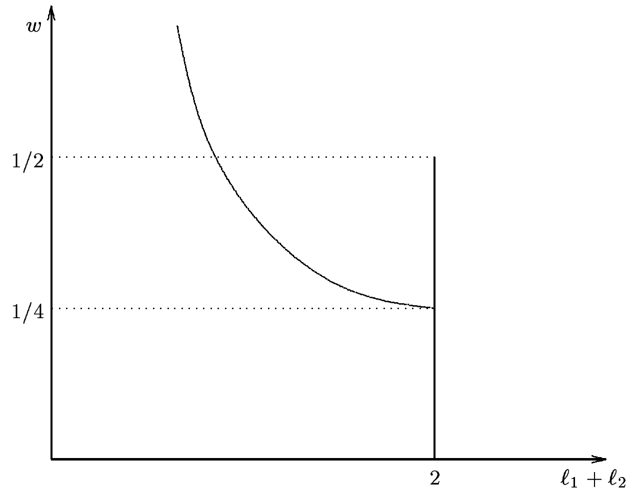


Figure 2: Aggregate employment in stage II equilibria after symmetric wages in stage I.

effect[6] leads to the following profit function of firm 2:

$$\pi_2 = \begin{cases} \dfrac{J_2}{J_1 + J_2} - wJ_2 & \text{if} \quad J_2 \leq 2 - J_1 \ , \\ \dfrac{J_2}{2} - wJ_2 & \text{if} \quad 2 - J_1 \leq J_2 \leq 2 \ , \\ 1 - 2w & \text{if} \quad J_2 \geq 2 \ . \end{cases} \tag{9}$$

π_2 is illustrated in Figure 3 when $w < 1/2$.

Therefore, it may become possible that firm 2 wants to expand employment to 2. In fact, it turns out that $J_1 = J_2 = 2$ is now a stage II Nash equilibrium. $J_2 = 2$ is a best response to $J_1 = 2$ when $w \leq 1/2$, and $J_1 = 2$ is a best response to $J_2 = 2$ since firm 1 cannot attract any workers no matter how much labour it demands. In particular, firm 1 makes zero profit and undercutting is unprofitable. Thus, we have

Lemma 3: If $w_2 = w \in (1/4, 1/2]$ and $w_1 < w$, there exists a stage II Nash equilibrium with $J_1 = J_2 = 2$, $\ell_1 = 0$, $\ell_2 = 2$, and profits $\pi_1 = 0$, $\pi_2 = 1 - 2w$. Thus, undercutting from (w, w) where $w \in (1/4, 1/2]$ is unprofitable.

Consider next an upward deviation of firm 1, $w_1 > w_2 = w \in (1/4, 1/2]$. The unique

[6]Note that the cause of the kinks is quite different from Benassy (1989) where raising of a firm's price increases demand signalled to rival firms producing differentiated, substitute goods. At some stage, rivals will cease to want to meet the increase in demand, kinking upwards the original firm's profit function at this point.

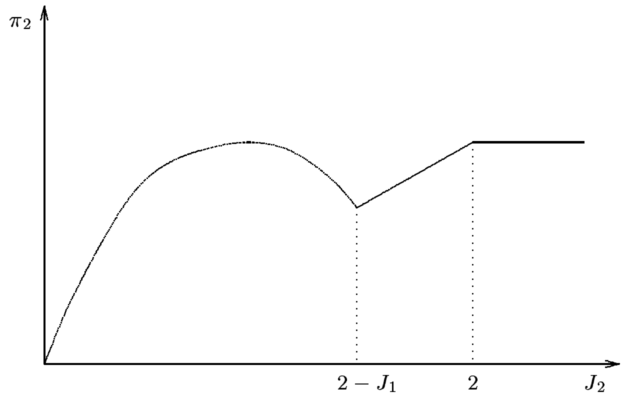


Figure 3: The profit function of the high wage firm 2.

positive intersection of URF_1 and URF_2 is given by

$$\ell_1 = \frac{w_2}{(w_1 + w_2)^2} \ , \ \ell_2 = \frac{w_1}{(w_1 + w_2)^2} \ ,$$

which satisfies $\ell_1 + \ell_2 < 2$ and results in profits

$$\pi_1 = \frac{w_2^2}{(w_1 + w_2)^2} \ , \ \pi_2 = \frac{w_1^2}{(w_1 + w_2)^2} \ .$$

It will now be shown that $J_i = \ell_i$, $i = 1, 2$, is indeed a stage II Nash equilibrium, provided that $w \geq 3/8$. Clearly, $J_2 = \ell_2$ is a best response to $J_1 = \ell_1$, since the low wage firm 2 can only expand employment up to $2 - J_1$, but not any further. Also $J_1 = \ell_1$ is a best response to $J_2 = \ell_2$ amongst all $J_1 \in [0, 2 - J_2]$. If firm 1 signals a higher labour demand, it can attract workers from firm 2 and receive a profit of

$$\pi_1 = \frac{J_1}{2} - w_1 J_1 \ , \quad 2 - J_2 \leq J_1 \leq 2 \ ,$$

which is non–decreasing in J_1 when $w_1 \leq 1/2$, and attains its maximum $\bar{\pi}_1 = 1 - 2w_1$ at $J_1 = 2$. $\bar{\pi}_1$ is lower than firm 1's profit at $J_1 = \ell_1$ if

$$1 < 2w_1 + \frac{w^2}{(w_1 + w)^2} \quad . \tag{10}$$

The r.h.s. of (10) is increasing in w_1 and is therefore greater than $2w + 1/4$ for all $w_1 > w$. Hence, (10) is fulfilled when $w \geq 3/8$, and this implies that $J_i = \ell_i$, $i = 1, 2$, is a stage II Nash equilibrium. In this equilibrium, firm 1 gets the profit $w^2/(w_1 + w)^2$ which is less than its profit at the involuntary unemployment stage II equilibrium at $w_1 = w_2 = w$. Hence, we have

Lemma 4: If $w_2 = w \in [3/8, 1/2]$ and $w_1 > w$, there exists a stage II Nash equilibrium with involuntary unemployment where

$$J_i = \ell_i = \frac{w_j}{(w_1 + w_2)^2} \quad ,$$

and with profits

$$\pi_i = \frac{w_j^2}{(w_1 + w_2)^2} \quad ,$$

$i = 1, 2,\ j \neq i$. Therefore, upward wage deviations are unprofitable.

Lemma 3 and 4 provide unprofitable stage II equilibrium continuations for upward and downward wage deviations from an unemployment stage II equilibrium at wages $w_1 = w_2 = w \in [3/8, 1/2]$. For all other wages $w_1 \neq w \neq w_2$, the existence of stage II Nash equilibria (at least in mixed strategies) follows from Dasgupta and Maskin (1986). These findings prove the following theorem.

Theorem 1: For the duempory model with unit elastic demand ($\eta = 1$) and constant returns ($\alpha = 1$), there exists a continuum of SPE with involuntary unemployment with wages $w_1 = w_2 = w \in [3/8, 1/2]$, employment levels $\ell_1 = \ell_2 = 1/(4w)$, and with profits $\pi_1 = \pi_2 = 1/4$, so aggregate employment is between 1 and 4/3.

Consider now the full employment stage II Nash equilibria of Lemma 2. It will be shown that none of them are SPE of the full game. Suppose first $w_1 = w_2 = w < 1/2$ and that firms coordinate in stage II on a full employment equilibrium with profits $\pi_1 = \pi_2 = 1/2 - w$. Now if firm 1 deviates upwards, $w_1 > w$, it can employ all workers and receive a profit of $1 - 2w_1$, irrespective of the employment decision of firm 2. Therefore, in each stage II continuation following $w_1 > w_2$ firm 1 receives an (expected) profit of at least $1 - 2w_1$. If w_1 is close to w, $1 - 2w_1 > 1/2 - w$ because of $w < 1/2$, and so an upward deviation must be profitable.[7]

Suppose second $w_1 = w_2 = 1/2$ and that firms coordinate in stage II on the Walrasian full employment equilibrium with zero profits. If firm 1 deviates upwards, $1/2 < w_1 < 3/2$, any stage II continuation gives firm 1 a positive (expected) profit, as the following Lemma shows.

Lemma 5: Suppose $1/2 = w_2 < w_1 < 3/2$. Then in each stage II Nash equilibrium, firm 1 makes a positive (expected) profit. Thus, an upward wage deviation from the Walrasian equilibrium wage is profitable.

[7] The same argument excludes wage announcements $w_1 = w_2 = w < 3/8$ followed by involuntary unemployment continuations as SPE, since $1 - 2w_1 > 1/4$.

Proof: Firm 1's profit function is

$$\pi_1 = \begin{cases} \dfrac{J_1}{J_1 + J_2} - w_1 J_1 & \text{if } \ J_1 \le 2 - J_2 \ , \\ J_1/2 - w_1 J_1 & \text{if } \ 2 - J_2 \le J_1 \le 2 \ , \\ 1 - 2w_1 & \text{if } \ 2 \le J_1 \ , \end{cases} \tag{11}$$

which is decreasing in J_1 when $J_1 > \mathrm{URF}_1(J_2)$. $\mathrm{URF}_1(.)$ is maximal at $J_2 = 1/(4w_1)$, so π_1 is decreasing in J_1 when $J_1 > \mathrm{URF}_1(1/(4w_1)) = 1/(4w_1)$ for all $J_2 \ge 0$. This implies that the support of each best response of firm 1 is contained in $[0, 1/(4w_1)]$. Firm 2's profit function is decreasing in J_2 when $J_2 > \mathrm{URF}_2(J_1)$. $\mathrm{URF}_2(.)$ is maximal at $J_1 = 1/(4w_2) = 1/2$, so π_2 is decreasing in J_2 when $J_2 > \mathrm{URF}_2(1/2) = 1/2$ for all $J_1 \ge 0$. Therefore, the support of each best response of firm 2 is contained in $[0, 1/2]$. But now for all

$$(J_1, J_2) \in [0, 1/(4w_1)] \times [0, 1/2] \ ,$$

$J_1 + J_2 < 1$ and (11) implies that firm 1 makes a positive (expected) profit since $J_1 + J_2 \le 1/(4w_1) + 1/2 < 1/w_1$ under the assumption $w_1 < 3/2$. □

Theorem 2: For the duempory model with unit elastic demand ($\eta = 1$) and constant returns ($\alpha = 1$), there exists no symmetric SPE with full employment.

4 Robustness

In this section we investigate how far the results of Theorem 1 and 2 are robust to changes of the model specification. For this purpose we consider unilateral variations of the basic model of the previous section to decreasing returns ($\alpha < 1$) in 4.1, to an uniformly elastic demand ($\eta \in (0,1)$) in 4.2 and to an oligempory of $n > 2$ firms in 4.3. Proofs of all Theorems in this section can be found in the Appendix.

4.1 Decreasing returns

The production function of firm $i = 1, 2$ is now $y_i = \ell_i^\alpha$, $0 < \alpha < 1$. (8) becomes

$$\max_{\ell_i \ge 0} \hat{\pi}_i = \frac{\ell_i^\alpha}{\ell_i^\alpha + \ell_j^\alpha} - w_i \ell_i \ .$$

$\hat{\pi}_i$ is strictly concave in ℓ_i and it is straightforward to define URF_i and to show the analogue of Lemma 1 (now $w_1 = w_2 = w > \alpha/4$ for an involuntary unemployment Nash equilibrium of the stage II game). The difficulty in generalising Theorem 1 is that π_2 (see equation (9) and Figure 3) is now non–linear on $[2-J_1, 2]$ which creates problems for Lemma 3 and 4. However, it turns out that π_2 is convex on $[2-J_1, 2]$ when $J_1 \leq 1$ and a version of Lemma 3 (with $w_2 = w \in (\alpha/4, \alpha/2]$) and Lemma 4 (with $w_2 = w \in [1/4 + \alpha/8, \alpha/2]$)) leads to:

Theorem 3: For the duempory model with unit elastic demand ($\eta = 1$) and decreasing returns ($2/3 < \alpha < 1$), there exists a continuum of SPE with involuntary unemployment with wages $w_1 = w_2 = w \in [1/4 + \alpha/8, \alpha/2]$, employment levels $\ell_1 = \ell_2 = \alpha/(4w)$, and with profits $\pi_1 = \pi_2 = 1/2 - \alpha/4$, so aggregate employment is between 1 and $4\alpha/(2+\alpha)$.

It is also possible to generalize exactly Theorem 2:

Theorem 4: For the duempory model with unit elastic demand ($\eta = 1$) and decreasing returns ($\alpha < 1$), there exists no symmetric SPE with full employment.

4.2 Uniformly elastic demand

With the inverse demand function $p = Y^{-\eta}$, $0 < \eta < 1$, (8) becomes

$$\max_{\ell_i \geq 0} \hat{\pi}_i = \frac{\ell_i}{(\ell_i + \ell_j)^\eta} - w_i \ell_i \ .$$

$\hat{\pi}_i$ is again strictly concave in ℓ_i and one can show immediately that there is a unique positive intersection of the URF$_i$ which gives the analogue of Lemma 1 (now $w_1 = w_2 = w > (2-\eta)/2^{1+\eta}$ for an involuntary unemployment Nash equilibrium of the stage II game). The analogue of π_2 as defined in equation (9) is now again linear on $[2-J_1, 2]$, and so the generalisation of Lemma 3 is straightforward (with $w_2 = w \in ((2-\eta/2^{1+\eta}, 2^{-\eta}])$. To extend Lemma 4, one has to show that upward wage deviations lead to an unprofitable involuntary unemployment equilibrium. This requires that the wage is not too low since otherwise the deviating firm would want to take up the whole labour force. In Lemma 4 this critical wage was 3/8, but it is now implicitly defined by the equation

$$2^{1-\eta} = 2w + \frac{\eta(2-\eta)^{1/\eta - 1}}{2^{1+1/\eta}} w^{1-1/\eta} \quad , \tag{12}$$

whose unique solution in the interval $((2-\eta)/2^{1+\eta}, 2^{-\eta})$ is denoted $w^*(\eta)$. This leads to the following generalization of Theorem 1:

Theorem 5: For the duempory model with uniformly elastic demand ($0 < \eta < 1$) and constant returns, there exists a continuum of SPE with involuntary unemployment with wages $w_1 = w_2 = w \in [w^*(\eta), 2^{-\eta}]$, employment levels $\ell_1 = \ell_2 = (2-\eta)/(w2^{1+\eta})^{1/\eta}$, and with profits $\pi_1 = \pi_2 = \eta(2-\eta)^{1/\eta-1}/(2^{1/\eta+1}w^{1/\eta-1})$.

In the limit of a perfectly competitive output market ($\eta \to 0$), both $w^*(\eta)$ and $2^{-\eta}$ converge to $w = 1$, i.e. to the outcome of the duopsony model ($\eta = 0$). However, while the employment level is completely indeterminate in the duopsony model (Proposition 1), it turns out that in the limit $\eta \to 0$ a positive employment level as well as a positive unemployment level persist. In fact, the limit of the lowest aggregate employment level at the wage $w = 2^{-\eta}$ is $2e^{-1/2} \approx 1.213$. The limit of the highest aggregate employment level at $w^*(\eta)$ turns out to be the unique solution of the equation $8(\ln(\bar{L}/2)) = \bar{L} - 4$ in the interval $(0, 2)$ which is approximately $\bar{L} \approx 1.456$ (a proof can be found in the appendix).

Corollary 1: In the limit of a perfectly competitive output market ($\eta \to 0$), the interval of aggregate employment levels in these SPE converges to the interval $[2e^{-1/2}, \bar{L}]$ with $\bar{L} \approx 1.456$.

Parallel to Theorem 2 we also have:

Theorem 6: For the duempory model with uniformly elastic demand ($0 < \eta < 1$) and constant returns, there exists no symmetric SPE with full employment.

4.3 Oligempory

To extend the model to $n > 2$ firms, the labour allocation function $J = (J_1, \ldots, J_n) \mapsto \ell = (\ell_1, \ldots, \ell_n)$ has to be generalized. We assume again that workers (a continuum of mass 2) allocate themselves symmetrically across those firms who announce the highest wage. If this symmetric allocation is not voluntary for the firms, workers re–allocate to the nearest efficient and symmetric labour allocation consistent with the labour demands. When not all workers are employed by the highest wage firms, workers supply labour to the firms setting the second highest wage and are allocated according to the same procedure.

More precisely, suppose first that $w_1 = w_2 = \ldots = w_n = w$ and w.l.o.g. $J_1 \geq J_2 \geq \ldots \geq J_n$. When $J_n \geq 2/n$, the symmetric allocation is voluntary and then

$$\ell_i = \frac{2}{n}\ , \ i = 1, \ldots, n, \quad \text{if} \quad J_i \geq \frac{2}{n}\ \forall\ i = 1, \ldots, n. \tag{13}$$

When $J_n < 2/n$, $\ell_n = 2/n$, and the remaining $2 - J_n$ workers allocate their labour supplies symmetrically to the remaining firms $i = 1, \ldots, n-1$. When $J_{n-1} \geq (2 - J_n)/(n-1)$, this symmetric allocation is voluntary and so $\ell_i = (2 - J_n)/(n-1)$

for all $i = 1, \ldots, n-1$. When $J_{n-1} < (2 - J_n)/(n-1)$, $\ell_{n-1} = J_{n-1}$ and the remaining $2 - J_n - J_{n-1}$ workers have to be allocated amongst the remaining $n-2$ firms. Generally,

$$\begin{aligned} &\ell_i = J_i \ , \ i = n, \ldots, n-k, \\ &\ell_i = \frac{2 - J_n - \ldots - J_{n-k}}{n-k-1} \ , \ i = n-k-1, \ldots, 1, \\ &\text{if for some } k \leq n-2, \ J_n < \frac{2}{n}, \ldots, J_{n-k} < \frac{2 - J_n - \ldots - J_{n-k+1}}{n-k} \\ &\text{and } J_{n-k-1} \geq \frac{2 - J_n - \ldots - J_{n-k}}{n-k-1} \ . \end{aligned} \tag{14}$$

Finally,

$$\ell_i = J_i \ , \ i = 1, \ldots, n, \quad \text{if} \quad \sum_{i=1}^{n} J_i < 2 \ . \tag{15}$$

If not all firms announce the same wage, workers are allocated to the firms setting the highest wages, according to the symmetric wage procedure (13), (14), (15). In case (15), the remaining workers are allocated to the firms setting the second highest wage according to the same procedure, and so on.

It is straightforward to show that there is a unique intersection of the URF$_i$ when firms announce equal wages at stage I, and so Lemma 1 generalizes immediately (now $w_1 = \ldots = w_n = w > (n-1)/(2n)$ for an involuntary unemployment stage II equilibrium). Undercutting turns out to be unprofitable, since then a stage II subgame equilibrium emerges in which all other firms employ the workers of the undercutting firm (now $(n-1)/(2n) < w \leq 1/2$). Upward wage deviations can be shown to lead to an involuntary unemployment stage II equilibrium, provided that the wage is not too low since otherwise the deviating firm would want to take up the whole labour force (i.e., $w \geq (1 - 1/n^2)/2$). Thus we obtain the following generalisation of Theorem 1:

Theorem 7: For the oligempory model ($n \geq 2$ firms) with unit elastic demand and constant returns, there exists a continuum of SPE with involuntary unemployment with wages $w_1 = \ldots = w_n = w \in [(1 - 1/n^2)/2, 1/2]$, employment levels $\ell_1 = \ldots = \ell_n = (n-1)/(n^2 w)$, and with profits $\pi_1 = \ldots = \pi_n = 1/n^2$.

Corollary 2: In the limit $n \to \infty$, all equilibrium wages converge to the Walrasian wage $w = 1/2$ and aggregate employment levels $\ell_1 + \ldots + \ell_n = (n-1)/(nw)$ converge to 2.

Unlike Theorem 1, Theorem 2 does not generalize to $n \geq 3$ firms. In fact, it turns out that there is a SPE in which all firms $i = 1, \ldots, n$ announce the Walrasian wage $w_i = 1/2$, signal labour demands $J_i = 2$, employ $\ell_i = 2/n$ workers and make zero profit. Undercutting is not profitable by the same argument as in the proof of Theorem 7 (see Lemma A.9 below). Announcing a higher wage $w_1 > w_2 = \ldots = w_n = 1/2$ would be profitable if it was followed by a stage II continuation with involuntary unemployment as in the proof of Theorem 7. However, there exists now another continuation in which firm 1 makes zero profit. This continuation is $J_1 = 0$ and $J_i = 2$, $i > 1$. $J_1 = 0$ is clearly a best response to $J_i = 2$, $i > 1$, since firm 1 makes negative profit for all $J_1 > 0$. For firm $i > 1$, $J_i = 2$ is a best response to $J_1 = 0$, $J_j = 2$, $j \neq 1, i$, since firm i makes zero profit for all J_i. Thus, Lemma 5 fails since there are now at least two firms $i > 1$ announcing $w = 1/2$ who can coordinate on a full employment stage II continuation, giving thereby the deviating firm 1 zero profit.

This full employment SPE is Pareto inferior to all the involuntary unemployment SPE of Theorem 7, and there are no other symmetric SPE with full employment. Moreover, this equilibrium is not robust to the change to decreasing returns.

Theorem 8: The oligempory model with $n \geq 3$ firms, unit elastic demand and constant returns has a unique symmetric full employment SPE in which $w_1 = \ldots = w_n = 1/2$, $\ell_1 = \ldots = \ell_n = 2/n$, and with profits $\pi_1 = \ldots = \pi_n = 0$. The oligempory model with unit elastic demand and decreasing returns ($\alpha < 1$) has no symmetric SPE with full employment.

5 Conclusions

The paper has shown how involuntary unemployment emerges as the typical subgame perfect equilibrium outcome of a 3 stage game in which all market power resides with firms (oligemporists) who set wages, then determine employment and output levels and finally set prices for the sale of the resulting output. This is quite different from the "no rationing" equilibria of the multi–stage game models of Stahl (1988) and Jones and Manuelli (1992) who exclude any strategic use of employment levels by firms to influence the output price. Full employment (i.e. "no rationing") equilibria typically disappear from our model since firms may raise the wage, ration the resulting labour supply and force up the output price. On the other hand involuntary unemployment equilibria emerge since, in particular, firms do not wish to undercut the wage as other firms would then increase their employment levels to squeeze the low wage firm out of both markets. This classical Bertrand mechanism emanates from a kinked demand effect. If a firm undercuts the wage, expansion of employment by rival firms will increase aggregate employment and reduce the output price whilst there remains involuntary unemployment; but once full employment is created, the rival firms now take labour from the low wage deviant, leaving aggregate employment unchanged, and the ensuing output price "kinks" upwards. In equilibrium the kink is enough to sustain the classical Bertrand response by the rival firms to the low wage deviation.

Our exposition has shown the existence of a continuum of involuntary unemployment equilibria and the non–existence of a full employment equilibrium first in a special case with constant returns, unit elastic demand and 2 firms. Unlike the well–known Bertrand paradox, these results are robust to decreasing returns, and the typically involuntary unemployment nature of equilibria also remains under either elastic demand or with n firms. We claim therefore that the strategic use of employment levels to influence output price by powerful firms who have precommitted to wages (but not prices), provides a quite new mechanism for the generation of involuntary unemployment as an equilibrium phenomenon.

Appendix

Proof of Theorem 3:

URF$_i$ is defined by the first order condition

$$\frac{\partial \hat{\pi}_i}{\partial \ell_i} = \frac{\alpha}{\ell_i} \cdot \frac{\ell_1^\alpha \ell_2^\alpha}{(\ell_1^\alpha + \ell_2^\alpha)^2} - w_i = 0 \quad . \tag{16}$$

If $w_1 = w_2 = w$ there is a unique positive intersection of URF$_1$ and URF$_2$ with

$$\ell_1 = \ell_2 = \frac{\alpha}{4w} \ , \ \hat{\pi}_1 = \hat{\pi}_2 = \frac{1}{2} - \frac{\alpha}{4} \quad ,$$

which is feasible if $w \geq \alpha/4$ and exhibits involuntary unemployment if $w > \alpha/4$. Exactly as with Lemma 1 we have

Lemma A.1: If $w_1 = w_2 = w > \alpha/4$, there is a Nash equilibrium of the stage II game with involuntary unemployment, where

$$J_1 = J_2 = \ell_1 = \ell_2 = \frac{\alpha}{4w} \ , \ y_1 = y_2 = \left(\frac{\alpha}{4w}\right)^\alpha$$

and with profits $\pi_1 = \pi_2 = 1/2 - \alpha/4$.

Consider now the symmetric wages of Lemma A.1 as a candidate SPE of the whole game, and consider first the case where firm 1 undercuts to $w_1 < w_2 = w$. Under constant returns we found that, in some cases, $J_1 = J_2 = 2$ was then a stage II Nash equilibrium leading to $\pi_1 = 0$ and so no incentive to undercut (Lemma 3).

The analogous argument is now as follows. Suppose $J_1 = 2$. Then the best response problem of firm 2 is

$$\max_{J_2 \geq 0} \pi_2 = \begin{cases} \dfrac{J_2^\alpha}{(2-J_2)^\alpha + J_2^\alpha} - w_2 J_2 & \text{if} \quad J_2 \leq 2 \ , \\ 1 - 2w_2 & \text{if} \quad J_2 > 2 \ . \end{cases}$$

Define $g : [0,2] \to \mathbb{R}$ by

$$g(J_2) = \frac{J_2^\alpha}{(2-J_2)^\alpha + J_2^\alpha} - w_2 J_2 \quad . \tag{17}$$

Under constant returns this was linear and increasing when $w_2 \leq 1/2$. We now have:

Lemma A.2:
(a) g is concave on $[0,1]$ and convex on $[1,2]$.
(b) $g'(1) = \alpha/2 - w_2$ and so g is increasing on $[0,2]$ if $w_2 \leq \alpha/2$.

Proof:
(a)

$$g'(J_2) = \frac{\alpha J_2^{\alpha-1}(2-J_2)^\alpha + \alpha J_2^\alpha (2-J_2)^{\alpha-1}}{(J_2^\alpha + (2-J_2)^\alpha)^2} - w_2 \tag{18}$$

Differentiating again, we find that $g''(J_2)$ has the sign of $A + B$ where

$$\begin{aligned} A &= \alpha(\alpha-1)J_2^{\alpha-2}(2-J_2)^{\alpha-2}((2-J_2)^2 - J_2^2)(J_2^\alpha + (2-J_2)^\alpha) \ , \\ B &= -4\alpha^2 J_2^{\alpha-1}(2-J_2)^{\alpha-1}(J_2^{\alpha-1} - (2-J_2)^{\alpha-1}) \ . \end{aligned}$$

Inspection reveals that, since $\alpha < 1$, $A < 0$ and $B < 0$ if $J_2 < 1$, $A = B = 0$ if $J_2 = 1$, and $A > 0$ and $B > 0$ if $J_2 > 1$.

(b) follows immediately from (18) inserting $J_2 = 1$. □

Lemma A.2 ensures that $J_2 = 2$ is a best response to $J_1 = 2$ if $w_1 < w_2 = w \leq \alpha/2$. Moreover, if $J_2 = 2$, the low wage firm 1 will get zero profits for all J_1, so $J_1 = 2$ is a best response to $J_2 = 2$. Hence, as in Lemma 3,

Lemma A.3: If $w_2 = w \in (\alpha/4, \alpha/2]$ and $w_1 < w$, there exists a stage II Nash equilibrium with $J_1 = J_2 = 2$, $\ell_1 = y_1 = 0$, $\ell_2 = 2$, $y_2 = 2^\alpha$ and profits $\pi_1 = 0$, $\pi_2 = 1 - 2w$. Thus, undercutting from (w, w) where $w \in (\alpha/4, \alpha/2]$ is unprofitable.

Consider now upward deviations by firm 1 from (w, w), $w \in (\alpha/4, \alpha/2]$, to $w_1 > w$. At (w, w) we know there is an intersection of URF_1 and URF_2 with involuntary unemployment and profits $\pi_1 = \pi_2 = 1/2 - \alpha/4$. We now show:

Lemma A.4: If $w_1 > w_2 = w \in (\alpha/4, \alpha/2]$, there is an intersection of URF_1 and URF_2 with involuntary unemployment and with $\pi_1 < 1/2 - \alpha/4$.

Proof:

It follows from (16) that any intersection (ℓ_1, ℓ_2) of URF_1 and URF_2 at arbitrary wages (w_1, w_2) satisfies $w_1\ell_1 = w_2\ell_2$. Substituting back into (16) gives the unique positive intersection

$$\ell_i = \frac{\alpha w_1^\alpha w_2^\alpha}{w_i(w_1^\alpha + w_2^\alpha)^2} \ , \ i = 1, 2 \ . \tag{19}$$

It is straightforward to check that

(i) $\partial\ell_1/\partial w_1$ has the sign of

$$w_1^\alpha((\alpha - 3)w_1^\alpha + (\alpha - 1)w_2^\alpha) < 0 \text{ since } \alpha < 1 \ .$$

(ii) $\partial\ell_2/\partial w_1$ has the sign of

$$w_1^\alpha((\alpha - 2)w_1^\alpha + \alpha w_2^\alpha) < 0 \text{ since } \alpha < 1 \text{ and } w_1 > w_2 \ .$$

Thus at any wages (w_1, w) in the statement of the Lemma, there is a positive intersection of URF_1 and URF_2 with involuntary unemployment since there is such an intersection at (w, w).

Using (19) it is also straightforward to check that firm 1's profits at this intersection are

$$\pi_1 = w_2^\alpha \frac{((1 - \alpha)w_1^\alpha + w_2^\alpha)}{(w_1^\alpha + w_2^\alpha)^2} \ ,$$

and so $\partial\pi_1/\partial w_1$ has the sign of

$$\alpha(\alpha - 1)w_1^{\alpha-1}(w_1^\alpha + w_2^\alpha) < 0 \quad \text{since} \quad \alpha < 1 \ .$$

So firm 1's profit at the intersection of URF_1 and URF_2 with (w_1, w) is less than the corresponding profit $1/2 - \alpha/4$ with (w, w). □

Now consider the intersection in Lemma A.4, denoted (ℓ_1, ℓ_2) with profits (π_1, π_2), as a candidate stage II Nash equilibrium following (w_1, w) where $w_1 > w \in (\alpha/4, \alpha/2]$. Since 2 is the low wage firm, $J_2 = \ell_2$ is a best response to $J_1 = \ell_1$. If $J_2 = \ell_2$, there is no benefit for firm 1 in deviating from $J_1 = \ell_1$ to any $J_1 \in [0, 2 - \ell_2]$. For $J_1 \in (2 - \ell_2, 2]$, firm 1's profit is $g(J_1)$ (where g is as defined in (17) with (w_1, J_1) replacing (w_2, J_2)). Since $\ell_2 < 1$, $2 - \ell_2 > 1$ and g is convex on $(2 - \ell_2, 2]$ from Lemma A.2 (a). Hence,

(a) $J_1 = \ell_1$ is a best response to $J_2 = \ell_2$ if $\pi_1 \geq g(2) = 1 - 2w_1$.

(b) $J_1 = 2$ is a best response to $J_2 = \ell_2$ if $\pi_1 \leq 1 - 2w_1$.

In case (a), $(J_1, J_2) = (\ell_1, \ell_2)$ is indeed a stage II Nash equilibrium after (w_1, w) giving firm 1 profits $\pi_1 < 1/2 - \alpha/4$ (from Lemma A.4). In case (b), with $J_1 = 2$, the low wage firm 2 gets zero profits for all $J_2 \geq 0$, so $J_2 = \ell_2$ is a best response. Thus $(J_1, J_2) = (2, \ell_2)$ is now a stage II Nash equilibrium after (w_1, w) giving firm 1 profits $1-2w_1 < 1-2w$ (since $w_1 > w$) and $1-2w \leq 1/2-\alpha/4$ when $w \geq 1/4+\alpha/8$ $(> \alpha/4)$. Thus, if $w \in [1/4+\alpha/8, \alpha/2]$, there is in case (b) also a stage II Nash equilibrium after (w_1, w) in which firm 1's profits are less than $1/2-\alpha/4$. The interval $[1/4+\alpha/8, \alpha/2]$ has a nonempty interior if $\alpha > 2/3$, which completes the proof of Theorem 3. □

Proof of Theorem 4:

Suppose $w_1 = w_2 = w$ is a full employment SPE followed by $J_1 = J_2 \geq 1$ in stage II. Then, $\ell_1 = \ell_2 = 1$ and $\pi_1 = \pi_2 = 1/2 - w$. It is immediately clear that $w \leq 1/2$ – otherwise both firms would make negative profits and would be better off by signalling zero labour demand. Suppose first $w < 1/2$ and that firm 1 deviates at stage I to $w_1 > w$. By naming $J_1 = 2$ in the ensuing stage II game, firm 1 can guarantee a payoff of $1 - 2w_1$ irrespective of J_2, and $1 - 2w_1 > 1/2 - w$ if $w_1 > w$ is sufficiently close to w. Thus in any (pure or mixed strategy) Nash equilibrium of the stage II game following a small upward deviation from (w, w), firm 1's (expected) profit increases. Hence there can be no full employment SPE with $w < 1/2$.

Finally, suppose $w = 1/2$, in which case both firms make zero profit in the conjectured full employment SPE. But in the stage II game after $(1/2, 1/2)$, $J_1 \geq 1$ cannot be a best response to $J_2 \geq 1$. Reducing J_1 to 1 has no effect on the output price or π_1. But, if $J_2 > 1$, reducing J_1 to lie in $(2 - J_2, 1)$ leaves full employment but increases the output price and gives profit

$$\pi_1 = \frac{J_1^\alpha}{J_1^\alpha + (2 - J_1)^\alpha} - \frac{1}{2}J_1 \geq \frac{J_1^\alpha}{2} - \frac{1}{2}J_1 \ .$$

If $J_2 = 1$, reducing J_1 to lie in $(0, 1)$ produces unemployment, again increasing the output price and giving profit

$$\pi_1 = \frac{J_1^\alpha}{J_1^\alpha + 1} - \frac{1}{2}J_1 \geq \frac{J_1^\alpha}{2} - \frac{1}{2}J_1$$

again. Now $J_1^\alpha/2 - J_1/2 = 0$ at $J_1 = 1$ but is always decreasing there. Thus, reducing J_1 to just below 1 always produces positive profits for firm 1, and $w = 1/2$ cannot be a full employment SPE. □

Proof of Theorem 5:

The first order condition of the unconstrained best response problem $\partial\hat{\pi}_i/\partial\ell_i = 0$ is

$$\ell_i(1 - \eta) + \ell_j = w_i(\ell_i + \ell_j)^{1+\eta} \quad ,$$

which has a unique solution $\ell_i \geq -\ell_j$ denoted $\ell_i = \Phi(w_i, \ell_j)$ and hence URF_i is $\ell_i = \max(0, \Phi(w_i, \ell_j))$.

If $w_1 = w_2 = w$, the unique positive intersection of URF_1 and URF_2 is

$$\ell_1 = \ell_2 = \Big(\frac{2-\eta}{w2^{1+\eta}}\Big)^{1/\eta} \quad \text{with} \quad \hat{\pi}_1 = \hat{\pi}_2 = \frac{\eta(2-\eta)^{1/\eta-1}}{2^{1/\eta+1}w^{1/\eta-1}} \quad ,$$

which is feasible if $w \geq (2-\eta)/2^{1+\eta}$ and exhibits involuntary unemployment if $w > (2-\eta)/2^{1+\eta}$. The analogue of Lemma 1 is now

Lemma A.5: If $w_1 = w_2 = w > (2-\eta)/2^{1+\eta}$, there is a Nash equilibrium of the stage II game with involuntary unemployment, where

$$J_1 = J_2 = \ell_1 = \ell_2 = \Big(\frac{2-\eta}{w2^{1+\eta}}\Big)^{1/\eta}$$

and with profits $\pi_1 = \pi_2 = \eta(2-\eta)^{1/\eta-1}/(2^{1/\eta+1}w^{1/\eta-1})$.

Consider again the symmetric wages of Lemma A.5 as a candidate SPE of the whole game, and suppose first that firm 1 undercuts to $w_1 < w_2 = w$. Now $J_1 = J_2 = 2$ is a stage II Nash equilibrium when $w \leq 2^{-\eta}$ since the profit function of firm 2, given $J_1 = 2$, is

$$\pi_2 = \begin{cases} \frac{J_2}{2^\eta} - w_2 J_2 & \text{if} \quad J_2 \leq 2 \ , \\ 2^{1-\eta} - 2w_2 & \text{if} \quad J_2 > 2 \ , \end{cases}$$

which is maximal at $J_2 = 2$, and since firm 1 gets zero profits for all $J_1 \geq 0$ when $J_2 = 2$. Hence, Lemma 3 generalizes to

Lemma A.6: If $w_2 = w \in ((2-\eta)/2^{1+\eta}, 2^{-\eta}]$ and $w_1 < w$, there exists a stage II Nash equilibrium with $J_1 = J_2 = 2$, $\ell_1 = 0$, $\ell_2 = 2$, and profits $\pi_1 = 0$, $\pi_2 = 2^{1-\eta} - 2w$. Thus, undercutting from (w, w) is unprofitable.

Consider next an upward deviation of firm 1, $w_1 > w_2 = w \in ((2-\eta)/2^{1+\eta}, 2^{-\eta}]$. There exists a unique positive intersection (ℓ_1, ℓ_2) of URF_1 and URF_2. When $w_1 \leq w_2/(1-\eta)$, this is

$$\ell_1 = \Big(\frac{2-\eta}{w_1+w_2}\Big)^{1/\eta} \frac{w_2 - w_1(1-\eta)}{\eta(w_1+w_2)} \ , \ \ell_2 = \Big(\frac{2-\eta}{w_1+w_2}\Big)^{1/\eta} \frac{w_1 - w_2(1-\eta)}{\eta(w_1+w_2)} \ , \tag{20}$$

and profits are

$$\pi_1 = \Big(\frac{2-\eta}{w_1+w_2}\Big)^{1/\eta} \frac{(w_2 - w_1(1-\eta))^2}{\eta(2-\eta)(w_1+w_2)} \ , \ \pi_2 = \Big(\frac{2-\eta}{w_1+w_2}\Big)^{1/\eta} \frac{(w_2 - w_1(1-\eta))^2}{\eta(2-\eta)(w_1+w_2)} \ . \tag{21}$$

When $w_1 > w_2/(1-\eta)$,

$$\ell_1 = 0 \ , \ \ell_2 = \Big(\frac{1-\eta}{w_2}\Big)^{1/\eta} \ , \ \pi_1 = 0 \ , \ \pi_2 = \ell_2^{1-\eta} - w_2 \ell_2 \ . \tag{22}$$

In both cases, $\ell_1 + \ell_2 < 2$. Furthermore, $\partial \pi_1 / \partial w_1 < 0$ when $w_1 < w_2/(1-\eta)$, so firm 1's profit is less than its profit at the unemployment equilibrium at $w_1 = w_2 = w$. It will now be shown that $J_i = \ell_i$, $i = 1, 2$, is indeed a stage II Nash equilibrium, provided that $w \geq w^*(\eta)$ for some $w^*(\eta) \in ((2-\eta)/2^{1+\eta}, 2^{-\eta})$.

Since firm 2 is the low wage firm, $J_2 = \ell_2$ is a best response to $J_1 = \ell_1$. $J_1 = \ell_1$ is also a best response to $J_2 = \ell_2$ amongst all $J_1 \in [0, 2 - J_2]$. However, for $J_1 \in [2 - J_2, 2]$, firm 1's profit is $\pi_1 = J_1(2^{-\eta} - w_1)$. When $w_1 > 2^{-\eta}$, π_1 is decreasing, and so $J_1 = \ell_1 \in \mathrm{BR}_1(\ell_2)$. When $w_1 \leq 2^{-\eta}$, π_1 attains its maximum on $[2 - J_2, 2]$ at $J_1 = 2$ which is $\bar{\pi}_1 = 2^{1-\eta} - 2w_1$. $w_1 \leq 2^{-\eta}$ also implies $w_1 \leq w_2/(1-\eta)$, and therefore $\bar{\pi}_1$ is not higher than firm 1's profit at $J_1 = \ell_1$ if

$$2^{1-\eta} \leq 2w_1 + \Big(\frac{2-\eta}{w_1 + w_2}\Big)^{1/\eta} \frac{(w_2 - w_1(1-\eta))^2}{\eta(2-\eta)(w_1+w_2)} \quad . \tag{23}$$

A straightforward calculation shows that the r.h.s. of (23) is increasing in w_1 (since $w_2 \geq (2-\eta)/2^{1+\eta}$), and so (23) is fulfilled for all $w_1 > w_2 = w$ if (23) holds at $w_1 = w_2 = w$ which is

$$2^{1-\eta} \leq 2w + \frac{\eta(2-\eta)^{1/\eta - 1}}{2^{1+1/\eta}} w^{1-1/\eta} \quad . \tag{24}$$

Observe that (24) holds strictly at $w = 2^{-\eta}$ and is not fulfilled at $w = (2-\eta)/2^{1+\eta}$, and that the r.h.s. of (24) is increasing in-between. Hence there exists a unique $w^*(\eta) \in ((2-\eta)/2^{1+\eta}, 2^{-\eta})$ that fulfils (24) with equality, and we have the following generalization of Lemma 4:

Lemma A.7: Let $w_2 = w \in [w^*(\eta), 2^{-\eta}]$ and $w_1 > w$, where $w^*(\eta)$ is the unique solution of (12) in $((2-\eta)/2^{1+\eta}, 2^{-\eta})$. Then there exists a stage II Nash equilibrium with involuntary unemployment with $J_i = \ell_i$ and with profits π_i, $i = 1, 2$, as in (20) and (21), when $w_1 \leq w_2/(1-\eta)$, and as in (22), when $w_1 > w_2/(1-\eta)$. Therefore, upward wage deviations are unprofitable.

Lemma A.5, A.6, and A.7 prove Theorem 5. □

Proof of Corollary 1:

Aggregate employment is

$$L_\eta(w) := \ell_1 + \ell_2 = \Big(\frac{2-\eta}{2w}\Big)^{1/\eta} \quad .$$

The limit of the lowest equilibrium employment level at $w = 2^{-\eta}$ is $\lim_{\eta\to 0} L_\eta(2^{-\eta}) = 2e^{-1/2} \approx 1.213$. The SPE with the highest employment level are at $w^*(\eta)$ which is defined by (12). This equation can be expressed equivalently in terms of aggregate employment $L = L_\eta(w)$:

$$2^{3-\eta} L^\eta - \eta L = 4(2-\eta) \quad . \tag{25}$$

The l.h.s. is concave in L, less than the r.h.s. at $L = 0$ and larger than the r.h.s. at $L = 2$. Thus, for each $\eta \in (0, 1]$, there is a unique solution $L(\eta) \in (0, 2)$. Let $\bar{L} := \lim_{\eta\to 0} L(\eta)$. Since $L(\eta)$ solves (25), it is true that for each $\eta > 0$

$$L(\eta) = \frac{2^{3-\eta}(L(\eta)^\eta - \bar{L}^\eta)}{\eta} + \frac{2^{3-\eta}\bar{L}^\eta - 4(2-\eta)}{\eta} \quad .$$

Taking $\lim_{\eta\to 0}$ and applying Hospital's rule twice gives

$$\begin{aligned} \bar{L} &= \lim_{\eta\to 0} \Big(- \ln 2 \cdot 2^{3-\eta}(L(\eta)^\eta - \bar{L}^\eta) + 2^{3-\eta}(L(\eta)^\eta(\frac{\eta L'(\eta)}{L(\eta)} + \ln L(\eta)) - \bar{L}^\eta \ln \bar{L}) \Big) \\ &\quad + \lim_{\eta\to 0} \Big(- \ln 2 \cdot 2^{3-\eta}\bar{L}^\eta + 2^{3-\eta}\bar{L}^\eta \ln \bar{L} + 4 \Big) \\ &= \frac{8}{\bar{L}} \lim_{\eta\to 0} \eta L'(\eta) + 8 \ln(\frac{\bar{L}}{2}) + 4 \quad . \end{aligned} \tag{26}$$

L' can be calculated from (25), and it is

$$\lim_{\eta\to 0} \eta L'(\eta) = \bar{L} \ \frac{\bar{L} - 4 - 8\ln(\bar{L}/2)}{8 - \bar{L}} \quad .$$

Plugging this into (26) yields

$$\bar{L} - 4 = 8(\ln(\frac{\bar{L}}{\ln 2}))$$

which has a unique solution $\bar{L} \approx 1.456$ in $(0, 2)$. □

Proof of Theorem 6:

Suppose that $w_1 = w_2 = w$ in stage I and $J_1 = J_2 \geq 1$ in stage II is a full employment SPE with profits $\pi_1 = \pi_2 = 2^{-\eta} - w$. First, when $w > 2^{-\eta}$, both firms would make negative profits and signalling zero labour demand would be profitable. Second, when $w < 2^{-\eta}$, firm 1 could increase w_1 slightly, employ all workers ($J_1 = 2$) and make profit $2^{1-\eta} - 2w_1 > 2^{-\eta} - w$. Thus, in any (pure or mixed strategy) stage II continuation, profits of firm 1 are higher.

Third, when $w = 2^{-\eta}$ both firms would make zero profit in the conjectured full employment SPE. But now, as in Lemma 5, an upward deviation guarantees the deviator a positive (expected) profit in any stage II continuation. To show this, suppose $w_2 = 2^{-\eta} < w_1 < (e/4)^\eta$. As in the proof of Lemma 5 it turns out that the support of any stage II best response of firm i must be contained in $[0, w_i^{-1/\eta}(1+\eta)^{-1-1/\eta}]$. Since $\lim_{\eta\to 0}(1+\eta)^{1+1/\eta} = e$ and since $(1+\eta)^{1+1/\eta}$ is decreasing in η,

$$J_1 + J_2 \le \tfrac{1}{e}\Big(\frac{1}{w_1^{1/\eta}} + \frac{1}{w_2^{1/\eta}}\Big) \le \tfrac{4}{e} < 2$$

for all (J_1, J_2) played in any stage II Nash equilibrium. In any such equilibrium, firm 1 makes a positive (expected) profit since $\pi_1 = J_1(J_1+J_2)^{-\eta} - w_1 J_1 > 0$ because of $J_1 + J_2 \le 4/e < w_1^{-1/\eta}$. $\square$

Proof of Theorem 7:

The unconstrained best response problem of firm $i = 1, \ldots, n$ is

$$\max_{\ell_i \ge 0} \hat{\pi}_i = \frac{\ell_i}{\ell_i + \ell_{-i}} - w_i \ell_i \ ,$$

where $\ell_{-i} = \sum_{j\neq i} \ell_j$, and URF_i is again

$$\mathrm{URF}_i(\ell_{-i}) = \max\Big(0, \sqrt{\frac{\ell_{-i}}{w_i}} - \ell_{-i}\Big) \quad .$$

When firms announce equal wages $w_1 = \ldots = w_n = w$, the unique positive intersection of $\mathrm{URF}_1, \ldots, \mathrm{URF}_n$ is, for $i = 1, \ldots, n$,

$$\ell_i = \ell = \frac{n-1}{n^2 w} \quad \text{with profits} \quad \hat{\pi}_i = \frac{1}{n^2} \quad ,$$

which is feasible if $w \ge (n-1)/(2n)$ and exhibits involuntary unemployment if $w > (n-1)/(2n)$.

Given $w > (n-1)/(2n)$, $J_i = \ell$, $i = 1, \ldots, n$, is a stage II Nash equilibrium, since no firm can expand employment beyond $2 - (n-1)\ell$ because of the above definition of the labour allocation function. Hence, as in Lemma 1 we have

Lemma A.8: If $w_1 = \ldots = w_n = w > (n-1)/(2n)$, there is a Nash equilibrium of the stage II game with involuntary unemployment, where

$$J_i = \ell_i = \frac{n-1}{n^2 w} \quad , \ i = 1, \ldots, n,$$

and with profits $\pi_i = 1/n^2$, $i = 1, \ldots, n$.

Consider first an undercutting of firm 1 to $w_1 < w_2 = \ldots = w_n = w > (n-1)/(2n)$. As in Lemma 3, if $w \le 1/2$ there exists a stage II Nash equilibrium in which all firms $i = 1, \ldots, n$ signal labour demands $J_i = 2$. Firm 1 gets zero profits for all J_1, so $J_1 = 2$ is a best response; firm $i > 1$ gets for $J_i \in [0, 2/(n-1)]$ a profit of $J_i(1/2 - w)$, and for $J_i > 2/(n-1)$ a profit of $2/(n-1) \cdot (1/2 - w)$, and therefore $J_i = 2$ is a best response.

Lemma A.9: If $w_2 = \ldots = w_n = w \in ((n-1)/(2n), 1/2]$ and $w_1 < w$, there exists a stage II Nash equilibrium with $J_1 = \ldots = J_n = 2$, $\ell_1 = 0$, $\ell_2 = \ldots = \ell_n = 2/(n-1)$, and profits $\pi_1 = 0$, $\pi_2 = \ldots = \pi_n = 2/(n-1) \cdot (1/2 - w)$. Thus, undercutting from $(w, \ldots, w)$ where $w \in ((n-1)/(2n), 1/2]$ is unprofitable.

When $w_1 > w_2 = \ldots = w_n = w > (n-1)/(2n)$, there is a unique positive intersection of $\mathrm{URF}_1, \ldots, \mathrm{URF}_n$ which is, when $w_1 \leq w(n-1)/(n-2)$,

$$\begin{aligned} \ell_1 &= \frac{(n-1)((n-1)w - (n-2)w_1)}{((n-1)w + w_1)^2}\,, \\ \ell_i = \ell &= \frac{(n-1)w_1}{((n-1)w + w_1)^2}\,, \quad i = 2, \ldots, n, \end{aligned} \tag{27}$$

and, when $w_1 > w(n-1)/(n-2)$,

$$\ell_1 = 0 \quad \text{and} \quad \ell_i = \ell = \frac{n-2}{(n-1)^2 w}\,, \quad i = 2, \ldots, n\,. \tag{28}$$

Aggregate employment in this intersection is

$$\ell_1 + \ldots + \ell_n = \begin{cases} \dfrac{n-1}{w(n-1) + w_1} & \text{if} \quad w_1 \leq w\dfrac{n-1}{n-2}\,, \\ \dfrac{n-2}{(n-1)w} & \text{if} \quad w_1 > w\dfrac{n-1}{n-2}\,, \end{cases}$$

which is non–increasing in w_1, feasible at $w_1 = w$ because of $w > (n-1)/(2n)$ and therefore feasible for all $w_1 > w > (n-1)/(2n)$. Firm 1's profit is

$$\pi_1 = \begin{cases} \dfrac{((n-1)w - (n-2)w_1)^2}{((n-1)w + w_1)^2} & \text{if} \quad w_1 \leq w\dfrac{n-1}{n-2}\,, \\ 0 & \text{if} \quad w_1 > w\dfrac{n-1}{n-2}\,. \end{cases} \tag{29}$$

π_1 is decreasing in w_1 when $w_1 \leq w(n-1)/(n-2)$ and thus π_1 is lower than firm 1's profit at $w_1 = w$. Analogous to Lemma 4, it turns out that $J_i = \ell_i$, $i = 1, \ldots, n$, is indeed a Nash equilibrium of the stage II game, provided that w is high enough, which completes the proof of Theorem 7:

Lemma A.10: If $w_2 = \ldots = w_n = w \in [(1 - 1/n^2)/2, 1/2]$ and $w_1 > w$, there exists a stage II Nash equilibrium with involuntary unemployment with $J_i = \ell_i$,

$i = 1, \ldots, n$, where ℓ_i are as in (27), when $w_1 \le w(n-1)/(n-2)$, and as in (28), when $w_1 > w(n-1)/(n-2)$, and in which firm 1's profit is (29). Therefore, upward wage deviations are unprofitable.

Proof: It has to be shown that $J_i = \ell_i$, $i = 1, \ldots, n$ is a stage II Nash equilibrium. ℓ_i is optimal for firm $i > 1$ in $[0, 2 - \ell_1 - (n-2)\ell]$, and firm i cannot expand employment beyond $2 - \ell_1 - (n-2)\ell$ by signalling a higher labour demand because of our specification of the labour allocation function. Thus, $J_i = \ell_i$ is a best response to $J_j = \ell_j$, $j \neq i$. $J_1 = \ell_1$ is best amongst all $J_1 \in [0, 2-(n-1)\ell]$. However, when $2 \ge J_1 > 2 - (n-1)\ell$, $\ell_1 = J_1$, and firm 1's profit is $J_1(1/2 - w_1)$. When $w_1 > 1/2$, firm 1's profit is decreasing for $J_1 > 2 - (n-1)\ell$, and $J_1 = \ell_1$ is a best response to $J_i = \ell$, $i > 1$. When $w_1 \le 1/2$, firm 1's profit attains its maximum on $[2-(n-1)\ell, 2]$ at $J_1 = 2$ which is $\bar{\pi}_1 = 1 - 2w_1$. $w_1 \le \frac{1}{2}$ also implies $w_1 \le w(n-1)/(n-2)$ because of $w \ge (n-1)/(2n)$. Thus, $\bar{\pi}_1$ is less than firm 1's profit at $J_1 = \ell_1$ if

$$1 \le f(w_1) := 2w_1 + \frac{((n-1)w - (n-2)w_1)^2}{((n-1)w + w_1)^2} \; .$$

Straightforward calculations show that $f(w) = 2w + 1/n^2 \ge 1$ and that $f'(w_1) \ge 2(1 - (n-1)^2/(wn^3)) > 0$ because of $w_1 > w \ge (1 - 1/n^2)/2$. Therefore, $f(w_1) \ge 1$, which implies that $J_1 = \ell_1$ is a best response to $J_i = \ell$, $i > 1$. □

Proof of Theorem 8:

It has to be shown that there are no symmetric full employment SPE when $\alpha < 1$ and that there are no symmetric full employment SPE with $w \neq 1/2$ when $\alpha = 1$. Consider first $\alpha \le 1$ and suppose that $w_1 = \ldots = w_n = w < 1/2$ followed by some $J_1 = \ldots = J_n \ge 2/n$ is a SPE. In such a SPE, each firm would make a profit of $(1-2w)/n$. But then a small upward deviation $w_1 > w_2 = \ldots = w_n = w$ followed by $J_1 = 2$, $\ell_1 = 2$, $y_1 = 2^\alpha$, $p = 2^{-\alpha}$ would give firm 1 a profit of $1 - 2w_1 > (1-2w)/n$. Second, wages $w_1 = \ldots = w_n > 1/2$ cannot be announced in a full employment SPE since then all firms would make negative profits. It remains to show that $w_1 = \ldots = w_n = 1/2$ followed by some $J_1 = \ldots = J_n = J \ge 2/n$ cannot be a SPE when $\alpha < 1$. In such a SPE all firms would make zero profits. But now in the stage II game after $(1/2, \ldots, 1/2)$, $J_1 \ge 2/n$ cannot be a best response to $J_2 = \ldots = J_n = J \ge 2/n$. Reducing J_1 to $2/n$ has no effect on the labour allocation and on π_1. But, if $J > 2/n$, reducing J_1 to lie in $(2-(n-1)J, 2/n)$ leaves full employment but increases output price and gives profit

$$\pi_1 = \frac{J_1^\alpha}{J_1^\alpha + (n-1)((2-J_1)/(n-1))^\alpha} - \frac{1}{2}J_1 \ge \frac{J_1^\alpha}{n(2/n)^\alpha} - \frac{1}{2}J_1 \quad .$$

If $J = 2/n$, reducing J_1 to lie in $(0, 2/n)$ produces unemployment, increases output price and gives profit

$$\pi_1 = \frac{J_1^\alpha}{J_1^\alpha + (n-1)(2/n)^\alpha} - \frac{1}{2}J_1 \ge \frac{J_1^\alpha}{n(2/n)^\alpha} - \frac{1}{2}J_1$$

again. Now $J_1^\alpha/(n(2/n)^\alpha) - J_1/2 = 0$ at $J_1 = 2/n$ and is decreasing there. Thus reducing J_1 to just below $2/n$ always produces positive profits for firm 1, and $w_1 = \ldots = w_n = 1/2$ cannot be a full employment SPE. $\square$

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